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Modeling the Contribution of Maintenance Manpower to Readiness and Sustainability

Glenn A. Gotz, Richard E. Stanton

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PREFACE

The research described in this report was conducted in The Rand Corporation's Defense Manpower Research Center, sponsored by the Office of the Assistant Secretary of Defense (Force Management and Personnel). The purpose of the project, "Manpower Impacts on Readiness," was to examine important methodological problems that must be solved to help defense policymakers make tradeoffs more effectively among manpower and other kinds of resources.

The authors are concerned with the consequences of accounting for uncertain wartime demands for resources. The report shows that the existence of these uncertainties increases the importance of modeling a richer mix of manpower than is currently modeled in capability assessment models. More generally, the report illustrates the importance of modeling the flexibility of support resources in capability assessment models when, in fact, real systems do have flexible resources. It should be of interest to those who must evaluate the wartime implications of alternative experience and skill mixes of maintenance personnel, as well as to those involved in broader questions of resource tradeoffs.

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SUMMARY

This report describes and simulates a simple queuing model relating the mix of maintenance personnel in a repair station to weapon system availability in dynamic, wartime scenarios. The mixes of manpower considered in the model are described by the number of personnel in each occupation, their skill levels, and their cross-training. The model does not simulate a specific unit nor is it exercised using real data. Anthony developed it to demonstrate the feasibility and importance of modeling a richer mix of manpower while explicitly considering the uncertainties about the true wartime demands for skills.

The most important model output is the expected number of weapon systems (aircraft in our applications) non-available (NA) due to missing reparable parts by day of the war. The model provides a framework for illustrating the effects of changes in the types and numbers of repairmen, spare parts, and job assignment rules on this output. Other outputs include the standard deviation of NA aircraft by day of the war and average time to repair broken parts.

Three mixes of manpower are analyzed, each under a base case scenario and two minor variations on that scenario. The first mix contains only one skill level per occupation and no substitutability among individuals in different occupations. The second mix assumes cross-training among personnel; individuals in different occupations are imperfectly substitutable for one another. The third mix introduces an additional skill level per occupation.

The base case scenario is what we *assume* to be the most likely scenario, and the first mix of manpower—no substitutability—was designed to do well in it. Introducing cross-training yields only a small improvement in NA aircraft. The second scenario presents a higher than expected failure rate of one type of part and a slower than expected repair rate for that part. The flexibility afforded by the cross-training reduces NA aircraft markedly compared with the no-substitutability case. The value of cross-trained personnel increases when the inventory of primary skills does not match up with the demands for those skills.

The third scenario introduces spare parts to offset the higher failure rate and slower repair rate presented by the second scenario. These spare parts bring the repair resources back into rough balance with resource demands and diminish the improvement yielded by cross-training. Since the higher failure rate was, by assumption, unexpected,

an alternative way of viewing the results is that the marginal value of the spare parts is lower in the presence of cross-trained personnel.

The simulations demonstrate the importance of focusing on measures directly related to generating wartime sorties. The transition from scenario 2 to scenario 3 results in fewer aircraft not flying due to maintenance and supply—a preferred outcome. However, because more aircraft are flying, more parts are subject to failure, and average time to repair increases as a result of queuing at the repair station. Thus, focusing on average time to repair as a performance measure can lead to inappropriate conclusions.

The simulations also demonstrate that uncertainty about the true wartime demands for resources makes it important to evaluate the contributions of each resource mix in a range of possible scenarios. Even the minor variations in failure rates and repair times examined in this study indicate how one's view of the relative importance of resources can change. Specifically, uncertainty about the scenario puts a premium on flexible assets. Because people can be more flexible than hardware or spares, inattention to scenario uncertainty means that people will be undervalued relative to these other assets. Formally cross-trained and retrained personnel are more flexible than single-skill specialists and, hence, will be relatively undervalued in requirements determinations unless their value in an uncertain environment is considered.

ACKNOWLEDGMENTS

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I. INTRODUCTION

Manpower is ultimately important to the military services because it contributes to readiness and sustainability. But the quantitative relationship between different manpower configurations and readiness or sustainability is not well understood. Data on manpower productivities by experience level and other characteristics are not routinely gathered. And of the models that relate spare parts, POL, and other support resources to weapon system availability in dynamic, wartime scenarios, few capture the richness and complexity of the contributions of people to this availability. Thus, the lack of data and shortage of tools preclude evaluating manpower's effects on the readiness and sustainability of units and examining manning alternatives.

Computer models that relate support resources to wartime weapon system availability are common in the Department of Defense. The models are used in two ways: to determine the weapon system availability associated with alternative mixes of support resources and to calculate the resources required to support specific weapon system availability. The Air Force's LCOM and the Rand-developed TSAR, and its derivative AURA, are Monte Carlo simulation models that include most of the resources required to generate tactical aircraft sorties (LCOM and TSAR) and combined arms unit maneuvers (AURA). Included are organizational and intermediate levels of maintenance and quite detailed rules by which maintenance and repair are accomplished.

Although the tasks to be performed may be described in great detail, the personnel performing these tasks are not. Manpower is usually modeled as having only one skill level for each occupation. Substitutability among personnel in different occupations is either perfect or absent. That is, an individual cross-trained[1] in tasks regularly performed by those in another occupation is assumed to do those tasks just as well and just as fast as those primarily trained for the tasks.

The probabilities of combat-essential equipment and parts failing or being damaged are key inputs to these models. Generally, requirements computations using these models treat these probabilities as known; the models estimate the resources required to satisfy demand for repair at a given confidence level. There are very great

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[1]Throughout this report we will use the term cross-training to denote either formal or informal training in other than the primary specialty that does not lead to changing the individual's primary specialty. We will use the term retraining for training into a new primary specialty.

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uncertainties about true failure probabilities, however, and treating them as if they are known can lead to setting requirements for overly specialized resources, i.e., resources that are inflexible in the face of unexpected demands.

This report demonstrates the feasibility of modeling wartime weapon system availability under alternative maintenance manpower mixes and the important policy implications of doing so. We describe and simulate a simple queuing model relating the mix of maintenance personnel in a repair station to weapon system availability in dynamic scenarios. The manpower considered in the model is described by the number of personnel in each occupation, their skill levels, and their cross-training.

In the following sections, we describe the model's inputs and outputs in nontechnical terms and simulate a small number of manpower mixes in three slightly different environments—different in terms of failure rates, repair rates, and numbers of spare parts. The manpower mixes consist of three increasingly complex mixes of personnel. The first mix has only one skill level per occupation and no cross-trained personnel. The second mix introduces personnel who are cross-trained, at a lower skill level, in another occupation. The third mix introduces a second skill level per occupation while retaining cross-training for personnel who have a high skill level in their primary occupation.

We examine the performance of the manpower mixes under different job assignment rules. A repairman qualified to repair more than one type of part presents a special problem: it must be decided which part he should repair next. We examine the consequences of using two different job assignment rules, both derived from objectives of maximizing the number of available weapon systems. One of the rules derives from a very short-term availability objective; the other accounts for the future consequences of the current job assignment decision.

The model does not simulate a specific unit nor is it exercised using real data. Rather, we developed it to demonstrate the feasibility and importance of modeling a richer mix of manpower while explicitly considering the uncertainties about the true wartime demands for skills.

II. MODELING MAINTENANCE MANNING

The model described in this section illustrates the feasibility of relating the skill levels and cross-training characteristics of maintenance personnel to an output-oriented measure of performance. The model simulates the number of weapon systems (e.g., aircraft) non-available because of limited repair capabilities. It does this by simulating arrivals of broken parts at a repair station and their repair by repairmen. The model keeps track of how many broken parts of each type are awaiting repair and how many are in repair by each type of repairman. When a broken part arrives at the repair station, it is assigned either to a qualified repairman if one is free, or to the awaiting repair queue. When a repairman completes a job, he is assigned another part to fix. The part he is assigned depends on his skills, the number of each type of part waiting to be fixed, and the job assignment rule specified by the user. Using this framework, we can simulate the effects of changes in the types and numbers of repairmen, spare parts, and job assignment rules on the number of weapon systems unavailable due to missing parts in a dynamic, wartime environment.

We adapted our model from the Dyna-Sim model developed at Rand for Project AIR FORCE.[1] Dyna-Sim simulates the effects of alternative priority-of-repair rules on various performance measures when there is a small number of identical machines that are used to repair a variety of types of incoming reparables. The arrival rate of incoming reparables changes over time—hence the "Dyna" in Dyna-Sim—but the arrival pattern of these reparables is assumed to be independent of past maintenance.

Our most significant revisions to Dyna-Sim were changing "identical machines" to "personnel with different skills" and causing the arrival rates of reparables to depend on previous maintenance decisions. That is, the number of aircraft parts at risk of failure depends on the number of available aircraft, which, in turn, depends on past maintenance decisions. In addition, we expanded the priority-of-repair rules to account not only for the number of each type of part needing repair, but also for who is free to begin a new repair job. We maintained the dynamic, discrete event Monte Carlo simulation structure of the model

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[1] See Miller, Stanton, and Crawford (1984) for a discussion of the simulation methodology.

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and the essential model logic. Thus, our revised model emphasizes
skill mixes of maintenance personnel in a dynamic environment.[2]

The model is a simple mathematical simulation of queuing and
repair at a repair station, e.g., an intermediate maintenance facility.
Because Dyna-Sim was developed for the analysis of aircraft com-
ponent repair, it is convenient to refer to aircraft when discussing
weapon systems. There is nothing that precludes using the model for
components of other weapon systems. We also believe the model could
be adapted to on-equipment or organizational maintenance, although
we have not yet done so.

Because the model is a Monte Carlo simulation, multiple trials must
be run and statistics averaged over the trials. The most useful statis-
tics are the daily averages, standard deviations, and histograms of the
numbers of back-ordered parts[3] and numbers of aircraft not available
due to missing parts. Summary statistics on the mean time to repair
for each type of part and the standard deviation around the mean are
also produced.

INPUTS

The important inputs to the model describe people and how long
they take to repair parts; parts, how often they break, and how many
spares are in the system; and the initial number of aircraft and how
rapidly they are lost in combat. Excluded from the model are POL,
munitions, transportation assets, and other support resources that
potentially influence weapon system availability or capability.

The model allows us to specify a total number of repairmen and to
divide these repairmen into groups according to skill (occupation), skill
level, secondary skills, and skill level in each secondary skill. In this
model, skills are identified with broken parts—if the individual can
repair part type I, then he holds skill I. His skill level in I is deter-
mined by how long, on the average, it takes him to repair a broken I.
A specialist at repairing part I may be cross-trained to repair part II.
Any pattern of cross-training may be assumed. For example, personnel
trained to repair I may be cross-trained to repair II, but personnel
trained to repair II may not be trained to repair I. Alternatively, spe-
cialists highly skilled at repairing part I may be cross-trained into
another skill, but low-skill-level personnel may not be. Because we do

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[2]Appendix A explains the technical assumptions we made about repair times and part
arrival rates when we revised Dyna-Sim.

[3]The number of back-ordered parts is equal to the number of parts in repair plus the
number awaiting repair less the number of spare parts.

not model repair jobs that call for teams of repairmen, we do not allow for task-assist-qualified training.

In the simulations below there are only two types of parts: *I* and *II*. To be mission-capable, an aircraft has to have one of each type of part. These parts fail randomly, depending on the user-supplied failure parameters and on the number of mission-capable aircraft. That is, we assume that parts only fail when flying. Thus, if maintenance policy or the provision of spare parts increases the number of mission-capable aircraft, then the expected number of failures will also increase. On the other hand, higher-than-expected failures early in the day imply lower-than-expected failures later in the day because the number of aircraft flying has decreased.

We assume that parts are freely cannibalized, i.e., failures are consolidated to create the largest number of mission-capable aircraft.[4] For example, if two aircraft have broken part *I*'s and one aircraft has a broken part *II*, these failures may be consolidated to create a mission-capable aircraft.

The number of mission-capable aircraft at any moment is equal to the beginning aircraft inventory less combat losses and the number missing one or both mission-essential parts. Aircraft are completely lost at a user-specified rate per day—we term these attrition losses. In the simulations presented below, we begin with a 72 aircraft inventory and a daily attrition rate of 6 percent. For simplicity, we assume that the percentage of aircraft lost in combat each day is fixed and not a random variable.[5]

We will focus on the number of aircraft missing at least one part and will denote these aircraft non-available (NA) because this is what maintenance and spare parts influence in our model. The number of NA aircraft at any given time is equal to the maximum of the number of missing part *I*'s and part *II*'s, following our assumption that failures are consolidated into the least number of NA aircraft.[6] The number of missing part *I*'s is equal to the number of *I*'s being repaired plus the

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[4] Assuming that cannibalization is not freely done would add realism to the model but would not change the conclusions of this study. When constructing a manning table for a unit, such realism might be necessary.

[5] We include combat attrition for the sake of realism, but do not simulate the interactions between friendly and enemy forces. Thus, we have no way of estimating how the combat attrition rate should change as the number of mission-capable aircraft changes. Since our principal interest is in constructing a simple model of maintenance manning, the assumption that combat attrition is independent of marginal changes in maintenance policy or manning should not affect our conclusions.

[6] The model does not represent individual airplanes. The assumption of full cannibalization is the only way to connect parts with available airplanes as the model is structured. See Miller, Stanton, and Crawford.

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number awaiting repair less the number of spare *I's*. This quantity is commonly called the back-order quantity, (BOQ); the larger of the BOQ for part *I* and the BOQ for part *II* is termed the maximum back-order quantity, to which we will occasionally refer.

JOB ASSIGNMENT RULES

When a repairman qualified to repair both *I's* and *II's* is ready to begin a new task and both parts are queued up waiting to be repaired, which should be assigned to the repairman? Application of simple rules like "first in/first to repair" is inappropriate because such rules are not related to what we care about—available aircraft. We examine two discretionary rules in this study, each derived from an objective of minimizing the number of NA aircraft.

The first rule we examine derives from the objective of maximizing the expected number of available aircraft in the next moment of time. This objective is equivalent to minimizing the expected maximum back-order quantity in the next moment of time, given our assumptions relating back-order quantities to non-available aircraft. The rule is to assign to the repairman whichever part is in greatest back-order if the repairman has been trained to fix that part. We will refer to this rule as the MB (minimize back-orders) rule.

The second rule derives from the objective of minimizing the number of NA aircraft from now until resupply,[7] a dynamic optimization. The dynamic optimization determines, for each possible state of the system, which part a free repairman should repair next. The state of the system is described by the numbers of each type part awaiting repair, in repair by a specialist at repairing that part, and in repair by a nonspecialist.

Job assignments follow critical value rules under this objective. That is, there is a critical number for each type of part, denoted below as Q_{crit}, against which back-orders are compared. The result of the comparison determines which part is assigned to a free repairman. The job assignment rules derived from the optimization have the following form. Assume that it is part *I* that is in greatest back-order.

- If the free repairman is a specialist at repairing *I*, then assign him a *I*.

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[7]Appendix B presents the objective functions from which both rules are derived.

- If the free repairman is a specialist at repairing II and if the number of back-ordered II's is greater than Q_{crit}, then assign him a II.
- If the free repairman is a specialist at repairing II and if the number of back-ordered II's is less than Q_{crit}, then assign him a I.

Q_{crit} is the smallest number of waiting II's such that a cross-trained II specialist would work on a II rather than a I. When part II is in greatest back-order, there is a Q_{crit} that is the smallest number of waiting I's such that a cross-trained I specialist would work on a I rather than a II. Thus, Q_{crit} is a critical value that is determined by the optimization; its value depends on the state of the system, the part arrival probabilities, repair times, and time to resupply. In the simulations below we assume that resupply of spare parts takes place during the fifteenth day of the war and maintenance no longer constrains available aircraft. A dynamic program was used to derive the Q_{crit} values and we will refer to the derived job assignment rules collectively as the DP rule.[8]

Why the two types of rules? Consider the following possible state of the system. There are no spares of either type of reparable and part I is in greatest back-order. A specialist at repairing part II, who is cross-trained at a low skill level to repair part I, has become free. Assigning him a I to repair will yield the quickest expected reduction in the number of NA aircraft. However, a specialist in repairing a I may become free soon and assigning the part II specialist a I means a higher probability that part II will become the constraining resource, i.e., the short-term reduction in NA aircraft may come at the expense of a longer term increase. Hence, there is a tradeoff between the speed with which the number of NA aircraft is reduced and the longer term levels of NA aircraft.

What determines the acceptability of a short-term loss in available aircraft in exchange for a longer run gain? It clearly depends on the sizes of the gains and losses. It also depends on the relative importance of being able to mass forces earlier rather than later. If the value of immediately adding an available aircraft is infinitely greater than the value of adding an aircraft at any later time, then no short-term loss is acceptable. Alternatively, if the value of an added aircraft is the same regardless of when it is added, then some short-term losses are acceptable. The importance of making aircraft available on one day versus the next can be expressed as a daily discount rate. For example,

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[8] The functional equation for the dynamic program is presented in App. B.

if the value of adding an available aircraft the first morning of the war is twice the value of adding an available aircraft on the second day, then the discount rate for the first day is 100 percent. Camm (forthcoming) discusses this issue in more depth. His analysis using nonstochastic Lanchester equations indicates that the daily discount rate for available aircraft is probably best taken to be equal to the daily combat attrition rate.

In fact, the MB and DP rules are derived from objective functions that differ only in the assumed discount rates. The objective function is the sum of discounted NA aircraft or, equivalently, the sum of the discounted maximum back-order quantities. Setting the discount rate to zero yields the DP rule used in this study. Setting the discount rate to infinity yields the MB rule.[9] Although we could have examined discount rates intermediate to zero and infinity, the job assignment rules that we examine in this study indicate the sensitivity of our results to assumptions about the daily discount rate.

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[9]See App. B.

III. BASE CASE

The utility of modeling cross-training and skill levels among maintenance personnel is most easily illustrated through a series of cases. This section develops a base case in which there are only two types of parts to repair. Each part type has an associated occupation dedicated to repairing it, with one skill level per occupation and no cross-training. We examine the pattern of non-available (NA) aircraft in what we *assume* to be the most likely scenario—where scenario refers to the parts failure rates, repair times, and stockage levels.[1] The daily pattern of NA aircraft in this base case/most likely scenario is then compared with the outcomes under two alternative scenarios. Since personnel in one occupation cannot substitute for personnel in another occupation and there are no skill level substitutions, we term this the no substitution (NS) rule. In Sec. IV we compare the NS rule outcomes in the three scenarios with cases in which personnel are cross-trained, and in Sec. V low-skill-level specialists are introduced and the results compared with the preceding cases. All of the results shown in the text are simple graphs; the numerical results are in App. C.

Before developing the base case, it is useful to develop a system for labeling cases. Each case we simulate is characterized by a manpower structure, a scenario, and a job assignment rule. We examine manpower structures with and without more than one skill level per occupation; these are labeled A and B, respectively. The scenarios are labeled 1, 2, and 3, with the most likely scenario labeled 1.[2] As described earlier, the job assignment rules are labeled NS for no substitution, MB for minimize the number of expected NA aircraft in the immediate future, and DP for minimize the number of expected NA aircraft from now until day 15 of the war. Our base case is labeled A/1/NS for manpower structure A (one skill level), scenario 1 (most likely scenario), and no substitutability between occupations.

To keep things simple and easy to track, the manpower structures in both occupations are the same. Thus,

- *Manpower Structure A:* There are two type *I* repairmen and two type *II* repairmen.

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[1] The scenarios in this report were constructed for the purpose of illustration. They are not drawn from any specific plans or data.

[2] Appendix C contains the parameters (i.e., failure rates, spares, etc.) for all the cases examined in the report.

Scenario 1 has been constructed to be perfectly symmetric in all dimensions so the effects of deviations from the scenario are readily apparent.

Scenario 1:

- A type *I* repairman takes 0.8 days, on the average, to repair a part, and a type *II* repairman takes the same time to repair his part.
- The daily probability that a part *I* fails on a mission-capable aircraft is 0.042 and the same is true for a part *II*.[3]
- There are no spares of either type part.

Because there is *no substitutability among types of repairmen in our base case*, as soon as a repairman completes a job he immediately begins on another of the part he specializes in repairing if one is awaiting repair.

The average number of NA aircraft, counted at the end of each day, is illustrated in Fig. 1. The graph is the result of 100 Monte Carlo trials using the input parameters above. The vertical axis measures the number of NA aircraft and the day of the war is measured on the horizontal axis.[4] The number of NA aircraft first rises and then falls because initially the expected arrival rate of broken parts exceeds the expected rate of repair. Aircraft are being lost in combat, however, and the arrival rate of broken parts is ultimately exceeded by the repair rate. That is, the number of NA aircraft does not include aircraft lost in combat and, eventually, the rate of arrival of broken parts is less than the rate of repair. The number of NA aircraft is still positive in this latter period because there are queues of broken parts that must be repaired.

Now, it is impossible to perfectly forecast wartime skill demands. Even peacetime failure and repair rates vary in unexplained ways among locations and over time at each location. It seems unreasonable to assume that we are more certain about wartime failure rates than peacetime rates. Thus, it is useful to analyze the performance of units whose manning and other resources were determined by balancing resources with demands in a specific scenario when it is some other scenario that actually occurs.

In scenario 2 we assume that the part *I* failure rate is 25 percent higher and the part *I* average repair time 33 percent longer than in

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[3] The daily per-aircraft probability is constant for the duration of the scenario. See App. A for how these probabilities are used to generate broken parts arriving at the repair station.

[4] Appendix D presents the daily averages, standard deviations, and maxima in tabular form for this and all other cases.
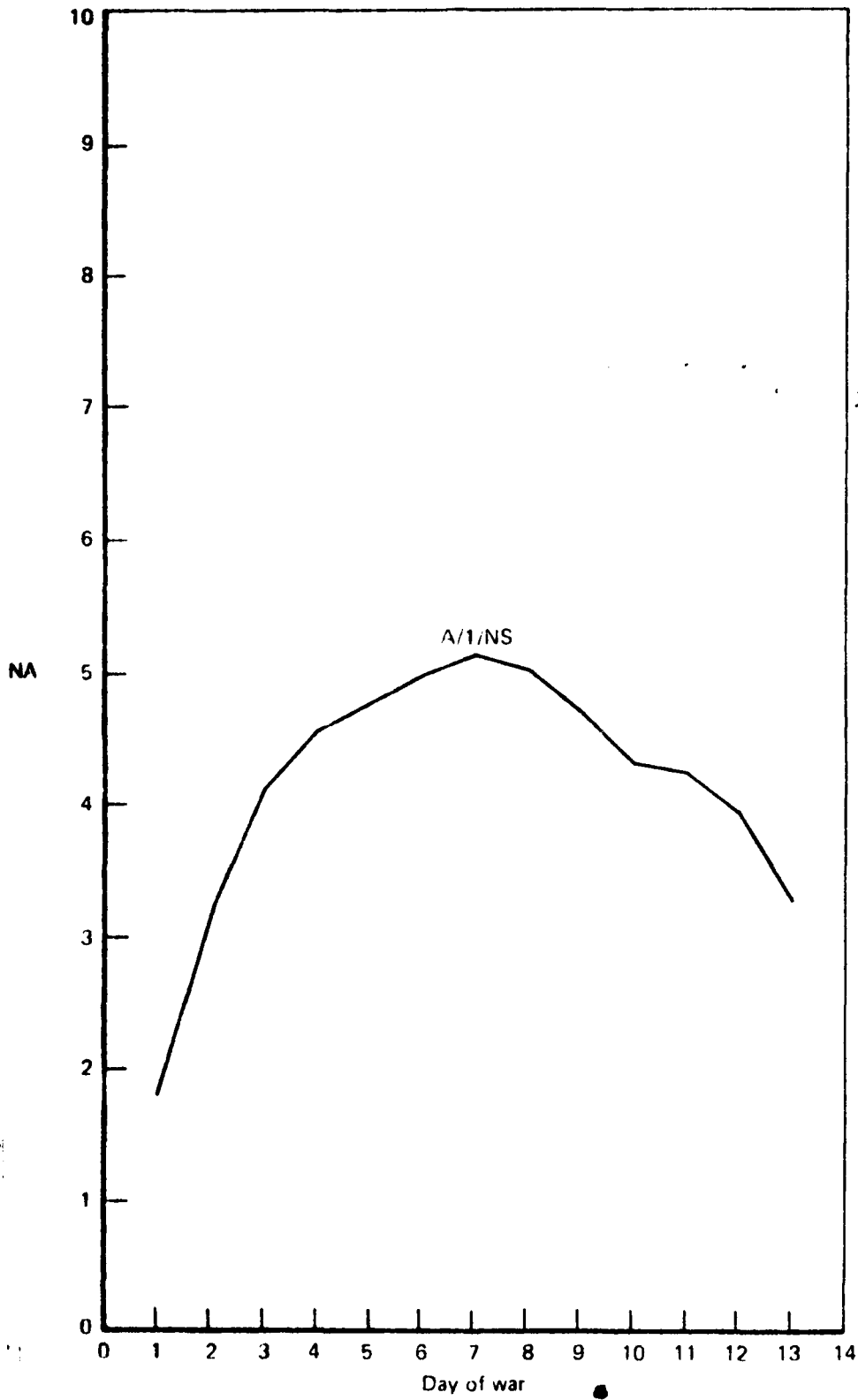
Fig. 1—No substitution/most likely scenario

scenario 1. The scenario 1 failure rate and repair time for part *II* are assumed to hold in scenario 2.

Scenario 2:

- A type *I* repairman takes 1.067 days, on the average, to repair a part, and a type *II* repairman takes 0.8 days to repair his part.
- The daily probability that a part *I* fails on a mission-capable aircraft is 0.052 and 0.042 for a part *II*.
- There are no spares of either type part.

Our new case is labeled A/2/NS and is shown in Fig. 2, with case A/1/NS for comparison. It is not surprising that the expected number of NA aircraft is uniformly greater under scenario 2 than under scenario 1. Total aircraft days lost over the first 13 days of the war are almost twice as high under scenario 2 as under scenario 1: 97 days versus 54.[5] However, in the following section we show that the negative effects of the increases in the failure rate and repair time for part *I* are reduced by the presence of cross-trained personnel in the unit.

If we knew in advance that the scenario 2 failure rates and repair times are the true parameters, the negative effects could be offset by the provision of spare type *I* parts to the unit. This gives rise to scenario 3, which differs from scenario 2 only in that the unit has been provided with five spare type *I* parts.

Scenario 3:

- A type *I* repairman takes 1.067 days, on the average, to repair a part, and a type *II* repairman takes 0.8 days to repair his part.
- The daily probability that a part *I* fails on a mission-capable aircraft is 0.052 and 0.042 for a part *II*.
- There are five spare type *I* parts and no spare type *II* parts.

The new case is labeled A/3/NS and is shown in Fig. 3 along with the graphs from the two preceding cases. A principal reason for examining A/3/NS is to illustrate the effect of accounting for the contribution to sustainability of an additional resource. That providing five spare part *I*'s yields an unambiguous improvement in the number of NA aircraft over case A/2/NS is no surprise. In fact, the number of NA aircraft in case A/3/NS is smaller early in the war than in the base case A/1/NS. However, the NA aircraft in A/3/NS exceeds those in A/1/NS later in the war.

There are performance measures in addition to available aircraft. For example, our model calculates a variant of repair cycle time, a

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We calculated total aircraft days lost as the sum of the NA aircraft from the first through the thirteenth day of the war.
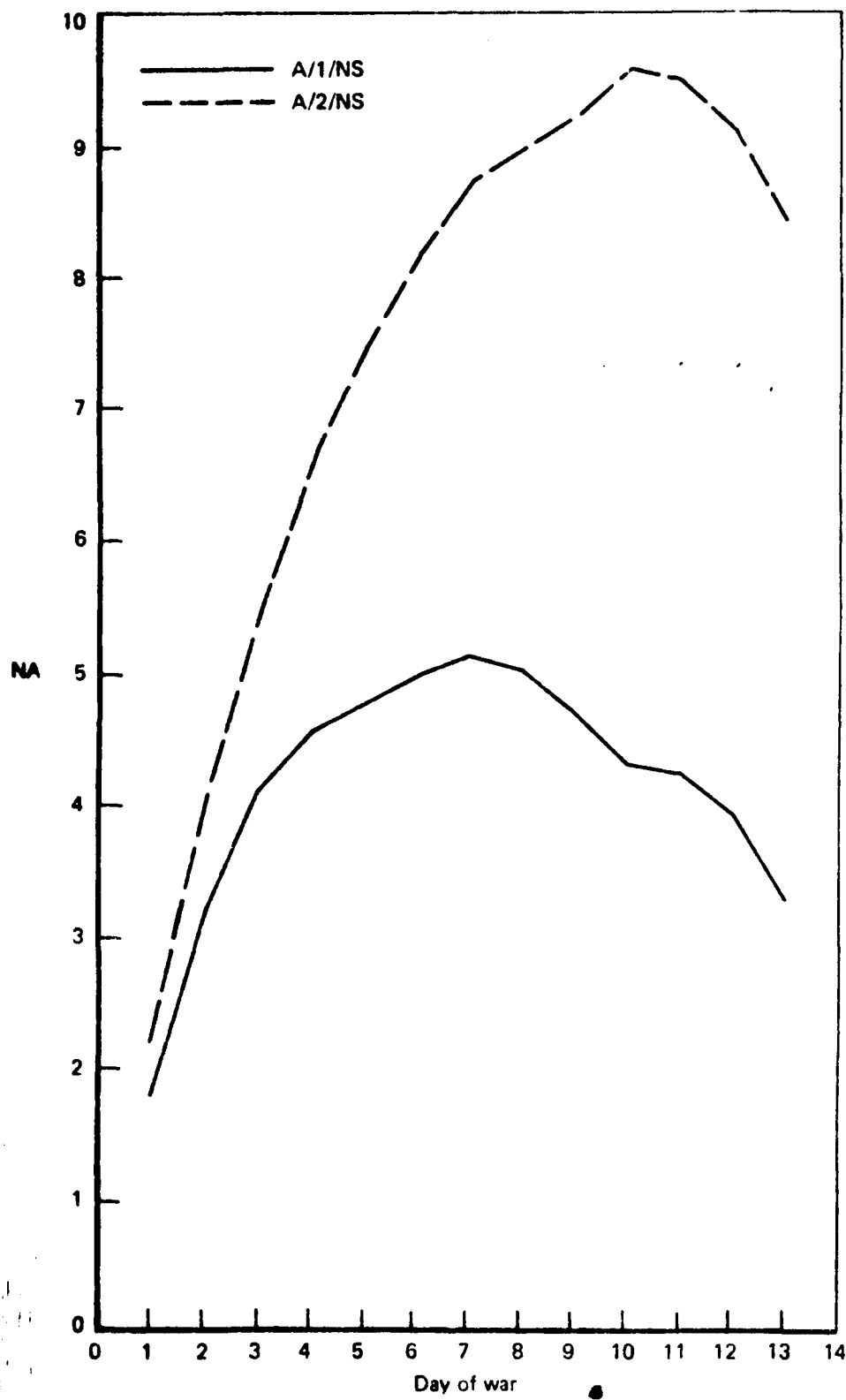
Fig. 2--No substitution/scenarios 1 and 2
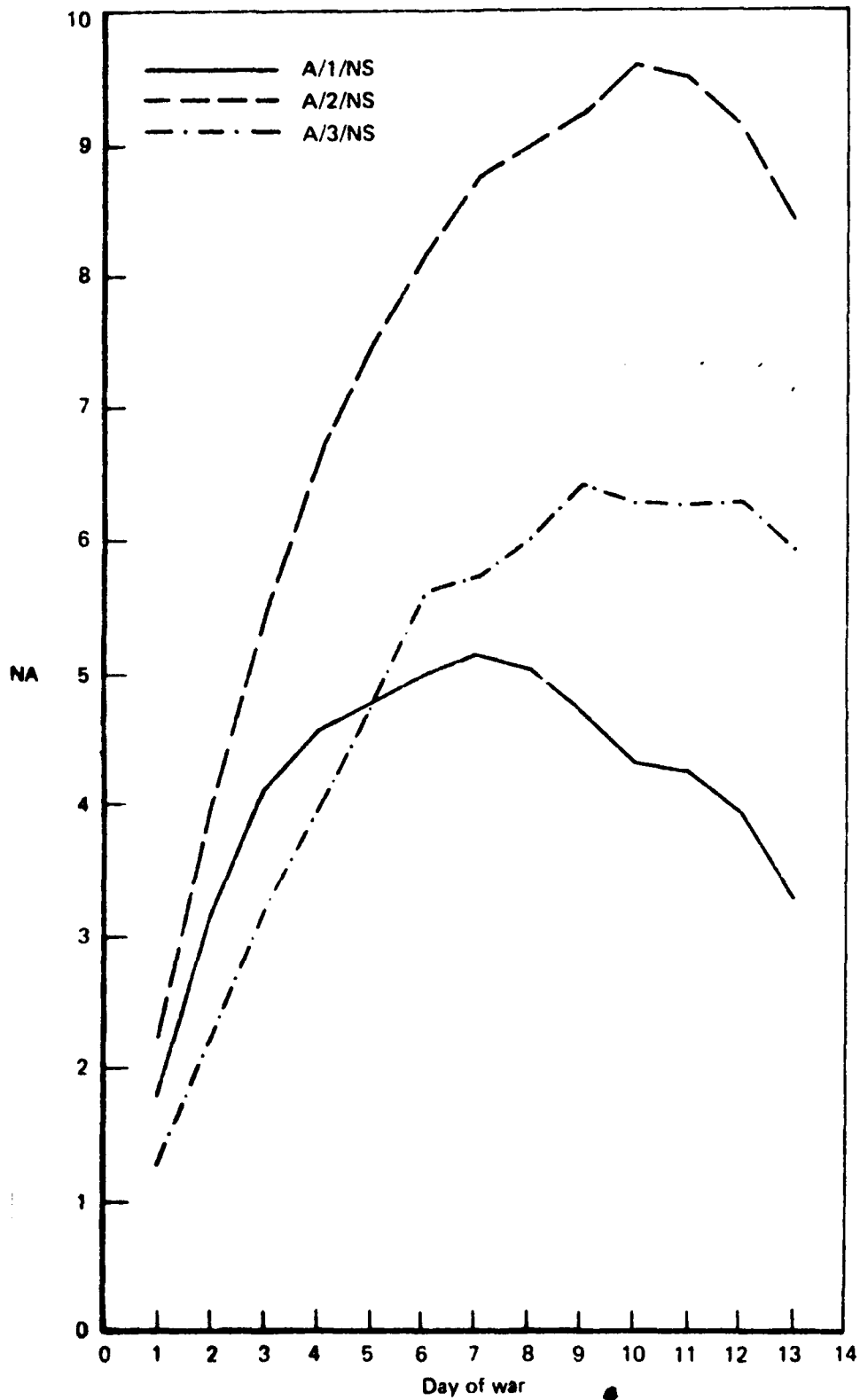
Fig. 3—No substitution/scenarios 1, 2, and 3

commonly used logistics performance measure. Since we do not include transportation assets or times, we calculate average time to repair from arrival of broken parts at the repair station to when their repair is completed. Therefore, the average time to repair is the sum of awaiting repair time and actual repair time. Table 1 presents the average time to repair for each of the two part types in the three cases above.

The reader will note that the average times to repair parts *I* and *II* are different under case A/1/NS although they could be expected to be the same. The difference is just sampling error; we also ran a 1000 trial case and the two averages were 1.50 for part *I* and 1.54 for part *II*. The standard deviations are roughly 1.30.

Recall that the differences between scenario 1 and scenario 2 are that part *I* failure rates are higher and average repair times are slower. Hence, the average time to repair increases for part *I* as we move from scenario 1 to scenario 2.

Scenario 2 and scenario 3 differ only in that we have added five spare part *I*'s to offset the high failure and slow repair rates of part *I*. *Yet the average time to repair part I increases by one half day.* This increase is not attributable to sampling error. The five spares allowed more aircraft to fly, thereby generating more failures. Since the part's repair rates do not change, the higher arrival rate of the part in scenario 3 causes longer times awaiting repair and, hence, longer times to repair.

The pattern of changes in the average times to repair clearly indicates that such an intermediate measure can be misleading when evaluating the potential performance of the maintenance system in a wartime setting.

Table 1

AVERAGE TIME TO REPAIR
(Waiting + repair)

Part	A/1/NS	A/2/NS	A/3/NS
I	1.60	3.83	4.32
II	1.42	1.37	1.44

IV. CROSS-TRAINING

We next examine the change in aircraft availability associated with having cross-trained personnel in the unit. We assume that every individual in the unit is able to repair both types of parts but not at the same pace. The type *I* repairman takes longer, on the average, to repair a type *II* part than does a type *II* repairman, and the same relationship is true for a type *II* repairing a type *I* part. In fact, we have introduced a second, lower skill level for each skill and trained the specialist in the other skill to that level. In Sec. V we introduce personnel who hold *only* the lower skill level for one or the other skill and are not cross-trained.

The consequences of having cross-trained personnel are presented below by first evaluating the outcome in each of our three scenarios and then comparing the results, scenario by scenario, with the no substitution (NS) case.

The number of repairmen by primary skill is the same as in the base case and we will refer to a repairman according to his primary skill. Thus, we remain with

- *Manpower Structure A*: There are two type *I* repairmen and two type *II* repairmen.

We must augment our description of scenario 1 to account for the cross-trained personnel. The scenario is unchanged other than for the cross-training.

Scenario 1:

- A type *I* repairman averages 0.8 days to repair a part *I* and 1.2 days to repair a part *II*.
- A type *II* repairman averages 0.8 days to repair a part *II* and 1.2 days to repair a part *I*.
- The daily probability that a part *I* fails on a mission-capable aircraft is 0.042 and the same is true for a part *II*.
- There are no spares of either type part.

Now that there is substitutability among types of repairmen we must adopt a rule for assigning broken parts to repairmen as repairmen become free. We will begin by assigning the repairman the type part that is currently constraining aircraft from flying, i.e., that part that is in greatest back-order. This short-term decision rule was termed the MB rule (for minimize maximum back-orders). Thus, the first cross-

training case we examine is labeled A/1/MB for manpower structure A, scenario 1, and the minimize maximum back-orders rule. Figure 4 is a graph of the number of NA aircraft, by day, for this case.

In scenarios 2 and 3 we assume that the part *I* failure rate is 25 percent higher than in scenario 1. We also assume that the part *I* average repair time is 33 percent longer for the type *I* repairman. We assume that there is no change in the average length of time spent by a type *II* repairman repairing type *I* parts. We do not change this latter repair time because we want to isolate the effects of changing the comparative advantages of type *I* and *II* repairmen. Scenarios 2 and 3 differ in that scenario 2 has no spare parts, whereas scenario 3 has five spare part *I*s. The descriptions of scenarios 2 and 3 are:

Scenarios 2 and 3:

- A type *I* repairman averages 1.067 days to repair a part *I* and 1.2 days to repair a part *II*.
- A type *II* repairman averages 0.8 days to repair a part *II* and 1.2 days to repair a part *I*.
- The daily probability that a part *I* fails on a mission-capable aircraft is 0.052 and 0.042 for a part *II*.
- *Scenario 2*: There are no spares of either type part.
- *Scenario 3*: There are five spare type *I* parts and no spare type *II* parts.

Our two new cases are labeled A/2/MB and A/3/MB and are graphed in Fig. 5 along with case A/1/MB. Because the principal interest in these cases is in the comparison with the NS and DP cases, discussion of the graphs is deferred until after the DP case is presented.

The DP rule derives from the longer term objective of minimizing the total number of NA aircraft-days from "now" until day 15 of the war. The three scenarios we examine using this rule are exactly those we used above for the short-term decision rule. The three cases are labeled A/1/DP, A/2/DP, and A/3/DP, and are shown in Fig. 6.

We now turn to comparison of the no substitution case, the MB cross-training case, and the DP cross-training case. Recall that scenario 1 was constructed to be perfectly symmetric in failure rates and repair rates and that there are two repairmen per type of part. Thus, the demands for repair are proportional to repair capability even in the no substitution case, and cross-training of personnel would not seem to confer significant benefits. Indeed, we see in Fig. 7 that cross-trained personnel do not augment the performance of the unit by very much.
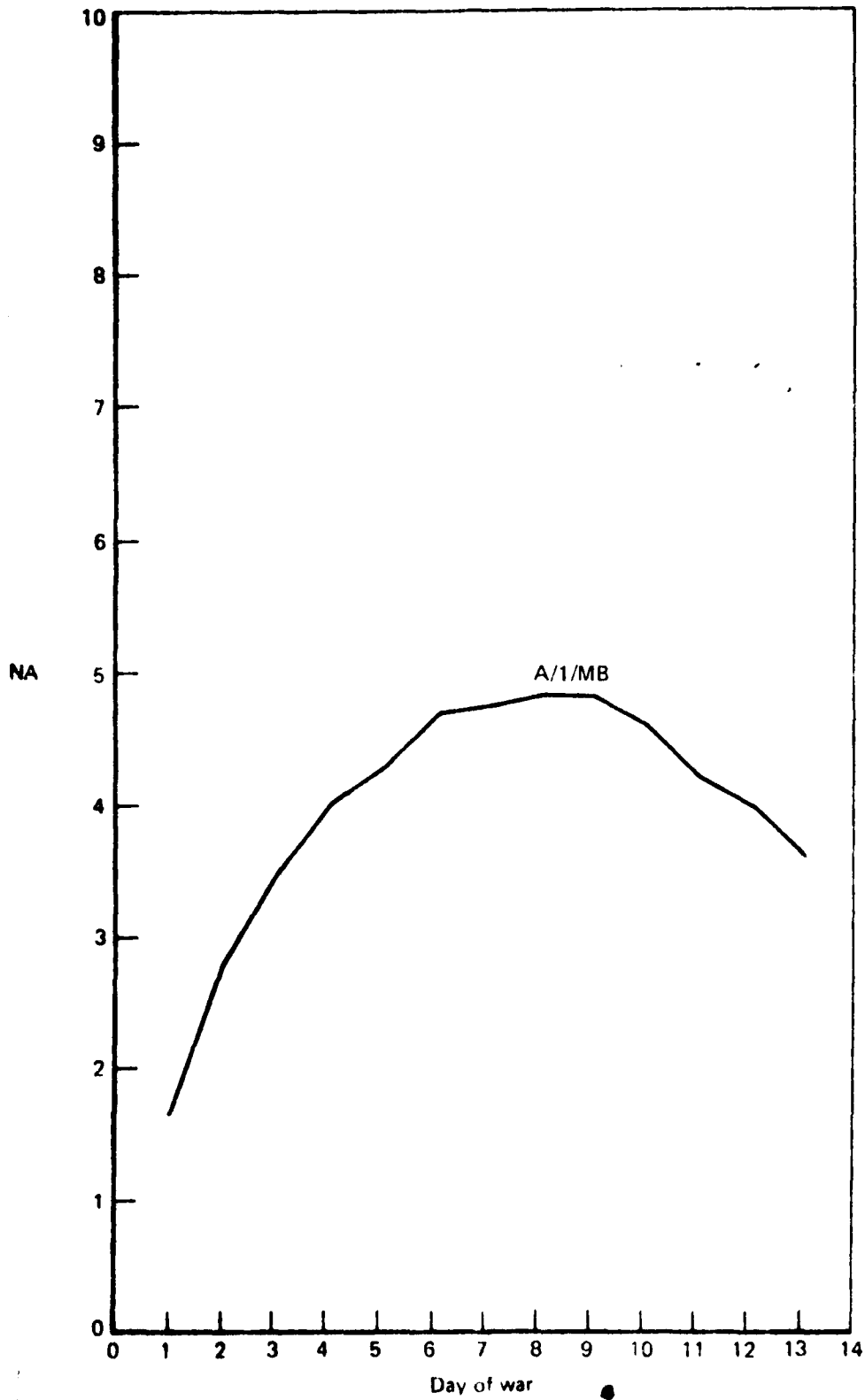
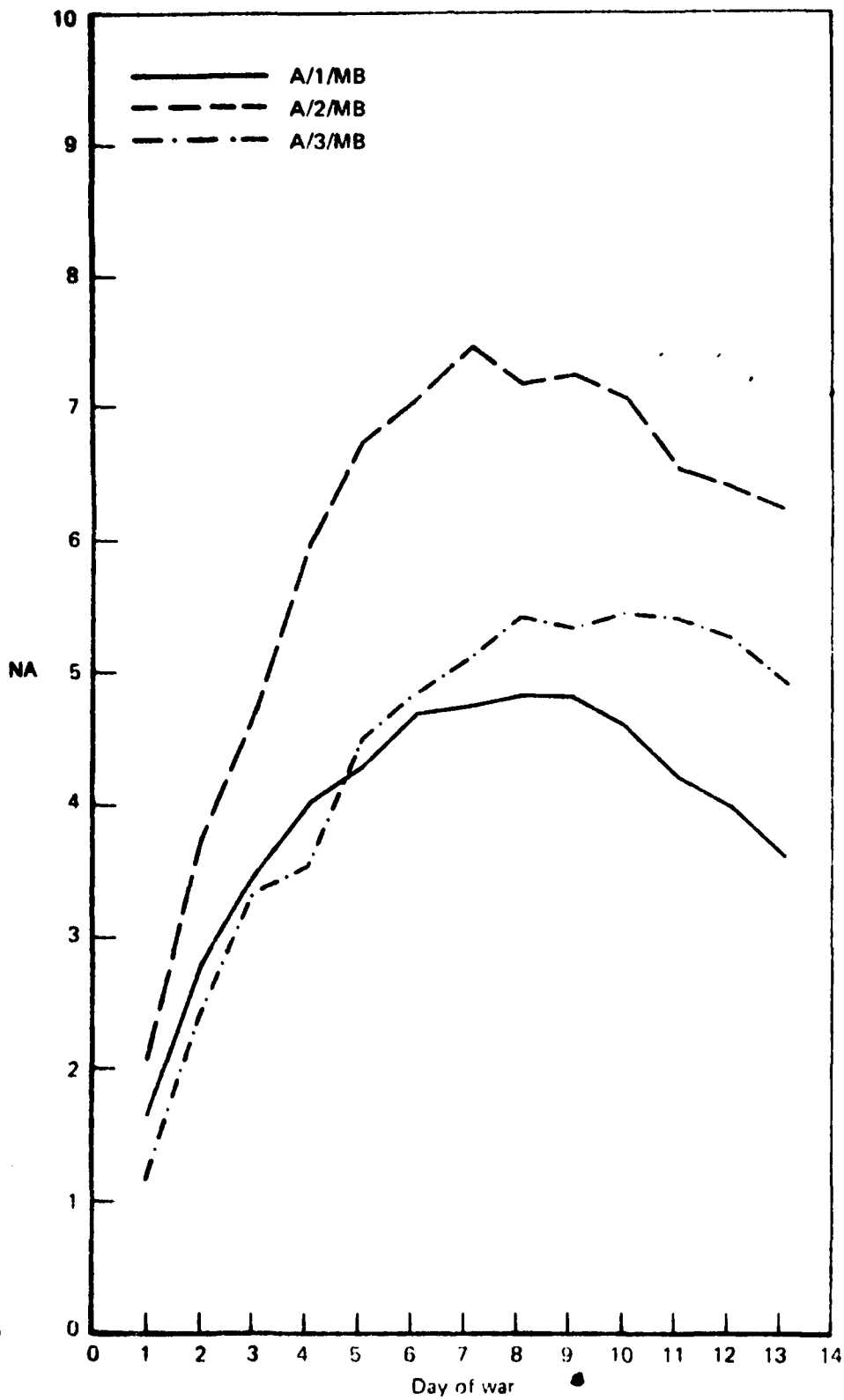**Fig. 4—Cross-training/most likely scenario/short-term decision rule**

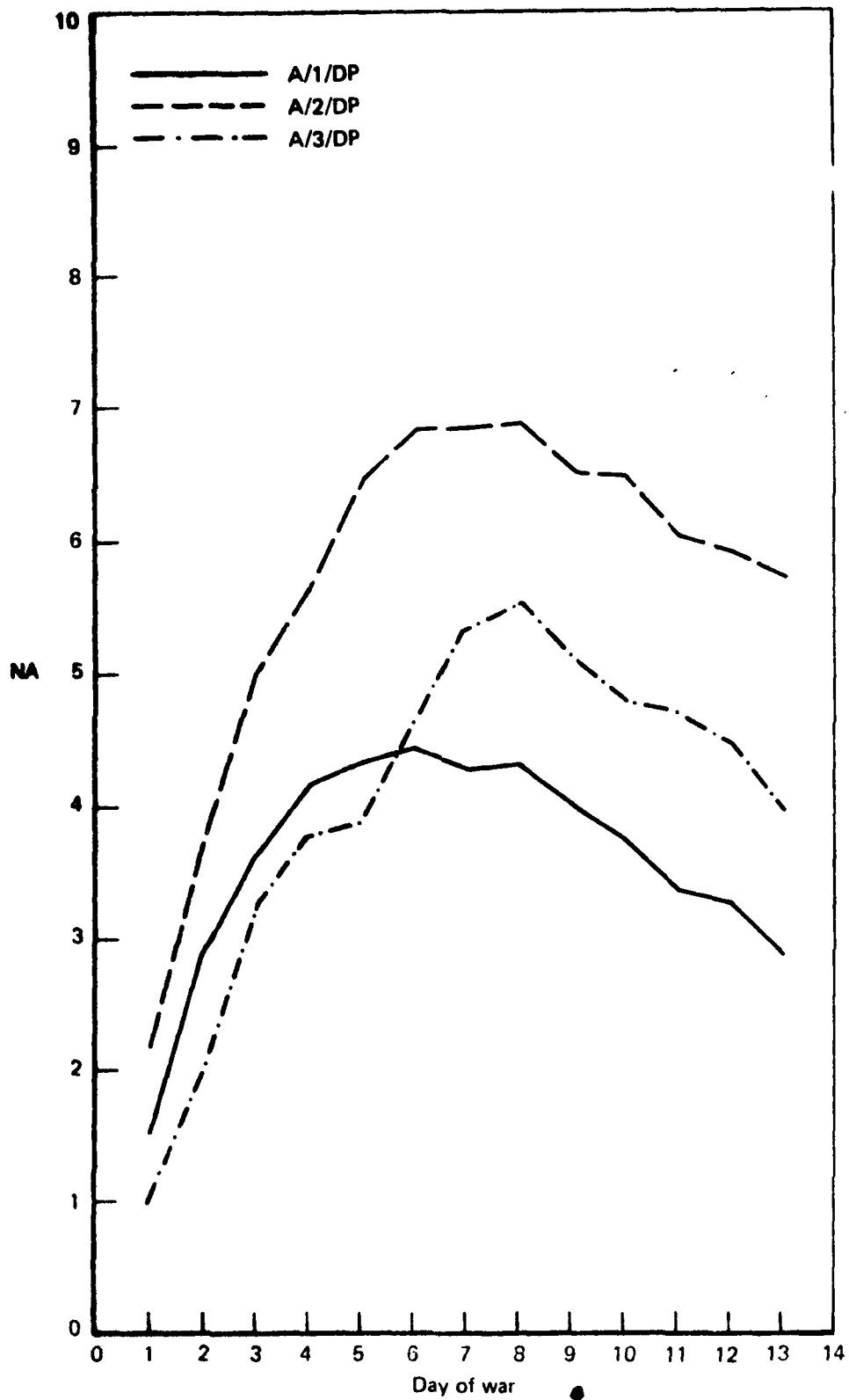Fig. 5—Cross-training/scenarios 1, 2, and 3/short-term
decision rule

Fig. 6—Cross-training/scenarios 1, 2, and 3/long-term decision rule
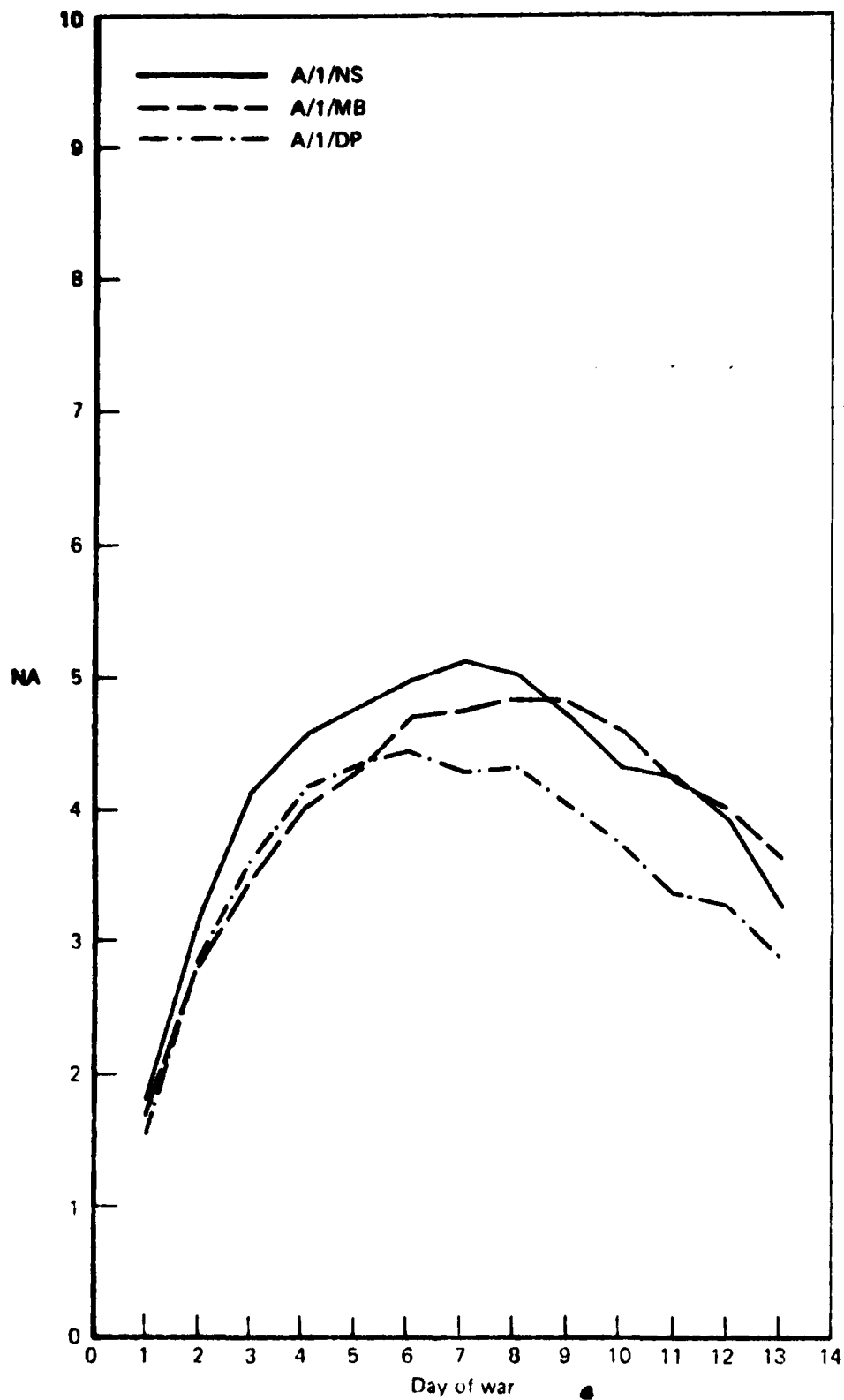
Fig. 7—Manpower structure A/scenario 1/all decision rules

The differences in numbers of NA aircraft among the three cases are small, but in the expected directions. The no substitution case (NS) has a greater number of NA aircraft-days over the thirteen day period (54) than do the short-term (MB) decision case (52) and the long-term (DP) case (47). Both MB and DP rules yielded fewer NA aircraft early in the war, an improvement over the NS rule. Thus, cross-training unambiguously improved the unit's performance in terms of wartime NA aircraft. In addition, the DP rule yielded fewer NA aircraft than the MB rule. That should be no surprise; the DP rule was explicitly designed to minimize the total number of NA aircraft. On the other hand, the MB rule has fewer NA aircraft in the first five days of the war.

When one accounts for the cost of cross-training personnel, scenario 1 seems to indicate little, if any, gain to cross-training. Alternatively, from the requirements modeling viewpoint, there is little benefit from accounting for cross-trained personnel even if such personnel are commonly found in the force. Scenario 2, a modest deviation from scenario 1, alters these conclusions.

Figure 8 is graphic evidence of the importance of looking across scenarios. There is a distinct difference in the pattern of NA aircraft between the NS case and the two cross-training cases under scenario 2. The flexibility afforded by the cross-training reduces the number of NA aircraft both early and late and allows the number of NA aircraft to start declining earlier in the war. Thus, the value of cross-trained personnel increases when the inventory of primary skills does not match up with the demands for those skills. Since it is impossible to perfectly forecast wartime skill demands, it is probable that the inventory of primary skills will never match skill demands.

Cross-trained repairmen fill in for the unexpectedly overloaded type I repairmen under scenario 2. Had the overload been predicted, spare parts could fill in for repairmen. In scenario 3, five spare part Is are added to the unit to offset the increased failure rate and reduced repair rate of part Is. Figure 9 summarizes the scenario 3 results for all decision rules. The spare parts bring the repair resources back into a rough balance with the demands and the marginal value of the cross-training is much less than under scenario 2. Alternatively, the marginal value of the five spare type I parts is much lower in the presence of cross-trained personnel than in the NS case.

The choice of job assignment rule, short-term versus long-term, does not appear to matter very much when contrasted with the choice of modeling or not modeling cross-training. This is a gratifying result. The MB rule is very easy to calculate and does not require predictions of failure rates; hence, it is easy to envision a unit using the rule.
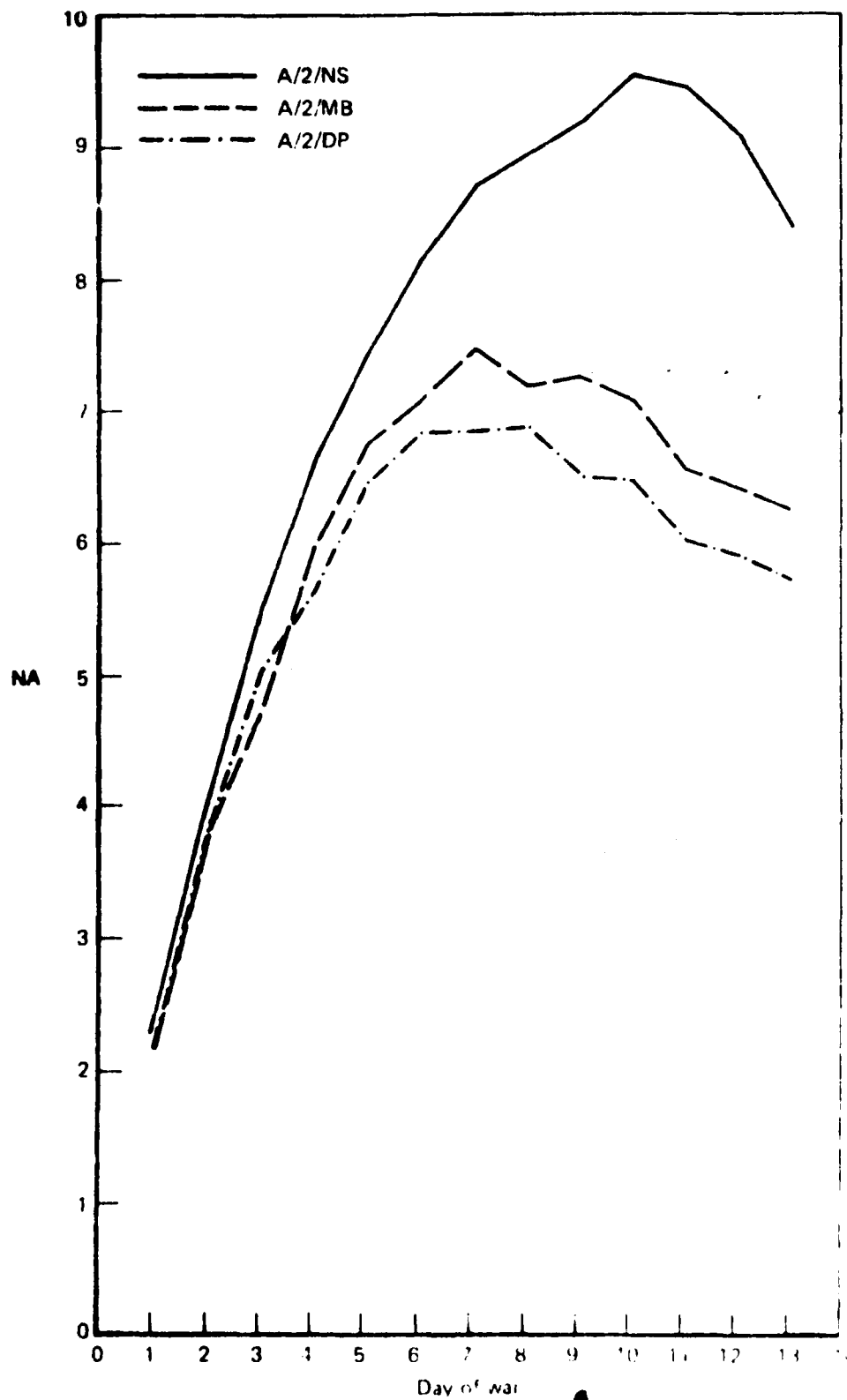
Fig. 8—Manpower structure A/scenario 2/all decision rules
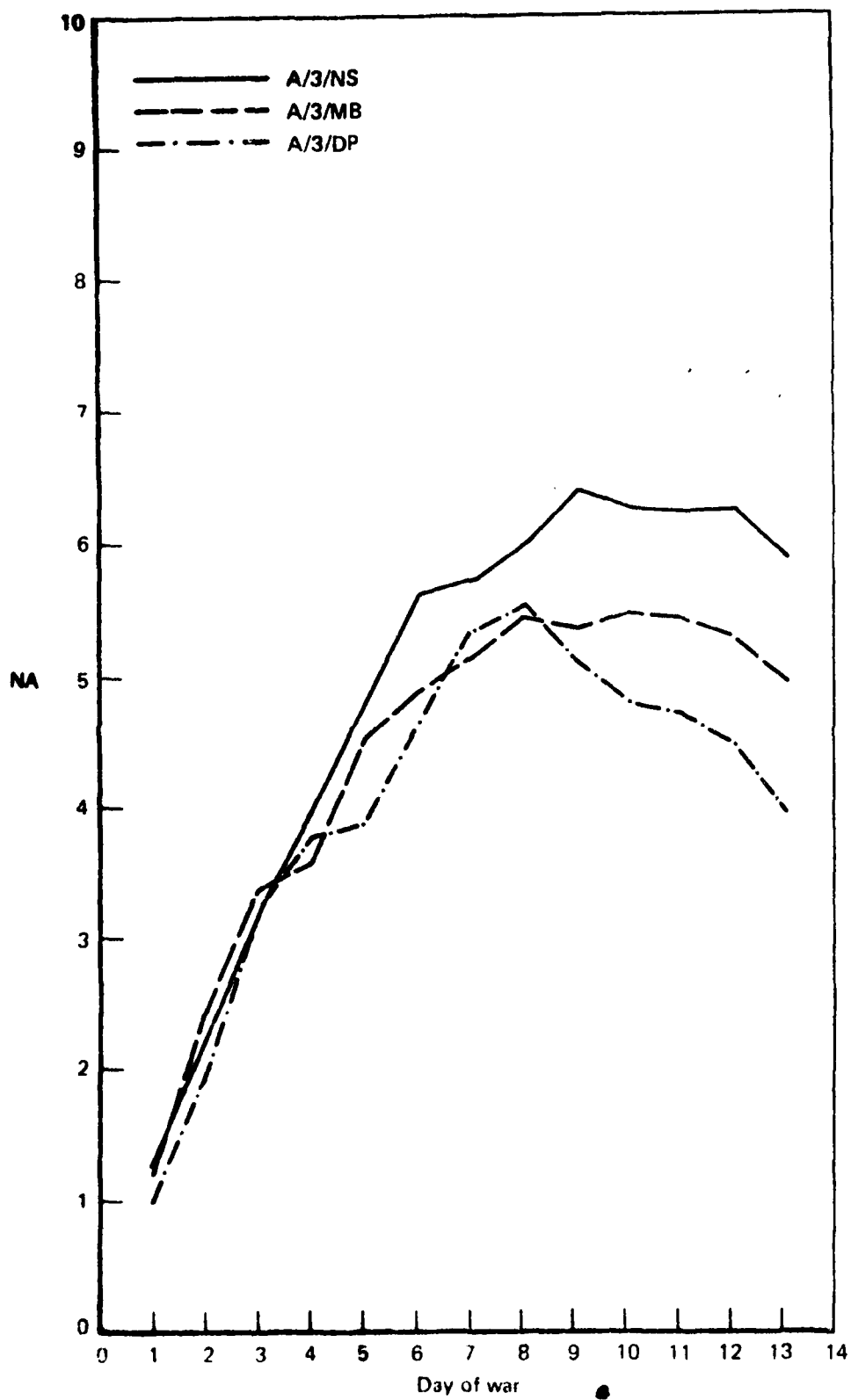
Fig. 9—Manpower structure A/scenario 3/all decision rules

The DP rule is more difficult to calculate and uses the predicted failure rates and repair rates in calculating job assignments. The MB rule depends only on the current state of the system—the number of each type part in repair and awaiting repair—and not on predicted rates. We have not yet examined the sensitivity of the DP rule to inaccurate predictions. We also do not know how much the two rules would diverge in a model with a richer description of the resources available to the unit and the demands facing it.

The outcomes associated with the two rules do not appear to differ significantly partly because both job assignment rules are output oriented and the end of the planning period—resupply on the fifteenth day—is not far off. Thus, the job assignments derived from the two rules differ only some of the time. If we had adopted a rule derived from minimizing average time to repair, the choice of rule might have been shown to be more important than is the case here. However, our results in Sec. III indicated that focusing on average time to repair can lead to inappropriate conclusions. And it is unlikely that a unit would focus on such a measure during a war.

A second reason the choice of job assignment rule does not seem to matter very much is that the flexibility afforded by cross-training is limited by having only two skills and two people in each skill who are cross-trained. If the unit were larger and had a more varied workload, we might see a greater difference in the outcomes under the two decision rules.

V. MULTIPLE SKILL LEVELS

The remaining dimension of manpower that we examine in this study is skill level. Each occupation's inventory is composed of personnel having many skill levels. The exact mix of skill levels within occupations is determined by personnel and compensation policies and external forces. Because the actual skill composition of units must reflect the skill composition of occupations, the ability to relate changes in unit capabilities to changes in available personnel is both useful and important.

The measure of skill level that we use is task time. Hence, the repairmen cross-trained into the "other" occupation in Sec. IV have a low skill level in that other occupation; in this section we add repairmen who have *only* low skill level in their own occupations and are not cross-trained.

We recognize that there are more dimensions of skill level in an occupation than task time. Low-skill-level personnel cannot do some tasks as well as higher skill personnel. There are other tasks that low-skill-level personnel do not know how to do at all. The latter problem is easily modeled by a finer partition of the repair work flowing into the repair station and specification of the skill levels competent for each type of repair. The *quality* of repair problem can be modeled in a variety of ways. For example, one could specify that the probability that a repaired part will fail at first use depends on the skill level of the person repairing it. Getting data on these probabilities would call for extraordinary data collection measures and experiments and still would be problematic. Obtaining failure rates for parts distinguished by symptom and task time estimates by skill level is a much smaller deviation from current data collection.

In the simulations below, we add low-skill personnel and have fewer high-skill personnel compared with manpower structure A. As before, we will refer to a high-skill, cross-trained repairman according to his primary skill. The manpower structure is:

- *Manpower Structure B:* There are one high-skill and two low-skill type *I* repairmen and one high-skill and two low-skill type *II* repairmen.

Our description of scenario 1 must be augmented to account for the repair times of the lower skill repairmen. The part failure

rates and number of spare parts are unchanged from earlier scenario 1 definitions.

Scenario 1:

- A high-skill type *I* repairman averages 0.8 days to repair a part *I* and 1.2 days to repair a part *II*.
- A high-skill type *II* repairman averages 0.8 days to repair a part *II* and 1.2 days to repair a part *I*.
- A low-skill type *I* repairman averages 1.2 days to repair a part *I* and a low-skill type *II* repairman averages 1.2 days to repair a part *II*.
- The daily probability that a part *I* fails on a mission-capable aircraft is 0.042 and the same is true for a part *II*.
- There are no spares of either type part.

Scenario 1 has cross-trained, high-skill personnel. Section IV contrasted the outcomes associated with cross-trained and perfectly specialized personnel in sufficient detail that we do not simulate the no substitution (NS) case in this section.

The scenario 1 average number of non-available (NA) aircraft, counted at the end of each day, is shown in Fig. 10. The job assignment rule is the short-term (MB) rule. Figure 11 graphs the NA aircraft under both the short-term and the long-term (DP) rules.

The numbers of NA aircraft are very close under the MB and DP rules, because there are not as many choices to make under manpower structure B as there are in structure A. In manpower structure A with cross-training, whenever a repairman becomes free he can be assigned either a type *I* or type *II* part to repair. In manpower structure B that choice exists for only two of the six repairmen. The other four can repair only one type of part. Thus, decision rules that differ in their job assignments for cross-trained personnel will yield smaller differences in NA aircraft when there are fewer cross-trained personnel.

The similarity in NA aircraft under the DP and MB rules carries through scenarios 2 and 3. Hence, we present no graphs of the DP rule in these scenarios. Appendix B contains the average NA aircraft for cases B/2/DP and B/3/DP, however.

In scenarios 2 and 3 we again assume that the part *I* failure rate is 25 percent higher than in scenario 1. We increase the average repair time for high-skill type *I* repairmen but do not change average repair times for low-skill and cross-trained repairmen.

Scenarios 2 and 3:

- A high-skill type *I* repairman averages 1.067 days to repair a part *I* and 1.2 days to repair a part *II*.
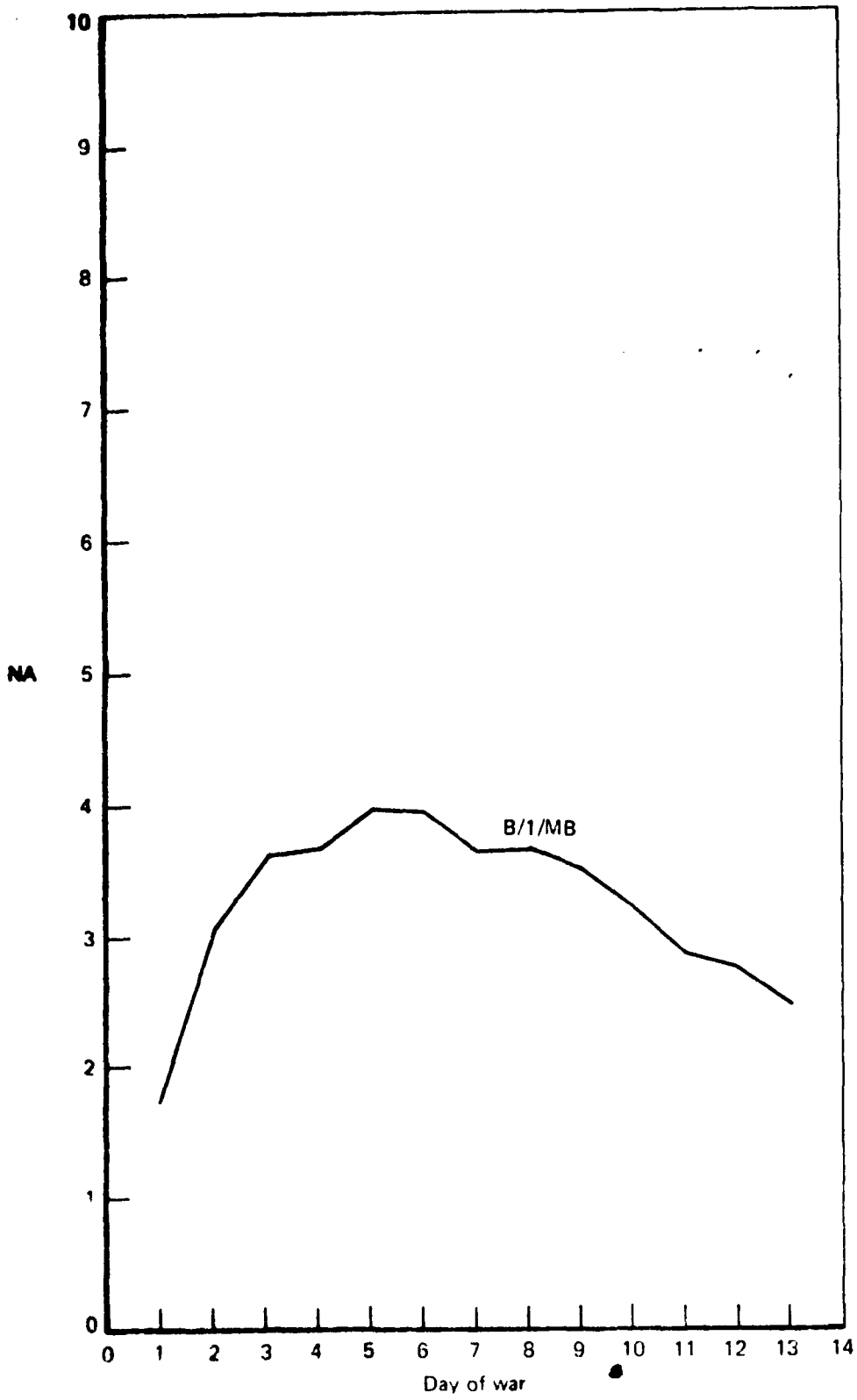
Fig. 10—Manpower structure B/scenario 1/short-term decision rule
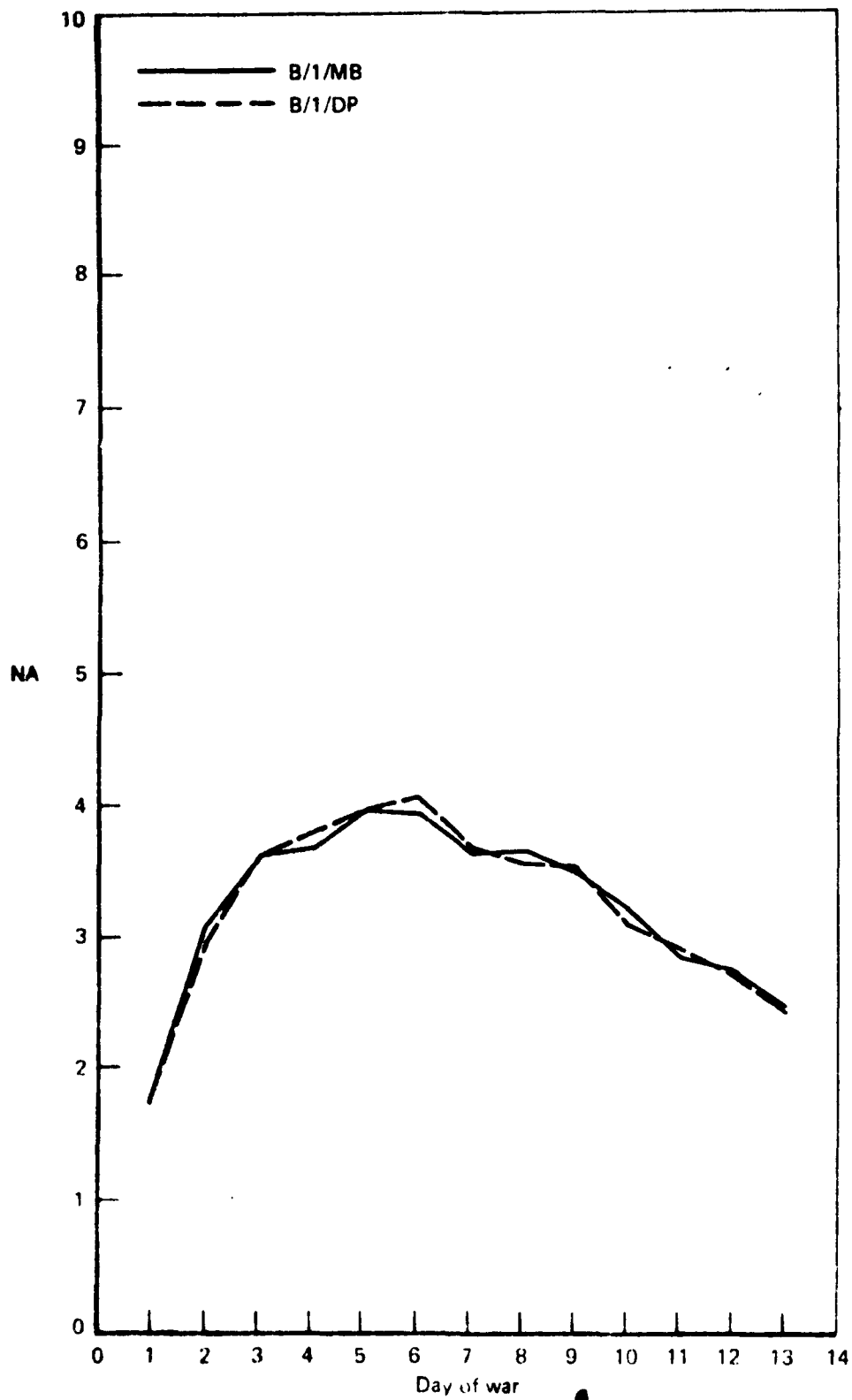
Fig. 11—Manpower structure B/scenario 1/short- and
long-term decision rules

- A high-skill type *II* repairman averages 0.8 days to repair a part *II* and 1.2 days to repair a part *I*.
- A low-skill type *I* repairman averages 1.2 days to repair a part *I* and a low-skill type *II* repairman averages 1.2 days to repair a part *II*.
- The daily probability that a part *I* fails on a mission-capable aircraft is 0.052 and 0.042 for a part *II*.
- *Scenario 2*: There are no spares of either type part.
- *Scenario 3*: There are five spare type *I* parts and no spares of type *II* parts.

The two new cases are labeled B/2/MB and B/3/MB and are shown in Fig. 12 along with case B/1/MB. The story told by Fig. 12 is qualitatively the same as the other comparisons of outcomes under the three scenarios. Increasing the failure rate of part *I*'s and reducing the repair rate (scenario 2) unambiguously makes the number of NA aircraft increase. Augmenting unit capability by adding five spare type *I* parts to offset the higher failure rate reduces NA aircraft.

The manpower structure B cases are not comparable with structure A cases because assumptions about task times under all the scenarios are artificial. We degraded repair rates by a much higher proportion in manpower structure A than in structure B under scenarios 2 and 3, for example. We could have attempted to achieve the same proportional decrease in repair capabilities under the two structures, but that would have been equally artificial without motivating the reduction in repair rates. Any conclusions we could draw from such a comparison would depend on specific numerical assumptions about the repair rates of cross-trained versus low-skill-level repairmen.
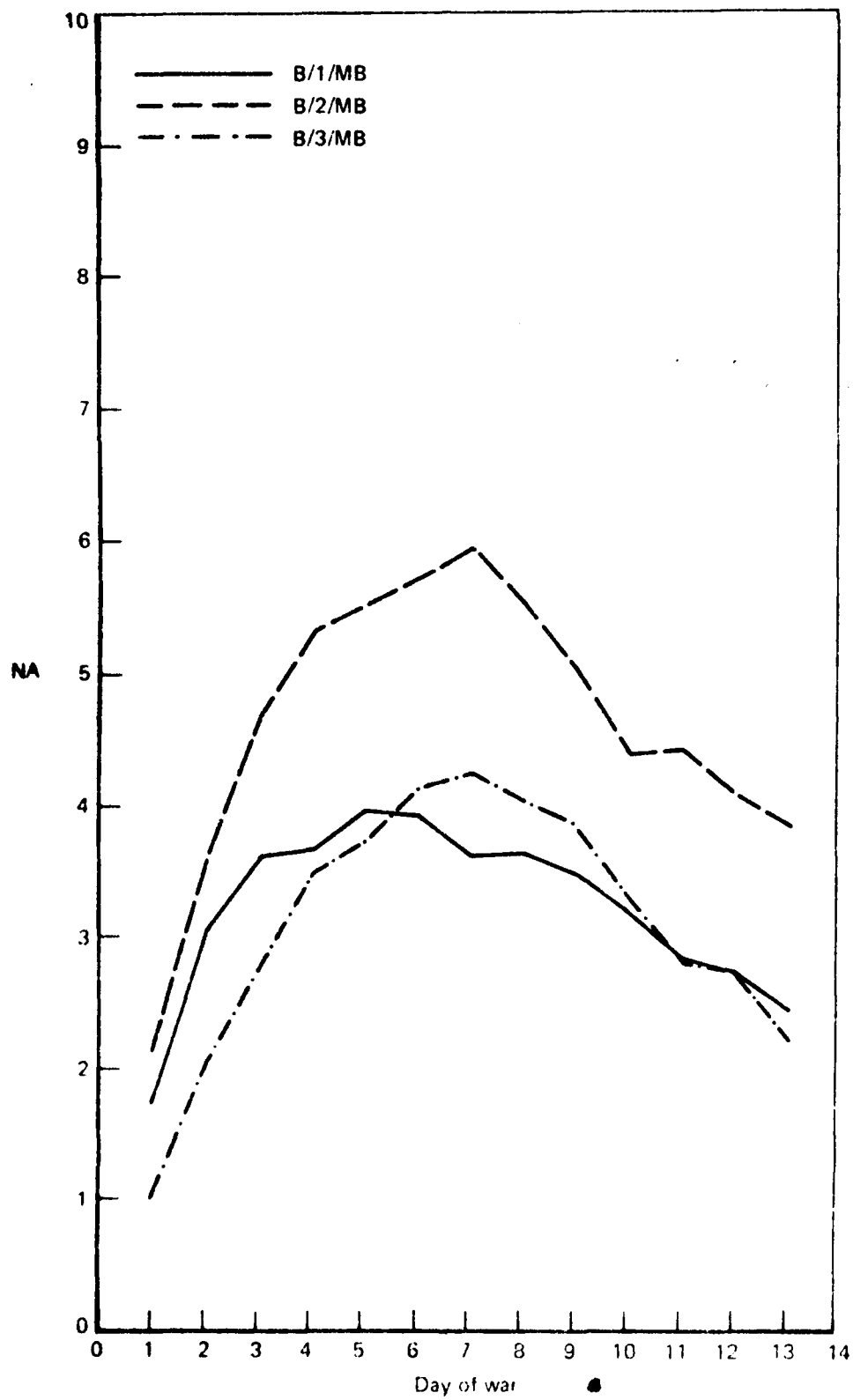
**Fig. 12—Manpower structure B/scenarios 1, 2, and
3/short-term decision rule**

VI. CONCLUSIONS

The simulations presented in the preceding sections demonstrate the feasibility and utility of modeling unit outputs under alternative mixes of maintenance manpower. We describe the mix of manpower by the number of people in each occupation, their skill levels, and their cross-training. Skill level, in turn, was described by task time. We did not attempt to achieve a realistic description of a real unit. Such a description would require a more complete inventory of the resources available to the unit and the demands on the unit and thus a much more comprehensive model.

A more realistic unit description would also require more detailed task time data than are normally available. Data on task times by skill type are routinely gathered, but not by skill level and not for cross-trained personnel. Collection of these additional data would significantly improve our ability to evaluate maintenance manpower's effects on readiness and sustainability of units and to evaluate manning alternatives.

The simulations also demonstrate the importance of focusing on measures directly related to generating wartime sorties. The transition from scenario 2 to scenario 3 results in fewer aircraft not flying due to maintenance and supply—a preferred outcome—but longer average time to repair. Thus, using average time to repair as a performance measure can lead to inappropriate conclusions if it is available aircraft that we care about.

Job assignment rules intended to maximize some measure of output lead to different conclusions about unit performance than other job assignment rules. One job assignment rule is to assign to the free repairman whatever he is best at fixing among those parts needing repair. The simulated output resulting from the use of this rule would be virtually the same as the no substitution (NS) cases because the rule does not take advantage of the flexibility afforded by cross-training. Hence, if units with cross-trained personnel actually use output-oriented rules during a war, performance *predictions* using other rules would underestimate performance. That is, models such as LCOM will underpredict the performance of units and, thereby, overestimate manning requirements on this account.

Wartime weapon system availability depends on the balance between the demands for specific resources and the number of those resources available. When comparing alternative resource re-

quirements, uncertainty about the true wartime demands for resources makes it important to evaluate the contributions of each resource mix in a range of possible scenarios. Even the minor variations in failure rates and repair times examined in this study indicate how one's view of the relative importance of resources can change. Specifically, uncertainty about the scenario puts a premium on flexible assets. Because people can be more flexible than hardware or spares, inattention to scenario uncertainty means that people may be undervalued relative to those other assets. Because formally cross-trained and retrained personnel are more flexible than single-skill specialists, they may be relatively undervalued in requirements determinations unless their value in an uncertain environment is considered.

Because secondary skills will contribute to the functioning of the unit in an uncertain wartime environment, they should be accounted for in readiness measures. Current measures of readiness undervalue flexibility because they do not account for the mix of secondary skills actually possessed by unit personnel. Unit readiness is generally evaluated against a specific set of requirements for primary skills; readiness measures take no account of the flexibility of resources. Indeed, these measures are not output-oriented. However, even counting the inventory of secondary skills and using the count to determine the number of critical jobs that are doubly or triply covered would improve these measures of personnel and unit readiness.

More completely modeling the richness of the maintenance manpower structure would yield benefits in addition to evaluating unit readiness and sustainability. It would become possible to specify the alternative combinations of skill mixes in units that would result in the same performance. Personnel planners would then determine the least costly skill mix satisfying performance requirements rather than satisfying demands for specific mixes of manpower, and programmers would have more flexibility in satisfying manpower requirements.[1]

Explicit modeling of skill mixes and cross-training, such as illustrated in this report would also tie training decisions more closely to output-oriented measures of performance. Simulation of differing mixes of specialization versus cross-training and, more generally, the bundling of skills in people would indicate the value of different training and occupational strategies. The feasibility and cost of such strategies would also need to be determined.

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[1] See Moore (1981) and Armstrong, Chapel, and Moore (1980) for closely related discussions on integrating manpower requirements and personnel management.

Appendix A

TECHNICAL ASSUMPTIONS

The key technical assumptions made in this analysis relate to rates of repair and arrivals of reparables at the repair station. We assume that repair time is exponential once a broken part is in the hands of a repairman. That is, the probability that a type i part will be repaired within t time units by a type j repairman is given by

$$1 - \exp[-t/\lambda(i,j)]$$

where $\lambda(i,j)$ is the mean repair time. In the simulations below we use mean repair times ranging from 0.8 to 1.2 days. The standard deviation of repair times around the mean is also equal to $\lambda(i,j)$ for the exponential probability distribution.[1] The mean and standard deviation are for repair time only; there is also a random delay between arrival of a part at the repair station and commencement of repair on that part.

If a type i part is being repaired by a type i repairman and another by a type j repairman, the probability that at least one of the parts will be repaired within t time units is given by

$$1 - \exp[-t/\lambda(i,i) - t/\lambda(i,j)]$$

The arrival probability of a specific type of reparable at the repair station in a given time interval depends on two factors: the number of mission-capable aircraft and the rate at which reparables on mission-capable aircraft fail. Denote this latter factor as γ. In the simulations below, the value of γ is either 0.042 or 0.052 failures per available aircraft per day. As we noted earlier, the number of mission capable aircraft, denoted MC, is equal to the beginning inventory of aircraft, 72, less the combat attrition losses and the number missing one or both mission-essential parts. We treat the combat attrition losses as a deterministic, continuous process. Thus, MC takes on fractional values. Also, when calculating the probability of an arrival in a given short-time interval, for computational simplicity we assume no addi-

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[1] We examined nonrandom repair times in some of our early simulations. In the limited cases we tried, the results were not very sensitive to the choice of exponential versus deterministic repair times. We believe that other repair time distributions that have different variances and/or different shapes would not change our qualitative conclusions.

tional combat attrition during the interval. This assumption slightly increases the number of arrivals over a day.

Conditional on MC, we assume that the arrival rate of each part is governed by a Poisson process. If the number of mission-capable aircraft at time t is MC_t, the intensity parameter of the process is $\theta_t(i) = \gamma(i)MC_t$. The probability that a broken part i will *not* arrive in the time interval t to $t + \Delta t$ is

$$\exp[-\theta_t(i)\Delta t]$$

Hence, the probability of an arrival in the period t to Δt is

$$1 - \exp[-\theta_t(i)\Delta t]$$

Now, these probabilities are used with random number generators to schedule arrivals of broken type i parts. Suppose that it is the "other" part that is in maximum back-order. If MC changes because of either the repair or the arrival of the "other" part, then a new random arrival time must be scheduled because the intensity parameter $\theta_t(i)$ has changed.

Appendix B

OBJECTIVE FUNCTION AND
DYNAMIC PROGRAM

An available aircraft's value tomorrow relative to today is the source of the difference between the short-term (MB) and long-term (DP) job assignment rules. That is, the objective functions differ only in the assumed value of the daily discount rate. In this appendix, we present an objective function with the daily discount rate as a parameter; we show how setting the parameter value to infinity and zero yields the short-term and long-term objective functions, respectively. We then present the dynamic programming functional equation we use to derive the long-term job assignment rules.

Objective Function

At any time t, the number of aircraft available for flying missions, MC_t, is given by

$$MC_t = 72 - CA_t - MAXBOQ_t \tag{1}$$

where 72 is the initial number of aircraft, CA_t is the cumulative number of combat losses at t, and $MAXBOQ_t$ is the number of aircraft not mission capable due to missing parts (i.e., the maximum back-order quantity). We divide the day into 40 equal intervals and the number of available aircraft is counted at the end of each interval. The per-period discount rate is a, where a is related to the daily discount rate d as follows:

$$(1 + a)^{40} = (1 + d) \tag{2}$$

The per-period discount factor is α, where

$$\alpha = (1 / 1 + a) \tag{3}$$

The objective function evaluated in the current period, t_o, is simply the discounted sum of available aircraft:

$$V_{t_o} - \sum_{t \geq t_o} \alpha^{t-t_o} MC_t$$

$$= 72(T - t_o) - \sum_{t \geq t_o} \alpha^{t-t_o} CA_t - \sum_{t \geq t_o} \alpha^{t-t_o} MAXBOQ_t \qquad (4)$$

T is the number of time intervals over which we count available aircraft. In the text we assumed 15 days before arrival of a more than adequate resupply of spares, which corresponds to 600 periods.

Maintenance policy cannot influence the first two terms in the second line of Eq. (4) by our assumptions, viz., $MC_o = 72$ and CA_t is deterministic. Thus, maximizing V_{t_o} is equivalent to minimizing B_{t_o}, the sum of discounted $MAXBOQ$s.

$$B_{t_o} - \sum_{t \geq t_o} \alpha^{t-t_o} MAXBOQ_t \qquad (5)$$

Setting α equal to one in Eq. (5) is equivalent to setting the daily discount rate a to zero and yields the long-term objective function:

$$BLT_{t_o} = \sum_{t \geq t_o} MAXBOQ_t \qquad (6)$$

The resupply of spares sets $MAXBOQ_t$ to zero for t greater than 600 periods (15 days).

The short-term objective function is given by:

$$BST_t = MAXBOQ_{t+1} \qquad (7)$$

BST_t is derived by first observing that there is nothing that maintenance policy can do to influence the current value of $MAXBOQ$; actions taken in the current period can only influence $MAXBOQ$ in the next and subsequent periods. Thus, the actions that minimize B_{t_o} are the same as the actions that minimize $B_{t_o} - MAXBOQ_{t_o}$. Subtracting $MAXBOQ_{t_o}$ from Eq. (5), dividing the result by α, and then taking the limit as α approaches zero yields the short-term objective function. Letting α approach zero is equivalent to letting a approach infinity.

The Dynamic Program

The best job assignment decision depends on the current state of the system, the future changes in the state of the system associated with the decision, and the discount rate.

The state of the system is described by the number of each type part awaiting repair and the number of each type part in repair by each type of repairman. Movements between states are influenced by the number of spare parts of each type, parts arrival rates and repair rates,

and job assignments. Denote the stocks of spare parts and manpower by:

$S(i)$ = initial stock of spares for type i parts
$N(i)$ = number of type i repairmen

The state variables are defined by:

$q_t(i)$ = number of type i parts awaiting repair at time t
$n_t(i,j)$ = number of type i parts being repaired by type j repairmen at time t

Back-orders for type i parts at time t are defined by:

$$BQ_t(i) = q_t(i) + \sum_j n_t(i,j) - S(i)$$

Although our definition allows back-orders to be negative, the functional equation sets the maximum back-order quantity to the maximum of back-orders of type I parts, back-orders of type II parts, and zero. Thus, negative back-orders do not mean extra aircraft flying.

Repairs, arrivals, and job assignments govern transitions from one state to another. Job assignments are governed by decision rules; arrivals and repairs are random events. The number of type i parts being repaired by type j repairmen at time $t + 1$ is given by:

$$n_{t+1}(i,j) = n_t(i,j) + \Delta n(i,j) - \begin{cases} 0 & \text{if event is not repair of type } i \text{ by repairman type } j \\ 1 & \text{if event is repair of type } i \text{ by repairman type } j \end{cases} \tag{8}$$

$\Delta n(i,j)$ is the job assignment. $\Delta n(i,j)$ is 1 if the decision at $t + 1$ is to assign a type i part to a type j repairman, and 0 otherwise. The number of broken type i parts awaiting repair at time $t + 1$ is:

$$q_{t+1}(i) = q_t(i) - \Delta n(i,1) - \Delta n(i,2) + \begin{cases} 0 & \text{if event is not arrival of type } i \text{ part} \\ 1 & \text{if event is arrival of type } i \text{ part} \end{cases} \tag{9}$$

The number of parts assigned to a type j repairman may not exceed the number of type j repairmen. Thus,

$$n_t(1,j) + n_t(2,j) \leq N(j) \qquad \text{all } j; \text{all } t \qquad (10)$$

A job assignment decision must be made whenever (1) a broken part arrives at the repair station and there is at least one repairman available to begin work on it, or (2) a repairman becomes available for another job and there are broken parts awaiting repair. If a broken part arrives and there are no free repairmen, the part joins the awaiting repair queue. Similarly, if a repairman becomes available but there are no parts awaiting repair, he must wait until a broken part arrives. In manpower structure A (Sec. III), there are individuals in each of the two primary occupations, and each person is cross-trained to a low skill level in the "other" occupation. Thus, one of seven mutually exclusive events will occur in a period:[1]

- *Event* 1: No parts arrive and no repairs are completed.
- *Event* 2: A type *I* part arrives.
- *Event* 3: A type *II* part arrives.
- *Event* 4: A type *I* repairman finishes repairing a type *I* part.
- *Event* 5: A type *I* repairman finishes repairing a type *II* part.
- *Event* 6: A type *II* repairman finishes repairing a type *I* part.
- *Event* 7: A type *II* repairman finishes repairing a type *II* part.

Each of these events has an associated probability derived from the technical assumptions in App. A.

For each possible state of the system and array of event probabilities, the dynamic program selects the job assignment decision that minimizes the expected present value of maximum back-orders in expression (6). Denote the minimum expected discounted value of maximum back-orders as the *optimal return*. There is an optimal return associated with each possible state of the system and time period. The optimal return at time t, given the numbers of parts in the awaiting repair queues and in repair, is given by:

$$B_t\left[q_t(1), q_t(2), n_t(1,1), n_t(1,2), n_t(2,1), n_t(2,2)\right]$$

The optimal return is a probability weighted average of seven *event returns*, each associated with the respective event listed above. Each event return is the sum of the current maximum back-order quantity and the discounted optimal return, both associated with the occurrence of the event. The discounted optimal return is the minimized expected present value of maximum back-order quantities in the next period. The event returns are given by:

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[1] The periods are short enough that the probability of more than one event occurring in a period is negligible.

Event 1 *Return* $= MAX[0, BQ_t(1), BQ_t(2)]$

$\qquad + \alpha\ MIN\ B_{t+1}[q_{t+1}(1), q_{t+1}(2), n_{t+1}(1,1), n_{t+1}(1,2), n_{t+1}(2,1), n_{t+1}(2,2)]$

$\qquad\qquad \Delta n(1,1)$

$\qquad\qquad \Delta n(1,2)$

$\qquad\qquad \Delta n(2,1)$

$\qquad\qquad \Delta n(2,2)$ \hfill (11a)

Event 2 *Return* $= MAX[0, BQ_t(1) + 1, BQ_t(2)]$

$\qquad + \alpha\ MIN\ B_{t+1}[q_{t+1}(1), q_{t+1}(2), n_{t+1}(1,1), n_{t+1}(1,2), n_{t+1}(2,1), n_{t+1}(2,2)]$

$\qquad\qquad \Delta n(1,1)$

$\qquad\qquad \Delta n(1,2)$

$\qquad\qquad \Delta n(2,1)$

$\qquad\qquad \Delta n(2,2)$ \hfill (11b)

$\qquad + \alpha\ MIN\ B_{t+1}[q_{t+1}(1), q_{t+1}(2), n_{t+1}(1,1), n_{t+1}(1,2), n_{t+1}(2,1), n_{t+1}(2,2)]$

$\qquad\qquad \Delta n(1,1)$

$\qquad\qquad \Delta n(1,2)$

$\qquad\qquad \Delta n(2,1)$

$\qquad\qquad \Delta n(2,2)$ \hfill (11c)

Event 4 *Return* $= MAX[0, BQ_t(1) - 1, BQ_t(2)]$

$\qquad + \alpha\ MIN\ B_{t+1}[q_{t+1}(1), q_{t+1}(2), n_{t+1}(1,1), n_{t+1}(1,2), n_{t+1}(2,1), n_{t+1}(2,2)]$

$\qquad\qquad \Delta n(1,1)$

$\qquad\qquad \Delta n(1,2)$

$\qquad\qquad \Delta n(2,1)$

$\qquad\qquad \Delta n(2,2)$ \hfill (11d)

Event 5 *Return* $= MAX[0, BQ_t(1), BQ_t(2) - 1]$

$\qquad + \alpha\ MIN\ B_{t+1}[q_{t+1}(1), q_{t+1}(2), n_{t+1}(1,1), n_{t+1}(1,2), n_{t+1}(2,1), n_{t+1}(2,2)]$

$\qquad\qquad \Delta n(1,1)$

$\qquad\qquad \Delta n(1,2)$

$\qquad\qquad \Delta n(2,1)$

$\qquad\qquad \Delta n(2,2)$ \hfill (11e)

Event 6 Return $= MAX[0, BQ_t(1) - 1, BQ_t(2)]$

$\quad + \alpha\ MIN\ B_{t+1}[q_{t+1}(1), q_{t+1}(2), n_{t+1}(1,1), n_{t+1}(1,2), n_{t+1}(2,1), n_{t+1}(2,2)]$

$\qquad \Delta n(1,1)$

$\qquad \Delta n(1,2)$

$\qquad \Delta n(2,1)$

$\qquad \Delta n(2,2)$ (11f)

Event 7 Return $= MAX[0, BQ_t(1), BQ_t(2) - 1]$

$\quad + \alpha\ MIN\ B_{t+1}[q_{t+1}(1), q_{t+1}(2), n_{t+1}(1,1), n_{t+1}(1,2), n_{t+1}(2,1), n_{t+1}(2,2)]$

$\qquad \Delta n(1,1)$

$\qquad \Delta n(1,2)$

$\qquad \Delta n(2,1)$

$\qquad \Delta n(2,2)$ (11g)

The functional equation for the dynamic program is:

$B_t[q_t(1), q_t(2), n_t(1,1)n_t(1,2), n_t(2,1), n_t(2,2)] =$

$\qquad\qquad$ Pr{*Event* 1} *Event* 1 Return +

$\qquad\qquad$ Pr{*Event* 2} *Event* 2 Return +

$\qquad\qquad$ Pr{*Event* 3} *Event* 3 Return +

$\qquad\qquad$ Pr{*Event* 4} *Event* 4 Return + (12a)

$\qquad\qquad$ Pr{*Event* 5} *Event* 5 Return +

$\qquad\qquad$ Pr{*Event* 6} *Event* 6 Return +

$\qquad\qquad$ Pr{*Event* 7} *Event* 7 Return $\quad t < 601$

and

$B_{601}[\quad] = 0$ (12b)

For ease of exposition, our notation does not show the dependence of the event probabilities on the state of the system. Our actual calculations include this dependence.

The q_{crit} values described in the text are constructed from the optimal $\Delta n(i,j)$ decisions determined by the dynamic program. Every state of the system that has at least one free repairman will have at least one non-zero $\Delta n(i,j)$. We will restrict our attention to cases in

which there is exactly one free repairman.[2] Assume that it is a type 2 repairman who is free. For all states in which it is part type 2 that is in maximum back-order, the repairman is assigned a type 2 part to repair. Thus, q_{crit}s are calculated only when the part type in maximum back-order differs from the part type that the repairman specializes in repairing. Now, assume that it is part type 1 that is in maximum back-order. For each set of values of $q_t(1)$, $n_t(1,1)$, $n_t(1,2)$, $n_t(2,1)$, and $n_t(2,2)$, we find the value of $q_t(2)$, denoted q_{crit}, below which the value of $\Delta n_t(1,2)$ is 1 and above which it is 0, and the value of $\Delta n_t(2,2)$ is 0 below and 1 above.[3] The logic is the same when determining the q_{crit} values when it is a type 1 repairman who is free and it is part type 2 that is in maximum back-order. The logic is also the same when optimizing with manpower structure B, but there are two additional state and job assignment variables corresponding to the low-skill repairmen.

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[2]It can be shown that there will be more than one free repairman only when there are no parts in the awaiting repair queue.

[3]The value of a job assignment variable changes once at most as $q_t(2)$ increases. If $q_{crit} = 0$, then the job assignment variable will take on only one value.

Appendix C

SCENARIO PARAMETERS

Base Case (Sec. III)		A/1		A/2		A/3	
Repairmen	Number	Part *I*	Part *II*	Part *I*	Part *II*	Part *I*	Part *II*
I	2	0.800	—	1.067	—	1.067	—
II	2	—	0.800	—	0.800	—	0.800
Failure rate		0.042	0.042	0.052	0.042	0.052	0.042
Spares		0	0	0	0	5	0

Cross-Training (Sec. IV)		A/1		A/2		A/3	
Repairmen	Number						
I	2	0.800	1.200	1.067	1.200	1.067	1.200
II	2	1.200	0.800	1.200	0.800	1.200	0.800
Failure rate		0.042	0.042	0.052	0.042	0.052	0.042
Spares		0	0	0	0	5	0

Multiple Skill Levels (Sec. V)		B/1		B/2		B/3	
Repairmen	Number						
High skill *I*	1	0.800	1.200	1.067	1.200	1.067	1.200
Low skill *I*	2	1.200	—	1.200	—	1.200	—
High skill *II*	1	1.200	0.800	1.200	0.800	1.200	0.800
Low skill *II*	2	—	1.200	—	1.200	—	1.200
Failure rate		0.042	0.042	0.052	0.042	0.052	0.042
Spares		0	0	0	0	5	0

Appendix D

DAILY AVERAGES, MAXIMA, AND STANDARD DEVIATIONS OF NA AIRCRAFT

	A/1/NS			A/2/NS			A/3/NS		
Day	Mean	Max	Std Dev	Mean	Max	Std Dev	Mean	Max	Std Dev
1	1.81	5	1.23	2.34	7	1.58	1.28	6	1.26
2	3.20	7	1.67	4.09	10	1.84	2.29	7	1.72
3	4.14	9	1.98	5.45	13	2.31	3.21	11	2.26
4	4.55	12	2.26	6.62	15	2.90	3.99	13	2.69
5	4.78	12	2.58	7.49	17	3.09	4.79	13	2.77
6	4.98	13	2.75	8.15	20	3.61	5.61	15	3.07
7	5.13	16	3.20	8.71	20	4.16	5.73	14	3.23
8	5.01	15	3.07	9.05	18	4.21	6.03	15	3.54
9	4.70	14	3.23	9.11	20	4.25	6.41	16	3.88
10	4.31	15	3.08	9.47	23	4.34	6.29	16	3.82
11	4.24	13	2.87	9.37	23	4.32	6.27	19	4.20
12	3.92	13	2.65	9.07	24	4.43	6.29	18	4.08
13	3.26	12	2.49	8.39	22	4.16	5.90	17	3.80

| | A/1/MB | | | A/2/MB | | | A/3/MB | | |
Day	Mean	Max	Std Dev	Mean	Max	Std Dev	Mean	Max	Std Dev
1	1.67	5	1.14	2.18	7	1.43	1.18	5	1.16
2	2.80	8	1.48	3.75	9	1.94	2.41	8	1.82
3	3.47	7	1.45	4.70	11	2.05	3.33	10	2.09
4	4.03	10	1.99	5.97	13	2.11	3.53	10	2.34
5	4.29	11	2.21	6.75	14	2.34	4.48	11	2.41
6	4.70	12	2.59	7.09	14	2.72	4.86	13	2.79
7	4.74	12	2.69	7.48	17	3.04	5.10	15	2.96
8	4.84	12	2.63	7.19	17	3.21	5.41	14	3.17
9	4.83	13	2.60	7.25	16	3.27	5.36	13	3.12
10	4.61	14	2.44	7.08	16	3.42	5.48	14	3.18
11	4.23	15	2.66	6.55	16	3.35	5.41	13	3.04
12	4.01	15	2.61	6.40	15	3.33	5.29	11	2.99
13	3.63	12	2.55	6.24	13	3.23	4.92	13	3.02

| | A/1/DP | | | A/2/DP | | | A/3/DP | | |
Day	Mean	Max	Std Dev	Mean	Max	Std Dev	Mean	Max	Std Dev
1	1.53	5	1.13	2.18	7	1.47	.98	3	.89
2	2.88	9	1.63	3.74	8	1.69	1.99	6	1.45
3	3.61	9	1.76	5.01	11	2.01	3.25	8	1.94
4	4.16	10	2.07	5.66	12	2.35	3.77	9	2.24
5	4.32	12	2.29	6.46	13	2.88	3.88	11	2.51
6	4.43	12	2.30	6.83	15	2.70	4.62	12	2.73
7	4.29	10	2.16	6.84	15	2.93	5.33	14	2.96
8	4.32	12	2.64	6.89	16	3.47	5.52	15	2.73
9	4.01	10	2.29	6.51	14	3.37	5.13	12	2.81
10	3.76	10	2.08	6.49	15	3.44	4.80	12	2.73
11	3.38	10	2.09	6.04	16	3.29	4.73	12	2.70
12	3.28	10	2.08	5.92	16	3.23	4.47	12	2.65
13	2.88	8	1.91	5.71	17	3.21	3.99	11	2.64

Table—continued

	B/1/MB			B/2/MB			B/3/MB		
Day	Mean	Max	Std Dev	Mean	Max	Std Dev	Mean	Max	Std Dev
1	1.75	5	1.16	2.14	7	1.29	1.01	6	1.04
2	3.06	9	1.70	3.63	8	1.71	2.04	8	1.65
3	3.60	9	1.81	4.70	11	2.14	2.77	9	1.80
4	3.68	9	1.94	5.32	11	2.50	3.49	14	2.28
5	3.98	9	1.87	5.52	12	2.49	3.73	13	2.38
6	3.93	10	1.97	5.73	13	2.68	4.16	15	2.92
7	3.64	14	2.02	5.96	15	3.06	4.26	13	2.79
8	3.67	9	1.93	5.50	17	3.02	4.04	14	2.79
9	3.51	11	1.97	5.06	17	3.08	3.89	11	2.74
10	3.21	8	1.76	4.41	15	2.69	3.32	11	2.51
11	2.88	9	1.64	4.44	15	2.81	2.86	13	2.24
12	2.76	7	1.46	4.10	10	2.33	2.74	11	1.95
13	2.47	7	1.62	3.88	12	2.23	2.24	9	1.77

	B/1/DP			B/2/DP			B/3/DP		
Day	Mean	Max	Std Dev	Mean	Max	Std Dev	Mean	Max	Std Dev
1	1.74	5	1.21	2.11	5	1.30	1.16	4	1.07
2	2.96	8	1.56	3.96	10	1.88	2.48	8	1.75
3	3.61	10	1.85	4.49	11	2.06	3.10	8	1.81
4	3.80	9	2.08	5.26	11	2.24	3.52	10	2.00
5	3.99	8	1.91	5.45	12	2.47	3.48	11	2.36
6	4.07	10	2.02	5.64	15	2.81	3.95	11	2.68
7	3.69	14	1.99	5.70	15	2.77	4.06	15	3.08
8	3.58	10	2.02	5.40	15	2.80	3.86	13	2.72
9	3.55	9	1.80	4.78	14	2.47	3.54	12	2.70
10	3.11	9	1.65	4.48	12	2.27	3.46	10	2.33
11	2.92	8	1.65	4.03	12	2.24	3.27	11	2.21
12	2.71	8	1.40	3.62	11	2.16	2.57	9	2.26
13	2.42	7	1.54	3.31	9	2.02	2.23	10	1.89

NOTES:

Mean: The number of non-mission-capable (NA) aircraft at the end of each day averaged over 100 trials. Equal to the average maximum back-order quantity at the end of each day.

Max: The largest number of NA aircraft in the 100 trials.

Std Dev: The standard deviation of the 100 values of NA aircraft.

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