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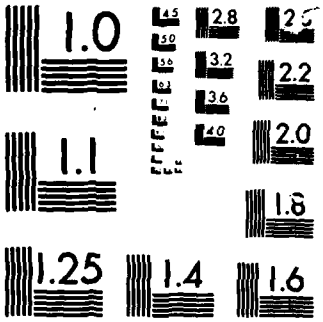

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ON L_1-CONTRACTION FOR SYSTEMS OF CONSERVATION LAWS

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MATHEMATICS RESEARCH CENTER

ON L_1-CONTRACTION FOR SYSTEMS OF CONSERVATION LAWS[1]

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ABSTRACT

We prove that for 2×2, strictly hyperbolic, genuinely nonlinear systems of conservation laws, there is no metric D such that

$$\int_{-\infty}^{\infty} D(u(x,t),c)dx$$

is a decreasing function of time for every weak solution u, $u_0(\pm\infty) = c$.

AMS (MOS) Subject Classifications: 35L65, 76N10

Key Words: Conservation laws, Decay of solutions, L_1-dependence

Work Unit Number 1 (Applied Analysis)

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[1]Work done while the author was a visiting member of the Mathematics Research Center, University of Wisconsin-Madison.
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SIGNIFICANCE AND EXPLANATION

The Cauchy problem for a 2×2 system of conservation laws in one dimension is

$$u_t + (f(u))_x = 0, \quad x \in \mathbb{R},\ t > 0$$

$$u(x,0) = u_0(x)$$

where $u = (u_1, u_2)$, $f = (f_1(u), f_2(u))$.

Such systems of equations usually come from the application of the laws of conservation for physical quantities like mass, momentum and energy, and arise in problems of gas dynamics, elasticity, oil reservoir simulation and other areas of engineering.

The questions of decay and continuous dependence with respect to the initial data are central issues in the study of the problem above. The result proved here rules out the use of certain functionals to study the decay of solutions and is relevant to the issue of L_1 continuity with respect to the data.

The reponsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

ON L_1-CONTRACTION FOR SYSTEMS OF CONSERVATION LAWS[1]

Jorge G. S. Patiño

For 2×2, strictly hyperbolic, genuinely nonlinear (cf. [1]) systems of conservation laws it was proved in [2] that there is no metric D, compatible with the state space, such that

$$I_D(u,v;t) \equiv \int_{-\infty}^{\infty} D(u(x,t),v(x,t))dx \tag{1}$$

is a decreasing function of time for any two weak solutions u,v whose initial conditions agree off a compact set.

In [2] a metric D is compatible with the state space Σ if

C1. $D : \Sigma \times \Sigma \to R$

is a symmetric function.

C2. $D(u,v) + D(v,w) \geq D(u,w) \quad \forall u,v,w \in \Sigma$

C3. $C_0^{-1}|u - v| \leq D(u,v) \leq C_0|u - v| \quad \forall u,v \in \Sigma$

with a *uniform constant* C_0.

Here we give an easier proof of this result, and give conditions that a metric must satisfy if condition C3 is to be relaxed.

We wish to point out that relaxing condition C3 is important since it rules out the use e.g. of entropies or quadratic functions to obtain certain integral decay estimates. It is also interesting to note that the solutions used in the

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construction of the counter examples below are the elementary "spikes" used frequently in decay arguments (cf. [3]).

Thus let

$$u_t + (f(u))_x = 0 \tag{2}$$

be any 2×2 system; strictly hyperbolic and genuinely nonlinear on a region $N \subset \mathbb{R}^2$. Let $\lambda_1(u)$ and $\lambda_2(u)$ be the eigenvalues of $df(u)$ with corresponding eigenvectors $r_1(u)$ and $r_2(u)$. Let $R_1(u,u^*)$ and $R_2(u,u^*)$ be respectively the integral curves of $r_1(u)$ and $r_2(u)$ passing through u^*. $R_1(u,u^*)$ and $R_2(u,u^*)$ are called rarefaction curves. Let $S_1(u,u^*)$ and $S_2(u,u^*)$ be the curves of states that can be joined by respectively a 1-shock and 2-shock to the right of u^*. These are called shock curves. Given a state u^* on N, shock and rarefaction curves exist locally [1].

We then have the following theorem

<u>Theorem 1</u>. Let u and v be weak solutions of (2) whose initial conditions agree off a compact set, <u>then</u> there exists no metric D, which is symmetric, such that $I_D(u,v;t)$ is a strictly decreasing function of time.

<u>Proof</u>. Take any states $u_L, u_R, \bar{u}$ and $\bar{\bar{u}}$ related in the following way (Figure 1)

i) u_R and u_L are joined by a 1-shock with speed s_1, with u_L on the left.

ii) u_R and $\bar{u}$ are joined by a 1-rarefaction.

iii) $\bar{u}$ and u_L are joined by a 2-rarefaction.

iv) $\bar{\bar{u}}$ and u_L are joined by a 2-rarefaction.

v) $\bar{\bar{u}}$ and u_L are joined by a 1-rarefaction.

(We assume here that $\lambda_2(u)$ increases from $\bar{u}$ to u_L. The case where $\lambda_2(u)$ decreases from $\bar{u}$ to u_L is discussed below.)

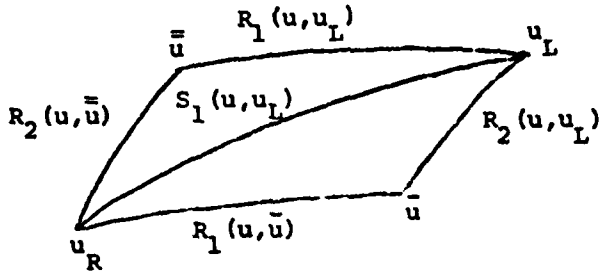

Figure 1

The system (2) with initial condition

$$u(x,0) = \begin{cases} u_R & \text{if } 0 < x < (s_1 - \lambda_1(u_R))T \\ u_L & \text{otherwise} \end{cases}$$

has, for $t < T$, the solution u shown in Figure 2.

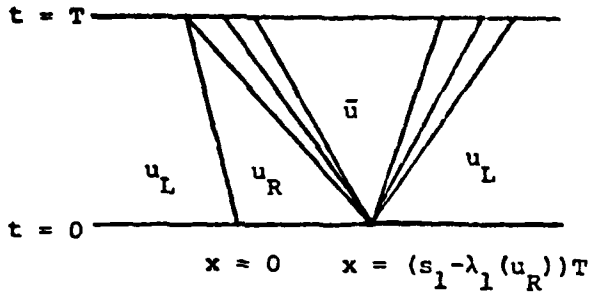

Figure 2. A line denotes a shock and a fan denotes a rarefaction.

Then

$$I_D(u,u_L;0) = D(u_R,u_L)(s_1 - \lambda_1(u_R))T$$

and

$$I_D(u,u_L;T) = T\int_{\lambda_1(u_R)}^{\lambda_1(\bar{u})} D(\mu(\lambda),u_L)d\lambda + D(u_L,\bar{u})(\lambda_2(\bar{u}) - \lambda_1(\bar{u}))T$$
$$+ T\int_{\lambda_2(\bar{u})}^{\lambda_2(u_L)} D(\nu(\lambda),u_L)d\lambda \ ,$$

where $\mu(\lambda)$ and $\nu(\lambda)$ denote parametrizations of $R_1(u,\bar{u})$ with respect to λ_1 and of $R_2(u,u_L)$ with respect to λ_2, respectively.

Now, with u_R, u_L and $\bar{u}$ denoting the same states as in Figure 1, consider the following initial condition:

$$v(x,0) = \begin{cases} u_L & \text{if } 0 < x < (\lambda_2(u_L) - s_1)T \\ u_R & \text{otherwise.} \end{cases}$$

The solution v of this problem, for $t < T$, is given by the waves in Figure 3.

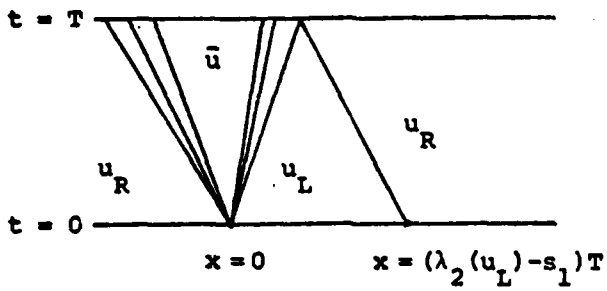

Figure 3

Then

$$I_D(v,u_R;0) = D(u_L,u_R)(\lambda_2(u_L) - s_1)T$$

and

$$I_D(v,u_R;0) = T\int_{\lambda_1(u_R)}^{\lambda_1(\bar{u})} D(\mu(\lambda),u_R)d\lambda + T\int_{\lambda_2(\bar{u})}^{\lambda_2(u_L)} D(\nu(\lambda),u_R)d\lambda$$

$$+ D(\bar{u},u_R)(\lambda_2(\bar{u}) - \lambda_1(\bar{u}))T .$$

To prove the theorem by contradiction, assume now that

$$I_D(v,u_R;T) + I_D(u,u_L;T) < I_D(v,u_R;0) + I_D(u,u_L;0) . \qquad (3)$$

Thus

$$\int_{\lambda_1(u_R)}^{\lambda_1(\bar{u})} \{D(\mu(\lambda),u_L) + D(\mu(\lambda),u_R)\}\,d\lambda$$

$$+ \int_{\lambda_2(\bar{u})}^{\lambda_2(u_L)} \{D(\nu(\lambda),u_L) + D(\nu(\lambda),u_R)\}\,d\lambda$$

$$+ D(u_L,\bar{u})(\lambda_2(\bar{u}) - \lambda_1(\bar{u})) + D(\bar{u},u_R)(\lambda_2(\bar{u}) - \lambda_1(\bar{u}))$$

$$\leq D(u_R,u_L)(s_1 - \lambda_1(u_R)) + D(u_L,u_R)(\lambda_2(u_L) - s_1) \ .$$

Now, adding and subtracting

$$D(u_L,u_R)(\lambda_1(\bar{u}) - \lambda_1(u_R)) + D(u_L,u_R)(\lambda_2(u_L) - \lambda_2(\bar{u}))$$

we get

$$\int_{\lambda_1(u_R)}^{\lambda_1(\bar{u})} \{D(\mu(\lambda),u_L) + D(\mu(\lambda),u_R) - D(u_L,u_R)\}\,d\lambda$$

$$+ \int_{\lambda_2(\bar{u})}^{\lambda_2(u_L)} \{D(\nu(\lambda),u_L) + D(\nu(\lambda),u_R) - D(u_R,u_L)\}\,d\lambda \tag{4}$$

$$+ (D(u_L,\bar{u}) + D(\bar{u},u_R) - D(u_R,u_L))(\lambda_2(\bar{u}) - \lambda_1(\bar{u})) \leq 0 \ .$$

By the triangle inequality the two integrands and the third line above are positive. Since $\lambda_1(u_R) < \lambda(\bar{u})$ and $\lambda_2(\bar{u}) < \lambda_2(u_L)$, equality holds in (4) if and only if equality holds in each of the triangle inequalities, in particular only if

$$D(u_L,\bar{u}) + D(\bar{u},u_R) = D(u_R,u_L) \ . \tag{5}$$

A similar construction as above using the initial conditions

$$u_0(x) = \begin{cases} u_L & \text{if } 0 < x < \lambda_1(u_L) - s \\ \bar{\bar{u}} & \text{otherwise} \end{cases}$$

and

$$u_0'(x) = \begin{cases} \bar{\bar{u}} & \text{if } 0 < x < \lambda_2(v) - \lambda_1(v) \\ u_L & \text{otherwise} \end{cases}$$

will, by the same argument, yield

$$D(u_R,\bar{\bar{u}}) + D(u_R,u_L) = D(\bar{\bar{u}},u_L) \ . \tag{6}$$

Now, (5) and (6) give

$$D(\bar{\bar{u}},U_L) - D(\bar{u},u_R) = D(u_L,\bar{u}) + D(u_R,\bar{\bar{u}}) \ . \tag{7}$$

This equation gives, using the compatibility condition C3. Lemma 4 in [2] after which the proof proceeds identically.

If one does not assume C3, (5) and (6) put restrictions on the metric D. For instance the Euclidean distance or a linear combination of changes in Riemann invariants will not work.

In the case where $\lambda_2(u)$ decreases from $\bar{u}$ to u_L a similar construction using the shock curves $S_2(u,\bar{\bar{u}})$ and $S_2(u,\hat{u})$ (see Figure 4) and the same initial value problems as above yield conditions similar to (5) and (6) from where the proof would proceed identically.

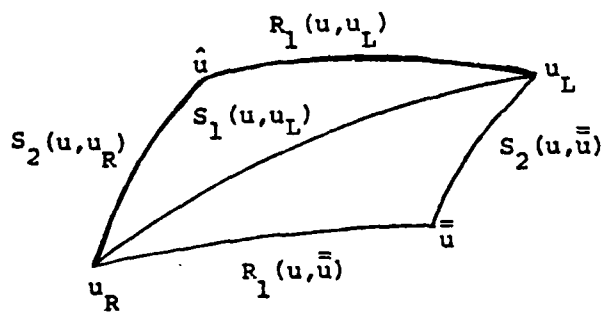

Figure 4

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