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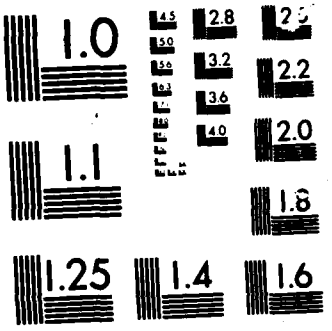

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SHEAR STRESS-STRAIN RELATION OBTAINED FROM TORQUE-TWIST DATA

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A standard technique for finding the shear stress-strain relation of a material employs a torsion test on a solid circular cylinder. It is shown that the exact solution is easily obtained as a correction to the sometimes-used quasi-elastic approximation. The solution is extended to the case of hollow cylinders. The analysis is shown to be applicable also to the case where a constant axial stress is present, and to a special case of planar isotropic materials.

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SHEAR STRESS-STRAIN RELATION OBTAINED FROM TORQUE-TWIST DATA

INTRODUCTION

One method of establishing the shear stress-strain relation of a material involves applying torsion to a circular cylinder. This approach has the advantage of experimental simplicity, but the disadvantage that the specimen is nonuniformly loaded, thus requiring a somewhat sophisticated data analysis. Fortunately the required analysis has already been worked out for the case of solid cylinders made of an isotropic elastic material. Such an analysis was furnished by Nadai [1]. Unfortunately, sometimes only an approximate analysis is performed, and results are only reported as the resulting approximate shear stress-strain relation. Fortunately however, as we shall show, this type of approximate result can be easily corrected to get the proper result without having to go back to the original torque-twist data, which may no longer be available.

A second contribution of the paper is to extend the analysis to the case of hollow cylinders. A third is to call attention to the fact that some cases of combined loads can also be analyzed in essentially the same manner. This is of particular interest in the case of highly anisotropic materials, in which compression-shear interaction may be large.

We first derive the standard result, using an approach that emphasizes the physical aspects of the problem, and serves as the basis of the extension to hollow cylinders.

SOLID ISOTROPIC CYLINDER

Consider first a solid circular cylinder in uniform torsion, in which the twist is θ radians per unit length. The shear strain is then given by

$$\gamma = r\theta \tag{1}$$

where r is the radius of the field point under consideration. If the cylinder is linearly elastic, the shear stress τ is proportional to the radius as shown in Fig. 1. If the material yields at a stress below the maximum shown in the figure, a plot of τ vs. r will be nonlinear. Also included in the figure is a nonlinear plot for the same applied torque.

In the elastic case, the analysis is quite straightforward. The torque is given by

$$M = \int_0^a 2\pi r \cdot r \cdot G r\theta \, dr = \frac{\pi a^4}{2} G\theta \tag{2}$$

where G is the shear modulus of the material, and a is the radius of the cylinder. The shear stress at the cylinder surface is given by

$$\tau(a) = G\theta a = 2M / \pi a^3 \tag{3}$$

An approximate shear stress-strain curve for the nonlinear case can be obtained by assuming that for any value of twist (3) applies. Since (3) is only valid in the elastic case, we shall refer to this as the quasi-elastic (QE) solution. It is equivalent to replacing the curve in Fig. 1 by the straight line and finding the shear stress-strain relation at the surface. It is clear that this overestimates the shear stress for a given strain, but it is not immediately obvious how big the error might be.

The standard result [1-3] is

$$\tau(a) = \frac{1}{2\pi a^3}\left[3M + \theta\frac{dM}{d\theta}\right] \tag{4}$$

One way to derive this result is as follows. In the nonlinear case, for twist θ the torque is

$$M(\theta) = \int_0^a 2\pi r^2 \tau(r\theta)dr = \int_0^a \Pi dr \tag{5}$$

where, for simplicity, Π denotes the integrand. When the torque is increased to M + dM the twist becomes $\theta + d\theta$. By virtue of (1), every stress contour moves toward the center a distance equal to $r\, d\theta / \theta$. This is depicted graphically in Fig.2.

It is convenient to consider the torque for M + dM to be composed of two parts, the part contributed by stresses below $\tau(a, \theta)$ and the part contributed by stresses larger than this value. Thus

$$M(\theta + d\theta) = \int_0^{a(1 - d\theta/\theta)} \Pi dr + \int_{a(1 - d\theta/\theta)}^{a} \Pi dr \tag{6}$$

Since each r and the dr in the first integral are decreased by the factor $(1 - d\theta / \theta)$ while the product $r\theta$ remains unaffected by the increase in θ, we can write

$$\int_0^{a(1 - d\theta/\theta)} \Pi dr = (1 - d\theta/\theta)^3 M(\theta)$$

$$= \left(1 - 3\frac{d\theta}{\theta}\right) M(\theta) \tag{7}$$

Note that this result holds regardless of the form of $\tau(\gamma)$. The second term is simply

$$\int_{a(1-d\theta/\theta)}^{a} \Pi dr \approx \tau(a)2\pi a^3 d\theta/\theta \tag{8}$$

Combining (5)-(8) we obtain

$$M(\theta + d\theta) - M(\theta) = dM(\theta)$$

$$= \left[-3M + 2\pi a^3 \tau(a)\right] d\theta/\theta \tag{9}$$

whence

$$\tau(a) = \frac{1}{2\pi a^3}\left[3M + \theta\frac{dM}{d\theta}\right] \tag{4}$$

From (4) it is evident that the proper reduction of the torque-twist data requires knowing both the ordinate and the slope of the torque-twist curve. What can we do about it if we only have the QE approximation referred to earlier? Fortunately the correction is very easily made.

The approximate result was obtained by assuming that (3) applies. Let us designate the resulting shear stress by the symbol $\breve{\tau}$. Thus

$$\breve{\tau} = 2M/\pi a^3 \tag{10}$$

Then

$$\tau(a) = \frac{1}{2\pi a^3}\left[-M + \theta\frac{dM}{d\theta}\right] + \frac{2M}{\pi a^3}$$

$$= \breve{\tau}(a) - \frac{1}{4}\left[\breve{\tau}(a) - \theta\frac{d\breve{\tau}(a)}{d\theta}\right] \tag{11}$$

Since $\gamma = a\theta$ we finally obtain

$$\tau = \breve{\tau} - \frac{1}{4}\left[\breve{\tau} - \gamma\frac{d\breve{\tau}}{d\gamma}\right] \tag{12}$$

It is readily seen from Fig. 3 that the term in the square bracket in (12) is simply the $\breve{\tau}$-intercept of the tangent to the approximate (QE) shear stress-strain curve taken at any chosen value of γ. Below the elastic limit the term is zero. The term increases as the slope decreases, becoming equal to the ordinate when the slope is zero. Assuming the slope never becomes negative (at least for values of γ that are of practical interest) the correction never exceeds 25%. The correct curve is thus bounded by the two curves $\breve{\tau}(a)$ and $0.75\,\breve{\tau}(a)$ as shown schematically in Fig. 4.

HOLLOW CYLINDER

From Eq. (9) we see that in the case of a solid cylinder the increment in torque resulting from an increment in twist is composed of two parts, one negative and one positive. The negative one is associated with the shrinking radius of each of the shear stress contours, while the positive one is the consequence of a higher stress being generated at the outer surface. The latter contains the surface stress $\tau(a,\theta)$ as a factor. On physical grounds one can anticipate the same or similar terms will be present in the case of a hollow cylinder; additionally there will be a negative term containing the factor $\tau(b,\theta)$ because when the twist increases the lowest stress zone moves out of the cylinder from the inner surface, r = b.

To make these qualitative considerations quantitative, consider a hollow cylinder with outer and inner radii equal to a and b respectively. The torque M required to twist the cylinder through an angle θ per unit length can be regarded as the difference in the torques $M_a(\theta)$ and $M_b(\theta)$ required to twist solid cylinders of radii a and b through the same angle. That is,

$$M(\theta) = M_a(\theta) - M_b(\theta) \tag{13}$$

Now from (4) the exterior surface stresses on the two solid cylinders are

$$\tau(a,\theta) = \frac{1}{2\pi a^3}\left[3M_a + \theta\frac{dM_a}{d\theta}\right] \tag{14}$$

$$\tau(b,\theta) = \frac{1}{2\pi b^3}\left[3M_b + \theta\frac{dM_b}{d\theta}\right] \tag{15}$$

Thus

$$a^3\tau(a,\theta) - b^3\tau(b,\theta) = \frac{1}{2\pi}\left[3M + \theta\frac{dM}{d\theta}\right] \tag{16}$$

or

$$\tau(a,\theta) = \frac{1}{2\pi a^3}\left[3M + \theta\frac{dM}{d\theta}\right] + \frac{b^3}{a^3}\tau(b,\theta) \tag{17}$$

Comparing (17) with (4) it is evident that, as expected, the surface stress $\tau(a)$ is a little larger for a hollow cylinder than for a solid one with the same torque applied. Also as $b \to a$, $\tau(b) \to \tau(a)$, as expected.

The torque-twist curve of the hollow cylinder enables us to evaluate the term in square brackets. But this yields only a single equation to evaluate the two unknown stresses $\tau(a, \theta)$ and $\tau(b, \theta)$. We can, nevertheless, solve for shear stress as a function of strain; but in contrast to the solid cylinder case, it must be done iteratively. We start by choosing a value of θ such that $\tau(b,\theta)$ is known, namely one at which it is at or below the elastic limit τ_e. In this case,

$$\tau(b,\theta) = Gb\theta \tag{18}$$

If G is not already known, it can be obtained by employing the equation found by changing the lower limit in (2) from 0 to b:

$$M = \frac{\pi}{2}(a^4 - b^4)G\theta \tag{19}$$

If the next angle of twist is chosen to be larger by the factor a/b, the corresponding value of τ (b) is equal to the initial value of τ (a), i.e.,

$$\tau(b,\theta_2) = \tau(a,\theta_1) \tag{20}$$

Repeating this process one will always know the value of τ (b) needed to evaluate τ (a), but only for the sequence of twist angles $\theta_i = (a/b)^{i-1}\theta_1$. If this procedure results in the calculated points on the shear stress-strain curve being too far apart and more points are desired, we can proceed as follows:

Choose θ_1 to be equal to or slightly below the twist at which the torque-twist curve becomes nonlinear. Then choose successive values of twist to satisfy the relation

$$\theta_{i+1} = \theta_i (a/b)^{1/j} \tag{21}$$

where j-1 is the number of points desired between successive pairs of points obtained by the previous procedure. Note that for $i \leq j$, $\tau(b,\theta_i)$ is at or below the elastic limit and thus is known from (18). The corresponding values of outer surface shear stress $\tau(a,\theta_i)$ can thus be found from (17). For i>j,

$$\tau(b,\theta_i) = \tau(a,\theta_{i-j}) \tag{22}$$

so that each τ (b) is equal to a previously calculated τ (a).

By following the above procedure, $\tau(a,\theta)$ can be evaluated over the entire range of twist θ. The corresponding shear strain, of course, is equal to $a\theta$.

DISCUSSION

The discussion up to this point has assumed that the material under consideration is isotropic. Actually, of course, the procedure is equally applicable to planar isotropic materials, provided that the cylinder axis is normal to the plane of isotropy. The shear stress-strain curve obtained in this case is τ_{zx} vs. γ_{zx} or alternatively τ_{zy} vs. γ_{zy} where the x and y axes lie in the plane of isotropy.

We note in closing that although the preceding analysis assumed that the applied axial stress is zero, it continues

to hold for any other constant value of axial stress. That is because in this case the combined stress contours move in toward the center of the cylinder in just the same way as they do for pure torsion. A cylinder torsion test under constant applied axial load thus offers a good experimental means for testing shear-compression coupling in planar-isotropic materials. This coupling is currently quite inadequately understood; but it can be very important when the compressive modulus is orders of magnitude greater than the shear modulus.

CONCLUSIONS

1. A new derivation has been given for the correct procedure for converting the torque-twist relation into a shear stress-strain relation in the case of a solid isotropic cylinder. While the result is not new, the derivation is based more on physical considerations and less on mathematical ones than most previous derivations, and therefore may be preferred by some readers.

2. It is shown that results sometimes reported using the quasi-elastic (QE) approximation for this purpose can be easily processed to give the correct result.

3. A procedure is described for similarly finding the shear stress-strain curve from the the torque-twist curve obtained using a hollow cylinder.

4. The procedure described is also applicable to the analysis of planar-isotropic materials provided the cylinder axis is normal to the plane of isotropy.

5. Cylinder torsion tests provide a potentially valuable means for studying shear-compression interaction in highly anisotropic planar-isotropic materials.

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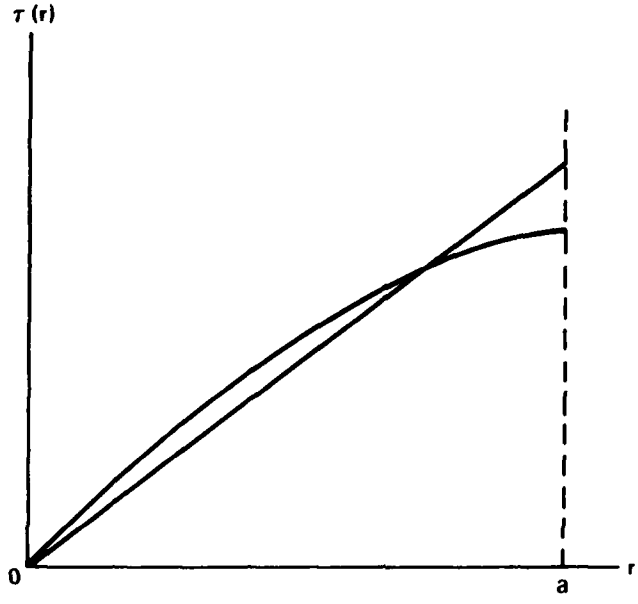

Fig.1 Elastic and inelastic shear stress for same applied torque.

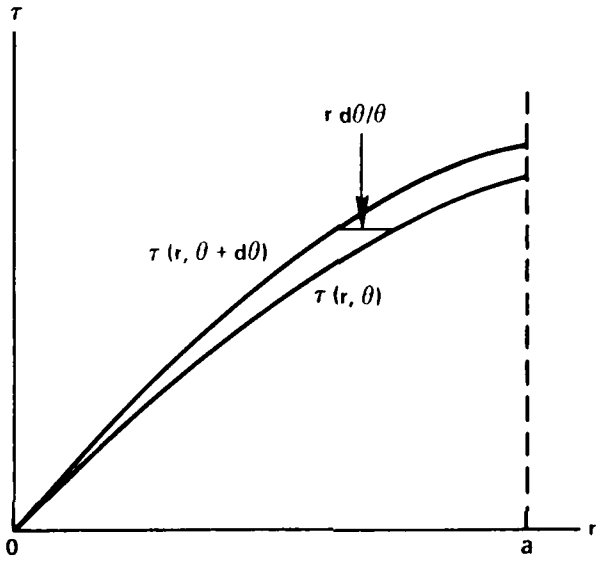

Fig.2 Shear stress contours move inward as twist angle θ increases.

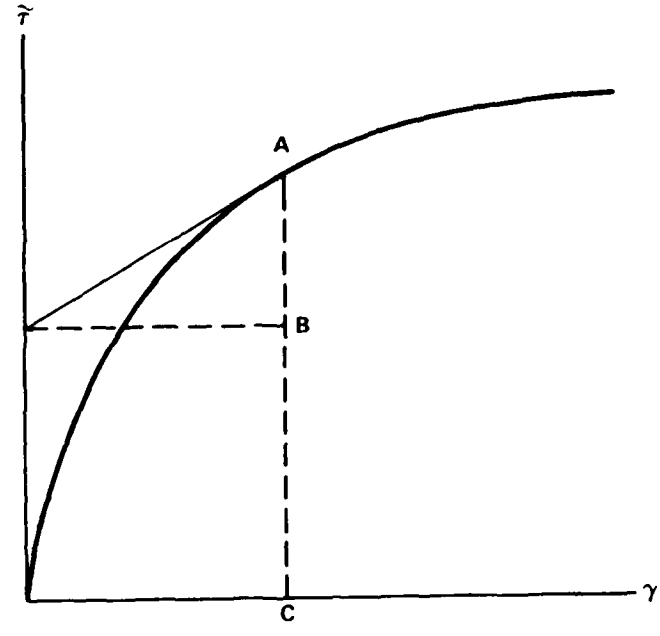

Fig.3 Diagram shows $\tilde{\gamma}$- intercept of the tangent to the approximate shear stress-strain curve. Note that $\overline{AC} - \overline{AB}$ equals square bracket term in Eqn. (12).

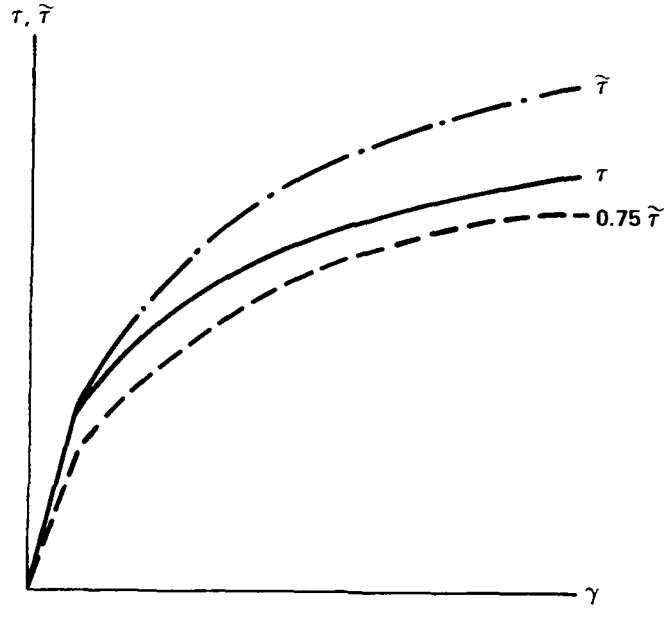

Fig.4 Note that correct shear stress-strain curve is bounded by quasi-elastic approximation $\tilde{\tau}$ and $0.75\tilde{\tau}$.

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