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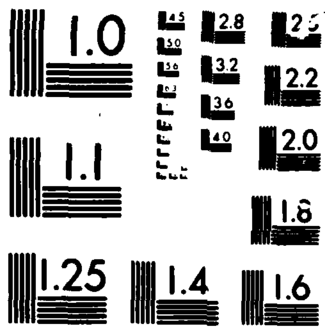

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This is the final report of a project carried out in the Applied Mathematics Group of the Department of Mathematics at Stanford University. Results were obtained on the reflection of waves from rough boundaries. Novel features are the calculation of second moments of the scattered field and the relation of the results to those of Twersky. In addition the effective viscosity tensor of a periodic suspension and the effective elasticity tensor of a periodic composite were calculated for all concentrations up to close packing. The acoustoelastic effect has also been analyzed. Various new results on inverse scattering have been obtained in two and three dimensions.

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Mathematical Problems of Nonlinear Wave Propagation and of Waves in Heterogeneous Media

Final Report

October 1, 1983 — September 30, 1984

Professor Joseph B. Keller

March 1985

Air Force Office of Scientific Research

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Stanford University

I. BRIEF OUTLINE OF RESEARCH FINDINGS

During the last year we have worked on problems of wave propagation in random media, reflection of waves from rough surfaces, effective parameters of heterogeneous materials, inverse scattering theory, acoustoelasticity, Hill's equation and some other topics. We shall describe some of this work in this section. Most of the rest of our completed work, which is not described here, is contained in the research papers which are listed in Section II. Some of the articles listed there have been published already, others have been accepted for publication and are now in press and some have been submitted for publication but have not yet been accepted. The status of each paper is indicated after its title, and the references to the published papers are also given.

Concerning waves in random media, Dr. Graham Eatwell and Professor J.B. Keller have treated waves in a medium of randomly distributed discrete scatterers. They have used a method like the path integral procedure used by Dashen to find the moments of a field in a continuous random medium. A paper on this work is in preparation.

Dr. John G. Watson and Professor Keller have published two papers, Nos. 83 and 112, on reflection of waves from rough boundaries. They are now completing a companion paper on rough interfaces. The novel features are the calculation of second moments of the scattered field, and the relation of the results to Twersky's method.

The effective viscosity of a periodic suspension and the effective elasticity tensor of a periodic composite have been calculated by Dr. Kevin C. Nunan and Professor Keller in papers Nos. 100 and 101. The new features are that all components of the viscosity and elasticity tensors are found at all concentrations up to close packing. Related work on effective conductivity is being done by Mr. John Nevard and Professor Keller. So far the reciprocal theorem has been extended to anisotropic media.

In inverse scattering theory Dr. Margaret Cheney has developed the two and three dimensional theory in a number of papers (86, 99, 125), and applied it in several others (104, 123, 124) to problems which are more realistic than many which were treated previously. Dr. John Fawcett has also developed some theoretical results on stability (116) and on inversion of spherical means (121) as well as several more applied results (117, 118, 120). Professor Stephanos Venakides has employed the inverse scattering method to solve the Korteweg-de Vries equaion in the zero dispersion limit (in paper No. 106) and in a manuscript in preparation. Mr. William Boyse and Professor Keller are analyzing the inverse problem for elastic media by the Born approximation.

Dr. Luis L. Bonilla has studied the effective elastic constants of aggregates of crystallites, each with cubic symmetry, by using invariance and symmetry methods (115). He and Professor Keller have also treated wave propagation and the acoustoelastic effect in such media (119).

Dr. Michael I. Weinstein and Professor Keller have successfully analyzed Hill's equation with a large potential and found the behavior of the stability bands in paper 122 and in a paper in preparation.

III. ABSTRACTS OF MANUSCRIPTS SUBMITTED DURING REPORT PERIOD

Newton's Second Law, by J.B. Keller

Newton's second law of motion is usually written in the form $\vec{f} = m\vec{a}$. Here $\vec{a}$ denotes the acceleration of a point particle, m denotes the mass of the particle, and $\vec{f}$ denotes the force acting on the particle. We wish to elucidate the empirical content of this law, and in particular to indicate whether it embodies a definition of force, a definition of mass, or a definition of both force and mass.

Effective Elastic Constants of Polycrystalline Aggregates, L.L. Bonilla

A method is presented for the determination of the effective elastic constants of a transversely isotropic aggregate of crystallites with cubic symmetry. The results obtained generalize those given in the literature for the second and third order elastic constants. In addition, the second moments and the binary angular correlations of the second order stiffnesses are obtained. It is also explained how these moments can be used to find the two-point correlations of the elastic constants.

On the Stability of Inverse Scattering Problems, J.A. Fawcett

Some aspects of the time-dependent inverse scattering problem are discussed. Then some simple stability results are presented. Several analytical and numerical examples are given to illustrate them.

Two Dimensional Modelling and Inversion of the Acoustic Wave Equation in Inhomogeneous Media, J.A. Fawcett

In this paper we outline a fast and efficient method for the finite difference calculation of solutions to the two dimensional acoustic wave equation in an inhomogeneous medium. We then implement our forward modeling scheme in an inversion procedure to determine the two dimensional velocity variation. A numerical example is given.

Three Dimensional Ray Tracing and Geophysical Inversion in Layered Media, J.A. Fawcett and H.B. Keller

In this paper the problem of finding seismic rays in a three dimensional layered medium is examined. The "layers" are separated by arbitrary smooth interfaces that can vary in three dimensions. The endpoints of each ray and the sequence of interfaces it encounters are specified. The problem is formulated as non-linear system of equations and efficient, accurate methods of solution are discussed. An important application of ray tracing methods, which is discussed, is the non-linear least squares estimation of medium parameters from observed travel times. In addition the "type" of each ray is also determined by the least squares process — this is in effect a deconvolution procedure similar to that desired in seismic exploration. It enables more of the measured data to be used without filtering out the multiple reflections that are not pure P-waves.

Acoustoelastic Effects and Sound Wave Propagation in Heterogeous Anisotropic Materials, L.L. Bonilla and J.B. Keller

A theoretical analysis of acoustoelastic effects is presented. It is based upon the theory of sound wave propagation in a stressed heterogeneous anisotropic elastic medium composed of grains. The effect of residual stress is included, and shown to be different from that of applied stress. The statistics of grain orientation and grain correlation are taken into account. The acoustoelastic coefficients and the effects of dispersion, attenuation and texture of the medium are determined.

Tomographic Reconstruction of Velocity Anomalies, J.A. Fawcett and R.W. Clayton

An approximate inversion formula is proposed for the reconstruction of slowness anomalies in a known depth varying background field. The data are observed traveltime perturbations for reflections from a known planar reflector. The limitations of the formula are discussed and numerical examples are given.

Inversion of N-dimensional Spherical Averages, J.A. Fawcett

In this paper we give a simple inversion formula for determining the even part of a function from its averages over the surfaces of spheres in $\mathbb{R}^n$, centred on a hyperplane $y = 0$. We first give the formula in Fourier space and then in physical space as a generalized convolutional back-projection.

Hill's Equation with a Large Potential, M.I. Weinstein and J.B. Keller

We obtain asymptotic expansions, for large values of the parameter λ, of the stability boundaries, the stability band widths, the Floquet multipliers and the solutions of Hill's equation

$$\left[-\frac{d^2}{dx^2} + \lambda^2 q(x)\right] u = Eu.$$

The potential $q(x)$ is assumed to be periodic and to have a unique global minimum within each period, at which $q'' > 0$. The results for the stability band widths show that they decay exponentially with λ as λ increases. These results generalize those for symmetric potentials due to Harrell, and that for the Mathieu equation due to Meixner and Schäfke.

Inversion of the 2.5-d Acoustic Equation, M. Cheney, S. Coen and A. Weglein

An inverse acoustic scattering theory and algorithm is presented for the reconstruction of a two dimensional inhomogeneous acoustic medium from surface measurements. The measurements of the surface pressure due to a harmonically oscillating surface point source at two arbitrary frequencies allows the separate reconstruction of the density and velocity of the subsurface. The solution of the inverse acoustic scattering problem is obtained by a transformation to an inverse quantum scattering problem in two dimension, the solution of which has been obtained recently by Cheney, who was inspired by Newton's solution in three dimensions. The solution presented is a generalization of Coen's and Stickler and Deift's solutions of the three dimensional acoustic problem for a plane stratified acoustic medium.

The Connection Between Time- and Frequency-domain Three Dimensional Inverse Scattering Methods, M. Cheney, J.H. Rose, and B. DeFacio

We relate three dimensional scattering theory for the time-independent Schrödinger equation without spherical symmetry to scattering theory for the plasma wave equation (PWE). We review a recent inverse scattering method for the PWE and find the corresponding method for the Schrödinger equation. We then review Newton's three dimensional Marchenko method for the Schrödinger equation and transform it to the corresponding

PWE method. The resulting time-domain hyperbolic method clarifies the role of causality in Newton's important recent work.

A Rigorous Derivation of the "Miracle" Identity of Three Dimensional Inverse Scattering, M. Cheney

The large-energy asymptotic behavior of scattering solutions of the three dimensional time-dependent Schrödinger equation is investigated. The second term of the expansion leads to the "*miracle*" of Newton's three dimensional inverse scattering theory.

Small Gap Flows, E.O. Tuck

This report consists of five chapters, namely:

1. Airfoils in Extreme Ground Effect, *pp*. 2-6.
2. Automobile Aerodynamics, *pp*. 7-11.
3. Shallow-water Waves at Discontinuities, *pp*. 12-16.
4. Sliding Sheets, *pp*. 17-20.
5. Wings Over Water, *pp*. 21-24.

These chapters are independent of each other, although linked by the "small-gap" theme. Chapters 1-4 were originally presented in the form of a lecture series in the Department of Naval Architecture and Offshore Engineering, University of California at Berkeley, during February and March 1984. Chapter 5 formed the basis for a presentation to the panel H5 (Analytical Ship-Wave Relations) of the Society of Naval Architects and Marine Engineers, on March 20th, 1984.

Research for and preparation of this report was carried out during a period spent by the author in the Department of Mathematics, Stanford University, January-May 1984.

Discriminant, Transmission Coefficient, and stability bands of Hill's equation, J.B. Keller

The discriminant $\Delta(k^2)$ of Hill's equation is shown to be related to the transmission coefficient $T(k)e^{i\theta(k)}$ of one period of the potential by $\Delta(k^2) = [2/T(k)]\cos[k\pi + \theta(k)]$. This result is used to find the boundaries of the stability bands.

One Hundred Years of Diffraction Theory, J.B. Keller

The development of diffraction theory in the last 100 years is discussed from a personal viewpoint, with emphasis on the geometrical theory of diffraction. First some early work of Kirchhoff, Rayleigh, Sommerfeld, MacDonald and others is mentioned to indicate the state of the field in the 1940's. Next the author's work during World War II is described. Then the considerations that led him to the geometrical theory of diffraction are explained, and the defects of that theory are outlined. Finally the advances in the theory since its introduction, which have remedied many of these defects, are mentioned.

Solition Generation and Nonlinear Wave Propagation, J.B. Keller

Solition generation by various means is described. First experimental results of Wehausen et al on solitons generated by a ship model in a towing tank are presented. Then T.Y. Wu's related Boussinesq system of equations for shallow water motion with a moving pressure distrubance on the free surface is introduced. Numerical solutions of this system by Wu and Wu are shown to compare well with the experimental results. Similar numerical results on an initial-boundary value problem for the KdV equation by Chu et al are presented which also yield soliton generation. Then Keener and Rinzel's analysis of pulse generation in the Fitzhugh-Naqumo model of nerve condutions is described. Next Whitham's modulation theory of nonlinear wave propagation is explained and the problem of relating its results to initial and boundary data is mentioned. Asymptotic methods for solving this problem for the KdV equation are described. They include the Lax-Levermore theory for the case of small dispersion, its extension by Venakides, and the centered simple wave solution of the modulation equations by Gurevich and Pitaevskii. Finally the theory of weakly non-linear waves of Choquet-Bruhat and of Hunter and Keller is described.

Computers and Chaos in Mechanics, J.B. Keller

Much recent progress in mechanics is related to the use of computers in the solution of problems, in the control of experiments, and in the analysis of data. These tools have made certain kinds of calculations and measurements easier. But they have also revealed the widespread occurrence of chaotic and stochastic behavior of mechanical systems, and have shown that there are certain regularities in this behavior. The understanding of the chaotic behavior and its regularites, and of how to analyze and control it, are unsolved problems. Many theoretical attacks have been made on these problems, but they have only touched the surface.

Semiclassical Mechanics, J.B. Keller

Classical mechanics and the quantum conditions of Planck, Bohr, Sommerfeld, Wilson and Einstein are presented. The virtues and defects of this "old quantum theory" are pointed

out. Its replacement by quantum mechanics is described, leading to the Schrödinger equation for the wave function and the corresponding energy eigenvalues. For separable systems, the reduction of this equation to ordinary differential equations and their asymptotic solution by the WKB method are described, as well as the resulting corrected quantum conditions with integer or half-integer quantum numbers. For nonseparable systems, the analogous asymptotic solution constructed by the author is described, together with the corrected quantum conditions to which it leads. Examples of the use of these conditions in the solution of eigenvalue problems are presented. It is explained that difficulties arise in using this method when the classical motion is stochastic or chaotic. Suggestions for overcoming these difficulties are mentioned.

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99.	M. Cheney	Inverse scattering in dimension two Pub: J. Math. Phys., 25 (1), 94-107, 1984.
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108.	J. B. Keller	Genetic Variability Due to Geographical Inhomogeneity Acc: J. Math. Biol., in press.
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114.	J.B. Keller	Newton's second law Sub: Am. J. Physics

115. L.L. Bonilla — Effective elastic constants of polycrystalline aggregates

Sub: J. Mech. and Phys. Solids

116. J. Fawcett — On the Stability of Inverse Scattering Problems

Acc: Wave Motion

117. J. Fawcett — Two Dimensional Modelling and Inversion of the Acoustic Wave Equation in Inhomogeneous Media

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118. J. Fawcett, H.B. Keller — Three Dimensional Ray Tracing and Geophysical Inversion in Layered Media

Sub: SIAM J. Appl. Math.

119. L.L. Bonilla, J.B. Keller — Acousto-elastic effects and sound wave propagation in heterogeous anisotropic materials

Sub: J. Mech. and Phys. Solids

120. J. Fawcett, R.W. Clayton — Tomographic Reconstruction of Velocity Anomalies

Sub: Bulletin of the Seismological Soc. Am.

121. J. Fawcett — Inversion of *N Dimensional Spherical Averages*

Sub: SIAM J. Appl. Math.

122. M.I. Weinstein, J.B. Keller — Hill's equation with a large potential

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123. M. Cheney, S. Coen, A. Weglein — Inversion of the 2.5-D Acoustic Equation

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124. M. Cheney, J.H. Rose, B. DeFacio — The connection between time- and frequency- domain three-dimensional inverse scattering methods

Acc: J. Math. Phys.

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Acc: J. Math. Phys.

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127. J.B. Keller — Discriminant, transmission coefficient and stability bands of Hill's equation

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Acc: Trans. IEEE PGAP

129. J.B. Keller — Soliton generation and nonlinear wave propagation

Acc: Phil. Trans. R. Soc.

130. J.B. Keller — Semiclassical Mechanics

Sub: SIAM Review

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