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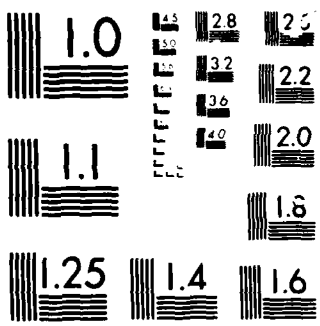

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POINTWISE A-PRIORI BOUNDS FOR STRONGLY COUPLED SEMILINEAR PARABOLIC SYSTEMS

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ABSTRACT

A-priori boundedness results for solutions of strongly coupled semilinear parabolic systems of second order under homogeneous linear boundary conditions are established. In contrast to [1], [2], [4] it is not supposed that the diffusion operator is self-adjoint or that the nonlinearity is of gradient-type.

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SIGNIFICANCE AND EXPLANATION

The prototype parabolic partial differential equation is the heat conduction equation. This paper deals with systems of parabolic equations. Such systems occur in many contexts in addition to heat conduction, e.g. biology, in nuclear reactor techniques, in economics, etc.

Let the n-vector u denote the (unknown) solution of a system of n parabolic partial differential equations. An important question in the study of these systems is the boundedness of u. Many techniques and criteria have been developed to solve this problem if the system is weakly coupled, i.e. if the k^{th} equation contains second order space derivatives of only u^k, the k^{th} component of u. If this is not the case, the system is said to be strongly coupled.

In the present paper for a broad class of strongly coupled parabolic systems pointwise boundedness of the solution u is established.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

POINTWISE A-PRIORI BOUNDS FOR STRONGLY COUPLED SEMILINEAR PARABOLIC SYSTEMS

Reinhard Redlinger

1. We consider strongly coupled semilinear parabolic systems of the form

$$\frac{\partial u^k}{\partial t} = \sum_{i,j,\ell} \frac{\partial}{\partial x_j}\left(a_{ij}^{k\ell} \frac{\partial u^\ell}{\partial x_i}\right) + \sum_{i,\ell} a_i^{k\ell} \frac{\partial u^\ell}{\partial x_i} + f^k(t,x,u) \tag{1}$$

$$\text{for} \quad 0 < t < T \quad , \quad x \in \Omega \quad , \quad k = 1,2,\ldots,n$$

under homogeneous linear boundary conditions. Here $u = (u^1,\ldots,u^n)$, Ω is a bounded domain in $\mathbb{R}^m$ and $0 < T \leq \infty$. It is the purpose of this paper to establish pointwise a-priori bounds for solutions u of (1). The same problem has also been treated by Cosner [4], Alikakos [1, §3] and Amann [2, §7]. In contrast to these authors, we do not suppose that f is a gradient or that the matrix $(a_{ij}^{k\ell})$ satisfies any symmetry conditions. Our main result is stated in section 2. Section 3 contains some examples.

2. Let $\Omega \subset \mathbb{R}^m$ be a bounded domain whose boundary $\partial\Omega$ is a $(m-1)$-dimensional C^2-manifold such that Ω lies locally on one side of $\partial\Omega$. Let $0 < T \leq \infty$ and set $J = (0,T)$ with $J_0 = [0,T)$. We wish to study the system of equations

$$u_t^k = L^k(t,x)u + f^k(t,x,u) \quad \text{for} \quad t \in J,\ x \in \Omega,\ k = 1,2,\ldots,n \quad , \tag{2}$$

where $u = (u^1,\ldots,u^n)$, $u_t^k = \partial u^k/\partial t$, $f : J \times \Omega \times \mathbb{R}^n \to \mathbb{R}^n$ is a given measurable function and

$$L^k(t,x)u \equiv \sum_{i,j,\ell} D_j(a_{ij}^{k\ell}(t,x)D_i u^\ell) + \sum_{i,\ell} a_i^{k\ell}(t,x)D_i u^\ell$$

with $D_i = \partial/\partial x_i$. Summation in i,j is from 1 to m and in k,ℓ from 1 to n. The coefficients of L^k are assumed to satisfy the following smoothness conditions: For all i, j, k, ℓ the partial derivatives $D_j a_{ij}^{k\ell}$ exist in $J_0 \times \Omega$ and $a_{ij}^{k\ell}$, $D_j a_{ij}^{k\ell}$, $a_i^{k\ell}$ are bounded continuous functions in $J_0 \times \Omega$. Moreover, these functions are Hölder continuous of exponent μ in t for some $0 < \mu < 1$, uniformly with respect to $x \in \Omega$. Also, the

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functions $a_{ij}^{k\ell}$ are uniformly continuous in x with a modulus of continuity independent of $t \in J_0$, and there is a function $c : J_0 \to (0,\infty)$ such that

(3) $$\sum_{i,j,k,\ell} a_{ij}^{k\ell}(t,x)q_j^k q_i^\ell \geq c(t) \sum_{j,k} (q_j^k)^2 \quad \text{in} \quad J_0 \times \Omega \quad \text{for all} \quad q_j^k \in \mathbb{R}^{mn} \quad .$$

We will show at the end of the paper how condition (3) can be relaxed.

We assume that u satisfies Dirichlet boundary conditions,

(4) $$u = 0 \quad \text{on} \quad J \times \partial\Omega \quad .$$

However, if the coefficients of L^k for all k are independent of t, then we also admit boundary conditions of the form $(k = 1,2,\ldots,n)$

(5) $$\delta^k \sum_{i,j,\ell} a_{ij}^{k\ell} D_i u^\ell \gamma^j + (1-\delta^k)u^k + \delta^k \sum_\ell b^{k\ell}(x)u^\ell = 0 \quad \text{on} \quad J \times \partial\Omega \quad .$$

Here $\delta^k \in C(\partial\Omega,\{0,1\})$, $\gamma = (\gamma^1,\ldots,\gamma^m)$ denotes the outer normal to $\partial\Omega$ and the $b^{k\ell}$ are continuously differentiable functions on $\partial\Omega$ satisfying

(6) $$\sum_{k,\ell} \delta^k \eta^k b^{k\ell} \delta^\ell \eta^\ell \geq 0 \quad \text{on} \quad \partial\Omega \quad \text{for all} \quad \eta \in \mathbb{R}^n \quad .$$

Of course, (4) is a special case of (5).

Let $X = L^p(\Omega,\mathbb{R}^n)$, $2 \leq p < \infty$, with $\|\cdot\|_{,p}$ denoting the usual norm. For $t \in J_0$ define the operator $A(t) : D(A) \subset X \to X$ by

$$A(t)u \equiv -(L^1(t,\cdot),\ldots,L^n(t,\cdot))u + d_p u \quad , \quad u \in D(A)$$

with $d_p > 0$ a real constant and

$$D(A) = \{u \in W^{2,p}(\Omega,\mathbb{R}^n) \quad , \quad u \text{ satisfies (4) or resp. (5)}\} \quad .$$

Note that $D(A)$ is independent of t. If we wish to stress the fact that $A(t)$ and X depend on p, we will write $A_p(t)$ and X_p in the sequel.

$D(A)$ is dense in X, the $A(t)$ are closed operators and, provided d_p is chosen sufficiently large, we have

$$\|(A(t)+\lambda)^{-1}\| \leq M_p(1+|\lambda|)^{-1} \quad \text{for all} \quad t \in J_0, \ \mathrm{Re}\,\lambda \geq 0$$

with a constant M_p independent of t, λ ($\|\cdot\|$ denotes the norm in $L(X)$, the space of bounded linear transformation on X). For proof see [5]. Furthermore, our assumptions imply that there are constants K, $P > 0$ such that

$$\|(A(t) - A(s))A^{-1}(\tau)\| \leq K|t-s|^\mu \quad , \quad t, s, \tau \in J_0$$

and

$$\|A(t)A^{-1}(s)\| \leq P \quad , \quad t, s \in J_0 \quad .$$

Hence, for each $t \in J_0$, $A(t)$ generates an analytic semigroup $\exp(-sA(t))$, $s \geq 0$, and the fractional power $A^{\alpha}(t)$ of $A(t)$ for any $\alpha > 0$ can be defined as the inverse of

$$A^{-\alpha}(t) = \frac{1}{\Gamma(\alpha)} \int_0^{\infty} e^{-sA(t)} s^{\alpha-1} ds$$

(see [9]). Set $X^{\alpha} = D(A^{\alpha})$, $A = A(0)$, with norm $\|x\|_{\alpha} = \|A^{\alpha}x\|$ for $x \in X^{\alpha}$. We then have the continuous imbeddings [6, §1.6]

(7a) $$X_p^{\alpha} \to X_q \qquad \text{for} \quad m/p < 2\alpha + m/q, \ q \geq p \quad ,$$

(7b) $$X_p^{\alpha} \to C^{\nu}(\overline{\Omega}, \mathbb{R}^n) \quad \text{for} \quad 0 \leq \nu < 2\alpha - m/p \quad .$$

Finally, denote by $W(t,s)$, $s \leq t \in J_0$ the linear evolution system generated by the operators $A(t)$, [9]. Then there is a $\delta > 0$ such that for any $0 \leq \beta \leq \alpha \leq 1$ we have

(8) $$\|A^{\alpha}(t)W(t,s)A^{-\beta}(s)\| \leq C(t-s)^{\beta-\alpha}e^{-\delta(t-s)} \quad , \quad s \leq t \in J_0$$

with a constant $C = C(\alpha,\beta)$ independent of s, t. If the operators $A(t)$ are independent of t, then $W(t,s) = \exp(-(t-s)A)$ and the estimate (8) is well-known [9, §2]. In the time-dependent case a detailed proof of (8) is given in [8].

Equations (2) with (4) or (5) can be summarized in the abstract equation

(9a) $$u_t + A(t)u = F(t,u) \quad , \quad t \in J$$

where

$$F(t,u) = f(t,\cdot,u) + d_p u \quad .$$

Together with

(9b) $$u(0) = u_0 \in X$$

(9) describes an initial value problem in X. Let us assume that f satisfies the following condition:

(10) There are constants $0 < \delta, \rho \leq 1$ such that for any $R > 0$ we have

$$|f(t,x,v) - f(s,x,w)| \leq C(|t-s|^{\delta} + |v-w|^{\rho})$$

for all $t, s \in J_0$, $x \in \Omega$, $v, w \in \mathbb{R}^n$ with a constant $C = C(R)$.

It then follows from [9, Sect. 2.5] that the initial value problem (9) has a local solution $u \in C([0,\tau], X_p) \cap C^1((0,\tau], X_p)$, defined on some interval $0 \leq t \leq \tau$, provided $u_0 \in X_p^{\beta}$

for some $\beta > m/2p$. Moreover, we have $u \in C([0,\tau],X_p^\alpha)$ for any $0 < \alpha < \beta$ and $u(t) \in D(A_p)$ for $0 < t < \tau$. We wish to establish a time-dependent bound for u in X_p^β. By [9, p. 57] this is sufficient to prove a global existence theorem.

Thus, let $u \in C(J_0,X_p) \cap C^1(J,X_p)$ denote from now on a fixed solution of (9) in X_p obtained by the method of proof used in [9, Sect. 2.5]. We assume that $p = 2$ in case $m < 4$ and that $p > m/2$ otherwise.

For $t \in J_0$ let

$$\alpha^{k\ell}(t) = \sup\{(\sum_i [a_i^{k\ell}(t,x)]^2)^{1/2} : x \in \Omega\}$$

and set

$$\alpha(t) = \sup\{\sum_{k,\ell} v^k\alpha^{k\ell}(t)w^\ell : v, w \in \mathbb{R}^n, |v| = |w| = 1\}$$

with $|\cdot|$ denoting the Euclidean norm in $\mathbb{R}^n$. We define

$$V(t) = \frac{1}{2}\int_\Omega \sum_k (u^k(t,x))^2 dx \quad , \quad t \in J_0$$

and introduce the following hypotheses:

(V_1) There is a continuous function $\Phi : J_0 \times [0,\infty) \to \mathbb{R}$ such that

$$\int_\Omega \sum_k u^k f^k(t,x,u)dx \le \Phi(t,V) \quad \text{in } J \quad .$$

(V_2) The maximal solution y_{max} of the ordinary differential equation

$$y_t = \Phi(t,y) + \frac{1}{2}\frac{\alpha(t)}{c(t)}\, y \quad , \quad y(0) = V(0)$$

with c given by (3) exists on J_0 and is bounded.

(V_3) There are constants $C_0 > 0$, $r > 1$ such that

$$|f(t,x,z)| \le C_0(1 + |z|^r) \quad \text{for } t \in J,\ x \in \Omega,\ z \in \mathbb{R}^n \quad .$$

<u>Theorem</u>. Let u be a solution of (9) in X_p as described above and assume $u_0 \in D(A_p)$. Let hypotheses (V_1), (V_2) and (V_3) with $r < 1 + 4/m$ be satisfied. Then, for any $\beta < 1$, $u(t)$ is bounded in X_p^β uniformly on J_0. In particular, $u(t)$ is uniformly bounded in $C(\bar\Omega,\mathbb{R}^n)$.

Remarks (i) Assume that f satisfies (10) and that $u_0 \in X_p^\beta$ for some $\beta > m/2p$. (This holds, if $u_0 \in C^\nu(\bar{\Omega}, \mathbb{R}^n)$ for some $\nu > 0$ with $u_0^k = 0$ on $\partial\Omega$ for all k with $\delta^k = 0$ in (5).) Then (9) will have a local solution u in X_p. As noted above, $u(t) \in D(A_p)$ for $t > 0$. We can thus use the assertion of the theorem to prove that this solution is global. However, in this case the initial condition on y in (V_2) has to be replaced by $y(0) = V(0) + \varepsilon$, where $\varepsilon > 0$ is arbitrarily small.

(ii) Uniform a-priori bounds on u in $L^\infty(\Omega, \mathbb{R}^n)$ with f satisfying (10) have also been obtained by Cosner [4] and Alikakos [1, §3] for the time-independent system (2) under Dirichlet boundary conditions and by Amann [2, §7] for the boundary value problem (2), (5). (In the last paper, time-dependent boundary conditions and higher order elliptic systems are also considered.) The bounds on r used in these papers are less restrictive than the one stated above. However, all these authors impose severe structure conditions on the system (2): It is assumed that $a_{ij}^{k\ell} = a_{ji}^{k\ell} = a_{ij}^{\ell k}$ in $J_0 \times \Omega$ for all i, j, k, ℓ and that f is of the form $f = g+h$, where g satisfies a linear growth condition and h is a gradient, $h = \text{grad}_u H(x,u)$. No such conditions are needed in the above theorem.

(iii) As the following proof shows it suffices to require $c(t) > 0$ on J_0 in (3) in case $\alpha(t) = 0$ on J_0.

Proof of the theorem. We first establish a time-independent bound for u on J_0 in X_2. In fact, since $u \in C^1(J, X_2)$, we get for $t \in J$

$$dV/dt = \int_\Omega \sum_k u^k u_t^k$$

$$= \int_\Omega \sum_k u^k \{ \sum_{i,j,\ell} D_j(a_{ij}^{k\ell} D_i u^\ell) + \sum_{i,\ell} a_i^{k\ell} D_i u^\ell + f^k \} \quad .$$

By partial integration the first summand is equal to

$$\int_{\partial\Omega} \sum_{i,j,k,\ell} u^k a_{ij}^{k\ell} D_i u^\ell \gamma^j - \int_\Omega \sum_{i,j,k,\ell} D_j u^k a_{ij}^{k\ell} D_i u^\ell$$

$$= -\int_{\partial\Omega} \sum_{k,\ell} \delta^k u^k b^{k\ell} \delta^\ell u^\ell - \int_\Omega \sum_{i,j,k,\ell} D_j u^k a_{ij}^{k\ell} D_i u^\ell$$

$$\leq 0 - c(t) \int_\Omega \sum_{k,j} |D_j u^k|^2 = -c(t) \int_\Omega |\text{grad } u|^2 \quad ,$$

where we have used (3) and (6). Also

$$\int_\Omega \sum_{i,k,\ell} u^k a_i^{k\ell} D_i u^\ell \leq \int_\Omega \alpha(t) \; |u| \; |\text{grad } u|$$

$$\leq \frac{1}{2} \frac{\alpha(t)}{c(t)} V(t) + c(t) \int_\Omega \; |\text{grad } u|^2 \quad .$$

Summing up and using (V_1) we thus get

$$dV/dt \leq \Phi(t,V) + \frac{1}{2} \frac{\alpha(t)}{c(t)} V \quad \text{in} \quad J \quad .$$

But this implies $V \leq y_{max}$ in J_0, where y_{max} is defined in (V_2). Hence V is bounded in J_0, i.e. u is bounded in X_2, uniformly on J_0.

Since u is a solution of (9) in X_2, it follows in particular that u satisfies in X_2 the integral equation

(11) $$u(t) = W(t,0)u_0 + \int_0^t W(t,s)F(s,u(s))ds \quad , \quad t \in J_0 \quad .$$

By (V_3) and (7a) we have

$$\|F(s,u(s))\|_{,2} \leq C_1(1 + \|u(s)\|^r_{,2r})$$

$$\leq C_2(1 + \|u(s)\|^r_\alpha)$$

for any $\alpha > m(r-1)/4r$ with constants C_1, C_2 independent of $s \in J$. We now make use of the inequality

$$\|u\|_\alpha \leq C\|u\|^\nu_{,2} \|u\|^{1-\nu}_\beta \quad \text{for} \quad u \in X_2^\beta, \; 0 < \alpha < \beta$$

where $\nu = 1 - \alpha/\beta$ and $C = C(\alpha,\beta)$ is a certain constant (see [9, (1.55)]). This gives

(12) $$\|F(s,u(s))\|_{,2} \leq C_3(1 + \|u(s)\|_\beta^{r(1-\nu)}) \quad \text{for} \quad s \in J$$

with $\beta > \alpha$ arbitrary and constant C_3. We choose β in such a way that

(13) $$\alpha < \beta < 1 \quad \text{and} \quad r(1-\nu) < 1 \quad .$$

This is equivalent to $r\alpha < \beta < 1$, i.e. to $m(r-1)/4 < \beta < 1$. Since $r < 1+4/m$, such a hoice is possible.

Define $z(t) = \|u(t)\|_\beta$. Using [9, (1.68)] one can show that $z \in C(J_0, X_2)$. Further, for $t \in J$ we have

$$z(t) \le C_4\|u_0\|_1 + \int_0^t C_5(t-s)^{-\tilde{\beta}} e^{-\delta(t-s)}(1 + z^{r(1-\nu)}(s))ds \tag{14}$$

by (8) and (12) with constants C_4, C_5 independent of t, s and arbitrary $\beta < \tilde{\beta} < 1$. Here, use is made of the estimate

$$\|A^\beta A^{-\tilde{\beta}}(t)\| \le \text{Const.} \quad \text{for} \quad \beta < \tilde{\beta},\ t \in J_0 \quad ,$$

which can be proved as [9, (1.59)].

But $r(1-\nu) < 1$ by (13) and hence (14) implies that $z(t) \le S$ in J_0 for some constant S. In case $m < 4$ this establishes the assertion of the theorem.

In case $m \ge 4$ we choose β close to 1 and apply (7a). It follows that $u(t)$ is bounded in X_q, uniformly on J_0, for any $q \ge 2$ satisfying $1/q > \frac{1}{2} - 2/m$. We can choose $q = q_1 > 2r$. Consider (9) in X_{p_1} with $p_1 = q_1/r$. Then $F(s,u(s))$ for $s \in J_0$ is bounded in X_{p_1} by (V_3) and using (11) it is easy to see that $\|u(t)\|_{\beta,p_1}$ is bounded on J_0 for any $0 \le \beta < 1$. Thus, again by (7a), u is bounded in X_{q_2} for any $q_2 \ge p_1$ satisfying $2 + m/q_2 > m/p_1$. Repeating this reasoning we get two sequences (q_ν), (p_ν) with

$$p_\nu = \frac{q_\nu}{r} \le q_{\nu+1} \quad , \quad -\frac{m}{q_{\nu+1}} < 2 - \frac{m}{p_\nu} \quad ,$$

where $q_1 > 2r$, $1/q_1 > \frac{1}{2} - 2/m$ and $m \ge 4$. It follows that after finitely many steps we can choose $p_\nu = p > m/2$. This proves the theorem.

3. Examples. α) Consider the system

$$\begin{aligned} u_t &= \lambda\Delta u - \rho_1\Delta v - v^2 - \gamma_1 u \\ v_t &= -\rho_2\Delta u + \mu\Delta v + uv - \gamma_2 v \end{aligned} \tag{15}$$

under boundary conditions (5). Let λ, μ, ρ_1, ρ_2, γ_1, γ_2 be real constants with $\lambda, \mu, \gamma_1, \gamma_2 > 0$ and $4\lambda\mu > (\rho_1 + \rho_2)^2$. Since $g(u,v) = (-v^2, uv)$ is not a gradient and since we do not assume $\rho_1 = \rho_2$, the results in [1], [2] and [4] cannot be applied to (15). To prove global boundedness for solutions of (15) one could also try to use comparison methods [3], [10]. However, apart from the structure of the nonlinearity in (15) this method will only be available if we additionally assume that $\rho_1 = \rho_2 = 0$ or that $4\rho_1\rho_2 + (\lambda-\mu)^2 > 0$. In this case, possible candidates for invariant sets are sets which at each boundary point have their normal vectors equal to left eigenvectors of the diffusion matrix (see [3]). But on the boundary of any such set (at least for small γ_1) there are points at which the nonlinearity in (15) does not point inward. Hence this method also cannot be applied to (15).

On the other hand, it is easy to see that (V_1) is satisfied with $\Phi(t,V) = -2\gamma V$, $\gamma = \min(\gamma_1,\gamma_2)$. Making use of the theorem we thus see that for any smooth initial data (15) will have a bounded solution existing for all time provided the space dimension m is 1, 2 or 3. Note that $y_{max} \to 0$ as $t \to \infty$ and hence $u, v \to 0$ as $t \to \infty$.

β) Let $\Omega = (0,L)$, $L > 0$ and consider

$$\begin{aligned} u_t &= u_{xx} + 3v_{xx} + v - u^3 \\ v_t &= v_{xx} + u - v^3 \end{aligned} \tag{16}$$

with $u_x = v_x = 0$ on $\partial\Omega$. Since the diffusion matrix $A = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$ is not symmetric, the hypotheses in [1], [2] and [4] are not satisfied. Comparison methods cannot be applied to (16), since A cannot be brought into diagonal form. Also, A is not positive definite, and hence condition (3) is not satisfied.

Define $w = \frac{1}{3}u$, $z = v$. This gives

$$\begin{aligned} w_t &= w_{xx} + z_{xx} + \frac{1}{3}z - 9w^3 \ , \\ z_t &= z_{xx} + 3w - z^3 \end{aligned} \tag{17}$$

with $w_x = z_x = 0$ on $\partial\Omega$. For the transformed boundary value problem, all assumptions of section 2 are satisfied. In particular, we can choose

$$\Phi(t,V) = \frac{10}{3}V - 2V^2/L \ .$$

Hence (17), and thus (16), has a global solution for any smooth initial values.

The transformation used in the last example indicates a way to get rid of condition (3). It is only necessary to assume that there are positive constants τ^k such that, with u^k replaced by $w^k = \tau^k u^k$, the transformed boundary value problem (2), (5) satisfies the assumptions of the theorem. In essence, this leads to the following problem: Let A be a real $(n \times n)$-matrix. Give sufficient general conditions on A under which there exists a diagonal matrix $D = (d_1, \ldots, d_n)$ with positive d_i such that DA is positive definite.

For $n = 2$ the answer can be given readily, but for higher dimensions the problem seems to be rather difficult. A recent discussion of this question and some results can be found in [7].

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18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Parabolic systems, boundedness of solution

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A-priori boundedness results for solutions of strongly coupled semilinear parabolic systems of second order under homogeneous linear boundary conditions are established. In contrast to [1], [2], [4] it is not supposed that the diffusion operator is self-adjoint or that the nonlinearity is of gradient-type.

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