

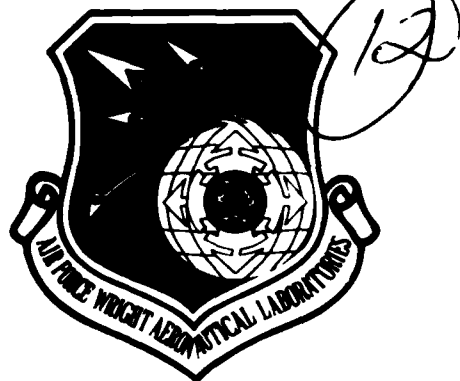
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DIFFUSE DISCHARGE SWITCH ANALYSIS

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FEBRUARY 1986

FINAL REPORT FOR PERIOD JULY 1982 - SEPTEMBER 1985

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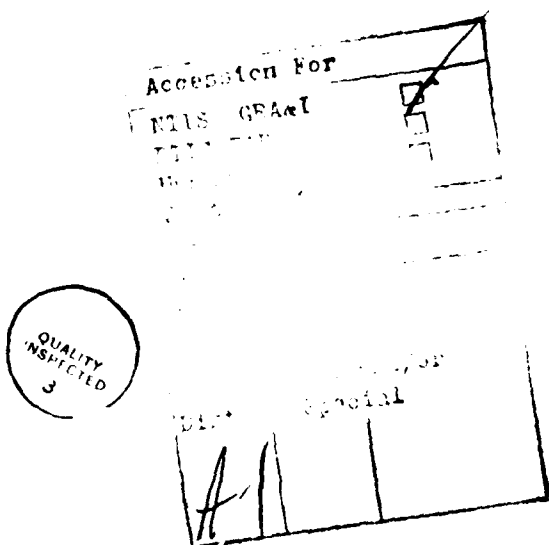
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1.0 INTRODUCTION

The technique of sustaining a diffuse discharge by electron beam irradiation of a gas has been known for more than 10 years.⁽¹⁻³⁾ In this type of discharge, a high energy (> 100 kV) electron beam introduced through a foil into a gas maintains a controlled level of ionization and, hence, a controlled conductivity. A voltage applied between electrodes draws a current through the gas, even though the electric field is below the gas breakdown strength.

The e-beam-sustained discharge has a number of advantages. Because the e-beam current and the discharge voltage can be independently controlled, the technique enables one to independently adjust the electron density and the electron temperature over a considerable range. Because, unlike a conventional glow discharge, this discharge is not dependent upon wall recombination for stability, it can be scaled to large volumes. Unlike a preionized, over-voltaged avalanche discharge, the e-beam-sustained system is not subject to rapid (< 1 μ s) arc formation, so the pulse can be extended for tens, or even hundreds, of microseconds. And, since a high energy electron beam can penetrate a considerable distance through gas at atmospheric pressure or above, the technique can be used to produce uniform, diffuse discharges in high pressure gas. There is an extensive literature on e-beam-sustained discharges, which is summarized in several reviews.⁽⁴⁻⁶⁾

The e-beam-sustained discharge has been used, or at least studied, for many applications, including gas discharge lasers, discharge chemical processing, the measurement of gas kinetic processes, and as an electrical switch that can be opened under load. Probably the most extensively explored application is the CO₂ infrared laser. The present study is based on a prior investigation of arcing in laser-type discharges.⁽⁷⁻⁸⁾ But the use of an e-beam-sustained discharge as a switch, which was proposed quite early in the history of this technology, has gained increasing attention in recent years. A discussion of this application, with references to the earlier work, is given in the recent review by Kline.⁽⁹⁾ The purpose of the present study was to extend the information about discharge arcing processes reported in Refs. 7 and 8 to the somewhat different operating regime required for a switch.

Switch development is of interest because a compact, efficient opening switch would allow one to store energy inductively, and the energy density--joules per m³ or joules per kilogram--that can be stored in a coil is more than an order of magnitude greater than the best that can be done with a capacitor bank.^(10,11) An opening switch would also be useful to the electric power industry because it would allow the interruption of high voltage dc transmission lines.

An e-beam-sustained discharge can be used as an opening switch. If it is run well below the breakdown voltage, termination of the e-beam interrupts the discharge current, even if the voltage rises because of the inductive nature of the storage. Indeed, for pulsed power applications, a voltage rise is desirable since the stored energy can then be delivered to the load at higher voltage. An e-beam-sustained discharge is always somewhat lossy while conducting, so most proposed applications envision this element used in parallel with another switch, such as a mechanical circuit breaker, with the e-beam discharge used to hold down the switch voltage during the time that the primary switch is being opened. Unless superconducting coils are used, inductive storage is also dissipative, and would be used only as a stage in the pulse-forming system. Typically, one might, over a period of minutes, accumulate energy in a flywheel; then, in a time ~ 1 sec, transfer this energy to an inductor; and, finally, in a time ~ 1 μ sec, open-circuit the switch, or system of switches, forcing the inductor current to flow through the load (or into a further pulse steepening network if submicrosecond pulses are required).

In an electric power dc transmission system, the e-beam-sustained switch would also be used in parallel with a circuit breaker to conduct the current until the main breaker was fully open.

The requirements for a switch discharge are quite different from a laser discharge since, in a switch, one wants to minimize the energy deposited in the discharge medium. The gas used in the switch should have a high breakdown strength so that it can stand off the voltage after the switch is opened, but also have a high conductivity when it is e-beam-ionized so that the dissipation will be low. One way to achieve the latter requirement is to use a gas with a Ramsauer minimum and operate the switch at an E/n such that electron energies are in the minimal region while the switch is conducting. To further

reduce the collision rate, the electron energies (i.e., the electron random velocities) should be low, which requires a gas in which a large fraction of the electron collisions are inelastic. For this cooling process, one wants a molecular gas in which there are vibrational and rotational modes that can be excited by low energy electrons. Hence, one expects that a molecular gas with a Ramsauer minimum can give a high electron drift velocity and, hence, a high conductivity. This has been found to be true of methane,⁽¹²⁻¹⁴⁾ and most presently envisioned switch designs are based on methane, or silane, or mixtures of these and noble gases.⁽¹⁵⁾ One problem with such gases is that a Ramsauer minimum can make the dependence of electron drift velocity upon electric field non-monotonic, which can make the discharge unstable.^(16,17) The discharge parameters must be chosen to avoid this instability.

An efficient switch would be recombination-dominated during the current conduction phase but, since recombination alone gives a long tail to the ionization after e-beam turnoff, it is usually proposed to add a small concentration of an attaching species to terminate more efficiently the switch conductivity. Since the electric field will increase as the switch is opened, it has also been suggested that one use an attachers, attachment rate of which increases with E/n ; i.e., with electron energy. (This is true of dissociative attachment, which has a threshold energy.) Some care should be taken, however; an attachment rate that increases with E/n can make the discharge unstable.⁽¹⁸⁾ This, however, is less of a concern than it would be in a discharge that was attachment-dominated during conduction.

To be efficient, the switch must have a high current gain. That is, the discharge current must be greater than the e-beam current needed to maintain the conductivity. In a recombination-dominated discharge at a given gas density, the electron density and, hence, the conductivity, increases as the square root of the e-beam current. This favors a design where both e-beam and discharge current densities are low. But a low current density means a large switch and, if the switch becomes too large, the advantage of inductive storage is nullified. An absolute limit on e-beam current and pulse length is imposed by the foil. If the current is too high, the absorbed energy will melt the foil.

A final, absolute requirement is that the switch discharge not arc either during the conduction phase or afterwards, when the voltage increases as the switch is opened. For this, it is obviously necessary that the peak voltage be kept below the breakdown threshold of the gas. But that is not sufficient. Although it is run below the breakdown voltage, an e-beam sustained discharge can still arc, through the mechanism of discharge streamers. These streamers are hot, filamentary structures that start at the electrodes and propagate across the discharge. This is the process that was studied in our earlier work on laser-type discharges. The same phenomenon has also been reported by others. ⁽⁵⁾

In discussions of e-beam switch capabilities, two alternative simple criteria are sometimes used to account for the danger of arcing. One approach is to work well below the arcing threshold; i.e., to allow a large "safety factor." Unfortunately, the data on these discharges show that streamers can still form and that, if the discharge lasts long enough and deposits sufficient energy in the gas, it can still arc.

An alternate approach is to place a limit on the gas heating, noting that laser-type discharges typically arc if the gas is heated by more than ~ 500 K. As a rough estimate, this is reasonable but, as a precise design criterion, it is inadequate. As discussed in Ref. 8, the specific energy that can be deposited in an e-beam sustained discharge before arc formation depends upon the discharge operating conditions. At high conductivity (i.e., in higher current, lower voltage discharges), the allowable gas heating is greater than for laser operating conditions. In fact, at low voltages, the discharge arcing limit was found to be insensitive to discharge power. Such effects have also been reported by others. ^(16,19)

For a switch, the implications are good. Not only does the higher conductivity operation imply that less energy is deposited than in a laser discharge with the same current density, but the allowable energy deposition may also be greater. Note, however, that this advantage is somewhat in conflict with the need for high current gain. The latter implies that low e-beam current is desirable, while the need for a compact, arc-free device inclines one towards a higher e-beam current. It is clear that a proper assessment must include all these requirements and, in particular, must be based on a more precise understanding of the arcing limit.

2.0 SWITCH SCALING

Before examining the arcing mechanisms in more detail, however, it is useful to consider the switch design problem more systematically. Suppose that one wishes to store a megajoule and then deliver this energy to a load at 200 kV in 10 μ s. If a simple inductor is the store and the load voltage is constant, the current would ramp down, starting at one megampere just after the switch opens. So the switch must conduct 10^6 A, with the voltage, of course, much less than 200 kV while the switch is closed. Note that the volume of the storage coil can be as small as 0.1 m³; a capacitor bank that can store 10^6 J would occupy at least 1 m³.

The e-beam-sustained switch must conduct for a time sufficient to open a parallel breaker, then open in a time much shorter than the time to deliver the energy to the load. Since we are postulating a 10- μ s load delivery, we assume that the switch must open in 1 μ s. To open a parallel breaker, one would like a conduction time of a few ms but, since this is probably impractical, we require that the e-beam switch conduct for 100 μ s, which is at least long enough to open an explosive switch,⁽¹¹⁾ and perhaps to blow out an arc or open a fast mechanical device.

Assume that the switch is to run on 90 percent CH₄, 10 percent A and, for efficiency, is to be recombination-dominated during the conduction phase. Then, just after the e-beam ceases, the electron density in the discharge evolves according to

$$\frac{1}{n_e} \frac{dn_e}{dt} = -\alpha n_e \quad (1)$$

where α is the recombination rate coefficient. Since the switch must open in 1 μ s, we require that the initial rate of decay be at least 2×10^6 /sec. This limits the discharge electron density: $n_e \geq 2 \times 10^6/\alpha$.

The recombination coefficient, α , for methane has been calculated to be $\sim 2 \times 10^{-7}$ cm³/sec,⁽⁹⁾ but recent measurements by Bletzinger⁽²⁰⁾ gave

a value an order of magnitude larger. Using this measured value, $2 \times 10^{-6} \text{ cm}^3/\text{sec}$, one concludes that the electron density must be $n_e \geq 10^{12} \text{ electrons/cm}^3$.

Note that this determines the e-beam power deposition in the discharge. If the electron density is 10^{12} and the plasma is recombining at a rate $d(\ln n_e)/dt = -2 \times 10^6/\text{sec}$, one must produce $2 \times 10^{18} \text{ electrons/cm}^3 \text{ sec}$ to keep the switch closed. If E_i , the effective ionization potential (the e-beam energy deposited per electron produced) is 35 eV (a typical number), then the e-beam must deposit 11.2 watts/cm^3 while the switch is closed.

If the switch is to run at an E/n which puts electrons in the Ramsauer minimum of the gas, the discharge current, j_D , is also fixed by n_e . For methane-argon, 9:1, the minimum starts at an E/n of 2.5 Td and gives an electron drift velocity of approximately 10^7 cm/sec .⁽²¹⁾ Hence, the discharge current density is $j_D = e n_e v_e = 1.6 \text{ A/cm}^2$. This, in turn, defines the area of the switch needed to carry 10^6 A : the discharge must cover 62.5 m^2 .

This is a major disadvantage. A switch this size would completely nullify the advantages of compact storage offered by inductors.

What can be done to improve the situation? There are two changes that will help. The first change is to deliver the energy at higher voltage and lower current. The second change one can effect is to run the discharge at a higher electron density to increase the current. In practice, one would almost certainly have to do both to make the switch small enough to be useful.

It is clear that the circuit parameters assumed above (10^6 A while closed, 200 kV after opening) are not appropriate to this type of energy storage. Suppose instead that we specify $2 \times 10^5 \text{ A}$ conduction current and 1 MV open circuit standoff. Suppose also that the switch is run at an electron density of $5 \times 10^{12}/\text{cm}^3$. Then the e-beam power deposition (assuming $\alpha = 2 \times 10^{-6}$) is 280 W/cm^3 , the discharge current density (at $E/n = 2.5 \text{ Td}$) is 8 A/cm^2 , and the area of the switch is 2.5 m^2 . Since one reason for using this switch is to replace a few m^3 of capacitors by a smaller inductor, this switch is still undesirably large, but at least we have reduced it to a size that begins to be practical.

The electric field, the discharge electrode spacing, the discharge power density, and the required e-beam current density all depend upon the switch

gas density. At the desired E/n , the electric field is $625 \rho/\rho_0$ V/cm, where ρ/ρ_0 is the ratio of switch gas density to the density at atmospheric conditions. The discharge power dissipation is

$$P = Ej = 5 \rho/\rho_0 \text{ kW/cm}^3 \quad (2)$$

For a high pressure discharge, this is much larger than the e-beam power deposition. Using the simple arcing criterion mentioned earlier - that the gas should not be heated by more than 500°K - gives, for typical molecular gases, an allowable energy deposition,

$$W \leq 0.6 \rho/\rho_0 \text{ J/cm}^3 \quad (3)$$

A significant amount of energy could be deposited when the switch opens, if the voltage rises while the switch is still conducting. Indeed, it can be shown⁽²²⁾ that the external circuit imposes an irreducible switch dissipation during opening. But, because the heating of the gas is a major design constraint and because heating during opening would be at high E , low j , where the allowable heating is less, it is unlikely that a large dissipation upon opening would be acceptable. It is more likely that the external circuit would be designed to prevent such heating. One could, for example, put an R-C, or a more complex waveform-shaping network, in parallel with the switch to hold down the voltage for $\sim 1 \mu\text{s}$ while the switch was opening. (If this unacceptably degraded the load voltage rise time, a spark gap or other closing switch could be added between this system and the load.) This requires some capacitive energy storage in the filter, but the stored energy would be only ~ 0.1 of the total in the coil, so the filter capacitor would still be much smaller than the capacitor that would be needed to store all the energy. Hence we assume that the significant dissipation in the switch occurs during the conduction phase.

The allowable energy dissipation and the switch power together put a limit on the time the switch can remain closed without arcing. For our example,

$$t = \frac{W}{P} \leq 120 \mu\text{s} \quad (4)$$

Note that this time is independent of the gas density. This pulse length meets the requirement of 100 μ s assumed earlier, but not by much. One would like a longer pulse and, if the arcing limit can be improved, that could be done. The overall scaling should be noted: since the electric field and the allowable energy deposition are both proportional to gas density, while the discharge current density, at fixed E/n , is proportional to n_e , the allowed pulse length varies inversely with the discharge electron density. This is the essential tradeoff: by increasing n_e at a given gas density one obtains a more compact device (and a more rapid opening), but at the price of a reduction in the time the switch can remain closed.

The e-beam current needed to maintain n_e is found from the equation

$$\alpha n_e^2 = \frac{j_{eb} \rho}{e E_i} \frac{dE}{dm} \quad (5)$$

where dE/dm is the mass stopping power of the gas - typically 2×10^6 eV cm^2/g , ρ is gas density, and E_i is the effective ionization potential used earlier. Taking $E_i = 35$ eV and $\alpha = 2 \times 10^{-6}$ cm^3/sec , one has

$$j_{eb} = 6.7 \times 10^{-27} \frac{n_e^2}{(\rho/\rho_0)} \text{ A/cm}^2 \quad (6)$$

For $n_e = 5 \times 10^{12}$, this gives

$$j_{eb} = \frac{167}{(\rho/\rho_0)} \text{ mA/cm}^2 \quad (7)$$

The current gain is

$$\frac{j_0}{j_{eb}} = \frac{e v_e n_e}{6.7 \times 10^{-27} \frac{n_e^2}{(\rho/\rho_0)}} = \frac{2.4 \times 10^{14}}{n_e} (\rho/\rho_0) \quad (8)$$

For the assumed n_e ,

$$\frac{j_0}{j_{eb}} = 48 (\rho/\rho_0) \quad (9)$$

At 10 atmospheres, the current gain is almost 500. To improve the gain, one can either reduce n_e - i.e., work at low j_{eb} and low j_0 -, or increase the density. Since lowering n_e makes the switch impractically large, one wants to work at higher density. The same is true of the foil-melting limit. If $n_e = 5 \times 10^{12}$ is needed to make a compact system or to reduce the switch-opening time, the foil-melting limit (which gives a maximum j_{eb} for a given pulse duration) then implies a minimum ρ/ρ_0 for the system. High pressure operation, of course, requires a strong foil to hold the gas without breaking, but this can be done. A recent study⁽²³⁾ showed that more than 35 atmospheres can be held by an 0.6 mil Ti alloy foil on a frame with 0.3 cm slots.

If j_{eb} is kept constant (e.g., held at the foil melting limit), the electron density scales as $\sqrt{\rho/\rho_0}$, so the recombination (switch-opening) time is inversely proportional to $\sqrt{\rho/\rho_0}$ and, if E/n is kept constant (i.e., if the gap spacing is decreased inversely with ρ), the current density, j_0 , varies as $\sqrt{\rho/\rho_0}$, so the current gain increases as $\sqrt{\rho/\rho_0}$ and the switch area varies inversely with this square root of the density. It is clear that the switch should be designed to use the highest practical pressure.

The gap spacing between switch electrodes, which, for a given ρ and E/n determines both the conduction voltage of the switch and the e-beam voltage needed to maintain a nearly uniform ionization, is set by the electric field which the switch can hold off after opening. The Paschen curve for methane shows a breakdown field of 2.5×10^4 V/cm atm.⁽⁹⁾ Assuming that, after a pulse, one can do half this well, 1.25×10^4 gives the gap spacing for a 1-MV system:

$$d = \frac{80}{(\rho/\rho_0)} \text{ cm.} \quad (10)$$

So, for 10 atmospheres, the anode-cathode spacing is 8 cm. To penetrate this gas, the e-beam must have an energy of ~ 250 keV. This is practical. One does not, however, want to use a much higher voltage. An e-beam with a few times 250 kV would produce hard X-rays, which would create a shielding problem. Hence the discharge voltage assumed here (1 MV open circuit) is an attractive design point.

For this gap spacing and E/n, the closed circuit voltage is 50 kV, the switch volume, A.d, is $2.0 (\rho_0/\rho) \text{ m}^3$ (at constant $n_e = 5 \times 10^{12}$), the total discharge power, I.V = 1.0×10^{10} W, and, for a 100 μs pulse, the dissipated energy, I.V.t = 1.0 MJ, equal to the energy to be delivered to the load.

Note that this last result is independent of most of the switch parameters. For a given ratio of open E to closed E in the switch, and a given ratio of switch conduction time, t, to load delivery time, t_L (5 μs in our example, since the current ramps down in 10 μs), one has a system "Q"

$$\frac{\text{energy to load}}{\text{switch dissipation}} = \frac{I E_{\text{open}} d t_L}{I E_{\text{closed}} d t} \quad (11)$$

$$= \frac{E_{\text{open}}}{E_{\text{closed}}} \frac{t_L}{t} = 20 \frac{1}{20} = 1$$

In fact, the efficiency is even worse than this. Since one initially must store 2 MJ, the coil -- and hence switch -- current is initially greater by 2 and the initial dissipation will be higher. Note that this also reduces the time that the switch can conduct without arcing.

For applications, it may be acceptable to dissipate half the energy in the switch, if the cost of spinning up a flywheel and then storing that energy in a coil is relatively small, but a higher efficiency is certainly preferable. The above simple result shows that the efficiency can be improved only by reducing the ratio of conduction time to load delivery time, or else by increasing the ratio of open-to-closed circuit voltage. The ratio of times is already inconvenient. One would like the switch to conduct for a longer time, and it may be necessary to deliver the energy to the load in a shorter time.

A better change is to increase $E_{\text{open}}/E_{\text{closed}}$. This may be possible. When open, the switch may be able to hold off more than half the breakdown voltage. More important, during conduction, it may be practical to use a lower E than assumed here. The plots of electron drift versus E/n in Ref. 20 have a plateau (actually a slight decline) above 2.5 Td, so this value was taken as the "Ramsauer minimum," the point where a switch should run. But below 2.5 Td, the curve looks linear. The drift velocity is just proportional to E . If true, this means that the gas conductivity is constant there. In a sense, 2.5 Td is the upper bound of the Ramsauer minimum. At higher fields, the drift velocity does not increase because electrons have energies higher than the minimum and hence are scattered more efficiently. So 2.5 Td is the highest field that one should consider for the conduction phase. To use a lower field, one must of course accept a lower j_D and, hence, a larger switch, but the tradeoff is an increase in $E_{\text{open}}/E_{\text{closed}}$; i.e., an increase in the switch efficiency.

The simple coil storage system assumed here may also not be the best choice, since the triangular load delivery waveform is unlikely to be what one wants. It may be better to use a lumped element transmission line in which the energy initially is stored inductively. Another possibility is to store more energy in a larger coil. Then the current would not droop significantly during either the conduction time of the switch or the load delivery interval, and one might be able to return the remainder to the flywheel or use it on successive pulses of the system. Clearly, switch development and more detailed work on the circuits in which such switches would be used should be done together.

Therefore, the foregoing results for a simple coil circuit are not final, but they are useful as a guide to the improvements needed for a practical switch. The principal limitations indicated by this analysis are the physical size and the energy efficiency of the switch. To reduce the size requires a high current density, which is best achieved by a high gas density in the device. The device also becomes more compact as one goes to higher voltage and lower current. But the useable open circuit voltage is limited by the need to keep the e-beam voltage below the level at which X-ray shielding problems would become a major consideration.

The other major limitation is the switch conduction time, which must be long enough to open a parallel, lower-resistance switch. Both the open-circuit voltage and the conduction time are limited by streamer growth, and there is probably a tradeoff: if the switch conducts for a shorter time, streamers will not grow as far, and the switch can probably then hold off more voltage after opening. The choice of closed circuit voltage (i.e., E/n) affects all three parameters: if the switch is run at a lower E/n , the area must be larger, but the conduction time, or the overall system efficiency, or both, can be increased.

Switch-opening time, on the other hand, does not appear to be a problem, at least if $1 \mu s$ is adequate. At the high gas density and relatively high electron density needed for a compact, efficient system, the recombination rate is rapid. A small amount of attenuator may be needed to truncate the recombination tail, but a larger quantity, which would accelerate the initial turnoff, is unnecessary.

The strongest conclusion of these design considerations is the need to go to higher pressures. Three major characteristics that a practical switch must have (high current density, high current gain, and rapid opening) are all improved by increasing the density of the discharge gas.

The essential unanswered questions that will determine the practicality of such a switch are: the realistic limit on the pressure at which the switch can be run; and the arcing limit, the energy deposition and the open circuit E/n which the switch can sustain without arcing. Of particular importance is the pressure-scaling of this limit. The gains apparently offered by an increase in the pressure are real only if the specific energy deposition can be maintained at higher densities.

3.0 THE STREAMER INSTABILITY

3.1 EXPERIMENTAL RESULTS AND DISCUSSION

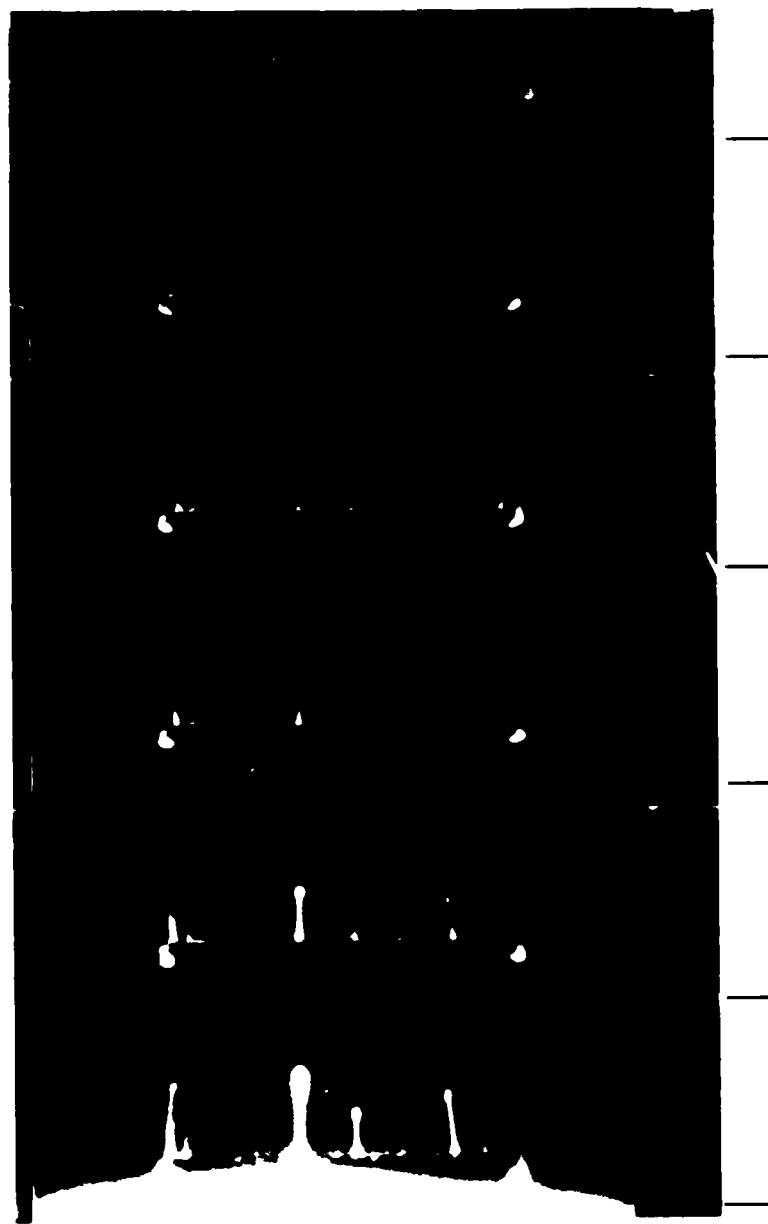
The theoretical description of arc formation in an e-beam-sustained discharge developed under this program is based on observations of streamer formation in a small discharge device. That work is described in Refs. 7 and 8 but, for completeness, it will also be summarized here.

High-speed framing camera photographs of the discharge showed that arcing results from discharge streamers, luminous filaments that start at the electrodes (usually the cathode, although anode streamers can also be produced) and grow into the discharge. An example of such a sequence of photos is shown in Figure 1. The same phenomenon had been seen earlier in open shutter photographs of pulsed discharges when the pulse length was short enough that the discharge didn't arc. Streamers in e-beam-sustained discharges have also been observed by others.^(6,24)

As can be seen in Figure 1, the streamer has a characteristic structure. It consists of a filamentary core with a glowing ball on the end. In color photographs (shown in Ref. 8), the core is seen to be brilliant white, resembling an arc, while the halo, or cap sphere, is, at least in nitrogen, a reddish or salmon color, more closely resembling a corona-type of discharge. As the streamer grows, the form remains the same, while the column and the halo are seen to grow proportionately.

When the length of the streamer is plotted against time, as in Figure 2, the growth is found to be exponential. This was always the case in our experiments, except in a few high current discharges where the voltage drooped during the pulse. Typically, the discharge arcs when the streamer has grown about two thirds of the way across the gap. The evident explanation is that the streamer is a highly conducting structure, essentially at cathode potential, and an arc forms when the field across the remaining gap between the streamer and the anode exceeds the threshold for rapid, avalanche-type breakdowns.

FILM 16 500 TORR N₂ 2.9 KV/CM
0.72 mA/CM² E-BEAM



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Figure 1 Streamer growth. The marks on the right denote successive frames. The framing rate is 22,800 frames/sec. The line of sight is parallel to the cathode surface. The anode to cathode spacing was 1 inch and the view here includes most of the discharge.

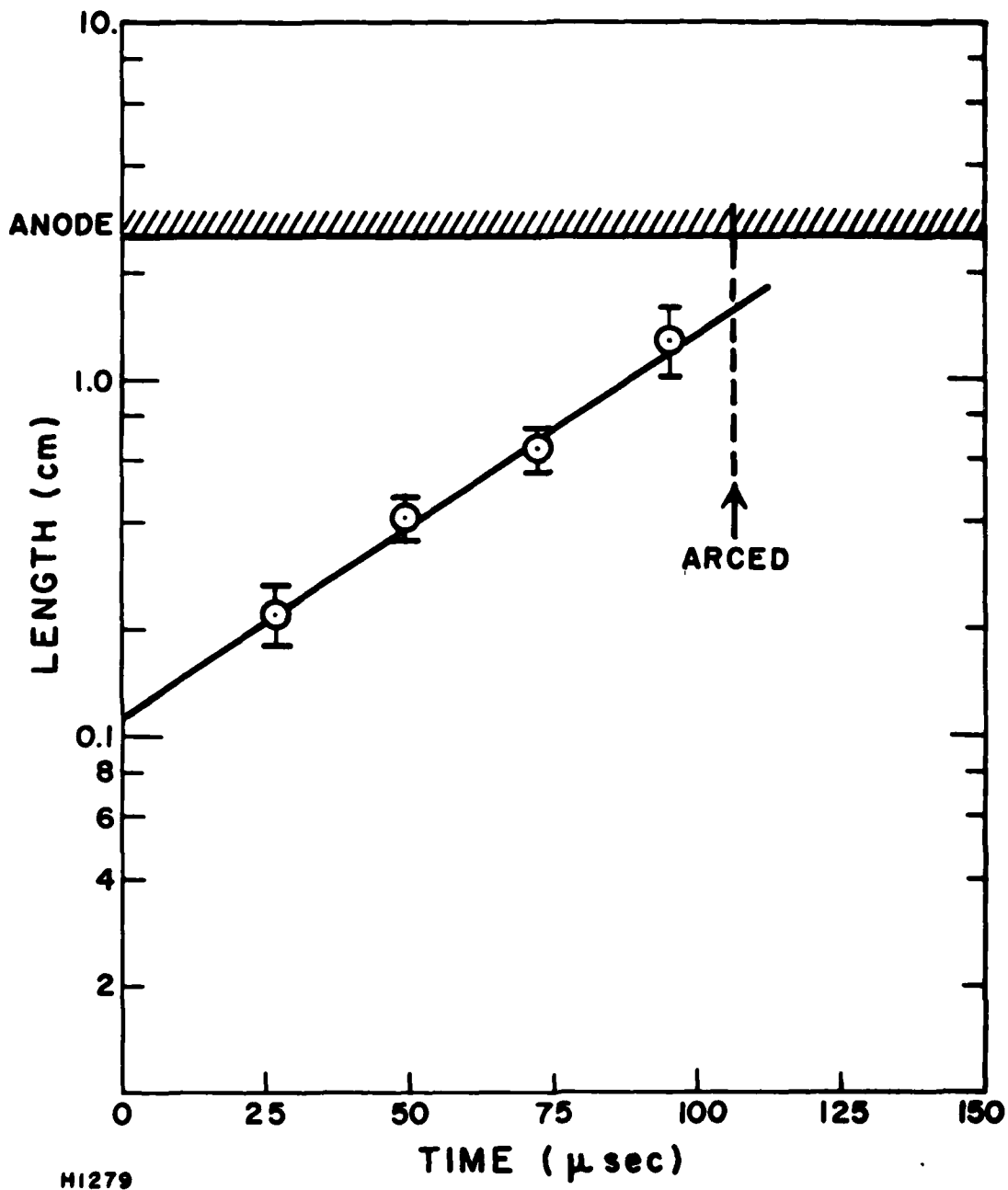


Figure 2. Growth of a typical streamer.

So streamer growth is characterized by a growth rate, or exponentiation time, rather than by a velocity. This behavior is different from the arcing of an overvoltaged, avalanche discharge, where the filamentary streamers that precede an arc seem to advance at a roughly constant, although much higher, velocity.⁽²⁵⁾ In our studies of the slower streamer propagation in e-beam-sustained discharges, no velocity effects were seen, even for streamers that became supersonic with respect to the background gas. There was no change in the exponential growth and, in shadowgraphs, which were also taken with the framing camera, there was no evidence of shock waves. Apparently the gas, at least near the streamer, was heated enough so that the gas motion was still subsonic. As discussed below, streamers in the experiments were seen to form rapidly--in the first few microseconds--with a size of about 1 millimeter, and the discharge arcing time was about 2.5 times the streamer exponentiation time seen in the photographic sequences.

The streamer growth rate was seen to depend upon both the voltage and the power of the discharge, as shown in Figure 3, where data in three different growth rate ranges are plotted against power and electric field. The solid line is a contour of constant growth rate, as determined from the data. What this shows is that one cannot avoid arcing simply by limiting the electric field or the gas heating. The correct criterion involves both. At low E field, one can allow more gas heating. Indeed, at low E, where a switch would operate while closed, the line in Figure 3 is nearly vertical, implying that in this regime the streamer growth rate is largely independent of discharge power. (Experimentally, at constant E the power is varied by changing the e-beam current, which changes the discharge gas ionization, and hence the conductivity, and hence the discharge current.) At high E, where laser discharges typically run, the line is nearly horizontal, implying that the streamer growth rate is primarily power-dependent in that regime. As noted above, such variation and, in particular, the fact that more gas heating is allowable at lower E, has also been seen by other investigators.^(16,19)

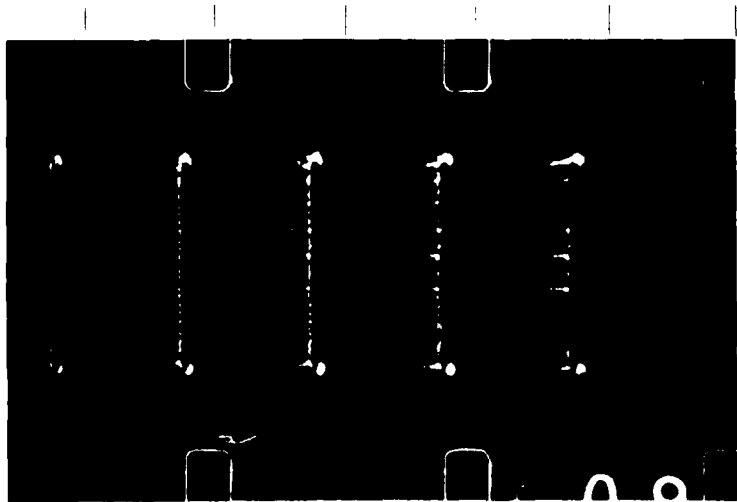
The evident explanation is that the streamer propagation involves both field ionization--i.e., avalanching--and gas heating, and that the growth rate is most strongly dependent upon whichever process is weaker. To be useful, a theoretical model must explain this behavior and predict the variation with pressure, gas composition, etc.

Streamers, once formed, can be maintained, and can continue to grow after the current is almost completely shut off. In the experiments, post arcs were seen to be due to streamers that continued growing slowly after the end of the discharge. (In this device, the main discharge ended when the e-beam pulse ended, but the discharge voltage remained on for a considerable time afterwards.) An example is shown in Figure 4, a framing sequence of a discharge that arced about a millisecond after the end of the e-beam pulse. At these voltages, the discharge never pre-arc'd, even though the voltage, applied by a mechanical switch, was present for several seconds before the pulse. The post arc was clearly caused by the discharge and, as can be seen, was due to a streamer, a phenomenon that must be considered if an e-beam sustained discharge is to be used as a repetitively pulsed switch.

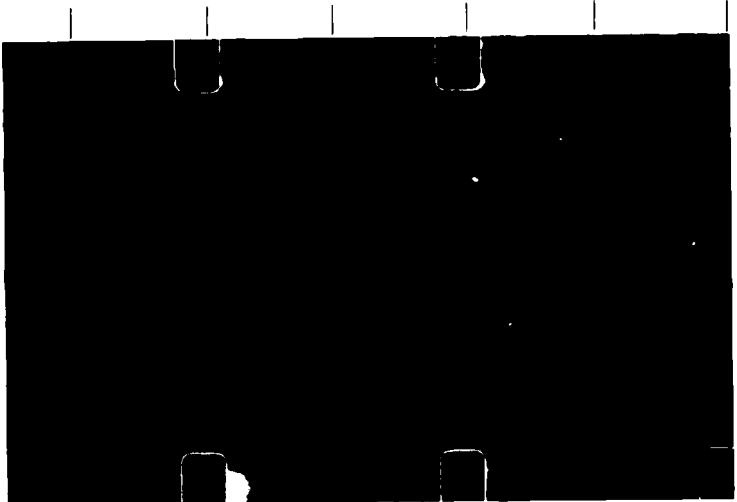
Qualitatively, we think that a streamer is a composite structure around a thermally ionized core, essentially an arc, which grows into the discharge by heating the gas ahead of it. Because the streamer column is a good conductor, it collects a concentrated current, as would a wire stuck into the discharge from the cathode. Indeed, in some of the early experiments, a wire loop about 1 mm in diameter was mounted on the cathode and the largest streamers always grew from there, which simplified the observations.

The concentrated current through the streamer maintains the temperature and hence conductivity of the core and also heats the gas around the tip. Because the streamer is a good conductor, it is close to cathode potential, so there is an enhanced electric field around the tip. One expects this field to produce non-thermal ionization, which is consistent with the glowing colored ball seen in the photographs. In this region one expects the field to be close to the gas breakdown strength. A higher field would give rapid buildup of the ionization, which would act to reduce the field; a lower field would allow this non-thermal ionization to decay, which would cause the field to increase. Since the voltage drop around the streamer tip is proportional to the length of the streamer (it is just the voltage that would have been dropped uniformly between that point and the cathode if there were no streamer), a clamped electric field in the halo implies that the halo radius should be proportional to streamer length, which agrees with the observations.

FILM 14 500 TORR N₂ 2.6 KV/CM
0.72 mA/CM² E-BEAM



FILM 14 CONTINUED (16 FRAMES OMITTED)



FILM 14 CONTINUED (THIRD SECTION)

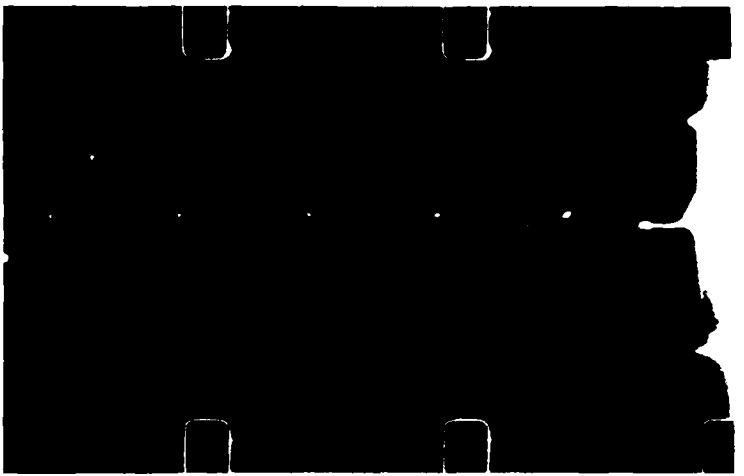


Figure 4. Framing camera photos of a discharge which post-arced ~ 1 ms after the pulse.

Note that this does not imply a constant E within the halo, since the gas temperature certainly varies there, and the field needed for avalanching is proportional to density. But if the temperature profile is self-similar as the streamer grows (and we think it is), the E field profile will be, too.

It is plausible that such a structure would grow exponentially. For most of its growth time the whole streamer is relatively close to the cathode, so it is effectively growing into a semi-infinite discharge. A good conductor, the streamer will collect roughly that current that would otherwise have gone to the area on the cathode that is "shadowed" by the streamer. That is, it will collect a current roughly equal to the background current density times an area with radius equal to the length of the streamer. Hence the streamer current grows as the square of the length of the streamer.

Since the radius of the cap sphere, or halo, is also proportional to streamer length, its surface area also grows as the streamer length squared, so the current density there--and at other regions within and around the halo--remains constant or, more precisely, self-similar as the streamer grows. Since this is also true of the electric field, it is also true of the power, or gas heating. If the heating profile within and around the halo is self-similar as the streamer grows, the time to heat the gas in the halo or, at least, in the leading part of the halo up to the core temperature, is constant as the streamer grows. In each such heating time, the streamer grows by a halo radius, a distance proportional to streamer length. A structure that grows by a certain fraction of its length in a fixed time, of course, grows exponentially, as streamers are consistently observed to do. A proper theoretical analysis should test and quantify this picture.

3.2 STREAMER INITIATION

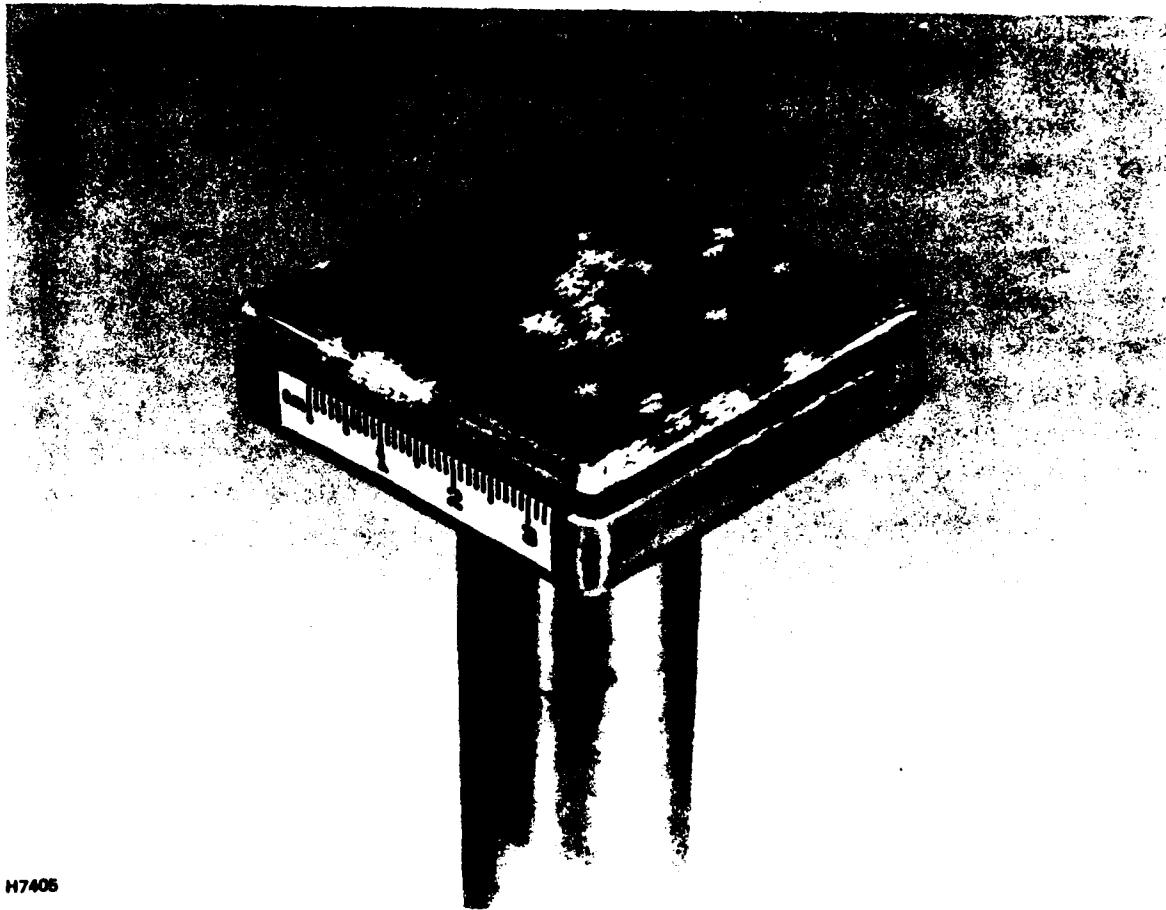
In short-pulse, high-voltage discharges, such as are used in TEA lasers, careful shaping and polishing of the electrodes are crucial to the prevention of premature arcing of the discharge. This suggests that similar techniques might be effective in preventing or at least delaying the formation of streamers in e-beam-sustained discharges. However, a more careful analysis of the problem, plus some observations made in our experimental work, make this solution seem less promising.

The cathodes used in our experiments were not carefully shaped for field uniformity, and the field was highest at the edges. Yet, as shown by burn spots left by arcs, streamers did not always start at the edges. Many of them did but, as can be seen from Figure 5, a photograph of a cathode after a series of discharges, there were also burn marks scattered over the flat central region where the cathode E field (or at least the vacuum field) would have been weakest. The cathode shows a tendency for the arcs to clump in the center, where the e-beam and, hence, the discharge current density, was highest.

Consideration of the structure and evolution of the cathode layer in a high-pressure discharge suggests two mechanisms by which streamers might be started: growth from cathode spots and an instability of a large-area cathode layer. There is evidence of both mechanisms in the experimental data. Work on low pressure discharges^(26,27) has provided a fairly complete picture of the cathode layer, the region where electron avalanching multiplies the positive ion driven cathode electron emission up to the level needed to carry the discharge current, which, at the cathode, is primarily an ion current. What the standard theory shows is that, for a given gas and cathode material, the cathode layer voltage drop is essentially fixed, independent of pressure (more precisely, of gas density). The sheath voltage drop is just that needed to give the required 10-to-20 fold electron multiplication in the sheath. In nitrogen this sheath voltage is about 500 V.

The electric field in the sheath is directly proportional to density and the sheath thickness is inversely proportional to density; i.e., there is a constant voltage drop per electron mean free path and a constant number of mean free paths within the sheath. Since the current at the cathode is mostly ion current, there is a net positive charge density within the sheath and, since this must satisfy Poisson's equation for the voltage drop, the sheath has a characteristic current density, which varies inversely as the square of the sheath thickness. So the current is proportional to the square of the gas density. At high pressures, this current density is greater than the discharge current, so the sheath breaks up into cathode spots.

The foregoing is all quite standard analysis but worth reviewing because it is crucial to the problem of streamer initiation. The formation of cathode spots gives an unavoidable nonuniformity that cannot be eliminated by contouring the electrodes. In some of the framing camera photographs, the spots are



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Figure 5. A view of the cathode after the experiments, showing burn marks.

clearly visible and it appears likely that they are the source of streamers that are then seen to develop. In a switch discharge, the current density will be higher, and one might think that this would give more complete coverage of the cathode by the sheath. But the scaling analysis in Section 2.0 showed that, to make a practical switch, one wants to work at higher pressures (10 atmospheres or more). Since the cathode layer current density scales as gas density squared, the concentration of the cathode current into spots can, unfortunately, be expected to be worse, not better, than in laser discharges.

The other difference between low- and high-pressure discharges is that heating of the cathode layer is a much stronger effect at high pressure. Since the current density increases as gas density squared, and the electric field is proportional to the gas density, the dissipation in the sheath increases as the cube of the room temperature pressure of the discharge. In high pressure discharges, the cathode layer rapidly heats and expands. The gas density, and hence current density, goes down, so the cathode spots can expand to cover the whole cathode. Indeed, some of the experiments showed evidence of this. In some shadowgraph framing camera sequences, one can see an expanding cathode layer which appears to cover the whole cathode. But streamers still formed and the discharge still arced.

The reason that streamers still formed is indicated by another shadowgraph sequence (shown in Ref. 8) where a sinusoidal perturbation of the cathode front is visible. Apparently the cathode layer expansion had become unstable, and an analysis of the sheath expansion explains why this happens. Since our analysis of sheath expansion is given in Ref. 8, it need only be summarized here. The essential point is that, as the layer expands, its impedance drops. When the cathode spots have merged to cover the whole cathode, the sheath current density is just that of the bulk discharge. Then, as the layer expands, the sheath electric field decreases, producing an increasing cathode layer conductivity. When the sheath conductivity exceeds that of the bulk discharge, which typically happens at a thickness of about 1 mm, the expansion becomes unstable. When the cathode layer is a better conductor than the bulk discharge, a bump on the cathode layer will draw an increased current, causing further heating and more rapid expansion at that location. This generates a streamer. In fact, this mechanism is quite similar to the process responsible for streamer growth.

There is no evident way to stabilize the cathode layer expansion. Even if streamer growth of cathode spots could be prevented, which in a high pressure switch discharge seems unlikely, one must still assume that streamers will be initiated at the cathode. The one thing that could be done is to use a porous or mesh cathode and flow gas into the cathode; i.e., suck the cathode layer and the associated small streamers into the cathode surface fast enough to prevent streamer growth into the discharge. The needed flow is just the velocity at which a streamer grows early in its development. This solution, however, would make the switch apparatus much more complicated and expensive. The velocity of a small, beginning streamer, while less than that of larger ones, is still considerable - of the order of 0.1 time the sound speed in the gas. This solution should be kept in mind, but only as a last resort. If possible, the switch should be designed to prevent arcing by keeping the pulse length shorter than the time needed for streamers to propagate across the discharge.

3.3 THEORIES OF STREAMER GROWTH

The formation of arcs in e-beam sustained discharges has not been studied nearly as intensively as arc formation in self-sustaining, overvoltaged discharges, such as preionized TEA lasers, spark gaps, lightning strokes, etc. But, as reported in Refs. 5 and 6 and the papers cited therein, some calculations have also been made in an attempt to understand the arcing of nonself-sustained discharges. However, comparison with the observations discussed above does not support these previous analyses.

The predictions of arcing time are in some cases of the right order, but the observed dependence upon discharge voltage and current is not predicted. The calculations often give a characteristic streamer velocity, analogous to the overvoltaged case, or else predict a collapsing current throughout the discharge, not the growth of filaments from the electrodes that is seen in actual devices.

A common procedure in stability analyses is to linearize the fundamental equations by assuming small perturbations of electron density, gas density, etc., within a uniform discharge. Then, in the same way that one analyzes

sound waves in a gas, or the various waves and instabilities in a fully ionized plasma, one can obtain a set of linear equations for the perturbation and study its stability. If small disturbances are found to grow in amplitude, the discharge is predicted to be unstable.

The problem with this technique is that it assumes that the perturbations are initially of small amplitude, so that terms quadratic in the variations in densities, field, etc., can be neglected. But the experiments on streamer growth do not support such a picture. The photographs show a large amplitude disturbance that grows in size. The variation in gas density, current density, etc., within the streamer is not a small perturbation, even early in the process. So amplitude linearization is not an appropriate way to describe this phenomenon, even approximately.

Existing analyses also tend to consider: increased local gas heating caused by current concentration, and increased local electron multiplication caused by E field enhancement as two different instability modes to be discussed separately. But, as explained above, both the visual appearance of streamers and the functional dependence of their growth rate upon the discharge parameters strongly suggest that gas heating and electron avalanching are both essential to streamer propagation. It appears that the two processes must be considered together to construct a meaningful theory.

Both effects were considered in an earlier theoretical model that we constructed to describe streamer growth in laser discharges. Since the work done under the present program is a generalization of that earlier analysis, we will first summarize the simple model. (It is discussed more fully in Refs. 7 and 8.) The basic idea is that a streamer is a cylinder of hot and consequently conductive gas that protrudes into the discharge. Near its head is both a strong field and a high current density, just as would occur near the tip of a wire protruding from the cathode. Both field and current act to raise the conductivity of the gas ahead of the streamer--the field by ionization and the current by heating.

To estimate the rate of growth of a streamer one must compare the energy input needed to make the gas ahead of the streamer highly conducting with the rate of heating caused by the concentrated current ahead of the streamer. For nitrogen, in which most of the experiments were done, the Saha equation predicts ionization and, hence, high conductivity, at $\sim 6200^\circ\text{K}$. At constant

pressure, the energy needed to heat a constant volume of nitrogen, from which the gas is permitted to expand to this temperature, is $e \sim 1 \text{ J/cm}^3 \text{ atm}$. (This may be an underestimate; in nitrogen, a significant fraction of the energy may be taken up by vibrational excitation, which is stored as internal energy.)

Around the tip of the streamer there is a halo, of radius a , where the conductivity is high due to nonthermal ionization by the strong electric field. In our original model we assumed that most of the heating therefore occurs at the halo edge. Modeling this by a conductivity that varies as

$$\sigma = \sigma_0 e^{k\left(\frac{a}{r} - 1\right)} \quad (12)$$

where r is the distance from the tip of the hot gas column, one can integrate j^2/σ over the halo from the radius of the column, b , to the radius of the halo, a , to obtain the total dissipated power in the halo,

$$\begin{aligned} Q &= \int_a^b \frac{I^2}{(4\pi r^2)^2} \frac{4\pi r^2}{\sigma_0} e^{k\left(1 - \frac{a}{r}\right)} dr \\ &= \frac{I^2}{4\pi\sigma_0 a k} \left[1 - e^{-k\left(\frac{a}{b} - 1\right)} \right] \text{ watts} \end{aligned} \quad (13)$$

Since the ratio of conductivities in the core at 6200°K and in the background gas under e-beam ionization is $\sim 10^3$ and $a/b \approx 3$, the value of k which matches the boundary condition on the conductivity is $k = 3.45$. Thus, the second term in the above equation is completely negligible and the mean power density within the halo is

$$q = \frac{3I^2}{16\pi^2 \sigma_0 a^4 k} \text{ W/cm}^3 \quad (14)$$

From this power density, one could compute the time required to heat the halo to the temperature of the core of the streamer. However, a heating time calculated in this way would be too long because only the center need be heated for the core to grow. One can show that, to be consistent with the assumed radial dependence of the conductivity, the ratio of the volume to be heated to the final temperature should be reduced by a factor $\lambda = 0.3$.

These results imply an instantaneous velocity for propagation of the streamer. It is the radius of the sphere a , divided by the time required to heat the gas in the halo interior to the core temperature, $t = \lambda c/q$. Thus we have

$$v = \frac{dc}{dt} = \frac{aq}{\lambda c} = \frac{3I^2}{16\pi^2 \sigma_0 \lambda a^3 k c} \quad (15)$$

where I is the total current through the streamer and c is the streamer length. This current can be estimated by assuming that the streamer is essentially at cathode potential, that the current is quasi-static (i.e., divergenceless), and that the conductivity outside the halo is uniform. Then the potential beyond the streamer obeys Laplace's equation,

$$\nabla \cdot j = \nabla(\sigma_0 E) = \sigma_0 \nabla^2 V = 0$$

Approximating V by the first few Legendre polynomials, matching the boundary conditions on V at points on the halo and the anode, assuming $j = \sigma_0 E$ at the halo edge, and integrating over the cap sphere gives a total current

$$I = 4\pi a^2 \sigma_0 E_0 \left[\frac{l}{4} + \frac{c}{a} \right] \quad (16)$$

where E_0 is the electric field far from the streamer. Using this in the expression for the velocity gives a propagation velocity for the streamer

$$v = \frac{3a \sigma_0 E_0^2}{\lambda k c} \left[\frac{l}{4} + \frac{c}{a} \right]^2 \quad (17)$$

The model predicts that c/a should remain roughly constant as the streamer grows and this is confirmed by the framing camera photographs. Putting the result in a form more appropriate for such a scaling, and using the above estimated values of λ , k , and ϵ , we have

$$\frac{dc}{dt} = \gamma c$$

where

$$\gamma = 2.07 \left(\frac{\sigma_0 E_0^2}{P_0} \right) \left(\frac{a}{c} \right) \left(\frac{1}{4} + \frac{c}{a} \right)^2 \quad (18)$$

Hence the model predicts that streamers should grow exponentially. This prediction is in good agreement with the observations. The predicted growth rate, γ , is primarily dependent upon the specific power loading, $\sigma_0 E_0^2 P_0$. This is also consistent with the data at the relatively high electric fields at which laser discharges run.

However, as discussed above, data taken at lower fields more typical of switch discharges show a different parametric variation. The most probable reason that the foregoing model fails there is that heating ahead of the halo was neglected, and such heating is more significant at lower fields, where the halo will be relatively small. To estimate the effect of heating gas outside the halo, one can consider the other extreme and assume that streamer growth is caused by this alone. Just beyond the halo, the electric field is E_c , the critical breakdown field, and the conductivity is σ_0 . So the power input is $\sigma_0 E_c^2$, which will heat the gas to the conductive temperature in a time $t = \epsilon / \sigma_0 E_c^2$. Since the scale length of this field is a , the halo radius, such heating alone would give a velocity $v = a/t$. Here a , the halo radius, can be estimated from the Legendre polynomial expression for the E-field mentioned earlier. The result is:

$$\frac{c}{a} \approx \frac{E_c}{E_0} - 1 \quad (19)$$

This gives a speed

$$v = \frac{dc}{dt} = \frac{c \sigma_0 E_0^2}{\left(\frac{E_c}{E_0} - 1\right) 1.4 P_0} \quad (20)$$

or

$$\frac{dc}{dt} = \gamma c$$

where

$$\gamma = \frac{\sigma_0 E_0 E_c}{\left(1 - \frac{E_0}{E_c}\right) 1.4 P_0} \quad (21)$$

So one again predicts exponential growth, but now the growth rate is primarily dependent upon $\sigma_0 E_0 = j_0$, the current density in the uniform background discharge. This is an extreme conclusion, since it ignores all heating within the halo, but it is worth noting that the data in Figure 3 shown earlier are not really inconsistent with a growth rate that is primarily current-dependent at low discharge voltages.

Clearly, what is needed is a more complete description that treats the whole region around the end of a streamer consistently. Before proceeding to that, however, we would like to review one other simple estimate that is useful as a guide to the analysis. As noted earlier, the electric field within the halo must be nearly equal to the breakdown threshold level. Since this field limit is proportional to gas density, n , and the current density must vary as $1/r^2$ if one assumes approximate spherical symmetry, the power deposition in the halo must have the radial dependence,

$$P = j \cdot E \propto \frac{n E_{c0}}{n_0 r^2} \quad (22)$$

where r is the distance from the streamer tip and n_0 and E_{c0} are the gas density and breakdown field of the background discharge.

Matching this to the boundary condition at the halo edge gives

$$P = \sigma_0 E_{co}^2 \frac{n}{n_0} \left(\frac{a}{r}\right)^2 \quad (23)$$

For nitrogen, an analysis of gas heating shows that,

$$\epsilon = \epsilon_0 \left(\frac{T}{T_1}\right)^{0.63} \quad (24)$$

Differentiating this gives,

$$j \cdot E \, dT = d\epsilon = \frac{0.63 \epsilon_0}{T_1} \left(\frac{T}{T_1}\right)^{-0.37} dT. \quad (25)$$

We also know that $n = n_0 T_0 / T$, so

$$-dT = \frac{n_0 T_0}{n^2} dn \quad (26)$$

Combining these results gives

$$\sigma_0 E_{co}^2 \left(\frac{a}{r}\right)^2 = \frac{0.63}{T_1} \left(\frac{T}{T_1}\right)^{-0.37} \frac{n_0^2 T_0}{n^3} \frac{dn}{dt} \quad (27)$$

Making the substitution $dn/dt = v \, dn/dr$, we obtain

$$n^{-2.63} \frac{dn}{dr} = \frac{K}{r^2} \quad (28)$$

where

$$K = \frac{T_1^{0.63} \sigma_0 E_{co}^2 a^2}{0.63 n_0^2 T_0 v}$$

This has the solution, for $n = n_0$ at $r = a$,

$$n = \left[1.63 K \left(\frac{1}{r} - \frac{1}{a}\right) + \frac{1}{n_0^{1.63}} \right]^{-0.61} \quad (29)$$

Well inside the halo, this reduces to

$$n = K' r^{0.61} \quad (30)$$

So one expects the gas density to vary roughly as the 0.6 power of the distance from the streamer tip in the interior of the halo. The point is that, from such an analysis, one can deduce dependences that had merely to be postulated in the earlier model.

The objective now is to start from the basic equations for gas motion, electric field, etc., and try to put the whole picture on a solid mathematical footing. Since one cannot linearize by assuming small amplitude perturbations, that might seem to be a hopeless undertaking. But the data suggest that streamer growth has a different kind of linearity. Streamers were seen to grow exponentially, which is a characteristic of a linear system. We expect that the equations will be at least linear in time.

The picture that emerges from the simple calculations and the data is that a streamer grows in size while retaining the same form. Such a structure can be described by a similarity solution, a technique which is used in fluid mechanics.⁽²⁸⁾ One assumes that the coordinates of the structure have an overall time dependence, in this case that they vary as $\exp(\gamma t)$. The idea is to try to account for all the time variation, or at least all the rapid variation, in this simple way. The framing camera photographs strongly suggest that such a solution to the equations should exist.

4.0 MATHEMATICAL MODEL OF A STREAMER

4.1 ASSUMPTIONS OF THE MODEL

As in our earlier work, a streamer in the new model consists of three regions: the core, the halo, and the background discharge. These three regions are distinguished by the source of their electrical conductivity. The core, essentially an arc, is a column of hot, low-density gas that is thermally ionized and highly conducting. In this region there is a high current density and a weak electric field. Around the tip of the core is the streamer halo, where the electric field is strong enough to produce nonthermal, electron avalanche ionization of the gas. Outside the halo is the background region where the conductivity is caused by e-beam ionization. The background, which has a lower conductivity than the core or halo, acts as a resistive ballast, limiting the current that can flow through the streamer.

As sketched in Figure 6, the core of the streamer is modeled as a cylinder of length c and radius b . The radius may vary along the length, since heat conduction will cause the core to expand with time, but this variation is so gradual that the dynamics are still those of a cylindrical structure. Moreover, all that is actually needed in the calculation is the radius near the tip of the column.

The halo is modeled as a region of radius a around the tip of the core. As shown by our earlier photographic studies, this region is not exactly spherical, but is nearly so, and all that is needed for the calculations is the structure of the region ahead of the streamer, where gas heating causes streamer growth.

Some distinctions in terminology should be noted. The halo discussed in the theory is the region of nonthermal ionization by the electric field. The glowing ball seen in the photographs is, presumably, a region of nonthermal luminosity. Since an electric field too weak to cause electron avalanching could still accelerate electrons to energies sufficient to excite optical transitions, the visible ball may be somewhat larger than the ionization halo. But the fact that the ball appears spherical is certainly a strong indication that this region is roughly spherical.

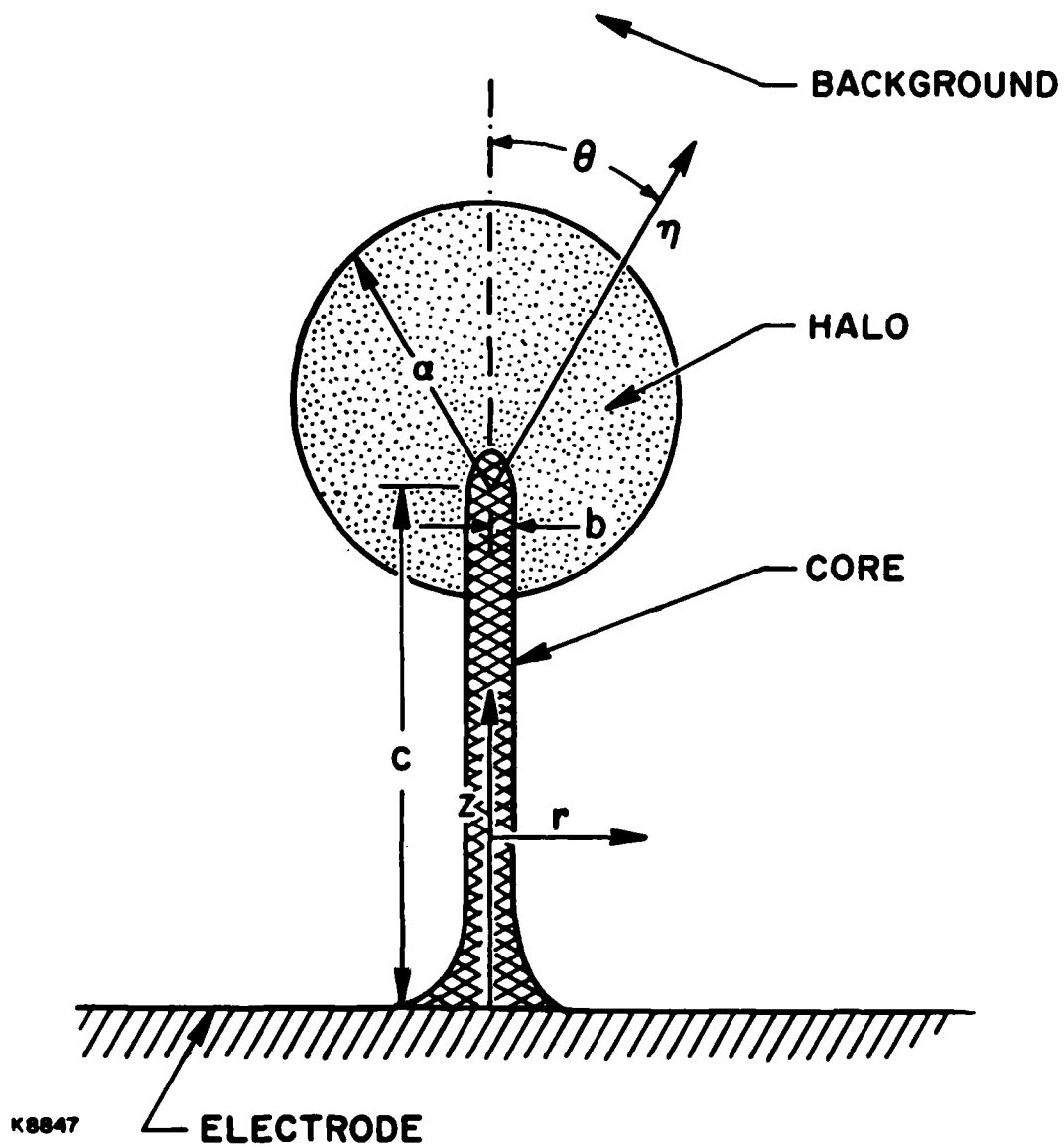


Figure 6. Streamer model.

Note also that the background discharge, as defined here, is not necessarily the same as the unperturbed discharge far from the streamer. The background is simply the region outside the halo, where ionization is caused by the e-beam. The gas in this region ahead of the streamer may be heated enough by the streamer current to significantly change its density and conductivity.

In the following analysis, the subscripts c, H, and B are used to denote core, halo, and background regions. Thus T_c and σ_c are the temperature and electrical conductivity of the core, $\sigma_H(|E|, T, P)$ is the conductivity of the halo, etc. The properties of the unperturbed discharge far from the streamer are denoted by the subscript naught. Thus ρ_0 , E_0 , and j_0 are the gas density, electric field, and current density of the unperturbed discharge. These quantities may be time varying, but they are not dependent on position. It is assumed that the whole pattern is cylindrically symmetric about the axis of the streamer. For pulsed, uniform discharges, this is well borne out by the data.

It is also assumed that the local properties are isotropic, that the pressure, p , electrical conductivity, σ , and thermal conductivity, K , are scalars, not tensors. Consistent with the use of a scalar pressure, the viscosity and the resultant viscous dissipation of energy are neglected. Because, as noted earlier, the data do not show any evidence of shock waves or other sonic effects as the streamer velocity exceeds the sound speed of the background gas, the gas flow away from the streamer is assumed to be subsonic. Consistent with subsonic flow, the pressure variations are expected to be relatively small, and the kinetic energy of flow of the gas is expected to be much less than the changes in internal energy caused by heating by the current.

Finally, the gas kinetic processes - ionization, free electron heating, etc., are assumed to be rapid in comparison with the streamer growth rate. Hence the electrical conductivity, for example, is considered to be a function of the instantaneous gas temperature, density, and electric field, but not to depend upon the history of the gas. This omits such effects as E-field overshoot when the ionization lags the current increase, or non-thermal storage of energy in vibrational modes, and subsequent abrupt heating when the gas temperature rises to the point where vibrations relax rapidly. Such effects could be included through additional variables, such as a vibration temperature, which could be different from the gas temperature, or by allowing rate

dependence, e.g., a conductivity dependent upon dE/dt as well as E , but this was considered to be an impractical complication. The rapid turn-off needed in a switch implies that kinetic rates will be rapid - submicrosecond - while the needed conduction time implies that streamer growth rates must be slower - tens of microseconds - in systems of practical interest.

4.2 BASIC EQUATIONS

4.2.1 Gas Dynamics

Since the system is cylindrically symmetric, it can be described in terms of three independent variables, r , z , and t , with r and z defined as sketched in Figure 6. Streamers are assumed to originate at an electrode and propagate into a uniform discharge. Because the opposing electrode affects the streamer only when it has nearly bridged the gap, when only a brief time remains before arcing, most of the streamer growth time can be explained without reference to an opposing electrode. Hence the discharge will be considered to be semi-infinite, with the boundary condition that the current, electric field, etc., must approach the uniform, unperturbed values at large distances from the streamer.

The success of our earlier theoretical estimates in predicting the right magnitude, and, at high electric fields, the right scaling for the growth rate of a streamer, is persuasive evidence for the assumption that streamer growth is due to gas heating. Because the data also show no evidence of shock waves, one expects that the pressure around a streamer will be nearly uniform. For these reasons, it is convenient to describe the state of the gas in terms of temperature, T , and pressure, p , and to describe the internal energy--heating, dissociation, etc.--by the internal enthalpy per unit mass, $h = e + p/\rho$, where e is the internal energy. The quantity, h , is assumed to be known as a function of T and p for the discharge gas mixture. The other relevant gas properties, the density, ρ , and thermal conductivity, K , are also assumed to be known or calculable in terms of T and p .

In addition to T and p (and ρ , h , and K), there are six other variables, the two components of velocity, v_r and v_z , current, j_r and j_z , and electric field, E_r and E_z . Eight equations are therefore needed to describe the dynamics of the system.

One knows that the mass and momentum of the gas are conserved.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{v}) = 0 \quad (31)$$

$$\rho \frac{Dv}{Dt} + \nabla p = 0 \quad (32)$$

Here D/Dt denotes the comoving derivative, $\partial/\partial t + \underline{v} \cdot \nabla$, and (32) includes the fact that there are no forces on the gas, and that the viscous drag is negligible.

The effect of gas heating by the current is described by the enthalpy equation,

$$\rho \frac{DH}{Dt} - \frac{\partial p}{\partial t} = \nabla \cdot (K \nabla T) + \underline{E} \cdot \underline{J} \quad (33)$$

Here viscous dissipation has been neglected and the equation has been written in terms of the total enthalpy, $H = v^2/2 + h$. Since the gas flow is subsonic and the gas heating increases h manyfold, one expects that the difference between H and h will probably be unimportant.

4.2.2 Electrodynamics

Because there are no significant magnetic fields, one has, from Faraday's law,

$$\nabla \times \underline{E} = 0 \quad (34)$$

Poisson's equation and charge conservation together imply that

$$\nabla \cdot \left[\underline{j} + \epsilon_0 \frac{\partial \underline{E}}{\partial t} \right] = 0$$

Because the charge transported by the current in these discharges is many orders of magnitude greater than the charge needed to produce the electric field, the second term is quite negligible, and one has, to good approximation,

$$\nabla \cdot \mathbf{j} = 0 \quad (35)$$

Since the momentum equation, (Eq. 32) has two components, Eqs. (31-35) provide six independent equations. The remaining two that are needed are, of course, the Ohm's law relation between \mathbf{j} and $\underline{\mathbf{E}}$. As discussed already, this dependence is different for the different regions of the streamer. In all three regions, however, the conductivity is scalar, so one always has

$$\frac{j_r}{j_z} = \frac{E_r}{E_z} \quad (36)$$

All that is still needed is the dependence of $|\mathbf{j}|$ upon $|\mathbf{E}|$.

4.2.3 Conductivity

a. Background

In the background region, the conductivity, $\sigma_B = n_e \mu_e e$ is proportional to the electron density, n_e , which is determined by the balance between e-beam ionization of the gas and electron loss through recombination or attachment. The electron loss rates may depend upon the electric field (through the free electron temperature). The mobility, μ_e , is also in general a function of E/ρ . Thus the only generally correct form is $\sigma_B = \sigma_B(T, p, \underline{\mathbf{E}})$, or, solving $\mathbf{j} = \sigma \underline{\mathbf{E}}$ for $\underline{\mathbf{E}}(\mathbf{j})$,

$$\sigma_B = \sigma_B(T, p, |\mathbf{j}|) \quad (37a)$$

In many cases of interest, however, the mobility and the recombination or attachment rates may be approximated as independent of E , because the electron temperature is clamped by the energy sinks provided by atomic or molecular transitions. If the gas is well below the ionization temperature, these rates may depend on T and p only through ρ . Then, since the rate of production of electrons by the e-beam is proportional to ρ , one has the simpler dependence,

$$\sigma_B = \sigma_B(\rho) \quad (37b)$$

Of course, a dependence on e-beam current is also implied here.

For the special case of a discharge satisfying Eq. (37b) and having as the dominant electron loss dissociative recombination, a process whose rate is proportional to ion density, one has

$$n_e = n_{e0} \sqrt{\frac{\rho}{\rho_0}}$$

$$\nu_e = \nu_{e0} \frac{\rho_0}{\rho}$$

and hence,

$$\sigma_B = \sigma_0 \sqrt{\frac{\rho_0}{\rho}} \quad (37c)$$

If the electron loss is dominated by attachment, whose rate is proportional to ρ , then, instead of Eq. (37c), one has $n_e = n_{e0}$, and hence

$$\sigma_B = \frac{\sigma_0 \rho_0}{\rho} \quad (37d)$$

b. Halo

In the halo, the ionization is enhanced by electron avalanching. Since, near the breakdown threshold, this rate is a rapidly increasing function of electric field, it is impractical to try to estimate n_e from the electron production and loss rates. Fortunately, however, it is not necessary to do that. Because the rate of electron production increases rapidly as E exceeds the breakdown strength of the gas, one expects that such an increase would rapidly (in a time short compared to streamer growth times) increase the conductivity, which would, in turn, cause the electric field to drop back towards the breakdown field. (Remember that the streamer current is ballasted by the resistance of the background discharge; so, near the streamer, the system is effectively driven by a current source.) Hence we hypothesize that, in the halo, the electric field is clamped at the breakdown threshold:

$$E_H = \frac{E_{b0} \rho_0}{\rho} \quad (38a)$$

where E_{bo} is the breakdown strength of the unperturbed discharge. If desired, Eq. (38a) can be expressed in terms of conductivity, $\sigma_H(\rho, j)$, by dividing j by E .

$$\sigma_H = \frac{j \rho}{E_{bo} \rho_0} \quad (38b)$$

This expression for the breakdown field ignores any gas temperature dependence that could arise if the gas were hot enough for a significant fraction of the atoms to be in excited states, but that is expected to be a small correction, especially since the system is at nearly constant pressure: as the gas is heated, its density decreases, so subsequent heating acts upon a smaller mass of gas. Thus, as the gas approaches the ionization temperature, the heating occurs more and more rapidly. Once the gas begins to ionize thermally, the temperature will plateau, as additional energy is invested, mostly in ionization.

There may, however, be a second, lower temperature plateau, probably within the halo, where the gas is dissociating. To include such effects one could add an explicit temperature dependence to σ_H . That could be done without changing the formulation developed here. In general, the division of the streamer into core, background, and halo is itself a simplifying approximation. Near the background-halo boundary, there must be a region where e-beam ionization and electron multiplication are both significant. Similarly, near the halo-core boundary there must be a region where both avalanche and thermal ionization are significant. The model could be generalized by constructing an overall conductivity $\sigma(T, p, j)$, which includes all these processes, but that has been left for later work.

c. Core

In the core of a streamer, the gas is thermally ionized, as in an arc. The Saha equation shows that this happens fairly abruptly, as the temperature reaches a threshold. Hence we expect that the core is at a well defined temperature

$$T_c = T_I$$

where T_I is the ionization temperature of the gas. That is from an estimate of the degree of ionization one can calculate the gas temperature from the saha equation. In general, this depends upon pressure, but, since pressure variations are expected to be small, it can, to reasonable accuracy, be evaluated at the ambient pressure of the discharge.

The degree of ionization within the core, which can vary with radius, is difficult to estimate. Hence the conductivity and electric field in the core are not as easily calculated as in the other regions. But since the core is essentially an arc, one can make some use of existing understanding of arc discharges and, moreover, the exact characteristics of this region do not strongly influence the conclusions about streamer growth. The core conductivity is certainly high enough so that the tip is close to cathode potential. One can simply take a rough estimate obtained, for example, from arc physics for this high conductivity

$$\sigma_c = \sigma_c(T_I) \quad (39)$$

It would also be useful to estimate the energy invested in ionization and the radius of the core, since this determines the current density. But, because the gas even in the core is only weakly ionized and because the final heating near the core tip is done on gas of much lower density from the background discharge gas, these parameters also have only a small effect on streamer dynamics.

This completes the formulation of the problem. To solve these equations, it is not actually necessary to assume a structure with a cylindrical core and a ball-shaped halo in a background discharge. One could regard Eqs. (37-39) as defining a conductivity $\sigma(T, p, j)$ according to the prescription: If $T \geq T_I$, Eq. (39) applies. For colder gas, Eq. (37) should be used, unless the predicted electric field would exceed the breakdown strength of the gas at the local density, in which case, Eq. (38) applies. If this model is correct, a structure like that seen in the framing camera photographs should emerge from a general solution of these equations.

Even more ideally, one might prefer to solve the equations numerically in three dimensions, to confirm the expectation of cylindrical symmetry. This could, in principle, be done. The addition of a third spatial dimension adds three variables, the third components of \underline{v} , \underline{E} , and \underline{j} , but it also adds three

equations, the third component of Ohm's law and two more components of Faraday's law. We have thus succeeded in formulating the basic physics of the streamer instability as a well posed mathematical problem, which had not been done prior to this program.

4.3 DESCRIPTION IN CYLINDRICAL COORDINATES

Restating the equations in cylindrical coordinates, with no angular dependence, and only r and z components of the vectors, one has the conservation of mass,

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \rho v_r) + \frac{\partial}{\partial z} (\rho v_z) = 0 \quad (40)$$

where $\rho = \rho(T, p)$, the conservation of momentum,

$$\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} + \frac{1}{\rho} \frac{\partial p}{\partial r} = 0 \quad (41)$$

$$\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} + \frac{1}{\rho} \frac{\partial p}{\partial z} = 0 \quad (42)$$

The enthalpy equation

$$\begin{aligned} & \rho \left(\frac{\partial H}{\partial t} + v_r \frac{\partial H}{\partial r} + v_z \frac{\partial H}{\partial z} \right) - \frac{\partial p}{\partial t} \\ & = \frac{1}{r} \frac{\partial}{\partial r} (r K \frac{\partial T}{\partial r}) + \frac{\partial}{\partial z} (K \frac{\partial T}{\partial z}) + E_r j_r + E_z j_z \end{aligned} \quad (43)$$

where

$$H = \frac{v_r^2}{2} + \frac{v_z^2}{2} + h(T, p)$$

Faraday's law,

$$\frac{\partial E_r}{\partial z} = \frac{\partial E_z}{\partial r} \quad (44)$$

charge conservation,

$$\frac{1}{r} \frac{\partial}{\partial r} (r j_r) + \frac{\partial j_z}{\partial z} = 0 \quad (45)$$

and Ohm's law

$$j_r = \sigma(T, \rho, j) E_r \quad (46)$$

$$j_z = \sigma(T, \rho, j) E_z \quad (47)$$

where

$$j = \sqrt{j_r^2 + j_z^2}$$

4.4 CONVERSION TO COMOVING, SPHERICAL COORDINATES

As explained earlier, we propose to look for a similarity solution to these equations, a solution describing a streamer that grows in size while retaining the same form. A convenient way to do this is first to transform to spherical coordinates (η, θ) centered on the tip of the streamer column, as shown in Figure 6. In these terms, the cylindrical coordinates (r, z) are given by

$$r = \eta f \cos \theta \quad (48a)$$

$$z = c + \eta f \sin \theta \quad (48b)$$

Here c is the length of the streamer column, f is a scale factor, and $c(t)$ and $f(t)$ will be chosen to simplify the equations. Of course, one also must transform the vector components according to

$$v_r = v_\eta \sin \theta + v_\theta \cos \theta \quad (49a)$$

$$v_z = v_\eta \cos \theta - v_\theta \sin \theta \quad (49b)$$

and similarly for the components of j and E .

After considerable algebra (omitted here), one obtains the following forms. The equation of mass conservation (40) becomes

$$\begin{aligned}
& f \frac{\partial p}{\partial t} - \frac{\partial p}{\partial n} \left(n \frac{df}{dt} + \cos \theta \frac{dc}{dt} \right) + \frac{\sin \theta}{n} \frac{dc}{dt} \frac{\partial p}{\partial \theta} \\
& + \frac{1}{2} \frac{\partial}{\partial n} \left(n^2 \rho v_n \right) + \frac{1}{n \sin \theta} \frac{\partial}{\partial \theta} \left(\rho v_\theta \sin \theta \right) = 0 \quad (50)
\end{aligned}$$

From Euler's Eqs. (41,42) one has the two components

$$\begin{aligned}
& \frac{\partial v_n}{\partial t} - \frac{v_\theta \sin \theta}{n f} \frac{dc}{dt} - \frac{v_\theta^2}{n f} \\
& + \frac{\partial v_n}{\partial n} \left[-\frac{1}{f} \left(n \frac{df}{dt} + \cos \theta \frac{dc}{dt} \right) + \frac{v_n}{f} \right] \\
& + \frac{\partial v_n}{\partial \theta} \left[\frac{\sin \theta}{n f} \frac{dc}{dt} + \frac{v_\theta}{nf} \right] + \frac{1}{\rho f} \frac{\partial p}{\partial n} = 0 \quad (51)
\end{aligned}$$

and

$$\begin{aligned}
& \frac{\partial v_\theta}{\partial t} + \frac{v_n \sin \theta}{n f} \frac{dc}{dt} + \frac{v_n v_\theta}{n f} \\
& + \frac{\partial v_\theta}{\partial n} \left[-\frac{1}{f} \left(n \frac{df}{dt} + \cos \theta \frac{dc}{dt} \right) + \frac{v_n}{f} \right] \\
& + \frac{\partial v_\theta}{\partial \theta} \left[\frac{\sin \theta}{n f} \frac{dc}{dt} + \frac{v_\theta}{nf} \right] + \frac{1}{\rho f} \frac{\partial p}{\partial \theta} = 0 \quad (52)
\end{aligned}$$

The enthalpy Eq. (43) becomes

$$\begin{aligned}
& \rho \left[f \frac{\partial H}{\partial t} + \frac{\partial H}{\partial n} \left(v_n - n \frac{df}{dt} - \cos \theta \frac{dc}{dt} \right) \right. \\
& \quad \left. + \frac{1}{n} \frac{\partial H}{\partial \theta} \left(v_\theta + \sin \theta \frac{dc}{dt} \right) \right] - f \frac{\partial p}{\partial t} \\
& + \frac{\partial p}{\partial n} \left(n \frac{df}{dt} + \cos \theta \frac{dc}{dt} \right) - \frac{\sin \theta}{n} \frac{dc}{dt} \frac{\partial p}{\partial \theta} \\
& = f \left(E_n j_n + E_\theta j_\theta \right) \\
& + \frac{1}{2f} \left[\frac{\partial}{\partial n} \left(n^2 k \frac{\partial T}{\partial n} \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(k \frac{\partial T}{\partial \theta} \sin \theta \right) \right] \quad (53)
\end{aligned}$$

where

$$H = \frac{v_n^2}{2} + \frac{v_\theta^2}{2} + h(T, p)$$

Faraday's law in the new coordinates is

$$\frac{\partial E_\theta}{\partial \eta} + \frac{E_\theta}{\eta} = \frac{1}{\eta} \frac{\partial E_\eta}{\partial \theta} \quad (54)$$

Charge conservation has the form

$$\frac{1}{\eta} \frac{\partial}{\partial \eta} (\eta^2 j_\eta) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (j_\theta \sin \theta) = 0 \quad (55)$$

Ohm's law now has the η and θ components

$$j_\eta = \sigma E_\eta \quad (56)$$

and

$$j_\theta = \sigma E_\theta \quad (57)$$

Since the gas density and thermal conductivity are determined by the temperature and pressure, $\rho = \rho(T, p)$, $K = K(T, p)$ and the electrical conductivity, as explained, is also a function of the other variables, $\sigma = \sigma(T, p, j)$, there are still eight dependent variables, T , p and the η and θ components of v , j , and E , which obey these eight equations.

From the data and the simple physical agreements discussed above, we expect that the problem should have a solution with an exponential time dependence. That is, the equation should be solved by defining

$$f(t) = e^{\gamma t} \quad (58a)$$

$$c(t) = c_0 e^{\gamma t} \quad (58b)$$

and assuming, as a trial form that

$$v_n = v_{n0} e^{\gamma t} \quad (59a)$$

$$v_\Theta = v_{\Theta 0} e^{\gamma t} \quad (59b)$$

Here, v_{n0} and $v_{\Theta 0}$ are functions of n and Θ , but not of T . We postulate that this is also true of the remaining dependent variables, i.e., that in these coordinates

$$\frac{\partial \rho}{\partial t} = 0 \quad (60)$$

and, likewise for p , T , j and E .

What one finds is that this form is a solution if the thermal conduction can be neglected.

$$K = 0 \quad (61)$$

This is a reasonable approximation beyond the streamer core. The arc-like core has a diameter that is determined by heat conduction, so that will have to be analyzed separately and probably, approximately. Analysis of heat flow in an arc is complicated, even for the standard, stationary arc, and the present, time-dependent problem may be even more complicated. We note, however, that arcs are known to run at a roughly constant current density over a considerable range of total currents. That is, as the current is increased, the arc cross section increases proportionately. (See, for example, Section 9.7 of the book by Cobine, Ref. 26). In a streamer, whose current is proportional to the square of its length, such a variation would make the core diameter proportional to streamer length; i.e., it would cause this part of the structure to obey a similarity solution, too.

In any case, outside the core, the thermal gradients are very much reduced, because the scale is larger and the temperature differences are smaller. So we can assume that, in the halo and the background the gas heating is balanced by absorption; i.e., an increasing gas temperature, and by

convection, as heat is carried off by the expanding gas, but that conduction is a minor effect, which can be neglected.

Under this assumption, one obtains a simpler set of equations. Conservation of mass, Eq. (50), becomes

$$\begin{aligned} & (-\gamma n - \gamma c_0 \cos \theta) \frac{\partial \rho}{\partial \eta} + \frac{\gamma c_0 \sin \theta}{\eta} \frac{\partial \rho}{\partial \theta} \\ & + \frac{1}{\eta} \frac{\partial}{\partial \eta} (\eta^2 \rho v_{\eta 0}) + \frac{1}{\eta \sin \theta} \frac{\partial}{\partial \theta} (\rho v_{\theta 0} \sin \theta) = 0 \end{aligned} \quad (62)$$

Here the time-dependence has completely disappeared, as desired.

The two components of Euler's Eq. (51, 52) become

$$\begin{aligned} & \gamma v_{\eta 0} + \frac{\partial v_{\eta 0}}{\partial \eta} (v_{\eta 0} - \gamma n - \gamma c_0 \cos \theta) \\ & + \frac{(v_{\theta 0} + \gamma c_0 \sin \theta)}{\eta} \left(\frac{\partial v_{\eta 0}}{\partial \theta} - v_{\theta 0} \right) + \frac{e^{-2\gamma t}}{\rho} \frac{\partial p}{\partial \eta} = 0 \end{aligned} \quad (63)$$

and

$$\begin{aligned} & \gamma v_{\theta 0} + \frac{\partial v_{\theta 0}}{\partial \eta} (v_{\eta 0} - \gamma n - \gamma c_0 \cos \theta) \\ & + \frac{(v_{\theta 0} + \gamma c_0 \sin \theta)}{\eta} \left(\frac{\partial v_{\theta 0}}{\partial \theta} + v_{\eta 0} \right) + \frac{e^{-2\gamma t}}{\rho} \frac{\partial p}{\partial \theta} = 0 \end{aligned} \quad (64)$$

These equations retain a time-dependence in the pressure terms. At first glance, this might appear to invalidate the approach, since we are assuming that the density, temperature, etc., depend only upon η and θ . Euler's equation shows that this cannot be true of pressure gradients. The reason is that the velocities, as functions of (η, θ) , have been assumed to increase as $\exp(\gamma t)$. Hence the acceleration of the gas must have a similar dependence. So the pressure gradients must also vary this way to produce the needed acceleration of the gas away from the streamer. Moreover, $\partial p / \partial \eta$ is not a gradient,

since the coordinate η is itself expanding. The result is that this derivative must increase as $\exp(2\gamma t)$ to conserve gas momentum.

Recall, however, that we are assuming that the system is nearly isobaric, because the flows are subsonic.

$$p = p_0 + p'$$

where

$$p' \ll p_0$$

and p_0 is just the constant, uniform background pressure of the discharge. This is very different from the behavior of the density, temperature, and internal enthalpy, which vary by large factors within a streamer. Therefore, in figuring such quantities as $\rho(T, p)$ one can to this approximation use p_0 and consider ρ to be a function only of T . Equations (63) and (64) then specify what the small variation in pressure must be. If it turns out to be comparable to p_0 , which, of course, it will eventually, since it grows as $\exp(2\gamma t)$, this calculation ceases to be valid. That is just the statement that eventually the streamer, growing exponentially, must become supersonic. At that time, the streamer growth will apparently cease to be a simple exponential. But the observations imply that this is unlikely to occur in systems of practical interest. Moreover, even if it did occur on the last e-folding, most of the streamer growth time is spent in the earlier stage, where the streamer is small and moving slowly. So the present calculation should still give a fairly accurate prediction of the discharge arcing time.

Regarding Eqs. (63) and (64) as specifying pressure variations that are so small they are unimportant eliminates one variable, the changing pressure, and one equation. The two equations still leave one, which can be obtained by taking the cross derivatives and differencing to eliminate p . This resulting equation, while quite complicated, and of second order, is time-independent as desired.

The enthalpy Eq. (53) becomes

$$\begin{aligned} & \frac{\partial H}{\partial \eta} (v_{\eta 0} - \gamma \eta - \gamma c_0 \cos \theta) \\ & + \frac{1}{n} \frac{\partial H}{\partial \theta} (v_{\theta 0} + \gamma c_0 \sin \theta) = \frac{1}{\rho} (E_n j_n + E_\theta j_\theta) \end{aligned} \quad (65)$$

Here heat flow and the enthalpy changes due to pressure variations have been omitted because they are small effects, and the result is an equation with no explicit time-dependence. Note also that if E/ρ is constant, which we believe it to be within the halo, where the E field is at the avalanche threshold, the right-hand side of this equation is just proportional to j , the current density.

The remaining Eqs. (54-57), namely Faraday's law, charge conservation, and Ohm's law, are unchanged because they were already time independent.

We have now obtained a set of seven time independent equations for the seven remaining variables. This confirms our original supposition that a similarity solution should exist. The next step is to simplify this set to derive a prediction for γ , the growth rate of the streamer.

4.5 THE DYNAMICS DIRECTLY AHEAD OF THE STREAMER

The remaining equations, as they stand, are still too complicated to solve, at least within the scope of the present program. Thus, although the most desirable result would be a solution of the whole set predicting the shape of the streamer, the current distribution within and around the streamer, etc., we will not be able to carry the calculation to that point. However, what is really needed for applications is a calculation of the growth rate γ and, for that purpose, it is sufficient to consider only the region along the axis ahead of the streamer.

Restricting the problem to the line $\theta = 0$ ahead of the streamer is a considerable simplification. There, the field, current, and gas flow must, by symmetry, be radial. E_θ , j_θ , $v_{\theta 0}$ and their η derivatives all vanish, which gives a much simpler set of equations. Conservation of mass, Eq. (62), becomes

$$\begin{aligned}
 -\gamma(\eta + c_0) \frac{\partial \rho}{\partial \eta} + \frac{1}{\eta} \frac{\partial}{\partial \eta} (\eta^2 \rho v_{\eta 0}) \\
 + \frac{2\rho}{\eta} \frac{\partial v_{\theta 0}}{\partial \theta} = 0 \quad \text{at } \theta = 0 \quad (66)
 \end{aligned}$$

The η component of Euler's Eq. (63) becomes

$$\gamma v_{\eta 0} + \frac{\partial v_{\eta 0}}{\partial \eta} (v_{\eta 0} - \gamma \eta - \gamma c_0) + \frac{e^{-2\gamma t}}{\rho} \frac{\partial p}{\partial \eta} = 0 \quad \text{at } \theta = 0 \quad (67)$$

The θ component of Euler's Eq. (64) is satisfied identically, because all the terms vanish along $\theta = 0$. The enthalpy Eq. (65) becomes

$$\frac{\partial H}{\partial \eta} (v_{\eta 0} - \gamma \eta - \gamma c_0) = \frac{1}{\rho} E_{\eta} j_{\eta} \quad \text{at } \theta = 0 \quad (68)$$

Faraday's law becomes

$$\frac{\partial E_{\eta}}{\partial \theta} = 0 \quad \text{at } \theta = 0 \quad (69)$$

Charge conservation becomes

$$\frac{1}{\eta} \frac{\partial}{\partial \eta} (\eta^2 j_{\eta}) + 2 \frac{\partial j_{\theta}}{\partial \theta} = 0 \quad \text{at } \theta = 0 \quad (70)$$

and the remaining single component of Ohm's law is

$$j_{\eta} = \sigma E_{\eta} \quad (71)$$

As they stand, these six equations (actually five, since the remaining Euler equation just gives the pressure variation, which is a higher-order quantity) are not a complete set, as a check of the number of variables easily shows. Indeed, one would not expect to be able to solve a set of partial differential equations by restricting to a line in this fashion. However, this set is sufficient for an estimate of the growth rate γ if one knows, or is willing to assume, enough about the form of the solution.

The needed additional assumption is that, ahead of the streamer, the system is nearly spherically symmetric. This is certainly the appearance conveyed by our photographs of streamers, and it is what one expects for the concentrated field and current around the tip of a narrow filament. To obtain a more complete, two-dimensional, solution, one would almost certainly use a series expansion with a spherically symmetric current as the leading term. One expects this to be dominant in the region so close to the streamer that E and j are well in excess of the background field and current of the unperturbed discharge. Since this is also the region where gas heating, which causes streamer growth, is concentrated, keeping only the radial current should give a reasonable estimate of the growth rate.

Assuming that the current, field, and gas flow are radial not just along $\theta = 0$ but, for some region around that line, one sees from Faraday's law Eq. (69) that the field E_η must be a function only of η , and from charge conservation Eq. (70) that the current density must vary as $1/\eta^2$.

$$j_\eta = \frac{j_0}{\eta^2} \quad (72)$$

Here J_0 is just a constant, independent of both time and position. From Ohm's law, one has

$$\frac{j_0}{\eta^2} = \sigma E_\eta \quad (73)$$

Since Euler's equation just specifies the negligible variations in the pressure, there remain only two equations: the continuity equation for the gas, which now has the simpler form,

$$-\gamma (\eta + c_0) \frac{\partial p}{\partial \eta} + \frac{1}{\eta^2} \frac{\partial}{\partial \eta} (\eta^2 \rho v_{\eta 0}) = 0 \quad (74)$$

and the enthalpy Eq. (68). Since j is known, and E is determined by σ , which is determined by the temperature of the gas, we have left two equations, (74)

and (68) for the two unknown functions, the gas velocity $v_{n0}(\eta)$ and the temperature, $T(\eta)$ which, at constant pressure, p_0 , determines the density, internal enthalpy, and, with j , the conductivity of the gas at points around the streamer.

Equation (74) can be rewritten as

$$\frac{\partial}{\partial \eta} \left[\rho \eta^2 (v_{n0} - \gamma \eta - \gamma c_0) \right] = -\gamma \rho \eta (3\eta + 2c_0)$$

Integrating along $\theta = 0$ from the edge of the core η_b to a point η in the halo or beyond, one has

$$\begin{aligned} \rho \eta^2 (v_{n0} - \gamma \eta - \gamma c_0) + \rho_c \gamma \eta_b^2 (\eta_b + c_0) \\ = -\gamma \int_{\eta_b}^{\eta} \rho (3\eta'^2 + 2c_0 \eta') d\eta' \end{aligned}$$

Here, ρ_c is the density of the gas in the core of the streamer, at points $\eta \leq \eta_b$, and we have used the fact that in the core heat is radiated or conducted away and the gas flow $v_{n0}(\eta = \eta_b)$ goes to zero. Remember that, in general, ρ and v_{n0} are both functions of the radial distance, η . Now notice that, since ρ_c is essentially constant within the arc-like core, because the temperature is roughly constant there, the second term on the left is just the integral from zero to η_b . So one can write the equation as

$$\rho \eta^2 (v_{n0} - \gamma \eta - \gamma c_0) = -\gamma \int_0^{\eta} \rho (3\eta'^2 + 2c_0 \eta') d\eta' \quad (75)$$

The assumption of spherical symmetry is not expected to be very accurate within the core, which is a cylindrical structure, although the cap of the core may have a somewhat spherical form. But formally one can still carry the

integral on in to $\eta = 0$ to obtain this simpler expression, which should only be used for points η outside η_b .

Combining this result with Eq. (68), the equation for the enthalpy, or gas heating, gives

$$-\gamma \frac{dH}{d\eta} \int_0^\eta \rho (3\eta'^2 + 2c_0\eta') d\eta' = \eta^2 E_\eta j_\eta \quad (76)$$

Here the gradient of H , the enthalpy, has been written as a total derivative, since everything varies only with radius in the spherically symmetric solution. This derivative can be written as

$$\frac{dH}{d\eta} = \frac{dH}{dT} \frac{dT}{d\eta}$$

because, at constant pressure, H depends only upon T , provided that the kinetic energy of gas motion is negligible, which it is, to good approximation for this subsonic flow. Then the equation becomes

$$\frac{dT}{d\eta} = \frac{-j_0^2}{\gamma \sigma \eta^2 \frac{dH}{dT} \int_0^\eta \rho (3\eta'^2 + 2c_0\eta') d\eta'} \quad (77)$$

Since ρ , H , and σ are all known functions of the temperature, T (σ also involves the current, but that is known, too) and γ , j_0 , c_0 , and η_c (which is needed to start a solution) are just numbers, we have now reduced the problem to a single ordinary differential equation for a single unknown function of one variable, $T(\eta)$. This equation can certainly be solved.

5.0 DISCUSSION OF RESULTS

Equation (77) can be solved numerically by stepping out a solution, $T(\eta)$. One must start such a calculation at the inner boundary, η_c , since the integral in the denominator involves ρ and, hence, T , at radii smaller than that of the point where one is calculating $dT/d\eta$. (Physically this occurs because the outward radial flow of gas is caused by heating interior to the region considered.) So one starts at the edge of the core, η_c , where the temperature is the ionization temperature of the gas, T_I , and steps outward in η . Since γ is not known, this must be done for a whole range of γ values. The choice of γ , which causes T to approach the ambient temperature of the background discharge at large distances from the streamer, is the predicted growth rate, the solution to the problem.

To do this calculation one needs j_0 , the constant that specifies the streamer current. This cannot be calculated exactly without solving the whole two dimensional problem, but it can be estimated. As discussed earlier, we expect the streamer to collect approximately that current that would otherwise have been drawn by the area of the cathode that is shadowed by the streamer.

$$I = A \pi c^2 \sigma_0 E_0 = A \pi c_0^2 \sigma_0 E_0 e^{2\gamma t} \quad (78)$$

Here, as before, σ_0 and E_0 are the conductivity and electric field of the unperturbed discharge and A is a constant near unity. In the absence of a two-dimensional analysis, A should be taken as one, unless comparison with data shows that a slightly different value is more accurate.

Assuming that this current is collected by the top half of the streamer halo, one has

$$I = 2\pi j_n n^2 e^{2\gamma t} = 2\pi j_0 e^{2\gamma t} \quad (79)$$

Combining these equations gives

$$j_0 = \frac{A c_0^2 \sigma_0 E_0}{2} \quad (80)$$

Using this in Eq. (77), it is convenient to define a dimensionless length variable

$$\lambda = \frac{\eta}{c_0} \quad (81)$$

Then Eq. (77) becomes

$$\frac{dT}{d\lambda} = \frac{-A^2 \sigma_0^2 E_0^2}{4\gamma \sigma \lambda^2 \frac{dH}{dT} \int_0^\lambda \rho (3\lambda'^2 + 2\lambda') d\lambda'} \quad (82)$$

In calculating solutions, this equation, not Eq. (77), is the most convenient form to use.

In stepping out a solution, one starts at the inner boundary

$$\lambda = \lambda_c \quad (83a)$$

$$T = T_I \quad (83b)$$

Here, λ_c is the ratio of the radius of the tip of core, $\eta_c \exp(\gamma t)$, to the length of the streamer, $c_0 \exp(\gamma t)$. In the photographs, the ratio appears to be of the order of 0.1 or less.

To do the numerical calculation, one needs to know the core radius. The core begins where the temperature reaches T_I , the ionization temperature given by the Saha equation for the discharge gas and pressure. At this point, the field will drop below the breakdown field, and the ionization will be

maintained by heating. At this radius, also, the structure makes the transition from the spherical symmetry of the halo to the cylindrical symmetry of the core. The reason is that, after the streamer has grown further and the tip has passed beyond this point, the gas will still be hot and hence conducting, a persistence not possessed by the halo conductivity, which depends upon the enhanced electric field around the tip of the streamer.

The core radius is, of course, related to the current density in the column, j_c . The total current in the cylindrical core is

$$I = \pi r_c^2 j_c e^{2\gamma t}$$

Using Eq.(78) for the total streamer current gives

$$j_c = \frac{A \sigma_0 E_0}{\lambda_c^2}$$

or,

$$\lambda_c = \sqrt{\frac{A \sigma_0 E_0}{j_c}} \quad (84)$$

The ratio of core radius to streamer length is, obviously enough, just the square root of the ratio of discharge current density to core current density. Since the λ_c seen in the photographs was about 0.1 and the discharge current density was an ampere per square centimeter or more, the filamentary core of the streamer must have carried a current of 100 A/cm² or more. The reason the current is this high is straightforward. The arc behind a streamer develops so fast that there is not time for thermal diffusion to spread the current very much.

There is one more equation or, rather, boundary condition, that must also be considered. This is the prescription that, since the streamer core is

a good conductor, the tip, at λ_c , is essentially at cathode potential. Hence the concentrated electric field around the tip must drop a voltage equal to $E_0 c$, the voltage that would have been dropped over the streamer length in the background discharge. So we have,

$$E_0 c_0 e^{\gamma t} = \int_{\lambda_c}^c \frac{j}{\sigma} d\lambda e^{\gamma t}$$

This reduces to

$$\frac{2}{\sigma_0 A} = \int_{\lambda_c}^1 \frac{d\lambda}{\lambda^2 \sigma} \quad (85)$$

Notice that, since this integral contains σ , the spatially varying conductivity, which depends upon the gas density, which depends upon T , this equation can only be evaluated after one has solved Eq.(82) for the temperature profile, $T(\lambda)$.

One cannot be completely sure until one has generated solutions and studied their form, and the present program did not get that far, but it appears that Eqs.(82) and (85) together are enough to allow calculation of both γ and λ_c . That is, for each choice of γ and λ_c , one obtains, from the resulting temperature profile, $T(\lambda)$, two numbers, the temperature at $\lambda = 1$ and the voltage dropped around the streamer, which, in these terms, is the integral in Eq.(85). The solution to the problem is that pair, γ and λ_c , which makes $T(1)$ equal to the ambient temperature and satisfies Eq.(85).

There is a way to look at the physics of this, which may make the mathematics more understandable. The streamer grows by heating the gas to the conduction temperature, T_I . It does this by concentrating the current, which, of course, gives a much higher heating ahead of the streamer than elsewhere in the discharge. One might then wonder why the streamer does not "neck down" even more, concentrating the current into an extremely tiny filament. Then it would have to heat even less gas, and the current density,

and hence the rate of heating at the tip would be even higher, so the streamer could grow even faster. What Eq.(85) says is that this does not happen because the streamer must concentrate the current, using only the available voltage. If the streamer made a narrower core and grew faster, it would be growing through colder gas, which would have a lower conductivity. To concentrate the current would then require a stronger electric field, which cannot be supported with the available voltage. The point is that the streamer must not only heat the gas that eventually becomes part of the core but it must also heat all the gas around the tip enough to increase the conductivity of the whole surrounding region, so that the current can all be funneled into the tip of the core without requiring a larger voltage drop than is available. By considering both requirements, it appears possible to calculate both the streamer growth rate and the radius of the tip of the core.

The remaining inputs needed, namely the functions $\rho(T)$ and $H(T)$ and the electrical conductivity σ , must be known for the discharge gas mixture. The density and enthalpy are standard thermodynamic functions. The conductivity, as explained earlier, is given in the halo by the assumption that the field is at the breakdown threshold, and beyond the halo by the e-beam-produced electron density, n_e , and the mobility, μ_e , which, of course, depends upon the gas density. Since one does not a priori know which form to use, the procedure is to calculate both, then use whichever conductivity is higher.

Finally, it is useful to consider the pressure-dependence of the result. The conclusion of the scaling analysis in Section 2.0 was that one wants to go to higher discharge pressure. Assume that the pressure is increased by a factor k . At constant E/n , the field E_0 will increase by the same factor. If the e-beam current is increased proportionally, the electron density in a recombination dominated discharge will also be increased by a factor k . This leaves the electrical conductivity σ_0 unchanged, since the mobility is reduced by k . So the discharge current is increased by k , keeping the current gain constant.

In the halo of a streamer, the current would also be higher by a factor of k , but so is the breakdown field. Hence the conductivity σ is unchanged (if the temperature is similar). From Eq. (82), one sees that, to keep the expression similar, one would have to increase γ by a factor of k also. The

unknown here is the change in λ_c , the streamer core diameter. But even allowing for this, the effect on the equation is worrisome.

Consider now a second case, where the e-beam current is left at the same value as in the lower pressure discharge. Then the electron density only increases as \sqrt{k} , the conductivity, σ_0 , decreases by \sqrt{k} and the discharge current density, $\sigma_0 E_0$, and hence the current gains of the system are increased by a factor \sqrt{k} . Here the halo conductivity is reduced by \sqrt{k} , because the breakdown field is still increased by k , but the current only by \sqrt{k} for streamers of similar geometry. Now, to keep the equation the same, one must again increase γ , but only by \sqrt{k} . The unknown factor is the same: the change, if any, in λ_c .

It appears that this higher pressure discharge would probably arc sooner. Basically, what the equation shows is that, if the discharge specific power is increased, the streamer growth rate should be higher. That was true in each of the above examples since the E field was increased by the same factor as the pressure, and the current density was also raised, so the dissipation, in watts per gram of gas, was higher.

This is not a firm conclusion, since the change in λ_c is not known, and we only considered the effect in the halo, not the lower field region beyond. Indeed, as discussed earlier, there is evidence that a switch type discharge has an arcing limit that depends more upon field than power. Still, it is plausible that if E/ρ is kept constant while $\sigma_0 E_0^2/\rho$ is raised, the allowable pulse length will be reduced.

The remaining possibility is to reduce E/ρ somewhat as the pressure is raised. That is, one can increase the voltage by a smaller factor than k . Then one could increase the discharge current density without a proportional increase in specific power. In terms of the foregoing analysis, the effect of a decrease in E/ρ is to decrease the size of the high field halo around the tip of a streamer. The calculations that we have made strongly suggest that one will want to do this.

The scaling analysis shows that this technology should go in the direction of higher pressure discharges. The streamer analysis shows that it will probably not be feasible to increase the voltage by as large a factor as the pressure.

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