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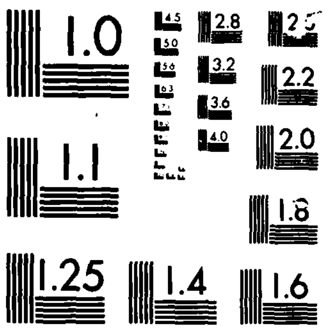

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OUTDOOR SCULPTURES

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ABSTRACT

The purpose of this paper, to be submitted to the "Topologie Structural" published by the School of Architecture at Québec, is to show by means of three examples, that some theorems of 3-dimensional Geometry suggest esthetically promising outdoor sculptures. The first two examples are obtained by the Harmonic Analysis, i.e. the Finite Fourier Series, applied to an arbitrary skew hexagon in R^3. The third example of a geometric outdoor sculpture is furnished by an Anti-cylinder. The author hopes that some architect will notice this paper and build an outdoor sculpture based on one of the three examples given here.

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Key Words: Outdoor Sculptures, Finite Fourier Series, Anti-cylinder

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SIGNIFICANCE AND EXPLANATION

The paper tries to show that some theorems of 3-dimensional Geometry do suggest outdoor sculptures that are esthetically satisfying and also appealing to non-mathematicians. Three examples of such theorems are given and illustrated in Figs. 1, 2, and 3.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

OUTDOOR SCULPTURES

I. J. Schoenberg

Outdoor sculptures have a long history, from ancient Egypt and Greece, to Michelangelo, Henri Moore and Picasso, not to forget their use to inspire religious feelings. Using three examples, I wish to show here that some theorems of 3-dimensional Geometry may lead to outdoor sculptures that might also appeal to the esthetic imagination of non-mathematical beholders, this appeal being the main point. Of course, a mathematical understanding of their construction would enhance their appreciation, just as religious feelings magnify the meaning of a religious work of art, whether it is a sculpture, a painting, or a musical composition.

Our three examples are illustrated in Figs. 1, 2, and 3, respectively, but these do not do them justice, because they are 2-dimensional representations of 3-dimensional figures. Much more suggestive are the three models made by the author, of about two to three feet in diameter. These models suggest that similar constructions made of anodised aluminum tubing, of from 15 to 20 feet in size, would provide attractive outdoor sculptures of general appeal.

Examples 1 and 2 (§§1,2) are based on the Harmonic Analysis, i.e. the Finite Fourier Series, of a skew pentagon, and a skew hexagon, respectively. Example 3 represents a novel figure called an Anti-cylinder.

In spite of the rigid geometric restrictions implied by our theorems, the constructor is left with some free choices characteristic of works of art: In Figures 1 and 2 it is the choice of the shapes of the pentagon and hexagon: Order out of chaos. In Example 3 it is mainly the orientation of the Figure.

1. Douglas' Skew Pentagon. About 35 years ago Jesse Douglas, of Plateau Problem fame, discovered the following beautiful

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Theorem 1. Let

$$\Pi = (Z_0, Z_1, Z_2, Z_3, Z_4)$$

be an arbitrary skew pentagon in R^3 viewed as a vector space. Let

$$Z'_k = \frac{1}{2}(Z_{k+2} + Z_{k-2}) \text{ , } [k = 0, 1, \ldots, 4 \text{ ; } Z_{k+5} = Z_k]$$

be the midpoint of the side $[Z_{k+2}, Z_{k-2}]$ which is opposite to the vertex Z_k. For each k we determine on the line joining Z_k to Z'_k the points f_k^1 and f_k^2 such that

$$f_k^1 - Z'_k = \frac{1}{\sqrt{5}}(Z'_k - Z_k) \text{ , } f_k^2 - Z'_k = -\frac{1}{\sqrt{5}}(Z'_k - Z_k) \text{ .}$$

Then the pentagon

$$\Pi^1 = (f_0^1, f_1^1, f_2^1, f_3^1, f_4^1)$$

is a plane pentagon and is an affine image of a regular pentagon. Also the pentagon

$$\Pi^2 = (f_0^2, f_1^2, f_2^2, f_3^2, f_4^2)$$

is a plane pentagon and is an affine image of a star-shaped regular pentagon (Fig. 1). See [1] and [2]. Also [3, Chap. 9, Sections 1 to 3].

Theorem 1 is not difficult to prove, but was not easy to discover. It seems that Douglas never built a model in R^3 illustrating his Theorem 1. The author constructed such a model and showed it to the late Allen McNab, a former director of the Chicago Art Institute. McNab placed the model as in Fig. 1 and said that it would make a fine outdoor sculpture. It would suggest the superstructure of a sailing ship.

2. The Skew Hexagon. From the skew pentagon we now pass to a skew hexagon. This has three harmonic components; however, the last two components are trivial: The second component is a triangle described twice, and the third is a segment described six times. For this reason we construct only the first harmonic component which is an affine image of a regular hexagon. Its construction is described by our

Theorem 2. Let

$$\Pi_x = (x_0, x_1, x_2, x_3, x_4, x_5)$$

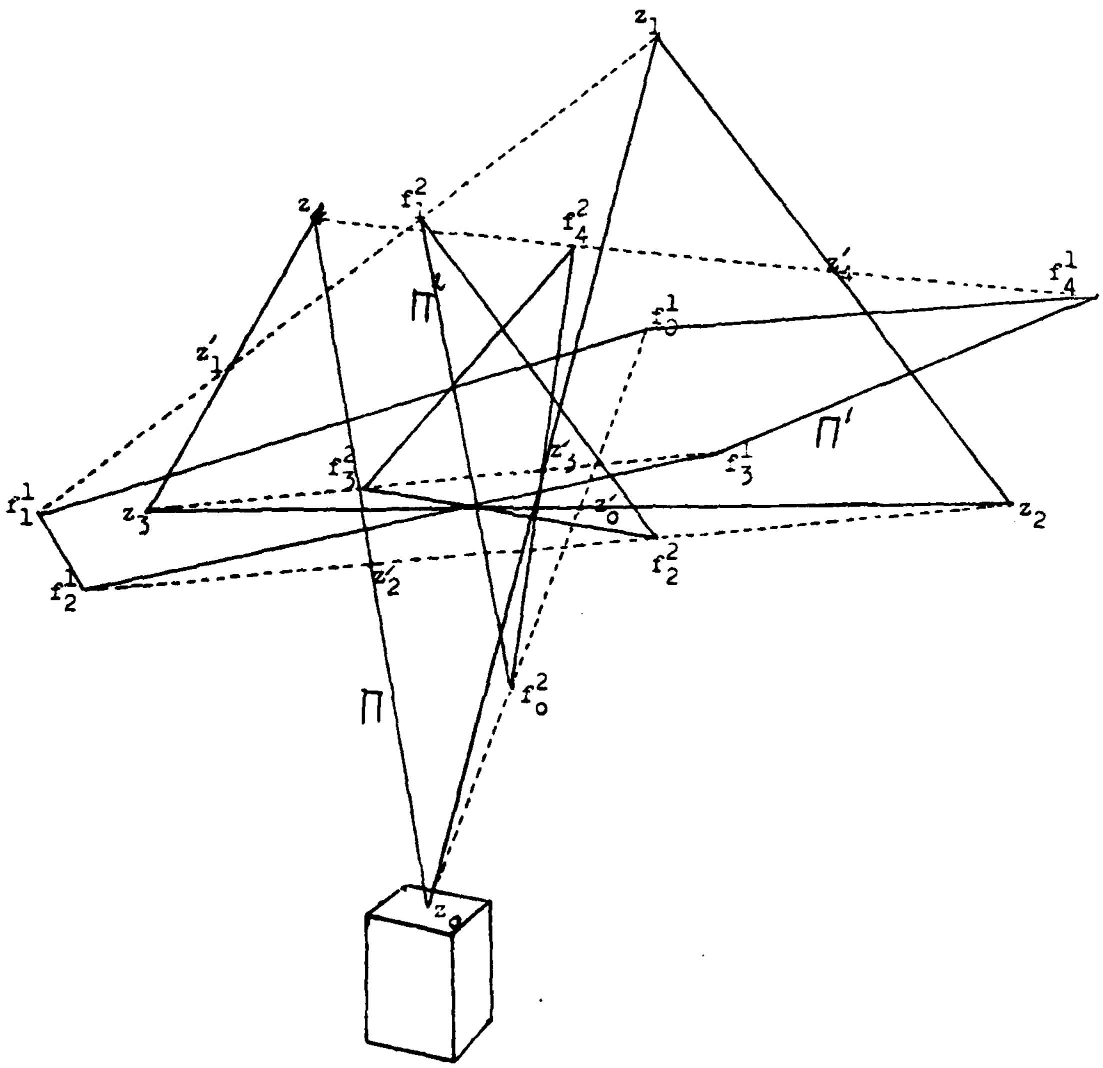


Fig.1
The two harmonic components of a skew pentagon

be an arbitrary closed skew hexagon in R^3, which we regard as a vector space. For convenience we write $\Pi_x = (x_k)$, where k runs from 0 to 5, mod 6, so that $x_0 = x_6$, $x_1 = x_5$ etc.

We define the new hexagon $\Pi_y = (y_k)$ by

$$\Pi_y : y_k = x_k + \frac{1}{2}(x_k - x_{k-3}) \quad .$$

Notice that the three vectors

$$\overrightarrow{y_0y_3}, \overrightarrow{y_1y_4}, \overrightarrow{y_2y_5}$$

are, respectively, on the same lines as the three diagonals

$$\overrightarrow{x_0x_3}, \overrightarrow{x_1x_4}, \overrightarrow{x_2x_5} \text{ of } \Pi_x ,$$

but have double their respective lengths, and have common midpoints with them.

With $z_k := \frac{1}{2}(x_{k+1} + x_{k-1})$, we define our last hexagon $\Pi_w = (w_k)$ by

$$\Pi_w : w_k = z_k + \frac{1}{3}(y_k - z_k) \quad .$$

Since $\overrightarrow{y_kz_k}$ is a median of the triangle $y_kx_{k-1}x_{k+1}$, we may also write

$$w_k = \frac{1}{3}(y_k + x_{k-1} + x_{k+1}) \quad ,$$

showing that w_k is the centroid of the triangle $y_kx_{k-1}x_{k+1}$.

The final result: The hexagon Π_w is in a plane π and is in π an affine image of a regular haxagon. (Fig. 2).

In Fig. 2 the arbitrary hexagon Π_x is drawn with full lines. The model constructed by the author looks like a large insect (Praying Mantis?) or a bird. A proof of Theorem 2 follows readily from the finite Fourier series for the sixth roots of unity.

3. The Anti-cylinder. Everybody knows what a cylinder is, one of the important figures of Solid Geometry. It has a bottom-circle and an equal top-circle, both having a common axis. Its lateral surface is made of infinitely many straight segments, all of equal length, which are the generatrices of the cylinder. Each generatrix cuts each of the two circles at right angle (90°). But what is an anti-cylinder?

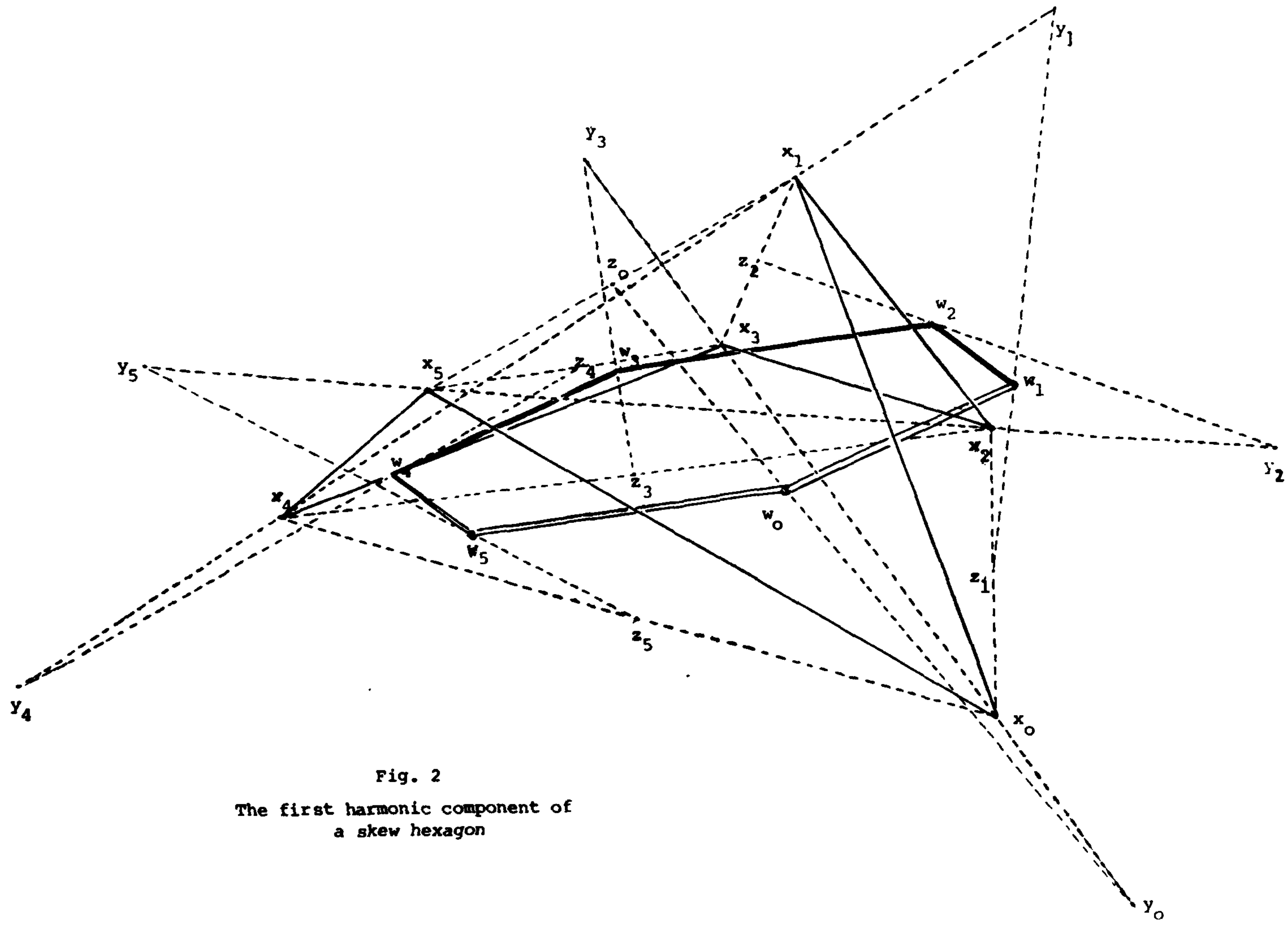


Fig. 2

The first harmonic component of
a *skew hexagon*

The upper part of Fig. 3 represents an anti-cylinder described by

Theorem 3. Let Γ be a circle of radius $OA = a$ which we see en face in Fig. 3. The segment $AC = 2a$ is its vertical diameter. We select the length $c < a$ and let

$AA' = c$. We perform on Γ in succession the following two rigid motions which move Γ to its final position Γ' :

(i) We translate (or lift) Γ vertically by the vector $\overrightarrow{AA'}$ obtaining the circle Γ_1 (not shown in Fig. 3) having the vertical diameter $A'C'$.

(ii) We turn Γ_1 around its diameter $A'C'$ clockwise (if seen from above) by an angle α defined by the equation

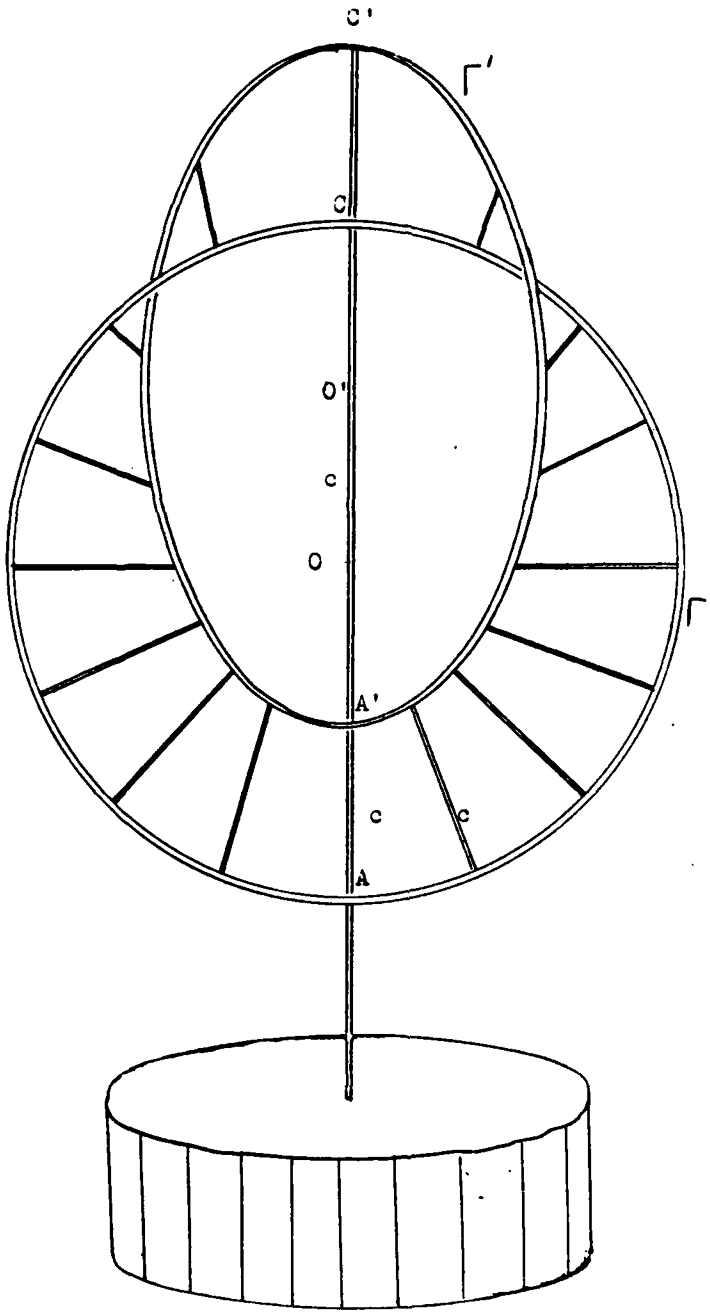
$$\sin \alpha = \frac{c}{a} .$$

This rotation changes Γ_1 into its new position Γ' .

The final result: The circles Γ and Γ' have an infinity of common normals all of the same length $= c$. These are the generatrices of the anti-cylinder.

Fig. 3 shows 16 generatrices of the anti-cylinder. See [4] where Theorem 3 is established without mentioning the name anti-cylinder. A final remark: Let $T(\Gamma,c)$ denote the torus enveloped by a moving sphere of radius c whose center describes the circle Γ . It is observed in the first two paragraphs of [5] that the circle Γ' of Theorem 3 is identical with one of the Villarceau circles of the torus $T(\Gamma,c)$. The anti-cylinder of Fig. 3 could be set up in a children's playground, the children climbing around the figure using its generatrices.

For his model of Fig. 3, the author chose in Theorem 3 the values $a = 10$ cm , $c = 6$ cm , the corresponding value of the angle α , obtained from $\sin \alpha = c/a$, being $\alpha = 36^\circ.8699$.



A cylinder carrying an anti-cylinder

Fig.3

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