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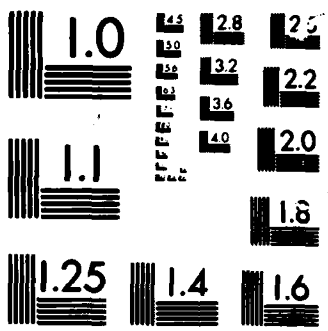
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AD-A167 145

AN APPLICATION OF RESPONSE SURFACE METHODOLOGY TO A MACROECONOMIC MODEL

THESIS

James L. Donovan
Captain, USAF

AFIT/GOR/ENS/85D-4

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SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) AFIT/GOR/ENS/85D-4	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION School of Engineering 6b. OFFICE SYMBOL (If applicable) AFIT/ENS	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State and ZIP Code) Air Force Institute of Technology Wright-Patterson AFB, Ohio 45433	7b. ADDRESS (City, State and ZIP Code)
8a. NAME OF FUNDING/SPONSORING ORGANIZATION 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State and ZIP Code)	10. SOURCE OF FUNDING NOS. PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO. / WORK UNIT NO.
11. TITLE (Include Security Classification) See Box 19	
12. PERSONAL AUTHOR(S) James L. Donovan, B.S., Capt, USAF	
13a. TYPE OF REPORT MS Thesis 13b. TIME COVERED FROM ______ TO ______	14. DATE OF REPORT (Yr., Mo., Day) 1985 December 15. PAGE COUNT 144
16. SUPPLEMENTARY NOTATION	

17. COSATI CODES

FIELD	GROUP	SUB. GR.
05	03	

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)

Economic Policy, Macroeconomic Model, Econometric, Response Surface

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Title: AN APPLICATION OF RESPONSE SURFACE METHODOLOGY TO A MACROECONOMIC MODEL

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Approved for public release: IAW AFR 190-1.
LYNN E. WOLAVER 13 Feb 86
Dean for Research and Professional Development
Air Force Institute of Technology (AFSC)
Wright-Patterson AFB OH 45433

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐	21. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL Palmer W. Smith, Lt Col, USAF	22b. TELEPHONE NUMBER (Include Area Code) 513-255-5533 22c. OFFICE SYMBOL AFIT/ENS

DD FORM 1473, 83 APR EDITION OF 1 JAN 73 IS OBSOLETE.

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The purpose of this investigation is to apply response surface methodology to a macroeconometric model to facilitate better analysis with the model. Second degree polynomial response functions are used to derive function multipliers for the Klein-Goldberger econometric model. The function multipliers show that the impact of changes in exogenous variables depends on the levels of one or more other exogenous variables. The function multipliers are used to conduct policy analysis and assess factor importance. As an extension, first degree polynomial response functions are used in an example problem to maximize gross national product subject to constraints on unemployment, inflation, and ranges of fiscal policy variables. The example problem demonstrates the flexibility and value of developing response surface equations for complex macroeconometric models.

The study concludes that a response surface can capture the complexity of macroeconometric models such as the Klein-Goldberger model. Results also show that the assumptions of linearity for developing multipliers can result in misleading values when non-linearity is present. Recommendations for further research include fitting a more nonlinear model with response surfaces, and including time as an independent variable in the response surface equations.

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Abstract

The purpose of ~~the investigation~~ this thesis is to apply response surface methodology to a macroeconometric model to facilitate better analysis with the model. First and second degree polynomial response surface equations express endogenous variables as functions of selected exogenous variables in the Klein-Goldberger econometric model.

Second degree polynomial response functions are used to derive function multipliers. The function multipliers show that the impact of changes in exogenous variables on endogenous variables depends on the levels of one or more other exogenous variables. The function multipliers are used to conduct policy analysis and assess factor importance. As an extension, first degree polynomial response functions are used in an example problem to maximize gross national product subject to constraints on unemployment, inflation, and ranges of fiscal policy variables. The example problem demonstrates the flexibility and value of developing a response surface equation for complex macroeconometric models.

The study concludes that a response surface can capture the complexity of macroeconometric models such as the Klein-Goldberger model. Results also show that the assumptions of linearity for developing multipliers can result in misleading values when nonlinearity is present. Recommendations for further research include fitting a more nonlinear model with response surfaces, and including time as an independent variable in the response surface equations.

AN APPLICATION OF RESPONSE SURFACE METHODOLOGY TO A MACROECONOMIC MODEL

I. The Research Problem

Introduction

This thesis deals with how macroeconomic models and response surface methodology (RSM) can be brought together to provide better analysis of a national economy. Its purpose is to demonstrate that RSM can reduce complex relationships embodied in macroeconometric models to simple equations. The thesis also interprets the simple equations and shows how they can be used for practical applications. This chapter puts the research effort into perspective by briefly describing macro-econometric models and response surface methodology suggesting possible ways to combine the two. The chapter then outlines the research plan including the research problem, research questions, research objectives, scope, and general methodology for attacking the problem.

Macroeconomic Models

Macroeconomic models are a set of economic relationships expressed in mathematical equations which allow economists to predict the performance of a national economy. Economists have developed several types of macroeconomic models. One type uses certain economic indicators which have historically led cyclical changes in the economy. Another type uses consumer attitudes and buying plans to predict economic performance. The type of particular interest to this thesis is the econometric model. Econometric models are systems of statistically derived simultaneous equations based on theory and historical data which predict

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AN APPLICATION OF RESPONSE SURFACE METHODOLOGY TO A MACROECONOMIC MODEL

THESIS

Presented to the Faculty to the School of Engineering

of the Air Force Institute of Technology

Air University

In Partial Fullfillment of the Requirements for the Degree of

Master of Science in Operations Research

James L. Donovan, B. S.

Captain, USAF

December 1985

Acknowledgements

First, I wish to thank God, who was always there to help during the very rare occasions when my thesis advisors were unavailable. I sincerely thank my two thesis advisors LtCol Palmer W. Smith and Dr Robert F. Allen for their many hours of help and constant encouragement. I have learned much from their technical expertise and personal examples.

I sincerely appreciate the patience and support of my family and especially Michele, who is soon to become my family. This thesis is a product of their sacrifice as well as mine.

Finally, I would like to thank Mrs. Joanne Crane, who graciously volunteered many long hours to help me prepare this document. Without her assistance this thesis would have been nearly impossible to complete on time.

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economic performance.

To formulate an econometric model, economists first hypothesize equations to describe theoretical economic relationships for sectors of the economy. For example, in one model, manufacturing output is a function of the amount of hours worked in the manufacturing sector, the amount of money invested in manufacturing, and average productivity. Money invested in manufacturing is a function of manufacturing capacity used in the previous period, manufacturing output in previous periods, cash flow in the manufacturing sector, and interest rates on bonds. The number of hours worked is a function of previous period manufacturing output, wage rates, and percent of manufacturing capacity used. Wage rates depend on past wage rates and the cost of living, and so on (Evans, 1969:433-442). Economists postulate the form of functions such as these and then use historical data to estimate unknown coefficients in the equations with statistical techniques. Finally, they put equations representing all sectors of the economy together and solve them simultaneously to obtain predictions.

Klein and Evans ennumerate three major uses for econometric models in their work entitled, The Wharton Econometric Forecasting Model (Klein and Evans, 1968:50). First, economists can use them for prediction. Second, econometric models can simulate the consequences of economic policies such as tax increases, government spending, and Federal Reserve actions for periods in the past. In many ways the most important use for econometric models, claim Klein and Evans, is computing multipliers for fiscal and monetary policy alternatives. Fiscal policy is manipulating the economy by government spending and taxation. Monetary policy is manipulating the economy through actions of the Federal Reserve such as selling government bonds, changing the required reserves that banks must hold against demand deposits, etc. A multiplier is a constant which, when multiplied by a change in an econometric input variable, gives the change in an output variable. Multipliers are especially important because of the increasing size and complexity of econometric models. For example, the Warton Econometric and Forecasting Unit has 53 equations and 29 identities (Evans, 1969:442). A model designed at Brookings has over 150 equations (Evans, 1969:503). The effects of changes in certain input variables are difficult to trace through to the final output of such large models unless multipliers are computed. Unfortunately, multiplier analysis does not account well for nonlinear systems with interactions among input variables. Response surface methodology might provide a way of overcoming these difficulties.

Response Surface Methodology

Response Surface Methodology (RSM) is an analytical tool for modelling a very complex or unknown process with a single mathematical equation and exploring the resulting relationship between the inputs and an output of the process. A response surface results from plotting output values obtained from the response surface equation against input variables as they vary over continuous ranges. Chemical reactions are classic RSM applications. Factors which affect the yield of a chemical reation are temperature, pressure, amount of reactants, and reaction time. If one were to set the amount of reactants at some set level and fix the time the reactants are allowed to react, a set of experiments could be conducted at various combinations of temperature and pressure. By recording the yield of each experiment and plotting that yield against temperature and pressure, the result would be something like the graph of the response surface in Figure 1.1.

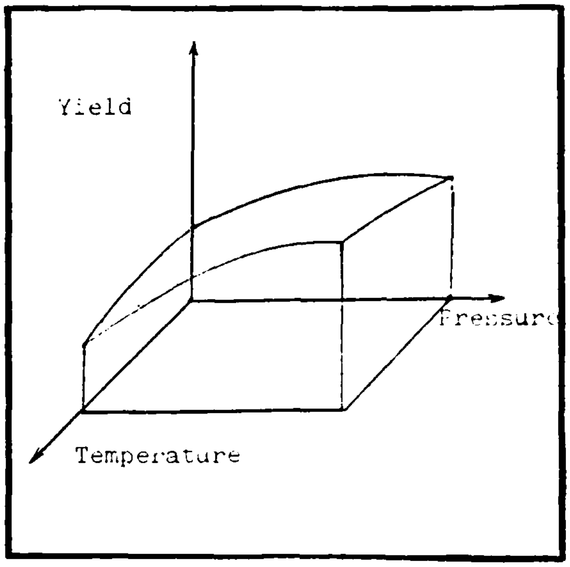

Figure 1.1. Example Response Surface.

Box and Wilson first introduced response surface methodology in their 1951 paper entitled, "On the Experimental Attainment of Optimum Conditions" (Box and Wilson, 1951). Since then, many researchers have profitably applied the technique to problems in chemistry, foodstuffs, tool life testing, and other areas (Hill and Hunter, 1966:576). Smith and Mellichamp first demonstrated that RSM could provide valuable insight into complex deterministic analysis models (Smith and Mellichamp, 1979). Based on Smith and Mellichamp's work, students at the Air Force Institute of Technology have applied RSM to several deterministic and probabilistic models (Manacapilli, 1984; Graney, 1984; Meitzler, 1984; Sparrow, 1984). Deterministic models are mathematical representations of an underlying process for which given inputs to the process yield the same output every time the model is run. The outputs of probabilistic models contain random variation. Burdick and Naylor suggested applying RSM to econometric models. They showed how to combine a simple six equation econometric model and a utility function for economic policy optimization by response surface techniques (Burdick and Naylor, 1969:29). However, they did not actually estimate response surfaces for this system. Their ideas merit a more thorough development.

Applying RSM to a problem involves several steps. First the problem is defined and variables of interest are specified. Next a response surface equation, usually a low order polynomial, is selected to model the process under study. Based on the response surface equation selected, an appropriate experimental design is chosen which compromises between economy of design points and orthogonality. Then the experiment or model is run repeatedly at factor levels specified by the experimental design. With the data collected from the experiment or model runs, a response surface equation is estimated using ordinary least squares. After checking for adequate fit, the response surface is ready for interpretation and analysis. These application steps are covered in much greater detail in Chapter III.

Several analysis methods are available for exploring response surfaces. Most of these methods are devised to enable the analyst to optimize the underlying process or model that the response surface represents. They include the method of steepest ascent, classical optimization using calculus, Lagrangian techniques, mathematical programming, and others. However, optimization is not the only use for response surfaces. Examining the response surface equation itself reveals characteristics about the underlying process or model. In addition, the impact of tradeoffs between inputs is easily assessed.

These analysis techniques have potential applications for studying macroeconometric models. For instance, constants or expressions similar to multipliers could be estimated. These constants or expressions would make the impact of changes in economic policy variables explicit. Multipliers implicitly assume that the changes in response are at least approximately linearly related to the change in input. Expressions derived to serve as "multipliers" using the response surface technique have no such implicit assumption. Thus alternative policy options for nonlinear models could be evaluated more accurately. In fact, given specific economic objectives, economic policy could be optimized. The potential applications for RSM in macroeconomics suggest several areas for research. Below is the specific plan for this thesis effort.

Research Plan

Problem Statement. Economists have developed large econometric models to predict the performance of national economies. Unfortunately, because of the complexity of these models, economists have difficulty investigating the effects of changing key input variables on economic performance. Response surface methodology may be able to reduce key relationships in the model to a single equation.

Research Question. How well can response surface methodology capture the predictive power of a large econometric model and can response surface methodology simplify sensitivity analysis of such a model?

Subsidiary Questions. Several issues related to the research deserve investigation.

1. Can a response surface based on a simple function accurately capture the relationships in a large macro-economic model?

2. Is there a limit to the size of model that response surface methodology can handle?

3. How can the coefficients of the response surface equation be interpreted?

4. Do the response surface coefficients identify economic inputs which are most influential in driving a national economy?

5. Can one use response surface methodology to determine fiscal and monetary policy which the model predicts will optimize particular measures of economic performance?

Research Objectives. To answer the research questions, several objectives must be met. They are:

1. Determine how well a response surface can fit the response of important economic variables to changes in fiscal and monetary variables for an actual macroeconometric model. The model should be moderatly sized, have some nonlinearities, and have characteristics which are well known from previous analysis.

2. Verify that response surface does in fact reflect model characteristics by comparing response surface equation parameters to multipliers computed for the model.

3. Interpret the response surface equations.

4. Develop applications of practical value for the response surface equations.

Scope. This study demonstrates feasibile applications for response surface methodology techniques in macroeconomics. To make the research effort manageable, several decisions are made. The study uses the Klein-Goldberger macroeconometric model for the investigation. Reasons for selecting the model are given in Chapter II. Based on previous experience with response surface methodology, a second order polynomial response surface equation is assumed. The model is constructed to mirror the characteristics of the national economy which changes slowly in response to changes in policy and so a second degree polynomial should adequately fit the model. Moreover, Goldberger argues that the

Klein-Goldberger model is nearly linear (Goldberger, 1959:136-138). Response surfaces are built for three output variables in terms of five input variables. Although response surfaces with more variables could be constructed, no particular advantage is seen in this. Although any econometric model is stochastic in nature, it is assumed that the model is deterministic. Finally it is assumed that the model is a reasonably valid representation of the economy's behavior with the exception of deficiencies in the monetary sector which will be discussed in Chapter II. No effort is made to evaluate the model's forecasting record.

General Methodology. The general plan of attack for accomplishing the research objectives is:

1. *Develop a computer program* which solves the Klein-Goldberger model for output variables in terms of given input variable values.

2. Solve the model for values of input variables reqiured by the experimental design selected.

3. Fit a second order polynomial response function to the data and check fit.

4. Fit a first order polynomial model to data for a direct comparison of response surface coefficients to multipliers computed by Goldberger.

5. Interpret response surface equations and develop ways for summarizing the information contained in the equations.

6. Develop practical applications for derived response surfaces. Specifically, develop optimization applications.

Thesis Overview. The chapters in this thesis follow the pattern in this chapter. Chapter II examines macroeconomic models in general and the Klein-Goldberger model in particular. It also describes solution techniques and methods for deriving multipliers. Chapter III describes the steps in applying response surface methodology and discusses how RSM can be applied to macroeconomic models. Chapter IV details methodology for this research effort. Chapter V addresses how well the response surfaces fit the model, and compares first order response surface coefficients to Goldberg's multipliers. Chapter VI interprets features of the derived response surfaces and develops an optimazation problem application for the response surfaces. Finally, Chapter VII summarizes findings and recommends further research.

II. Macroeconomic Models

Introduction

Chapter I described what macroeconometric models are and how they are used. The chapter explained that macroeconometric models are a set of simultaneous equations based on theory and historical data which allows economists to predict the performance of the national economy. Uses for econometric models include forecasting, policy simulation in historical periods, and most importantly, for computing multipliers which relate changes in fiscal and monetary policy to changes in economic performance. This chapter explains aspects of a simple macroeconomic model, and uses this model to analyze how changes in fiscal policy affect economic performance. Next the Klein-Goldberger (KG) model is introduced. After discussing macroeconomic models, this chapter discusses methods for solving macroeconometric models and deriving multipliers.

Describing how macroeconomic models are built is often the subject of an entire college course. The discussion of macroeconomics here is merely meant to be a quick, simplified review of points relevant to the research effort. The material presented is condensed from Baird and Cassuto's introductory text entitled Macroeconomics: Monetary, Search, and Income Theories (Baird and Cassuto, 1984). The text is thorough yet extremely readable with plenty of helpful examples and illustrations. The reader who is unfamiliar with macroeconomics is highly encouraged to consult the Baird and Cassuto text or a similar text. The discussion below begins with economic equalibrium. Then the national income identity is used to develop the commodities market of a general equalibrium macroeconomic model.

A Simple Equalibrium Model

The primary assumption behind equalibrium macroeconomic models is that the economy always seeks an equalibrium. A rigorous argument supporting this assertion's truth will not be attempted; however, the proposition makes intuitive sense. Nature is full of examples of systems which seek equalibrium. A dislodged boulder rolls down the mountainside until is finds a valley to rest in. Chemicals react until they reach equalibrium. Economic theory assumes that the economy will also seek equalibrium in the absence of external disturbances. With this assumption this in mind, the macroeconomic model building discussion may begin.

The National Income Accounting Identity. Perhaps the most widely used performance measure of an economy is the gross national product (GNP). This number is a measure of national income. It is defined as the dollar value of all final goods and services produced for final consumption during a calendar year. Mathematically GNP can be defined as:

$$Y = C + I_A + G + F_E \qquad (2.1)$$

where

Y = gross national product,

C = consumption of goods and services,

I_A = actual investment,

G = government expenditures, and

F_E = net exports to foreign nations.

In words, the equation says GNP is the total of a country's expenditures on final goods and services plus the value of net exports.

Double entry accounting procedures for producing firms require that the expenditures for producing those final goods and services be equal to the income from the sales of those goods and services. Mathematically,

$$C + I_A + G + F_E = Y_d + S_b + T \qquad (2.2)$$

where

Y_d = disposable income,

S_b = business savings, and

T = taxes.

The left side of Eq (2.2) is total expenditures and the right side is total income. Eq (2.2) is known as the national income accounting identity and it forms the basis for macroeconomic model building. On the right side, disposable income, Y_d, is after tax after business saving income. Business savings, S_b, are the portion of business net income which firms do not distribute to owners. For this discussion, business savings will be assumed set a fixed level, $\bar{S}_b$. Finally taxes, T, are income appropriated by the government. For this simple model, it will be assumed that taxes are fixed at $\bar{T}$.

On the left side of Eq (2.2) are the components of total expenditure including consumption, investment, government spending, and net foreign exports. If each of the components of GNP can be determined, GNP can be computed. The commodities market models the relationship between components of national income.

The Commodities Market. All of the analysis which follows assumes fixed prices. A variable price model requires development of other markets in the economy. The first component of the expenditures side of the national income accounting identity is consumption. Consumption is in large part based on disposable income. The more income a person receives, the more that person usually spends. Mathematically, this is written $C = C(Y_d)$. A simple consumption function is $C = \bar{C} + bY_d$, where C is autonomous consumption that occurs without income, b is the marginal propensity to consume, and Y_d is disposable income. In general b is between zero and one because individuals divide their disposable income between savings and consumption.

The next component of expenditures is investment. Planned investment is also a function of income because the more sales a firm receives the more it will want to expand operations through capital investment. A simple investment function is $I = \bar{I} + vY$, where I is the autonomous part of investment and v is the marginal propensity for firms to invest. v is between zero and one because firms expend income received on profits, operating costs, etc., as well as investment.

The next component of expenditures is government spending. Government spending is set by the government instead of market forces. Government expenditures is assumed to be set at a particular level, say $\bar{G}$.

The last component of expenditures is net foreign exports which is the difference between exports and imports. For the purposes of this analysis net foreign exports is assumed zero.

Figure 2.1 shows the relationships between components in the national income accounting identity which together comprise the commodities market. The vertical axis measures total expenditures, E, and the horizontal axis measures total income, Y.

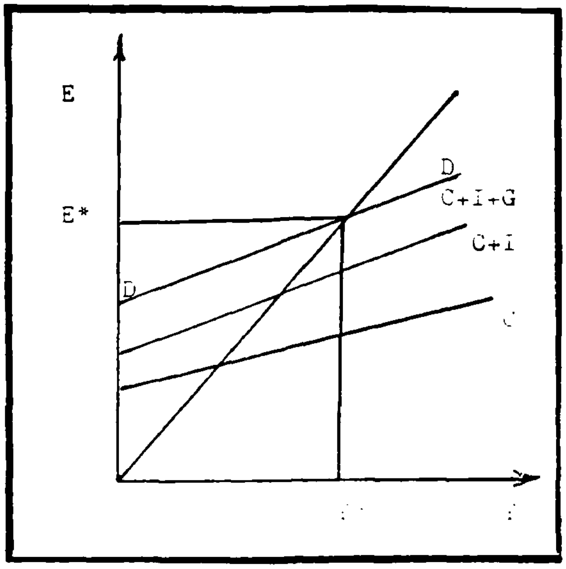

Figure 2.1. The Commodities Market

The 45° line is the national income accounting identity and must always hold, Line DD represents the sum of consumption, expected investment, and government expenditures known as aggregate demand. Even though aggregate demand is the sum of consumption, investment, and government spending, it is different from GNP because it includes planned investment by firms instead of actual investment. It is possible that planned investment will not equal actual investment. Included in investment are inventories of goods produced for sale. If demand for goods is lower than expected, inventories will increase in the short run and actual investment will be higher than planned. The difference between expected investment and actual investment is the unplanned increase in inventories. Expected investment equals actual investment only at equilibrium in the commodities market (ie., when unplanned changes in inventories are zero and the 45° line and the aggregate demand line intersect). In Figure 2.1 Y^* is the equalibrium income and E^* is the equalibrium total spending. The system of equations which describes economic equalibrium in the hypothetical economy is

$$Y = C + I_a + G \qquad (2.3)$$

$$Y = Y_d + S_b + T \qquad (2.4)$$

$$C = \bar{C} + bY \qquad (2.5)$$

$$I = \bar{I} + vY \qquad (2.6)$$

$$G = \bar{G} \qquad (2.7)$$

$$T = \bar{T} \qquad (2.8)$$

$$S_b = \bar{S}_b \qquad (2.9)$$

G, T, and S_b are assumed fixed. In the system of equations above, Y, C, I, and Y_d are known as endogenous variables. Endogenous variable are variables which have values determined within the system. G, T, and S_b are exogenous variables. Exogenous variables are assumed to have a value determined outside the system. Although the system above is a complete system with the number of unknowns equal to the number of equations, it is complete only because of the simplifying assumptions used to formulate the equations. For instance, prices are assumed to be constant. Since $Y = PQ$ where P is the price level and Q is the real output of the economy, any increases in Y are assumed to be increases in real output, Q. Also it has been assumed that consumption and investment are functions only of income, and that business savings are constant.

The system of equations may be solved for any of the endogenous variables in terms of constants and exogenous variables. Solving the equations in this way gives the explicit effect of exogenous variables on an endogenous variable. Since the analysis has centered on determinants of national income, Y is solved for. The solution is

$$Y = \frac{1}{(1 - b - v)} (\bar{C} + \bar{I} + \bar{G} - b\bar{T} - b\bar{S}_b) \qquad (2.10)$$

Eq (2.10) is known as the reduced form for Y since Y is expressed in terms of known, exogenous quantities. The quantity $1/(1-b-t)$ is known as a multiplier because is tells how many times larger the change in Y will be in response to a change in $\bar{G}$, $\bar{C}$, or $\bar{I}$. For instance, if b = 0.70 and v = 0.05 and G changes by ten billion dollars, then Y increases by

$$\frac{(\$10 \text{ billion})}{(1-0.70-0.05)} = \$40 \text{ billion}$$

under the assumptions set forth above. The multiplier, $1/(1-b-v)$ will henceforth be denoted as m. Eq (2.10) makes the effects of changing exogenous variables clear. For instance, if government expenditures increase by ΔG, Y will increase by $m\Delta G$. On the other hand, if T is decreased by ΔT, Y will increase by $-bm\Delta T$. Since b is less than one, bm is less than m. An increase in government spending changes total income by more that the same decrease in taxes. T is a lump sum tax. A tax rate function could be introduced into the model, but it is not necessary for this discussion and will be omitted for simplicity. Changes in Y due to changes in the autonomous components of C and I are also easily determined. Figure 2.2 shows the increase in Y due to an increase in G.

Increasing government spending shifts the aggregate demand line from DD to DD′which causes income to increase from Y* to Y′. In fact, any change which causes a shift in $\bar{C}$, $\bar{I}$, or $\bar{G}$ will shift the aggregate demand curve in a similar manner.

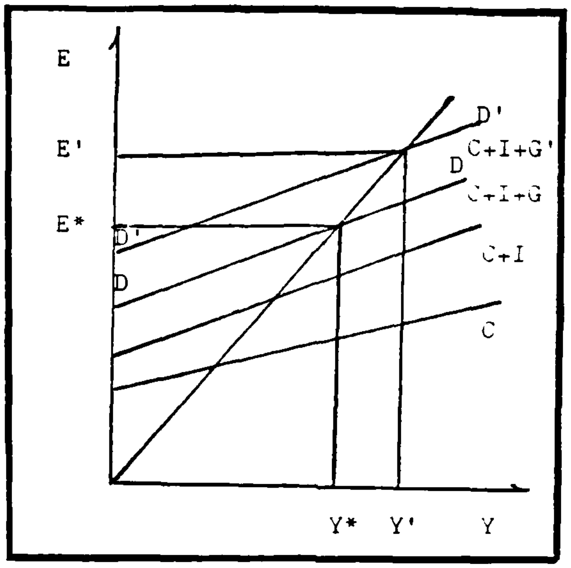

Figure 2.2. Income Effects of Increased Government Spending

Changes in government spending affect more in the economy than just the commodities market. Increased government spending increases demand for output and labor in the short run and eventually raises prices and interest rates in the longer term. These effects are not seen in the simplified commodities market presented here because of simplyfying assumptions. Most macroeconomic models develop other interrelated markets to model the behavior of interest rates, prices, money suply, and labor. Multipliers for larger systems are much more complex than the simple multiplier in Eq (2.10). Multipliers must capture the effects of all variables in the model.

The Klein-Goldberger Model

Econometric models like the Klein-Goldberger model are formulated to include many variables in the economy. The KG model consists of 21 simultaneous equations, 15 of which are behavioral and 6 of which are identities relating variables. Behavioral equations such as Eq (2.5) and (2.6) have parameters which must be estimated from historical economic data. Parameters in the KG model were estimated using economic data from several sources, primarily the United States Department of Commerce. The KG model breaks the economy into separate government, corporate, labor, and agricultural sectors. The model characterizes the dynamic nature of the economy through a system of lagged variables.

Lagged variables give the value of a variable in previous years. For instance investment lagged one year is the value of investments one year prior to the current year. Lagged variables are denoted by a subscripted negative number which represents the number of periods the variable is lagged. Investment lagged five years is denoted I_{-5}. Lagged variables together with exogenous variables are know as predetermined variables. Understanding lagged variables is essential to understanding a central feature of the KG model.

Table 2.1 lists the 21 equations in the KG model and Table 2.2 defines the variables in the model. For a detailed discussion of the model's theoretical development, one can consult Klein and Goldberger's original work presenting the model (Klein and Goldberger, 1955). Another description of the model appears in Theil's Econometrics (Theil, 1971).

Table 2.1. The Klein-Goldberger Model (Adapted from Goldberger, 1959:4-7)

$$C = -22.26+0.55(W_1+W_2-T_W)+0.41(P-T_C-T_N-S_C)$$

$$+0.34(R_1+R_2-T_R)+0.26C_{-1}+0.072(L_1)_{-1}+0.26N_P \quad (2.1.1)$$

$$I = -16.71+0.78(P-T_C-T_N+R_1+R_2-T_R+D)_{-1}-0.073K_{-1}+0.14(L_2)_{-1} \quad (2.1.2)$$

$$S_C = -3.53+0.72(P_C-T_C)+0.076(P_C-T_C-S_C)_{-1}-0.028(S_B)_{-1} \quad (2.1.3)$$

$$P_C = -7.60+0.68P \quad (2.1.4)$$

$$D = 7.25+0.10\frac{(K+K_{-1})}{2}+0.044(Q-W_2) \quad (2.1.5)$$

$$W_1 = -1.40+0.24(Q-W_2)+0.24(Q-W_2)_{-1}+0.29t \quad (2.1.6)$$

$$(Q-W_2) = -26.08+2.17[h(N_W-N_G)+N_E]+0.16\frac{(K+K_{-1})}{2}+2.05t \quad (2.1.7)$$

$$w-w_{-1} = 4.11-0.74(N_L-N_W-N_E)+0.52(p_{-1}-p_{-2})+0.54t \quad (2.1.8)$$

$$F_I = 0.32+0.0060(M-T_W-T_C-T_N-T_R)(p/p_F)+0.81(F_I)_{-1} \quad (2.1.9)$$

$$R_1(p/p_R) = -0.36+0.054(W_1+W_2-T_W+P-T_C-T_N-S_C)(p/p_R)$$

$$-0.007[(W_1+W_2-T_W+P-T_C-T_N-S_C)(p/p_R)]_{-1}+0.012F_R \quad (2.1.10)$$

$$p_R = -131.17+2.32p \quad (2.1.11)$$

$$L_1 = 0.14(M-T_W-T_C-T_N-S_C-T_R)+76.03(i_L-2.0)^{-0.84} \quad (2.1.12)$$

$$L_2 = -0.34+0.26W_1-1.02i_s-0.26(p-p_{-1})+0.61(L_2)_{-1} \quad (2.1.13)$$

$$i_L = 2.58+0.44(i_s)_{-3}+0.26(i_s)_{-5} \quad (2.1.14)$$

$$100\frac{[i_s-(i_s)_{-1}]}{i_s} = 11.17-0.67L_B \quad (2.1.15)$$

$$K-K_{-1} = I-D \quad (2.1.16)$$

$$S_B-(S_B)_{-1} = S_C \quad (2.1.17)$$

$$W_1+W_2+P+R_1+R_2 = M \quad (2.1.18)$$

$$C+I+G-F_I = M+T_E+D \quad (2.1.19)$$

$$h(w/p)N_W = W_1+W_2 \quad (2.1.20)$$

$$Q = M+T_E+D \quad (2.1.21)$$

Table 2.2. Glossary of Variables for the Klein-Goldberger Model (Adapted from Goldberger, 1959:5-6)

New Symbol	Brief Definition	Category
C	Consumption	Endogenous
D	Depreciation	Endogenous
F_I	Imports	Endogenous
F_R	Farm exports	Exogenous
G	Government expenditures and exports	Exogenous
h	Hours of work	Exogenous
I	Investment	Endogenous
i_L	Long-term interest rate	Endogenous
i_S	Short-term interest rate	Endogenous
K	Capital stock	Endogenous
L_1	Household liquid assets	Endogenous
L_2	Business liquid assets	Endogenous
L_B	Percentage excess reserves	Exogenous
M	National income	Endogenous
N_E	Entrepreneurs	Exogenous
N_G	Government employees	Exogenous
N_L	Labor force	Exogenous
N_P	Population	Exogenous
N_W	Employees	Endogenous
P	Nonwage nonfarm income	Endogenous
P_C	Corporate profits	Endogenous

p	Price level	Endogenous
p_F	Import price level	Exogenous
p_R	Farm price level	Endogenous
Q	Gross national product	Endogenous
R_1	Farm income	Endogenous
R_2	Farm subsidies	Exogenous
S_P	Corporate surplus	Endogenous
S_C	Corporate savings	Endogenous
t	Time trend	Exogenous
T_C	Corporate taxes	Exogenous
T_E	Indirect taxes	Exogenous
T_N	Nonwage nonfarm noncorporate taxes (less transfers)	Exogenous
T_R	Farm taxes (less transfers)	Exogenous
T_W	Wage taxes (less transfers)	Exogenous
w	Wage rate	Endogenous
W_1	Private wage bill	Endogenous
W_2	Government wage bill	Exogenous

The text below briefly describes each equation in the model.

Eq (2.1.1) is the consumption function. It gives consumption as a function of labor, corporate, and agricultural disposable income. The equation includes factors from the money market through the household liquid assets (L_1) term. Also included is a population trend.

Eqs (2.1.2) through (2.1.5) model the behavior of the corporate sector. The investment function, Eq (2.1.2) is similar in form to the consumption function. The investment function depends on corporate and agricultural disposable income. The business liquid asset term relates invenstment to factors in the money market. Investment also depends on capital which is the accumulation of undepreciated capital as stated in Eq (2.1.16). Eq (2.1.3) links corporate savings to business income from the current and previous year and business suplus from the previous year. Corporate savings are also defined as the change in business suplus in Eq (2.1.17). Eq (2.1.4) relating corporate profits to nonwage nonfarm income is really just an empirical relationship used to close the system. Depreciation naturally depends on the existing capital stock in Eq (2.1.5). The second term in the equation shows that depreciation increases when there is a high degree of capacity utilization.

Eq (2.1.6), (2.1.7), and (2.1.8) model the behavior of the labor market. Eq (2.1.6) is the labor demand function which gives private demand for labor as a function of private sector output. Eq (2.1.7) is the production function which shows how labor and capital combine to produce private output. Eq (2.1.8) is the labor supply function, but is expressed in terms of unemployment. The lagged prices in the equation indicate that wages are slow to change in response to price changes.

Foreign imports in Eq (2.1.9) increase when national disposable income is high and foreign prices are low.

Eqs (2.1.10) and (2.1.11) model the agriculture sector of the economy. Eq (2.1.10) relates farm income to domestic customer prosperity and foreign exports. The ratio between the general price index and the agricultural price index accounts for the terms of trade of agriculture. Eq (2.1.11) relates farm prices to the general price level.

Eqs (2.1.13) through (2.1.15) comprise the money market. Eqs (2.1.12) and (2.1.13) are the household and business demand for money equations. The demand for money has two components: speculative demand which is related to interest rates and prices, and transaction demand which is related to income. Both components appear in Eqs (2.1.12) and (2.1.13). Eqs (2.1.14) and (2.1.15) show the relationship between short term interest rates, long term interest rates, and bank excess reserves. Long term interest rates are merely a weighted average of past period short term interest rates. The percent change in short term interest rates depends on bank excess reserves which are supposedly determined outside the system. Most equilibrium macroeconomic models relate bank excess reserves (which determine the supply of money) to other markets in the economy through prices. The Klein-Goldberger model does not. Goldberger suggests that this is a deficiency in modeling the link between the money market and the commodities market. When the KG model was linked together to make extended period forecasts with excess bank reserves set at a constant level, interest rates increased without bound. This caused investment, consumption, and GNP to diminish to zero. To remedy this deficiency, Goldberger set liquid reserves (L_1 and L_2) equal to a constant for all studies of the dynamic nature of the model. This measure effectively deletes the money market from the model (Goldberger, 1959:84-85). Consequently, interest rates must be ignored and the commodity and labor markets are linked directly through output and prices.

Eqs (2.1.18) through (2.1.21) are identities relating variables in the model. Eqs (2.1.19) and (2.1.21) together are KG model version of the national income accounting identity.

In order to use the Klein-Goldberger model for the current state of the economy, economists must solve the system of simultaneous equations in Table 2.1. The following section shows how econometric models are solved and linked together to provide extended period forecasts. In addition the section discusses how the forecasts are used to compute multipliers.

<u>Solving Econometric Models and Computing Multipliers</u>

Macroeconometric models are classified as linear or nonlinear. Linear models are solved with different methods than nonlinear models. The KG model is an example of a nonlinear model. Nonlinearities appear when current endogenous variables are raised to powers other than one or are multiplied together. Nonlinearities appear in KG model Eqs (2.1.7), (2.1.9), (2.1.10), (2.1.12), (2.1.15), and (2.1.20).

<u>Solving Linear Econometric Models</u>. An example of a linear model is the Klein Model I shown in Table 2.3 below.

Table 2.3. The Klein Model I Linear Econometric Model
(Adapted from Theil, 1971:432-435)

$$C = 16.78+0.02+0.23P_{-1}+0.80(W+W')+e$$

$$I = 17.79+0.23P+0.55P_{-1}-0.15K_{-1}+e'$$

$$W = 1.60+0.42X+0.16X_{-1}+0.13(T-1931)+e''$$

$$X = C+I+G$$

$$P = X-W-T$$

$$K = K_{-1}+I$$

where

C = consumption,

P = profits,

W = wage bill paid by private industry,

I = net investment,

K = capital stock,

X = total production of private industry, and

e, e′, and e′′ = random error terms.

The model predicts current endogenous variable values given current exogenous variables and lagged endogenous values.

Linear models can be solved with matrix algebra. Any linear econometric model can be put into the form

$$Gy + Bz = E$$

where

y = the m element column vector of m endogenous variables,

G = the m x m coefficient matrix with a coefficient for each of m endogenous variables for each of the m equations

z = the n element column vector of n predetermined (lagged endogenous, exogenous, and lagged exogenous) variables,

B = the m x n coeficient matrix with one coefficient for each of n predetermined variables for each of the m equations, and

E = the m element error vector.

For the Klein Model I,

$$y^T = [C, P, W, I, K, X]$$

$$G^T = \begin{bmatrix} 1.00 & -0.02 & -0.80 & 0.00 & 0.00 & 0.00 \\ 0.00 & -0.23 & 0.00 & 1.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & -0.42 \\ 1.00 & 0.00 & 0.00 & 1.00 & 0.00 & -1.00 \\ 0.00 & 1.00 & 1.00 & 0.00 & 0.00 & -1.00 \\ 0.00 & 0.00 & 0.00 & 1.00 & -1.00 & 0.00 \end{bmatrix}$$

$$z^T = [1, P_{-1}, K_{-1}, X_{-1}, W', T, G, t]$$

$$
B_T = \begin{bmatrix} -16.78 & -0.23 & 0.00 & 0.00 & -0.80 & 0.00 & 0.00 & 0.00 \\ -17.79 & -0.55 & 0.15 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\ 249.43 & 0.00 & 0.00 & -0.16 & 0.00 & 0.00 & 0.00 & -0.13 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 0.00 \\ 0.00 & 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix}
$$

and

$$E_T = [e,\ e',\ e'',\ 0.00,\ 0.00,\ 0.00]$$

For the deterministic case, it can be assumed that $E = 0$. Using matrix algebra one can solve for y.

$$y = -G^{-1}Bz \qquad (2.12)$$

Letting $D = -G^{-1}Bz$, Eq (2.12) may be expressed as

$$y = Dz \qquad (2.13)$$

Eq (2.13) gives the unknown endogenous variables in terms of linear functions of known predetermined variables. The m by n D matrix contains constants which, when multiplied by a predetermined variable, give the level of an endogenous variable. These constants are known as multipliers. Multipliers will be discussed more fully later.

The Klein Model I forecasts only the present period from past periods and current policy. It would be useful to forecast a future period based on the current state of the economy and the expected external forces influencing the economy. Figure 2.3 shows how the state of the economy evolves from period to period as influenced by exogenous forces.

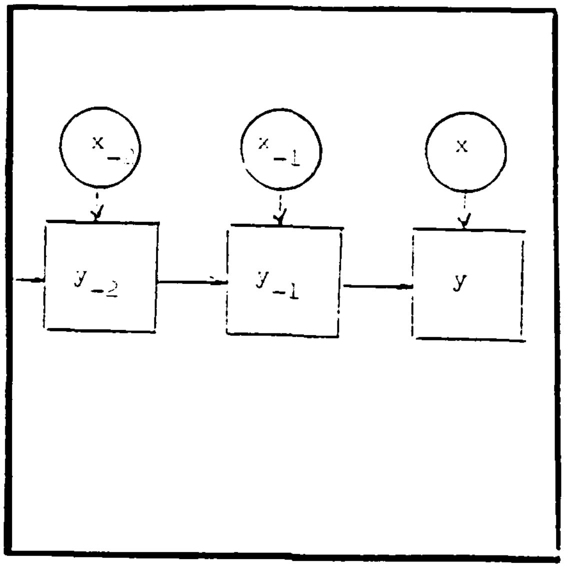

Figure 2.3. The Evolving Economy

The solution for an extended forecast can be found by decomposing Eq (2.13) into its components. For example, in the case where there are only one period lags, Eq (2.13) can be rewritten,

$$y = d_0 + D_1 y_{-1} + D_2 x_0 + D_3 x_{-1}$$

where

y_0 = vector of current endogenous variables to be solved for,

y_{-1} = vector of endogenous variables lagged one year,

x_0 = vector of current exogenous variables,

x_{-1} = vector of exogenous variables lagged one year, and

d_0, D_1, D_2, and D_3 = coefficient matricies.

D_2 contains multipliers which give the impact of current exogenous variables on current endogenous variables and are known as impact multipliers.

It is assumed that x_0 and x_{-1} are determined outside the system by uncontrollable events or by policy. However, y_{-1} is the result of previous period activity. Writing this explicitly,

$$y_0 = d_0+D_1(d_0+D_1y_{-2}+D_2x_{-1}+D_3x_{-2})+D_2x_0+D_3x_{-1}$$
$$= (I+D_1)d_0+D_1^2y_{-2}+D_2x_0+(D_1D_2+D_3)x_{-1}+ D_1D_3 \qquad (2.14)$$

By decomposing the y_{-2} into its components, one can express the current state of the economy in terms of exogenous variables and endogenous variables lagged three periods. The process is repeated to obtain forecasts for any number of periods in the future.

The coefficient matricies for the lagged exogenous variables [e.g.,x_{-1}, x_{-2} in Eq (2.14)] are especially important for policy analysis because they give the impact of exogenous variables, including fiscal and monetary variables. They are different for each lagged period [e.g., D_2, $(D_1D_2+D_3)$ in Eq (2.14)] indicating the changing influence of the exogenous variable over time. The numbers in the D_2 and $D_1D_2+D_3$ matricies are known as interim multipliers.

<u>Solving Nonlinear Econometric Models</u>. Multipliers computed by matrix algebra completely characterize both dynamic and static aspects of a linear system. They give the impact of exogenous changes on the state of the economy at any point in time. Unfortunately, the national economy cannot be accurately described by a linear system. Economic theory prescribes inherently nonlinear functions. For example, nominal endogenous variables are often divided by a price index to obtain real

values. The price index is itself an endogenous variable, and so equations containing nominal values converted to real values are nonlinear. Another example of inherent nonlinearity is production functions. Functions such as the Cobb-Douglas production function and the constant elasticity of substitution production function have proven to characterize real world economics quite well. They are nonlinear.

Solving and analyzing nonlinear systems is not as simple as solving linear systems. Nonlinear systems cannot be solved by simple matrix algebra. Usually some numerical technique must be used. However, if the system is <u>approximately</u> linear, a derivative technique may be used to linearize the system. Goldberger used this technique to linearize the KG model. The derivations which follow are a condensation of Goldberger's work. (Goldberger, 1959:17-20)

A Nonlinear equation in a single y and a single z can be written in the form,

$$f(y,z) = 0$$

The total differential of the function f is also equal to zero.

$$df = (\partial f/\partial y)dy + (\partial f/\partial z)dz$$

Solving for dy,

$$dy = -\frac{(\partial f/\partial z)dz}{(\partial f/\partial y)} \qquad (2.15)$$

Eq (2.15) gives the explicit dependence of changes in y on changes in z. The expression

$$\frac{(\partial f/\partial z)}{(\partial f/\partial y)}$$

may not be a constant, but if sample means are substituted for variables, the expression can be evaluated at a point. If the equation is approximately linear, this constant will be approximately correct for a large range of z. Next, consider the nonlinear system of equations written in the operator form,

$$F(y,z)=0$$

where F is a matrix of functional operators. Taking the total differential and solving for dy,

$$dF = (\partial F/\partial y)dy + (\partial F/\partial z)dz = 0$$

$$-(\partial F/\partial y) = -(\partial F/\partial z)dz$$

$$dy = -(\partial F/\partial y)^{-1}(\partial F/\partial z) \qquad (2.16)$$

Eq (2.16) is like the solution to the linear system in Eq (2.12) except it is expressed in terms of differentials. Also, $\partial F/\partial y$ and $\partial F/\partial z$ are not always constant matricies but can be function matricies. By evaluating these matricies at some value, say at the sample mean of each predetermined variable, these matricies can be converted to constant matricies. The elements of the constant matricies are multipliers for changes in predetermined variables. In general, they are guaranteed to be valid only for small changes about the point at which they are evaluated. However, if the system is approximately linear, the multipliers may be approximately correct over a wide range of values. If so, they may be used in a manner similar to the D matricies computed for linear systems. Extended period forecasts and interim multipliers are computed in a manner analagous to the linear econometric model. Unfortunately, one cannot always count on the econometric model being even approximately linear.

If the model cannot be linearized, then a numerical technique for solving the model is usually used. There are several numerical techniques available including Newton-Raphson, Gauss-Sidel, and others. The method chosen for this study is the Gauss-Sidel. Klein recommended the method over the Newton-Raphson method because although the Newton-Raphson method usually converges in fewer iterations, each iteration requires significantly more computation than each iteration of the Gauss-Sidel method (Klein, 1974:238-240). The Gauss-Sidel method is also easy to program and debug and does not require the computation of a derivative. Appendix A describes the Gauss-Sidel method in detail.

Solving the nonlinear econometric model for current endogenous in terms of predetermined variables produces a forecast of current endogenous variables. To be even more useful, a method must be devised to produce extended period forecasts. Extended period forecasts can be computed from current period forecasts by setting lagged endogenous variables equal to the current endogenous solution, updating the exogenous variables, and resolving the system. For example, after each solution is computed, current values of consumption, investment, etc., are determined. To extend the forecast, the consumption, investment etc., variables lagged one year are set equal to the current solution for consumption, investment, etc. Lagged exogenous variables are updated in a similar manner. Current exogenous variables are set to whatever the policy under investigation requires. Then the model is resolved. In this way, the model can be linked together to obtain forecasts for any number of periods in the future.

Because multipliers are so valuable for policy analysis, a method of computing multipliers for models solved by numerical techniques is needed. Evans and Klein describe a more general method of determining multipliers which can be used with models solved by numerical techniques (Evans and Klein, 1968:48-49). To calculate the multipliers, a controlled solution, y_c, is computed with all predetermined variables (exogenous and lagged endogenous) at a given level, and with the input variable of interest set at, say, x_c. Next, a new solution, y_d, is computed with x at a disturbed level, x_d. The multiplier, m, is then:

$$m = \frac{y_d - y_c}{x_d - x_c} \qquad (2.17)$$

A generalized multiplier such as m can be computed for a "package" of changes in predetermined variables. However, they are valid only for the changes and variable levels used to estimate them. A separate run for each combination of input variable changes must be made to estimate each multiplier.

Limitations of Econometric Models and Multiplier Analysis.

The limitations of econometric models and multiplier analysis are summarized below.

1. Linear econometric models can be constructed which are easily solved and analyzed; however, they do not accurately reflect the national economy in theory or in practice.

2. Near linear models more accurately predict the performance of the national economy, and they may be anlayzed with minor accuracy degadation, but they may not adequately model the inherent nonlinearities of the actual economy.

3. Less aggragated, more nonlinear models may be devised which accurately predict the economy, but such models are difficult to analyze. Multipliers may be calculated by computing control and disturbed solutions and then dividing the difference in these two solutions by the difference between input variables. However these multipliers are good only for small changes about the specific disturbed solution for which they were computed. The model must be rerun for each policy alternative is examined.

Response surface methodology is one way of overcoming some of these difficulties. Response surface methodology accomodates nonlinearities. By fitting a response surface to an econometric model, one could investigate the effects of varying one or more key input variables singly or jointly over their entire ranges. The next chapter describes response surface methodology and explains how it might be applied to econometric models.

III. Applying Response Surface Methodology

Introduction

Chapter I describes what response surface methodology is and how it can be used. This chapter discusses the steps in applying response surface methodology to a problem in general to provide a background for the methodology developed in Chapter IV. This chapter also suggests specific ways to apply response surface methodology to the analysis of econometric models.

Applying RSM can be divided into eleven distinct steps. The steps are

1. Define the problem and determine that RSM is an appropriate analysis technique.

2. Determine the input and output variables of interest.

3. Determine the operating region of interest.

4. Select a response surface equation.

5. Select an experimental design.

6. Translate the coded design points to actual factor levels.

7. Run the experiment or model to obtain responses for each set of factor levels.

8. Regress the coded experimental design on the responses.

9. Check the response equation fit.

10. Decode the response surface coefficients.

11. Perform analysis on the fitted response surface.

The discussion below amplifies each step.

Defining the Problem

The first step in applying RSM is to define the problem and to decide that RSM is an appropriate method for analysis. Not all problems lend themselves to analysis by RSM. RSM relates multiple inputs to a single output. Meyers further points out fundamental assumptions underlying RSM in his text, Response Surface Methodology (Meyers, 1976:62). RSM is appropriate for problems in which the relationship between input variables and output variables is either very complex or unknown, but the variables are quantitative and continuous. Also, the functional relationship between inputs and the response must be approximated by a low order polynomial or other simple function whose parameters are estimable. Finally, the input variables must be controllable and all variables must be measured with negligible error.

Determining Variables of Interest

The second step in applying RSM is to determine the input and output variables of interest. The input variables selected for the analysis must include all the important factors which bear on the problem. Properly defining the problem should make these important factors obvious. However, the size of the experimental design required to estimate response function coefficients increases rapidly with the number of factors (more on this below). All important factors should be included in the response surface equation, but the number of experimental design points required must be considered.

Determining the Operating Region

Once the variables of interest have been selected, their ranges must be specified. The ranges of the input variables must be feasible and independent of one another. In addition, the ranges should be narrow enough so that the response does not contain too many inflection points. Too many inflection points in the response require a complicated response function with higher order terms and a large experimental design to accurately capture the input output relationship.

Selecting the Response Surface Equation

The response surface equation selected in the fourth step is usually a first or second degree polynomial. An example of a first degree polynomial response surface equation with two input variables is

$$y = B_0 + B_1x_1 + B_2x_2$$

and an example of a second degree polynomial response surface equation with two input variables is

$$y = B_0 + B_{11}x_1^2 + B_1x_1 + B_{12}x_1x_2 + B_2x_2 + B_{22}x_2^2$$

where

y = output (response) variable,

x_1, x_2 = input variables, and

B_i = response surface coefficients to be determined

(i = 0, 1, 2.).

There are a number of advantages to using a low order polynomial as a response surface equation. First, the coefficients of a polynomial are estimable by the method of least squares, the most commonly used regression technique. Also, experience has shown that a first or second degree polynomial works well as a response surface function because a polynomial is a truncated form of the Taylor series (Meyers, 1976:62). Another argument for the use of low order polynomials is that many experimental designs have been developed for collecting data to estimate the coefficients of polynomials. However, functions other than polynomials can serve as the response surface equation. Theoretical considerations may dictate the use of a certain type of mathematical function. Such functions may approximate the response very closely and should be used; however, these functions should remain simple so that aspects of the underlying process can be easily explored and interpreted (Hill and Hunter, 1966:573).

Selecting the Experimental Design

The next step in applying RSM is selecting an experimental design. An experimental design is a set of specifications of input variable levels for repeated experimental runs of the process under study. Each combination of input levels is called a design point. Table 3.1 contains an example of a three level three factor experimental design with 27 design points.

The design in Table 3.1 is in coded form. Factor levels are represented by 1, 0, and -1 for three level experimental designs and 1 and -1 for a two level design. For a three level design, a 1 represents the factor high level, a -1 represents a low factor level, and a 0 represents the average of the high and low values. In a two level design the 1 represents the high level, and the -1 represents the low level.

Table 3.1. Three Factor Three Level Factorial Experimental Design

Factor 1	Factor 2	Factor 3
-1	-1	-1
-1	-1	0
-1	-1	1
-1	0	-1
-1	0	0
-1	0	1
-1	1	-1
-1	1	0
-1	1	1
0	-1	-1
0	-1	0
0	-1	1
0	0	-1
0	0	0
0	0	1
0	1	-1
0	1	0
0	1	1
1	-1	-1
1	-1	0
1	-1	1
1	0	-1
1	0	0
1	0	1
1	1	-1
1	1	0
1	1	1

By using an appropriate experimental design, one can estimate the coefficients in the response function with a minimum number of experimental runs. Economy of runs is an important criteria for choosing an experimental design. However, the minimum number of runs required to accurately estimate response surface coefficients depends on the type of response surface equation. Equations with higher powers of variables and interaction terms (products of input variables) require more runs to estimate the unknown coefficients. As noted in the paragraph on selecting variables of interest, the size of the experimental design limits the number of variables in the response surface equation. For example, if k is the number of factors, a three level factorial experimental design requires 3^k experimental runs to estimate the coefficients of main, inteaction, and squared factor effects. A 3^6 factorial design requires 729 experimental runs. If it is known that some coefficients are insignificant, some experimental runs may be eliminated. In addition, some runs may be eliminated at the expense of having some of the variation attributed to the wrong term.

Attributing some of the variation to the wrong term is caused by multicolinearity in the experimental design. Multicolinearity occurs because of correlation between input variables. Ideally, the experimental design should be orthogonal, which means that there is zero correlation between input variables. Most orthogonal designs require numerous design points. Consequently, selecting an experimental design involves a tradeoff between economy of experimental runs and orthogonality. Box and Benkhen devised some three level designs which make a very reasonable tradeoff between orthogonality and economy. These designs can be found in the paper entitled "Some New Three Level Designs" (Box and Behnken, 1960:460-463).

One additional point worth mentioning is that most experimental designs are devised for experiments in which there is random variation in the response due to uncontrollable factors. They contain extra points, usually center points, to estimate the size of this random variation. If the response has no random variation, then these extra points are redundant and may be eliminated. Work in this study assumes that there is no random variation in the Klein-Goldberger econometric model (i.e., it is a deterministic model), and so repetitive center points are not needed.

Decoding the Experimental Design

Before the experiment can be run to obtain data for estimating the response surface, the experimental design must be decoded from 1's, 0's, and -1's to actual factor levels. The range of the variables of interest determines the factor levels for the experiment. For example, if the range of a variable is 10 to 60, then a 1 in the experimental design represents a factor level of 60, a -1 represents 10 and a 0 represents the average of the two factors, i.e., 35.

Running the Experimental Design

To obtain the input and output data necessary to estimate the response surface coefficients, the experiment must be run at the levels specified in the experimental design and the response recorded. This step is straight forward. When running an actual experiment with random variation, it is advisable to randomize the order in which the experiments are run. However, when obtaining data from a deterministic mathematical model, order is unimportant.

Fitting the Response Surface

Once an appropriate experimental design has been selected and the experiment run to collect data, the response surface must be fit to the data. The surface is fit to the data by computing the response surface coefficients. The method of least squares regression is the usual method for fitting a surface to data. This method computes coefficients which minimize the distance from the observed responses to the response surface. For a more thorough discussion of least squares estimation, one can consult a statistics textbook. One excellent source is <u>Mathematical Statistics with Applications</u> by Mendenhall, Scheaffer, and Wackerly

(Mendenhall, Scheaffer, and Wackerly, 1981:425). Response surface coefficients are estimated by regressing the coded experimental design matrix on the response variable. Using the coded design matrix preserves orthogonality.

Checking Fit

Once the response surface has been constructed, it must be checked for proper fit. There are at least four ways to check the fit of the response surface. They include checking the R^2 values, checking the sum of squares error (SSE), checking the residuals of the design points, and checking the residuals of random points. The R^2 value gives the fraction of total variation explained by the response surface equation. R^2 always increases with the number of factors in the response surface equation. SSE gives much the same information as R^2, except it may increase with the number of factors after a certain point. The R^2 and SSE criteria are the easiest and quickest way to check for fit.

Another way to check fit is to examine residuals. By dividing the residual by the actual response value, a measure of the error can be computed. These errors can be averaged for all the design points and then subtracted from one to give a value similar to R^2. Mathematically, this relation is

$$\text{percent fit} = 100\ \% \left[1 - \frac{1}{n} \sum_{i=1}^{n} \left| \frac{y_i - \hat{y}_i}{y_i} \right| \right]$$

where

y_i = macroeconometric model output for input combination i,

$\hat{y}_i$ = response surface output for input combination i, and

n = number input combinations.

It is perhaps even more useful to look at the largest errors to determine where the greatest lack of fit occurs. Residual plots are also helpful in determining where lack of fit occurs.

As a final check of the response surface fit, experiments with random factor levels can be run and the responses compared with responses predicted by the response surface. An error measure similar to the error measure based on residuals can also be computed. The advantage of checking random points is that it may reveal anomalies in the response which were missed by the experimental design.

The four measures of response surface fit mentioned here are used to determine whether or not the response surfce fits well enough for the purpose intended. If the response surface does not fit well enough, a new response surface equation is usually postulated. Steps 4 through 9 are repeated until the fit meets requirements.

<u>Decoding the Response Surface Equation</u>

Once an acceptable response surface is obtained, the response surface equation is decoded. The equation requires decoding because the response equation computed in step eight was computed from coded input variables. To make the response surface equation interpretable it must be expressed in terms of the original variables. Let x_{imax} be the high factor level for the ith factor, x_{imin} be the low factor level for the ith factor level, and x_{iD} be the decoded factor level for the ith factor, and x_{ic} be the coded factor level for the ith factor. By substituting the expression

$$x_{ic} = \frac{x_{iD} - (x_{imax}+x_{imin})/2}{(x_{imax}-x_{imin})/2} \qquad (3.2)$$

in the coded response equation for the ith coded variable and collecting terms, the equation is decoded. Decoding equations in this manner by hand is tedious and prone to error. To make the job easier, a second order polynomial response surface equation can be decoded using matrix algebra. (See Appendix C)

Analyzing the Response Surface

The final step in applying RSM is analyzing the response surface. The methods available for analysis are discussed in detail in the section below. But first, a few prelimiary comments are in order. Obtaining a good response surface equation fit implies that the response surface is an accurate representation of the underlying process or model. More faith can be placed in the validity of the response surface if the check of random points confirms a good fit. However, the response surface is only valid for input variable values within the ranges originally used to estimate the response surface. A response surface fit to a deterministic model may fit well, but the response surface is only as good as the underlying model used to construct it. If the model does not capture the process it is supposed to represent, the response surface will not either.

There are several well developed uses for response surfaces. The most commonly used technique is optimization. The explicit form of the response surface equation giving the response variable as a function of known, controllable input variables lends itself to optimization problems. Moreover, Graney showed how several response surfaces could be combined for constrained optimization problems (Graney, 1984). Another use of response surfaces is for performing "what if" analysis. One can determine the impact of changing input variables individually or jointly without running the model or experiment. In addition, tradeoffs between factors can be displayed to decision makers graphically for valuable insights into the problem. Response surfaces can also be used to make predictions. One must remember that the predictions made are only valid if independent variables not in the response surface equations actually assume the levels that they were at when the response surface was estimated.

Applications for Macroeconometric Models

These uses suggest several applications for macroeconomic models. Multiplier analysis completely captures the input-output relationship of linear models well, but it cannot handle nonlinear models as well. RSM should be able to capture input-output relationships in nonlinear models for key variables easily. Response surfaces describing key economic performance variables such as gross national product, inflation, and unemployment could be used to assess the impact of changing fiscal or monetary policy variables such as government spending, taxes, and money supply. The response surface could be used to answer "what if" questions in policy simulation. Optimal policy for obtaining specific economic goals could be determined. All of this analysis could be done using a limited number of experimental runs. A response surface captures the relationship between variables of interest over the entire region of interest. The controldisturbed solution method of computing multipliers only characterizes how specific changes in input variables affect output variables. Every time a new combination of policies is considered, the model must be rerun and a new multiplier computed.

This chapter describes the general procedures for generating response surfaces and suggests ways of applying RSM to the analysis of macroeconomic models. Chapter IV describes how the general application steps are actually applied to the Klein-Goldberger econometric model in this study.

IV. Methodology

Introduction

This thesis effort proposes to bring together concepts from two areas, response surface methodology and macroeconometric modeling. Chapter I lists the specific objectives to be met by this research effort. They include determining whether RSM can accurately fit a macroeconometric model, reproducing multipliers for the macroeconometric model via RSM, interpreting the response surface equations, and developing RSM applications for the response surface equations. This chapter develops the methodology by which these objectives are achieved.

To accomplish the research objectives, a scheme must be developed for generating and checking response surface equations from the Klein-Goldberger model. The scheme used here follows the general steps for applying RSM to any problem as discussed in Chapter III. This chapter discusses each step in detail.

Generating and Checking Response Surfaces

Defining the Problem and Selecting Variables of Interest. Determining which variables to use in the response function depends on the specific purpose for which the response surface is to be used. Because applications developed are geared toward determining the best economic policy for the federal government, the exogenous variables selected for manipulation are corporate taxes (T_c), wage taxes (T_w), government nonwage spending (G), government wage bill (W_2), and number of government employees (N_G). These variables are instruments of federal policy which are broken out in the KG model. It is also desirable to see how the economy can be manipulated through monetary policy as well.

Unfortunately, Goldberger admitted that while the KG model did an adequate job of forecasting current monetary variables (such as short term interest rates, long term interest rates and liquid assets), it did a poor job of simulating economic changes further in the future (Goldberger, 1959:84-85). Goldberger claimed that short term interest rates would increase without bound. In addition, the model failed to capture the relationships between the model's monetary sector and the other sectors adequately. Consequently, the monetary sector of the model is omitted from further consideration.

There are several possible endogenous variables for which response surfaces could be built. Economic performance indicators commonly used to asses the health of the national economy include percent growth in gross national product, percent unemployment, and percent inflation. Other measures are interest rates and federal deficits. The KG model does not include federal deficits. Because of limitations mentioned in the preceding paragraph, the model cannot be used to study the dynamic properties of interest rates. In this study, response surfaces are built for gross national product (Q), total number of workers employed (N_w), and the price index (p).

Selecting the Response Surface Equation. Once the variables of interest have been selected, the next step, selection of the response surface equation can be accomplished. Because of the arguments enumerated in Chapter III, a second order polynomial has been selected as the response surface equation. Since Goldberger argued that the model was nearly linear (Goldberger, 1959:136-138), a second order polynomial should have no difficulty approximating model ouputs. In addition, if there are any significant interactions between variables or second order effects, the second order equation will pick them up. If the second order polynomial does not fit well enough, a higher order polynomial can be used.

Because one of the objectives of the study is to reproduce multipliers, the first order polynomial is also of interest. It can be shown that coefficients computed for a first order response surface for linear econometric models are mathematically equivalent to the multipliers derived in Chapter II (see Appendix B). Because the KG model is nearly linear, the coefficients of the variables in the first order decoded response surface should compare quite closely with multipliers computed by Goldberger.

Selecting the Experimental Design. Selecting the variables of interest and the form of the response equation narrows the choices of experimental designs. A three level full factorial design with five factors would require $3^5 = 243$ runs of the KG model just to build response functions for period zero. For each succeeding period, another set of 243 runs would have to be made. A more economical design is found in Box and Behnken's 1960 paper entitled, "Some New Three Level Designs for the Study of Quantitative Variables" (Box and Behnken, 1960:460). Box and Behnken's five factor design has only 46 design points. Of these, five are redundant center points which can be eliminated for a deterministic model. The design is highly orthogonal with only a slight correlation between the squared and intercept terms. The Box and Behnken design works especially well for variables with small variance. Since the K-G model is deterministic without error terms, the output has no variance. Consequently, this design is especially appropriate. The

Box and Behnken design is meant for use with a second degree polynomial, but the design can be used for the first order model because the main effects are uncorrelated. Using a three level design to estimate a first order model has the added advantage of enabling assessment of second order effects not accounted for in the first order model. If other models with terms higher than second degree are used as response equations, another design must be selected. Appendix D shows the five factor Box and Behnken design with extra center points deleted.

Determining Range of Independent Variables. The next step in RSM application is to determine the range of the independent variables. In this study, several different ranges are appropriate. To check whether a second order response function can fit the model output, the entire range of data for the years over which the model was estimated is appropriate. For those exogenous variables in the response surface equation, the maximum, minimum, and average of maximum and minimum sample values comprise the three levels used in model runs. Variables not in the response equation are set at the sample mean. The objective of these runs is to get the best fit possible.

Goldberger's multipliers were computed at the sample mean for unit changes in the predetermined variables. For runs reproducing multipliers, the sample mean plus or minus one unit are used as factor levels.

For runs used to evaluate specific policies, response surfaces should be constructed for exogenous variable levels which are considered politically feasible by the decision makers using the analysis. All predetermined variables not included in the response surface equation should be set to current or forecast values so that predicted values for endogenous variables will be reasonable.

Decoding the Experimental Design. Once the factor levels have been specified, the coded experimental design must be decoded. The decoded design specifies the actual exogenous variable values for which a solution is to be computed. To reduce drugery, save time, and decrease arithmetic errors, a FORTRAN program was developed to automate the task. The program reads a coded design file and writes a decoded version of the experimental design to a new file. The actual code for this program is in Appendix D.

Completing Model Runs. Creating a file with factor levels specified is one prerequisite for the next step in developing a response surface, solving the model for each design point. Also needed is a file specifying the values of the other predetermined variables. Once these are specified, the model becomes a system of twenty-one equations in twenty-one unknown endogenous variables. Since the system is large and nonlinear, a numerical approach to solving the system is used. Because of the arguments set forth in Chapter III regarding the best method to solve a maroeconometric model, the Gauss-Sidel method is selected as the algorithm for solving the KG model. Appendix A discusses the mathematical aspects of the method and contains the FORTRAN implementation of the method as applied to the KG model. The program reads data from five data files including the coded design file, the decoded design file, a file containing predetermined variable values not included in the experimental design, an initial trial solution file, and a file containing control language for the program. The program has the capability to link solutions together to produce extended period forecasts. The program produces a data file which includes the coded experimental design, a sequential case number for each design point, the period number for each solution, and the solution for all twenty-one endogenous variables. This output file may be read directly by the BMDP statistical package.

Fitting the Data. Once the data sets are generated, response functions are fit using BMDP's stepwise regression routine. The BMDP package is used for this research project because it has all the capabilities needed for fitting response surfaces, is familiar to the researcher, and is available to the researcher. Stepwise regression is used because it brings in independent variables one at a time in order of influence on the dependent variable.

The second order response surface equations have higher order terms. To estimate the coefficients for these terms, appropriate transformations are made in BMDP's control language. One point to reiterate is that regressions are made in terms of the coded variables to preserve orthogonality. The coefficients computed by BMDP must therefore be decoded (except in the case where multipliers are reproduced). Before decoding the coefficients, however, it is convenient to check the fit of the response surface.

Checking the Fit. The four methods of checking the response function fit, R^2, SSE, residuals, and random points are all useful for this study. The R^2 and SSE values are given automatically for each step in the stepwise regression procedure. The R^2 value is the primary criteria for deciding which variables to keep in the response surface equation used in this study. However the SSE is checked to insure that it is not increasing as more variables are brought into the response surface equation. Variables which reduce the R^2 negilgibly (less than 0.0001) can be omitted from the response surface equation.

While the R^2 value gives a good overall average measure of fit, it is also useful to examine the percent deviation of the response surface equation from the KG model solution for each design point. This quantity is computed from the residuals by dividing the residual by the actual response value and multiplying by 100 percent. Of interest are the largest percent deviations and where those deviations occur.

Finally, the most stringent test of response surface fit is the percent deviation between the model responses and response surface responses for random points. If a coded random experimental design with random values on the interval (-1,1) for design points is created, the random design can be treated just like a regular experimental design. This random design can be used to compute new data points. The resulting data file can then be appended to the data used to fit the response surface. If the random points are given a weight of zero in BMDP, residuals are computed for the random points, but the points are not used to compute response surface coefficients. The percent error is computed from the random point residuals to provide another assessment of response function fit.

If the response surface fits well, (within 98 percent) the coefficients may be decoded and analysis can begin. If not, a new response surface function and experimental design must be selected and the fitting procedure repeated until a satisfactory fit is obtained. Decoding the response surface coefficients is simply a straight application of the procedure discusssed in Chapter III.

Once response surfaces have been fit satisfactorily and decoded, the response surface equations must be interpreted. Some important questions about the response surface equations follow.

1. What does the response function imply about the relative contribution of each input and the relationship between inputs? Do these implications make economic sense?

2. If there are significant higher order or interaction terms, why do these occur? Can economic theory explain?

3. How do the response surface coefficients compare with multipliers?

4. The K-G model is composed of mostly linear equations with some products of input variables. How does this affect the response function?

If the response surface equations appear valid, applications for the equations may be developed. Chapter III suggests several uses for response surfaces fit to a macroeconomic model. They include policy simulation, trade off analysis, and optimization. Describing the details of these applications is deferred until Chapter VI.

The methodology outlined in this chapter describes what steps must be taken to meet the research objectives set forth in Chapter I. The Gauss-Sidel numerical method of solving simultaneous nonlinear equations is implemented in a computer program to solve the KG model. Second order polynomial response surfaces are built for important economic indicators to asses how well response surface can fit the macroeconomic model. A first order response surface is estimated and the coefficients compared to Goldberger's multipliers. Finally, the response surfaces are interpreted and applied.

V. Results

Introduction

Chapter IV described what data are required to achieve the research objectives and outlined analysis to be performed with the data. Computer runs were made on the Air Force Institute of Technology's VAX 11/780 computer to collect required data. Appendix A contains the actual FORTRAN code used to obtain the data. This chapter summarizes results of the regression and comments on significant aspects of the results. It also discusses research objectives one and two in light of the results.

Second Order Model Fit

The first research objective is to see how well a second degree polynomial can approximate the output of the KG model when five factors are changed jointly. To satisfy this objective, the KG model is solved for period zero and period five at factor levels required by the Box and Bennken experimental design. (For a discussion of the Gauss-Sidel numerical technique used to solve the Klein-Goldberger model, see Appendix A.) Second order polynomial coefficients are estimated for number of workers employed (N_w), price index (p), and gross national product (Q) in terms of wage taxes (T_w), corporate taxes (T_c), government nonwage spending (G), government wage bill (W_2), and number of government workers (N_G) using the BMDP 2R program, stepwise regression. The general form of each equations is

$$Q = a_{00}+a_{01}T_c+a_{02}T_w+a_{03}G+a_{04}W_2+a_{05}N_G+a_{12}T_cT_w+a_{13}T_cG+a_{14}T_cW_2$$
$$+a_{15}T_cN_G+a_{23}T_wG+a_{24}T_wW_2+a_{25}T_wN_G+a_{34}GW_2+a_{35}GN_G+a_{45}W_2N_G+a_{11}T_c^2$$
$$+a_{22}T_w^2+a_{33}G^2+a_{44}W_2^2+a_{55}N_G^2$$

$$N_w = b_{00}+b_{01}T_c+b_{02}T_w+b_{03}G+b_{04}W_2+b_{05}N_G+b_{12}T_cT_w+b_{13}T_cG+b_{14}T_cW_2$$
$$+b_{15}T_cN_G+b_{23}T_wG+b_{24}T_wW_2+b_{25}T_wN_G+b_{34}GW_2+b_{35}GN_G+b_{45}W_2N_G+b_{11}T_c^2$$
$$+b_{22}T_w^2+b_{33}G^2+b_{44}W_2^2+b_{55}N_G^2$$

$$p = c_{00}+c_{01}T_c+c_{02}T_w+c_{03}G+c_{04}W_2+c_{05}N_G+c_{12}T_cT_w+c_{13}T_cG+c_{14}T_cW_2$$
$$+c_{15}T_cN_G+c_{23}T_{Gw}G+c_{24}T_wW_2+c_{25}T_wN_G+c_{34}GW_2+c_{35}GN_G+c_{45}W_2N_G+c_{11}T_c^2$$
$$+c_{22}T_w^2+c_{33}G^2+c_{44}W_2^2+c_{55}N_G^2$$

where a_{ij}, b_{ij}, and c_{ij} are the coeffecients to be determined. The following conditions are applied in estimating the coefficients.

1. The three factor levels for corporate taxes (T_c), wage taxes (T_w), government nonwage spending (G), government wage bill (W_2), and number of government employees (N_G) are the maximum sample value, the minimum sample value, and the average of the maximum and minimum sample values.

2. The Box and Behnken five factor three level design discussed in the methodology chapter with redundant center points deleted is used.

3. All other predetermined variables are set at sample mean values. In computing sample means for lagged variables, the appropriate data values from the periods 1923-1951 are used. (e.g., the sample mean for the price index lagged one year includes the price indicies for 1928 and 1944, but excludes the price indicies from 1940 and 1952.)

4. For each design point, all current (nonlagged) exogenous variables are held fixed for extended period forecasts (beyond period zero). Lagged variables are updated with new values after each period's forecasts are computed.

5. The monetary sector is suppressed by excluding the liquidity forecasting equations, Eqs (2.2.12) and (2.2.13). This step is necessary to match Goldberger's analysis.

6. The time trend variable is updated by one each year.

Tables 5.1a-f summarize the results of stepwise regression for each response function. They include step number, entering variable, multiple R and R^2, and change in R^2.

Table 5.1a. Summary Table of Stepwise Regression Results for Number of Workers Employed in Period Zero

Step No.	Variable Entered	Multiple R	Multiple R^2	Change in R^2
1	3 G	.9091	.8264	.8264
2	5 N_G	.9681	.9372	.1108
3	1 T_w	.9843	.9689	.0317
4	4 W_2	.9993	.9985	.0296
5	2 TC	1.0000	1.0000	.0015
6	41 GW_2	1.0000	1.0000	.0000
7	31 G^2	1.0000	1.0000	.0000
8	42 GN_G	1.0000	1.0000	.0000
9	35 T_wG	1.0000	1.0000	.0000
10	36 T_wW_2	1.0000	1.0000	.0000
11	38 T_cG	1.0000	1.0000	.0000

Table 5.1b. Summary Table of Stepwise Regression Results for the Price Index in Period Zero

Step No.	Variable Entered	Multiple R	Multiple R^2	Change in R^2
1	3 G	.6919	.4787	.4787
2	4 W_2	.9107	.8294	.3507
3	5 N_G	.9887	.9775	.1481
4	1 T_w	.9979	.9959	.0184
5	32 $W_2{}^2$	.9991	.9981	.0023
6	2 T_c	.9995	.9990	.0008
7	43 W_2N_G	.9997	.9994	.0004
8	31 G^2	.9999	.9998	.0004
9	41 GW_2	1.0000	.9999	.0002
10	35 T_wG	1.0000	1.0000	.0000
11	33 $N_G{}^2$	1.0000	1.0000	.0000
12	42 GN_G	1.0000	1.0000	.0000
13	36 T_wW_2	1.0000	1.0000	.0000
14	38 T_cG	1.0000	1.0000	.0000

Table 5.1c. Summary Table of Stepwise Regression Results for Gross National Product in Period Zero

Step No.	Variable Entered	Multiple R	R^2	Change in R^2
1	3 G	.9738	.9484	.9484
2	1 T_w	.9923	.9847	.0363
3	4 W_2	.9992	.9983	.0136
4	2 T_c	1.0000	1.0000	.0017
5	5 N_G	1.0000	1.0000	.0000
6	41 GW_2	1.0000	1.0000	.0000
7	31 G^2	1.0000	1.0000	.0000
8	42 GN_G	1.0000	1.0000	.0000
9	35 T_wG	1.0000	1.0000	.0000
10	36 T_wW_2	1.0000	1.0000	.0000
11	38 T_cG	1.0000	1.0000	.0000
12	37 T_wN_G	1.0000	1.0000	.0000
13	30 T_c^2	1.0000	1.0000	.0000

Table 5.1d. Summary Table of Stepwise Regression Results for Number of Workers Employed in Period Five

Step No.	Variable Entered	Multiple R	R^2	Change in R^2
1	3 G	.9286	.8622	.8622
2	2 T_c	.9605	.9226	.0604
3	1 T_w	.9903	.9807	.0581
4	5 N_G	.9951	.9903	.0096
5	4 W_2	.9990	.9981	.0078
6	31 G^2	.9996	.9993	.0012
7	35 T_wG	.9997	.9995	.0002
8	38 T_cG	.9998	.9997	.0002
9	41 GW_2	.9999	.9998	.0001
10	42 GN_G	1.0000	.9999	.0001
11	36 T_wW_2	1.0000	.9999	.0000
12	39 T_cW_2	1.0000	.9999	.0000
13	34 T_wT_c	1.0000	1.0000	.0000
14	40 T_cN_G	1.0000	1.0000	.0000

Table 5.1e. Summary Table of Stepwise Regression Results for the Price Index in Period Five

Step No.	Variable Entered	Multiple R	R^2	Change in R^2
1	3 G	.8852	.7836	.7836
2	4 W_2	.9268	.8590	.0754
3	5 N_G	.9554	.9128	.0538
4	1 T_w	.9806	.9617	.0489
5	2 T_c	.9991	.9982	.0365
6	32 W_2^2	.9993	.9986	.0004
7	43 W_2N_G	.9995	.9990	.0004
8	41 GW_2	.9997	.9994	.0004
9	42 GN_G	.9998	.9996	.0002
10	33 N_G^2	.9999	.9997	.0001
11	38 T_cG	.9999	.9998	.0001

Table 5.1f. Summary Table of Stepwise Regression Results for Gross National Product in Period Five.

Step No.	Variable Entered	Multiple R	R^2	Change in R^2
1	3 G	.9305	.8658	.8658
2	2 T_c	.9701	.9411	.0753
3	1 T_w	.9990	.9980	.0569
4	31 G^2	.9995	.9990	.0010
5	5 N_G	.9997	.9993	.0003
6	35 T_wG	.9997	.9995	.0002
7	38 T_cG	.9998	.9996	.0001
8	41 GW_2	.9999	.9998	.0002
9	4 W_2	.9999	.9999	.0001
10	42 GN_G	1.0000	.9999	.0000
11	36 T_wW_2	1.0000	.9999	.0000
12	39 T_cW_2	1.0000	1.0000	.0000
13	34 T_wT_c	1.0000	1.0000	.0000
14	40 T_cN_G	1.0000	1.0000	.0000
15	37 T_wN_G	1.0000	1.0000	.0000
16	43 W_2N_G	1.0000	1.0000	.0000

The data in the Tables 5.1a-f yield two important conclusions. First, for both period zero and period five, the response surfaces fit the data well as reflected by the R^2 column. In no case does the R^2 exceed 0.9983. A second check of fit is the percent error for the design points. Table 5.2 lists the design points with the largest percent error for each response surface as computed from the residuals.

Table 5.2. Design Point Fit Check for
Second Order Response Surface.

	Resid-ual	Predict Value	Pct Error	Factor Levels T_w	T_c	G	W_2	N_G
Period Zero								
N_w	-0.0041	42.69	0.0097	1.0000	-1.0000	0.0000	0.0000	0.0000
p	0.1631	95.00	0.1717	0.0000	0.0000	-1.0000	1.0000	0.0000
Q	-0.0059	88.30	0.0067	1.0000	0.0000	-1.0000	0.0000	0.0000
Period Five								
N_w	-0.1745	14.06	1.2567	1.0000	0.0000	-1.0000	0.0000	0.0000
p	3.1580	35.53	8.1627	0.0000	0.0000	-1.0000	1.0000	0.0000
Q	-0.4030	46.91	0.8140	1.0000	0.0000	-1.0000	0.0000	0.0000

The data in Table 5.2 imply good fit. With the exception of price index in period five, design point error is less than 1.3 percent. Price index in period five has a larger percent error of 8.16 percent. Fifty random points in the operating region were also run to check fit. The points with the largest percent error for each surface are shown in Table 5.3. The lack of fit is extremely pronounced for period five price index (26.22 percent).

Table 5.3. Random Point Fit Check for
Second Order Response Surface

	Resid-ual	Predict Value	Pct Error	Factor Levels T_w	T_c	G	W_2	N_G
Period Zero								
N_w	-0.0053	19.38	0.0116	0.9154	-0.8486	-0.2112	-0.8846	0.8269
p	-0.2690	111.7	0.2291	0.1516	0.9346	-0.8707	-0.8653	-0.8905
Q	0.1082	91.54	0.0118	0.1516	0.9346	-0.8707	-0.8653	-0.8905
Period Five								
N_w	0.8136	19.38	4.0290	0.8024	-0.7953	-0.9008	0.9026	-0.7324
p	6.2620	17.62	26.2206	0.2778	0.1031	-0.9972	0.8670	-0.4157
Q	2.0090	74.96	2.6101	0.8024	-0.7953	-0.9008	0.9026	-0.7324

The large percent errors are evidence that the response surface does not fit the price index response for the entire operating region specified. Of all the response surfaces, one might expect the response surface for p to be the most difficult to fit. Of the six nonlinear equations in the KG model, p appears multiplied with other endogenous variables in Eqs (2.1.9), (2.1.10), and (2.1.20). If one were to solve for p in terms of the other endogenous variables, an endogenous variable would be in the denominator. Perhaps a higher order polynomial or logrithmic function can provide a closer approximation for the p response surface. Although the response surface theoretically should fit the response throughout the whole region, it is interesting to note that large errors for p occured at small values of p. The design points and the random points with the largest percent error also had the smallest values for p. In fact, for the point with the largest percent error, the value for p was 23.88, but the smallest sample value for p from the 1929-1952 data was 90.7. In addition, other values of exogenous variables were extremely far removed from the sample data for this case. It seems unlikely that real world analysis would be conducted in this region of the response surface. The largest percent error for any design or random point with a p value over 90.0 was 0.83 percent indicating a good fit in the range of real world response.

Apparently, the output of the KG model can indeed be approximated by a low order polynomial. To be absolutely certain on this point, response surfaces would have to be built for all endogenous variables which included all predetermined variables for all periods. For practical applications, however, if closely fitting response surfaces can be built for the endogenous variables of interest which include the predetermined variables of interest and which cover the time frame of interest, this is all that is necessary to proceed with analysis. Furthermore, there is no reason to believe that other closely fitting response functions cannot be developed for any endogenous variables in terms of any predetermined variables.

The second major conclusion to be drawn from Tables 5.1a-f is that first order terms account for most of the variation in the data. Table 5.4 lists the percent of variation explained by first order terms for each response surface equation. These values were obtained by fitting a first order model to a second order Box and Behnken experimental design (Box and Behnken, 1960:460)

Table 5.4. R^2 Values for The First Order Response Surface Equation.

	Period Zero	Period Five
N_w	1.0000	0.9981
p	0.9967	0.9982
Q	1.0000	0.9985

The high R^2 values indicate that the model is very nearly linear. One might expect price index to show significant nonlinearity. Of the six equations in the KG model which contain nonlinearities, price is involved in three. Although the response function for price index in periods zero and five are more nonlinear than either number of employed workers or gross national product, 99.7 percent of the variation is explained by linear terms. These observations are consistent with Goldberger's argument that the model is very nearly linear and that multipliers that he computed at the sample mean are accurate for a large range of time series data.

Multipliers and Response Surface Coefficients Compared

The second research objective is to reproduce Goldberger's multipliers using a first order response surface equation. To do this, runs are made with the following conditions:

1. The three factor levels of corporate taxes, wage taxes, government nonwage spending, government wage bill, and number of government employees are the sample means plus or minus one unit.

2. The experimental design used is the Box and Behnken five factor three level design discussed in the methodology chapter with redundant center points deleted.

3. All other predetermined variables are set at sample mean values. For lagged variables, the appropriate data values from before the periods 1929-1940, and 1945-1952 are used. (e.g., the sample mean for the price index lagged one year includes the price indicies for 1928 and 1944 but excludes the price indicies from 1940 and 1952.)

4. For each design point, all current (nonlagged) variables are held fixed for extended period forecasts (beyond period zero). Lagged variables are updated with new values after each period's forecasts are computed.

5. The monetary sector is suppressed by excluding the liquidity forecasting equations. This step was necessary to match *Goldberger*'s analysis.

6. The time trend variable is not updated since Goldberger computes a separate multiplier to account for the time trend.

The conditions were applied to correspond to the assumptions made by Goldberger in developing his multipliers. Tables 5.5a and 5.5b summarize the results. They show multipliers computed by Goldberger for a unit increase in government spending and the corresponding response surface equation coefficients. Multipliers are taken from Table 5.2 of Impact Multipliers and Dynamic Properties of the Klein-Goldberger Model (Goldberger, 1959:88).

Table 5.5a. Multipliers for Unit Increase in Government Spending.

	Period 0	1	2	3	4	5
N_w	0.611	1.214	1.628	1.842	1.899	1.835
p	1.500	3.134	4.631	5.911	7.043	8.023
Q	1.386	2.807	3.884	4.565	4.887	4.992

Table 5.5b. Response Surface Coefficients for a Unit Increase in Government Spending.

	Period 0	1	2	3	4	5
N_w	0.611	1.214	1.621	1.829	1.862	1.772
p	1.505	3.147	4.673	6.0734	7.302	8.312
Q	1.385	2.804	3.871	4.521	4.795	4.766

The two sets of numbers compare quite closely for period zero, but diverge somewhat for extended period forecasts. Goldberger used the linearized model to generate interim multipliers. Since the model does have some nonlinearity, the linearized model used by Goldberger would tend to accumulate error as each subsequent solution is computed based on previous approximate solutions.

The results presented in this chapter satisfy research objectives one and two. The data indicate that the KG model can be approximated by a low order polynomial. Furthermore, linear response surface coefficients are approximately equivalent to multipliers computed by linearizing the model. Further analysis requires that the coded response surface coefficients be decoded. Appendix E contains tables listing coded and decoded response surface coefficients of the first and second order response surfaces fit in this chapter. The next chapter addresses the final two research objectives. The chapter includes coefficients for any response surface used in analysis.

VI. Response Surface Analysis

Introduction

Chapter V demonstrates that a response surface can indeed fit the output of the KG model with a low order polynomial with a high degree of accuracy. Further, RSM verifies that the model is very nearly linear with first order terms accounting for over 99 percent of the total variation for all response surfaces generated. Since these surfaces fit so well, one may conclude that they are accurate representations of the model's characteristics and may be used as an approximation to the model for analysis. This chapter examines what the response surfaces mean and explores some analysis possibilities emphasizing practical applications.

Response Surface Interpretation

The response surfaces generated for the Klein-Goldberger Model summarize relationships in the model presenting the impacts of predetermined variable changes on current endogenous variables explicitly. An example serves to illustrate. For period five, the decoded response surface equation for gross national product (Q) in terms of wage taxes (T_w), corporate taxes (T_c), government nonwage spending (G), government wage bill (W_2), and number of government employees (N_G) is

$$Q_5 = 54.4254 - 4.0951T_w - 4.3058T_c + 5.0849G - 0.5864W_2 - 0.4145N_G - 0.0134G^2 + 0.0244T_wG + 0.0226T_cG + 0.0175GW_2 \quad (6.1)$$

The coefficients in Eq (6.1) are computed by stepwise regression. It is assumed that the accuracy afforded by Eq (6.1) is sufficient for purposes of discussion. The R^2 value for this response surface is 0.9998.

What is the significance of Eq (6.1)? First, the equation gives what the KG model prediction will be for any combination of T_w, T_c, G, W_2, and N_G in the operating range of interest. The equation does not claim to make any predictions about the economy that the original model could not make. The response surface is only as good as the underlying model. The response surface also only purports to characterize the gross national product in terms of the predetermined variables in Eq (6.1) with other predetermined variables at their sample means. There very well could be interactions between variables not included in Eq (6.1) (e.g., gross national product, investment, and prices from the previous period) and the variables appearing in Eq (6.1) (i.e., T_w, T_c, G, W_2, and N_G). For analysis using a response surface, the predetermined variables not included in the response surface equation should be set at values close to what they would be for the particular economic simulation under study. For instance, if a study is to be made of the effects of government spending and taxes on gross national product two years in the future, then the macroeconomic model used to generate the response surface should have lagged endogenous and exogenous variables set at their appropriate current levels or what they are expected to be. What Eq (6.1) does give is the relationship between T_w, T_c, G, W_2, and N_G, and Q in the Klein-Goldberger model in the operating region of interest for a period five years in the future with all other predetermined variables at sample means.

Eq (6.1) contains interaction and squared terms. These terms suggest that the change in Q due to a change in a particular predetermined variable is dependent on its own or another variable's level. For example, one may want to know the effect of increasing corporate taxes on gross national product five years in the future. Taking the first partial derivative of Eq (6.1) with respect to T_c yields an expression relating the change in Q to a change in T_c.

$$\frac{\partial Q}{\partial T_c} = -4.3058 + 0.0226G \qquad (6.2)$$

The right hand side of Eq (6.2) is a nonconstant "multiplier". Eq (6.2) suggests that the change in Q due to a change in T_c is dependent on the level of G as shown in Figure 6.1. It is important to note that Eq (6.2) is valid only for the ranges of T_c and G used to build the response surface (T_c ranges from $ 0.40 to $ 11.88 billion and G ranges from $ 11.5 billion to $ 41.7 billion).

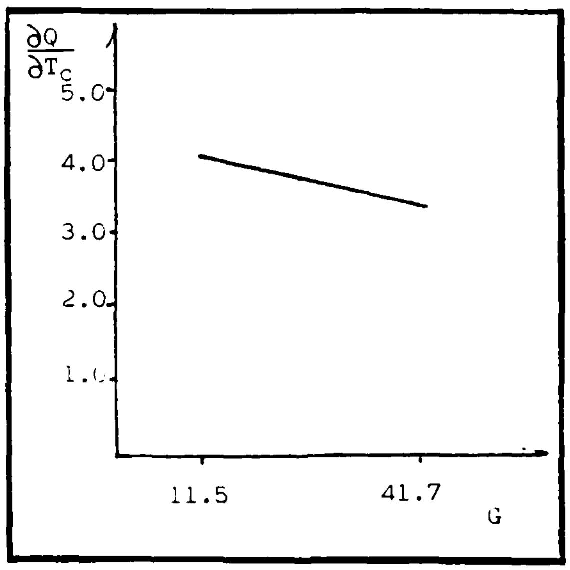

Figure 6.1. Dependence of $\partial Q/\partial T_c$ on G in Five Years

It is of interest to determine why the change in Q due to a change in T_c should depend on G. Corporate taxes directly effect gross national product through consumption and investment in Eqs (2.1.1), (2.1.2), (2.1.19), and (2.1.21). However, corporate taxes also affect another element of gross national product in Eq (2.1.9), foreign imports. It is important to note that it is not corporate taxes alone that affect foreign investment, but the product of the price index and corporate taxes. Economic theory asserts that government spending has a strong effect on prices. Consequently, there is an interaction between corporate taxes and government spending in determining foreign imports and hence gross national product. Eq (2.1.10), which models the determinants of farm income, also has a similar interaction between corporate taxes and prices. One way to visualize the magnitude of the T_cG interaction term is to plot Q versus T_c at different levels of G. Figure 6.2 shows Q, at the five year point, as a function of T_c for three levels of G. The T_cG term causes a change in slope at different G levels. The change in slope is barely discernable.

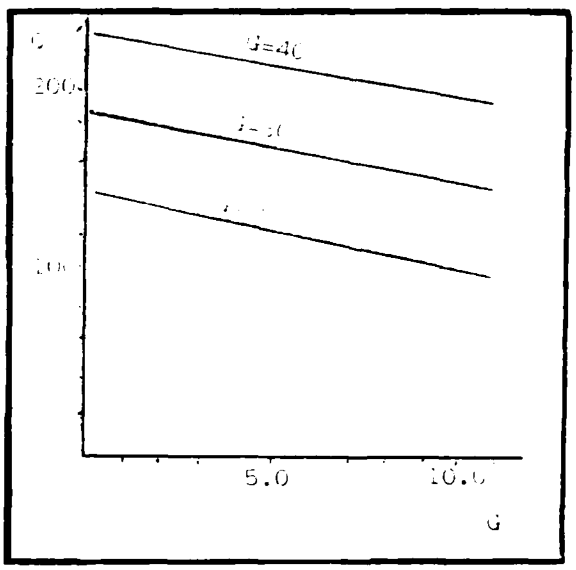

Figure 6.2. Relationship Between Q_5 and T_c at Different Levels of G in Period Five.

To generate Figure 6.2, all predetermined variables except T_c and G are set at sample means.

Computing Multipliers

Chapter Five shows the close correspondence between multipliers and decoded first order response surface coefficients. In fact, Appendix B shows that they are equivalent for linear systems. Since both RSM and the derivative method yield essentially the same numerical values for multipliers, Goldberger's extensive analysis applies to RSM derived multipliers as well.

If the KG model were more nonlinear, interaction and squared terms would become more significant. It is here that response surface methodology provides an advantage over multiplier analysis. By using response surfaces, one can detect interactions between predetermined variables as noted in the last section. To generate "multipliers" from response surface equations, one computes the partial derivative of the response surface equation with respect to the variable of interest. The last section computed a multiplier for changes in Q due to changes in T_c. This multiplier together with other multipliers computed from Eq (6.1) are listed below.

$$\frac{\partial Q}{\partial T_c} = -4.3058 + 0.0226G \qquad (6.2)$$

$$\frac{\partial Q}{\partial T_w} = -4.0951 + 0.0224G \qquad (6.3)$$

$$\frac{\partial Q}{\partial G} = 5.0894 + 0.0244T_w + 0.0226T_c - 0.0268G + 0.0175W_2 \qquad (6.4)$$

$$\frac{\partial Q}{\partial W_2} = -0.5847 + 0.0175G \qquad (6.5)$$

$$\frac{\partial Q}{\partial N_G} = -0.4145 \tag{6.6}$$

Eqs (6.2) through (6.6) give "function" multipliers which capture the relationship between Q and T_w, T_c, G, W_2, and N_G more accurately than traditional multipliers.

Policy Simulation

Using the multipliers computed above, economists can answer "what if" questions easily. For instance, if an economist wants to know the impact on gross national product in five years of increasing government spending by five billion dollars and paying for it with a five billion dollar increase in wage taxes, he can use the multipliers to forecast the answer. Assuming, for illustration purposes, that

T_w = \$8 billion

T_c = \$10 billion

G = \$40 billion

W_2 = \$16 billion

and all other predetermined variables are at sample means, then the multiplier relating changes in T_w to changes in Q is

$$-4.0951 + 0.0244(40) = -3.1191$$

from Eq (6.3). The multiplier relating changes in G to changes in Q is

$$5.0894 + 0.0244(8) + 0.0226(10) - 0.0268(40) + 0.0175(16) = 4.7186$$

from Eq (6.4). The assumed values for T_w, T_c, G, and W_2 are close to 1952 sample values from the data used to estimate the model (Klein and Goldberger, 1955:131-132) and all other predetermined values are at sample means. A five billion dollar increase in wage taxes changes gross national product by ($5 billion)(-3.1191) = -$15.60 billion. A five billion dollar increase in government spending increases gross national product by ($5 billion)(4.7186) = $23.59 billion. The net change is $23.59 billion - $15.60 billion = $7.99 billion.

It is interesting to compare the multiplier computed above to Goldberger's multipliers and the corresponding first order response surface coefficients (see Table 5.5). As an example, Table 6.1 compares the three types of multipliers for changes Q due to changes in G in period 5.

Table 6.1. Multiplier Comparisons for Changes in Q Due to Changes in G in Period 5.

Goldberger's Multiplier	4.922
First Order Response Surface Multiplier	4.766
Second Order Response Surface Multiplier	4.718

The difference shown between Goldberger's multiplier and the first order response surface coefficient is the accumulated error from the way in which Goldberger linearized the model. The difference between the first order response surface coefficient and the second order multiplier is that the first order multiplier is computed at the sample mean but, the second order multiplier is computed at values given in the example above. If the other variables in Eq (6.1) had been at different levels, the second order response surface multiplier would also have been different. If the KG model were more nonlinear the difference would have been more pronounced. Figure 6.3 shows the difference between the multipliers graphically. Depicted are multipliers for the full range of G with other variables fixed at the levels specified above.

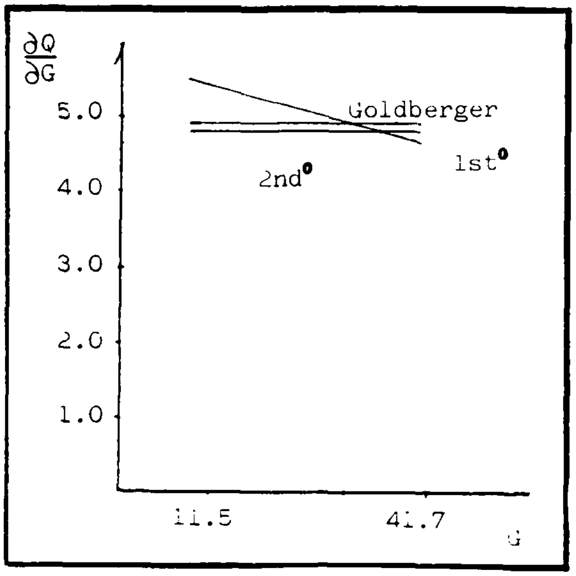

Figure 6.3. A Graphical Comparison of Multipliers for Changes in Q Due to Changes in G in Period Five.

By using response surfaces and multipliers generated from multipliers, economists can answer many questions without repeated runs of the macroeconomic model. Furthermore, interaction and squared terms are identified with response surfaces, but not with traditional multiplier analysis. There are still other valuable uses for response surfaces.

Factor Importance

Response surface equations can be used to evaluate factor importance in determining the response variable value. P.W. Smith and J. M. Mellichamp show how to evaluate factor importance for a nuclear exchange model in their paper entitled "A Methodology for Multidimensional Impact Analysis for Military Problems" (Smith and Mellichamp, 1979). In this paper the authors point out that the size of the factor coefficient gives the relative impact on response per unit of factor. The factor with the largest coefficient has the most influence per unit of factor.

The relative magnitude of coefficients in the response surfaces generated for the KG model do give some measure of the influence of factors which are measured in the same units. For instance, the first order coefficients presented in Appendix E indicate that for each dollar increase in government spending, gross national product increases by 4.5208, but for each dollar decrease in wage taxes, gross national product increases by 3.1234 in period five. Goldberger pointed out that determining which factors are most influential in causing endogenous variable changes from an historical point of view also involves the amount by which the factor changes from period to period. Two factors with the same response surface coefficient or multiplier do not have the same influence on an endogenous variable if one changes by only a small increment and the other changes by a large increment. To measure the relative importance of predetermined variables in determining endogenous variable values Goldberger formulated an index which was equal to the appropriate multiplier multiplied by the sum of the absolute values of the changes from one period to the next during the sample period and divided by the number of periods (Goldberger, 1959:72-73). Computing an equivalent index with response surface coefficients is certainly possible. Such an influence index is useful in quantifying the historical impact of predetermined variables on endogenous variables. However, from a policy simulation point of view, another measure might provide more useful information.

If a policy maker has influence to change economic policy variables over a limited, politically feasible range, then the policy maker would be interested in which policy variable at his disposal would be most influential in bringing about desired objectives. If the policy maker built a response surface using the maximum and minimum politically feasible values as factor levels in the experimental design, then the coded coefficient gives another measure of influence of that policy variable. For example, if a policy maker feels the maximum government expenditures that Congress will approve is $200 billion, while the minimum is $170 billion, then he could build a response surface using an experimental design with $200 billion and $170 billion as factor levels for running design points through his econometric model. Factor levels for other policy variables of interest would be formulated in the same way. The resulting coded coefficients give the amount of change that could be brought about by varying the policy variable over its politically feasible range.

Optimization Applications

The explicit form of the response surface equation with the unknown endogenous variable on one side of the equation and known predetermined variables on the other side of the equation suggests further applications. Because response surface equations have the form that they do and are expressed in terms of actual levels instead of changes (as in Goldberger's linearized KG model) economic optimization problems can be easily formulated and solved. An example serves to illustrate.

Suppose the year is 1952. The Klein-Goldberger model has just been estimated and an elected policy maker wishes . know what combination of fiscal policies will maximize economic growth (GNP), while holding inflation and employment at or below acceptable levels. The official would like these conditions to be realized in about three years. The KG RSM model can provide some guidance.

To solve this problem, several response surfaces must be generated. As shown in Chapter II, the KG model can be linked together to obtain forecasts for several periods in the future by solving the model, setting lagged variables equal to the current variable values, and then resolving the model. It is assumed that changes in fiscal policy variables made in period zero are sustained until period three. Solving the problem requires construction of three response surfaces, one for gross national product (Q), one for price level (p), and one for number of workers employed (N_w) for a time period three years in the future. The fiscal policy variables available for manipulation are government nonwage spending (G), wage taxes (T_w), corporate taxes (T_c), and government wage bill (W_2).

To generate the response surfaces needed, all predetermined variables except the four fiscal policy variables are set at expected constant levels, then the policy variables are set at the levels required by an experimental design, and the forecasts are computed. From the resulting data, stepwise linear regression is used to estimate the coefficients of first order response equations for each economic performance indicator. Shown below are the equations generated from the KG model with the predetermined variables set at selected levels, based on 1952 data which were the most current data used to estimate the model (Klein and Goldberger, 1955:131-133).

$$Q = 51.4040 - 3.1087T_w - 3.3323T_c + 4.5025G - 0.2854W_2 \quad (6.7)$$

$$NW = 13.1882 - 1.2541T_w - 1.2567T_c + 1.8021G - 0.50208W_2 \quad (6.8)$$

$$p = 143.6616 - 4.6051T_w - 4.1155T_c + 6.6816G - 5.6552W_2 \quad (6.9)$$

Since the quantitative relationships in the KG model are approximated quite adequately by linear functions (See Table 5.4.), a two level four factor factorial design was used to estimate the coefficients in Eq (6.7), (6.8), and (6.9). Design variables were varied over a limited politically feasible range ($5.63-11.63 billion for T_w, $7.14-13.14 billion for T_c, and $37.7-61.7 billion for G, and $13.82 to $21.82 billion for W_2.) These ranges were set by looking at the historical record of change over the sample period and then making a reasonable guess as to possible ranges of change.

From the feasible ranges and response surface equations above, one can formulate a linear programming problem as follows.

Maximize

$$Q = 51.4040 - 3.1087T_w - 3.3323T_c + 4.5025G - 0.2854W_2 \quad (6.10)$$

Subject to

$$NW = 13.1882 - 1.2541T_w - 1.2567T_c + 1.8021G - 0.50198W_2 = 58.71 \quad (6.11)$$

$$p = 143.6616 - 4.6051T_w - 4.1155T_c + 6.6816G - 5.6552W_2 \leq 207.714(1.05)^3 \quad (6.12)$$

$$T_w \leq 11.63 \quad (6.13)$$

$$T_w \geq 5.63 \quad (6.14)$$

$$T_c \leq 13.14 \quad (6.15)$$

$$T_c \geq 7.14 \quad (6.16)$$

$$G \leq 61.7 \quad (6.17)$$

$$G \geq 37.7 \quad (6.18)$$

$$W_2 \leq 21.82 \qquad (6.19)$$

$$W_2 \geq 13.82 \qquad (6.20)$$

The objective function, Eq (6.10), is simply the response surface for Q. The first constraint, Eq (6.11), is derived as follows. 1952 data indicate the number of workers in the labor force (N_L) is 66.6 million, the number of workers employed (N_W) is 56.0,the number of self employed workers (N_E) is 6.3 million, and the number of farm workers (N_F) is 4.0 million. Klein and Goldberger define the number of unemployed persons (N_U) to be (Klein and Goldberger, 1955:19):

$$N_U = N_L - (N_W + N_E + N_F)$$

For the 1952 data N_U is 0.3 million workers. This translates to an unemployment rate of 0.45 percent (this figure is clearly unrealistic). The number of self employed and farm workers together have been decreasing by about one percent per year, and the total labor force has been growing by about one and one-half percent per year. Projecting these trends forward three years,

$$(N_E + N_F)_3 = (6.3 + 4.0)(.99)^3 = 9.99$$

$$(N_L)_3 = (66.6)(1.015)^3 = 69.64$$

where the subscript 3 denotes three years in the future. If the acceptable rate of unemployment is set (arbitrarily for this example) at one percent, an expression for the unemployment rate in three years can be written

$$\frac{(N_L) - (N_E + N_F)_3 - (N_W)_3}{(N_L)_3} = 0.01$$

Solving for $(N_W)_3$ yields

$$(N_W)_3 = (1-0.01)(N_L)_3 - (N_E + N_F)_3$$

$$= (1-0.01)(69.64) - (9.99)$$

$$= 58.71$$

Setting the response surface equation for N_W equal to this value yields Eq (6.8).

The left side of the second constraint, Eq (6.12), is the response surface for the price index. The right side of the inequality is the currently forecast price index multiplied by a five percent per year increase for each of three years. This constraint keeps inflation below an average of five percent per year. The remaining constraints, Eqs (6.13) through (6.20), are the political constraints on fiscal policy variables. The right hand side values of the inequalities are 1952 levels of the exogenous variables plus or minus the amount by which the variables can be feasibly changed.

Solving this linear programming problem gives the optimal fiscal policy to be followed by the policy maker. Table (6.2) shows the solution.

Table 6.2. Optimal Fiscal Policy for the Example Problem

Maximum Attainable Q:

$185.8 billion

Fiscal Policy Variable Values

$T_W = 11.63$

$T_C = 7.14$

$$G = 44.55$$

$$W_2 = 21.82$$

The model forecasts gross national product to be $172.0 billion at the end of the current year. The maximum attainable gross national product, 185.8, translates to an average growth rate of 2.6 percent over three years. The average inflation rate is 4.2 percent, which means that there is "slack" in the inflation constraint. Appendix F contains the output from the linear programming computer routine for this problem.

This solution suggests that the best fiscal policy is to cut corporate taxes, raise wage taxes, hire more government employees (or just pay them more) and increase government expenditures slightly. This solution sounds fairly plausible, but one might wonder why this particular solution is optimal. Furthermore, one might wonder if the optimal solution for the linear programming problem is in fact the optimal solution for the actual Klein-Goldberger model.

The answer to the first question requires an examination of the coefficients in Eqs (6.7), (6.8), and (6.9). The employment constraint is always binding because it is met with equality. Economic theory and Eqs (6.7) through (6.9) indicate that either decreasing taxes or increasing government spending raises gross national product, employment, and prices. Increasing government wage bill decreases gross national product, employment, and prices according to Eqs (6.7) through (6.9). This is counter intuitive, but Goldberger explained somewhat unconvincingly that increases in W_2 with G constant represented "a shift in the composition of government expenditures from business produced goods to purchases of labor services." (Goldberger, 1959:30) Because the inflation constraint is not the limiting factor, it is ignored for the moment. The objective, then, is to find the feasible combination of fiscal policy vaiables which maximizes GNP for a given level of employment. To do this one would want to change the fiscal policy variable with the largest increase in gross national product per unit increase in employment, the fiscal policy variable with the next largest increase, and so on until the required employment level is reached. For example, the change in Q per change in N_w brought about by changes in government spending is

$$\frac{\partial Q}{\partial N_w} = \frac{\partial Q}{\partial G} \frac{\partial G}{\partial N_w} \tag{6.21}$$

Earlier in this chapter it was shown that the quantity Q/ G is simply the coefficient of G in the Q linear response function, Eq (6.7). In addition, G/ N_w is the reciprocal of the G coefficient in the N_w response surface equation, Eq (6.8). For example the ratio between the G coefficient in Eq.(6.7) and the G coefficient in Eq (6.8) is

$$\frac{\partial Q}{\partial G} \frac{\partial G}{\partial N_w} = \frac{4.5025}{1.8021} = 2.4985 \tag{6.22}$$

The number 2.4985 gives the increase in Q which occurs when G increase enough to raise N_w by one unit. Similar ratios can be computed for the other factors.

$$\frac{\partial Q}{\partial T_w} \frac{\partial T_E}{\partial N_w} = \frac{-3.1087}{-1.2541} = 2.4788$$

$$\frac{\partial Q}{\partial T_c} \frac{\partial T_c}{\partial N_w} = \frac{-3.3323}{-1.2567} = 2.6516$$

$$\frac{\partial Q}{\partial W_2} \frac{\partial W_2}{\partial N_w} = \frac{-0.2854}{-0.5019} = 0.5633$$

Gross national product increases most for a given level of employment by a cut in corporate taxes, then by an increase in government spending, then by a cut in wage taxes, and finally by a cut in the government wage bill. The optimal solution sets corporate taxes at the lower limit, government spending at an intermediate level, and wage taxes and government wage bill at the high limits. Thus the given optimal solution for the linear programming problem does seem reasonable. However, a question still remains as to whether the optimal solution for the linear programming problem is optimal for the actual Klein-Goldberger model.

Chapter V shows that the response surfaces do in fact closely approximate what is going on in the model over the entire range of data. Furthermore, higher order terms are not necessary to obtain a good fit. Therefore, what is optimal for the response surface model of the economy should be optimal for the KG model. Verifying this assertion requires searching the area around the alleged optimal solution to see if further gains might be made with an alternate policy. This search is to be done with the original model. If the solution given for the linear programming problem is not the optimal then one should be able to increase gross national product and satisfy the constraints by adjusting T_w, T_c, G, or W_2. The table below shows the results of running the KG model with the fiscal policy variables set at values slightly different than the optimal policy determined by the response surface derived linear programming problem.

Table 6.3. Klein-Goldberger Model Solutions in the Area of the Alleged Optimal Solution

				58.71 max	240.5 max	
T_w	T_c	G	W_2	N_w	p	Q
11.63	7.14	44.54	21.82	56.5	222.5	186.2
10.63	7.14	44.54	21.82	57.7	226.6	189.3*
11.63	8.14	44.54	21.82	55.2	217.6	182.8
11.63	7.14	43.54	21.82	57.3	215.0	181.6
11.63	7.14	45.54	21.82	58.3	228.7	190.8*
11.63	7.14	44.54	20.82	57.0	222.5	186.6*
10.63	8.14	44.54	21.82	56.5	222.5	186.0
10.63	7.14	43.54	21.82	55.9	219.9	184.8
11.63	8.14	45.54	21.82	57.0	224.5	187.4*
11.63	8.14	45.54	20.82	57.6	229.5	187.7
5.63	13.14	44.54	21.82	56.5	224.6	185.0
11.63	13.14	61.70	16.82	82.4	337.3	245.0

The first line in Table 6.3 is the alleged optimal solution. However, the starred solutions yield greater gross national product values than the supposed optimal solution. The solution computed by the linear programming algorithm is not optimal because the value of N_w forecast by the response surface function was four percent too low. The four percent error is not unreasonable because although the linear approximation to the KG model is good, it is not perfect.

All is not lost, however, because the response surface coefficients can be used to "tweek" the solution to optimality. As pointed out above, economic theory and Eqs (6.7), (6.8), and (6.9) indicate that when corporate or wage taxes decrease, gross national product increases, prices increase, and employment increases. The effect of increased government spending is the same. Increased wage bill has a small downward effect on gross national product and employment, and a substantial downward effect on prices according to Eqs (6.7), (6.8), and (6.9).

To increase Q, one could decrease taxes or government wage bill or increase government spending. However, the same measures which raise Q also raise N_w and p. N_w must increase by 58.71 - 56.50 = 2.21 to satisfy the employment constraint with equality and p may increase by 240.5 - 222.5 = 18. The best variable to alter is the variable which increases Q the most for the required change in N_w without violating the price index constraint. The discussion above shows how to compute the ratios for tradeoffs between employment and gross national product. For example, if N_w must increase by 2.21 to meet the employment constraint, then the total change in Q due to a change in G is (2.4985)(2.21) = 5.5217 using the ratio computed in Eq (6.22). To find how much G must increase to raise N_w to the required level, one can divide the required change by the G coefficient in Eq (6.8) (i.e., 2.21/1.8021 = $1.226 billion). By Eq (6.9) this increase in G induces a price index increase of (1.226)(6.6816) = 8.194. Since this increase in G would only increase the price index to 222.5 + 8.194 = 230.694, this solution is feasible because this price index is below the 240.5 value allowed by Eq (6.12).

To find the best factor to change, tradeoff ratios for T_w and W_2 must be compared to the G tradeoff ratio. The factor with the largest tradeoff ratio which does not cause the price index to exceed its maximum is the best. Because T_c is already at the lower limit, it need not be investigated for alteration. Table 6.4 summarizes the data required to select the best factor to adjust. The first column is the tradeoff ratio. The next column is the change in N_w required to satisfy the employment constraint. The Q column is simply the product between the first and second columns. It is the change in Q resulting when the factor is altered enough to bring about the required change in N_w. The F column is the change in the factor required to increase N_w by 2.21. It is equal to the N_w column divided by the factor's coefficient in Eq (6.8). The p column is the increase in p caused by the increase in the factor. It is equal to the F column times the factor's coefficient in Eq (6.10). Finally the p column is the new price index brought about by changing the factor to its new level. If the figure in the p column exceeds 240.5, the solution is infeasible.

Table 6.4. Data for Selecting the Best Variable to Alter

	Q/ N_w	N_w	Q	F	P	p
T_w	2.4788	2.21	5.4781	-1.7622	8.1151	230.6
G	2.4985	2.21	5.5217	1.2263	8.1936	230.7
W_2	0.5685	2.21	1.2563	-4.4026	24.897	247.4

A quick scan of Table 6.4 reveals that increasing G by \$1.2263 billion increases Q by \$5.5217 billion while a decrease in T_w of \$1.7622 billion increases Q by only \$5.4781 billion. Decreasing W_2 by the amount required to increase N_w by 2.21 million workers causes the price index to exceed the maximum. Thus, the adjusted optimal solution is

$$T_w = 11.63$$

$$T_c = 7.14$$

$$G = 45.78$$

$$W_2 = 21.82$$

A single solution for a linear programming problem is rarely very useful without sensitivity analysis. Fortunately, sensitivity analysis for linear programming problems is very well developed. For instance the shadow prices tell how much the objective function will change if the right hand side of a constraint is changed. Table 6.5 shows shadow prices for each binding constraint.

Table 6.5. Shadow Prices for the Fiscal Policy Problem.

Constraint	Objective Function Change
Maximum T_w	0.0254
Maximum W_2	0.9688
Minimum T_w	0.1925
Minimum W_2	2.4985

Another option for conducting sensitivity analysis for this example linear programming problem is (believe it or not) response surface methodology. A new response surface can be built for Q in terms of T_w, T_c, G, W_2, inflation and unemployment by varying right hand sides of the constraints in accordance with an experimental design and solving the linear programming problem (Smith and Mellichamp, 1979).

Because the KG model is nearly linear, first order response surface equations fit the model fairly well. The linear objective function and constraints make it possible to formulate an optimization problem as a linear program. If the model were not so linear and the response surfaces had higher order terms, an optimization problem could still be formulated and solved using nonlinear optimization techniques available. One computer implementation of nonlinear techniques is the Sequential Unconstrained Minimization Technique (SUMT) package. The program handles nonlinear objective functions and constraints with inequalities. The

AD-A167 145 AN APPLICATION OF RESPONSE SURFACE METHODOLOGY TO A MACROECONOMIC MODEL(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOOL OF ENGINEERING J L DONOVAN DEC 85 AFIT/GOR/ENS/85D-4 2/2

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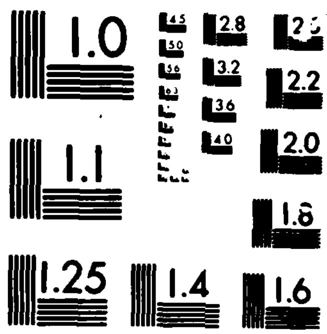

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SUMT program allows one to exploit a key advantage of RSM in studying macroeconomic models, which is the capability to derive a reduced form equation (one endogenous variable expressed as an explicit function of predetermined variables) equation for nonlinear macroeconometric models. For a description of the SUMT package, the reader may consult Mylander's text, A Guide to SUMT-Version 4.

This chapter interprets and applies response surfaces derived from the Klein-Goldberger econometric model. Computing the partial derivatives of response surfaces with respect to variables of interest yields multiplier functions for those variables. These multiplier functions may characterize nonlinear functions better than the traditional constant multipliers over a wide range of data. Both the coded and decoded response function coefficients give information on the relative importance of factors. Finally, this chapter shows that response functions readily adapt themselves to optimization problems.

VII. Concluding Remarks

Research Summary

This study has uncovered much about what can and cannot be done in applying response surface methodology to a macroeconomic model. Response surface methodology is a useful tool which can be used to investigate the properties of a macroeconomic model as long as the limitations of the method are kept in mind.

The research demonstrates how to generate response surfaces from a macroeconomic model. First, the problem to be addressed is defined and a determination is made that response surface methodology is the appropriate tool for solving the problem. Next, variables of interest are selected and their operating ranges specified. The form of the response surface equations is decided upon and an appropriate experimental design selected. After translating the coded experimental design to actual predetermined variable values, the model is solved for each combination of predetermined variable values specified by the experimental design. The data generated are used to estimate response surface coefficients in terms of the coded experimental design to preserve orthogonality. The response surface fit is checked and the response surface equation is decoded so that it is expressed in terms of the original variables. After generating the response surface, analyses may begin.

This study shows that a low order polynomial can indeed fit the responses of the Klein-Goldberger econometric model. The near linearity of the model is confirmed. The coefficients from a decoded first order response surface fit to the Klein-Goldberger model are compared to multipliers computed by Goldberger and found to correspond quite closely.

Economists use multipliers extensively to characterize the static and dynamic properties of econometric models and to conduct policy simulations. Techniques for generating multipliers include linearization of the econometric model through the derivative method and computing control and disturbed solutions. Response surface methodology offers another alternative for computing multipliers.

Multipliers are derived from response surface equations by taking the partial derivative of the response surface equation with respect to the variable of interest. The result may or may not be a constant. Higher order terms in the response surface equation cause multipliers to be dependent on the level of one or more variables. Consequently, multipliers obtained from response surfaces are most useful for investigating nonlinear econometric models.

The advantage of response surface derived multipliers over multipliers derived by model linearization is that the model does not have to be linear or near linear for multipliers to be valid over a wide range of variable values. In addition, significant interactions and higher order effects can be identified. The advantage of response surface multipliers over control-disturbed multipliers is that they more completely characterize the relationships in the model and reduce the number of runs required to estimate multipliers. Also, multipliers computed by the control-disturbed method do not identify interactions and higher order effects. These multipliers are only valid for small ranges of the predetermined variables.

Response surface multipliers can be used in the same ways that multipliers derived by other techniques are used. Uses include policy simulation and determining most influential factors in the economy.

Response surfaces can be used for more than just computing multipliers. They can also be used to formulate optimization problems. The explicit nature of the response surface equation giving endogenous variables in terms of predetermined variables facilitates optimization problem formulation. Chapter VI gives an example problem in which gross national product is maximized while holding unemployment and inflation at or below acceptable levels. The problem is formulated as a linear programming problem and solved. Optimization problems with nonlinear response surface derived constraints and objective functions can also be formulated and solved using optimization packages such as SUMT.

Applying response surface methodology to macroeconomic models is not without limitations. Computing multipliers using response surface methodology is more cumbersome than existing methods for deriving multipliers from linear or near linear models. Separate response surfaces must be computed for each response variable for each time period for each subset of predetermined variables. For nonlinear models, response surface multipliers which are functions better characterize input-output relationships than traditional multipliers. However, special care must be taken to insure response surface fit before drawing inferences about the model based on the response surface generated.

A limitation which detracts from using response surfaces for prediction is the few number of factors which can be inducted in the response surface function. The number of variables which can be included in the response surface equation is limited by the size of the experimental design. Of particular concern are interactions between exogenous variables included in the response surface and lagged endogenous variables not included in the response surface. It is quite possible that the effect of exogenous changes depends on the current state of the economy. If variables are omitted from the response surface equation, then the response surfaces only capture model relationships at the specific levels assumed in generating the response surface. The other methods of computing multipliers suffer from this deficiency too.

On balance, the limitations of response surface methodology do not preclude it from being a valuable tool for analyzing certain aspects of macroeconomic models.

Recommended Further Research

There are several areas available for further research. First, it has been assumed that a low order polynomial could adequately fit an econometric model more nonlinear than the Klein-Goldberger model. This assumption needs testing. Second, including more variables in the model by using larger experimental designs has yet to be explored. Large experimental designs could be developed by computer algorithm. A final improvement of the research presented in this thesis would be to include time as an independent variable in the response surface equation. Including time in the response surface equation would eliminate the need to generate response surfaces for each period. Preliminary attempts to fit response functions with time as an independent variable to the Klein-Goldberger model yielded R^2 values of 0.9610 for number of workers employed, 0.9858 for price index, and 0.9630 for gross national product. To generate these response surfaces, a second order polynomial including wage taxes, corporate taxes, government nonwage expenditures, government wage bill, number of government employees and time was fit to data from periods zero through five. Developing response surfaces with time as an independent variable would reveal time delay aspects of the econometric model which could prove quite valuable.

Appendix A: Solving the Klein-Goldberger Model

Introduction

Because the Klein-Goldberger model is not linear and has no simple analytical solution, solving the model requires an solution approximation technique. Goldberger used a derivative method to obtain a linear approximation to the model formulated in terms of variable changes. He then derived impact and interim multipliers from the linear approximation. This thesis requires a method for solving the Klein-Goldberger model without linearizing it. Economists commonly use some sort of numerical technique for solving nonlinear econometric models. Klein recommended the Gauss-Sidel numerical method for solving econometric models (Klein, 1974:238). The method is a simple iterative procedure which does not require derivative computations. This appendix describes the method, illustrates it with an example, and shows how the method was applied to solve the Klein-Goldberger model for this research effort.

Gauss-Sidel Method Description

Klein describes the Gauss-Sidel method in his text, A Textbook of Econometrics (Klein, 1974:238-239). The material below restates Klein's description. An econometric model with n current endogenous variables, n endogenous variables lagged up to p periods, and m exogenous variables lagged up to p periods can be written in the form

$$y_{i,t} = g_i(y_{1,t},\ldots,y_{i-1,t},y_{i+1,t},\ldots,y_{n,t},y_{1,t-1},\ldots,y_{n,t-p},\ldots,$$
$$x_{1,t},\ldots,x_{m,t-p})+e_{i,t} \qquad (A.1)$$

$i = (1,2,\ldots,n)$

where

$y_{i,t}$ = one of n current endoenous variables,

$y_{i,t-p}$ = one of n endogenous variables lagged p periods,

$x_{i,t}$ = one of m current exogenous variables,

$x_{i,t-p}$ = one of m exogenous variables lagged p periods, and

$e_{i,t}$ = a random error term.

In words, Eq (A.1) says that each equation in the model gives a single current endogenous variable (which must be solved for) as a function of the _other_ current endogenous variables, lagged endogenous variables, and exogenous variables. Lagged endogenous variables together with current and lagged exogenous variables are known as predetermined variables. With a few exceptions, the Klein-Goldberger model in Table 2.1 has the form of Eq (A.1). It is usually possible to rewrite Eq (A.1) in the form

$$y_{i,t} = g_i(y_{1,t},\ldots,y_{n,t}, y_{1,t-1},\ldots,y_{n,t-p},\ldots,x_{1,t},\ldots,x_{m,t-p}) + e_{i,t} \qquad (A.2)$$

$i = (1,2,\ldots,n)$

Eq (A.2) is the same as Eq (A.1) except in Eq (A.2) $y_{i,t}$ appears on both sides of the equation. Omitting the error term, e_i, and inserting superscripts in accordance with the Gauss-Sidel method converts Eq (A.2) to an algorithm.

$$y_{i,t}^{(r+1)} = g_i(y_{1,t}^{(r+1)},\ldots,y_{i-1,t}^{(r+1)}, y_{i,t}^{(r)},\ldots y_{n,t}^{(r)}, y_{1,t-1},\ldots,y_{n,t-p}, x_{1,t},\ldots,x_{m,t-p}) \qquad (A.3)$$

$i = (1,2,\ldots,n)$

where

$y_{i,t}^{(r+1)}$ = the value of the ith current endogenous variable from the (r+1)th iteration of the method, and

$y_{i,t}^{(r)}$ = the value of the ith current endogenous variable from the rth iteration.

Iterations are performed until

$$\left|\frac{y_{i,t}^{(r+1)} - y_{i,t}^{(r)}}{y_{i,t}^{(r)}}\right| < \text{tolerance}$$

A simple example illustrates the confusing notation in Eqs (A.1), (A.2), and (A.3). The system of nonlinear equations,

$$x = -4z + 2w + 6 \tag{A.4}$$

$$y = 4x^{-1/3} + 8 \tag{A.5}$$

$$z = x/y - 2w \tag{A.6}$$

where

x, y, z = variables to be solved for, and

w = a variable whose value is known,

is in the form of Eq (A.1). Eqs (A.4) through (A.6) can be rewritten in the form of Eq (A.2) by multiplying both sides of the equations by a constant, say 0.5, and then adding (1-0.5) times the left-hand side variables to both sides of the equations.

$$x = 0.5(-4z + 2w + 6) + 0.5x \tag{A.7}$$

$$y = 0.5(4x^{-1/3} + 8) + 0.5y \tag{A.8}$$

$$z = 0.5(x/y - 2w) + 0.5z \tag{A.9}$$

By arranging terms and inserting superscripts denoting iterations, Eqs (A.7) through (A.9) become algorithms for computing a solution.

$$x^{(r+1)} = 0.5x^{(r)} - 2z^{(r)} + 1w + 3 \qquad \text{(A.10)}$$

$$y^{(r+1)} = 2x^{-1/3(r+1)} + 0.5y^{(r)} + 4 \qquad \text{(A.11)}$$

$$z^{(r+1)} = 0.5(x^{(r+1)}/y^{(r+1)}) - w + 0.5z^{(r)} \qquad \text{(A.12)}$$

The Gauss-Sidel method requires an initial solution, a specification for w, and a specification of the error tolerance. If $x^{(0)} = 0$, $y^{(0)} = 0$, $z^{(0)} = 0$, $w=5$, and tolerance=.01 then the first iteration of algorithm Eqs (A.10) through (A.12) is

$$x^{(1)} = (0.5)(0) - 2(0) + 1(5) + 3 = 8$$

$$y^{(1)} = 2(8)^{-1/3} + (0.5)(0) + 4 = 4$$

$$z^{(1)} = 0.5(8/4) - 5 + (0.5)(0) = -4$$

$$x^{(1)} = 8 \quad y^{(1)} = 4 \quad z^{(1)} = -4$$

The second iteration is

$$x^{(2)} = (0.5)(8) - 2(4) + 5 + 3 = 4$$

$$y^{(2)} = 2(-8)^{1/3} + (0.5)(4) + 4 = 2$$

$$z_{(2)} = 0.5(-8/-1) - 5 + (0.5)(-4) = -3$$

$$x^{(2)} = 4 \quad y^{(2)} = 2 \quad z^{(2)} = -3$$

Iterations continue until

$$\left|\frac{x^{(r+1)} - x^{(r)}}{x^{(r)}}\right| < 0.01 \quad \left|\frac{y^{(r+1)} - y^{(r)}}{y^{(r)}}\right| < 0.01$$

and

$$\left|\frac{z^{(r+1)} - z_{(r)}}{z^{(r)}}\right| < 0.01$$

The algorithm is not guaranteed to converge for all forms of Eq (A.2) and for all trial solutions. There are convergence conditions for the Gauss-Sidel method, but often the convergence conditions are extremely difficult to compute. In practice, trial and error usually reveals simple forms that converge.

The method described above computes a solution for current endogenous variables given lagged endogenous variables and exogenous variables. The method easily adapts to compute extended period forecasts. Once the method yields a solution, lagged variables are updated with current variable values and exogenous variables are set to values dictated by the policy under investigation. Then the model is resolved. In the notation of Eq (A.1),

$$y_{i,t-(k)} = y_{i,t-(k+1)} \quad \text{for all } i=1,2,\ldots,n \text{ and } k=1,2,\ldots,p$$

$$x_{i,t-(k)} = x_{i,t-(k+1)} \quad \text{for all } i=1,2,\ldots,m \text{ and } k=1,2,\ldots,p$$

The Klein-Goldberger model was solved using the Gauss-Sidel method. The Klein-Goldberger model equations in Table 2.1 were put in the form of Eq (A.1) with one current endogenous variable expressed as a function of the other current endogenous variables, lagged endogenous variables, and exogenous variables. For the sake of simplified discussions, Eq (A.1) can be abbreviated,

$$y_i = g_i \tag{A.13}$$

where g_i is a function of the other current endogenous variables, lagged endogenous variables, and exogenous variables. Performing some simple algebraic manipulation converts Eq (A.13) to the form of Eq (A.2). If a is a constant,

$$ay_i = a\,g_i \tag{A.14}$$

$$ay_i + (1-a)y_i = ag_i + (1-a)y_i \tag{A.15}$$

and

$$y_i = ag_i + (1-a)y_i \qquad (A.16)$$

The approach given in Eqs (A.14), (A.15), and (A.16) was used to convert Klein-Goldberger model equations in the form of Eq (A.1) to the form of Eq (A.2). For a=2, and a starting solution equal to the endogenous variable sample means, the method diverged. The method converged for a=0.5. The number of iterations required to reach a solution appears to depend on the value of the constant a. Runs with other forms of the model were not attempted, but the number of runs required for solution might be considerably reduced by using another form of the model.

Shown below is a FORTRAN program for solving the Klein-Goldberger model with the Gauss-Sidel method. Program inputs are files containing control language, an initial trial solution, values for predetermined variables to be included in the response surface equation, values for predetermined variables not to be included in the response surface equation, and coded values (-1, 0, or 1) for the variables to be included in the response surface equation. The program outputs a file containing the coded variable values, a case number, the forecast period number, and the solution for endogenous variables. The output file may be read directly by the BMDP statistical package for response surface coefficient estimation.

The program first reads and echoes the control language contained in the file "kg.ctl." After initializing arrays, the program enters a loop which solves the Klein-Goldberger model for each set of variable levels specified case by case for the number of periods specified. The program first reads one set of coded values (-1, 0, or 1) for variables to be included in the response surface equation (variables which will henceforth be referred to as design variables). Subroutine setbas reads predetermined variable values into the arrays P, IS, and PI. Array P contains exogenous variable values and endogenous variable values lagged one period. IS and PI contain values of short term interest rates and prices lagged more than one period. Setbas also reads a trial solution into X0. Next, subroutine setdes resets the design variables to the levels specified in the file "design.cod." Subroutine solve calls subroutine iterate in a loop to compute Gauss-Sidel iterations of the Klein-Goldberger model until the solution converges within tolerance. If the control language specifies that intermediate period solutions are to be printed, the program writes the coded design variable levels, case number, period number, and the intermediate solution to the file "kg.out." The last period's coded design variable levels, case number, period number, and solution are always printed. If the control language specifies that extended period forecasts are to be made, subroutine update updates lagged variables and a new solution is computed. The program stops when solutions are computed for the number of periods specified for each set of design variable level specifications.

```
**********************************************************************

        program kgsolv

* Solves the Klein-Goldberger model using the Gauss-Sidel numerical
* technique

        double precision X0(21),X1(21),P(44),IS(5),PI(2)
        real tol, CODDES(5)
        integer d1, d2, i, icase, ipriod, iiter, ncase, npriod, itmax,
     &          prtall, guess1, bascas, design, kgout, kgctl, codes
        open(1,file='guess1.dat')
        rewind(1)
        open(2,file='bascas.dat')
```

```
        rewind(2)
        open(3,file= 'design.dat')
        rewind(3)
        open(4,file= 'kg.out',status='new')
        open(8,file= 'kg.ctl')
        rewind(8)
        open(9,file= 'design.cod')
        rewind(9)
        guess1=1
        bascas=2
        design=3
        kgout=4
        kgctl=8
        codes=9

* Initialization

*    Read and echo control language

        write(*,*)' CONTROL DATA'
        read(kgctl,*)prtall
        write(*,*)' PRINT EACH PERIOD DATA? (1=YES)  ',prtall
        read(kgctl,*)ncase
        write(*,*)' # CASES= ',ncase
        read(kgctl,*)npriod
        write(*,*)' # PERIODS= ',npriod
        read(kgctl,*)tol
        write(*,*)' TOLERANCE= ',tol
        read(kgctl,*)itmax
        write(*,*)' MAX ITERATIONS= ',itmax
        icase=1
        ipriod=0
        iiter=1

*    Initialize arrays

        do 100 i=1,21
            X0(i)=0
            X1(i)=0
100     continue
        do 200 i=1,44
            P(i)=0
200     continue

*    Strip off data dimensions with dummy variables.

        read(codes,*)d1, d2

* Main Program

*    While the current case is less than the last case

400     if (icase.le.ncase) then
```

```
          rewind(guess1)
          rewind(bascas)

*    Read coded design variable levels

          read(codes,*)(CODDES(i),i=1,5)

*    Read predetermined variable levels

          call setbas(guess1,bascas,X0,P,IS,PI)

*    Read design variable levels

          call setdes(design,P,IS,PI)

          ipriod=0

*    while current period is less than or equal to the forecast period

300       if (ipriod.le.npriod) then
              call solve(X0,X1,P,icase,ipriod,itmax,tol,iiter)
              if (prtall.eq.1) then
                  write(kgout,1020)(CODDES(i),i=1,5)
                  write(kgout,*) icase, ipriod
                  write(kgout,1020)(X1(i),i=1,21)
              else if(ipriod.eq.npriod) then
                  write(kgout,1020)(CODDES(i),i=1,5)
                  write(kgout,*) icase, ipriod
                  write(kgout,1020)(X1(i),i=1,21)
              endif
1020          format(1x,5F12.6)


              call update(X0,X1,P,IS,PI)

              ipriod=ipriod+1
              goto 300
          endif
*         end while (ipriod)
          icase=icase+1
          goto 400
      endif
*     end while (icase)
      write(*,*)
      write(*,*)' PROGRAM COMPLETE, RESULTS IN KG.OUT'

      stop

      end

*****************************************************************
*****************SUBROUTINES*************************************
*****************************************************************
```

```
      subroutine setbas(guess1,bascas,X0,P,IS,PI)

* Reads initial guess solution and base case predetermined data.

      double precision X0(21), P(44), IS(5), PI(2)
      integer i, guess1, bascas

*    Read initial guess at solution.

      read(guess1,*)(X0(i),i=1,21)

*    Read base case predetermined data

      read(bascas,*)(P(i),i=1,44)

*    Set up variables with more than one year lag.

      IS(1)=P(29)
      IS(2)=(P(29)+P(30))/2
      IS(3)=P(30)
      IS(4)=(P(30)+P(31))/2
      IS(5)=P(31)
      PI(1)=P(35)
      PI(2)=P(36)

      return
      end

*****************************************************************

      subroutine setdes(design,P,IS,PI)

* Reads changed predetermined data for a new case

      double precision P(44), IS(5), PI(2)
      integer design

*    Read in design variable values

      read(design,*)P(2),P(3),P(6),P(7),P(13)

*    P(2)=TW, P(3)=TC, P(6)=G, P(7)=W2, P(13)=NG

*    Set up variables with more than one year lag

      IS(1)=P(29)
      IS(2)=(P(29)+P(30))/2
      IS(3)=P(30)
      IS(4)=(P(30)+P(31))/2
      IS(5)=P(31)
      PI(1)=P(35)
      PI(2)=P(36)
```

```
      return
      end

******************************************************************

      subroutine solve(X0,X1,P,icase,ipriod,itmax,tol,iiter)

* Computes a numerical solution to the Klein-Goldberger model

      double precision X0(21) ,X1(21) , P(44)
      real tol, ERROR(21),error0
      integer i, icase, ipriod, itmax, iiter

* Compute numerical solution

      iiter=0
*     Repeat until error < tolerance
400   continue
         if (iiter.lt.itmax) then
* call iterate subroutine
            call iterat(X0, P, X1)
            iiter=iiter+1
         else
            write(*,*)'CASE ',icase,' PERIOD ',ipriod,
     &                  ' FAILED TO CONVERGE AFTER '
            write(*,*)iiter, ' ITERATIONS. PROGRAM STOPPED.'
            stop
         endif
* Check current iteration for tolerance
         error0=0
         do 200 i=1,21
            ERROR(i)=abs((X1(i)-X0(i))/X0(i))
            if (ERROR(i).gt.error0) then
               error0=ERROR(i)
            endif
200      continue
         if (error0.gt.tol) then
            do 300 i=1,21
               X0(i)=X1(i)
300         continue
            goto 400
*        (Do another iteration until error below tolerance)
         else
            write(*,*)' CASE ',icase,' PERIOD ',ipriod,
     &                  ' - ',iiter,' ITERATIONS'
         endif

      return
      end

******************************************************************
```

```
      subroutine iterat(X0, P, X1)

* Performs Gauss-Sidel iterations

      integer j

      double precision  C,I,SC,PC,D,W1,NW,WR,FI,R1,PR,L1,L2,IL,IS,
     & K,SB,PY,M,PI,Q

*     double precision  C1,I1,SC1,PC1,D1,W11,NW1,WR1,FI1,R11,PR1,
*     & L11,L21,IL1,IS1,K1,SB1,PY1,M1,PI1,Q1

      double precision  TE,TW,TC,TN,TR,G,W2,R2,T,H,NP,NL,NG,NE,FR,
     & PF,LB

      double precision CL1,SCL1,PCL1,DL1,W1L1,WRL1,FIL1,R1L1,PRL1,
     & L1L1,L2L1,ISL1,ISL3,ISL5,KL1,SBL1,PL1,PIL1,PIL2,QL1

      double precision TEL1,TWL1,TCL1,TNL1,TRL1,W2L1,R2L1

      double precision X0(21),P(44),X1(21)


* Initialize variables in the Klein-Goldberger model

* ENDOGENOUS VARIABLES

      C =X0(1)
      I =X0(2)
      SC=X0(3)
      PC=X0(4)
      D =X0(5)
      W1=X0(6)
      NW=X0(7)
      WR=X0(8)
      FI=X0(9)
      R1=X0(10)
      PR=X0(11)
      L1=X0(12)
      L2=X0(13)
      IL=X0(14)
      IS=X0(15)
      K =X0(16)
      SB=X0(17)
      PY=X0(18)
      M =X0(19)
      PI=X0(20)
      Q =X0(21)

* EXOGENOUS VARIABLES

      TE=P(1)
      TW=P(2)
```

```
        TC=P(3)
        TN=P(4)
        TR=P(5)
        G =P(6)
        W2=P(7)
        R2=P(8)
        T =P(9)
        H =P(10)
        NP=P(11)
        NL=P(12)
        NG=P(13)
        NE=P(14)
        FR=P(15)
        PF=P(16)
        LB=P(17)

* LAGGED ENDOGENOUS VARIABLES

        CL1 =P(18)
        SCL1=P(19)
        PCL1=P(20)
        DL1 =P(21)
        W1L1=P(22)
        WRL1=P(23)
        FIL1=P(24)
        R1L1=P(25)
        PRL1=P(26)
        L1L1=P(27)
        L2L1=P(28)
        ISL1=P(29)
        ISL3=P(30)
        ISL5=P(31)
        KL1 =P(32)
        SBL1=P(33)
        PL1 =P(34)
        PIL1=P(35)
        PIL2=P(36)
        QL1 =P(37)

* LAGGED EXOGENOUS VARIABLES

        TEL1=P(38)
        TWL1=P(39)
        TCL1=P(40)
        TNL1=P(41)
        TRL1=P(42)
        W2L1=P(43)
        R2L1=P(44)

* THE KLEIN-GOLDBERGER MODEL
*1
        C=0.5*(-22.26+0.55*(W1+W2-TW)+0.41*(PY-TC-TN-SC)
     &          +0.34*(R1+R2-TR)+0.26*CL1+0.072*L1L1+0.26*NP)+0.5*C
```

```
*2
        I=0.5*(-16.71+0.78*(PL1-TCL1-TNL1+R1L1+R2L1-TRL1+DL1)
     &          -0.073*KL1+0.14*L2L1)+0.5*I
*3
        SC=0.5*(-3.53+0.72*(PC-TC)+0.076*(PCL1-TCL1-SCL1)-0.028*SBL1)
     &  +0.5*SC
*4
        PC=0.5*(-7.60+0.68*PY)+0.5*PC
*5
        D=0.5*(7.25+0.10*(K+KL1)/2+0.044*(Q-W2))+0.5*D
*6
        W1=0.5*(-1.40+0.24*(Q-W2)+0.24*(QL1-W2L1)+0.29*T)+0.5*W1
*7
        NW=0.5*((((Q-W2)+26.08-0.16*(K+KL1)/2.-2.05*T)/2.17-NE)/H+NG)
     &  +0.5*NW
*8
        WR=0.5*(4.11-0.74*(NL-NW-NE)+0.52*(PIL1-PIL2)+0.54*T+WRL1)
     &  +0.5*WR
*9
        FI=0.5*(0.32+0.006*(M-TW-TC-TN-TR)*PI/PF+0.81*FIL1)+0.5*FI
*10
        R1=0.5*(PR/PI)*(-0.36+0.054*(W1+W2-TW+PY-TC-TN-SC)*PI/PR
     &          -0.007*((W1L1+W2L1-TWL1+PL1-TCL1-TNL1-SCL1)*PIL1/PRL1)
     &          +0.012*FR)+0.5*R1
*11
        PR=0.5*(-131.17+2.32*PI)+0.5*PR
**********************************************************************

* The monetary sector is omitted (See Chapter IV)
*12
C       L1=0.5*(0.14*(M-TW-TC-TN-SC-TR)+76.03*((IL-2.0)**(-0.84)))
C     &  +0.5*L1
*13
C        L2=0.5*(-0.34+0.26*W1-1.02*IS-0.26*(PI-PIL1)+0.61*L2L1)+0.5*L2
**********************************************************************

*14
        IL=0.5*(2.58+0.44*ISL3+0.26*ISL5)+0.5*IL
*15
        IS=0.5*(100*ISL1/(100-11.17+0.67*LB))+0.5*IS
*16
        K=0.5*(I-D+KL1)+0.5*K
*17
        SB=0.5*(SC+SBL1)+0.5*SB
*18
        PY=0.5*(M-W1-W2-R1-R2)+0.5*PY
*19
        M=0.5*(C+I+G-FI-TE-D)+0.5*M
*20
        PI=0.5*((H*NW*WR)/(W1+W2))+0.5*PI
*21
        Q=0.5*(M+TE+D)+0.5*Q
```

```
C SET X1 = NEW VALUES

      X1(1)=C
      X1(2)=I
      X1(3)=SC
      X1(4)=PC
      X1(5)=D
      X1(6)=W1
      X1(7)=NW
      X1(8)=WR
      X1(9)=FI
      X1(10)=R1
      X1(11)=PR
      X1(12)=L1
      X1(13)=L2
      X1(14)=IL
      X1(15)=IS
      X1(16)=K
      X1(17)=SB
      X1(18)=PY
      X1(19)=M
      X1(20)=PI
      X1(21)=Q


      RETURN
      END

****************************************************************

      subroutine update(X0, X1, P, IS, PI)

* Updates values for linking forecasts together

      double precision X0(21), X1(21), P(44), IS(5), PI(2)

* Update lagged endogenous variables


      IS(5)=IS(4)
      IS(4)=IS(3)
      IS(3)=IS(2)
      IS(2)=IS(1)
      IS(1)=X1(15)

      PI(2)=PI(1)
      PI(1)=X1(20)

*CL1
      P(18)=X1(1)
*SCL1
      P(19)=X1(3)
```

```
*PCL1
        P(20)=X1(4)
*DL1
        P(21)=X1(5)
*W1L1
        P(22)=X1(6)
*WRL1
        P(23)=X1(8)
*FIL1
        P(24)=X1(9)
*R1L1
        P(25)=X1(10)
*PRL1
        P(26)=X1(11)
*L1L1
        P(27)=X1(12)
*L2L1
        P(28)=X1(13)
*ISL1
        P(29)=IS(1)
*ISL3
        P(30)=IS(3)
*ISL5
        P(31)=IS(5)
*KL1
        P(32)=X1(16)
*SBL1
        P(33)=X1(17)
*PL1
        P(34)=X1(18)
*PIL1
        P(35)=PI(1)
*PIL2
        P(36)=PI(2)
*QL1
        P(37)=X1(21)

* Update lagged exogenous variables

*TEL1
        P(38)=P(1)
*TWL1
        P(39)=P(2)
*TCL1
        P(40)=P(3)
*TNL1
        P(41)=P(4)
*TRL1
        P(42)=P(5)
*W2L1
        P(43)=P(7)
*R2L1
        P(44)=P(8)
```

```
*TIME TREND (Suppressed for some runs)
C         P(9)=P(9)+1

* Use X1 as new starting guess

*       do 100 i=1,21
*           X0(i)=X1(i)
*100    continue

        return
        end

****************************************************************
```

Appendix B. Equivalence of Multipliers and Least Squares Coefficients for Linear Systems.

Chapter II showed how the coefficients of a first order response surface equation could estimate multipliers for a linear system. Here least squares coefficient will be shown to be equivalent to the multipliers.

Let

Y = the n by m response matrix containing n observations on m endogenous variables

X = the n by k predetermined variable data matrix with k predetermined variables and n observations.

D = the k by m multiplier matrix

Then the linear system can be written

$$Y = XD$$

Let B be the k by m least squares coefficient matrix. B is defined as

$$\begin{aligned} B &= (X'X)^{-1}X'Y \\ &= (X'X)^{-1}(X'X)D \\ &= ID \\ B &= D \end{aligned}$$

Therefore, the least squares coefficients computed for a first order response surface fit to a linear model are identical to the multipliers for the same period.

Appendix C. Decoding Second Order Response Surface Coefficients

After estimating response surface coefficients in terms of the coded experimental design, the response surface equation must be re-expressed in terms of the original variables. Decoding the coded response surface coefficients for a second order response surface equation is time consuming, tedious, and prone to errors. This appendix outlines a method for decoding coded coefficients using matrix algebra which simplifies the decoding process. If a computer with routines capable of matrix inversion and multiplication is available, decoding can be made much easier.

Chapter II gave the formula for translating the ith original decoded variable to coded form.

$$x_{Ci} = \frac{x_{Di} - (x_{i\,max}+x_{i\,min})/2}{(x_{i\,max}-x_{i\,min})/2} \qquad (3.2)$$

where

x_{Ci} = the coded x value,

x_{Di} = the original, decoded x value,

x_{max} = the maximum factor level, and

x_{min} = the minimum factor level.

Let

$\bar{x}_i = (x_{i\,max}+x_{i\,min})/2$

$\Delta x_i = (x_{i\,max}-x_{i\,min})/2$

Eq (3.2) can be rewritten

$$x_{Ci} = \frac{x_{Di} - \bar{x}_i}{\Delta x_i} \qquad (C.1)$$

The coded second order response surface quation is a quadratic form which can be written

$$y = \sum_{i=0}^{k} \sum_{j=0}^{k} b_{cij} x_{ci} x_{cj} \qquad (C.2)$$

where

y = the response variable,

x_{ci} and x_{cj} = the ith and jth coded independent variables,

b_{cij} = the coefficient of the product of the ith and jth coded variables,

k = the number of factors,

x_{co} = 1, and

b_{coo} = the intercept term.

In regression program outputs b_{cij} and b_{cji} are summed because x_{ci} = x_{cj}. Consequently b_{cij} in Eq (C.2) is half the value appearing as a regression coefficient in a regression package output.

By defining appropriate vectors and matricies, Eq (C.2) can be written in matrix form. If k is the number of factors (independent variables) in the response surface equation, then let $\underline{x}_D$ be the k+1 element column vector whose first element is one and the remaining elements are the k decoded independent variables. Similarly, let $\underline{x}_c$ be the k+1 element column vector whose first element is one and the remaining elements are the k coded independent variables. For example,

$$\underline{x}_D = \begin{bmatrix} 1 \\ x_{D1} \\ x_{D2} \\ \cdot \\ \cdot \\ \cdot \\ x_{Dk} \end{bmatrix} \qquad \underline{x}_c = \begin{bmatrix} 1 \\ x_{c1} \\ x_{c2} \\ \cdot \\ \cdot \\ \cdot \\ x_{ck} \end{bmatrix}$$

Let B_C be the matrix whose elements are b_{Cij} and B_D be the matrix whose elements are b_{Dij}. Then Eq (C.2) can be rewritten

$$y = \underline{x}_C B_C x_C$$

Similarly,

$$y = \underline{x}_D^T B_C \underline{x}_D$$

It follows that

$$\underline{x}_C^T B_C \underline{x}_C = \underline{x}_D^T B_D \underline{x}_D \qquad \text{(C.3)}$$

B_D contains the coefficients of the decoded independent variables which are desired. It is convenient to solve for B_D in terms of B_C, x_i, and x_i. Let A be the matrix which transforms $\underline{x}_D$ to $\underline{x}_C$.

$$Ax_D = \underline{x}_C$$

then Eq (C.3) can be rewritten

$$\begin{aligned} \underline{x}_C^T B_C \underline{x}_C &= (Ax_D)^T B_D \underline{A}\underline{x}_D \\ &= \underline{x}_D^T (A^T B_C A) \underline{x}_D \\ &= \underline{x}_D^T B_D \underline{x}_D \end{aligned}$$

Then it follows that

$$B_D = A^T B_C A$$

It will be demonstrated but not proven that

$$A = C^{-1}(I - \bar{\underline{x}}\underline{u}^T) \qquad \text{(C.4)}$$

where

A = the transformation matrix,

C = a k+1 by k+1 diagonal matrix whose ith diagonal element is x_i (define $x_0 = 1$),

I = a k+1 by k+1 identity matrix.

$\bar{\underline{x}}$ = a k+1 element column vector whose first element is zero and the remaining elements are $\bar{\underline{x}}_i$, and

$\underline{u}$ = a k+1 element column vector whose first element is one and the remaining elements are zero.

An example demonstrates the validity of Eq (C.4). If k=2 then

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \Delta x_1 & 0 \\ 0 & 0 & \Delta x_2 \end{bmatrix} \qquad \bar{\underline{x}} = \begin{bmatrix} 0 \\ \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} \qquad \underline{u}^T = [1\ 0\ 0]$$

$$\bar{\underline{x}}\underline{u}^T = \begin{bmatrix} 0 \\ x_1 \\ x_2 \end{bmatrix} [1\ 0\ 0] = \begin{bmatrix} 0 & 0 & 0 \\ \bar{x}_1 & 0 & 0 \\ \bar{x}_2 & 0 & 0 \end{bmatrix}$$

$$I - \underline{x}\underline{u}^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ \bar{x}_1 & 0 & 0 \\ \bar{x}_2 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\bar{x}_1 & 1 & 0 \\ -\bar{x}_2 & 0 & 1 \end{bmatrix}$$

$$C^{-1} = \begin{bmatrix} 1/\Delta x_0 & 0 & 0 \\ 0 & 1/\Delta x_1 & 0 \\ 0 & 0 & 1/\Delta x_2 \end{bmatrix}$$

$$C^{-1}(I - \bar{\underline{x}}\underline{u}^T) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\Delta x_1 & 0 \\ 0 & 0 & 1/\Delta x_2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ -\bar{x}_1 & 1 & 0 \\ -\bar{x}_2 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -\bar{x}_1/\Delta x_1 & 1/\Delta x_1 & 0 \\ -\bar{x}_2/\Delta x_2 & 0 & 1/\Delta x_2 \end{bmatrix}$$

$$C^{-1}(I - \underline{\bar{x}}\underline{u}^T)\underline{x}_D = \begin{bmatrix} 1 & 0 & 0 \\ -\bar{x}_1/\Delta x_1 & 1/\Delta x_1 & 0 \\ -\bar{x}_2/\Delta x_2 & 0 & 1/\Delta x_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & + & 0 & + & 0 \\ -\bar{x}_1/\Delta x_1 & + & x_{D1}/\Delta x_1 & + & 0 \\ -\bar{x}_2/\Delta x_2 & + & 0 & + & x_{D2}/\Delta x_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ (x_{d1} - \bar{x}_1)/\Delta x_1 \\ (x_{d2} - \bar{x}_2)/\Delta x_2 \end{bmatrix}$$

$$= \quad \underline{x}_C$$

by Eq (C.1). Then

$$B_D = [C^{-1}(I-\underline{\bar{x}}\underline{u}^T)]^T B_C [C^{-1}(I-\underline{\bar{x}}\underline{u}^T)] \tag{C.5}$$

which is the desired result. Eq (C.5) is used to decode coded response surface coefficients in this thesis. It saves time and effort.

Appendix D. Decoding the Experimental Design

Coded experimental designs are used to collect data for fitting response surfaces. In order to determine what factor levels to run an experiment or model at, the experimental design must be decoded from 1's, 0's, and -1's to actual factor levels. Below is a FORTRAN program which accomplishes this task. The program reads the coded experimental design and factor levels from user specified files, and writes the coded experimental design, factor levels, and decoded experimental design to a user specified file. At the end of this section is sample output. The output contains the Box and Behnken three level five factor coded experimental design used so extensively in this research effort. The output also contains the factor levels which are the design variable high and low sample values. After the factor levels is the decoded experimental design.

```
************************************************************************

        PROGRAM DECODE

* Translates a coded two level or three level experimental
* design matrix to a design matrix with actual factor levels.
* The factor level file must have MAXIMUM values followed by
* MINIMUM values.IMPORTANT! The data in each of the input
* design files must have the array dimensions (m n) as the
* first line.


*DECLARATIONS*

        integer mD, nD, mF, nF, maxm, maxn,
     &          runit1, runit2, wunit
        parameter(maxm=200,maxn=50)
        double precision D(maxm,maxn), F(maxm,maxn)
        double precision Dc(maxm,maxn)
        character*24 expdes, faclev, outfil
        runit1=1
        runit2=2
```

```
      wunit=3

*USER INPUT*

      call getdat(expdes, faclev, outfil)
      open (runit1, file = expdes, status = 'old' )
      rewind (runit1)
      open (runit2, file = faclev, status = 'old')
      rewind (runit2)
      open (wunit, file = outfil, status = 'new')

*END OF USER INPUT*

      call mread(runit1, D, mD, nD, maxm, maxn)
      write(wunit,1)
1     format(' EXPERIMENTAL DESIGN MATRIX')
      call mwrite(wunit, D, mD, nD, maxm, maxn)
      call mread(runit2, F, mF, nF, maxm, maxn)
      write(wunit,*)' FACTOR LEVEL MATRIX'
      call mwrite(wunit, F, mF, nF, maxm, maxn)
      call switch(D, mD, nD, F, maxm, maxn, Dc)
      write(wunit,*)' DECODED MATRIX'
      call mwrite(wunit, Dc, mD, nD, maxm, maxn)
      write(*,*)' All done!'
      stop
      end

******************SUBROUTINES*****************************************


      subroutine getdat(expdes, faclev, outfil)


*     DESCRIPTION: Requests experimental design file name,
*                  factor level file name, and output file
*                  name from user at terminal.
*
*     INPUT: Input and output file names supplied by the user.
*     OUTPUT: Variable values for expdes, faclev, and outfil.

      character*24 expdes, faclev, outfil

      write(*,*)' Enter coded experimental design file name.'
      read(*,10) expdes
10    format(a24)
      write(*,*)' Enter factor level file name.
      read(*,10) faclev
      write(*,*)' Enter output file name.'
      read(*,10) outfil
50    format(I1)
      return
```

```
      end

**********************************************************************

        subroutine mread(runit, A, m, n, maxm, maxn)
*         DESCRIPTION: Reads dimensions and values for matrix
*                A.
*         INPUT: Values for dimensions and values for elements
*                of matrix A from a file. Max size from calling
*                program.
*         OUTPUT: Matrix A, dimensions m, n to calling program.
*
          integer runit, m, n, maxm, maxn, i, j
          double precision A(maxm, maxn)
*
          read (runit, *) m, n, ((A(i, j), j = 1, n), i = 1, m)
        return
        end
*
*
**********************************************************************

        subroutine mwrite(wunit, A, m, n, maxm, maxn)
*       DESCRIPTION: Writes m, n, and then the matrix A in
*                rows.
*       INPUT: Array A with dimensions m, n and max size maxm, maxn
*              from calling program.
*       OUTPUT: Values for m, n, and elements of A to
*               standard output.
*
          integer m, n, maxm, maxn, i, j, wunit
          double precision A(maxm, maxn)
*
          write(wunit, 1000) m, n
1000      format(' ',I2,' BY ', I2)
          do 1150 i = 1, m
              write(wunit, 1100) (A(i, j), j = 1, n)
1100          format(' ',5E14.5)
1150        continue
          write(wunit,*)
        return
        end
*
*
**********************************************************************

    subroutine switch(Dc, mD, nD, F, maxm, maxn, Dd)
```

```
*       DESCRIPTION: Creates a new array containing factor levels in
*                    place of coded values from the design matrix.
*       INPUT: Coded design matrix, Dc, with dimensions mD by nD,
*              maximum array dimensions maxm, and maxn
*       OUTPUT: Decoded design matrix, Dd


        integer mD, nD, maxm, maxn, i, j
        double precision Dc(maxm,maxn), F(maxm,maxn), Dd(maxm,
     +                   maxn)


        do 40 i = 1, mD
            do 30 j = 1, nD
                Dd(i,j)=(F(1,j)+F(2,j))/2+Dc(i,j)*(F(1,j)-F(2,j))/2
30          continue
40      continue
        return
        end

* End of FORTRAN code.
*************************************************************************
```

Sample Output Of DECODE (Box and Behnken Three Level
Five Factor Design.

EXPERIMENTAL DESIGN MATRIX
41 BY 5

0. e+00	0. e+00	0. e+00	0. e+00	0. e+00
0.10000e+01	0.10000e+01	0. e+00	0. e+00	0. e+00
0.10000e+01	-0.10000e+01	0. e+00	0. e+00	0. e+00
-0.10000e+01	0.10000e+01	0. e+00	0. e+00	0. e+00
-0.10000e+01	-0.10000e+01	0. e+00	0. e+00	0. e+00
0. e+00	0. e+00	0.10000e+01	0.10000e+01	0. e+00
0. e+00	0. e+00	0.10000e+01	-0.10000e+01	0. e+00
0. e+00	0. e+00	-0.10000e+01	0.10000e+01	0. e+00
0. e+00	0. e+00	-0.10000e+01	-0.10000e+01	0. e+00
0. e+00	0.10000e+01	0. e+00	0. e+00	0.10000e+01
0. e+00	0.10000e+01	0. e+00	0. e+00	-0.10000e+01
0. e+00	-0.10000e+01	0. e+00	0. e+00	0.10000e+01
0. e+00	-0.10000e+01	0. e+00	0. e+00	-0.10000e+01
0.10000e+01	0. e+00	0.10000e+01	0. e+00	0. e+00
0.10000e+01	0. e+00	-0.10000e+01	0. e+00	0. e+00
-0.10000e+01	0. e+00	0.10000e+01	0. e+00	0. e+00
-0.10000e+01	0. e+00	-0.10000e+01	0. e+00	0. e+00
0. e+00	0. e+00	0. e+00	0.10000e+01	0.10000e+01
0. e+00	0. e+00	0. e+00	0.10000e+01	-0.10000e+01
0. e+00	0. e+00	0. e+00	-0.10000e+01	0.10000e+01
0. e+00	0. e+00	0. e+00	-0.10000e+01	-0.10000e+01
0. e+00	0.10000e+01	0.10000e+01	0. e+00	0. e+00
0. e+00	0.10000e+01	-0.10000e+01	0. e+00	0. e+00
0. e+00	-0.10000e+01	0.10000e+01	0. e+00	0. e+00
0. e+00	-0.10000e+01	-0.10000e+01	0. e+00	0. e+00
0.10000e+01	0. e+00	0. e+00	0.10000e+01	0. e+00
0.10000e+01	0. e+00	0. e+00	-0.10000e+01	0. e+00
-0.10000e+01	0. e+00	0. e+00	0.10000e+01	0. e+00
-0.10000e+01	0. e+00	0. e+00	-0.10000e+01	0. e+00
0. e+00	0. e+00	0.10000e+01	0. e+00	0.10000e+01
0. e+00	0. e+00	0.10000e+01	0. e+00	-0.10000e+01
0. e+00	0. e+00	-0.10000e+01	0. e+00	0.10000e+01
0. e+00	0. e+00	-0.10000e+01	0. e+00	-0.10000e+01
0.10000e+01	0. e+00	0. e+00	0. e+00	0.10000e+01
0.10000e+01	0. e+00	0. e+00	0. e+00	-0.10000e+01
-0.10000e+01	0. e+00	0. e+00	0. e+00	0.10000e+01
-0.10000e+01	0. e+00	0. e+00	0. e+00	-0.10000e+01
0. e+00	0.10000e+01	0. e+00	0.10000e+01	0. e+00
0. e+00	0.10000e+01	0. e+00	-0.10000e+01	0. e+00
0. e+00	-0.10000e+01	0. e+00	0.10000e+01	0. e+00
0. e+00	-0.10000e+01	0. e+00	-0.10000e+01	0. e+00

FACTOR LEVEL MATRIX
2 BY 5

0.11630e+02	0.13140e+02	0.61700e+02	0.21820e+02	0.10400e+02
0.56300e+01	0.71400e+01	0.37700e+02	0.13820e+02	0.10400e+02

DECODED MATRIX

41 BY 5

0.86300e+01	0.10140e+02	0.49700e+02	0.17820e+02	0.10400e+02
0.11630e+02	0.13140e+02	0.49700e+02	0.17820e+02	0.10400e+02
0.11630e+02	0.71400e+01	0.49700e+02	0.17820e+02	0.10400e+02
0.56300e+01	0.13140e+02	0.49700e+02	0.17820e+02	0.10400e+02
0.56300e+01	0.71400e+01	0.49700e+02	0.17820e+02	0.10400e+02
0.86300e+01	0.10140e+02	0.61700e+02	0.21820e+02	0.10400e+02
0.86300e+01	0.10140e+02	0.61700e+02	0.13820e+02	0.10400e+02
0.86300e+01	0.10140e+02	0.37700e+02	0.21820e+02	0.10400e+02
0.86300e+01	0.10140e+02	0.37700e+02	0.13820e+02	0.10400e+02
0.86300e+01	0.13140e+02	0.49700e+02	0.17820e+02	0.10400e+02
0.86300e+01	0.13140e+02	0.49700e+02	0.17820e+02	0.10400e+02
0.86300e+01	0.71400e+01	0.49700e+02	0.17820e+02	0.10400e+02
0.86300e+01	0.71400e+01	0.49700e+02	0.17820e+02	0.10400e+02
0.11630e+02	0.10140e+02	0.61700e+02	0.17820e+02	0.10400e+02
0.11630e+02	0.10140e+02	0.37700e+02	0.17820e+02	0.10400e+02
0.56300e+01	0.10140e+02	0.61700e+02	0.17820e+02	0.10400e+02
0.56300e+01	0.10140e+02	0.37700e+02	0.17820e+02	0.10400e+02
0.86300e+01	0.10140e+02	0.49700e+02	0.21820e+02	0.10400e+02
0.86300e+01	0.10140e+02	0.49700e+02	0.21820e+02	0.10400e+02
0.86300e+01	0.10140e+02	0.49700e+02	0.13820e+02	0.10400e+02
0.86300e+01	0.10140e+02	0.49700e+02	0.13820e+02	0.10400e+02
0.86300e+01	0.13140e+02	0.61700e+02	0.17820e+02	0.10400e+02
0.86300e+01	0.13140e+02	0.37700e+02	0.17820e+02	0.10400e+02
0.86300e+01	0.71400e+01	0.61700e+02	0.17820e+02	0.10400e+02
0.86300e+01	0.71400e+01	0.37700e+02	0.17820e+02	0.10400e+02
0.11630e+02	0.10140e+02	0.49700e+02	0.21820e+02	0.10400e+02
0.11630e+02	0.10140e+02	0.49700e+02	0.13820e+02	0.10400e+02
0.56300e+01	0.10140e+02	0.49700e+02	0.21820e+02	0.10400e+02
0.56300e+01	0.10140e+02	0.49700e+02	0.13820e+02	0.10400e+02
0.86300e+01	0.10140e+02	0.61700e+02	0.17820e+02	0.10400e+02
0.86300e+01	0.10140e+02	0.61700e+02	0.17820e+02	0.10400e+02
0.86300e+01	0.10140e+02	0.37700e+02	0.17820e+02	0.10400e+02
0.86300e+01	0.10140e+02	0.37700e+02	0.17820e+02	0.10400e+02
0.11630e+02	0.10140e+02	0.49700e+02	0.17820e+02	0.10400e+02
0.11630e+02	0.10140e+02	0.49700e+02	0.17820e+02	0.10400e+02
0.56300e+01	0.10140e+02	0.49700e+02	0.17820e+02	0.10400e+02
0.56300e+01	0.10140e+02	0.49700e+02	0.17820e+02	0.10400e+02
0.86300e+01	0.13140e+02	0.49700e+02	0.21820e+02	0.10400e+02
0.86300e+01	0.13140e+02	0.49700e+02	0.13820e+02	0.10400e+02
0.86300e+01	0.71400e+01	0.49700e+02	0.21820e+02	0.10400e+02
0.86300e+01	0.71400e+01	0.49700e+02	0.13820e+02	0.10400e+02

Appendix E. Response Surface Coefficients

Tables E.1a-k below contain both coded and decoded coefficient matricies for the second degree polynomial response surface equations described in Chapter V. The matrix algebra method in Appendix D was used to decode the coded coefficients. Consequently, off diagonal elements in Tables E.1a-k are half of the value normally given as a coefficient for a polynomial (See Appendix D.). The column and row marked with a 1 contain first degree term coefficients. The upper right hand corner element in the tables is the intercept term.

Table E.1a. Coded Second Order Polynomial Response Surface Coefficients for Number of Workers Employed in Period Zero

(R^2 = 1.0000, Adjusted R^2 = 1.0000)

	1	T_w	T_c	G	W_2	N_G
1	44.1080	-0.9029	-0.1937	4.6116	-0.8731	1.6890
T_w	-0.9029	0.0000	0.0000	0.0039	-0.0033	0.0000
T_c	-0.1937	0.0000	0.0000	0.0026	0.0000	0.0000
G	4.6116	0.0039	0.0026	-0.0088	0.0069	-0.0042
W_2	-0.8731	-0.0033	0.0000	0.0069	0.0000	0.0000
N_G	1.6890	0.0000	0.0000	-0.0042	0.0000	0.0000

Table E.1b. Decoded Second Order Polynomial Response Surface Coefficients for Number of Workers Employed in Period Zero

(R^2 = 1.0000, Adjusted R^2 = 1.0000)

	1	T_w	T_c	G	W_2	N_G
1	25.2089	-0.1686	-0.0345	0.3059	-0.1366	0.4989
T_w	-0.1686	0.0000	0.0000	0.0000	-0.0001	0.0000
T_c	-0.0345	0.0000	0.0000	0.0000	0.0000	0.0000
G	0.3059	0.0000	0.0000	0.0000	0.0001	-0.0001
W_2	-0.1366	-0.0001	0.0000	0.0001	0.0000	0.0000
N_G	0.4989	0.0000	0.0000	-0.0001	0.0000	0.0000

Table E.1c. Coded Second Order Polynomial Response Surface Coefficients for Price Index in Period Zero

(R^2 = 1.0000, Adjusted R^2 = 1.0000)

	1	T_w	T_c	G	W_2	N_G
1	134.5839	-2.1629	-0.4619	11.0416	-9.4508	6.1408
T_w	-2.1629	0.0000	0.0000	0.1592	0.0776	0.0000
T_c	-0.4619	0.0000	0.0000	0.0398	0.0000	0.0000
G	11.0416	0.1592	0.0398	-0.7819	-0.4222	-0.1191
W_2	-9.4508	0.0776	0.0000	-0.4222	1.8014	-0.6453
N_G	6.1408	0.0000	0.0000	-0.1191	-0.6453	0.1668

Table E.1d. Decoded Second Order Polynomial Response Surface Coefficients for Price Index in Period Zero

(R^2 = 1.0000, Adjusted R^2 = 1.0000)

	1	T_w	T_c	G	W_2	N_G
1	100.3344	-0.4786	-0.0927	0.8742	-1.5949	2.0708
T_w	-0.4786	0.0000	0.0000	0.0020	0.0022	0.0000
T_c	-0.0927	0.0000	0.0000	0.0005	0.0000	0.0000
G	0.8742	0.0020	0.0005	-0.0034	-0.0043	-0.0023
W_2	-1.5949	0.0022	0.0000	-0.0043	0.0431	-0.0294
N_G	2.0708	0.0000	0.0000	-0.0023	-0.0294	0.0144

Table E.1e. Coded Second Order Polynomial Response Surface Coefficients for Gross National Product in Period Zero

(R^2 = 1.0000, Adjusted R^2 = 1.0000)

	1	T_w	T_c	G	W_2	N_G
1	113.3169	-2.0454	-0.4398	10.4517	1.2531	-0.0251
T_w	-2.0454	0.0000	0.0000	0.0088	-0.0073	0.0044
T_c	-0.4398	0.0000	-0.0008	0.0051	0.0000	0.0000
G	10.4517	0.0088	0.0051	-0.0186	0.0151	-0.0094
W_2	1.2531	-0.0073	0.0000	0.0151	0.0000	0.0000
N_G	-0.0251	0.0044	0.0000	-0.0094	0.0000	0.0000

Table E.1f. Decoded Second Order Polynomial Response Surface Coefficients for Gross National Product in Period Zero

(R^2 = 1.0000, Adjusted R^2 = 1.0000)

	1	T_w	T_c	G	W_2	N_G
1	70.6007	-0.3836	-0.0780	0.6933	0.1904	-0.0033
T_w	-0.3836	0.0000	0.0000	0.0001	-0.0002	0.0002
T_c	-0.0780	0.0000	0.0000	0.0001	0.0000	0.0000
G	0.6933	0.0001	0.0001	-0.0001	0.0002	-0.0002
W_2	0.1904	-0.0002	0.0000	0.0002	0.0000	0.0000
N_G	-0.0033	0.0002	0.0000	-0.0002	0.0000	0.0000

Table E.1g. Coded Second Order Polynomial Response Surface Coefficients for Number of Workers Employed in Period Five

(R^2 = 1.0000, Adjusted R^2 = 0.9999)

	1	T_w	T_c	G	W_2	N_G
1	49.4412	-3.4778	-3.5475	13.4030	-1.2750	1.4146
T_w	-3.4778	0.0000	-0.0939	0.4135	-0.1006	0.0000
T_c	-3.5475	-0.0939	0.0000	0.4108	-0.0934	0.0750
G	13.4030	0.4135	0.4180	-1.2752	0.3521	-0.2912
W_2	-1.2750	-0.1006	-0.0934	0.3521	0.0000	0.0000
N_G	1.4164	0.0000	0.0750	-0.2912	0.0000	0.0000

Table E.1h. Decoded Second Order Polynomial Response Surface Coefficients for Number of Workers Employed in Period Five

(R^2 = 1.0000, Adjusted R^2 = 0.9999)

	1	T_w	T_c	G	W_2	N_G
1	10.3560	-0.7353	-0.7372	0.9930	-0.2682	0.5433
T_w	-0.7353	0.0000	-0.0030	0.0051	-0.0029	0.0000
T_c	-0.7350	-0.0030	0.0000	0.0047	-0.0025	0.0038
G	0.9925	0.0051	0.0048	-0.0056	0.0036	-0.0057
W_2	-0.2682	-0.0029	-0.0025	0.0036	0.0000	0.0000
N_G	0.5439	0.0000	0.0038	-0.0057	0.0000	0.0000

Table E.1i. Coded Second Order Polynomial Response Surface Coefficients for Price Index in Period Five

(R^2 = 0.9998, Adjusted R^2 = 0.9997)

	1	T_w	T_c	G	W_2	N_G
1	185.6951	-14.8952	-12.8819	59.6286	-18.4974	15.2612
T_w	-14.8952	0.0000	0.0000	0.0000	0.0000	0.0000
T_c	-12.8819	0.0000	0.0000	0.8976	0.0000	0.0000
G	59.6286	0.0000	0.8976	0.0000	-2.5389	1.9044
W_2	-18.4974	0.0000	0.0000	-2.5389	4.0012	-2.5593
N_G	15.6212	0.0000	0.0000	1.9044	-2.5593	2.3429

Table E.1j. Decoded Second Order Polynomial Response Surface Coefficients for Price Index in Period Five

(R^2 = 0.9998, Adjusted R^2 = 0.9997)

	1	T_w	T_c	G	W_2	N_G
1	23.1609	-2.7764	-2.5197	3.8950	-2.3456	3.2888
T_w	-2.7764	0.0000	0.0000	0.0000	0.0000	0.0000
T_c	-2.5197	0.0000	0.0000	0.0104	0.0000	0.0000
G	3.8950	0.0000	0.0104	0.0000	-0.0260	0.0371
W_2	-2.3456	0.0000	0.0000	-0.0260	0.0957	-0.1164
N_G	3.3947	0.0000	0.0000	0.0371	-0.1164	0.2027

Table E.1k. Coded Second Order Polynomial Response Surface Coefficients for Gross National Product in Period Five

(R^2 = 1.0000, Adjusted R^2 = 1.0000)

	1	T_w	T_c	G	W_2	N_G
1	141.3628	-9.2409	-10.6323	36.0425	-0.3852	0.7046
T_w	-9.2408	0.0000	-0.2276	0.9903	-0.2246	0.1707
T_c	-10.6323	-0.2276	0.0000	0.9796	-0.2342	0.1769
G	36.0425	0.9903	0.9796	-3.0659	0.8543	-0.7019
W_2	-0.3852	-0.2446	-0.2342	0.8543	0.0000	0.1568
N_G	0.7046	0.1707	0.1769	-0.7019	0.1568	0.0000

Table E.11. Decoded Second Order Polynomial Response Surface Coefficients for Gross National Product in Period Five

(R^2 = 1.0000, Adjusted R^2 = 1.0000)

	1	T_w	T_c	G	W_2	N_G
1	43.4207	-1.9947	-2.1269	2.6404	-0.2824	0.4108
T_w	-2.0001	0.0000	-0.0074	0.0122	-0.0065	0.0094
T_c	-2.1269	-0.0074	0.0000	0.0113	-0.0063	0.0091
G	2.6404	0.0122	0.0113	-0.0134	0.0088	-0.0137
W_2	-0.2805	-0.0071	-0.0063	0.0088	0.0000	0.0071
N_G	0.4108	0.0094	0.0091	-0.0137	0.0071	0.0000

Tables E.2a-e contain decoded coefficients for first order polynomial response surfaces. They may be compared directly to multipliers computed by Goldberger (Goldberger, 1959).

Table E.2a. First Order Response Surface Coefficients for a Unit Increase in T_w.

	Period					
	0	1	2	3	4	5
N_w	-0.3367	-0.7538	-0.1081	-1.2729	-1.3347	1.2937
p	-0.8293	-1.9435	-3.0677	-4.1201	.0475	-5.8117
Q	-0.7631	-0.1738	-2.5663	-3.12342	-3.4011	-3.4391

Table E.2b. First Order Response Surface Coefficients for a Unit Increase in T_c.

	Period					
	0	1	2	3	4	5
N_w	-0.6769	-0.5968	-0.1038	-1.2756	-1.3243	-1.2333
p	-0.1671	-1.4552	-2.6269	-3.5301	-4.1661	-4.5498
Q	-0.1532	-0.1418	-2.5847	-3.3451	-3.6914	-3.7003

Table E.2c. First Order Response Surface Coefficients for a Unit Increase in G.

	Period					
	0	1	2	3	4	5
N_w	0.6110	0.1214	0.1621	1.8268	1.8623	1.7715
p	1.5050	3.1437	4.6727	6.0734	7.3022	8.3124
Q	0.13848	0.2804	3.8707	4.5208	4.47952	4.7662

Table E.2d. First Order Response Surface Coefficients for a Unit Increase in W_2.

	Period					
	0	1	2	3	4	5
N_w	-0.2705	-0.4465	-0.5115	-0.5069	-0.4599	-0.3919
p	-2.9658	-4.5079	-5.3932	-5.9814	-6.4147	-6.7606
Q	+0.3867	-0.3642	-0.2389	-0.2863	-0.2314	-0.1149

Table E.2e. First Order Response Surface Coefficients for a Unit Increase in N_G.

	Period					
	0	1	2	3	4	5
N_W	0.9940	0.9776	0.9500	0.9127	0.8679	0.8218
p	3.6989	5.8267	7.3436	8.5786	9.6579	10.6483
Q	-0.0137	-0.5117	-0.1159	-0.2063	-0.3170	-0.4369

Apendix F. Optimization Problem Solution

Shown below is the output file of the linear programming package for the optimization problem formulated in Chapter VI. The output includes a problem specification, optimal basic variable values, shadow prices, and the objective function value.

```
 Problem Specified for Solution

 Maximize

          TW       TC       G        W2
          X  1    X  2    X  3    X  4    B 19
  Obj
           -3.11    -3.33     4.50    -0.29     54.40

 Constraint   1 - max TW   type is le
            1.00     0.       0.       0.       11.63
 Constraint   2 - max TC   type is le
            0.       1.00     0.       0.       13.14
 Constraint   3 - max G    type is le
            0.       0.       1.00     0.       61.70
 Constraint   4 - max W2   type is le
            0.       0.       0.       1.00     21.82
 Constraint   5 - inflatn  type is le
           -4.61    -4.12     6.68    -5.66     96.79
 Constraint   6 - unemploy type is eq
           -1.25    -1.26     1.80    -0.50     45.77
 Constraint   7 - min TW   type is gt
            1.00     0.       0.       0.        5.63
 Constraint   8 - min TC   type is gt
            0.       1.00     0.       0.        7.14
 Constraint   9 - min G    type is gt
            0.       0.       1.00     0.       37.70
 Constraint  10 - min W2   type is gt
            0.       0.       0.       1.00     13.82

 Activity variables 1 through  4
   Slack variables (S)   5 through   9
   Surplus variables (P)  10 through  13
   Artificial variables (A)  14 through  18
```

Answers:

Basic Variables		Value
X 1 : TW	=	11.6300
X 2 : TC	=	7.1400
X 3 : G	=	44.5489
X 4 : W2	=	21.8200
S 6 : max TC	=	6.0000
S 7 : max G	=	17.1511
S 9 : inflatn	=	5.4738
S10 : unemploy	=	6.0000
S12 : min TC	=	6.8489
S13 : min G	=	8.0000

Increase in Obj. Function for unit increase in
right hand side of constraints

Shadow Prices		Value
Y 1 : max TW	=	0.0246
Y 4 : max W2	=	0.9688
Y 7 : min TW	=	0.1925
Y10 : min W2	=	2.4985

The value of the objective function is: 185.8114

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Vita

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