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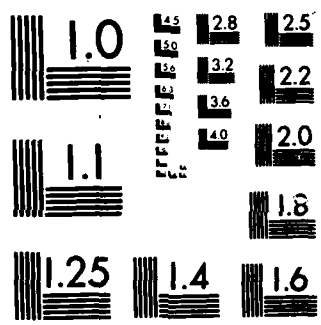

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Small-Signal Gain of a FEL in a Resonator
Gaussian Mode

Ilario Boscolo and Juan Gallardo

Applied Physics B
1984

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ABSTRACT

We present an analytical expression for the small-signal gain of a Free Electron Laser (FEL) in the presence of a gaussian mode. To describe the electron beam evolution we use the one-dimensional (1-d) Vlasov equation. Our perturbative result is valid for small values of the parameter q (length of the undulator L divided by the Rayleigh range z_R).

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I. INTRODUCTION

The 1-d theory of FEL has been thoroughly studied[1] and by now we have a good body of theoretical knowledge to be used in the design of actual experiments. All these papers assumed a plane wave as input field, as differentiated with the experimental situation where we have gaussian modes. This is not a trivial approximation considering that the bunching in the electron beam, and consequently the loss of gain of energy by it, depends critically on the relative phase between the electrons and the electromagnetic wave.

Colson and Elleaume[2] have published a complete treatment of this problem for several magnet designs including the varying phase and amplitude characteristics of a gaussian mode. Also, recently[3] analytical expressions of the gain have been obtained using the single particle theory as described by the Lorentz equations.

In this paper we used the Vlasov equation[4] to find the evolution of the electron beam in the combined fields of a copropagating azimuthal symmetric gaussian mode and the undulator magnet. The radiation field is described by the Maxwell equations in the slow amplitude and phase approximation[5] (S.V.P.A.).

The FEL dynamics is properly represented by the coupled system of self-consistent Vlasov and Maxwell equations. We solve it using perturbation theory in powers of the amplitude of the input field.

II. THEORY

The single particle equations of motion of an electron in the combined wiggler and co-propagating gaussian radiation field are

$$\dot{\vec{\beta}} = \frac{e}{mc\gamma_o} \vec{\beta} \wedge \vec{B}_\omega \tag{1}$$

$$\dot{\gamma} = \frac{e}{mc} \vec{\beta} \cdot \vec{E}$$

so the permanent magnet field $\vec{B}$ determines the electron trajectory and the phase of the gaussian field the energy exchange.

The magnetic field of the permanent magnet is

$$\vec{B} = \vec{e}_2 B_o \sin k_o z \quad , \tag{2}$$

the fundamental gaussian field is

$$\vec{E} = \vec{e}_1 E_o \frac{e^{-r^2/\omega(z)^2}}{\omega(z)/\omega_o} \cos(kz-\omega t+\phi(r,z) \tag{3}$$

where

$$\omega(z) = \omega_o \sqrt{1 + \left(\frac{z-L/2}{Z_R}\right)^2}$$

is the beam waist, ω_o is the spot size at half-way the wiggler; and Z_R is the Rayleigh range; the phase ϕ is

$$\phi(r,z) = -\tan^{-1}\left(\frac{z-L/2}{Z_R}\right) + \frac{kr^2}{2R(z)}$$

and

$$R(z) = (z-L/2)\left(1 + \frac{Z_R^2}{(L-1/2)^2}\right)$$

is the curvature radius. Proceeding a la Colson[5] we introduce the electron phase

$$\zeta = (k+k_o)z - \omega t$$

the dimensionless time

$$\tau = \frac{ct}{L} \qquad (0 \leq \tau \leq 1)$$

and the velocity

$$\nu = \dot{\zeta} = \frac{d\zeta}{d\tau} = L[(k+k_o)\beta - k]$$

We also define the total phase variable as

$$\psi = \zeta + \phi(r,z) \tag{4}$$

In terms of these quantities the single particle equations of motion read

$$\ddot{\psi} = a(z)\cos\psi + \ddot{\phi}$$

$$\mu \equiv \dot{\psi} = \nu + \dot{\phi} \tag{5a}$$

where μ is the generalized velocity and

$$a(z) = \frac{e^2 B_o L^2 E(z)}{(\gamma_o mc^2)^2}$$

is the dimensionless field amplitude. To these we append the Vlasov equation expressed in terms of τ, μ, and ψ variables

$$\frac{\partial\rho}{\partial\tau} + \dot{\psi}\frac{\partial\rho}{\partial\psi} + \dot{\mu}\frac{\partial\rho}{\partial\mu} = 0 \qquad (5b)$$

Now we turn to the Maxwell's wave equation, that governs the evolution of the radiation field in the presence of the electron current described by Eqs. 5a) and 5b).

Employing the S.V.A.P.-approximation and following Colson's procedure,[5,6,7] the resulting wave equation can be written as a pair of equations for the amplitude $a(r,z,t)$ and the phase $\psi(r,z,t)$,

$$\dot{a} = -j \langle\cos(\zeta + \phi)\rangle$$

$$a\dot{\psi} = j \langle\sin(\zeta + \phi)\rangle \qquad (6)$$

The dimensionless current density is

$$j = \frac{e^4 B_o^2 L^3 \lambda_o n_e}{(\gamma_o mc^2)^3}$$

where n_e the particle density in the electron beam. The complete set of equations that describe self-consistently the physics of FELs is given by Eq. 5 and 6. At this point we restrict ourselves to small signal, low gain regime where the amplitude and phase of the optical waves seen by the electrons do not change with time except for the "intrinsic variation", due to the gaussian nature of the radiation field. We wish to point out that it is here where we depart from the plane wave calculation. Therefore,

$$\dot{a}(z) = a(z)\,\frac{(\tau-1)}{Z_R}\,\frac{L^2}{Z_R}\,\beta \Bigg/ \left(1+\left(\frac{z-L/2}{Z_R}\right)^2\right)$$

$$a\dot{\psi}(z) = -a(z)\,\frac{L}{Z_R}\,\beta \Bigg/ \left[\left(1+\frac{z-L}{Z_R}\right)^2\right]^{3/2} \tag{7}$$

where we have neglected the transverse r-dependence of the gaussian field.

III. PERTURBATIVE SOLUTION OF THE VLASOV EQUATION

We seek a perturbative solution of Eq. 5b in powers of the amplitude of the input optical wave a(z). In first order we have

$$\rho(\psi,\mu,\tau) = \rho_o(\mu) + a(\tau)\rho^{(1)}(\psi,\mu,\tau) \tag{8}$$

Since the coefficient of the expansion is not constant, this problem should be more properly treated as a time-dependent perturbation problem. However, we observe that in the small signal, low-gain regime, $a(\tau)$ can be assumed $a(\tau) < 1$ and to be a slowly varying function during the interaction; hence, we are justified to take the average of the amplitude over the interaction length $\langle a(z)\rangle$ as the small parameter in the perturbative expansion. Then Eq. 8 can be rewritten as

$$\rho(\psi,\mu,\tau) = \rho_o(\mu) + \langle a(\tau)\rangle\,\rho^{(1)}(\psi,\mu,\tau) \tag{8'}$$

Substituting into (5b) we find for the perturbative term, the equation

$$\frac{\partial \rho^{(1)}}{\partial \tau} + \dot{\psi} \frac{\partial \rho^{(1)}}{\partial \psi} + \cos \psi \frac{\partial \rho_o}{\partial \mu} = 0 \qquad (9)$$

The solution is

$$\rho^{(1)} = - \frac{\partial \rho_o}{\partial \mu} \int d\psi \frac{d\tau}{d\psi} \cos \psi \qquad (9')$$

Using the second relation in Eq. (5a), we can perform the integral obtaining

$$\rho(\psi,\mu,\tau) = \rho_o - \langle a \rangle \frac{\partial \rho_o}{\partial \mu} \frac{1}{\mu} \left\{ \sin(\psi + \mu\tau) - \sin \psi \right\} \qquad (10)$$

The electron distribution function in first order in $\langle a(z) \rangle$.

IV. SMALL SIGNAL GAIN

By definition the small-signal gain is the relative power variation

$$G = \frac{P_{out} - P_{in}}{P_{in}} = \frac{\int a_o^2 \, dS - \int a_i^2 \, dS}{\int a_i^2 \, dS} \qquad (11)$$

where a_o and a_i are respectively the dimensionless output and input field. To calculate the final field a_o we use Eq. (6) obtaining

$$a_o = a_i - j \int_o^1 d\tau \, \langle \cos \psi \rangle \, . \qquad (12)$$

We assume the initial electron beam to be monochromatic in energy; therefore the zero-order distribution function ρ_o must be a constant, sharp-defined function of the generalized velocity μ along the interaction region. We write the initial electron distribution as,

$$\rho_o(\mu) = \frac{1}{2\pi} \ \delta(\mu - \mu_o) \tag{13}$$

with

$$\mu_o = \nu_o + \langle\dot{\phi}(\tau)\rangle = \nu_o - 2 \tan^{-1} \frac{q}{2} \tag{14}$$

where we have approximated μ_o taking the average value of the time derivative of the gaussian phase $\langle\dot{\phi}\rangle$ over the length of the wiggler. This approximation is consistent with our previous discussion for handling the time-dependent gaussian amplitude. Since we know the electron distribution function, it is straightforward to compute the phase distribution

$$\langle\cos \psi\rangle = \int\int d\psi \ d\mu \ \cos \psi \ \rho(\psi,\mu,\tau) \tag{15}$$

The variation in the field amplitude due to the perturbed part of the electron distribution, is δa; hence we can write

$$a_o^2 \simeq a_i^2 + 2a_i \ \delta a \tag{16}$$

with

$$\delta a = j \frac{\langle a \rangle}{2\mu_o^2} \int_o^1 d\tau \left\{ \mu_o \tau \cos \mu_o \tau - \sin \mu_o \tau \right\} \tag{17}$$

After performing the integration we get

$$\delta a = -\frac{1}{2} j \langle a \rangle \quad (2 - 2 \cos \mu_o - \mu_o \sin \mu_o)/\mu_o^3 \quad = -\frac{1}{2} j \langle a \rangle g(\mu_o) \tag{18}$$

where $g(\mu_o)$ is the usual lineshape function for the gain. We recall that the variation of the field δa is defined over the transverse area Σ_e of the electron beam. Combining (16) with (11), we find for the optical gain

$$G = \frac{2a_i \ \delta a \quad \Sigma_e}{\int dS \ a_i^2} \tag{19}$$

Introducing explicitly the electron current I

$$j \ \Sigma_e = \frac{e^3 B_o^2 L^3 \lambda_o}{(\gamma_o m)^3 c^7} I$$

the averaged amplitude

$$\langle a \rangle = 2 \frac{a_o}{q} \sinh^{-1} \frac{q}{2}$$

and the input flux $\pi\omega_o^2/2 \sim 1/q$. We obtain the final expression for the small signal gain.

$$G = \frac{32\pi^2 e N^2 K^2}{\gamma_o mc^2(1+K^2)} \; I \left[\frac{\sinh^{-1}\frac{q}{2}}{\sqrt{1+\left(\frac{q}{2}\right)^2}}\right] g(\mu_o) \tag{20}$$

We point out the following features in Eq. 20,

a) the functional form of the gain is given by $g(\mu_o)$, the usual lineshape function; the gaussian beam only introduces a shift of the entire curve towards higher values of the detuning parameter ν_o, as shown by Eq. 14.

b) the function in square brackets is a slowly varying function of q (Figure 2) reaching a maximum for $q \sim 3$. This value of q will optimize the design of an FEL resonator. The shift of the gain curve toward higher values of ν_o is approximately linear with q.

We remark that the classical gain formula

$$g(\mu) = -\frac{1}{2}\frac{d}{d\nu}\left(\frac{\sin\left(\frac{\nu}{2} - \tan^{-1}\frac{q}{2}\right)}{\frac{\nu}{2} - \tan^{-1}\frac{q}{2}}\right)^2 \tag{21}$$

in Eq. 20 is the result of our approximation of replacing the amplitude and phase of the radiation field by their averages $\langle a(z)\rangle$ and $\langle\phi(z)\rangle$ over the length of the undulator.

These results coincide with those of references 2 and 3. In addition in those papers it was shown that the gain curve is slightly distorted and that the absorption maximum is larger than the gain maximum.

V. CONCLUSIONS

We have presented an analytical expression for the FEL small-signal gain in the presence of a gaussian mode. The gain curve is shown to be shifted and distorted with respect to the, by now, classical 1-d formula. The maximum gain increases (linearly for $q \approx 0$) with q reaches a maximum and then decreases; $q \simeq 3$ is the optimum value to be used in the design of FEL resonators. The shift of the gain curve is also linear with q and changes substantially its rate of change at $q \simeq 3$. All these results are in fairly good agreement with previous works.

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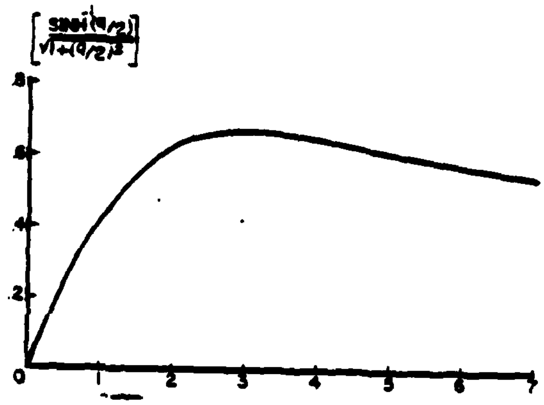
FIGURE CAPTION

Figure 1. $\frac{\sinh^{-1}(q/2)}{\sqrt{1+(q/2)^2}}$ vs. q. As shown in Eq. 20, the maximum gain is proportional to this amplitude factor.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