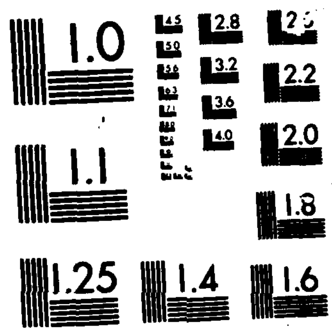
AD-A165 837 ANALYTICAL METHODS FOR CHARACTERIZATION OF NONLINEAR DEVICES AND NETWORKS..(U) UNIVERSITY OF SOUTH FLORIDA TAMPA DEPT OF ELECTRICAL ENGINEER.. V K JAIN ET AL. 1/1

UNCLASSIFIED DEC 85 RADC-TR-85-243-VOL-1 F/G 17/2 NL



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AD-A165 837

RADC-TR-85-243, Vol I (of three)
Final Technical Report
December 1985

ANALYTICAL METHODS FOR CHARACTERIZATION OF NONLINEAR DEVICES AND NETWORKS

University of South Florida

V. K. Jain, S. J. Garrett and A. R. Gondeck

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ROME AIR DEVELOPMENT CENTER
Air Force Systems Command
Griffiss Air Force Base, NY 13441-5700

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RADC-TR-85-243, Vol I (of three) has been reviewed and is approved for publication.

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UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE

ADA165837

REPORT DOCUMENTATION PAGE

1a REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS N/A
2a SECURITY CLASSIFICATION AUTHORITY N/A 2b DECLASSIFICATION / DOWNGRADING SCHEDULE N/A	3 DISTRIBUTION / AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
4 PERFORMING ORGANIZATION REPORT NUMBER(S) N/A	5 MONITORING ORGANIZATION REPORT NUMBER(S) RADC-TR-85-243, Vol I (of three)
6a NAME OF PERFORMING ORGANIZATION University of South Florida 6b OFFICE SYMBOL (If applicable)	7a NAME OF MONITORING ORGANIZATION Rome Air Development Center (RBCT)
6c. ADDRESS (City, State, and ZIP Code) Department of Electrical Engineering Tampa FL 33620	7b ADDRESS (City, State, and ZIP Code) Griffiss AFB NY 13441-5700
8a. NAME OF FUNDING / SPONSORING ORGANIZATION Rome Air Development Center 8b OFFICE SYMBOL (If applicable) RBCT	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER F30602-82-C-0135
8c. ADDRESS (City, State, and ZIP Code) Griffiss AFB NY 13441-5700	10 SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO 62702F · PROJECT NO 2338 · TASK NO 03 · WORK UNIT ACCESSION NO 42

11 TITLE (Include Security Classification)
ANALYTICAL METHODS FOR CHARACTERIZATION OF NONLINEAR DEVICES AND NETWORKS

12 PERSONAL AUTHOR(S)
V. K. Jain, S. J. Garrett, A. R. Gondeck

13a. TYPE OF REPORT	13b TIME COVERED	14 DATE OF REPORT (Year, Month, Day)	15 PAGE COUNT
Final	FROM Apr 83 TO Apr 84	December 1985	60

16 SUPPLEMENTARY NOTATION
N/A

17 COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	
09	03		Nonlinearities With Memory; Input Scaling; Frequency Dependent Nonlinearities, Nonlinear Circuits
09	01		Coupling Paths; Nonlinear Devices

19 ABSTRACT (Continue on reverse if necessary and identify by block number)
Multichannel communication systems are often mildly nonlinear and can be characterized by a truncated Volterra series. The purpose of this report is to present, in a practical way, techniques for effective representation of nonlinear circuits/systems by Volterra NLFTs. The "nonlinear current method" is applied to various communication type problems. These include systems of nonlinear differential equations, nonlinear devices, and nonlinear circuits.

20 DISTRIBUTION / AVAILABILITY OF ABSTRACT ☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT ☐ DTIC USERS	21 ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED
22a NAME OF RESPONSIBLE INDIVIDUAL Daniel J. Kenneally	22b TELEPHONE (Include Area Code) (315) 330-2519 22c OFFICE SYMBOL RADC (RBCT)

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete

SECURITY CLASSIFICATION OF THIS PAGE
UNCLASSIFIED

17. COSATI CODES (Continued)

Field	Group
17	04

18. SUBJECT TERMS (Continued)

Diode Model
Transistor Model
Nonlinear Compensation
Junction Capacitance

ACKNOWLEDGMENTS

The authors wish to thank Mr. D. J. Kenneally of RADC for his helpful criticism and suggestions throughout the duration of this project. His comments on the research initiatives at RADC for C^3I system interference-suppression, based on Volterra identification and compensation, have lent invaluable insights to this research effort.

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I. INTRODUCTION

Most circuits in a typical C^3I communication system are nonlinear to some degree. Examples include preamplifiers, mixers, frequency-converters, channel paths containing metal-to-metal-oxide junctions, and in particular power amplifiers. In addition to these inherent nonlinearities, there may be nonlinearities deliberately introduced for the purpose of minimizing the effects of the inherent ones. These circuits usually fall into a class which may be described as "mildly nonlinear" [4] circuits. Since these circuits generally have memory, a simple power series characterization is usually inadequate. However, a Volterra series expansion [1]-[6], which is a generalization of the power series, provides a very versatile characterization of a nonlinear circuit, subsystem, or an entire system [9]-[14]. Furthermore, the Volterra characterization is compact for mild nonlinearities in the sense that a truncated Volterra series can adequately describe both the amplitude and memory behavior of the system.

To the reader familiar with Volterra expansions, the basic system entity is the Volterra kernel $h_k(\tau_1,\tau_2,\cdots,\tau_k)$. Its Fourier transform $H_k(f_1,f_2,\cdots,f_k)$ is known as the k-th order nonlinear transfer function (NLTF) [4],[8]. The analyst of a C^3I communication system (and the EMC engineer responsible for the design and implementation of the nonlinear compensators for these systems) should acquire familiarity with techniques for deriving the Volterra NLTFs from the circuit or its equivalent description. The purpose of this report is to present, in a practical way, some techniques for effective representation of nonlinear circuits/systems by Volterra NLTFs.

II. BACKGROUND

Numerous alternative representations are available in the literature for characterizing and analyzing nonlinear electronic systems. Of these, the Volterra nonlinear transfer functions (NLTFs) description [1],[2] is particularly attractive since it lends itself to convenient frequency-domain interpretation. As such, it enables straightforward computations of such quantities as a) linear and higher order nonlinear responses [2], b) harmonic distortion, c) intermodulation distortion [7], and d) cross-modulation distortion [7]. Recent research has shown that these NLTFs are also well suited for compensator design [15],[16] to minimize intermodulation effects. In order to familiarize the reader with this analytical and design technique, this study briefly introduces the Volterra expansion and then uses this expansion to analyze a series of nonlinear phenomena.

To introduce the analytical technique, consider the input-output relationship

$$y(t) = \mathbf{T}[x(t)] \tag{1}$$

where **T** is the system operator. This study will be restricted to relationships which are time-invariant and only "mildly nonlinear." For such systems, the output may be expressed as,

$$y(t) = \sum_{k=1}^{\infty} y_k(t) \tag{2}$$

$$= \sum_{k=1}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_k(\tau_1, \cdots, \tau_k) x(t-\tau_1) \cdots x(t-\tau_k) d\tau_1 \cdots d\tau_k$$

where $y_k(t) = \mathbf{H}_k[x(t)]$ is referred to as the k-th order response and $\mathbf{H}_k$ is referred to as the k-th order system-operator. These various notations are consistent so that

$$y_k(t) = \mathbf{H}_k[x(t)] \tag{3}$$

$$= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_k(\tau_1, \cdots, \tau_k) x(t-\tau_1) \cdots x(t-\tau_k) d\tau_1 \cdots d\tau_k$$

This expansion can also be described diagrammatically as appears in Fig. 1.

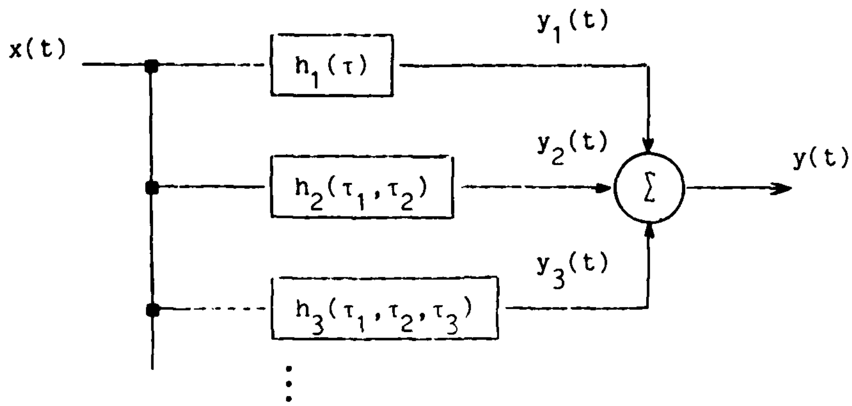

Fig. 1 Volterra System Representation

This expansion of y(t) was originally described by Vito Volterra and later named the Volterra expansion by Wiener [2] who applied it to nonlinear noise problems. It is analogous to a power series expansion. As with a power series expansion, this "Volterra expansion" is practically useful only if the series converges quickly as k increases. For the midly nonlinear relationships of interest in this report, only the first three responses h_1, h_2, and h_3 are considered significant.

While the overall nonlinear relationship of equation (1) is nonhomogeneous, equation (3) reveals that there is a simple relationship between the input and output of the individual k-th order responses when the input is scaled by a constant ε. Specifically,

$$y_{k\varepsilon}(t) = \mathbf{H}_k[\varepsilon x(t)] \tag{4}$$

$$= \varepsilon^k \mathbf{H}_k[x(t)]$$

Thus by scaling the input, the factor ε^k will appear as a multiplier and can be used to identify the order of a particular response [1]. This observation will be useful in the subsequent sections of the report.

The time-domain integration associated with these expansions are operationally complex. This complexity can be alleviated by use of the Fourier or Laplace transformations. In the image-space, convolution is isomorphic to multiplication. To demonstrate this fact, and to determine the proper product form, a multi-dimensional response $y_{(k)}(t_1,t_2,\cdots,t_k)$ can be postulated

$$y_{(k)}(t_1,\cdots,t_k) \triangleq \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty} h_k(\tau_1,\cdots,\tau_k)x(t_1-\tau_1)\cdots x(t_k-\tau_k)d\tau_1\cdots d\tau_k \tag{5}$$

where $y_{(k)}$ is referred to as the k-th order associated response [4]. It is apparent that this associated response reduces to $y_k(t)$ if $t_1=t_2=\cdots=t_k=t$. But the associated response is simple to Fourier transform to

$$Y_{(k)}(f_1,f_2,\cdots,f_k)= H_k(f_1,f_2,\cdots,f_k)X(f_1)X(f_2)\cdots X(f_k)$$

Then $y_k(t)$ is simply the inverse Fourier transform of $Y_{(k)}(f_1,f_2,\cdots,f_k)$ with $t_1=t_2=\cdots=t_k=t$, or

$$y_k(t)= \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\{H_k(\nu_1,\cdots,\nu_k)X(\nu_1)\cdots X(\nu_k)\}e^{j2\pi(\nu_1+\cdots+\nu_k)t}d\nu_1\cdots d\nu_k \tag{6}$$

Finally, the Fourier transform of $y_k(t)$ becomes

$$Y_k(f)= \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\{H_k(\nu_1,\cdots,\nu_k)X(\nu_1)\cdots X(\nu_k)\}\ \delta(f-(\nu_1+\cdots+\nu_k))\ d\nu_1\cdots d\nu_k \tag{7}$$

This equation allows easy determination of the k-th order response. For example, suppose that an input x(t) equal to $e^{j2\pi F_1 t}+e^{j2\pi F_2 t}$ is applied to a second order system $H_2(f_1,f_2)$. Then equation (7) yields

$$Y_2(f)= H_2(F_1,F_1)\delta(f-2F_1) +H_2(F_1,F_2)\delta(f-F_1-F_2)$$
$$+H_2(F_2,F_1)\delta(f-F_1-F_2) +H_2(F_2,F_2)\delta(f-2F_2)$$

or in the time-domain

$$y_2(t) = H_2(F_1,F_1)e^{j4\pi F_1 t} + H_2(F_1,F_2)e^{j2\pi(F_1+F_2)t}$$

$$+H_2(F_2,F_1)e^{j2\pi(F_2+F_1)t} + H_2(F_2,F_2)e^{j4\pi F_1 t}$$

In general, $H_2(F_1,F_2)$ may not equal $H_2(F_2,F_1)$. But often it is convenient to have functions which have this "symmetry" so that $H_k(f_1,\cdots,f_k)$ equals H_k with all possible permutations of the independent variable. This can be guaranteed by defining a symmetrized H_k as

$$H_k(f_1,\cdots,f_k) = \frac{1}{k!} \mathcal{P} \tilde{H}_k(f_1,\cdots,f_k) \tag{8}$$

where the tilde indicates that the function is unsymmeterized and the script-p $\mathcal{P}$ denotes the summation of the $\tilde{H}_k$'s over the k-factorial permutations of the independent variables [1].

In the above example, we used $H_2(s_1,s_2)$ in the abstract form. A particular realization (although not the most general one) is shown in Fig.

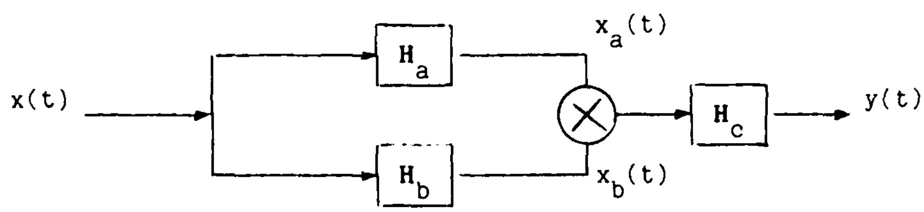

Fig. 2 A Simple Second-Order Volterra System

2. Note that each of the blocks H_a, H_b and H_c is linear. For this structure, it can be shown that

$$H_2(s_1,s_2) = H_a(s_1)\, H_b(s_2)\, H_c(s_1+s_2).$$

If H_a equals H_b, then the block diagram of Fig. 2 can be more concisely depicted as in Fig. 3.

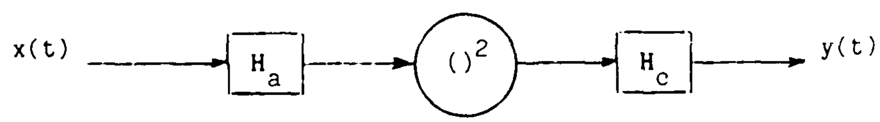

Fig. 3 Compact Representation of a Symmetric Second-Order System

Although we have employed the Fourier transform for the derivations in this section, we could equally well have used the two-sided Laplace transform.

III. OVERVIEW OF EXAMPLES

The purpose of this report is to provide techniques for deriving the Volterra NLTFs from other available descriptions. To this end, the subsequent sections will introduce the method of analysis through a collection of nonlinear electronic device and circuit examples. These "available descriptions" constitute a mix of differential equations, device representation, circuits, and system representation by block diagrams.

The first two examples consider nonlinear differential equations. The first example examines a nonlinear second-order differential equation; the coefficient of the first derivative is a variable and dependent upon the output. The second example is a generalization of the first equation; it is an n-th order differential equation where all of the coefficients are variable and dependent upon the output.

The third example analyzes a nonlinear device wherein the current can be expressed as a power series expansion of the voltage; a circuit including this device is analyzed using nodal analysis. A forward biased diode has a nonlinear current-voltage relationship as described; but the diode is further complicated by the voltage-dependent capacitance associated with its junction. Therefore, example four analyzes the diode as a device outside a circuit. This example is particularly important because of the wide spread use of diodes, and because the nonlinear characteristics of diodes can be employed in circuits designed to act as "nonlinear compensators."

The fifth example is the complementary dual to example three. Here, the nonlinear devices have a current-voltage relationship such that the voltage can be expressed as a power series expansion of the current. Also, the analysis employs loop equations rather than node equations.

The sixth example employs a more complicated nonlinear device, the transistor. Here the transistor current is modeled as the "product-power" series expansion of two voltages.

The seventh example analyzes a cascade of two nonlinear systems. This example is particularly important because this is a configuration which can be employed to eliminate the nonlinear output of the first system. The example explicitly describes how to develop a nonlinear cascade compensator to eliminate the nonlinear response of the first system. The eighth, and final, example uses the nonlinear characteristics of a diode to implement a nonlinear cascade compensator.

IV. NLTFs FROM NONLINEAR DIFFERENTIAL EQUATIONS

Example 1: Simple Nonlinear Differential Equation.

Volterra nonlinear transfer functions can be employed to characterize, in the frequency-domain, certain classes of nonlinear differential equations. As a specific example, consider the equation

$$\frac{d^2}{dt^2} y + \frac{d}{dt} \{y\ f(y)\} + by = x(t) \tag{9}$$

If f(y) can be expanded in a power series which converges quickly, then y may be expanded in a Volterra expansion which also converges quickly.

Solution.

It was stated that f(y) can be expanded as a power series; i.e.,

$$f(y) = \sum_{n=0}^{\infty} a_n y^n \tag{10}$$

Now form a Volterra series expansion for y as

$$y = H[x] = \sum_{k=1}^{\infty} H_k[x] = \sum_{k=1}^{\infty} y_k \tag{11}$$

Substituting these expansions into the differential equation, one obtains

$$\frac{d^2}{dt^2} \{ \sum_{k=1}^{\infty} y_k \} + \frac{d}{dt} \left\{ \{ \sum_{j=1}^{\infty} y_j \} \{ \sum_{n=0}^{\infty} a_n \ (\sum_{k=1}^{\infty} y_k)^n \} \right\} + b\{ \sum_{k=1}^{\infty} y_k \} = x(t)$$

For this example, the first three Volterra NLTFs for y_1, y_2, and y_3 will be formed. This is accomplished by explicitly writing the individual terms of the various orders for the above differential equation. To keep track of the order, scale the forcing function x to εx.

ε-Scaling.

$$y_\varepsilon = \sum_{k=1}^{\infty} H_k[\varepsilon x] = \sum_{k=1}^{\infty} \varepsilon^k H_k[x]$$

The differential equation then becomes

$$\frac{d^2}{dt^2}\{\varepsilon y_1+\varepsilon^2 y_2+\varepsilon^3 y_3+\cdots\} \tag{12}$$

$$+\frac{d}{dt}\{a_0+\varepsilon y_1+a_1\varepsilon^2 y_1{}^2+\varepsilon^3\{a_0y_3+2a_1y_1y_2+a_2y_1{}^3\}+\cdots\}$$

$$+b\{\varepsilon y_1+\varepsilon^2 y_2+\varepsilon^3 y_3+\cdots\} = \varepsilon x$$

Now, like powers of ε can be collected and equated.

<u>ε^1:</u>

$$\frac{d^2}{dt^2}y_1 + a_0\frac{d}{dt}y_1 + by_1 = x$$

Or more concisely,

$$G[y_1] = x,$$

where G is the linear differential operator $\{\frac{d^2}{dt^2} + a_0\frac{d}{dt} y_1 + b\}$

Then

$$y_1 = G^{-1}[x] = H_1[x]. \tag{13}$$

This equation can be Laplace transformed to

$$y_1(s) = \frac{1}{s^2+a_0s+b}x(s)$$

It therefore follows that H_1 is G^{-1} or

$$H_1(s) = \frac{1}{s^2+a_0s+b} \tag{14}$$

<u>ε^2:</u>

$$\frac{d^2}{dt^2}y_2 + a_0\frac{d}{dt}y_2 + by_2 = G[y_2] = -\frac{d}{dt}a_1y_1{}^2$$

So that

$$y_2 = -G^{-1}\left[\frac{d}{dt}\, a_1 y_1^{\,2}\right] \tag{15}$$

$$= -a_1 G^{-1}\left[\, \frac{d}{dt}\, \{H_1[x]\; H_1[x]\}\right]$$

Now the Laplace domain formulation of H_2 can be performed [4]. The determination of this NLTF is particularly simple if y_2 is diagrammed as described in Section II.

$$H_2(s_1, s_2) = -a_1 H_1(s_1+s_2)\{s_1+s_2\} H_1(s_1) H_1(s_2) \tag{16}$$

$\underline{\varepsilon^3:}$

$$\frac{d^2}{dt^2}\, y_3 + a_0\, \frac{d}{dt}\, y_3 + by_3 = \; G[y_3] = -\, \frac{d}{dt}\, \{2a_1 y_1 y_2 + a_2 y_1^{\,3}\}$$

and therefore

$$y_3 = -G^{-1}\left[\frac{d}{dt}\, \{2a_1 y_1 y_2 + a_2 y_1^{\,3}\}\right] \tag{17}$$

Here again, H_3 may be more apparent from the block diagram of Fig. 4.

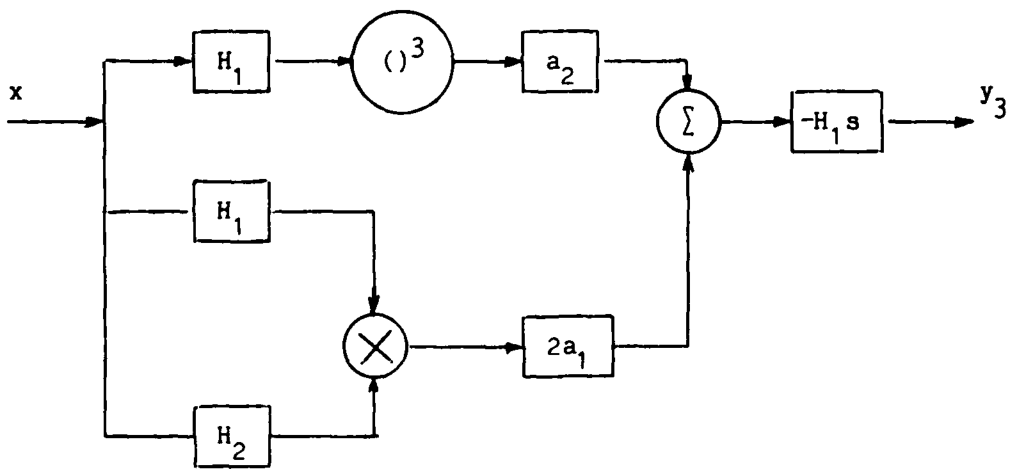

Fig. 4 Block Diagram of the Third-Order NLTF

From the diagram, H_3 can be written as

$$\tilde{H}_3(s_1,s_2,s_3) = -H_1(s_1+s_2+s_3)\{s_1+s_2+s_3\} \{a_2H_1(s_1)H_1(s_2)H_1(s_3) + 2a_1H_1(s_1)H_2(s_2,s_3)\} \quad (18)$$

where the tilde has been included over H_3 to indicate that this is an unsymmetrized transfer function. The symmetrized H_3 is obtained from $\tilde{H}_3$ as described in Section II as $H_3 = \frac{1}{3!}\mathscr{P}\,\tilde{H}_3(s_1,s_2,s_3)$ [1]. And therefore, the symmetrized H_3 is

$$H_3(s_1,s_2,s_3) = -H_1(s_1+s_2+s_3)\{s_1+s_2+s_3\}\Big\{a_2H_1(s_1)H_1(s_2)H_1(s_3) + \frac{2a_1}{3}\{H_1(s_1)H_2(s_2,s_3) + H_1(s_2)H_2(s_1,s_3) + H_1(s_3)H_2(s_1,s_2)\}\Big\}$$

Here it is noted that the y_k's are recursive in the sense that y_k is only a function of y_1, y_2, $\cdots$, y_{k-1}. Thus the truncation of the original equation to the third order terms leads to no error in the development of the first three NLTF's. Also, the process could be continued to obtain as many of the NLTF's as required. Indeed, there is no mathematical reason that all the y's could not be formed. But in most applications, this Volterra expansion is only practical for analysis and design if only the first few terms are significant.

Example 2: General Differential Equation.

The first example was a differential equation which was "nonlinear in the second term." This is a special case of the nonlinear differential equation

$$\frac{d^n}{dt^n}\{\sum_{\ell=1}^{\infty} a_{n,\ell} y^\ell\} + \frac{d^{n-1}}{dt^{n-1}}\{\sum_{\ell=1}^{\infty} a_{n-1,\ell} y^\ell\} + \cdots + \{\sum_{\ell=1}^{\infty} a_{0,\ell} y^\ell\} = x \tag{19}$$

This is a rather general nonlinear differential equation since it can represent a n-th order differential equation where each coefficient of the differential terms are dependent on the output, but can be expressed as power series expansions. The approach to solving this equation is identical to the previous example.

Solution.

Expand y as a Volterra series

$$y = H[x] = \sum_{k=1}^{\infty} H_k[x] = \sum_{k=1}^{\infty} y_k$$

As before this expansion is substituted into the differential equation and the input or forcing function x is scaled by ε to keep track of the order.

ε-Scaling.

$$\frac{d^n}{dt^n}\{\sum_{\ell=1}^{\infty} a_{n,\ell}\{\sum_{k=1}^{\infty}\varepsilon^k y_k\}^\ell\} + \frac{d^{n-1}}{dt^{n-1}}\{\sum_{\ell=1}^{\infty} a_{n-1,\ell}\{\sum_{k=1}^{\infty}\varepsilon^k y_k\}^\ell\} + \cdots$$

$$+\{\sum_{\ell=1}^{\infty} a_{0,\ell}\{\sum_{k=1}^{\infty}\varepsilon^k y_k\}^\ell\} = \varepsilon x$$

This equation can be rewritten as

$$\frac{d^n}{dt^n} \sum_{\ell=1}^{\infty} a_{n,\ell} \sum_{k_1=1}^{\infty} \cdots \sum_{k_\ell=1}^{\infty} \varepsilon^{k_1+\cdots+k_\ell} y_{k_1} \cdots y_{k_\ell} \tag{20}$$

$$+ \frac{d^{n-1}}{dt^{n-1}} \sum_{\ell=1}^{\infty} a_{n-1,\ell} \sum_{k_1=1}^{\infty} \cdots \sum_{k_\ell=1}^{\infty} \varepsilon^{k_1+\cdots+k_\ell} y_{k_1} \cdots y_{k_\ell}$$

$$\vdots$$

$$+ \sum_{\ell=1}^{\infty} a_{0,\ell} \sum_{k_1=1}^{\infty} \cdots \sum_{k_\ell=1}^{\infty} \varepsilon^{k_1+\cdots+k_\ell} y_{k_1} \cdots y_{k_\ell} = \varepsilon x$$

Now, by defining the linear operator L_ℓ as

$$L_\ell = \sum_{i=1}^{n} \frac{d^i}{dt^i} a_{i,\ell}$$

the differential equation (20) can be rewritten as

$$\sum_{\ell=1}^{\infty} L_\ell \sum_{k_1=1}^{\infty} \cdots \sum_{k_\ell=1}^{\infty} \varepsilon^{k_1+\cdots+k_\ell} y_{k_1} \cdots y_{k_\ell} = \varepsilon x \qquad (21)$$

Now as before, this expression will be evaluated for various powers of ε, and like powers will be collected.

$\underline{\varepsilon^1:}$

$$L_1[y_1] = x,$$

so that H_1 is L_1^{-1}; or in the complex-domain

$$H_1(s) = \frac{1}{a_{n,1}s^n + a_{n-1,1}s^{n-1} + \cdots + a_{0,1}} \qquad (22)$$

$\underline{\varepsilon^2:}$

$$L_1[y_2] = -L_2[y_1^2]$$

so that

$$y_2 = -H_1\left[L_2\left[H_1[x]H_1[x]\right]\right]$$

The Laplace domain formulation of H_2 produces

$$H_2(s_1,s_2) = -H_1(s_1)H_1(s_2)L_2(s_1+s_2)H_1(s_1+s_2) \qquad (23)$$

$\underline{\varepsilon^3:}$

$$L_1H_3 = -2L_2[H_1H_2] - L_3[H_1H_1H_1]$$

then the Laplace transform is

$$\tilde{H}_3(s_1,s_2,s_3) = -2L_2(s_1+s_2+s_3)H_1(s_1+s_2+s_3)H_1(s_1)H_2(s_2+s_3) \tag{24}$$

$$- L_3(s_1+s_2+s_3)H_1(s_1+s_2+s_3)H_1(s_1)H_1(s_2)H_1(s_3)$$

where the tilde has been included as a reminder that this is an unsymmetrized transfer function. The symmetrization of this transfer function is straight forward as described in Section 1 and as applied in example 1.

$\underline{\varepsilon^4:}$

$$L_1[H_4] = -L_2[H_2H_2+2H_1H_3] - 3L_3[H_1H_1H_2] - L_4[H_1H_1H_1H_1]$$

So that H_4 can be formed in the complex-domain directly or from a diagram as

$$\tilde{H}_4(s_1,s_2,s_3,s_4) = \tag{25}$$

$$-L_2(s_1+s_2+s_3+s_4)\{H_2(s_1,s_2)H_2(s_3,s_4)+2H_1(s_1)H_3(s_2,s_3,s_4)\}$$

$$-3L_3(s_1+s_2+s_3+s_4)H_1(s_1)H_1(s_2)H_2(s_3,s_4)$$

$$-L_4(s_1+s_2+s_3+s_4)H_1(s_1)H_1(s_2)H_1(s_3)H_1(s_4)$$

As stated earlier, these transfer functions generally are recursive so that the complexity of the function grows with the order. This is apparent in this example. It is for this reason that this analysis is usually applied only to slightly nonlinear systems where it is likely that the Volterra series expansion will converge rapidly.

V. NLTFs OF SIMPLE CIRCUITS AND DEVICES

Example 3: Single Nonlinearity.

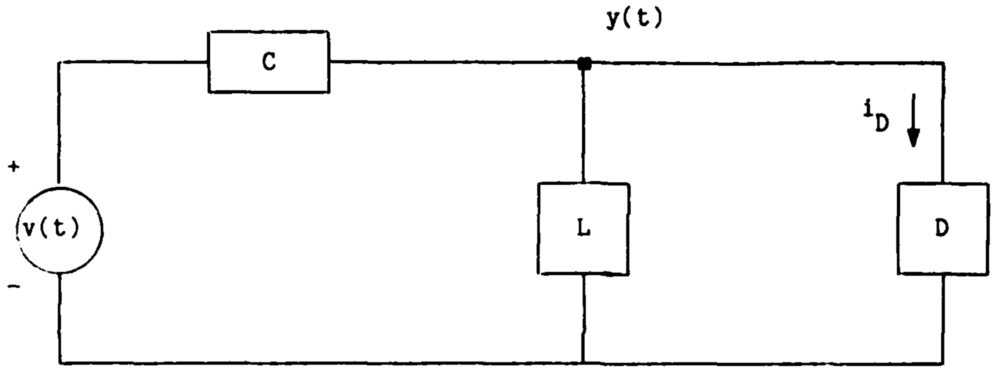

Fig. 5 Circuit with a Static Nonlinear Device

The circuit of Fig. 5 contains a single static nonlinear device D. The node equation for this circuit can be written as

$$C \frac{d}{dt} v = C \frac{d}{dt} y + \frac{1}{L} \int y \, dt + i_D \tag{26}$$

It is assumed that the nonlinear current-voltage relationship can be adequately modeled with a power series; i.e.,

$$i_D = \sum_{n=1}^{\infty} a_n y^n \tag{27}$$

The objective of this analysis is to find the form of the Volterra NLTFs H_k which relate the node voltage to the input voltage; i.e.,

$$y = \sum_{k=1}^{\infty} H_k[v] = \sum_{k=1}^{\infty} y_k$$

Solution.

If the input v is scaled to εv, then a new node voltage y_ε results

$$y_\varepsilon = \sum_{k=1}^{\infty} H_k[\varepsilon v] = \sum_{k=1}^{\infty} \varepsilon^k H_k[v] = \sum_{k=1}^{\infty} \varepsilon^k y_k$$

Substituting this into the node equation,

$$\varepsilon C \frac{d}{dt} v = C \frac{d}{dt} \sum_{k=1}^{\infty} \varepsilon^k y_k + \frac{1}{L} \int \sum_{k=1}^{\infty} \varepsilon^k y_k \, dt + \sum_{n=1}^{\infty} a_n \{ \sum_{k=1}^{\infty} \varepsilon^k y_k \}^n \tag{28}$$

$$= G[\sum_{k=1}^{\infty} \varepsilon^k y_k] + \sum_{n=2}^{\infty} a_n \{ \sum_{k=1}^{\infty} \varepsilon^k y_k \}^n$$

$$= G[\sum_{k=1}^{\infty} \varepsilon^k y_k] + \sum_{n=2}^{\infty} a_n \sum_{k_1=1}^{\infty} \sum_{k_2=1}^{\infty} \cdots \sum_{k_n=1}^{\infty} \varepsilon^{k_1+k_2+ \cdots +k_n} y_{k_1} y_{k_2} \cdots y_{k_n}$$

where G is the linear operator with the Laplace transform $Cs+a_1+\frac{1}{Ls}$; i.e., G(s) is the linear admittance.

Now as stated earlier, we are only interested in the first three NLTFs, and therefore only terms that contribute to ε^1, ε^2, and ε^3 need be explicitly expressed in the equations (28). So equation (28) can be expressed as

$$\varepsilon C \frac{d}{dt} v = \tag{29}$$

$$\varepsilon^1 G[y_1] + \varepsilon^2 \{G[y_2] + a_2 y_1^2\} + \varepsilon^3 \{G[y_3] + 2a_2 y_1 y_2 + a_3 y_1^3\} + \text{HOT}$$

where HOT are the higher order terms in ε. Notice that, this finite expansion of equation (29) is more readily obtained from the double sum form of equation (28) rather than the last form. While the last "infinite sums" form of equation (28) is consistent with the general theory of Volterra functions, for this example, and for all the future examples, this infinite sums form is operational less efficient then the double sum form. Therefore, in the future, this more complicated form will not be employed.

Using equation (29), the terms associated with the powers of ε can be collected.

$\underline{\varepsilon^1:}$

$$C \frac{d}{dt} v = G[y_1] \tag{30}$$

This equation can be Laplace transformed; and then by defining a linear impedance Z(s) as $G^{-1}(s)$, y_1 becomes

$$y_1(s) = Z(s)\ Cs\ v(s) \tag{31}$$

and therefore

$$H_1(s) = Cs\ Z(s) = Cs \frac{Ls}{LCs^2 + La_1 s + 1} \tag{32}$$

<u>ε^2:</u>

$$G[y_2] = -a_2 y_1^{\ 2} \tag{33}$$

The Laplace transform of H_2 can then be formed from the above equation, or a block diagram of the above equation, as

$$H_2(s_1,s_2) = -a_2\ Z(s_1+s_2)\ \{H_1(s_1)H_1(s_2)\} \tag{34}$$

<u>ε^3:</u>

$$G[y_3] = -2a_2 y_1 y_2 - a_3 y_1^{\ 3} \tag{35}$$

And therefore,

$$\tilde{H}_3(s_1,s_2,s_3) = \tag{36}$$

$$-Z(s_1+s_2+s_3)\{2a_2 H_1(s_1)H_2(s_2,s_3) + a_3 H_1(s_1)H_1(s_2)H_1(s_3)\}$$

where the tilde has been included over H_3 as a reminder that this expression is unsymmetrized. The symmetrized H_3 is obtained from $\tilde{H}_3$ as described in Section 1 as $H_3 = \frac{1}{3!} \cdot \tilde{H}(s_1,s_2,s_3)$. The symmetrized expression is therefore,

$$H_3(s_1,s_2,s_3) = \tag{37}$$

$$-Z(s_1+s_2+s_3)\{a_3 H_1(s_1)H_1(s_2)H_1(s_3)$$

$$+(a_2/3)\{H_1(s_1)H_2(s_2,s_3)+H_1(s_2)H_2(s_1,s_3)+H_1(s_3)H_2(s_1,s_2)\}\}$$

These three NLTFs can be depicted graphically as appears in Fig. 6.

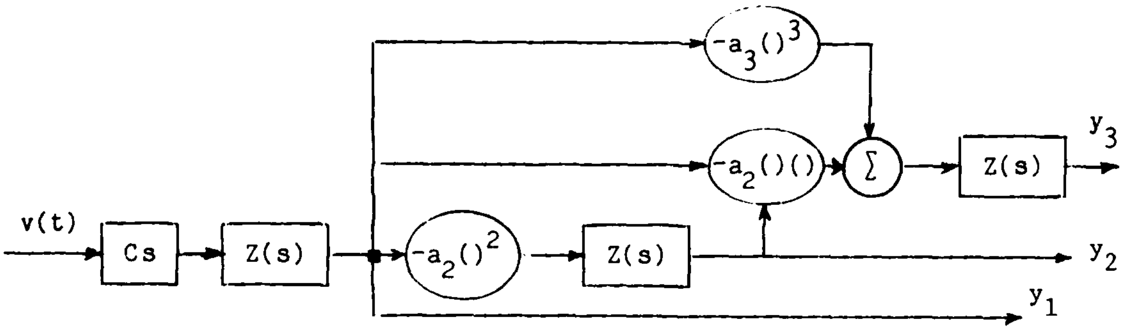

Fig. 6 First Three NLTFs of the Circuit of Fig. 5

The stated objective of this example was to determine the first three H's and that has been accomplished.

Example 4: Diode.

The previous example included nonlinear devices such that the current through them can be expanded as a power series of the voltage across them. One example of this type of nonlinear device is a forward biased diode. But the diode is more complicated since there are capacitances and resistances also associated with it. The capacitance C_D is a function of thickness of the junction regions, and this thickness is itself a function of the voltage across the diode; so that the capacitance is not a constant but depends upon the junction voltage. Since the diode is such a common nonlinear electronic device, it will be analyzed in this section as an isolated device.

As stated above, a power series representation will be used to describe the voltage-current relationship of the diode. The order of this power series expansion generally depends upon the quiescent point and operating range. In many practical applications, the operating conditions are selected so that only the dc, linear, and quadratic terms are significant. But for the present analysis, the cubic term will also be included.

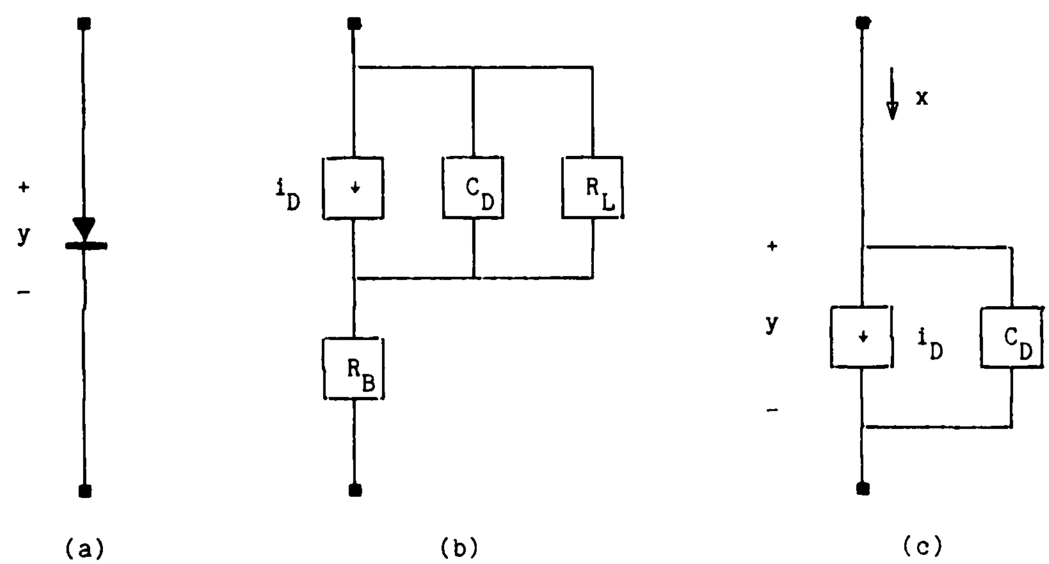

Fig. 7 Diode Models

Fig. 7(b) depicts a standard model for a diode. Typically, R_B is of the order of a few tenths of an ohm, and R_L is several hundred kilo-ohms. For the applications of interest to this study, these resistances are inconsequential. Therefore the simpler model of Fig. 7(c) will be employed.

The objective of this analysis is to determine the first three Volterra NLTF's P_1, P_2 and P_3 which relate the voltage to the current.

Solution.

If the diode of Fig. 7(c) is driven by a current source x(t), a voltage y(t) will result. Furthermore, the nonlinear current source and capacitance of the diode model will have the forms

$$i_D = \sum_{n=1}^{\infty} a_n y^n \tag{38}$$

$$C_D = \sum_{n=0}^{\infty} c_n y^n$$

where the voltage y can be expanded as a Volterra series

$$y = \sum_{k=1}^{\infty} P_k[x] = \sum_{k=1}^{\infty} y_k$$

The node equation for the diode and external current source is

$$\frac{d}{dt} C_D y + i_D - x = 0$$

$$= \frac{d}{dt} \sum_{n=0}^{\infty} c_n y^{n+1} + \sum_{n=1}^{\infty} a_n y^n - x$$

$$= \sum_{n=1}^{\infty} \{c_{n-1} \frac{d}{dt} + a_n\}[y^n] - x$$

ε-Scaling.

Now the driving current source x can be scaled by ε to obtain

$$\sum_{n=1}^{\infty} \{c_{n-1} \frac{d}{dt} + a_n\}[\{\sum_{k=1}^{\infty} \varepsilon^k y_k\}^n] - \varepsilon x = 0 \tag{39}$$

$$= \{c_0 \frac{d}{dt} + a_1\}[\varepsilon y_1 + \varepsilon^2 y_2 + \varepsilon^3 y_3] + \{c_1 \frac{d}{dt} + a_2\}[\varepsilon^2 y_1{}^2 + 2\varepsilon^3 y_1 y_2]$$

$$+ \{c_2 \frac{d}{dt} + a_3\}[\varepsilon^3 y_1] - \varepsilon x + \text{HOT}$$

where HOT denotes the higher order terms in ε. The powers of ε can now be collected and equated.

$\underline{\varepsilon^1:}$

$$\{c_0 \frac{d}{dt} + a_1\}[y_1] = x$$

Here it is convenient to introduce the notation $L_n(s) = c_{n-1} s + a_n$. Then the Laplace transform of the above equation becomes

$$L_1(s)\ y_1(s) = x(s)$$

From this equation, the Laplace transform of P_1 is found to be

$$P_1(s) = L_1{}^{-1}(s) = \frac{1}{c_0 s + a_1} \tag{40}$$

$\underline{\varepsilon^2:}$

$$L_1[y_2] = -\{c_1 \frac{d}{dt} + a_2\}[y_1{}^2]$$

$$= -L_2[y_1{}^2]$$

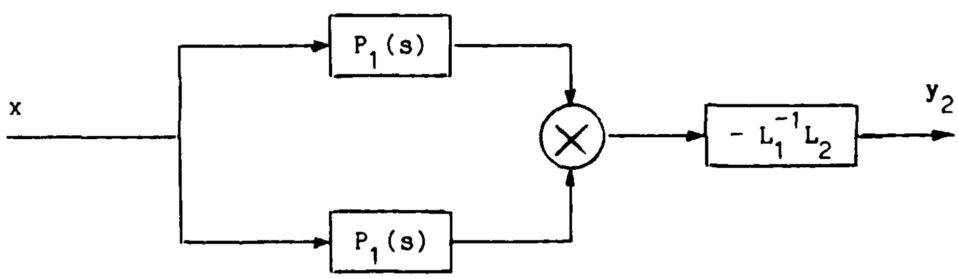

Fig. 8 Second-Order NLTF of a Diode

The form of the NLTF is apparent from Fig. 8 as

$$P_2(s_1,s_2) = -P_1(s_1)\ P_1(s_2)\ P_1(s_1+s_2)\ L_2(s_1+s_2) \tag{41}$$

$\underline{\varepsilon^3:}$

$$L_1[y_3] = -\{c_1 \frac{d}{dt} + a_2\}[2y_1y_2] - \{c_2 \frac{d}{dt} + a_3\}[y_1{}^3]$$

$$= -L_2[2y_1y_2] - L_3[y_1{}^3]$$

The multidimensional Laplace transform of P_3 can then be formed as

$$\tilde{P}_3(s_1,s_2,s_3) = -P_1(s_1+s_2+s_3)\{2L_2(s_1+s_2+s_3)P_1(s_1)P_2(s_2,s_3)$$
$$+L_3(s_1+s_2+s_3)P_1(s_1)P_1(s_2)P_1(s_3)\} \tag{42}$$

where the tilde has been included over P_3 as a reminder that this is a unsymmetrized operator as discussed in Section 1.

This then completes the analysis of the diode.

VI. NLTFs OF MULTI-LOOP AND DEPENDENT SOURCE CIRCUITS

Example 5: Nonlinear Multi-loop Circuits.

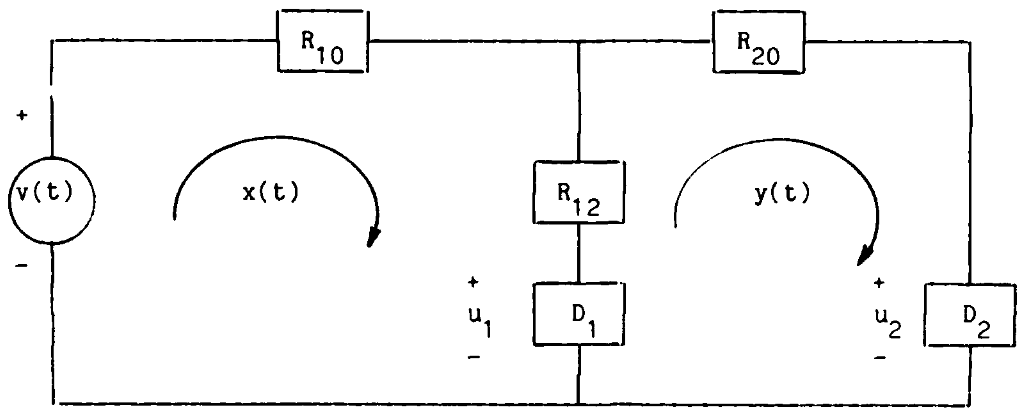

Fig. 9 Multi-Loop Circuit with Nonlinear Devices

The previous two circuit examples dealt with nonlinear devices such that the voltage across those devices could be described as a power series expansion of the current through them. For this example, the opposite is true; the nonlinear devices are such that the current through them can be described as a power series expansion of the voltage across them.

Fig. 9 depicts a network with two static nonlinear devices which are represented as D_1 and D_2. Here the R's represent "linear impedance operators" and actually reflect the presence of any linear "sub-network." The loop equations for Fig. 9 can be written

$$v = \mathbf{R}_{11}[x] - \mathbf{R}_{12}[y] + u_1 \tag{43}$$

$$0 = -\mathbf{R}_{21}[x] + \mathbf{R}_{22}[y] - u_1 + u_2$$

where $\mathbf{R}_{21}=\mathbf{R}_{12}$, $\mathbf{R}_{11}=\mathbf{R}_{10}+\mathbf{R}_{12}$, $\mathbf{R}_{22}=\mathbf{R}_{20}+\mathbf{R}_{12}$, and the voltages u_1 and u_2 are nonlinearly related to the currents through the devices D_1 and D_2 respectively.

The general objective of this analysis is to relate the currents x(t) and y(t) to the input voltage v(t). But more specifically, the objective is to find the form of Volterra transfer functions P_k and Q_k which relate the k-th nonlinear current $\underline{i}_k = [x_k \; y_k]^T$ to the input v(t); viz.,

$$x(t) = \sum_{k=1}^{\infty} x_k(t) = \sum_{k=1}^{\infty} P_k[v(t)] \tag{44}$$

$$y(t) = \sum_{k=1}^{\infty} y_k(t) = \sum_{k=1}^{\infty} Q_k[v(t)]$$

For efficient notation, define $\underline{H}_k$ as a column vector

$$\underline{H}_k = \begin{bmatrix} P_k \\ Q_k \end{bmatrix}$$

Solution.

We begin by requiring that the voltages u_1 and u_2 can be adequately modeled as a power series expansion where only the first three terms are significant; i.e.,

$$u_1 = \sum_{n=1}^{\infty} b_n (x-y)^n = \sum_{n=1}^{\infty} b_n \xi^n \tag{45}$$

$$u_2 = \sum_{n=1}^{\infty} a_n (y)^n$$

where the current through the common branch is $\xi = x-y$.

ε-Scaling.

Now replace v(t) with a scaled voltage $\varepsilon v(t)$ and thus generate new currents

$$x_\varepsilon = \sum_{k=1}^{\infty} P_k[\varepsilon v] = \sum_{k=1}^{\infty} \varepsilon^k P_k[v] = \sum_{k=1}^{\infty} \varepsilon^k x_k$$

$$y_\varepsilon = \sum_{k=1}^{\infty} Q_k[\varepsilon v] = \sum_{k=1}^{\infty} \varepsilon^k Q_k[v] = \sum_{k=1}^{\infty} \varepsilon^k y_k$$

Substituting this back into the loop equations:

$$\varepsilon v = \sum_{k=1}^{\infty} \varepsilon^k \{ R_{11}[x_k] - R_{12}[y_k]\} + \sum_{n=1}^{\infty} b_n \{ \sum_{k=1}^{\infty} \varepsilon^k \xi_k \}^n \tag{46}$$

$$0 = \sum_{k=1}^{\infty} \varepsilon^k \{ -R_{21}[x_k] + R_{22}[y_k]\} - \sum_{n=1}^{\infty} b_n \{ \sum_{k=1}^{\infty} \varepsilon^k \xi_k \}^n + \sum_{n=1}^{\infty} a_n \{ \sum_{k=1}^{\infty} \varepsilon^k y_k \}^n$$

As stated earlier, we are only interested in the first three NLTFs, and therefore only terms that contribute to ε^1, ε^2, and ε^3 need be explicitly expressed in the equations (46). So the first equation of (46) can be expressed as

$$\varepsilon v = \sum_{k=1}^{3} \varepsilon^k \{ R_{11}[x_k] - R_{12}[y_k]\} + \sum_{n=1}^{3} b_n \{ \sum_{k=1}^{3} \varepsilon^k \xi_k \}^n + \mathrm{HOT}$$

where HOT represent the higher order terms of ε as k runs from four to infinity, and also as n runs from 4 to infinity. Expanding the summations and regrouping the terms results in

$$\begin{aligned} \varepsilon v = \ & \varepsilon^1 \{R_{11}[x_1] - R_{12}[y_1] + b_1 \xi_1\} \\ & + \varepsilon^2 \{R_{11}[x_2] - R_{12}[y_2] + b_1 \xi_2 + b_2 \xi_1{}^2\} \\ & + \varepsilon^3 \{R_{11}[x_3] - R_{12}[y_3] + b_1 \xi_3 + 2b_2 \xi_1 \xi_2 + b_3 \xi_1{}^3\} + \mathrm{HOT} \end{aligned} \tag{47}$$

where HOT now includes the previous higher order terms and also the higher order terms formed from the square and cube processes. It is emphasized that the <u>expressed terms</u> are exact. The above manipulations have merely ignored terms of ε which have power greater than three.

The second equation of (46) can be manipulated to result in

$$\begin{aligned} 0 = \ & \varepsilon^1 \{-R_{21}[x_1] + R_{22}[y_1] - b_1 \xi_1 + a_1 y_1\} \\ & + \varepsilon^2 \{-R_{21}[x_2] + R_{22}[y_2] - b_1 \xi_2 - b_2 \xi_1{}^2 + a_1 y_2 + a_2 y_1{}^2\} \\ & + \varepsilon^3 \{-R_{21}[x_3] + R_{22}[y_3] - b_1 \xi_3 - 2b_2 \xi_1 \xi_2 - b_3 \xi_1{}^3 + a_1 y_3 + 2a_2 y_1 y_2 + a_3 y_1{}^3\} \\ & + \mathrm{HOT} \end{aligned} \tag{48}$$

Now the terms associated with the powers of ε can be collected and equated.

$\underline{\varepsilon^1:}$

$$v = R_{11}[x_1] - R_{12}[y_1] + b_1\xi_1$$

$$0 = -R_{21}[x_1] + R_{22}[y_1] - b_1\xi_1 + a_1 y_1$$

or more concisely,

$$\begin{bmatrix} v \\ 0 \end{bmatrix} = \underline{\psi} = \begin{bmatrix} (R_{11}+b_1) & -(R_{12}+b_1) \\ -(R_{21}+b_1) & (R_{22}+b_1+a_1) \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = [Z]\ \underline{i}_1 \qquad (49)$$

where [Z] represents the "linear network impedance."

Taking the Laplace transform of equation (49) yields

$$\underline{\psi}(s) = [Z(s)]\ \underline{i}_1(s)$$

The "admittance matrix" [G(s)] can now be formed as the inverse of the matrix [Z(s)]:

$$[G(s)] = [Z(s)]^{-1} = \frac{1}{\Delta(s)} \begin{bmatrix} (R_{22}(s)+b_1+a_1) & (R_{12}(s)+b_1) \\ (R_{21}(s)+b_1) & (R_{11}(s)+b_1) \end{bmatrix} \qquad (50)$$

where $\Delta(s)$ is the determinant of the matrix [Z(s)]. Then it follows that

$$\underline{i}_1(s) = [G(s)]\ \underline{\psi}(s)$$

$$= \frac{1}{\Delta(s)} \begin{bmatrix} (R_{22}(s)+b_1+a_1) \\ (R_{12}(s)+b_1) \end{bmatrix} v(s)$$

But since $\underline{H}_1$ operating on v(t) is defined as i_1, it follows that

$$\underline{H}_1(s) = \begin{bmatrix} P_1(s) \\ Q_1(s) \end{bmatrix} = \frac{1}{\Delta(s)} \begin{bmatrix} (R_{22}(s)+b_1+a_1) \\ (R_{12}(s)+b_1) \end{bmatrix} \qquad (51)$$

The relationship between $\underline{i}_1$ and v can be depicted graphically as appears in Fig. 10.

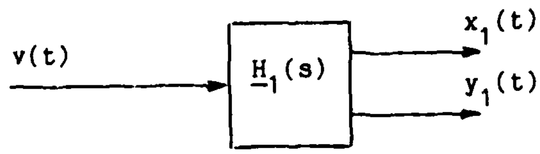

Fig. 10 First-Order NLTF

$\underline{\varepsilon^2:}$

$$0 = R_{11}[x_2] - R_{12}[y_2] + b_1\xi_2 + b_2\xi_1^2$$

$$0 = -R_{21}[x_2] + R_{22}[y_2] - b_1\xi_2 - b_2\xi_1^2 + a_1y_2 + a_2y_1^2$$

or more concisely,

$$\underline{0} = [Z]\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} + \begin{bmatrix} b_2\xi_1^2 \\ -b_2\xi_1^2 + a_2y_1^2 \end{bmatrix}$$

or

$$\underline{i}_2 = [G]\begin{bmatrix} -b_2\xi_1^2 \\ b_2\xi_1^2 - a_2y_1^2 \end{bmatrix} \tag{52}$$

This relationship can be depicted graphically as appears in Fig. 11.

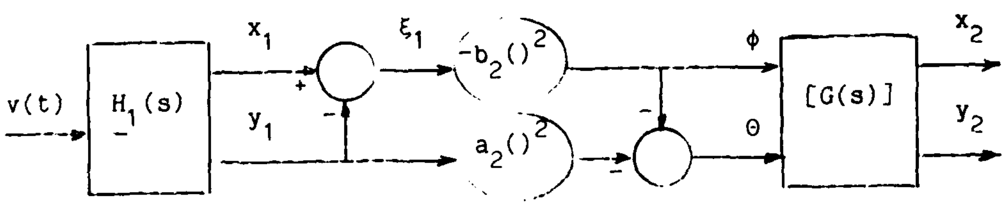

Fig. 11 Second-Order NLTF

But to solve for the form of P_2 and Q_2, equation (52) should be Laplace transformed. Since these equations involve a squaring process, the multidimensional Laplace transforms are employed. For convenience, define

$$M_k = P_k - Q_k$$

Then the transfer function from v to ϕ in Fig. 11 is $-b_2M_1(s_1)M_1(s_2)$; the transfer function from v to Θ is $b_2M_1(s_1)M_1(s_2) - a_2Q_1(s_1)Q_1(s_2)$. And therefore, the transfer function from input to output is

$$\underline{H}_2(s_1,s_2)= \quad [G(s_1+s_2)]\begin{bmatrix} -b_2M_1(s_1)M_1(s_2) \\ b_2M_1(s_1)M_1(s_2) \; -a_2Q_1(s_1)Q_1(s_2) \end{bmatrix} \tag{53}$$

$\underline{\varepsilon^3:}$

$$0= \; R_{11}[x_3] \; -R_{12}[y_3] \; +b_1\xi_3 \; +2b_2\xi_1\xi_2 \; +b_3\xi_1{}^3$$

$$0= -R_{21}[x_3] \; +R_{22}[y_3] \; -b_1\xi_3 \; -2b_2\xi_1\xi_2 \; -b_3\xi_1{}^3 \; +a_1y_3 \; + \; 2a_2y_1y_2 \; +a_3y_1{}^3$$

or more concisely,

$$\underline{0}= [Z]\begin{bmatrix} x_3 \\ y_3 \end{bmatrix} + \begin{bmatrix} 2b_2\xi_1\xi_2 \; +b_3\xi_1{}^3 \\ -2b_2\xi_1\xi_2 \; -b_3\xi_1{}^3 \; +2a_2y_1y_2 \; +a_3y_1{}^3 \end{bmatrix}$$

and therefore,

$$\underline{i}_3= \; [G]\begin{bmatrix} -2b_2\xi_1\xi_2 \; -b_3\xi_1{}^3 \\ 2b_2\xi_1\xi_2 \; +b_3\xi_1{}^3 \; -2a_2y_1y_2 \; -a_3y_1{}^3 \end{bmatrix} \tag{54}$$

This relationship can be depicted as appears in Fig. 12.

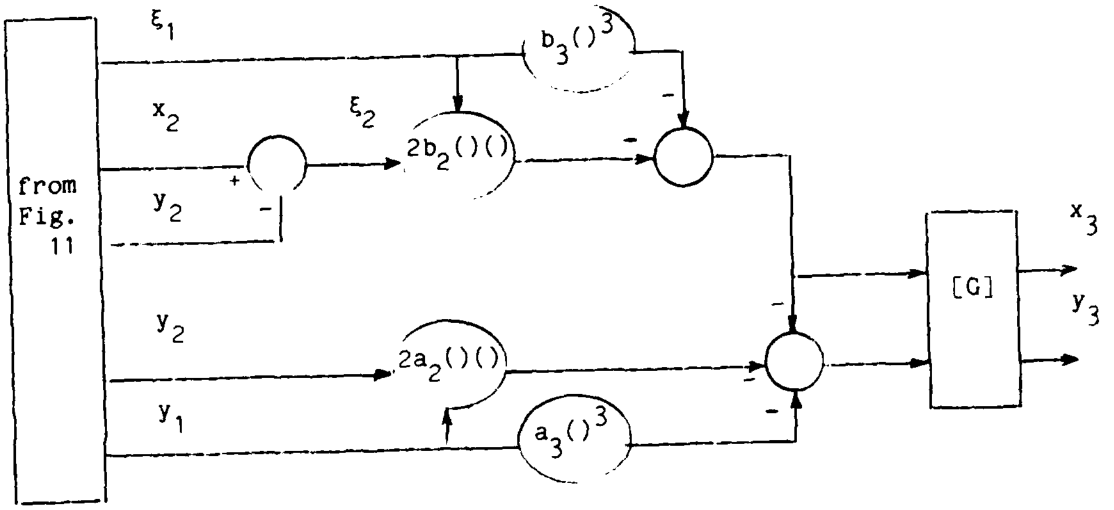

Fig. 12 Third-Order NLTF

The transform function from v to x_3 and y_3 is apparent from equation (54) as

$$\tilde{\underline{H}}_3(s_1,s_2,s_3) \tag{55}$$

$$= [G(s_1+s_2+s_3)] \begin{bmatrix} -2b_2M_1(s_1)M_2(s_2,s_3) - b_3M_1(s_1)M_1(s_2)M_1(s_3) \\ \\ 2b_2M_1(s_1)M_2(s_2,s_3) + b_3M_1(s_1)M_1(s_2)M_1(S_3) \\ -2a_2Q_1(s_1)Q_2(s_2,s_3) - a_3Q_1(s_1)Q_1(s_2)Q_1(s_3) \end{bmatrix}$$

where the tilde has been included over $\underline{H}_3$ as a reminder that this expression is unsymmetrized. Again, the symmetrized $\underline{H}_3$ is obtained as described in Section 1 as $\underline{H}_3 = \frac{1}{3!}\mathcal{P}\ \tilde{\underline{H}}(s_1,s_2,s_3)$. The stated objective of this example was to determine the first three P's and Q's and that has been accomplished.

Example 6: Dependent Sources (Transistor Model).

This example analyzes a circuit with a nonlinear transistor model. Here the transistor current is modeled as a function of two voltages u and w; viz.,

$$I = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} g_{nm} u^n w^m \tag{56}$$

where $g_{00}=0$. The circuit of interest appears below in Fig. 13.

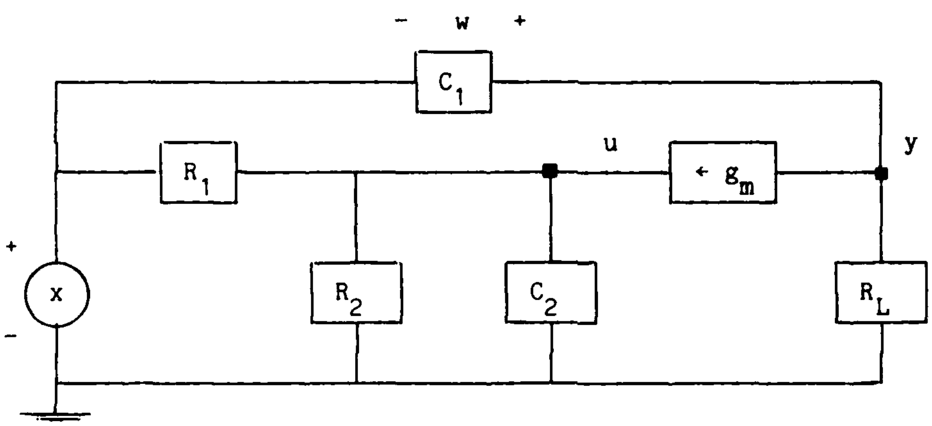

Fig. 13 Transistor Model

Generally, the objective of this example is to determine the output voltage y as a function of the input voltage x. More specifically, the objective is to determine the first three Volterra NLTF's.

Solution.

The node equations for the circuit can be written in terms of u and w as

$$C_2 \frac{du}{dt} = -\left(\frac{1}{R_1} + \frac{1}{R_2}\right)u + \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} g_{nm} u^n w^m + \frac{x}{R_1}$$

$$C_1 \frac{dw}{dt} = -\frac{w}{R_L} - \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} g_{nm} u^n w^m - \frac{x}{R_L}$$

These equations can be rewritten in vector for as

$$\begin{bmatrix} C_2 \frac{d}{dt} + (\frac{1}{R_1} + \frac{1}{R_2}) - g_{10} & -g_{01} \\ g_{10} & C_1 \frac{d}{dt} + \frac{1}{R_L} + g_{01} \end{bmatrix} \begin{bmatrix} u \\ w \end{bmatrix} = \tag{57}$$

$$\begin{bmatrix} 1/R_1 \\ -1/R_L \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \end{bmatrix} \sum_{\substack{n=0 \\ n+m>1}}^{\infty} \sum_{m=0}^{\infty} g_{nm} u^n w^m$$

where the linear part of the transistor model has been moved to the left side of the equation.

ε-Scaling.

The voltages u and w can be expanded as

$$u = P[\varepsilon x] = \sum_{k=1}^{\infty} P_k[\varepsilon x] = \sum_{k=1}^{\infty} \varepsilon^k u_k \tag{58}$$

$$w = Q[\varepsilon x] = \sum_{k=1}^{\infty} Q_k[\varepsilon x] = \sum_{k=1}^{\infty} \varepsilon^k w_k$$

where the scaling constant ε has been included to keep track of the order of the Volterra operators. These expansions for u and w can now be substituted into the node vector equation; viz.,

$$[L] \begin{bmatrix} \sum_{k=1}^{\infty} \varepsilon^k u_k \\ \\ \sum_{k=1}^{\infty} \varepsilon^k w_k \end{bmatrix} = \begin{bmatrix} 1/R_1 \\ -1/R_L \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \end{bmatrix} \sum_{\substack{n=0 \\ n+m>1}}^{\infty} \sum_{m=0}^{\infty} g_{nm} (\sum \varepsilon^k u_k)^n (\sum \varepsilon^k w_k)^m \tag{59}$$

where

$$[L] = \begin{bmatrix} C_2 \frac{d}{dt} + (\frac{1}{R_1} + \frac{1}{R_2}) - g_{10} & -g_{01} \\ g_{10} & C_1 \frac{d}{dt} + \frac{1}{R_L} + g_{01} \end{bmatrix}$$

Now like powers of ε can be collected and equated.

$\underline{\varepsilon^1:}$

$$[L]\begin{bmatrix} u_1 \\ w_1 \end{bmatrix} = \begin{bmatrix} 1/R_1 \\ -1/R_L \end{bmatrix} x$$

or

$$\begin{bmatrix} u_1 \\ w_1 \end{bmatrix} = [L]^{-1}\begin{bmatrix} 1/R_1 \\ -1/R_L \end{bmatrix} x \tag{60}$$

$$= \begin{bmatrix} P_1 \\ Q_1 \end{bmatrix} x = \underline{H}_1 x$$

where

$$[L]^{-1} = \frac{1}{\Delta}\begin{bmatrix} C_1 \frac{d}{dt} + \frac{1}{R_L} + g_{01} & g_{01} \\ -g_{10} & C_2 \frac{d}{dt} + (\frac{1}{R_1} + \frac{1}{R_2}) - g_{10} \end{bmatrix}$$

$$\underline{H}_1 = \frac{1}{R_L}[L]^{-1}\begin{bmatrix} 1/R_1 \\ -1/R_L \end{bmatrix} \tag{61}$$

and Δ is the determinant of [L].

$\underline{\varepsilon^2:}$

$$[L]\begin{bmatrix} u_2 \\ w_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}(g_{20}u_1^{\ 2} + g_{11}u_1w_1 + g_{02}w_1^{\ 2})$$

or

$$\begin{bmatrix} u_2 \\ w_2 \end{bmatrix} = [L]^{-1}\begin{bmatrix} 1 \\ -1 \end{bmatrix}(g_{20}u_1^{\ 2} + g_{11}u_1w_1 + g_{02}w_1^{\ 2}) \tag{62}$$

and therefore $\underline{H}_2$ can be described in "operator notation" as

$$\underline{H}_2 = R_L \underline{H}_1 \{g_{20} P_1^{\,2} + g_{11} P_1 Q_1 + g_{02} Q_1^{\,2}\} \tag{63}$$

$\underline{\varepsilon^3:}$

$$\begin{bmatrix} u_3 \\ w_3 \end{bmatrix} = [L]^{-1} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \{g_{30} u_1^{\,3} + g_{21} u_1^{\,2} w_1 + g_{12} u_1 w_1^{\,2} + g_{03} w_1^{\,3} + 2g_{20} u_1 u_2 + g_{11}(u_1 w_2 + u_2 w_1) + 2g_{02} w_1 w_2\} \tag{64}$$

$$\underline{H}_3 = \frac{1}{R_L} \underline{H}_1 \{g_{30} P_1^{\,3} + g_{21} P_1^{\,2} Q_1 + g_{12} P_1 Q_1^{\,2} + g_{03} Q_1^{\,3} + 2g_{20} P_1 P_2 + g_{11}(P_1 Q_2 + P_2 Q_1) + 2g_{02} Q_1 Q_2\} \tag{65}$$

where $\underline{H}_3$ is expressed in operator form.

As in all the previous examples, $\underline{H}_\ell$ is recursive in the sense that it is a function of only previous $\underline{H}$'s.

Now the operators can be Laplace transformed to

$$\underline{H}_1(s) = \frac{1}{\Delta(s)} \begin{bmatrix} C_1 s + 1/R_L \\ C_2 s + \{1/R_1 + 1/R_2\} \end{bmatrix} \tag{66}$$

$$\underline{H}_2(s_1,s_2) = R_L \underline{H}_1(s_1+s_2)\{g_{20} P_1(s_1) P_1(s_2) + g_{11} P(s_1) Q_1(s_2) + g_{02} Q_1(s_1) Q_1(s_2)\},$$

$$\begin{aligned} \tilde{\underline{H}}_3(s_1,s_2,s_3) = \; & R_L \underline{H}_1(s_1+s_2+s_3)\{ g_{30} P_1(s_1) P_1(s_2) P_1(s_3) \\ & + g_{21} P_1(s_1) P_1(s_2) Q_1(s_3) + g_{12} P_1(s_1) Q_1(s_2) Q_1(s_3) \\ & + g_{03} Q_1(s_1) Q_1(s_2) Q_1(s_3) + 2g_{20} P_1(s_1) P_2(s_2,s_3) \\ & + g_{11}\{P_1(s_1) Q_2(s_2,s_3) + P_2(s_1,s_2) Q_1(s_3) \\ & + 2g_{02} Q_1(s_1) Q_2(s_2,s_3))\} \end{aligned}$$

where

$$\begin{aligned} \Delta(s) = \; & C_1 C_2 s^2 + \{C_2(g_{01} + 1/R_L) + C_1(1/R_1 + 1/R_2 - g_{10})\} s \\ & + \{(g_{01} + 1/R_L)(1/R_1 + 1/R_2) - g_{10}/R_L\} \end{aligned}$$

The stated objective of this example was to determine $\underline{H}_1$, $\underline{H}_2$, and $\underline{H}_3$. Since this has been accomplished, this example is complete.

Example 7: Nonlinear Cascade.

In this example, we wish to design a cascade filter or compensator P which will eliminate some of the nonlinearities of a system H, as depicted in Fig. 14.

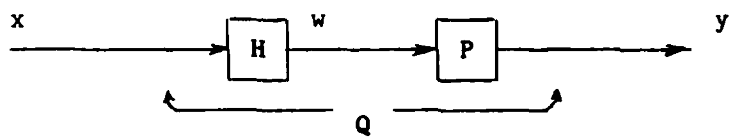

Fig. 14 Cascade of Two Nonlinear Subsystems

Here it is assumed that the form of the nonlinear system has already been determined as

$$w = H[x] = \sum_{n=1}^{\infty} H_n[x] = \sum_{n=1}^{\infty} w_n \tag{67}$$

Indeed, after the form of the compensator has been determined, the H's for an earlier example (viz., the single nonlinearity example) will be used to implement part of a specific compensator.

Solution.

Form the output y as

$$y = P[w] = \sum_{k=1}^{\infty} P_k[w] \tag{68}$$

$$= \sum_{k=1}^{\infty} P_k\Big[\sum_{n=1}^{\infty} H_n[x]\Big]$$

Then replace x with εx to form a new output;

$$y_\varepsilon = \sum_{k=1}^{\infty} P_k\Big[\sum_{n=1}^{\infty} H_n[\varepsilon x]\Big]. \tag{69}$$

$$= \sum_{k=1}^{\infty} P_k\Big[\sum_{n=1}^{\infty} \varepsilon^n H_n[x]\Big]$$

This relationship can be reexpressed in terms of the "x-y transfer function" Q as

$$y_\varepsilon = \sum_{m=1}^{\infty} \varepsilon^m Q_m[x]$$

Now if powers of ε are collected, the Q's can be determined using "operator notation" as

$$Q_1 = P_1H_1 \tag{70}$$

$$Q_2 = P_1H_2 + P_2H_1$$

$$Q_3 = P_1H_3 + 2P_2\{H_1,H_2\} + P_3H_1$$

$$Q_4 = P_1H_4 + 2P_2\{H_1,H_3\} + P_2H_2 + 3P_3\{H_1,H_1,H_2\} + P_4H_1$$

$$Q_5 = P_1H_5 + 2P_2\{H_1,H_4\} + 2P_2\{H_2,H_3\} + 3P_3\{H_1,H_2,H_2\} + 4P_4\{H_1,H_1,H_1,H_2\} + P_5H_1$$

$$\vdots$$

These equations can be expressed in the complex-domain as

$$Q_1(s) = P_1(s)H_1(s)$$

$$Q_2(s_1,s_2) = P_1(s_1+s_2)H_2(s_1,s_2) + H_1(s_1)H_1(s_2)P_2(s_1,s_2)$$

$$Q_3(s_1,s_2,s_3) =$$

$$P_1(s_1+s_2+s_3)H_3(s_1,s_2,s_3) + \frac{1}{3}\,[H_1(s_1)H_2(s_2,s_3)P_2(s_1,s_2+s_3) + H_1(s_2)H_2(s_1,s_3)P_2(s_2,s_1+s_3) + H_1(s_3)H_2(s_1,s_2)P_2(s_3,s_1+s_2)]$$

$$\vdots$$

The stated objective of this exercise is to reduce the nonlinearities without distorting the linear characteristics of **H**. Therefore, the form of $\mathbf{Q}_1$ should be $\mathbf{H}_1$; and therefore,

$$\mathbf{P}_1 = I \tag{71}$$

where I is the identity operator. Furthermore, $\mathbf{Q}_2$ and $\mathbf{Q}_3$ can be made zero by choosing $\mathbf{P}_2$ and $\mathbf{P}_3$ as

$$\mathbf{P}_2 = -\mathbf{H}_2\mathbf{H}_1^{-1} \tag{72}$$

$$\mathbf{P}_3 = -(\mathbf{H}_3 + 2\mathbf{P}_2\mathbf{H}_1\mathbf{H}_2)\mathbf{H}_1^{-1}$$

Now it was assumed that only the first three **H**'s were significant. So choose the remaining **P**'s as the null operators. Then the remaining Q's are only functions of the insignificant **H**'s and "cross-products" of the first three **H**'s. Thus by including only the first three **P**'s in the compensator, the first two "high-order" nonlinearities are eliminated, and the remaining nonlinear terms are small.

In order to be more specific concerning the implementation of a compensator, let us implement the $\mathbf{P}_2$ part of the compensator using the **H**'s of an earlier model. Specifically, the H's of the "single nonlinearity example" is employed.

$$H_1(s) = Cs \frac{Ls}{LCs^2 + Lg_1 s + 1}$$

$$H_2(s_1, s_2) = -Z(s_1+s_2)\ \lambda_2\ \{H_1(s_1)H_1(s_2)\}$$

Then P_2 becomes

$$P_2(s_1, s_2) = -H_2(s_1, s_2)H_1(s_1+s_2)^{-1} \tag{73}$$

$$= g_2 \frac{LC(s_1+s_2)^2}{LC(s_1+s_2)^2 + Lg_1(s_1+s_2) + 1}$$

The algebraic expression for $P_3(s_1,s_2,s_3)$ is determined by evaluating the second expression of equation (72). While this evaluation is mathematically straight forward, the resulting expression is involved. Therefore, the specific form of P_3 will not be determined. This then completes this example.

VIII. NLTFs OF A PROPOSED COMPENSATION NETWORK

Example 8: Diode Compensator.

The previous example dealt with the general design of a cascade compensator to reduce nonlinearities. This example analyzes a specific circuit (viz., a Balanced Diode Squarer (BDS) as depicted in Fig. 15) which can be employed as part of such a compensator. Specifically, it can be used to remove the second order nonlinear part of the signal. Since this circuit uses diodes, the results of the diode analysis of Example 4 can be employed to facilitate this BDS analysis. The dc-bias required to operate at the proper diode quiescent point has not been depicted since it adds nothing to the analysis and is blocked by the output capacitor.

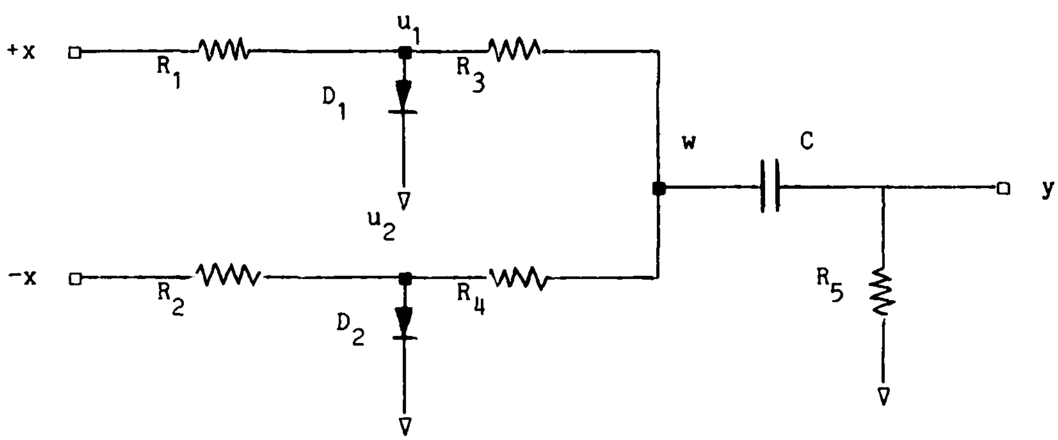

Fig. 15 Balanced Diode Squarer Circuit

For the purpose of analysis, the BDS circuit can be redrawn as appears in Fig. 16 The resistors R_1 and R_2 and the resistors R_3 and R_4 have been matched in Fig. 16. Furthermore, the two diodes are also matched; if the circuit is constructed of discrete components, this diode matching can be accomplished with additional external resistors and capacitors "hung-on" the diodes.

The loops of the circuit have been chosen so that only a single loop current passes through a diode; this choice simplifies the algebra of the analysis. The system or loop equations, and the output equation for the circuit of Fig. 16 can be written as

$$
\begin{array}{llllll}
R_1 I_1 & & +R_1 I_3 & & +u_1 & = x \\
& R_1 I_2 & & +R_1 I_4 & +u_2 & =-x \\
R_1 I_1 & & +Z_a I_3 & +Z_s I_4 & & = x \\
& R_1 I_2 & +Z_s I_3 & +Z_a I_4 & & =-x \\
y= & R_5\{I_3+I_4\} & & & &
\end{array}
\tag{74}
$$

where $Z_s = \dfrac{R_5 Cs+1}{Cs}$, and $Z_a = Z_s + R_1 + R_3$. Now by using the results of the diode analysis of Example 2, the voltages u_1 and u_2 can be written as a function of the loop currents. Then I_3 and I_4 can be expressed as Volterra NLTFs operating on x. Then the first three Volterra NLTFs of the output are simply formed.

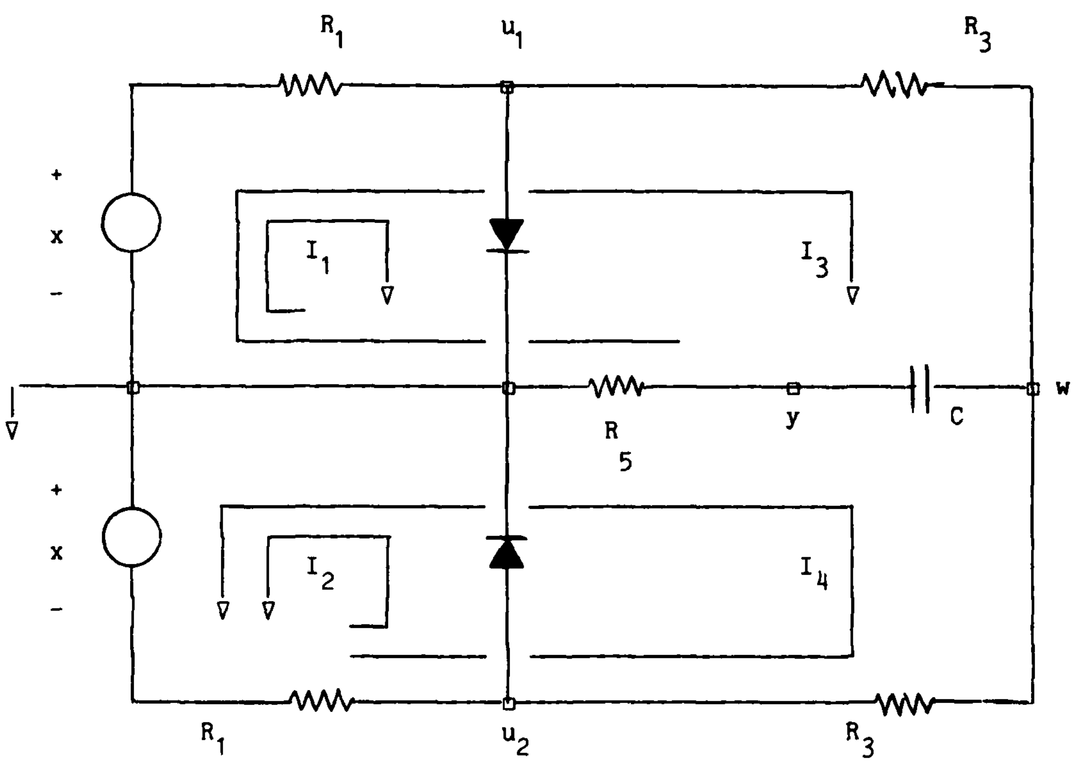

Fig. 16 BDS Circuit Redrawn

Solution.

Here the loop currents can be expanded as

$$I_1 = \sum_{k=1}^{\infty} I_{1k} = \sum_{k=1}^{\infty} H_1[x]$$

$$\vdots$$

$$I_4 = \sum_{k=1}^{\infty} I_{4k} = \sum_{k=1}^{\infty} H_4[x]$$

where the second index on the I's indicate the order of the Volterra term. These expressions can be restated more concisely as

$$\underline{I} = \sum_{k=1}^{\infty} \underline{I}_k = \sum_{k=1}^{\infty} \underline{H}_k[x]$$

From example 4, the voltage u_1 can be expanded as

$$u_1 = P_1[I_1] + P_2[I_1] + P_3[I_1] + \text{HOT} \tag{75}$$

where HOT are higher order terms which will be ignored in this analysis. There is a similar expansion for the voltage u_2. These expressions for u_1 and u_2 and the Volterra expansions of the loop currents can be substituted into the loop equations of equation (74). Then the input voltage can be scaled to εx.

$$R_1 \sum_{k=1}^{\infty} \varepsilon^k I_{1k} + R_1 \sum_{k=1}^{\infty} \varepsilon^k I_{3k} + \sum_{n=1}^{\infty} P_n[\sum_{k=1}^{\infty} \varepsilon^k I_{1k}] = \varepsilon x$$

$$R_1 \sum_{k=1}^{\infty} \varepsilon^k I_{2k} + R_1 \sum_{k=1}^{\infty} \varepsilon^k I_{4k} + \sum_{n=1}^{\infty} P_n[\sum_{k=1}^{\infty} \varepsilon^k I_{2k}] = -\varepsilon x$$

$$R_1 \sum_{k=1}^{\infty} \varepsilon^k I_{1k} + Z_a \sum_{k=1}^{\infty} \varepsilon^k I_{3k} + Z_s \sum_{k=1}^{\infty} \varepsilon^k I_{4k} = \varepsilon x$$

$$R_1 \sum_{k=1}^{\infty} \varepsilon^k I_{2k} + Z_s \sum_{k=1}^{\infty} \varepsilon^k I_{3k} + Z_a \sum_{k=1}^{\infty} \varepsilon^k = -\varepsilon x$$

The left-side of the first equation above can be written as

$$R_1\varepsilon^1 I_{11}+R_1\varepsilon^2 I_{12}+R_1\varepsilon^3 I_{13}+R_1\varepsilon^1 I_{31}+R_1\varepsilon^2 I_{32}+R_1\varepsilon^3 I_{33}+P_1[\varepsilon^1 I_{11}+\varepsilon^2 I_{12}+\varepsilon^3 I_{13}]$$

$$+P_2[\varepsilon^1 I_{11}+\varepsilon^2 I_{12}+\varepsilon^3 I_{13}]+P_3[\varepsilon^1 I_{11}+\varepsilon^2 I_{12}+\varepsilon^3 I_{13}]+\mathrm{HOT}$$

$$=R_1\varepsilon^1 I_{11}+R_1\varepsilon^2 I_{12}+R_1\varepsilon^3 I_{13}+R_1\varepsilon^1 I_{31}+R_1\varepsilon^2 I_{32}+R_1\varepsilon^3 I_{33}$$

$$+\varepsilon^1 P_1[I_{11}]+\varepsilon^2 P_1[I_{12}]+\varepsilon^3 P_1[I_{13}]$$

$$+\varepsilon^2 P_2[I_{11}{}^2]+\varepsilon^3 2P_2[I_{11}I_{12}]$$

$$+\varepsilon^3 P_3[I_{11}{}^3]+\mathrm{HOT}$$

where liberal operator notation is employed. There is a similar expansion for the second equation of the above loop equations.

And finally, the powers of ε can be collected.

$\underline{\varepsilon^1:}$

$$\begin{aligned}
&R_1 I_{11} && +R_1 I_{31} && && +P_1[I_{11}] = x \\
&&R_1 I_{21} && && +R_1 I_{41} & +P_1[I_{21}] = -x \\
&R_1 I_{11} && +Z_a I_{31} && +Z_s I_{41} && = x \\
&&R_1 I_{21} & +Z_s I_{31} && +Z_a I_{41} && = -x
\end{aligned} \tag{76}$$

where the diode Volterra relationship P_1 is a linear relationship ; viz., $P_1(s)= L_1{}^{-1}(s)= 1/\{c_0 s+\lambda_1\}$.

In this particular case, it is not necessary to solve for the first order loop currents to obtain the first order output y_1. From inspection of this system of equations, because of the symmetry of the equations it is apparent that $I_{21}= -I_{11}$ and $I_{41}= -I_{31}$. And therefore,

$$y_1 = R_5\{I_{31} + I_{41}\} \tag{77}$$
$$= 0.$$

Nevertheless, the loop equations must be solved to obtain $\underline{H}_1$ which will be required in the evaluation of the higher order transfer functions.

To this end, equations (76) can be restated as

$$\begin{bmatrix} \{R_1+P_1\} & 0 & R_1 & 0 \\ 0 & \{R_1+P_1\} & 0 & R_1 \\ R_1 & 0 & Z_a & Z_s \\ 0 & R_1 & Z_s & Z_a \end{bmatrix} \underline{I}_1 = [A]\,\underline{I}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} x \tag{78}$$

It therefore follows that

$$\underline{H}_1 = [A]^{-1} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \tag{79}$$

$\underline{\epsilon^2:}$

$$\begin{aligned} R_1I_{12} + R_1I_{32} + P_1[I_{12}] &= P_2[I_{11}^2] \\ R_1I_{22} + R_1I_{42} + P_1[I_{22}] &= P_2[I_{21}^2] \\ R_1I_{12} + Z_aI_{32} + Z_sI_{42} &= 0 \\ R_1I_{22} + Z_sI_{32} + Z_aI_{42} &= 0 \end{aligned} \tag{80}$$

which can be restated as

$$[A]\,\underline{I}_2 = \begin{bmatrix} P_2[I_{11}^2] \\ P_2[I_{21}^2] \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{H}_2 = [A]^{-1}\begin{bmatrix} P_2[I_{11}^{2}] \\ P_2[I_{21}^{2}] \\ 0 \\ 0 \end{bmatrix}$$

The second order output y_2 can now be formed as

$$y_2 = R_5\{I_3 + I_4\}$$

But this expression is unduly complicated. To gain more insight into this second order output, reexamine Fig. 15. The actual squaring of this BDS circuit is performed by the resistor-diode combination R_1-D_1 and R_2-D_2. The resistors R_3 and R_4 are employed to sum the voltages u_1 and u_2, and therefore these two resistors usually have impedance values which do not "load" the preceeding diode resistor combination. Furthermore, the combination C and R_5 are added to eliminate any DC-term; and it is designed so as not to load the circuit, and to pass all frequencies of x. Therefore, the output y_2 can be essentially described as a filtered square term as depicted in Fig. 17.

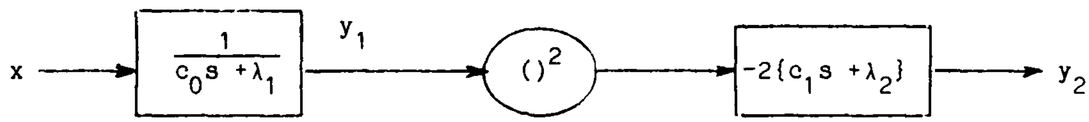

Fig. 17 BDS Second-Degree NLTF

$\underline{\varepsilon^3}$:

Here it is not necessary to solve explicitly for $\underline{I}_3$. Look again at Fig. 15; because of the symmetry of the circuit,

$$I_{1n} = -I_{2n}$$

Now look at the voltages u_1 and u_2;

$$u_1 = P_1[I_{11}] + P_2[I_{12}] + P_3[I_{13}] + P_4[I_{14}] + \cdots$$

$$u_2 = P_1[I_{21}] + P_2[I_{22}] + P_3[I_{23}] + P_4[I_{24}] + \cdots$$

$$= P_1[-I_{11}] + P_2[-I_{12}] + P_3[-I_{13}] + P_4[-I_{14}] + \cdots$$

$$= -P_1[I_{11}] + P_2[I_{12}] - P_3[I_{13}] + P_4[I_{14}] + \cdots$$

So that the voltage w and y consists only of the even orders. All odd orders are zero; viz.,

$$y_3 = 0$$

This same result can be obtained by recognizing the symmetry in the equations used to solve for $\underline{I}_3$ as was done in solving for $\underline{I}_1$.

Summary.

As stated earlier, usually the effect of R_3, R_4, R_5, and C are of no consequences to the application. But more often the "filters" of P_1 and P_2 are significant to the application. If a cascade filter requires a component $K[x^2]$, then the BDS circuit can be used as

$$K_c[y] = K[x^2]$$

where K_c includes K operator and the inverse filter operations of P_1 and P_2.

REFERENCES

[1] J. J. Bussgang, L. Ehrman and J. W. Graham, "Analysis of Nonlinear Systems with Multiple Inputs", Proceedings of the IEEE, Vol 62, August 1974.

[2] N. Wiener, Nonlinear Problems in Random Theory, M.I.T. Press, 1959.

[3] E. Bedrosian and S. O. Rice, "The Output Properties of Volterra Systems (Nonlinear Systems with Memory) Driven by Harmonic and Gaussian Inputs", Proceeding of IEEE, Vol 59, December 1971.

[4] M. Schetzen, The Volterra & Wiener Theories of Nonlinear Systems, John Wiley & Sons, 1980.

[5] V. K. Jain and A. M. Bush, "Nonlinear Representation and Pulse Testing of Communication Subsystems", RADC-TR-82-138 Technical Report, Rome Air Development Center, Griffiss Air Force Base, N.Y., May 1982.

[6] V. K. Jain, A. M. Bush and D. J. Kenneally,"Volterra Transfer Functions from Pulse Tests for Mildly Nonlinear Channels" RADC-TR-83-157 Technical Report, Rome Air Development Center, Griffiss Air Force Base, N.Y., July 1983.

[7] S. Narayanan, "Application of Volterra Series to Intermodulation Distortion Analysis of Transistor Feedback Amplifiers", IEEE Transaction on Circuit Theory, Vol CT-17, November 1970.

[8] D. D. Weiner and J. F. Spina, Sinusoidal Analysis and Modeling of Weakly Nonlinear Circuits with Application to Nonlinear Interference Effects, Van Nostrand Reinhold, 1980.

[9] Y. L. Kuo, "Frequency-Domain Analysis of Weakly Nonlinear Networks, 'Canned' Volterra Analysis" part 1 and part 2, Newsletter IEEE Circuits and Systems, Vol 11, August 1977 and Vol 11, October 1977.

[10] R. G. Meyer, M. J. Shensa and R. Eschenbach, "Cross Modulation and Intermodulation in Amplifiers at High Frequencies", IEEE Journal of Solid State Circuits, Vol SC-7, February 1972.

[11] S. Narayanan and H. C. Poon, "An Analysis of Distortion in Bipolar Transistors Using Integral Charge Control Model and Volterra Series", IEEE Transactions on Circuit Theory, Vol CT-20, July 1973.

[12] A. Javed, P. A. Goud and B. A. Syrett, "Analysis of a Microwave Feedforward Amplifier Using Volterra Series Representation", IEEE Transactions on Communication, Vol COM-25, March 1977.

[13] W. Reiss, "Nonlinear Distortion Analysis of p-i-n Diode Attenuators Using Volterra Series Representation, IEEE Transactions on Circuits and Systems, Vol CAS-31, June 1984.

[14] K. Y. Chang, "Intermodulation Noise and Products Due to Frequency-Dependent Nonlinearities in CATV Systems", IEEE Transactions on Communications, Vol COM-23, 1975.

]15] V. K. Jain and T. E. McClellan, "Stable Compensation of Nonlinear Communications Systems (Using Volterra Systems Characterization)", RADC-TR-85-243, Vol II, Technical Report, Rome Air Development Center, Griffiss AFB NY, 13441-5700, December 1985.

* [16] V. K. Jain and T. E. McClellan, "A Computer Program for the Design of Compensators for Nonlinear Communications Systems", RADC-TR-85-243, Vol III, Technical Report, Rome Air Development Center, Griffiss AFB NY, 13441-5700, December 1985.

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