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QUASI-TEM ANALYSIS OF MICROWAVE TRANSMISSION LINES BY THE FINITE ELEMENT METHOD

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This paper describes a finite element approach to quasi-TEM analysis of several different types of isolated and coupled microwave transmission lines. Both the first and higher-order ordinary elements, as well as singular and infinite elements are used to solve for the potential and field distribution in the cross-section of the line. Next, the cross-sectional field distribution is inserted in a variational expression to compute the capacitance per unit length of the line and the effective permittivity and characteristic impedance of the line are obtained from the capacitance value. A perturbational approach is developed for estimating the losses due to conductor and dielectric dissipation and computing the attenuation constant.

Lines treatable by this method may contain an arbitrary number of arbitrarily shaped conductors, including a system of conductors either placed above a single ground plane or between two parallel ground planes, and inhomogeneous dielectric regions that can be approximated locally by a number of homogeneous subregions. (over)

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Quasi-TEM Analysis of Microwave Transmission Lines

by the Finite Element Method

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ABSTRACT

This paper describes a finite element approach to quasi-TEM analysis of several different types of isolated and coupled microwave transmission lines. Both the first and higher-order ordinary elements. as well as singular and infinite elements are used to solve for the potential and field distributions in the cross-section of the line. Next. the cross-sectional field distribution is inserted in a variational expression to compute the capacitance per unit length of the line and the effective permittivity and characteristic impedance of the line are obtained from the capacitance value. A perturbational approach is developed for estimating the losses due to conductor and dielectric dissipation and computing the attenuation constant.

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I. INTRODUCTION

The accurate prediction of the characteristic impedance. attenuation. coupling. crosstalk. etc.. in microstrips. striplines and similar transmission lines is important in microwave and millimeter wave integrated circuits. digital circuit design. communication and other applications. The objective of this paper is to consider microwave transmission lines with rather arbitrary configuration and to present a computer-aided analysis that allows simple and accurate calculation of its design parameters.

In the past. microwave transmission lines have been thoroughly investigated by many authors and many approaches to analyzing them have been devised. e.g.. the Green's function techniques [1] - [6]. conformal mapping [7] - [9]. variational methods [10]. [11]. Fourier transform method [12]. [13]. Fourier integral method [14]. spectral-domain method [15] - [17]. boundary element method [18]. [19] and finite element method [20]. All of the above methods. with the exception of the last two are restricted in their application to transmission lines in which the conductors are thin strips or the dielectrics inserts have planar interfaces or both restrictions apply. In contrast. the finite element method (FEM) is capable of handling transmission lines with rather arbitrary configurations. since the lines treatable by this method may contain an arbitrary number of conductors of arbitrary shape and inhomogeneous dielectric regions that can be approximated locally by a number of homogeneous subregions. Because of the generality of the FEM approach the finite element method is employed in this paper for the analysis of several representative microwave transmission lines of practical interest. The analysis is based on a quasi-TEM model which is often adequate for microwave frequencies in communication applications and for typical pulse rise times of interest in high speed digital circuit design.

Both the first- and higher-order ordinary elements. as well as singular and infinite elements. are used in the FEM algorithm to solve for the quasi-static potential and the corresponding field distribution in a microwave transmission line. The capacitance per unit length of the line is obtained from a variational expression. and the effective dielectric permittivity. characteristic

impedance and phase velocity are also calculated. Finally, a perturbation method is used to compute the losses due to both the conductor and dielectric dissipations.

The characteristic impedance and loss characteristics have been calculated for various transmission lines of interest and very good agreement with available theoretical and experimental data has been obtained.

II. FINITE ELEMENT MODEL OF A MICROWAVE TRANSMISSION LINE

Consider a microwave transmission line with an arbitrary cross section consisting of a number of arbitrarily shaped conductors and inhomogeneous dielectric regions which can be approximated locally by homogeneous subregions (Fig. 1). Let us assume that the line is uniform along its longitudinal (z) axis and let ϵ and μ $(= \mu_o)$ denote the permittivity and permeability, respectively, of the medium in each homogeneous subregion.

We assume a quasi-TEM model, i.e., that the dominant mode propagating along the line is a TEM-mode. Under this approximation, the problem is reduced to that of finding the scalar potential function, ϕ, satisfying the Poisson's equation

$$\Delta \phi = - \frac{\rho}{\epsilon}. \tag{1}$$

subject to the (appropriate) boundary conditions

$$\phi = \phi, \ on \ \Gamma_1 \ \frac{\partial \phi}{\partial n} = 0 \ on \ \Gamma_2. \tag{1a}$$

where ρ is the volume charge distribution, ϵ is the dielectric permittivity and Γ_1 and Γ_2 are complementary portions of the boundary Γ of the domain S in which Eqn. (1) is defined.

In order to apply the finite element approach to the problem at hand we formulate it in variational terms. The correct solution of Eqn. (1) is one that minimizes the energy functional

$$F = \frac{1}{2} \int_S \epsilon |\nabla \phi|^2 \, dS - \int_S \rho \phi dS \tag{2}$$

Eqn. (1) is the Euler equation of the functional F [27] satisfying the essential boundary condition of Dirchlet type, viz.,
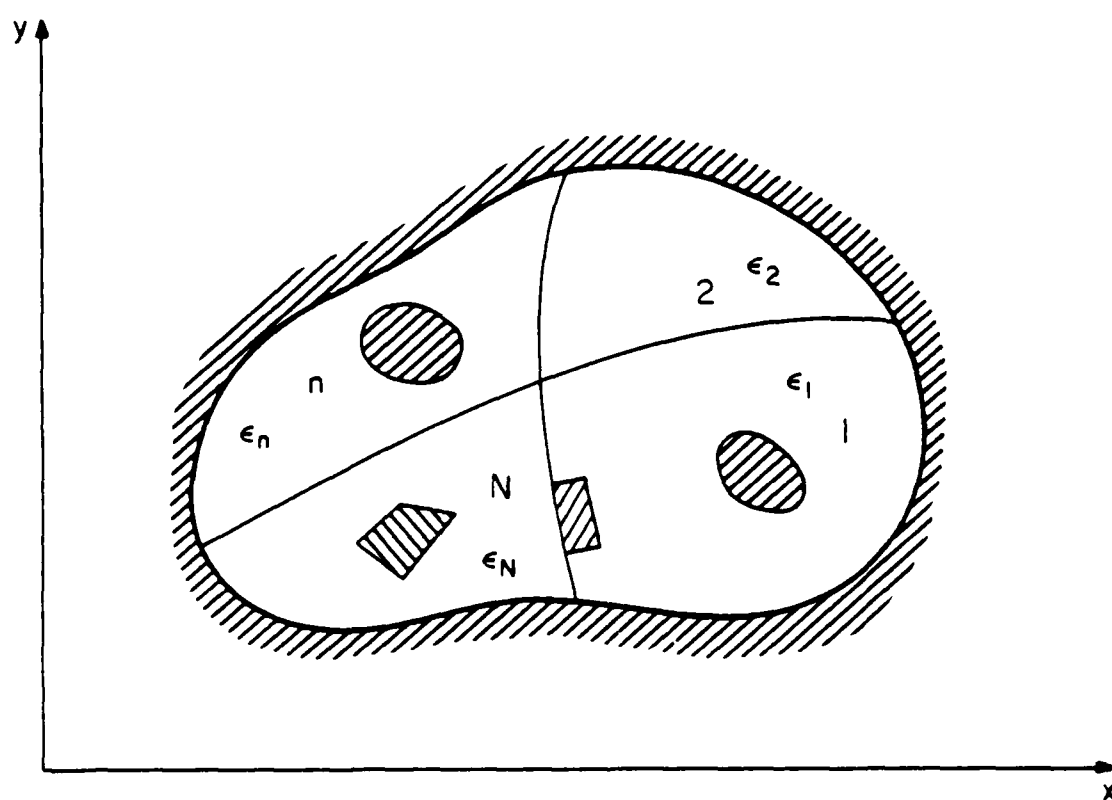
Figure 1. Cross section of a microwave transmission line.

$$\phi = \phi_o, \ on \ \Gamma_1. \tag{2a}$$

and the natural boundary condition of Newman type. i.e..

$$\frac{\partial \phi}{\partial n} = 0, \ on \ \Gamma_2. \tag{2b}$$

According to the FEM procedure. the cross section of the line. domain S. is subdivided into finite elements in an arbitrary manner provided that all the dielectric interfaces coincide with the element sides. Although a variety of different elements can be chosen. the triangular first- or higher-order [21] elements are adopted in this study. It has been shown [21] that the accuracy and efficiency of computation can be substantially enhanced via the use of the high-order elements as compared to the case where only the first-order elements are employed. if the same number of nodes is used. On the other hand. a large number of simple elements is clearly advantageous where a complicated boundary shape needs to be modeled.

However. when there are field singularities caused by edges in the cross section of the line. a very fine mesh of first- or high-order elements is required to obtain an accurate solution. However. in order to improve accuracy and reduce the number of necessary nodes. singular elements [22] are used in this study. A singular element with labeled nodes and associated triangular polar coordinate system (ρ, σ) is shown in Figure 2.

The global Cartesian coordinates are then

$$x = x_1 + \rho \left[(x_2 - x_1) + \sigma (x_3 - x_2) \right] \tag{3a}$$
$$y = y_1 + \rho \left[(y_2 - y_1) + \sigma (y_3 - y_2) \right]$$

The scalar potential distribution function that takes into account the field singularity at the node 1 $(\rho = 0)$ has the following form:

$$\phi = \Phi_1 (1 - \rho^\lambda) + \Phi_2 \rho^\lambda (1 - \sigma) + \Phi_3 \rho^\lambda \sigma. \ \frac{1}{2} < \lambda < 1. \tag{3b}$$

where coefficient λ is chosen in accordance with the Meixner edge condition [23]. and Φ_1. Φ_2. Φ_3 are corresponding nodal potentials. The linear variation of the scalar potential along the side 2-3 is provided so that the element is compatible with first-order ordinary elements.
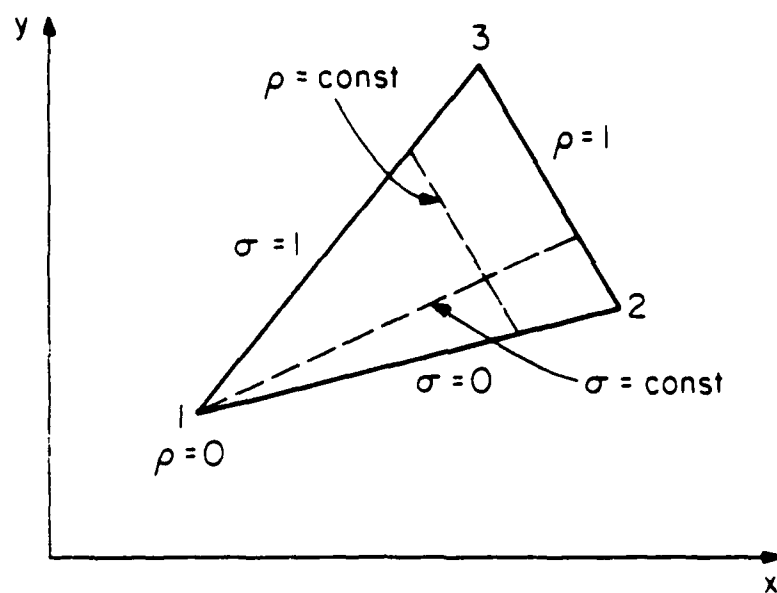
Figure 2. Singular element.

Although open-type transmission lines can be treated by the conventional finite element method with the shielding far away from the region of interest. the computation efficiency can be substantially improved if infinite elements [24] are used. Consider first the line consisting of a number of conductors arbitrarily placed between two parallel ground planes (Fig. 3a).

The entire domain can be divided into the near-field region (n.f.). which is the region of interest and the far-field region (f.f.) which is unbounded. They have a common boundary referred to as the far-field boundary (f.f.b). The near field (n.f.) region is then divided into finite triangular elements in the usual manner. while the f.f. region is divided into infinite elements. Each infinite element has two common nodes (lying on f.f.b) with an ordinary first-order element and two sides parallel to the x-axis.

Consider the infinite element with nodes 1 and 2 (Fig. 3b) and introduce normalized coordinates

$$\xi = \frac{x}{x_1}, \quad \eta = \frac{y - y_1}{y_2 - y_1}, \quad x_1 < x < \infty, \quad y_1 \leqslant y \leqslant y_2 \tag{4a}$$

Since in the first approximation the far-field of the system is equivalent to a dipole field. the electric scalar potential within the element is chosen in the following way:

$$\phi = \frac{1}{\xi^2} \left[\Phi_1(1 - \eta) + \Phi_2\eta \right] \tag{4b}$$

where Φ_1 and Φ_2 are potentials of nodes 1 and 2. respectively.

The next type of open region problems is a system of conductors above a ground plane. The procedure is similar to that in the previous case. The entire domain is divided into n.f. and f.f. regions (Fig. 4a) The n.f. region is divided into the usual triangular mesh and the f.f. region is divided into infinite elements of two different types. I and II. The infinite element type I is the element previously discussed. Consider now the infinite element of type II with nodes 1. 2 and radial sides intersecting at point (x_c, y_c) (Fig. 4b) and introduce triangular polar coordinates ρ, σ which are related to the global Cartesian coordinates by the relations
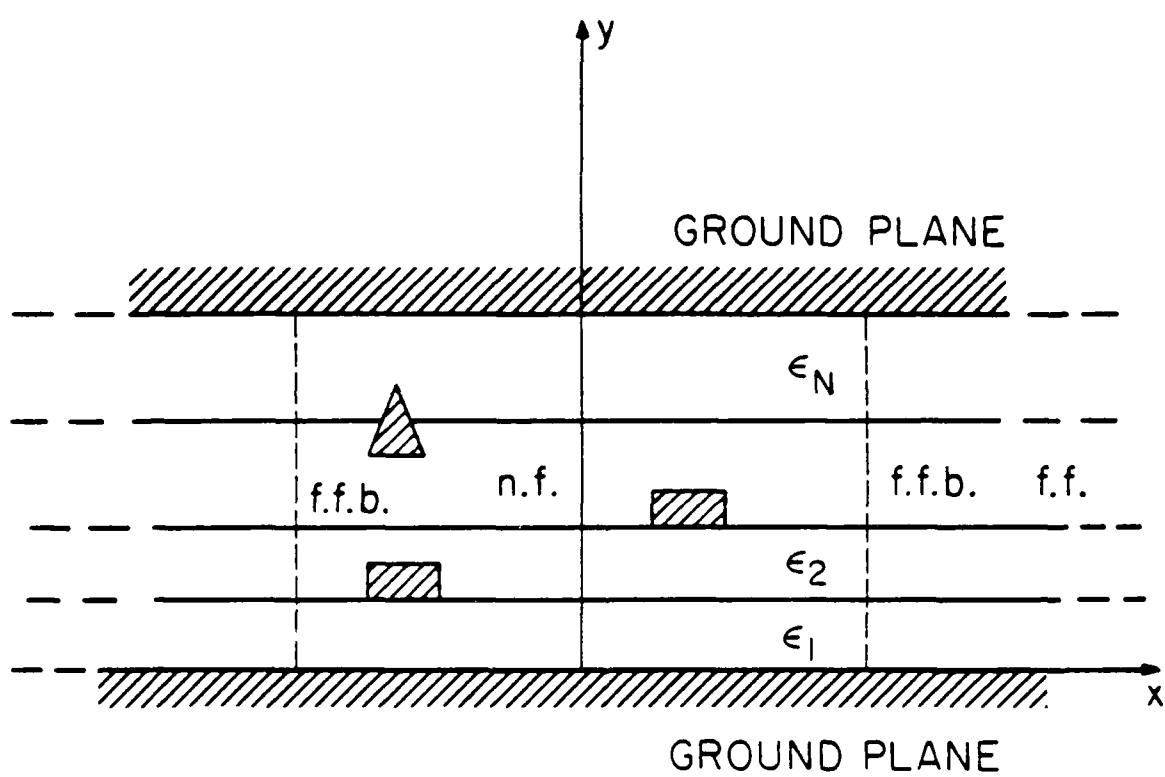
Figure 3a. Line consisting of a system of conductors placed
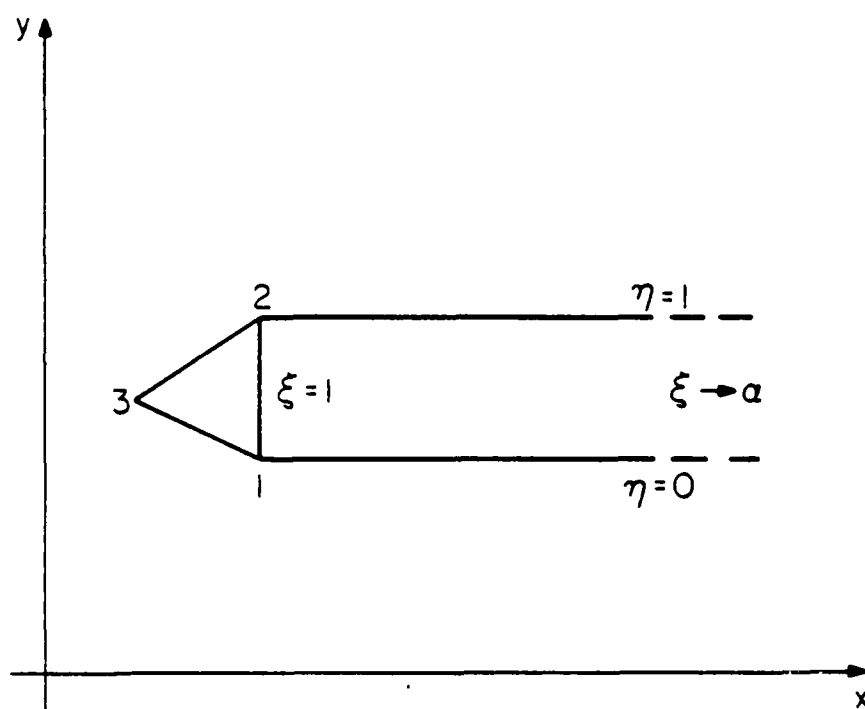
between two ground planes.
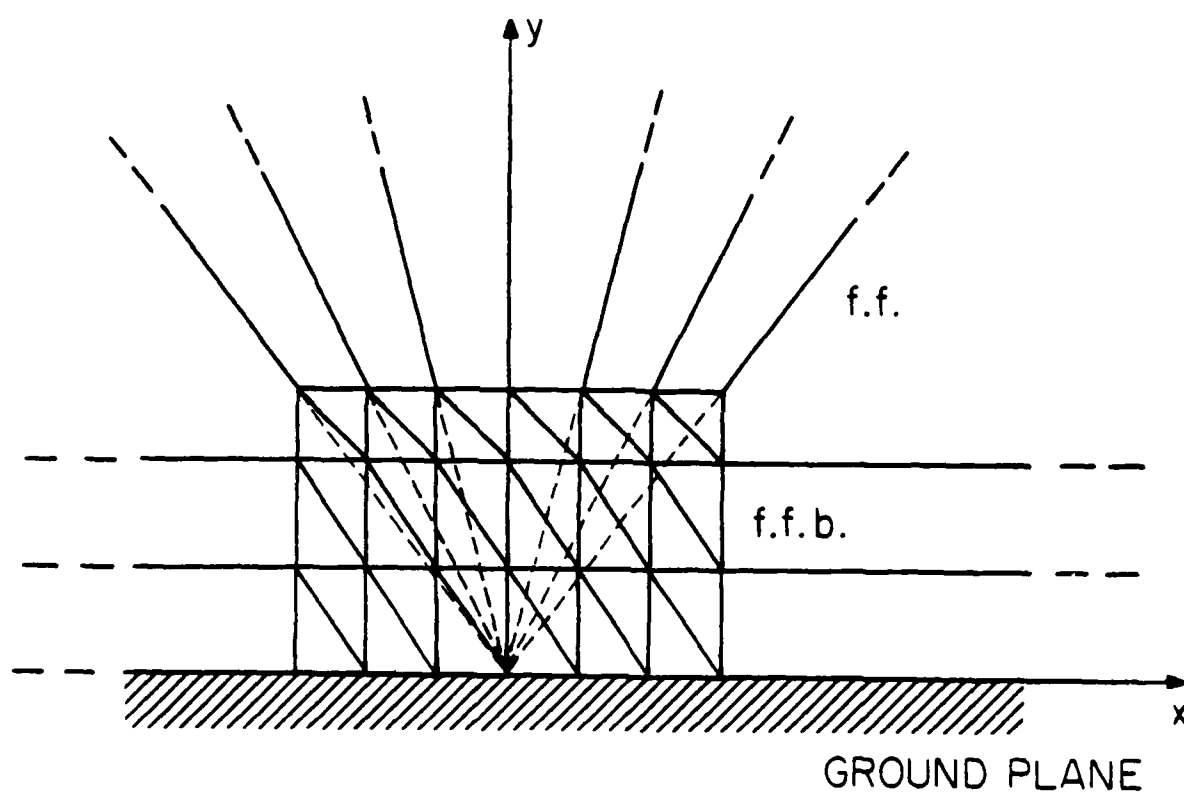
Figure 3b. Infinite element type I.

Figure 4a. Line consisting of a system of conductors placed above ground plane.
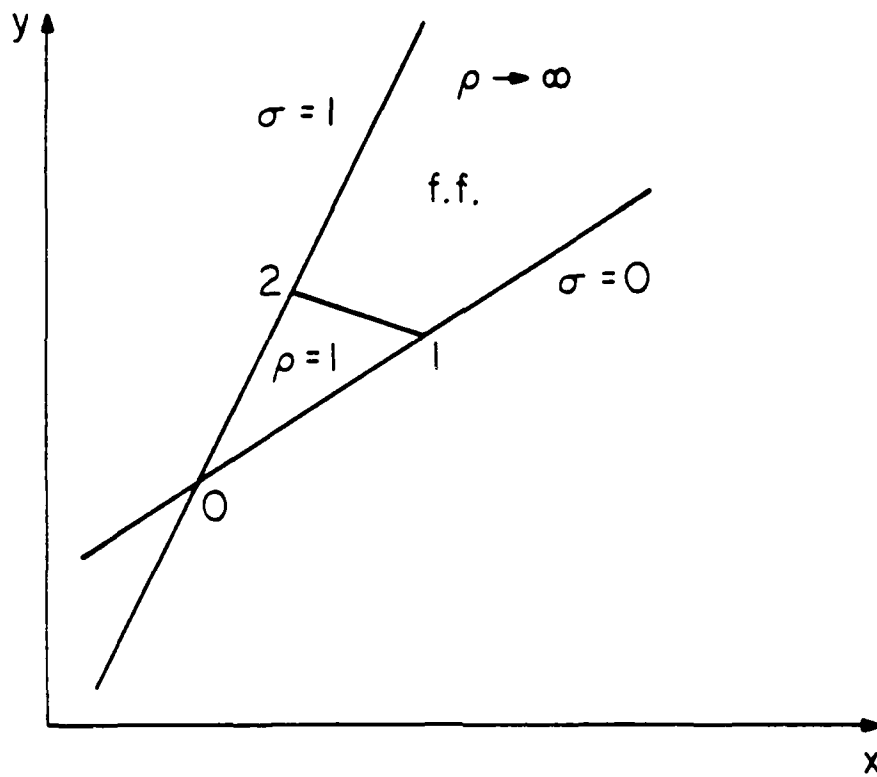
Figure 4b. Infinite element type II.

$$\iota = x_0 + \rho[(x_1 - x_0) + \sigma(x_2 - x_1)]$$ (5a)

$$y = y_0 + \rho[(y_1 - y_0) + \sigma(y_2 - y_1)]$$

The scalar potential distribution within the element is approximated by

$$\phi = \frac{1}{\rho}[\Phi_1(1-\sigma) + \Phi_2\sigma]$$ (5b)

where Φ_1, Φ_2 are nodal potentials. for element type IIa x_0, $y_0 = 0$

For both types of infinite elements. the linear variation of the potential function along the side 1-2 (common side with a corresponding ordinary element) is provided. These elements are compatible with ordinary first-order elements and satisfy completeness. finiteness and radiation conditions [24].

Once the potential distribution is known. the capacitance per unit length. ϕ. is readily obtained from the variational expression.

$$C = \frac{2W}{V^2}$$ (6a)

where W is the energy per unit length of the line and V is the conductor potential. Then the effective (dielectric) permittivity is

$$\epsilon_r = \frac{C}{C_0}$$ (6b)

where C is the capacitance of the actual line. C_0 is the free-space capacitance of the line. The phase velocity is

$$v_\rho = \frac{c}{\sqrt{\epsilon_r}}$$ (6c)

where c is the velocity of light in free space The characteristic impedance is

$$Z = \frac{1}{v_\rho C}$$ (6d)

The expressions given in (6a)-(d) are for a single line, but the same procedure can be applied to find the capacitance and characteristic impedance for even- and odd-modes on two coupled lines or to find capacitance and impedance matrices for a system of n-lines. The electromagnetic field distribution can also be obtained:

$$\vec{E}_o = -\nabla\phi, \quad \vec{H}_o = -\frac{1}{\eta}\vec{E} \times \hat{z} \tag{7a}$$

where $\hat{z}$ is unit vector in the z direction and η is the intrinsic impedance of the medium

$$\eta = (\mu_o/\epsilon_o\epsilon_{eff})^{1/2} \tag{7b}$$

Next, we employ a perturbational approach to solve for the attenuation constants due to dielectric and conductor losses

$$\alpha_d = \frac{P_d}{2P_o}, \quad \alpha_c = \frac{P_c}{2P_o} \tag{8a}$$

where P_o is the time-averaged power flow along the line, and P_d and P_c are the time-averaged powers dissipated in the dielectrics and conductors, respectively.

Dielectric losses are calculated using the formula

$$P_d = \omega\epsilon\tan\delta \int |E|^2 dS \tag{8b}$$

where the loss tangent, $\tan\delta$, is assumed to be sufficiently small so that the perturbed fields can be approximated by the fields for the lossless condition $\vec{E}_o, \vec{H}_o$. $\omega = 2\pi f$ is the angular frequency and S is the area of cross section covered by the dielectric.

Losses due to the imperfect conductors are obtained via the conventional perturbation formula [25]:

$$P = R_s \int |H_{tan}|^2 dl \tag{8c}$$

where R_s is the surface resistance, and H_{tan} is the magnitude of the tangential magnetic field at the conducting surfaces (line C for the lossless case.

The average power propagating along the line is given by

$$P_o = \text{Re} \int_S (\vec{E}_o \times \vec{H}_o{}^*) \cdot \hat{z} \, dS \tag{8d}$$

where S is the complete cross section of the line.

For the conductor loss calculation, special finite element matrices have been derived [26]. Consider a high-order element where vertices are labeled as nodes i, j, k (Fig. 5).

Suppose that the side j-k is an imperfect conductor of surface resistance R_s. Then the losses in that side can be expressed as

$$P_c^l = \frac{R_s}{\eta^2} \left| P_i \cot \theta_i + P_j \cot \theta_j + P_k \cot \theta_k \right| \left| \Phi \right|$$

where $[\Phi]$ is a column vector of nodal potentials for the element. θ_i, θ_j, θ_k are the included angles at vertices i, j, k, respectively and P_i, P_j, P_k are matrices of the numerical coefficients which do not depend on the triangle shape.

III. NUMERICAL AND EXPERIMENTAL RESULTS

On the basis of described finite element procedures, two different computer software packages have been developed for solving the problem at hand. The first of these is employed for semiautomatic mesh generation of first- or high-order ordinary elements including singular and infinite elements if necessary. The second package calculates the potential and field distributions, capacitance per unit length, characteristic impedance, effective permittivity, and attenuation due to conductor and dielectric losses. The linear system of equations derived from the application of the FEM method is rather sparse and special techniques for the solution of such equations can be employed to achieve enhanced computational efficiency. Two such techniques are direct envelope and band method, with node reordering, which are useful for smaller problems with a few hundred nodes. For large systems of equations (a few thousand nodes), the conjugate gradient method is suggested because it requires the storage of only non-zero elements. Preconditioning is
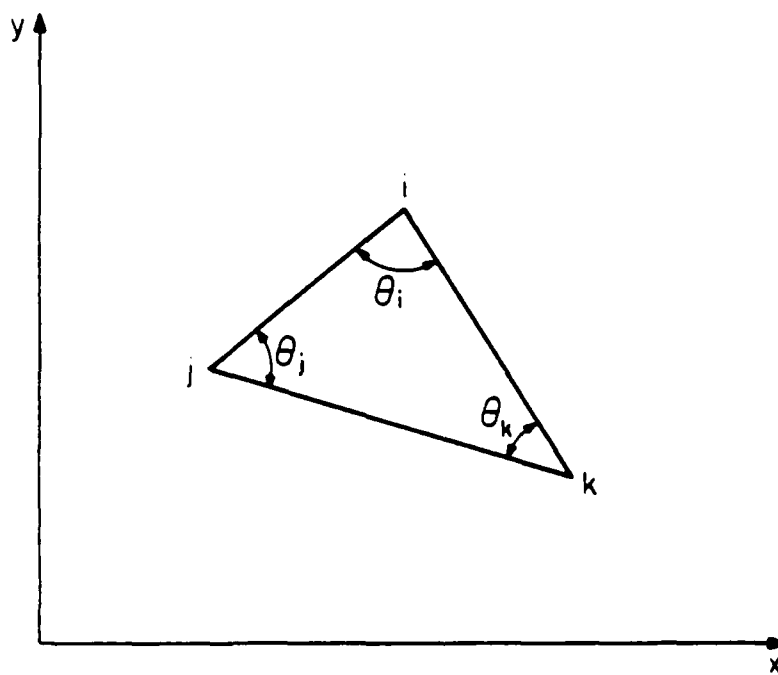
Figure 5. Ordinary triangular element.

accomplished via incomplete Cholesky decomposition [27] which leads to the acceleration of the iteration procedure.

To demonstrate the quality and the accuracy of the finite element method analysis of the previous section, the solutions for sample problems are given and are compared with available theoretical and experimental data. as well as with measured results.

The first example considered is the problem of a shielded multilayered thick strip transmission line shown in Fig. 6. The corresponding triangular mesh for one half of the line is presented in Fig. 7. The characteristic impedance of the line is calculated for a few different thicknesses of the strip. t/b. as a function of ratio $2a/b$ (width and height of the shielding) and is represented by the solid line in Fig. 8. Corresponding results obtained by the Green's function method [3] are denoted by the dashed line and good agreement between the two sets of results is evident. It is found that the influence of the side walls on the characteristic impedance is negligible when the walls are sufficiently removed from the strip, $2a/b > 5$.

The next example is an inverted microstrip whose generic cross section (see [17, 19]) is shown in Fig. 9. Calculated results for the characteristic impedance as a function of aspect ratio $2w/h$. denoted by the solid line. are compared with the numerical results obtained via the boundary element [BEM] method [19], shown by the dashed line in Fig. 10. Good agreement. within 5%. is obtained. Data for $\sqrt{\epsilon_{eff}}$ of the line as a function of aspect ratio $2w/h$ are presented in Fig. 11. Finite element results are denoted by solid lines and BEM results by dashed lines. The measured points shown in the same figure were obtained via time-domain reflectometer measurements [28]. It can be noted that FEM results are closer to the experimental results than the BEM data. For the same example. attenuation constants due to conductor. α . and dielectric. α_d . losses are calculated as functions of aspect ratio $2w/h$ and shown in Fig. 12 as solid curves. These data are compared with BEM [19] and spectral-domain method [17] results shown in the same figure and a very good agreement is found.
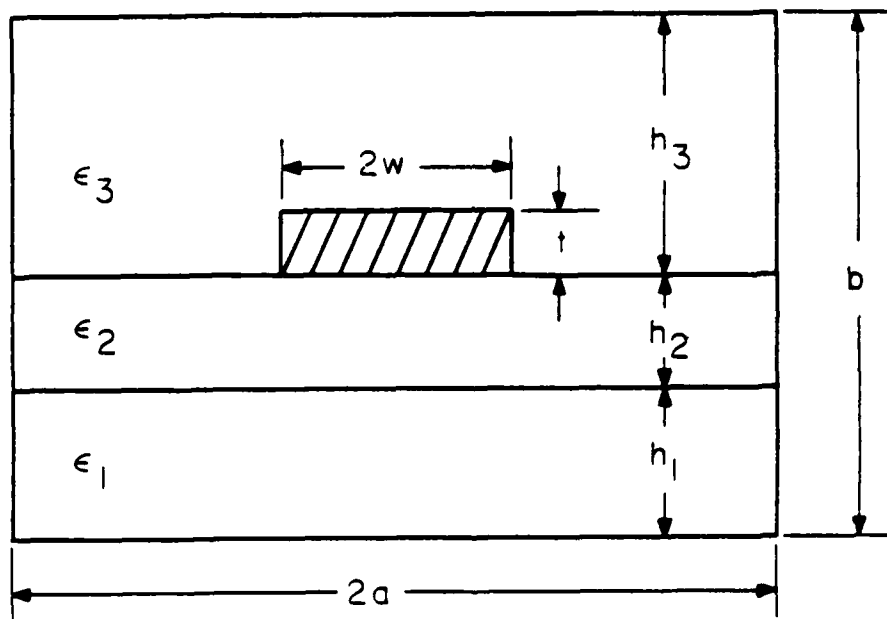
Figure 6. Thick-strip transmission line with multidielectric

layers and shielding structure

$h_1 = h_3 = 0.4b$, $h_2 = 0.2b$, $2w = 0.5b$,

$\epsilon_1 = \epsilon_3 = \epsilon_o$, $\epsilon_2 = 9.35\epsilon_o$
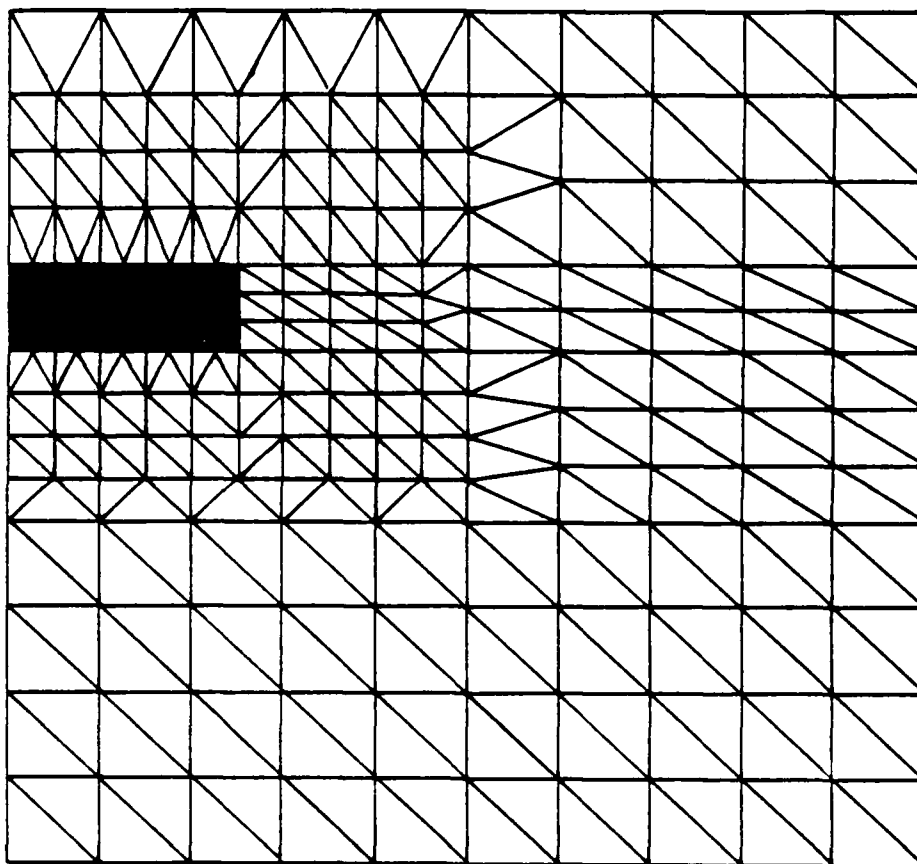
Figure 7. Triangular mesh for thick-strip transmission line
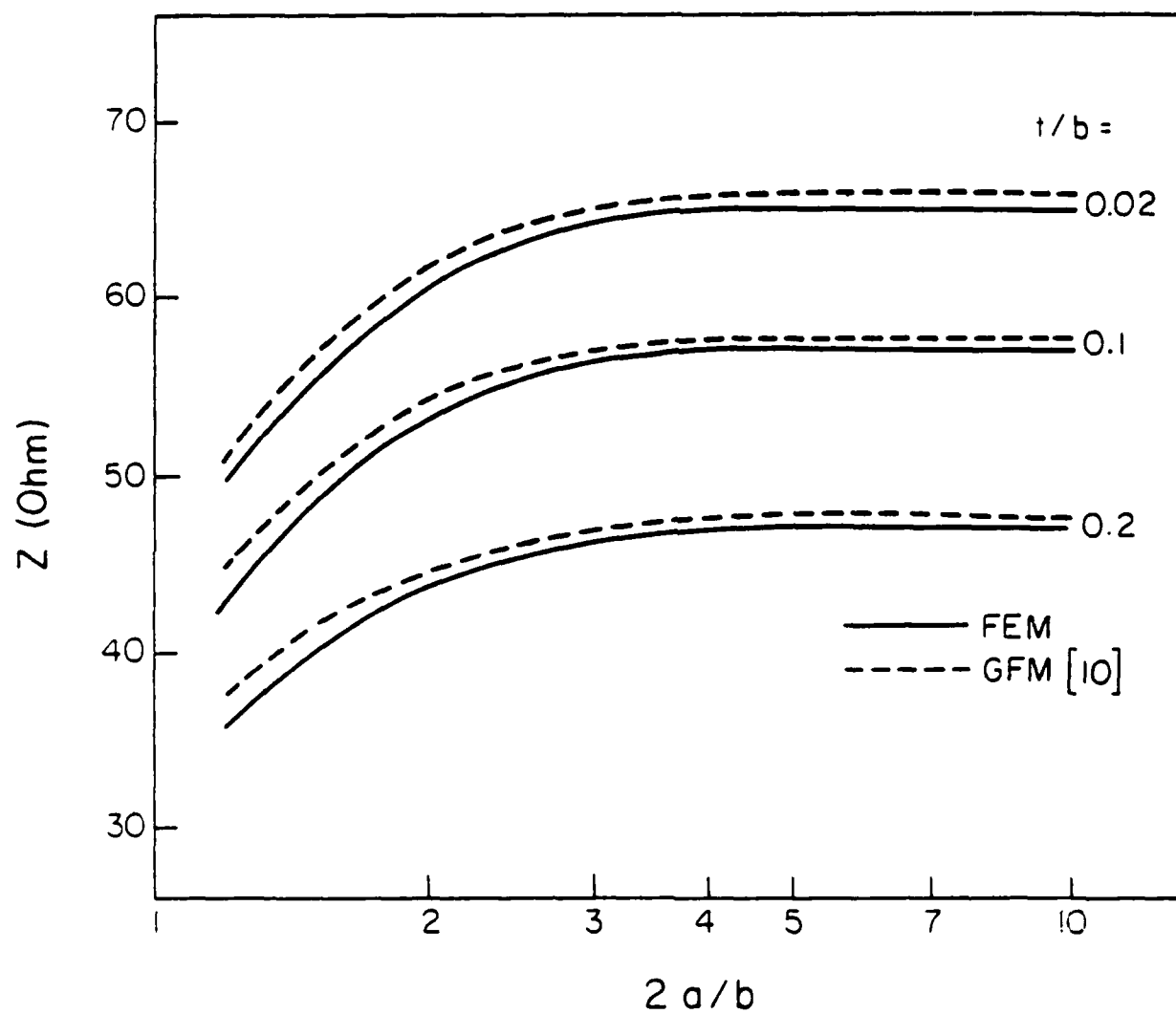with 153 nodes and 351 elements.

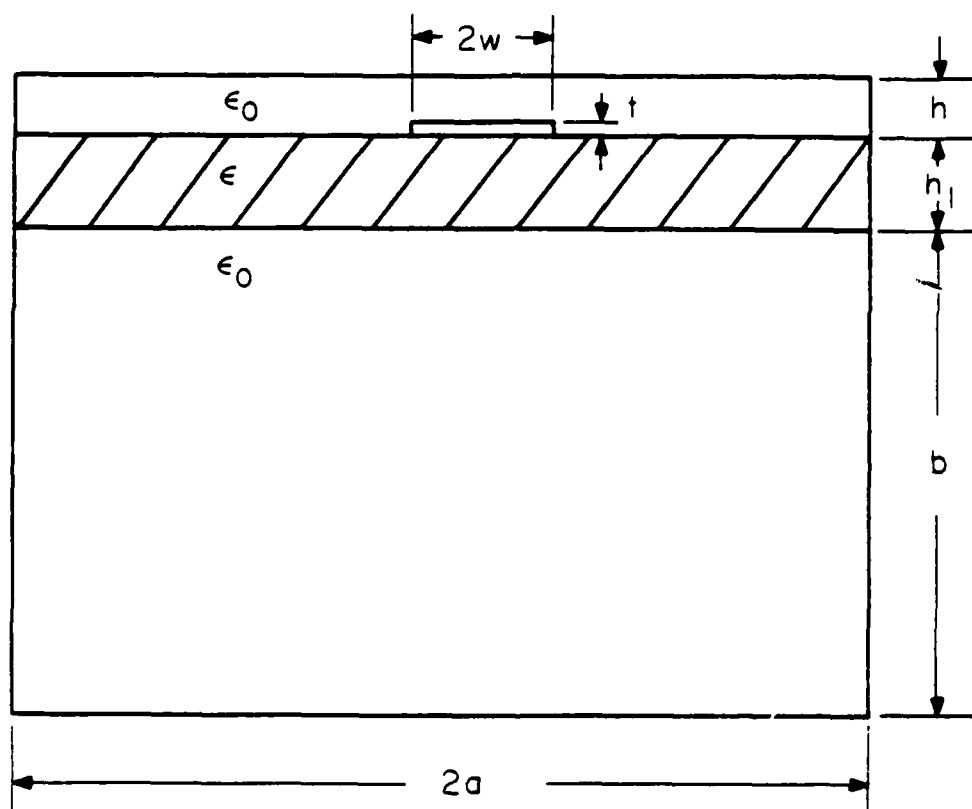Figure 8. Characteristic impedance Z of thick-strip transmission line.

Figure 9. Generic cross-section of inverted microstrip with

shielding structure

$h = 0.381mm$, $h_1 = 0.508mm$, $t = 0$.
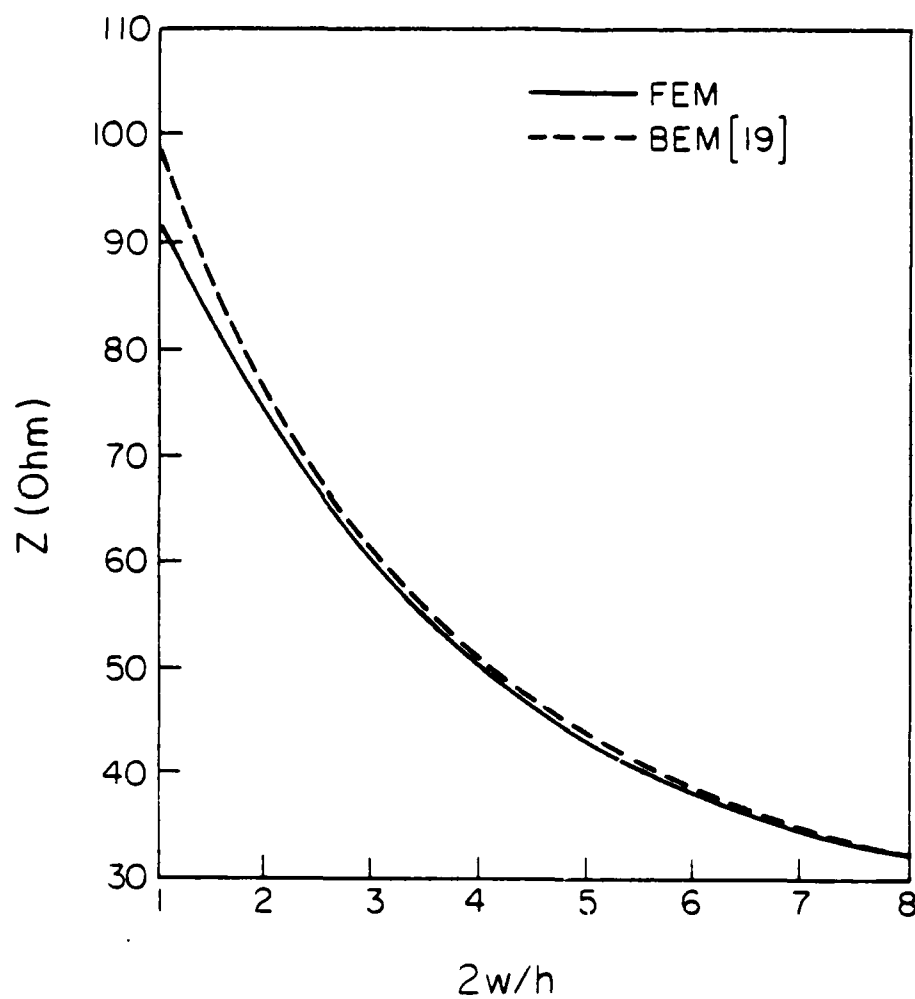
$2a = 10mm$, $b = 10mm$, $\epsilon = 3.78\epsilon_0$.

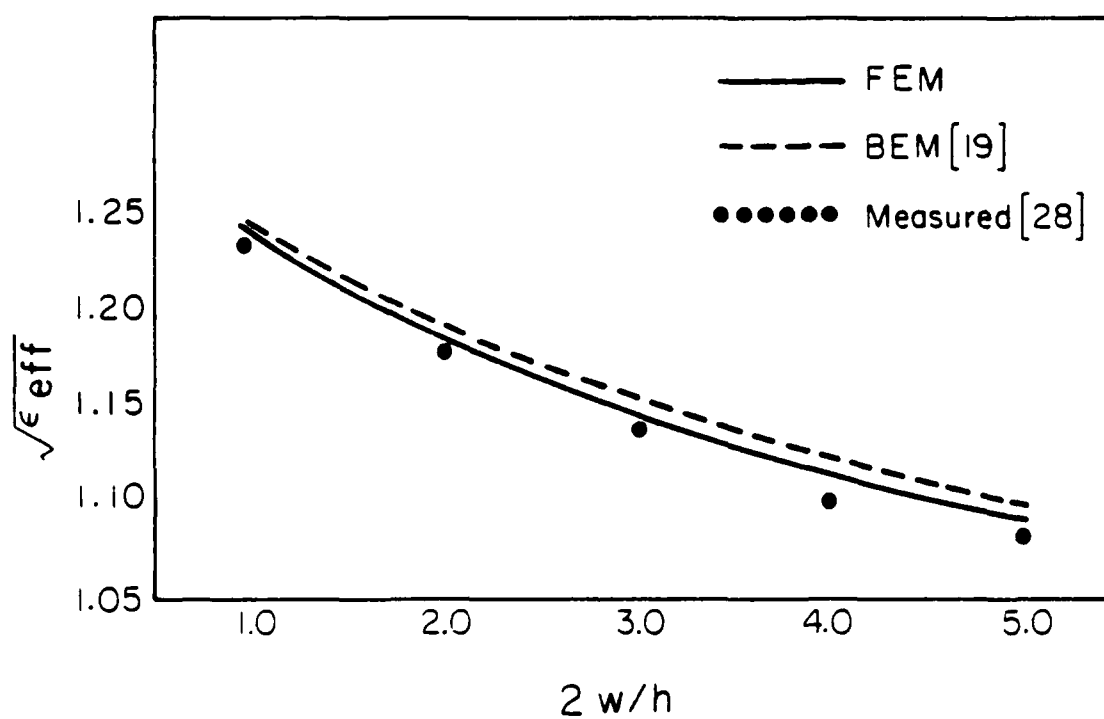Figure 10. Characteristic impedance Z of shielded inverted microstrip line versus shape ratio 2w/h.

Figure 11. $\sqrt{\epsilon_f}$ of shielded inverted microstrip line
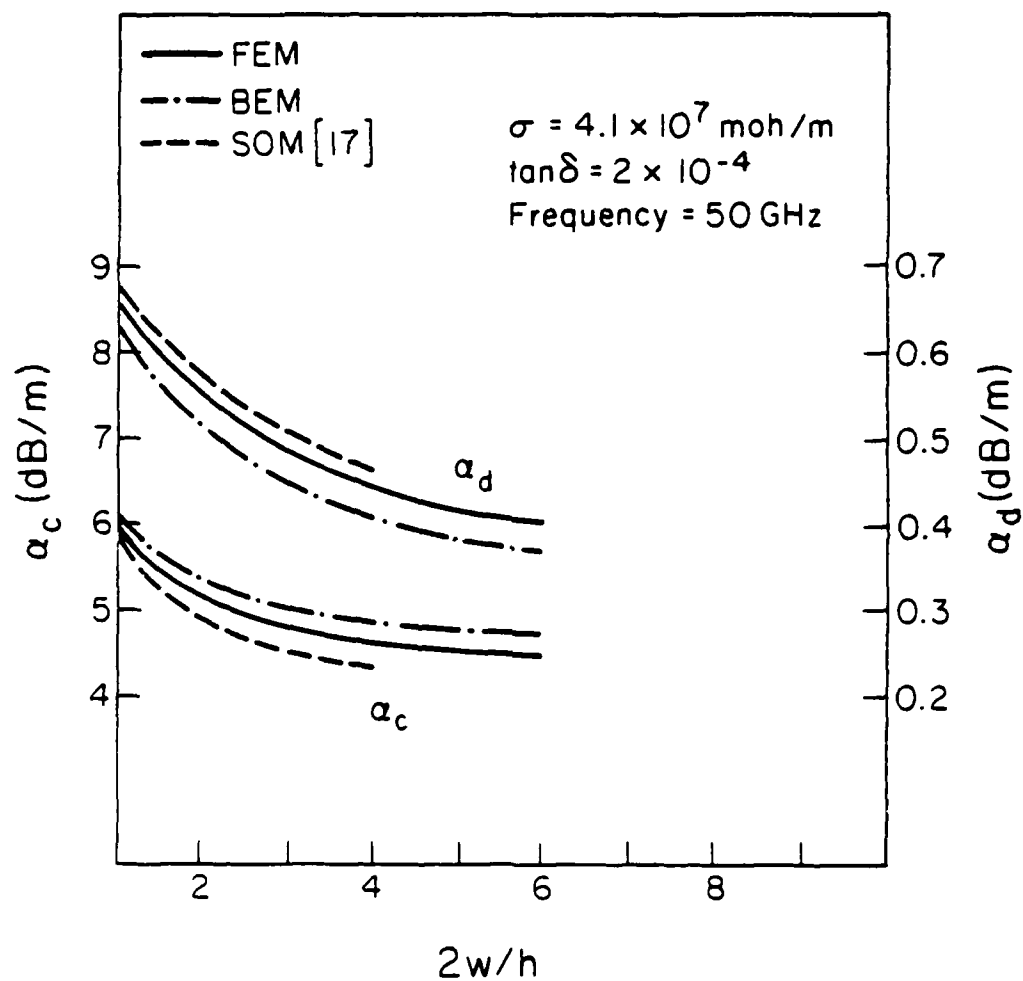
versus shape ratio $2w/h$.

Figure 12. Dielectric and conductor losses of shielded inverted microstrip line versus shape ratio 2w/h.

It should be pointed out that the accuracy of the calculation. especially of the loss calculations. can be significantly enhanced via the use of singular elements. The odd- and even-modes propagating on coupled lines can be treated by the same finite element procedure. Some of the results for coupled lines are presented in [29].

In order to verify the accuracy of the finite element procedures, the characteristic impedance and losses of a shielded microstrip line have been measured. The experiment has been done on the line whose cross section and the dimensions are shown in Fig. 13. By using a HP 8510 Network Analyzer. time-domain reflectometer measurements of characteristic impedance (Fig. 14), and frequency-domain measurements of the S_{11} parameter of open- and short-circuited lines (Fig. 15) have been done. The measured value of characteristic impedance is 48 ohms while the calculated value is 48.1 ohms. Excellent agreement between measured and calculated values of the S_{11} parameter of the line is found.

IV. CONCLUSIONS

In this study. a finite-element procedure for quasi-TEM analysis of microwave transmission lines is presented. The method is capable of handling microwave transmission lines with rather arbitrary configurations. Both first- and high-order triangular elements are employed. In the case of open region problems (conductors over one or between two parallel ground planes) and edge singularity problems. accuracy and efficiency of the method are enhanced via the use of infinite and singular elements. For the solution of finite element linear equations, the band or envelope matrix method (with reverse Cuthill-McKee reordering) is used for smaller systems and the incomplete Cholesky-conjugate gradient method for large systems.

Computer software for semiautomatic mesh generation as well as for calculation of capacitance per unit length. effective permittivity, characteristic impedance and attenuation due to conductor and dielectric losses has been developed.

Results obtained for various types of microwave transmission lines are in very good agreement with available theoretical and experimental data.
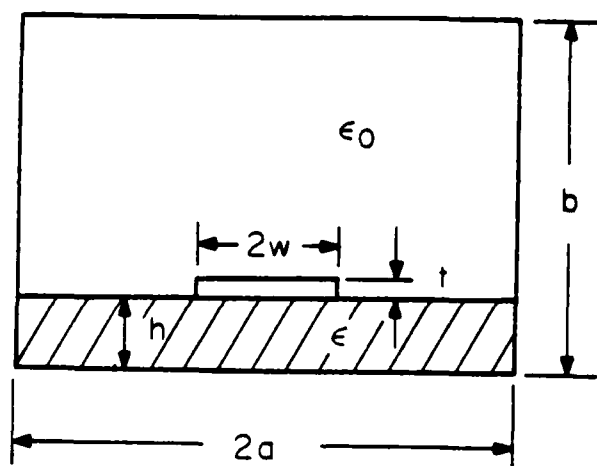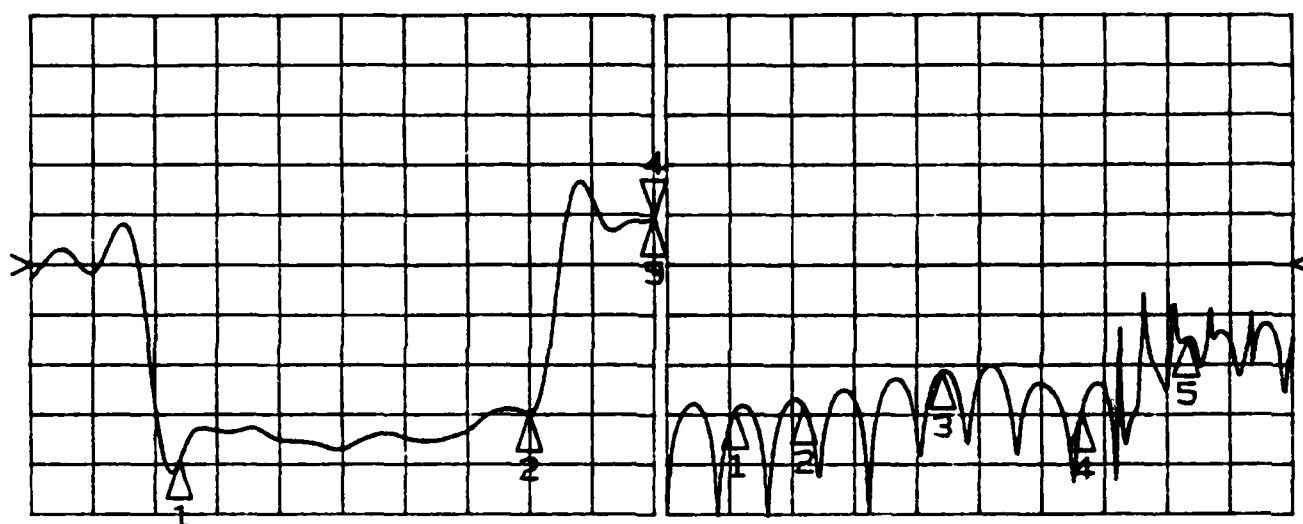
Figure 13. Shielded microstrip line.

2a = 480 mils. b = 200 mils. h = 30 mils. 2w = 93 mils. t = 0.8 mils,

$\epsilon = 2.2\epsilon_{..}$ $\tan\delta = 9 \times 10^{-4}$, $\rho = 1.7 \times 10^{-8}$ Ohm m, $\mathcal{l} = 2.97$ $inches$

S11   Re
REF 0.0 Units
4   5.0 mUnits/
▽   4.6911 mU.
50 OHM LINE MAY 21

C
A

S11   log MAG
REF 0.0 dB
△   10.0 dB/
4 −29.749 dB

START   150.0 ps   START   0.045000000 GHz
STOP   1.2 ns   STOP   18.045000000 GHz

Figure 14. Time-domain measured results.

S$_{11}$　　　log MAG
REF 0.0 dB
　　0.25 dB/

50 OHM LINE (2.97 INCHES)

C
A
S

START　　0.045000000 GHz
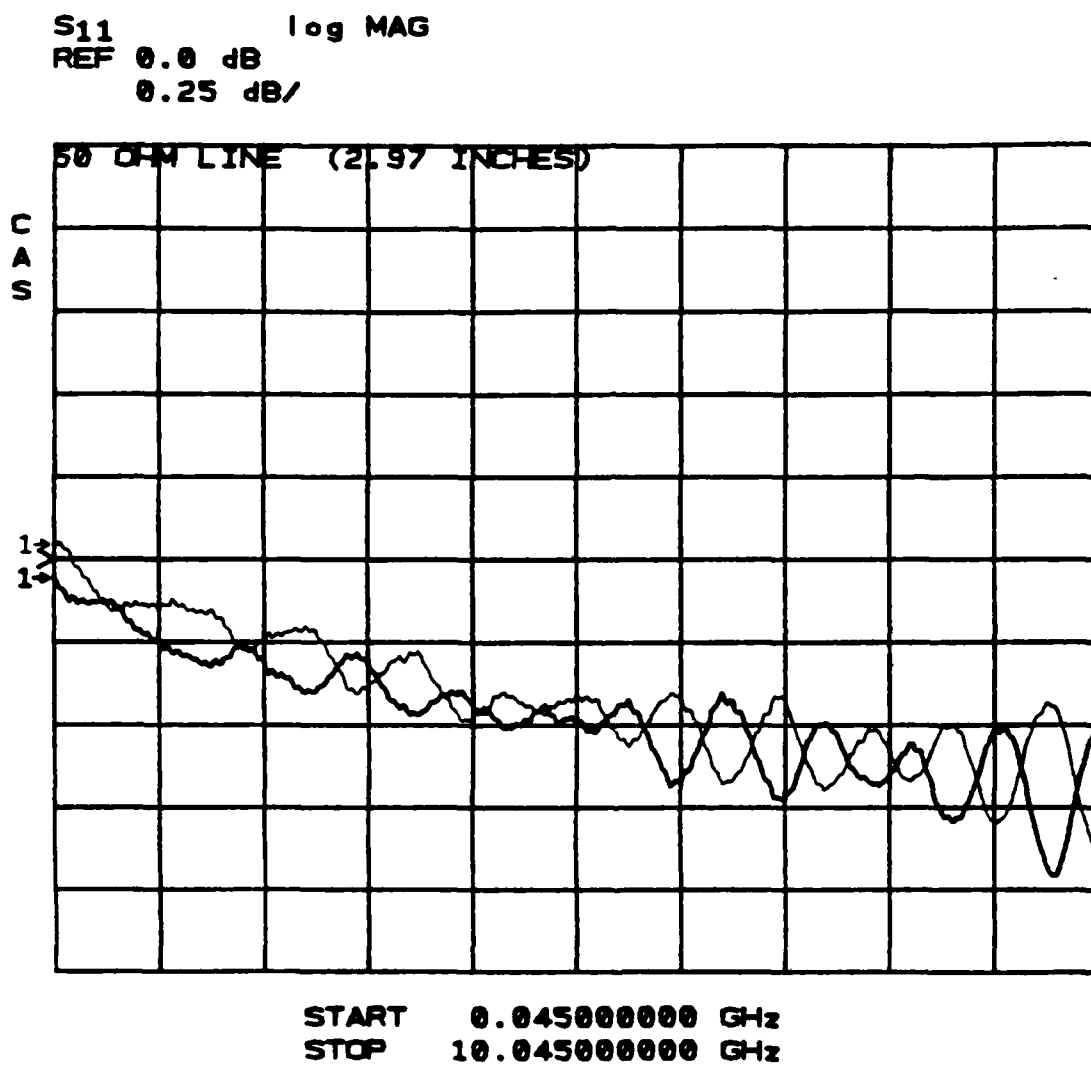STOP　　10.045000000 GHz

Figure 15. Frequency-domain measured results of S_{11} parameter.

The method presented is both versatile and accurate. It can be readily extended to the lines with anisotropic media. Similar 3-D finite element analysis would be a powerful tool for calculation and design of connectors or lines with discontinuities. In future work, the authors plan to extend the FEM procedure to a full-wave analysis of microwave transmission lines.

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