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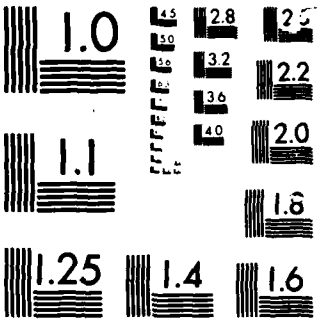

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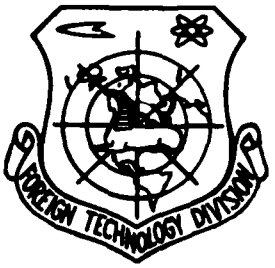
MAGNETIC SOLITON, HOMOTOPY AND HIGGS THEORY

by

Li Yuanjie and Lei Shizu



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By: Li Yuanjie and Lei Shizu

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Magnetic soliton, Homotopy and Higgs theory

Li Yuanjie Lei Shizu

Abstract

The two- and three-dimensional nonlinear Higgs field ϕ is used as an example to investigate the theory of magnetic soliton and the connection between the homotopy theory and Higgs theory. It is verified that, in a spontaneous and symmetrical breaking, because of the symmetry of electromagnetic gauge subgroup, there exists correspondence between the conservation of magneton and the incomplete symmetry of secondary order.

A. Soliton solution and Homotopy theory

Soliton is a stable finite energy solution of the classical Lagrange equation of motion. There are four kinds of stable soliton and their corresponding dimensions are D=1, 2, 3, 4, respectively. Soliton is arising from nonlinear field theory. We studied and discussed the 2- and 3-dimensional ϕ^4 theory in this paper.

(1). Spinor

The Lagrangian density is represented by

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(D_\mu\phi)^*D^\mu\phi - \frac{1}{4}\lambda(\phi\phi^* - \frac{m^2}{\lambda})^2,$$

and the field equation is

$$\partial^\nu F_{\mu\nu} = j_\mu = -\frac{1}{2}ie(\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*) + e^2A_\mu\phi\phi^*, \tag{1}$$

$$D_\mu D^\mu\phi = -\lambda\phi(\phi\phi^* - \frac{m^2}{\lambda}). \tag{2}$$

The condition of finite energy requires $D_\mu\phi \to 0$ at $r\to\infty$. Equation (2) yields a cylindrical symmetry solution:

$$\phi = f(r)e^{in\theta},\quad f(r)\to(1 - Ce^{-\sqrt{2}mr})\frac{m}{\sqrt{\lambda}}.$$

The existence of a stable soliton is indicated by a localization of energy around r=0 (see Figure 1). Examine

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu , \qquad \text{A.1.3}$$

where $F_{\mu\nu}$ is an antisymmetry tensor. F_{12} is the only independent nonzero term and its resulting flux on the $(x_1\ x_2)$ plane is

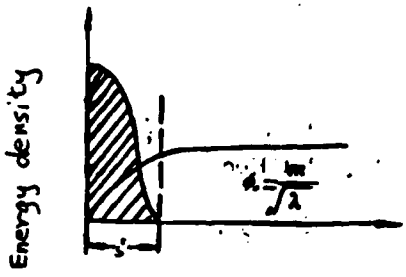

Figure 1.

$$\Phi = \int_\sigma F_{12} dx_1 dx_2 = \int_\sigma (\partial_1 A_2 - \partial_2 A_1) dx_1 dx_2$$

$$= \int_{S_-} A_k dx^k = \frac{1}{e}\int_{S_-} \partial_k \theta\, dx^k = \frac{1}{e}\int_{S_\infty} d\theta .$$

where $A_k = \frac{1}{e}\partial_k \theta$ is derived from the boundary condition $D_\mu \phi \to 0$. In fact, substituting $\phi = |\phi| e^{i\theta}$ into $\partial_\mu |\phi| = 0, A_\mu = \frac{1}{e}\partial_\mu \theta$, one obtains

$$e^{i\theta}\partial_\mu |\phi| + |\phi| i e^{i\theta} \partial_\mu \theta = ieA_\mu |\phi| e^{i\theta} .$$

Because of the cylindrical symmetry in the S_- annulus, $\partial_\mu |\phi| = 0$, we have $A_\mu = \frac{1}{e}\partial_\mu \theta$. Then,

$$\Phi = \frac{1}{e} 2n\pi = n\frac{2\pi}{e}, \quad (n = 0, \pm 1, \pm 2, \cdots).$$

In Homotopy theory, change of variable by the following means

$$\int_{S_-} A_k dx^k = \frac{1}{e}\int_{S_\infty} d\theta$$

can be regarded as a mapping $\phi:\ x \in S_- \to \phi(x) \in S$, . ϕmaps annulus S_∞ in two-dimensional space onto n multiple annuli in ϕ configuration. Its corresponding Homotopy set is $[n]$, therefore, there is topological charge n. It is a measure of the magnitude of the magnetic

flux (in units of $\frac{2\pi}{e}$). The topological invariance of the field configuration can be described by the Homotopy group $(U(1)) = n$ $(n=0, \pm 1, \pm 2, \ldots)$.

(2). Magnetic monopole

In this case, the Lagrangian density of the field is written as

$$L = -\frac{1}{4} F_{\mu\nu}^{a} F^{\mu\nu a} + \frac{1}{2} D_{\mu}\phi^{a} D^{\mu}\phi^{a} - \frac{1}{4}\lambda\left(\phi^{a}\phi^{a} - \frac{m^2}{\lambda}\right)^2,$$

the field equation is

$$D^{\mu}F_{\mu\nu}^{a} = -e\varepsilon^{abc}\phi^{b} D_{\nu}\phi^{c}, \tag{3}$$

$$D^{\mu}D_{\mu}\phi^{a} = -\lambda\phi^{a}\left(\phi^{2} - \frac{m^2}{\lambda}\right). \tag{4}$$

If we choose a stable finite energy solution, imposing boundary condition $D_{\mu}\phi^{a} \to 0$. Equation (4) yields a spherical symmetry solution:

$$\phi^{a} = -\frac{x_a}{r}\frac{m}{\sqrt{\lambda}}, \qquad \text{A.1.9}$$

and there is a stable soliton—magnetic monopole at $r=0$ (see Figure 2). Following t'Hooft method to construct a free gauge electromagnetic field, we obtain

$$F_{\mu\nu} = \hat{\phi}^{a} F_{\mu\nu}^{a} - \frac{1}{e}\varepsilon_{abc}\hat{\phi}^{a} D_{\mu}\hat{\phi}^{b} D_{\nu}\hat{\phi}^{c}, \qquad \text{A.1.10}$$

$F_{\mu\nu}$ can be written as

$$F_{\mu\nu} = M_{\mu\nu} + H_{\mu\nu}, \qquad \text{A.1.11}$$

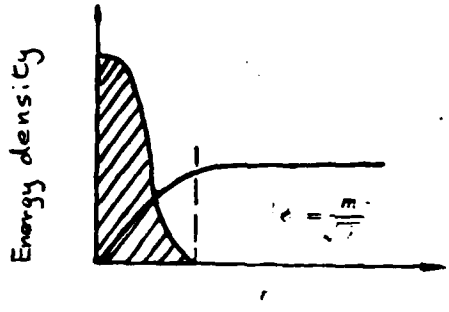

Figure 2.

where

$$M_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu; \quad \hat{\phi}^a = \phi^a/(\phi^b\phi^b)^{1/2};$$

$$B_\mu = \hat{\phi}^a A^a_\mu; \quad H_{\mu\nu} = \frac{1}{e}\epsilon_{abc}\hat{\phi}^a\partial_\mu\hat{\phi}^b\partial_\nu\hat{\phi}^c,$$

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + \frac{1}{e}\epsilon_{abc}\hat{\phi}^a\partial_\mu\hat{\phi}^b\partial_\nu\hat{\phi}^c.$$

then

The antisymmetry of $M_{\mu\nu}$ and $H_{\mu\nu}$ can easily be proved.

and

$$H_{\mu\nu} = \frac{1}{e}\in_{abc}\hat{\phi}^a\partial_\mu\hat{\phi}^b\partial_\nu\hat{\phi}^c = \frac{1}{e}\epsilon_{acb}\hat{\phi}^a\partial_\mu\hat{\phi}^a\partial_\nu\hat{\phi}^b$$

$$= -\frac{1}{e}\epsilon_{abc}\hat{\phi}^a\partial_\nu\hat{\phi}^b\partial_\mu\hat{\phi}^c = -H_{\nu\mu},$$

hence, the conserved current is

$$K_\mu = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial^\nu F^{\rho\sigma} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial_\nu H^{\rho\sigma},$$

and charge conservation is

$$M = \frac{1}{4\pi}\int k_0 d^3x.$$

The condition $\vec{K}\to 0$ as $x\to\infty$ leads to

$$\frac{\partial}{\partial t}M = \frac{1}{4\pi}\int\frac{\partial}{\partial t}k_0 d^3x = \frac{1}{4\pi}\int_V -\nabla\cdot\vec{K}d^3x = \frac{1}{4\pi}\int_{S^2_\infty}\vec{K}d^2x = 0,$$

Obviously, M is conserved.

We use the Homotopy theory to further study the meaning of M and start with

$$M = \frac{1}{4\pi}\int_V \epsilon_{ijk}\epsilon_{abc}\partial_i(\hat{\phi}^a\partial_j\hat{\phi}^b\partial_k\hat{\phi}^c)d^3x$$

$$= \frac{1}{8\pi e}\lim_{R\to\infty}\oint_{S^2_R}\epsilon_{ijk}\epsilon_{abc}\hat{\phi}^a\partial_j\hat{\phi}^b\partial_k\hat{\phi}^c(d^2x)_i,$$

x is parametrized in the S^2_R annulus such that $x = x(\xi_1,\xi_2)$

$$(d^2x)_i = \frac{1}{2}\epsilon_{imn}dx^m dx^n$$

$$= \frac{1}{2}\epsilon_{imn}\frac{\partial x^m}{\partial\xi^\alpha}\frac{\partial x^n}{\partial\xi^\beta}\epsilon_{\alpha\beta}d^2\xi,$$

then

$$4\pi eM = \lim_{R\to\infty}\frac{1}{2}\int_{S_R^2} \varepsilon_{ijk}\varepsilon_{abc}\hat{\phi}^a \frac{\partial\hat{\phi}^b}{\partial\xi^\alpha}\frac{\partial\xi^\alpha}{\partial x^j}\frac{\partial\hat{\phi}^c}{\partial\xi^\beta}\frac{\partial\xi^\beta}{\partial x^k}\frac{1}{2}\frac{\partial x^m}{\partial\xi^\alpha}\frac{2x^n}{\partial\xi^\beta}\varepsilon_{imn}\varepsilon_{\alpha\beta}d^2\xi$$

$$= \lim_{R\to\infty}\frac{1}{2}\int_{S_R^2}\varepsilon_{\alpha\beta}\varepsilon_{abc}\hat{\phi}^a\partial_\alpha\hat{\phi}^b\partial_\beta\hat{\phi}^c d^2\xi,$$

where, $\varepsilon_{ijk}\varepsilon_{imn}\delta_j^m\delta_k^n = 2.$

Note $g = \det(\partial_\alpha\hat{\phi}^a\partial_\beta\hat{\phi}^b)$ is the square of the integrated function.

Hence

$$4\pi eM = \lim_{R\to\infty}\int_{S_\phi^2}\sqrt{g}\,d^2\xi = 4\pi d,$$

or

$$M = d\frac{1}{e},$$

d is the number of times the unit sphere S_R^2 is covered by $\hat{\phi}(\xi)$ when S_R^2 is covering once by (ξ_1, ξ_2)).

Corresponding to mapping $\hat{\phi}$, we have

$$\xi\in S_R^2 \to \hat{\phi}(\xi)\in S_\phi^2,$$

let us define the Homotopy set $[\hat{\phi}]$ of ϕ as $[d]$. Then d is the topological chargeand M is the topological charge in units of e.

From previous discussion, we found that the stability of soliton assures the conservation of magneton. Magneton M is determined by the free gauge electromagnetic field $F_{\mu\nu}$ and the topological charge d is the magneton number in units of $\frac{1}{e}$. Both Homotopy theory and classical soliton solution provide description of magneton and its conservation. The former is simple and conclusive. It indicates that conservation is related to the topology of the field configuration rather than the symmetry of L. In addition, there is no continuous transformation amongst the Homotopy set $[\phi]$.. This is an indication of topological invariance which consists of some form of quantization. In fact, the isolated topological charge is a phenomenon of quantization.

B. Soliton solution and Higgs theory

Higgs theory treats magneton and its conservation. It describes the mechanism of obtaining a Higgs particle and a massive gauge vector Boson through the process of vacuum spontaneous symmetry breaking.

(1). Spinor

For convenience, L is written in the following manner

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^*D^\mu\phi + m^2\phi^*\phi - \lambda(\phi^*\phi)^2,$$

where the potential energy is

$$V(\phi) = -m^2\phi^*\phi + \lambda(\phi^*\phi)^2$$

At $|\phi| = \frac{m}{\sqrt{2\lambda}}$, the expectation value of the quantum field is nonzero.

$$\langle 0|\phi|0\rangle = \frac{m}{\sqrt{2\lambda}},$$

hence, vacuum spontaneous symmetry breaking occurs.

Rewriting $$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2),$$

let $$\langle 0|\phi_1|0\rangle = \frac{m}{\sqrt{\lambda}}, \quad \langle 0|\phi_2|0\rangle = 0.$$

Define a field such that $\phi_1' = \phi_1 - \frac{m}{\sqrt{\lambda}}$, $\phi_2' = \phi_2$, in vacuum

$$\langle\phi_1'\rangle_0 = 0, \quad \langle\phi_2'\rangle_0 = 0,$$

The transformed L' contains covariant derivative, square and quadratic part of ϕ:

$$\begin{aligned}
&|D_\mu\phi|^2 + \frac{m^2}{2}(\phi_1^2 + \phi_2^2) - \frac{\lambda}{4}(\phi_1^2 + \phi_2^2)^2 \\
&= \frac{1}{2}(\partial^\mu\phi_1' + eA^\mu\phi_2')^2 + \frac{1}{2}(\partial^\mu\phi_2' - eA^\mu\phi_1')^2 \\
&\quad - e\frac{m}{\sqrt{\lambda}}A^\mu(\partial_\mu\phi_2' - eA_\mu\phi_1') + \frac{1}{2}e^2\frac{m^2}{\lambda}A_\mu A^\mu \\
&\quad - m^2\phi_1'^2 - \sqrt{\lambda}m\phi_1'(\phi_1'^2 + \phi_2'^2) - \frac{\lambda}{4}(\phi_1'^2 + \phi_2'^2)^2.
\end{aligned}$$

In this equation, there is no $\phi_2'^2$ term. The gauge field A_μ has mass, however, its physical meaning is still not clear because of the presence of A and ϕ terms. Let

$$\phi'(x) = \exp\left(-i\xi/\frac{m}{\sqrt{\lambda}}\right)\phi(x) = \frac{1}{\sqrt{2}}\left(\frac{m}{\sqrt{\lambda}} + \eta(x)\right),$$

$$B_\mu(x) = A_\mu - \frac{\sqrt{\lambda}}{em}\partial_\mu\xi(x).$$

where $$\phi(x) = \frac{1}{\sqrt{2}}\left[\frac{m}{\sqrt{\lambda}} + \eta(x)\right]\exp\left(i\xi(x)/\frac{m}{\sqrt{\lambda}}\right)$$

Under the transformation $\phi \to \phi'$, $D_\mu\phi \to D_\mu'\phi'$ 下,

$$L' = \frac{1}{2}|D_\mu'\phi'|^2 + \frac{1}{2}m^2\phi'^2 - \frac{\lambda}{4}\phi'^4 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Note $L \to L'$ as $\phi \to \phi'$. SU(2) symmetry of L is breaking into U(1) symmetry associated with L'. Symmetry of system's Lagrangian is broken. However a clear physical meaning of the scalar Higgs fields φ' and free gauge potential B_μ is assured. This is because vacuum spontaneous symmetry breaking and gauge field assure the direction of φ' be fixed in dual space. $\xi(x)$ is absorbed by $\eta(x)$ and the remaining term is $\eta(x)$ or ϕ'. Also B_μ is a physically observable free gauge potential.

Let us discuss the relation between Higgs theory and classical soliton solution. We know the soliton solution is determined by equation of motion and boundary condition. On the other hand, Higgs theory states that equation of motion and boundary condition are invariant under $SU(2) \to U(1)$ symmetry breaking.

In fact, when the equation of motion is under transformation $\phi \to \phi'$,

$D_\mu\phi \to D_\mu'\phi'$ let $U = \exp\left(-i\xi/\frac{m}{\sqrt{\lambda}}\right)$, $\phi = U^*\phi'$, therefore

$$D_\mu\phi = D_\mu\left(\exp\left(i\xi/\frac{m}{\sqrt{\lambda}}\right)\phi'\right) = \partial_\mu\left(\exp\left(i\xi/\frac{m}{\sqrt{\lambda}}\right)\phi'\right) - ieA_\mu\exp\left(i\xi/\frac{m}{\sqrt{\lambda}}\right)\phi'$$

$$= \exp\left(i\xi/\frac{m}{\sqrt{\lambda}}\right)\partial_\mu\phi' + \exp\left(i\xi/\frac{m}{\sqrt{\lambda}}\right)i\frac{\sqrt{\lambda}}{m}\phi'\frac{\partial\xi}{\partial x^\mu} - ieA_\mu\exp\left(i\xi/\frac{m}{\sqrt{\lambda}}\right)\phi'$$

$$= \exp\left(i\xi/\frac{m}{\sqrt{\lambda}}\right)D_\mu'\phi' = U^*D_\mu'\phi',$$

from Equation (2), we have

$$D_\mu D^{\mu\nu}\phi' = \lambda\phi'\left(\phi'\phi'^{*} - \frac{m^2}{\lambda}\right),$$

Since $D_\mu\phi \to 0$, it can be seen that

$$D'_\mu\phi' = UD_\mu\phi \to 0.$$

As a result, the equation of motion and boundary condition are invariant under Higgs mechanism. This assures the invariance of the soliton solution. Based on a particular L, Reference 1 discussed the correspondence between magneton conservation and incomplete symmetry of secondary order. In this section, we started with Higgs theory and obtained the same conclusion. Similar results may be obtained in D=3 dimensional case of magnetic monopole theory.

(2). Magnetic monopole

L is written as:

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}D_\mu\phi^* D^\mu\phi^* - \frac{1}{4}\lambda\left(\phi^*\phi^* - \frac{m^2}{\lambda}\right)^2,$$

Based on the gauge condition of Higgs mechanism, the third component of $\hat{\phi}^a$ in isospin space is chosen as the direction of Higgs field. That is, $\hat{\phi}=(0,0,1)$ then $B_\mu = \hat{\phi}^a A^a_\mu = A^3_\mu$, also

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + \frac{1}{e}\varepsilon_{abc}\hat{\phi}^a\partial_\mu\hat{\phi}^b\partial_\nu\hat{\phi}^c$$
$$= \partial_\mu B_\nu - \partial_\nu B_\mu,$$

B_μ and $F_{\mu\nu}$ are the potential and free gauge electromagnetic field. $F_{\mu\nu}$ satisfies the Maxwell's equations. The physical meanings of $\hat{\phi}$, B_μ and $F_{\mu\nu}$ are assured by Higgs mechanism and they are physically observable particles. L' is written

$$L' = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}D_\mu\delta D^\mu\delta - \frac{1}{4}\lambda\left(\delta^2 - \frac{m^2}{\lambda}\right)^2,$$

Transformation L→L' is a form of SO(3)→U(1) symmetry breaking. It can be proved that due to Higgs mechanism, equation of motion Eq. (4) and boundary condition are invariant, thus

$$D_i D'^{+}\phi = -\lambda\phi\left(\phi^2 - \frac{m^2}{\lambda}\right)D_i\phi \to 0.$$

Hence, the corresponding classical field soliton solution is also invariant. Once again, the connection between soliton solution and Higgs theory is observed.

References

(1). R. A. Brandf and Guonkei Yang, Coupling Symmetry and Magneton Conservation, Sun Yetsin University Report (1978).

Abstract

The two- and three-dimensional nonlinear Higgs field ϕ^4 is used as an example to investigate the theory of magnetic soliton and the connection between the homotopy theory and Higg's theory. It is verified that, in a spontaneous and symmetrical breaking, because of the symmetry of electromagnetic gauge subgroup, there exists correspondence between the conservation of magneton and the incomplete symmetry of secondary order.

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