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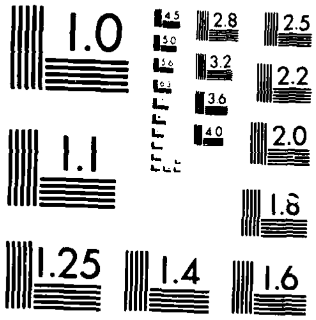
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Signal-to-Noise Ratio and Central Limit Theorem Considerations in Non-Gaussian Detection

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ABSTRACT

A generalized signal-to-noise ratio for detectors is defined and its relationship to quality of detection is investigated. Specific attention is given to log-likelihood ratio detectors. The question is discussed as to whether a Central Limit Theorem approximation for the output of the detector gives results which are close to those of the actual detector and is illustrated with examples.

I. INTRODUCTION

In this paper we will be looking at the discrete time problem of detection of a constant signal in the presence of independent, identically distributed noise; that is,

$$\text{Under } H:\ X_i = N_i \quad i = 1,2,\ldots,n$$

and

$$\text{Under } K:\ X_i = N_i + s \quad i = 1,2,\ldots,n$$

where X_i is the observation, N_i is the noise and s is a positive constant. The only limitations that we will place on the possible noise distributions will be that they are symmetric, bounded, unimodal and have zero means.

If the detector is linear, we can define a signal-to-noise ratio (SNR) in terms of s, n, and the noise variance σ^2. In this case the SNR is clearly a useful quantity: for a constant false alarm probability the probability of detection is a monotonic function of the SNR. If the detector is nonlinear, not only is the relationship between SNR and detector performance less clear but it is no longer immediately apparent what definition of SNR would be useful.

A generalized SNR has been proposed and it has been shown to be maximized when the nonlinearity is a monotonic function of the likelihood ratio[1,2]. This is an encouraging result since any monotonic function of the likelihood ratio is also the Neyman-Pearson optimal detector. Following some unpublished work of Poor, Chiang and Wise[3], we show later that, if the sample size is very large, this SNR functions much as does the conventional SNR for the linear detector. In fact, it can be shown that, asymptotically, the relationship between SNR and the probability of detection is once again monotonic.

As with any asymptotic result there arises here the basic problem of determining whether the Central Limit Theorem holds and the more obscure question of deciding how many samples are needed before the implications of the theorem merge with reality. Obviously, the theorem is useless as a practical tool if it is necessary to collect an infinite number of samples before it can be invoked and as a result it is desirable to know just how much is enough. In what follows we show that, under some mild restrictions, the output of a log-likelihood ratio detector obeys the Central Limit Theorem and we look at some examples in order to illustrate the rate of convergence.

The paper is composed of three parts and an appendix containing proofs. In Part I we define and discuss some relevant properties of the generalized SNR and the log-

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likelihood ratio. In Part II we investigate the relationship between SNR and traditional measures of detector performance. In Part III we look at some numerical examples concerning the convergence of the log-likelihood ratio to a Gaussian random variable. Part IV contains conclusions and a discussion of topics for further study.

I. PRELIMINARY PROPERTIES

The log-likelihood ratio. It is well known[4] that the likelihood ratio

$$L(\mathbf{X}) = \frac{f_K(\mathbf{X})}{f_H(\mathbf{X})} ,$$

maximizes the probability of detection β for a given false alarm rate α. Here f_K and f_H are the densities of the observation vector $\mathbf{X}$ under the hypotheses K and H respectively. If we are attempting to detect a shift to the right by a constant vector $\mathbf{s}$ and f is the density of the noise we have

$$L(\mathbf{X}) = \frac{f(\mathbf{X}-\mathbf{s})}{f(\mathbf{X})} .$$

It is also well known that any monotonically increasing function of the likelihood ratio will have the same receiver operating characteristic (ROC, that is, β vs. α) as that given by the likelihood ratio. The logarithm is, of course, one such monotonically increasing function. For the case of i.i.d. noise we have

$$L(\mathbf{X}) = \prod_{i=1}^{n} \frac{f(X_i - s)}{f(X_i)} .$$

By using $\log L$ rather than L we not only simplify our computation by converting the product to a sum

$$l(\mathbf{X}) = \log L(\mathbf{X}) = \sum_{i=1}^{n} \log \frac{f(X_i - s)}{f(X_i)} ,$$

but for large enough n we can approximate the test statistic $l(\mathbf{X})$ by using the Central Limit Theorem and thereby further simplify the problem.

In this paper we will discuss the log-likelihood ratio for unimodal, bounded, symmetric densities having zero means. We now list some properties which are relevant to this discussion:

1) The log-likelihood ratio is odd symmetric about $s/2$ [that is, $l(x) = log[f(x-s)/f(x)] = -l(s-x)$].
2) $E_H\ l(X) = -E_K\ l(X)$ where the subscripts indicate expectation under the given hypotheses.
3) $var_H\ l(X) = var_K\ l(X)$
4) Under certain mild restrictions, the variance of the test statistic $l(X)$ is finite.

Proofs:

1) $l(s-x) = \log\frac{f(x)}{f(s-x)} = \log\frac{f(x)}{f(x-s)} = -l(x)$ by the symmetry of f.
2) $E_H\ l(X) = \int l(X) f(X) dX = -\int l(s-X) f(X) dX = -\int l(Y) f(Y-s) dY = -E_K\ l(X)$
3) $E_H\ l^2(X) = \int l^2(X) f(X) dX = \int l^2(s-X) f(X) dX = \int l^2(Y) f(Y-s) dY = E_K\ l^2(X)$
4) proven in the Appendix.

Generalized signal-to-noise ratio (SNR). The motivation for defining a SNR for detectors is the same as that for doing so with any device; we want to know how strongly the output is affected by the signal and by the noise. For the linear detector the two effects are completely separable and the resulting SNR is only a function of the input SNR and the gain of the processor. For example, if the noise is Gaussian with zero mean and variance σ^2, then

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$$\beta = \Phi\,[\Phi^{-1}(\alpha) + \frac{\sqrt{n}\, s}{\sigma}]$$

where Φ is the standard normal cumulative distribution function. Since the Φ function is monotonic in its argument it is clear that for fixed α, β is a monotonic function of $\sqrt{n}\, s / \sigma$. However, for nonlinear detectors we no longer have this separation property and the intuition which lends itself to the linear case is no longer evident. What is clear is that if we are to use the SNR as a measurement of detector performance then we want a higher SNR detector to have a higher probability of detection at a given false alarm rate than a detector with a lower SNR.

The following definition has been suggested[1] for a generalized SNR for detectors:

$$\text{SNR}\,[g(X)] = \frac{E_K\; g(X) - E_H\; g(X)}{[\; var_H\; g(X)]^{1/2}} \;,$$

where g is the processing non-linearity. The numerator represents the shift in the mean between the two hypotheses while the denominator represents the noise power at the output under the hypothesis H (we have shown earlier that the output variance is equal under both hypotheses for the log-likelihood detector). The definition has an intuitive feel to it and while it is not always monotonic in β for a fixed α we will show that, for a large enough sample size, it will be. Hence, asymptotically, this SNR is equivalent to the Neyman-Pearson criterion for quality of detection.

Some properties of the SNR[3].

1) SNR (k *samples*) $= \sqrt{k}\; \times$ SNR (1 *sample*).

2) SNR is unaffected by gain or shift. That is,

$$\text{SNR}\,[a g(X) + b\,] = \text{SNR}\,[g(X)] \qquad\qquad a > 0.$$

3) For the log-likelihood ratio:

$$\text{SNR}\,[l(X)] = \frac{2E_K\, l(X)}{[\; var\;\; l(X)]^{1/2}} \;.$$

II. SNR AND THE CENTRAL LIMIT THEOREM FOR LOG-LIKELIHOOD RATIOS

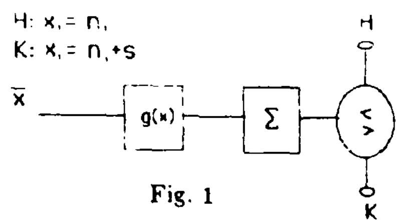

Fig. 1

The detector of Fig. (1) is in the form of a sum and hence if $\log(f\,(X_i - s)/f\,(X_i)$ can be shown to have finite variance we can use the Central Limit Theorem to determine the asymptotic properties of $l(\mathbf{X})$. In this section we investigate the relationship between SNR and detector performance for sample sizes which are large enough so that the Central Limit Theorem can be invoked.

In [3] it is shown that if the two hypotheses are equally likely and $\alpha = 1\text{-}\beta$ then, asymptotically, the probability of error is monotonically decreasing in SNR. We generalize this statement with the following theorem.

Theorem 1: *Given a false alarm rate, α, a symmetric, bounded unimodal density, f , and a detector structure as shown in Fig. 1. Suppose we are comparing any detector g for which $var_K\, g(X) \geq var_H\, g(X)$ with the linear detector, $g(X) = X$, and suppose the sample size is large enough that the Central Limit Theorem can be invoked. Then a sufficient* *condition for higher probability of detection of a positive constant in noise with density f is higher signal-to-noise ratio. If $var_K\, g(X) = var_H\, g(X)$ then the condition is also necessary. Furthermore, if we are comparing two log-likelihood detectors and the Central Limit Theorem can be invoked then higher SNR is a necessary and sufficient condition for higher probability of detection.*

Proof (see Appendix A)

In [3] it is shown that for $P_H = P_K$ and $\alpha = 1-\beta$ the magnitude of the difference in the probability of error of the actual detector and the CLT approximation to it is $O(n^{-1/2})$ and that the relative error can actually grow exponentially in n. Here P_H and P_K are the prior probabilities under the two hypotheses. This is a somewhat discouraging result and elucidates the need for care when invoking limit theorems in finite sample problems. In Part III we will look at some graphical comparisons between the operating characteristics for some detectors and their CLT approximations.

The Central Limit Theorem and the log-likelihood ratio. We now must determine whether the log-likelihood ratio obeys the CLT so that we can apply the above theorem. We will show that for symmetric, unimodal and bounded densities having zero mean, if $\lim_{x\to\infty} g'(x) < \infty$, then the log-likelihood ratio has finite variance and the CLT holds. More formally we have the following theorem:

Theorem 2:

Suppose X has the density $f(x)$, where f is symmetric, unimodal, bounded and has zero mean. Then the variance of the log-likelihood ratio $g(x) = \log[f(x-s)/f(x)]$ is finite if $\lim_{x\to\infty} g'(x) < \infty$. Furthermore, if X has density h where h is also symmetric, unimodal, bounded and has zero mean then $g(x)$ has finite variance if there exists Y such that $h(y) \le k_1 e^{-k_2 y}$, $k_1, k_2 > 0$ for all $y > Y$.

Proof (see Appendix B)

III. NUMERICAL RESULTS

Rate of convergence to CLT. In order to study the rate of convergence to the results predicted by the CLT we have computed the ROC curves for the actual detector and for a CLT approximation of its output for a variety of cases. In all cases the detector was optimal for the noise input density, that is, $g(x) = \log[f(x-s)/f(x)]$ where f is the density of the noise. The densities used were three members of the Johnson System family[5] and Laplace, or double exponential noise. Some of our results are shown in Figs. 2-8.

Figs. 2 and 3 show the actual detector ROC and the CLT approximation to it for Johnson Noise with parameter δ=3 for two and four samples respectively. There is a marked improvement in the quality of the approximation with the increase in the number of samples. Figs. 4 and 5 show the same quantities except δ was changed to 10. Again there is improvement although it is not as dramatic. It should be noted that in the limit, as $\delta\to\infty$, the distribution becomes normal. Hence we would expect the CLT approximation to improve as δ is increased. Fig. 6 shows the ROC and its approximation for δ=50. It is obvious that the approximation in this case is very good.

In [8] a functional expression is given for the distribution of the output of the log-likelihood detector for Laplace Noise. From this expression we have calculated the ROC curves for this detector and the CLT approximation to it. The results are shown in Figs. 7 and 8 for five and ten samples, respectively. Here again we see improvement with an increase in the number of samples although it is not as large as in Figs. 1 and 2. This might be due to the fact that the log-likelihood ratio for Laplace noise places a finite mass at $\sqrt{2}s/\sigma$ which causes the output distribution to be discontinuous.

IV. CONCLUSIONS AND TOPICS FOR FURTHER STUDY

Conclusions. It is clear that in the limit SNR and the Neyman-Pearson criterion for detector performance are equivalent. The question of whether SNR is an appropriate measure for finite sample size detection is much murkier. As was mentioned, an upper bound on the magnitude of the error in predicting the probability of error is derived in [3]; however, as was demonstrated by the numerical examples, good results can be obtained in certain cases when the CLT is used, even in the case of very small sample size. From this we conclude that care should be taken when using SNR as a measure of detector performance because the accuracy of the resulting predictions is application dependent.

Topics for further study. In [6], Czarnecki designed a simple piece-wise linear processor for small-signal detection by maximizing the efficacy over a class of such processors. Owing to the similar functional forms of efficacy and SNR an analogous design might be produced by maximizing the SNR over some class of simple detectors.

Another course of action which might yield interesting results is attempting to find an error bound for CLT approximation for a given class of densities processed by a given class of detectors. Both the Johnson System[5] and the Generalized Gaussian[9] families would lend themselves to this endeavor.

APPENDIX A

Proof of Theorem 1: Given two detectors, g_1 and g_2. Let

$$m_{iJ} = \int g_i(X) f_J(X) dX \qquad i = 1,2 \qquad J = H,K ,$$

$$\sigma_{iJ}{}^2 = \int [g_i(X) - m_{iJ}]^2 f_J(X) dX ,$$

where $f_H(x) = f(x)$ and $f_K(x) = f(x-s)$. Since we assume the CLT holds, the density, under the hypothesis J, of the test statistic,

$$s_i(\mathbf{X}) = \sum_{m=1}^{n} g_i(X_m),$$

is normal with mean nm_{iJ} and variance $n\sigma_{iJ}{}^2$. We then have:

$$\alpha_i = \frac{1}{\sqrt{2\pi}} \int_{t_i}^{\infty} e^{\frac{(z-nm_{iH})^2}{2n\sigma_{iH}{}^2}} dz = \Phi[\frac{nm_{iH} - t_i}{\sqrt{n}\,\sigma_{iH}}]$$

and

$$\beta_i = \frac{1}{\sqrt{2\pi}} \int_{t_i}^{\infty} e^{\frac{(z-nm_{iK})^2}{2n\sigma_{iK}{}^2}} dz = \Phi[\frac{nm_{iK} - t_i}{\sqrt{n}\,\sigma_{iK}}] \tag{A.1}$$

where once again Φ is the standard normal cumulative function.

Equating the expressions for α_1 and α_2 and solving for t_2 yields:

$$t_2 = (t_1 - nm_{1H})\frac{\sigma_{2H}}{\sigma_{1H}} + nm_{2H} .$$

Substituting for t_2 in the expression for β_2 yields:

$$\beta_2 = \Phi\left(\frac{nm_{2K} - (t_1 - nm_{1H})\frac{\sigma_{2H}}{\sigma_{1H}} - nm_{2H}}{\sqrt{n}\,\sigma_{2K}} \right) \tag{A.2}$$

We now turn our attention to the case where g_1 is the linear detector; that is, $g_1(X) = X$. We then have:

$$m_{1K} = s, \quad m_{1H} = 0, \quad \sigma_{1H} = \sigma_{1K} = \sigma_1, \quad t_2 = t_1\frac{\sigma_{2H}}{\sigma_1} + nm_{2H},$$

$$\beta_1 = \Phi\left(\frac{ns - t_1}{\sqrt{n}\ \sigma_1}\right) \quad ,\beta_2 = \Phi\left(\frac{nm_{2K} - t_1\frac{\sigma_{2H}}{\sigma_1} - nm_{2H}}{\sqrt{n}\ \sigma_{2K}}\right) .$$

Then $\beta_2 > \beta_1$ if and only if

$$\frac{nm_{2K} - t_1\frac{\sigma_{2H}}{\sigma_1} - nm_{2H}}{\sigma_{2K}} > \frac{ns - t_1}{\sigma_1} . \tag{A.3}$$

After rearranging we have:

$$\mathrm{SNR}_2 = \frac{m_{2K} - m_{2H}}{\sigma_{2H}} \geq \frac{m_{2K} - m_{2H}}{\sigma_{2K}} > \frac{s}{\sigma_1} - \frac{t_1}{n}\frac{(\sigma_{2K} - \sigma_{2H})}{\sigma_1\sigma_{2K}} ,$$

where the second term on the right side is non-negative. Hence the condition

$$\mathrm{SNR}_2 > \frac{s}{\sigma_1} \tag{A.4}$$

is sufficient for $\beta_2 > \beta_1$. If $\sigma_{2K} = \sigma_{2H}$ then Eq. (A.3) reduces to

$$\frac{m_{2K} - m_{2H}}{\sigma_{2H}} = \mathrm{SNR}_2 > \frac{s}{\sigma_1} ,$$

and the condition in Eq. (A.4) is also necessary.

For the log likelihood ratio we have $\sigma_{iH} = \sigma_{iK}$. From Eqs. (A.1) and (A.2) we have $\beta_2 > \beta_1$ if and only if

$$\frac{nm_{2K} - (t_1 - nm_{1H})\frac{\sigma_{2H}}{\sigma_{1H}} - nm_{2H}}{\sigma_{2K}} > \frac{nm_{1K} - t_1}{\sigma_{1K}} .$$

After some rearrangement we have

$$\frac{m_{2K} - m_{2H}}{\sigma_{2K}} > m_{1H}\frac{\sigma_{2H}}{\sigma_{1H}\sigma_{2K}} - \frac{t_1}{n}\left(\frac{1}{\sigma_{1K}} - \frac{\sigma_{2H}}{\sigma_{1H}\sigma_{2K}}\right) .$$

Under the equality constraint on the variances this reduces to

$$\mathrm{SNR}_2 = \frac{m_{2k} - m_{2H}}{\sigma_2} > \frac{m_{1K} - m_{1H}}{\sigma_1} = \mathrm{SNR}_1 .$$

APPENDIX B

Proof of Theorem 2: We are interested in showing that the following two integrals converge

$$E\ g^2 = \int_{-\infty}^{+\infty} g^2(X)f(X)dX \tag{B.1}$$

$$E\ g = \int_{-\infty}^{+\infty} g(X)f(X)dX \tag{B.2}$$

If $|g(x)| = |\log[f(x-s)/f(x)]|$ is bounded ($\leq M$) then we have

$$E\ g^2 = \int g^2 f \leq M^2 \int f = M^2$$

and

$$E\ g = \int gf \leq M\int f = M .$$

Hence for bounded g we have bounded variance.

We now look at the case where g is not bounded. In order to show that the integrals in Eqs. (B.1) and (B.2) converge we want to bound g and f with known functions for which convergence can be demonstrated. Since the problem is entirely symmetric we will concern ourselves only with integration from $+\frac{s}{2}$ to infinity. For a Gaussian density with variance σ^2

$$g_{Gau_\sigma}(x) = \frac{xs}{\sigma^2} - \frac{s^2}{2\sigma^2}$$

and for a double exponential density $f(x) = \gamma e^{-\gamma|x|}$ with variance $\sigma^2 = \frac{2}{\gamma^2}$

$$g_{Lap_\sigma} = \begin{cases} -\sqrt{2}\frac{s}{\sigma} & \text{if } x < 0 \\ \frac{\sqrt{2}}{\sigma}(2x-s) & \text{if } 0 \le x < s \\ \sqrt{2}\frac{s}{\sigma} & \text{if } x \ge s \end{cases}$$

Hence for any unbounded log-likelihood ratio with $\lim_{x\to\infty} g'(x) < \infty$ and for all $x \ge P$ we can say

$$g_{Lap_{\sigma_2}}(x) \le g(x) \le g_{Gau_{\sigma_1}}(x)\,, \tag{B.3}$$

where σ_1, σ_2 and P are positive constants. That is, g is bounded above by a straight line with positive slope and below by a positive constant. We then have

$$g(x) \le \frac{xs}{\sigma_1^2} - \frac{s^2}{2\sigma_1^2} \tag{B.4}$$

and

$$g(x) \ge \frac{\sqrt{2}s}{\sigma_2} \tag{B.5}$$

for all $x \ge P$. From Eq. (B.5) we have

$$\frac{f(x-s)}{f(x)} \ge e^{\frac{\sqrt{2}s}{\sigma_2}}$$

or

$$f(x) \le e^{\frac{-\sqrt{2}s}{\sigma_2}} f(x-s)\,. \tag{B.6}$$

We then have

$$f(s) \le e^{\frac{-\sqrt{2}s}{\sigma_2}} f(0)$$

$$f(2s) \le e^{\frac{-\sqrt{2}s}{\sigma_2}} f(s) \le e^{\frac{-2\sqrt{2}s}{\sigma_2}} f(0)$$

$$f(ks) \le e^{\frac{-\sqrt{2}s}{\sigma_2}} f[(k-1)s] \le e^{\frac{-k\sqrt{2}s}{\sigma_2}} f(0)$$

where due to the previously mentioned constraints on f we know it has a finite maximum at $x=0$. We now define the step function f_1 as follows

$$f_1(x) = e^{\frac{-(k-1)\sqrt{2}s}{\sigma_2}} f(0) \qquad (k-1)s < x \le ks \qquad k = 1,2,3,\ldots$$

$$f_1(0) = f(0)\,.$$

This function dominates f. We now go one step further and define f_2 as

$$f_2(x) = e^{-\frac{\sqrt{2}}{\sigma_2}(x-s)} f(0) \qquad\qquad x \ge 0\,.$$

This expression can be written as

$$f_2(x) = c_1 e^{-c_2 x} \qquad x \geq 0 . \tag{B.7}$$

where c_1 and c_2 are positive constants. It is this function, which dominates both f_1 and f, that we use to prove the integrals in Eqs. (B.1) and (B.2) converge.

Since both f and g_P are continuous they attain a maximum in any closed and bounded set[7]. Hence both $\int_{\frac{s}{2}} g^2(x) f(x) dx$ and $\int_{\frac{s}{2}} g(x) f(x) dx$ are finite. From Eq. (B.3) it is clear that if we can show $var\ g_{Ga\sigma_1}$ and $E\ g_{Ga\sigma_1}$ are finite then the proof is complete. This is implied by showing that $\int_P^\infty x f_2(x) dx$ and $\int_P^\infty x^2 f_2(x) dx$ converge. But

$$\int_P^\infty c_1 x e^{-c_2 x} dx = \frac{c_1}{c_2} e^{-c_2 P} (P + \frac{1}{c_2}) < \infty$$

and

$$\int_P^\infty c_1 x^2 e^{-c_2 x} dx = \frac{c_1}{c_2} e^{-c_2 P} (P^2 + \frac{2P}{c_2} + \frac{2}{c_2^2}) < \infty$$

and the theorem is proved.

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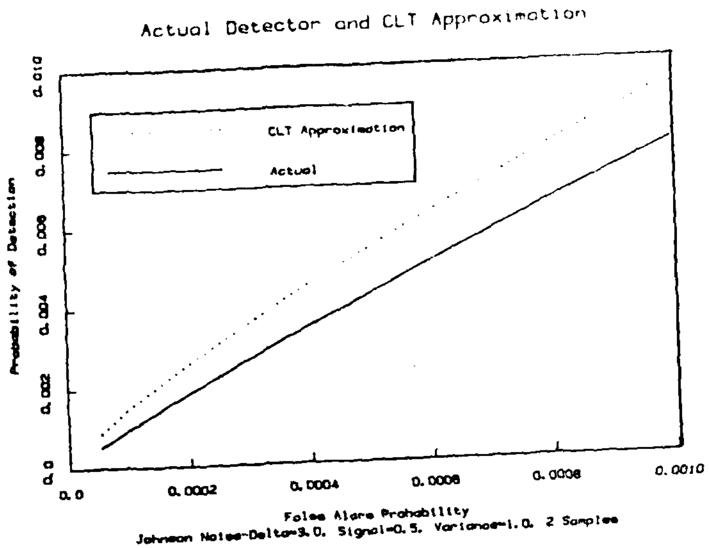
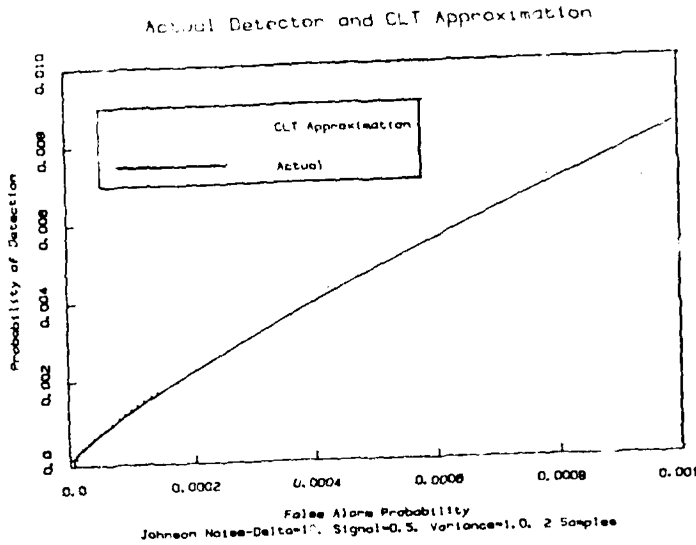


Fig. 2



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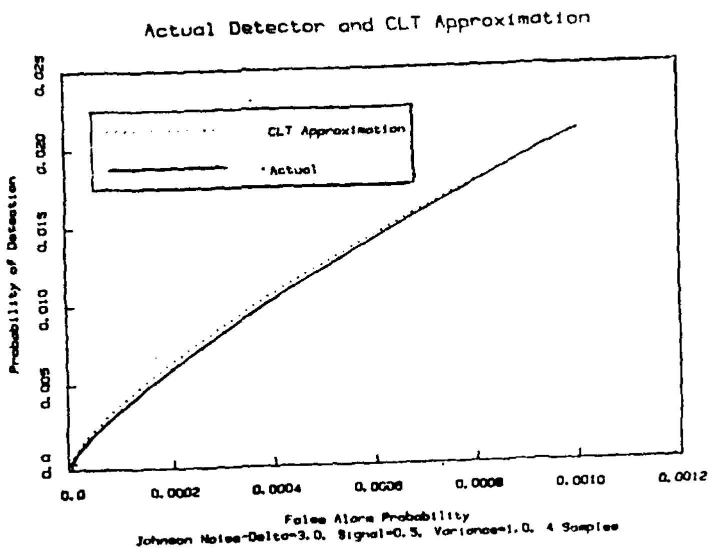

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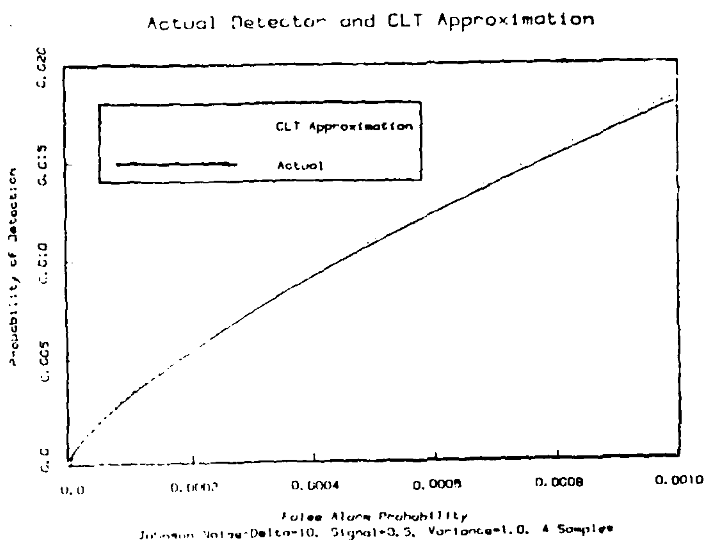

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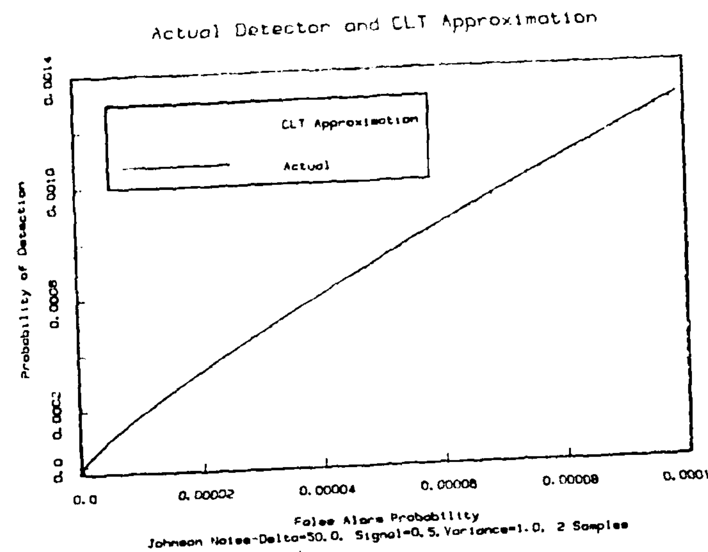

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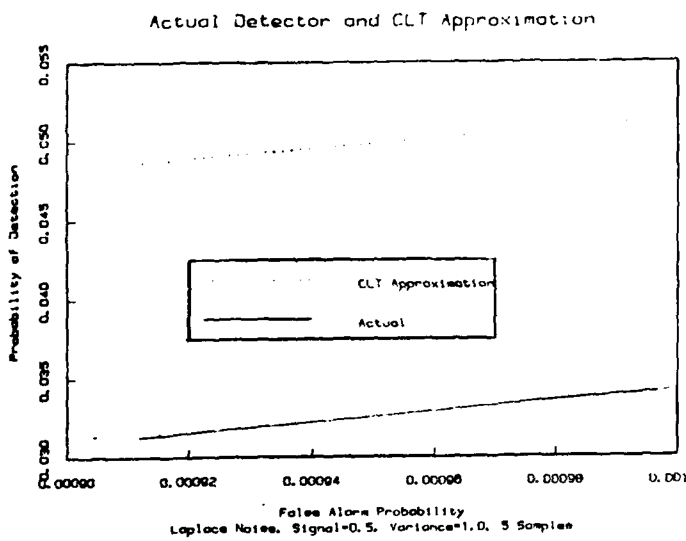

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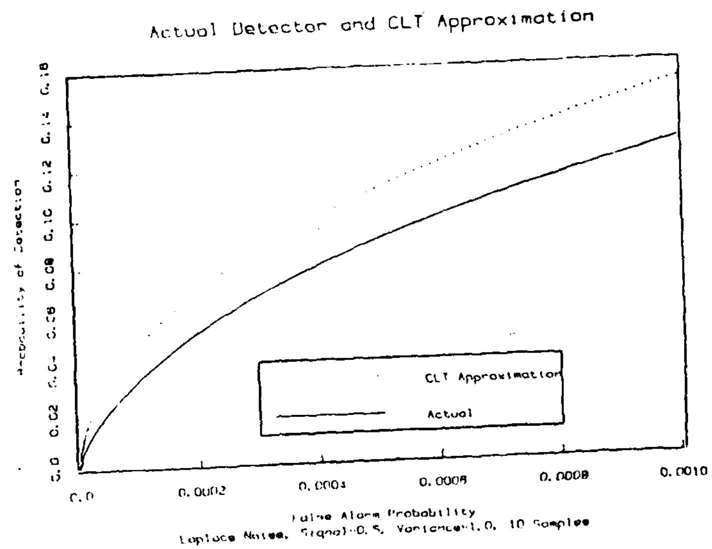

Fig. 8

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A generalized signal-to-noise ratio for detectors is defined and its relationship to quality of detection is investigated. Specific attention is given to log-likelihood ratio detectors. The question is discussed as to whether a Central Limit Theorem approximation for the output of the detector gives results which are close to those of the actual detector and is illustrated with examples.

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