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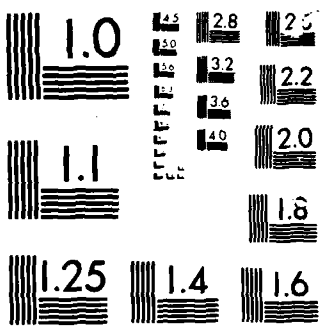

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A NEW RESULT IN THE THEORY AND COMPUTATION OF THE LEAST NORM SOLUTION OF A LINEAR PROGRAM

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ABSTRACT

By perturbing properly a linear program to a separable quadratic program it is possible to solve the latter in its dual variable space by iterative techniques such as sparsity-preserving SOR (successive overrelaxation) algorithms. The main result of this paper gives an effective computational criterion to check whether the solutions of the perturbed quadratic programs provide the least 2-norm solution of the original linear program.

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SIGNIFICANCE AND EXPLANATION

A novel way is presented in which the smallest solution of a linear programming problem can be determined. This result leads to an effective computational way for solving very large sparse linear programs.

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A NEW RESULT IN THE THEORY AND COMPUTATION OF THE
LEAST NORM SOLUTION OF A LINEAR PROGRAM

Stefano Lucidi

1. INTRODUCTION

Recently, it has been recognized that one of the most promising approach for solving very large linear programs is based on iterative SOR (successive overrelaxation) methods.

It was shown in [1] and [2] that the least 2-norm solution of a linear program can be obtained by perturbing "properly" the linear program to a separable quadratic program and by solving the latter in its dual variable space by iterative techniques such as SOR methods.

In this context some very effective algorithms were proposed in [3]. The principal and computationally-distinguishing features of these SOR algorithms are that they preserve the sparsity structure of the problem and require only simple operations, and, hence, very large problems can be tackled.

The main difficulty encountered by this approach for solving linear programs appears to be the difficulty of knowing "a priori" if the perturbed quadratic is a "proper" (according to the results of [1] and [2]) perturbation of the original linear program.

The main result of this paper tries to overcome this difficulty, in fact it gives an effective computational criterion to check whether the solutions of the perturbed quadratic programs provide the least 2-norm solution of the original linear program. We describe this result in section 3. In section 2 we give a new convergent result of a sparsity preserving SOR algorithm (proposed in [3]) for the solution of a class of quadratic programming problems.

In section 4 we present some algorithms for solving the linear problem with their convergent results. We briefly describe the notation used. All the matrices and vectors are real. For the $m \times n$ matrix A we denote row i by A_i, column j by $A_{\cdot j}$ and the

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element in row i and column j by A_{ij}. For x in the real n-dimensional Euclidean space R^n, x_i denotes element i for $i = 1,\ldots,n$ and x_+ denotes the vector with components $(x_+)_i = \max\{x_i, 0\}$ $i = 1,\ldots,n$. All vectors are column vectors unless transposed by T. $\|x\|$ will denote the 2-norm, $(x^Tx)^{1/2} = \left(\sum_{j=1}^{n} x_j^2\right)^{1/2}$. R_+^n will denote the nonnegative orthant $\{x : x \in R^n, x \geq 0\}$. For a point c in R^n and a closed set X in R^n the 2-norm projection $p_2(c,X)$ of the point c on X is defined by

$$\|c - p_2(c,X)\| = \min_{x \in X} \|c-x\| \quad .$$

2. SOR ALGORITHM FOR A CLASS OF QUADRATIC PROGRAMMING PROBLEMS

We consider the following separable quadratic program

$$\begin{aligned} &\text{Min } \frac{1}{2} x^T Dx + c^T x \\ &\text{s.t. } Ax \le b \\ &\qquad x \ge 0 \end{aligned} \tag{1}$$

where D is a positive diagonal matrix in $R^{n \times n}$, $A \in R^{m \times n}$, $c \in R^n$, $b \in R^m$ and $X = \{x : Ax \le b, x \ge 0\} \ne \phi$. Associated with this quadratic program is the dual quadratic program [4]

$$\begin{aligned} &\text{Max } -\frac{1}{2} x^T Dx - b^T u \\ &\text{s.t. } Dx + c + A^T u - v = 0 \\ &\qquad (u,v) \ge 0 \end{aligned}$$

which upon elimination of x by using the constraint relation

$$x = -D^{-1}(A^T u - v + c) \tag{2}$$

gives

$$\begin{aligned} &\text{Min } \frac{1}{2} (A^T u - v + c)^T D^{-1} (A^T u - v + c) + b^T u \\ &\text{s.t. } (u,v) \ge 0 \ . \end{aligned} \tag{3}$$

Problem (3) can be solved by using a sparsity-preserving SOR algorithm introduced in [3]. More specifically we have the following algorithm where we have assumed that $A_j \ne 0$, $\forall j = 1,\ldots,m$.

QPSOR algorithm

Choose $(u^0, v^0) \in R_+^{m+n}$, $\omega \in (0,2)$.

Having (u^k, v^k) compute (u^{k+1}, v^{k+1}) as follows:

$$u_j^{k+1} = \left(u_j^k - \frac{\omega}{\|A_j D^{-1/2}\|^2}\left(A_j D^{-1}\left(\sum_{\substack{\ell=1 \\ \text{for } j>1}}^{j-1} (A^T)_{\cdot\ell} u_\ell^{k+1} + \sum_{\ell=j}^{n} (A^T)_{\cdot\ell} u_\ell^k - v^k + c\right) + b_j\right)\right)_+ \qquad j = 1,\ldots,m$$

$$v^{k+1} = (v^k - \omega(-A^T u^{k+1} + v^k - c))_+ \quad .$$

REMARK. The algorithm works with the rows of A only and is well suited for matrices A which have a pronounced row structure. We refer to [3] for a similar algorithm which works with the columns of A.

Some convergence theorems of the preceding algorithm were given in [3] and [5]. Here we give a new convergence result under a mild regularity assumption on the constraints.

PROPOSITION 1. Let the gradients of the active constraints of problem (1) at the optimal point $\bar{x}$ be linearly independent. Then

a) The sequence $\{(u^k, v^k)\}$ generated by QPSOR algorithm converges to a point $(\bar{u}, \bar{v})$ which solves problem (3) and the corresponding x, determined by (2), is the unique solution of problem (1).

b) If, moreover, the strict complementarity holds at the optimal point $\bar{x}$ of (1) (that is $u_i > 0$ if $A_i x = b_i$ and $v_i > 0$ if $x_i = 0$) then, there exists an $\bar{k}$ such that for $k > \bar{k}$ the QPSOR algorithm becomes

$$u_j^{k+1} = 0 \quad , \quad j \in I_1$$

$$v_j^{k+1} = 0 \quad , \quad j \in I_2$$

$$u_j^{k+1} = u_j^k - \frac{\omega}{\|A_j D^{-1/2}\|^2}\left(A_j D^{-1}\left(\sum_{\substack{\ell=1 \\ j>1}}^{j-1} (A^T)_{\cdot\ell} u_\ell^{k+1} + \sum_{\ell=j}^{m} (A^T)_{\cdot\ell} u_\ell^k - v^k + c\right) + b_j\right) , \quad j \in \tilde{I}_1$$

$$v_j^{k+1} = (v_j^k - \omega(-A_j^T u^{k+1} + v_j^k - c_j)) \quad , \quad j \in \tilde{I}_2$$

where

$$I_1 = \{i : A_i \bar{x} < b_i\} \quad , \quad I_2 = \{i : \bar{x}_i > 0\}$$

$$\tilde{I}_1 = \{i : A_i \bar{x} = b_i\} \quad , \quad \tilde{I}_2 = \{i : \bar{x}_i = 0\}$$

and the sequence $\{u^k,v^k\}$ converges to the optimal point $(\bar{u},\bar{v})$ at linear root rate.

PROOF

a) The problem (1) has a unique solution $\bar{x}$ and by the linear independence assumption of the gradients of the active constraints it has a unique optimal multiplier $(\bar{u},\bar{v})$ which is also the unique solution of problem (3). Now from part iii) of Theorem 1 of [3] it follows that the $\{u^k,v^k\}$ is bounded and every accumulation point solves problem (3). But problem (3) has a unique solution and, therefore, the bounded sequence $\{u^k,v^k\}$ has only one accumulation point and, hence, it converges.

b) The proof of part b) follows easily by repeating the same steps of the proof of Theorem 2.5 of [6] by taking into account, that from part a) we have that the entire sequence converges and by noting that, from the linear independence assumption of the gradients of the active constraints, we obtain that, also in this case, the matrix M_{RR} (see page 481 of [6]) is nonsingular.

□

REMARK. Part b) of the preceding proposition says that the algorithm, after a finite number of steps, identifies which variables will be zero at the solution.

3. PERTURBATION OF LINEAR PROGRAMS

We consider the linear program

$$\begin{aligned} &\text{Max } c^T x \\ &\text{s.t. } Ax \leq b \\ &\quad\; x \geq 0 \end{aligned} \tag{4}$$

where $A \in R^{m\times n}$, $c \in R^n$, $b \in R^m$ and $X = \{x : Ax \leq b,\ x \geq 0\} \neq \phi$. Let $\bar{X}$ denote the (possibly empty) optimal solution set of (4). First of all we recall the following fundamental result given in [1].

THEOREM 1. Let the linear program (4) be feasible. Then

a) i) $\max_{x\in X} c^T x$ has a solution $\Longrightarrow \exists \varepsilon^* > 0 : p_2(\frac{c}{\varepsilon}, X) = p_2(0, \bar{X})$ for all $\varepsilon \in (0, \varepsilon^*]$

ii) $\left.\begin{array}{l} \max_{x\in X} \text{ has a solution} \\ \text{and } x^* = p_2(0,\bar{X}) \end{array}\right\} \Longleftarrow \exists \varepsilon^* > 0,\ x^* : p_2(\frac{c}{\varepsilon}, X) = x^*$ for all $\varepsilon \in (0, \varepsilon^*]$

where $p_2(x,X)$ denotes the 2-norm projection of x on X.

b) $\sup_{x\in X} c^T x = \infty \Longleftrightarrow \| p_2(\frac{c}{\varepsilon}, X)\| \to \infty$ as $\varepsilon \to 0^+$.

PROOF. See [1].

We can note that $p_2(\frac{c}{\varepsilon}, X)$ is also a solution of the problem

$$\begin{aligned} &\text{Min } \frac{\varepsilon}{2} x^T x - c^T x \\ &\text{s.t. } Ax \leq b \\ &\quad\; x \geq 0 \end{aligned} \tag{5}$$

and that the quadratic programming dual [3] to (5) is

$$\begin{aligned} &\text{Min } \frac{1}{2} \| A^T u - v - c\|^2 + \varepsilon b^T u \\ &\text{s.t. } (u,v) \geq 0 \end{aligned} \tag{6}$$

where the primal and dual variable x and (u,v) are related by

$$x = \frac{1}{\varepsilon}(-A^T u + v + c) \quad . \tag{7}$$

Unfortunately the parameter ε^* in Theorem 1 is not easy to compute (see [2]) and therefore, in general, we can not be sure that the point $x(\varepsilon)$ obtained by solving (5) or (6) is the optimal solution of the linear program (4). Even if we repeat the computation of points $p_2(\frac{c}{\varepsilon^j}, X)$ for decreasing values of ε^j until the condition $p_2(\frac{c}{\varepsilon^{j+1}}, X) = p_2(\frac{c}{\varepsilon^j}, X)$ is verified we can not conclude that the point $p_2(\frac{c}{\varepsilon^j}, X)$ is the optimal solution of the linear problem (4).

The following theorem allows to overcome this last difficulty and it will be useful in the next section where some algorithms for solving the linear program (4) will be proposed.

THEOREM 2. Assume that $\bar{X} \neq \emptyset$ and let the gradients of the active constraints of the linear program (4) at the optimal point $x' = p_2(0, \bar{X})$ be linearly independent.

Let $(\bar{x}, \bar{u}, \bar{v})$ and $(\tilde{x}, \tilde{u}, \tilde{v})$ be two points that satisfy the KKT conditions for problem (5) with two different values for ε, namely $\varepsilon = \bar{\varepsilon}$ and $\varepsilon = \tilde{\varepsilon}$ respectively where $\bar{\varepsilon} = \theta\tilde{\varepsilon}$ with $\theta \in (0,1)$. If $\bar{x} = \tilde{x} = x^*$ then it follows:

$$\left\langle x^* = p_2(0, \bar{X}) \right\rangle \Longleftrightarrow \left\langle \begin{array}{l} \bar{u} \geq \theta\tilde{u} \quad , \quad \bar{v} \geq \theta\tilde{v} \\ c^T x^* = b^T\left(\frac{\bar{u} - \theta\tilde{u}}{1-\theta}\right) \end{array} \right\rangle \quad .$$

Furthermore, if $x^* = p_2(0, \bar{X})$, $\left(\frac{\bar{u} - \theta\tilde{u}}{1-\theta}, \frac{\bar{v} - \theta\tilde{v}}{1-\theta}\right)$ is the optimal solution of the dual of the linear program (4).

PROOF

($\Longleftarrow$)

Since $(x^*, \bar{u}, \bar{v})$ and $(x^*, \tilde{u}, \tilde{v})$ satisfy the KKT conditions for (5) with $\bar{\varepsilon}$ and $\tilde{\varepsilon}$ we have

$$\tilde{\varepsilon} x^* - c + A^T \tilde{u} - \tilde{v} = 0 \tag{8}$$

$$\bar{\varepsilon} x^* - c + A^T \bar{u} - \bar{v} = 0 \tag{9}$$

$$\tilde{u}^T (Ax^* - b) = 0 \tag{10}$$

$$\bar{u}^T (Ax^* - b) = 0 \tag{11}$$

$$\tilde{v}^T x^* = 0 \tag{12}$$

$$\bar{v}^T x^* = 0 \tag{13}$$

$$Ax^* - b \le 0 \tag{14}$$

$$x^* \ge 0 \tag{15}$$

$$(\tilde{u},\tilde{v}) \ge 0$$

$$(\bar{u},\bar{v}) \ge 0 \quad .$$

By taking into account that $\tilde{\varepsilon} = \bar{\varepsilon}/\theta$, from (8) it follows

$$\frac{\bar{\varepsilon}}{\theta} x^* - c + A^T\tilde{u} - \tilde{v} = 0$$

which implies

$$\bar{\varepsilon}\, x^* - \theta c + A^T\theta\tilde{u} - \theta\tilde{v} = 0 \quad . \tag{16}$$

Now, by subtracting (16) from (9) and by dividing by $(1-\theta)$ we obtain

$$-c + A^T\left(\frac{\bar{u}-\theta\tilde{u}}{1-\theta}\right) - \frac{\bar{v}-\theta\tilde{v}}{1-\theta} = 0 \quad . \tag{17}$$

By using (10) - (13) we have

$$\left(\frac{\bar{u}-\theta\tilde{u}}{1-\theta}\right)^T (Ax^*-b) = 0 \tag{18}$$

$$\left(\frac{\bar{v}-\theta\tilde{v}}{1-\theta}\right)^T x^* = 0 \quad . \tag{19}$$

Therefore from (17), (18), (19), (14), (15) and the nonnegativity assumption on $\left(\frac{\bar{u}-\theta\tilde{u}}{1-\theta}\right)$ and $\left(\frac{\bar{v}-\theta\tilde{v}}{1-\theta}\right)$ it follows that the pair $\left(\frac{\bar{u}-\theta\tilde{u}}{1-\theta}, \frac{\bar{v}-\theta\tilde{v}}{1-\theta}\right)$ is dual feasible and that x^* is primal feasible for the linear problem (4). By assumption we have also that

$$c^T x^* = b^T\left(\frac{\bar{u}-\theta\tilde{u}}{1-\theta}\right)$$

which implies that x^* is an optimal solution for the linear program (4) and that $\left(\frac{\bar{u}-\theta\tilde{u}}{1-\theta}, \frac{\bar{v}-\theta\tilde{v}}{1-\theta}\right)$ is optimal for the dual of the linear program (4).

Moreover we can observe that both $\left(x^*, \frac{\bar{u}}{\bar{\varepsilon}}, \frac{\bar{v}}{\bar{\varepsilon}}, \frac{1}{\bar{\varepsilon}}\right)$ and $\left(x^*, \frac{\tilde{u}}{\tilde{\varepsilon}}, \frac{\tilde{v}}{\tilde{\varepsilon}}, \frac{1}{\tilde{\varepsilon}}\right)$ satisfy the KKT conditions for the problem

$$\text{Min } \frac{1}{2} \|x\|^2$$

$$\text{s.t.} \quad Ax \leq b$$
$$x \geq 0$$
$$cx \geq \gamma$$

where γ is the maximum value of (4), from which it follows that $\tilde{x} = \bar{x} = x^* = p_2(0,\bar{X})$. Now from the linear independence assumptions of the gradients of the active constraints we have that x^* has a unique optimal multiplier and, therefore, we can conclude that $(\frac{\bar{u}-\theta\tilde{u}}{1-\theta}, \frac{\bar{v}-\theta\tilde{v}}{1-\theta})$ is the unique solution of the dual of (4). In fact, if it had a different solution $(\hat{u},\hat{v})$, the pair $(\hat{u},\hat{v})$ should be another optimal multiplier for point x^* (see [4] cap. 5 and 8).

(==>)

For simplicity we can introduce the vector $\bar{\lambda}$, $\tilde{\lambda}$, λ^*

$$\bar{\lambda} = \begin{bmatrix} \bar{u} \\ \bar{v} \end{bmatrix}, \quad \tilde{\lambda} = \begin{bmatrix} \tilde{u} \\ \tilde{v} \end{bmatrix}, \quad \lambda^* = \begin{bmatrix} u^* \\ v^* \end{bmatrix}$$

where (u^*,v^*) are the KKT multipliers of the linear problem associated to the optimal point $x^* = p_2(0,\bar{X})$.

Under the assumption that the gradients of the active constraints are linearly independent we can use the multiplier function $\lambda(x)$ (see [7] and [8]) to compute the values of $\bar{\lambda}$, $\tilde{\lambda}$ and λ^* in function of x^*

$$\bar{\lambda} = \bar{\lambda}(x^*) = -D(x^*)^{-1}B^T[\bar{\varepsilon}x^*-c] \tag{20}$$

$$\tilde{\lambda} = \tilde{\lambda}(x^*) = -D(x^*)^{-1}B^T[\tilde{\varepsilon}x^*-c] = -D(x^*)^{-1}B^T[\frac{\bar{\varepsilon}}{\theta} x^* - c] \tag{21}$$

$$\lambda^* = \lambda^*(x^*) = -D(x^*)^{-1}B^T[-c] \tag{22}$$

where

$$D(x^*) = \left[\begin{pmatrix} AA^T & -A \\ -A^T & I \end{pmatrix} + \begin{pmatrix} \underset{1\le i\le m}{\text{Diag}} [(A_i x^* - b_i)^2] & 0 \\ 0 & \underset{1\le i\le n}{\text{Diag}} [(x_i^*)^2] \end{pmatrix}\right] =$$

$$= \begin{bmatrix} A & \underset{1\le i\le m}{\text{Diag}} [A_i x^* - b_i] & 0 \\ -I & 0 & \underset{1\le i\le n}{\text{Diag}} [x_i^*] \end{bmatrix} \begin{bmatrix} A & \underset{1\le i\le m}{\text{Diag}} [A_i x^* - b_i] & 0 \\ -I & 0 & \underset{1\le i\le n}{\text{Diag}} [x_i^*] \end{bmatrix}^T$$

and

$$B = [A^T, -I] \quad .$$

From (20) - (22) it follows

$$\begin{bmatrix} \frac{\bar{u}-\theta\tilde{u}}{1-\theta} \\ \\ \frac{\bar{v}-\theta\tilde{v}}{1-\theta} \end{bmatrix} = \frac{\bar{\lambda}-\theta\tilde{\lambda}}{1-\theta} = D(x^*)^{-1}B^T c = \lambda^* = \begin{bmatrix} u^* \\ v^* \end{bmatrix}$$

namely the pair $\left(\frac{\bar{u}-\theta\tilde{u}}{1-\theta}, \frac{\bar{v}-\theta\tilde{v}}{1-\theta}\right)$ is the optimal KKT multiplier of the linear problem (4).

□

REMARK. i) From the proof of the Theorem we can note that the regularity assumption on the active constraints is not needed for the implication ($\Leftarrow$).

From Theorem 1 and Theorem 2 we can also state the following results:

COROLLARY 1. Assume that $\bar{X} \neq \emptyset$ and let the gradients of the active constraints of the linear program (4) at the optimal point $x' = p_2(0,\bar{X})$ be linearly independent. Then there exist an $\varepsilon^* > 0$ such that for all $\varepsilon \in (0,\varepsilon^*]$ we have

$$u(\varepsilon) = u^* + \varepsilon\gamma$$

$$v(\varepsilon) = v^* + \varepsilon\gamma$$

where (u^*,v^*) is the optimal solution of the dual of the linear program (4) and $(u(\varepsilon),v(\varepsilon))$ is the unique solution of (6), and γ is a vector independent of ε.

PROOF. The proof follows from Theorem 1 and (20) - (22).

□

The next result gives a particular characterization of the solvability of a linear program.

COROLLARY 2. Let $(\bar{x},\bar{u},\bar{v})$ and $(\tilde{x},\tilde{u},\tilde{v})$ be two points that satisfy the KKT conditions for problem (5) with two different values for ε, namely $\varepsilon = \bar{\varepsilon}$ and $\varepsilon = \tilde{\varepsilon}$ respectively where $\bar{\varepsilon} = \theta\tilde{\varepsilon}$ with $\theta \in (0,1)$.

i) Suppose that

a) $\bar{x} = \tilde{x} = x^*$

b) The gradients of the active constraints of problem (5) are linearly independent at the points x^*

c) $\bar{u} > \theta\tilde{u}$, $\bar{v} > \theta\tilde{v}$

d) $c^T x^* = b^T(\frac{\bar{u}-\theta\tilde{u}}{1-\theta})$

then the linear program (4) is solvable (namely $\bar{X} \neq \emptyset$) and $x^* = p_2(0,\bar{x})$.

ii) Conversely, suppose that the linear program (4) is solvable and the gradients of the active constraints of linear program (4) at the optimal point $x^* = p_2(0,\bar{x})$ are linearly independent, then there exist two values for ε such that the conditions a) - d) of part i) hold.

PROOF. The proof of part i) follows by repeating the same steps of the first part of proof of Theorem 2.

The proof of part ii) follows from Theorem 1 and Theorem 2.

□

4. SOR ALGORITHM FOR LINEAR PROGRAMMING

The quadratic programming problem (6) can be solved by a sparsity-preserving algorithm which follows directly from the QPSOR algorithm of section 3 by replacing D by εI (also in this case we have assumed that $A_j \neq 0\ \forall j = 1,\dots,m$).

LPSOR algorithm

Choose $(u^0, v^0) \in R_+^{m+n}$, $\omega \in (0,2)$ and $\varepsilon > 0$.

Having (u^k, v^k) compute (u^{k+1}, v^{k+1}) as follows

$$u_j^{k+1} = \Big(u_j^k - \frac{\omega}{\|A_j\|^2}\Big(A_j\Big(\sum_{\substack{\ell=1\\ j>1}}^{j-1} (A^T)_{\cdot\ell} u_\ell^k + \sum_{\ell=j}^{m} (A^T)_{\cdot\ell} u_\ell^k - v^k - c\Big) + \varepsilon b_j\Big)\Big)_+ \qquad j = 1,\dots,m$$

$$v^{k+1} = (v^k - \omega(-A^T u^{k+1} + v^k + c))_+ .$$

By using the results of sections 2 and 3 we can state the following convergence result which sharpens previous LPSOR convergence results given in [1] and [5].

PROPOSITION 2. Assume that $\bar{X} \neq \emptyset$ and let the gradients of the active constraint of the linear program (4) at the optimal point $x^* = p_2(0,\bar{X})$ be linearly independent. Then

a) There exists a real positive number ε^* such that for each $\varepsilon \in (0,\varepsilon^*]$, the sequence $\{(u^k, v^k)\}$ generated by the LPSOR algorithm converges to a point $(u(\varepsilon), v(\varepsilon))$ which solves problem (6) and the corresponding $x(\varepsilon)$ determined by (7) is independent of ε and $x(\varepsilon) = x^* = p_2(0,\bar{X})$.

b) If, moreover, the strict complementarity holds at the optimal point $x^* = p_2(0,\bar{X})$ then there exists a real positive number ε^{**}, $\varepsilon^{**} < \varepsilon^*$, such that for each $\varepsilon \in (0,\varepsilon^{**}]$ there exists an $\bar{k}$ such that for $k > \bar{k}$ the LPSOR algorithm becomes

$$u_j^{k+1} = 0 \quad , \quad j \in I_1$$

$$u_j^{k+1} = 0 \quad , \quad j \in I_2$$

$$u_j^{k+1} = u_j^k - \frac{\omega}{\|A_j\|^2}\Big(A_j\Big(\sum_{\substack{\ell=1\\ j>1}}^{j-1} (A^T)_{\cdot\ell} u_\ell^{k+1} + \sum_{\ell=j}^{m} (A^T)_{\cdot\ell} u_\ell^k - v^k - c\Big) + \varepsilon b_j\Big), \qquad j \in \tilde{I}_1$$

$$v_j^{k+1} = v_j^k - \omega(-A_j^T u^{k+1} + v_j^k + c_j) \quad , \quad j \in \tilde{I}_2$$

where

$$I_1 = \{i : A_i x^* < b_i\} \quad , \quad I_2 = \{i : x_i^* > 0\}$$

$$\tilde{I}_1 = \{i : A_i x^* = b_i\} \quad , \quad \tilde{I}_2 = \{i : x_i^* = 0\}$$

and the sequence $\{(u^k,v^k)\}$ converges to the optimal point $(u(\varepsilon),v(\varepsilon))$ at linear root rate.

PROOF

a) It follows directly from Proposition 1 and Theorem 1.

b) If $(u(\varepsilon),v(\varepsilon))$ is a solution of problem (6) for a fixed ε, $\varepsilon < \varepsilon^*$, we have by using Corollary 1

$$\lim_{\varepsilon \to 0} (u(\varepsilon),v(\varepsilon)) = (u^*,v^*)$$

where (u^*,v^*) is the KKT multiplier of the linear problem (4).

Now by using the strict complementarity assumptions and theorem 1, we have that there exists an ε^{**}, $\varepsilon^{**} < \varepsilon^*$, such that for all $\varepsilon \in (0,\varepsilon^{**}]$

$$\{i : u_i(\varepsilon) > 0\} = \{i : u_i^* > 0\} \quad , \quad \{i : v_i(\varepsilon) > 0\} = \{i : v_i^* > 0\}$$

$$\{i : u_i(\varepsilon) = 0\} = \{i : u_i^* = 0\} \quad , \quad \{i : v_i(\varepsilon) = 0\} = \{i : v_i^* = 0\} \quad .$$

Therefore, by taking into account that $\varepsilon^{**} < \varepsilon^*$ and Theorem 1, it results that the strict complementarity assumption holds also for the problem (5) for all $\varepsilon \in (0,\varepsilon^{**}]$ and, again, the result follows from Proposition 1.

□

In applying part i) of Proposition 2 we must be able to select a value of ε such that $\varepsilon < \varepsilon^*$. In order to ensure that $\varepsilon < \varepsilon^*$ we may have to choose very small values for ε. Computational results have shown that very small values for ε yield a very slow convergence of the LPSOR algorithm when applied to the problem (6).

In order to overcome the lack of a practical "a priori" selection procedure for the parameter ε^* we can propone two algorithms which are based on the results of the preceding section.

ALGORITHM I

Choose $\theta \in (0,1)$, and $\varepsilon^0 > 0$.

Let $\varepsilon^i = (\theta)^i \varepsilon^0$, $i = 0,1,2,\ldots$ and let $(u(\varepsilon^i), v(\varepsilon^i))$ be a solution of problem (6) with $\varepsilon = \varepsilon^i$, and $x(\varepsilon^i)$ defined by (7) with $\varepsilon = \varepsilon^i$ and $u = u(\varepsilon^i)$, $v = v(\varepsilon^i)$.

By using Theorem 1 and Theorem 2 we can give the following convergence result for algorithm I.

PROPOSITION 3. Assume that $\bar{X} \neq \emptyset$ and let the gradients of the active constraints of the linear program (4) at the optimal point $x' = p_2(0,\bar{X})$ be linearly independent. Then for some integer $\bar{i}$

$$x(\varepsilon^{\bar{i}}) = x(\varepsilon^{\bar{i}-1}) = x^* = p_2(0,\bar{X})$$

is optimal for the linear program (4) and

$$\left(\frac{u(\varepsilon^{\bar{i}}) - \theta u(\varepsilon^{\bar{i}-1})}{1-\theta}, \frac{v(\varepsilon^{\bar{i}}) - \theta v(\varepsilon^{\bar{i}-1})}{1-\theta}\right)$$

is the optimal solution of the dual of the linear program (4).

The next algorithm contains an automatic adjustment rule for the parameter ε based on the abstract models considered in [8]. It carries out the minimization of two problem (6) for two different values of ε and uses the obtained points to adjust the value of ε during the minimization.

In the sequel we assume that we have an algorithm defined by an iteration map $F : R^{m+n} \times R^n \times R \rightarrow 2^{R^{m+n} \times R^n}$, such that for any fixed value of ε, any accumulation point of the sequence produced by F gives a point $x(\varepsilon)$ and a pair $(u(\varepsilon), v(\varepsilon))$ which solve the problems (5) and (6) respectively for the given ε.

For simplicity we introduce

$$B = [A^T, -I],\ M = B^T B,\ d = \binom{-Ac}{c},\ q = \binom{b}{0},\ \lambda = \binom{u}{v},$$

where $B \in R^{n\times(m+n)}$, $M \in R^{(m+n)\times(m+n)}$, $d, q, \lambda \in R^{m+n}$, and, in the sequel, we denote the scalar product with xy. The algorithm model described below makes use of two preselected sequences $\{\varepsilon^j\}$ and $\{N^k\}$, where $0 < \varepsilon^{j+1} < \theta\varepsilon^j < \varepsilon^j$, $j = 0,1,\ldots$, $\{\varepsilon^j\} \to 0$ as $j \to \infty$ and $\{N^k\} \to \infty$ as $k \to \infty$.

ALGORITHM II

Data : $\theta \in (0,1)$, $k_1,k_2,k_3,k_4,k_5 > 0$, $(\lambda^0,x^0,\bar\lambda^0,\bar x^0)$.

Step 0: Set $k = 0$, $j = 0$.

Step 1: Set $i = 0$ and $(\lambda^i,x^i,\bar\lambda^i,\bar x^i) = (\lambda^k,x^k,\bar\lambda^k,\bar x^k)$.

Step 2: Compute $(\lambda^{i+1},x^{i+1}) \in F(\lambda^i,x^i,\varepsilon^j)$, $(\bar\lambda^{i+1},\bar x^{i+1}) \in F(\bar\lambda^i,\bar x^i,\theta\varepsilon^j)$ and set $i = i+1$.

Step 3: If $i < N^k$ go to step 2, else go to step 4.

Step 4: Set $T^i = \lambda^i - (\lambda^i - (M\lambda^i + d + \varepsilon^j q))_+$, $\bar T^i = \bar\lambda^i - (\bar\lambda^i - (M\bar\lambda^i + d + \theta\varepsilon^j q))_+$.

If $\|T^i\| = 0$, $\|\bar T^i\| = 0$ go to step 5, else go to step 6.

Step 5: If $\|\bar x^i - x^i\| = 0$, $(\bar\lambda^i - \theta\lambda^i) \geq 0$ and $|c^T(\bar x^i - \theta x^i) - b^T(\bar u^i - \theta u^i)| = 0$ set $(\lambda^{k+1},x^{k+1},\bar\lambda^{k+1},\bar x^{k+1}) = (\lambda^i x^i,\bar\lambda^i,\bar x^i)$, $k = k+1$ stop;

else go to step 9.

Step 6: If $K_1\left(\left|\lambda^i\left(\frac{M\lambda^i+d}{\varepsilon^j} + q\right)\right| + \left|\bar\lambda^i\left(\frac{M\bar\lambda^i+d}{\theta\varepsilon^j} + q\right)\right|\right) + K_2((Ax^i - b)_+ + (A\bar x^i - b)_+)$

$+ K_3((-x^i)_+ + (-\bar x^i)_+) > (\varepsilon^j)^2\|\bar x^i - x^i\|^2$ go to step 7; else go to step 9.

Step 7: If $K_4(\theta\varepsilon^j \mid \|\bar x^i\|^2 - \|x^i\|^2| + |\theta\varepsilon^j(\|\bar x^i\|^2 - \|x^i\|^2) - c^T(\bar x^i - \theta x^i)$

$+ b^T(\bar u^i - \theta u^i)|) > \varepsilon^j|c^T(\bar x^i - \theta x^i) - b^T(\bar u^i - \theta u^i)|$ go to step 8;

else go to step 9.

Step 8: If $K_5(\|T^i\| + \|\bar T^i\|) > (\theta\lambda^i - \bar\lambda^i)_+$ go to step 10;

else go to step 9.

Step 9: Set $j = j+1$.

Step 10: Set $(\lambda^{k+1},x^{k+1},\bar\lambda^{k+1},\bar x^{k+1}) = (\lambda^i,x^i,\bar\lambda^i,\bar x^i)$, $j(k+1) = j$, $k = k+1$ and go to step 1.

The algorithm II is sketched in figure 1.

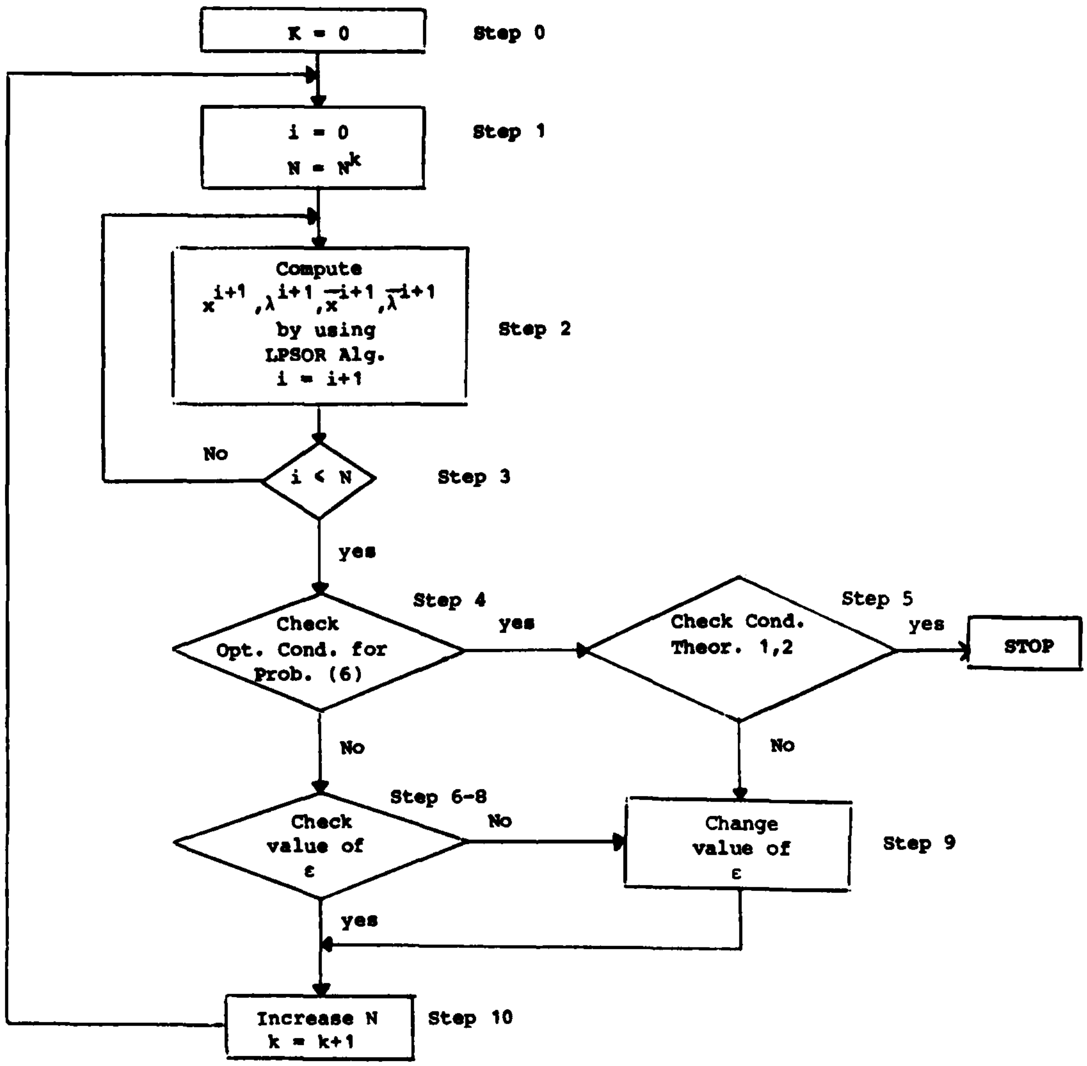
K = 0
Step 0
i = 0
N = N^k
Step 1
Compute
x^{i+1}, λ^{i+1}, x̄^{i+1}, λ̄^{i+1}
by using
LPSOR Alg.
i = i+1
Step 2
No
i < N
Step 3
yes
Step 4
Check
Opt. Cond. for
Prob. (6)
yes
Check Cond.
Theor. 1,2
Step 5
yes
STOP
No
No
Step 6-8
Check
value of
ε
No
Change
value of
ε
Step 9
yes
Increase N
k = k+1
Step 10

Figure 1

THEOREM 3. Assume that:

a) The linear program (4) is solvable.

b) The gradients of the active constraints are linearly independent at the optimal point $x' = p_2(0,\bar{x})$.

c) The strict complementarity holds at the optimal point $x' = p_2(0,\bar{x})$.

d) For a fixed ε the iteration map F is constituted by an iteration of the LPSOR algorithm and by the relation (7).

Then, i) either the algorithm terminates at some $(\lambda^{\nu}, x^{\nu}, \bar{\lambda}^{\nu}, \bar{x}^{\nu})$ where $\bar{x}^{\nu} = x^{\nu} = p_2(0,\bar{x})$ is optimal for the linear program (4) and $\left(\frac{\bar{\lambda}^{\nu} - \theta\lambda^{\nu}}{1-\theta}\right)$ the optimal solution of the dual of the linear program (4);

ii) or produces infinite sequences $\{\lambda^k, x^k, \bar{\lambda}^k, \bar{x}^k\}$ and $\{j(k)\}$ such that $\{j(k)\}$ is bounded (namely ε^j is changed only a finite number of times), $\{x^k\}$ and $\{\bar{x}^k\}$ converge to the optimal solution $x' = p_2(0,\bar{x})$ of the linear problem (4) and the sequence $\left\{\frac{\bar{\lambda}^k - \theta\lambda^k}{1-\theta}\right\}$ converges to the optimal solution of the dual of the linear program (4).

PROOF. First of all we prove that the sequence $\{j(k)\}$ is bounded.

We prove this by contradiction. Therefore we suppose that the sequence $\{j(k)\}$ is unbounded and consider the tests at step 5, step 6, step 7 and step 8 that should have increased the value of parameter j at step 9.

Let $\varepsilon^* > 0$ be the number considered in Theorem 1 then we must have for some k^*, $\varepsilon^{j(k)} < \varepsilon^*$ for all $k > k^*$. By using Theorem 1 and Theorem 2 it follows that the algorithm could not have increased the value of parameter j on account of a failure to satisfy the test in step 5. In fact if $\|\lambda^k - (\lambda^k - (M\lambda^k + d + \varepsilon^{j(k)}q))_+\| = 0$ and $\|\bar{\lambda}^k - (\bar{\lambda}^k - (M\bar{\lambda}^k + d + \theta\varepsilon^{j(k)}q))_+\| = 0$ for $k > k^*$ we have that λ^k and $\bar{\lambda}^k$ solve (see [6] page 472) problem (6) with $\varepsilon = \varepsilon^{j(k)}$ and $\varepsilon = \theta\varepsilon^{j(k)}$ respectively. Then Theorem 1 implies that $\|\bar{x}^k - x^k\| = 0$ and Theorem 2 that

$$(\bar{\lambda}^k - \theta\lambda^k) > 0 \quad \text{and} \quad \|c(\bar{x}^k - \theta x^k) - b(\bar{u}^k - \theta u^k)\| = 0$$

and hence, the algorithm should have terminated at step 5. Now we consider the test at step 6 for $k > k^*$. If we denote with $x(\varepsilon)$, $u(\varepsilon)$, $v(\varepsilon)$ the solutions of problems (5) and

(6), it follows from Lemma 2.1 of [10] that

$$\|x^k - x(\varepsilon^{j(k)})\|^2 \le \frac{1}{\varepsilon^{j(k)}}\Big(|\lambda^k(\frac{M\lambda^k + d}{\varepsilon^{j(k)}} + q)| + \|u(\varepsilon^{j(k)})\|\ \|(Ax^k - b)_+\| + \|v(\varepsilon^{j(k)})\|\ \|(-x^k)_+\|\Big) \tag{23}$$

$$\|\bar{x}^k - x(\theta\varepsilon^{j(k)})\|^2 \le \frac{1}{\theta\varepsilon^{j(k)}}\Big(|\bar{\lambda}^k(\frac{M\bar{\lambda}^k + d}{\theta\varepsilon^{j(k)}} + q)| + \|u(\theta\varepsilon^{j(k)})\|\ \|(A\bar{x}^k - b)_+\| + \|v(\theta\varepsilon^{j(k)})\|\ \|(-\bar{x}^k)_+\|\Big) \quad . \tag{24}$$

Since for $k \ge k^*$ (and, hence, $\varepsilon^{j(k)} \le \varepsilon^*$) we have that

$$x(\varepsilon^{j(k)}) = x(\theta\varepsilon^{j(k)}) = x^* = p_2(0,\bar{x})$$

by using (23) and (24) we obtain for $k \ge k^*$

$$\begin{aligned}
&K_2\Big(|\lambda^k(\frac{M\lambda^k + d}{\varepsilon^{j(k)}} + q)| + |\bar{\lambda}^k(\frac{M\bar{\lambda}^k + d}{\theta\varepsilon^{j(k)}} + q)|\Big) + K_2(\|(Ax^k - b)_+\| + \|(A\bar{x}^k - b)_+\|) + \\
&+ K_3(\|(-x^k)_+\| + \|(-\bar{x}^k)_+\|) - (\varepsilon^{j(k)})^2\|\bar{x}^k - x^k\|^2 \ge (k_1 - \varepsilon^{j(k)})|\lambda^k(\frac{M\lambda^k + d}{\varepsilon^{j(k)}} + q)| \\
&+ \Big(k_1 - \frac{\varepsilon^{j(k)}}{\theta}\Big)\ |\bar{\lambda}^k(\frac{M\bar{\lambda}^k + d}{\theta\varepsilon^{j(k)}} + q)| + (k_2 - \varepsilon^{j(k)}\|u(\varepsilon^{j(k)})\|)\|(Ax^k - b)_+\| \\
&+ \Big(k_2 - \frac{\varepsilon^{j(k)}}{\theta}\|u(\theta\varepsilon^{j(k)})\|\Big)\|(A\bar{x}^k - b)_+\| + (k_3 - \varepsilon^{j(k)}\|v(\varepsilon^{j(k)})\|)\|(-x^k)_+\| \\
&+ (k_3 - \frac{\varepsilon^{j(k)}}{\theta}\ \|v(\theta\varepsilon^{j(k)})\|)\|(-\bar{x}^k)_+\| \quad .
\end{aligned}$$

From assumption b) it follows that $\{u(\varepsilon^{j(k)})\}$, $\{u(\theta\varepsilon^{j(k)})\}$, $\{v(\varepsilon^{j(k)})\}$, $\{v(\theta\varepsilon^{j(k)})\}$ are bounded for all $k \ge k^*$ and therefore we can observe that the test at step 6 is satisfied for sufficiently large values of k (and hence for sufficiently small values of $\varepsilon^{j(k)}$).

The same arguments hold for test at step 7, in fact we have

$$\begin{aligned}
&K_4(\theta\varepsilon^{j(k)}|\|\bar{x}^k\|^2 - \|x^k\|^2| + |\theta\varepsilon^{j(k)}(\|\bar{x}^k\|^2 - \|x^k\|^2) - c(\bar{x}^k - \theta x^k) + b(\bar{u}^k - \theta u^k)|) \\
&- \varepsilon^{j(k)}|c(\bar{x}^k - \theta x^k) - b(\bar{u}^k - \theta u^k)| \ge (K_4 - \varepsilon^{j(k)})|c(\bar{x}^k - \theta x^k) - b(\bar{u}^k - \theta u^k)|
\end{aligned}$$

and, hence, the test is satisfied for suffieciently large values of k (hence for sufficiently small values of $\varepsilon^{j(k)}$).

Finally we verify the test at step 8. We can note that, for sufficiently large values of k, (and, hence, for sufficiently large values of N^k and sufficiently small values of $\varepsilon^{j(k)}$). Theorem 1, Corollary 1, Proposition 1 and Proposition 2 imply

$$J = \{i : \lambda_i^* = 0\} = \{i : \lambda_i^k = 0\} = \{i : \bar{\lambda}_i^k = 0\} = \{i : \lambda_i(\varepsilon^{j(k)}) = 0\} = \{i : \lambda_i(\theta\varepsilon^{j(k)}) = 0\}$$
$$I = \{i : \lambda_i^* > 0\} = \{i : \lambda_i^k > 0\} = \{i : \bar{\lambda}_i^k > 0\} = \{i : \lambda_i(\varepsilon^{j(k)}) > 0\} = \{i : \lambda_i(\theta\varepsilon^{j(k)}) > 0\} \tag{25}$$

where $\lambda^* = (u^*,v^*)$ is the KKT multiplier of the linear problem (4) and $\lambda(\varepsilon^{j(k)}) = (u(\varepsilon^{j(k)}),v(\varepsilon^{j(k)}))$ and $\lambda(\theta\varepsilon^{j(k)}) = (u(\theta\varepsilon^{j(k)}),v(\theta\varepsilon^{j(k)}))$ are, again, the solution of problem (6) with $\varepsilon = \varepsilon^{j(k)}$ and $\varepsilon = \theta\varepsilon^{j(k)}$ respectively.

Next we have $\forall i \in I$

$$\bar{\lambda}_i^k - \theta\lambda_i^k = (1-\theta)\lambda_i^* + (\bar{\lambda}_i^k - \lambda_i(\theta\varepsilon^{j(k)})) + (\lambda_i(\theta\varepsilon^{j(k)}) - \lambda_i^*) - \theta(\lambda_i^k - \lambda_i(\varepsilon^{j(k)}))$$
$$- \theta(\lambda_i(\varepsilon^{j(k)}) - \lambda_i^*) \geq |(1-\theta)\lambda^*_i| - |\bar{\lambda}_i^k - \lambda_i(\theta\varepsilon^{j(k)})| - |\lambda_i(\theta\varepsilon^{j(k)}) - \lambda_i^*|$$
$$- \theta|\lambda_i^k - \lambda_i(\varepsilon^{j(k)})| - \theta|\lambda_i(\varepsilon^{j(k)}) - \lambda_i^*| \quad .$$

Then by construction of $\{\lambda^k\}$ and $\{\bar{\lambda}^k\}$ in step 2 and step 3, by recalling Corollary 1 and by using that $\lim_{k\to\infty} N^k = \infty$ and $\lim_{k\to\infty} \varepsilon^{j(k)} = 0$ we obtain

$$\lim_{k\to\infty} \max\{|\bar{\lambda}_i^k - \lambda_i(\theta\varepsilon^{j(k)})|,\ |\lambda_i(\theta\varepsilon^{j(k)}) - \lambda_i^*|,\ |\lambda_i^k - \lambda_i(\varepsilon^{j(k)})|,\ |\lambda_i(\varepsilon^{j(k)}) - \lambda_i^*|\} = 0$$

and, for sufficiently large values of k,

$$\bar{\lambda}_i^k - \theta\lambda_i^k \geq 0 \quad \forall i \in I$$

and, by taking into account (25),

$$\bar{\lambda}^k - \theta\lambda^k \geq 0 \quad .$$

Hence, we have that, for sufficiently large values of k, the test at step 8 is satisfied.

Therefore we can conclude that j can not be increased an infinite number of times as hypothesised, but there exists a k^{**} such that $j(k) = j^*$ for all $k \geq k^{**}$.

Now if the algorithm terminates at step 5 part i) of the Theorem follows from Theorem 1 and Theorem 2.

If the algorithm produces infinite sequences $\{\lambda^k\}$ and $\{\bar{\lambda}^k\}$ then, by Proposition 2, these sequences converge to two points $\hat{\lambda}*$ and $\bar{\lambda}*$ which solve the problem (6) with two different values of ε (that is $\varepsilon = \varepsilon^{j*}$ and $\varepsilon = \theta\varepsilon^{j*}$) and $\hat{x}*$ and $\bar{x}*$ solve the primal problem (5).

Therefore we have

$$|\hat{\lambda}*(\frac{M\hat{\lambda}* + d}{\varepsilon^{j*}} + q)| = 0 , \qquad |\bar{\lambda}*(\frac{M\bar{\lambda}* + d}{\theta\varepsilon^{j*}} + q)| = 0 , \tag{26}$$

$$(A\hat{x}* - b)_+ = 0 , \qquad (A\bar{x}* - b)_+ = 0 , \tag{27}$$

$$(-\hat{x}*)_+ = 0 , \qquad (-\bar{x}*)_+ = 0 , \tag{28}$$

$$\|\hat{\lambda}* - (\hat{\lambda}* - (M\hat{\lambda}* + d + \varepsilon^{j*}q))_+\| = 0 , \quad \|\bar{\lambda}* - (\bar{\lambda}* - (M\bar{\lambda}* + d + \theta\varepsilon^{j*}q))_+\| = 0 , \tag{29}$$

and from equality between the primal objective function and the dual objective function of problem (5) it follows:

$$\frac{\varepsilon^{j*}}{2}\|\hat{x}*\|^2 - c^T\hat{x}* = -\frac{\varepsilon^{j*}}{2}\|\hat{x}*\|^2 - b^T\hat{u}*, \qquad \frac{\theta\varepsilon^{j*}}{2}\|\bar{x}*\|^2 - c^T\bar{x}* = -\frac{\theta\varepsilon^{j*}}{2}\|\bar{x}*\|^2 - b^T\bar{u}* ,$$

$$\varepsilon^{j*}\|\hat{x}*\|^2 - c^T\hat{x}* + b^T\hat{u}* = 0 , \qquad \theta\varepsilon^{j*}\|\bar{x}*\|^2 - c^T\bar{x}* + b^T\bar{u}* = 0 \tag{30}$$

Now by (26), (27), (28) and step 6 we have

$$\|\bar{x}* - \hat{x}*\| = 0 . \tag{31}$$

From (31), (30) and step 7 it results

$$|(1-\theta)c^T\hat{x}* - b^T(\bar{u}* - \theta\hat{u}*)| = 0 .$$

By (29) and step 8 we obtain also

$$\bar{u}* > \theta\hat{u}* , \quad \bar{v}* > \theta\hat{v}*$$

so that the second part ii) of Theorem follows, again, from Theorem 1 and Theorem 2.

□

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By perturbing properly a linear program to a separable quadratic program it is possible to solve the latter in its dual variable space by iterative techniques such as sparsity-preserving SOR (successive overrelaxation) algorithms. The main result of this paper gives an effective computational criterion to check whether the solutions of the perturbed quadratic programs provide the least 2-norm solution of the original linear program.

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