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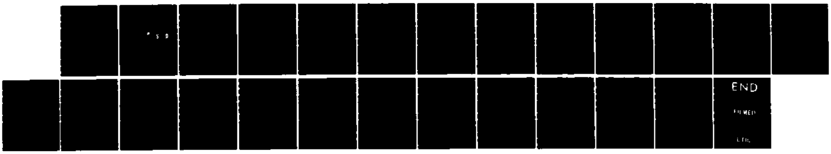

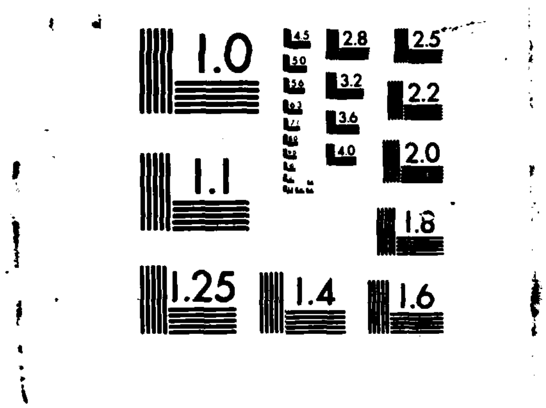

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NAVAL POSTGRADUATE SCHOOL

Monterey, California

PROJECT REPORT

A SEVEN POINT SMOOTHING PROGRAM

FOR NUWES DATA

J. B. TYSVER

JANUARY 1986

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Prepared for:

Naval Undersea Warfare Engineering Station
Keyport, WA 98345

NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA

Rear Admiral R. H. Shumaker
Superintendent

D. A. Schrady
Provost

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This report was prepared for Naval Undersea Warfare Engineering Station by:

J. B. TYSVER
Adjunct Research Professor
Department of Operations Research

Reviewed by:

ALAN R. WASHBURN
Chairman
Department of Operations Research

Released by:

KNEALE T. MARSHALL
Dean of Information
and Policy Sciences

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The BASIC program presented in this report uses the Least-Squares Method to select a polynomial of order 1, 2, or 3 to fit a 7-point segment of NUWES data. It establishes an estimate for the actual value of a coordinate at the midpoint of the data segment and a Figure of Merit to provide an indication of the quality of that estimate.

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I. INTRODUCTION

The proposed BASIC program for smoothing NUWES data uses the Least-Squares Method to fit a polynomial to a 7-point data segment. It determines the best fitting polynomial of order 1, 2, or 3 and establishes an estimate for the coordinate at the time in the center of the data segment. It also establishes a Figure-of-Merit (FM) for that estimate.

This program is designed specifically for treatment of outliers and missing points but it can also be used for smoothing of other observations. It is intended to be used as the basic smoothing program for NUWES data and to provide a value of FM for each observation time for feedback to the Instrumentation Department. Values of the FM's also provide information for other users of the smoothed data on it's quality (i.e., how well the smoothed values can be expected to represent the actual path of a vehicle.).

This report is intended to supplement the Project Report entitled "A Figure of Merit for NUWES Data". (Ref. 7)

II. DATA PREPARATION

It is assumed that the input data has been listed in the following format

Time, Array, x comp, y comp, z comp.

Each component (x,y,z) is smoothed separately and the Array identification is not used in the smoothing process so a separate file can be constructed for each component with the Array column replaced by a column for identification of Questionable Points (QP), i.e., for identification of Missing Points and Outliers. The identification numbers are specified with Array A(6,5) which is presented in the next section of this report. The proposed input format is

Time, QP, XO

where XO is the observed value of x at the corresponding observation time. Two preliminary steps must be taken before the smoothing can be performed. These are:

A. Treatment of Missing Points. Since the program is designed to treat 7-point data segments containing no missing points, the data must be screened and temporary values provided at each missing point. These temporary values are established by taking a linear average of adjacent observational values. The following formulae are used to supply the temporary values:

1. For a single missing value at time t when there are observed values xo(t-1) and xo(t+1) the temporary value is

$$xo(t) = (xo(t-1) + xo(t+1))/2.$$

2. For adjacent missing values xo(t) and xo(t+1) when there are observed values xo(t-1) and xo(t+2) the temporary values are

$$xo(t) = (2*xo(t-1) + xo(t+2))/3,$$
$$xo(t+1) = (xo(t-1) + 2*xo(t+2))/3.$$

3. For three adjacent missing values xo(t-1), xo(t), and xo(t+1) when there are observed values xo(t-2) and xo(t+2) the temporary values are

$$xo(t-1) = (3*xo(t-2) + xo(t+2))/4,$$
$$xo(t) = (xo(t-2) + xo(t+2))/2,$$
$$xo(t+1) = (xo(t-2) + 3*xo(t+2))/4.$$

4. For four adjacent missing values xo(t-1), xo(t), xo(t+1), and xo(t+2) when there are observations xo(t-2) and xo(t+3) the temporary values are

$$xo(t-1) = (4*xo(t-2) + xo(t+3))/5,$$
$$xo(t) = (3*xo(t-2) + 2*xo(t+3))/5,$$
$$xo(t+1) = (2*xo(t-2) + 3*xo(t+3))/5,$$
$$xo(t+2) = (xo(t-2) + 4*xo(t+3))/5.$$

5. When there are more than four adjacent missing values in a 7-point data segment the Least-Squares Method cannot be used to establish an estimate for the midpoint of the segment. (It could still be possible, in some cases, to smooth at other points than the midpoint of the segment. This has not been considered as it would require considerable extension of the smoothing program and its desirability would be questionable.)

B. Identification of Outliers. The procedure for identification of Outliers involves the use of Sequential Differences (Ref.2 and Ref.5). It will not be discussed in detail here. A Fourth Order difference of magnitude 50 or greater is considered an indicator that the corresponding value of xo is a potential outlier. When there are adjacent potential outliers, the one with the greatest magnitude of Fourth Order difference is identified as the actual outlier and assigned the appropriate number in the second column of Array A(6,5). After smoothing this outlier, it might be prudent to recalculate the Sequential Differences with the outlier replaced by its smoothed value xe(t) to give assurance that the adjacent potential outliers are no longer identified as such. Cases where adjacent potential outliers will also be identified as outliers are, hopefully, rare and should be flagged for special treatment.

III. THE SMOOTHING PROGRAM

A. ARRAYS

The program presented in this section was written in the BASIC Language for use on an IBM PC. It uses 6 Arrays which will be discussed in the order that they appear in the program.

The first Array to be used is A(6,5). It is shown below

ARRAY A(6,5)

i\j	0	1	2	3	4	5
	T	QP	XO	XM	XE	RES
0	-3	(0)	xo(-3)	xm(-3)	xe(-3)	r(-3)
1	-2	(-1)	xo(-2)	xm(-2)	xe(-2)	r(-2)
2	-1	(-2)	xo(-1)	xm(-1)	xe(-1)	r(-1)
3	0	(-10)	xo(0)	xm(0)	xe(0)	r(0)
4	+1	(-11)	xo(+1)	xm(+1)	xe(+1)	r(+1)
5	+2	(-12)	xo(+2)	xm(+2)	xe(+2)	r(+2)
6	+3	-	xo(+3)	xm(+3)	xe(+3)	r(+3)

QUESTIONABLE POINTS (QP)

0	xo is legitimate observed value
-1	xo is unscheduled missing point
-2	xo is scheduled missing point
-10	xo is identified as an outlier
-11	xo is an unscheduled missing point and an outlier
-12	xo is a scheduled missing point and an outlier

Smoothing of a 7-point data segment is performed to establish the estimate and FM for the midpoint of the segment. The times in the first column of Array A is obtained by subtracting the time of this midpoint from the observation times of the data segment so that column 1 of this array will always be the same for all data segments. The second column of Array A provides the identification of the QP's. A separate identification for scheduled missing points may be omitted but is included for potential future use. The XO's (column 3) are the input observed values including the temporary values for the missing points. The other columns of this array will be discussed as they appear in the smoothing program.

Array T(5) provides the table values of the Student-T distribution that are needed to establish the Figure-of-Merit (FM) which accompanies the estimate xe at the midpoint of the data segment. As discussed in Reference 7, the number 'infinity' cannot be used by a computer and hence it is arbitrarily replaced by the number 99.99 for T(0). This value is large enough to produce a large value of FM and hence to indicate low quality of the estimate it accompanies. The argument j in T(j) ranges from 0 to 5 and is the appropriate number of 'degrees-of-freedom' produced in Array B(2,3). This will be discussed later.

Array T(5) is shown below.

ARRAY T(5)

j	0	1	2	3	4	5
T(j)	99.99	6.314	2.920	2.353	2.132	2.105

ARRAY B(2,3)

i\j	0	1	2	3
	K	DFK	SDRK	FMK
0	1	DF1	SDR1	FM1
1	2	DF2	SDR2	FM2
2	3	DF3	SDR3	FM3

In this array K is the order of the polynomial being considered (k=1 for linear, k=2 for quadratic, and k=3 for cubic). DFK is the 'degrees of freedom' available for fitting a polynomial of degree k and is established by first determining the number NS (element C(3) in Array C(8)) and then reducing this number by the number of parameters (k+1) in the polynomial. Thus

NS = N - (number of QP's in column A(1,j))
 = C(3) - (number of negative entries in column A(1,j))

with, initially,

C(3) = 7 = number of points in the data segment.

Columns 2 and 3 are

B(i,2) = estimate of the standard deviation of the noise in the observations (the xo's) when a polynomial of order k is fitted to the data segment

and

B(i,3) = the Figure-of-Merit for the estimate xe(0).

ARRAY S(5)

j	0	1	2	3	4	5
S(j)	xav	ssx	s1x	s2x	s3x	s3x1

The element S(0)=xav is the average of the xo's. It is needed to produce the xm's (column 3 in Array A). The other elements of S are used to calculate the values for the SDRK's and the FMK's in Array B. The elements of this array must be zeroed at the start of each smoothing cycle (line 295 of the program) since they contain cumulations of product of the data.

ARRAY D(3)

j	0	1	2	3
D(j)	bk0	bk1	bk2	bk3

This array is for the coefficients of the polynomial with the order k selected to fit the data segment. Its elements must be zeroed before each iteration.

ARRAY C(8)

j	0	1	2	3	4	5	6	7	8
C(j)	T	QP	XO	NS	ITER	K	XE	RES	FM

This array provides the final output of the smoothing program. In addition to the time (C(0)) for which the smoothing was performed and the observed or temporary value of xo (C(2)) at that time, it identifies the nature of the QP (C(1)) and the number of legitimate observations in tahe data segment. It reports the number of iterations of the smoothing process (C(4)) and the order of the polynomial used to fit the data. It contains the smoothed value XE (C(6)) and the accompanying value of FM(C(8)). Finally, it includes the residual error (C(7)) which is the difference between the original value of xo (C(2)) and the estimated value (C(6)). Note that this residual error can differ from the value of the residual error r(0) in cell A(3,5) of Array A when smoothing is iterated. For example, if A(3,1) (and C(1)) has the value -10, then the residual in C(7) is the difference between the outlier value of xo in C(2) and the estimated value xe in C(6) whereas the residual error in A(3,5) is the difference between the estimated value xe(0) in cell A(3,4) after the last iteration and the value of xo(0) in cell A(3,2) which is the value of xe(0) in cell A(3,4) from the preceding iteration.

B. THE PROGRAM

A listing of the BASIC smoothing program follows.

BASIC PROGRAM LS7T3.DOC

```
05 REM LS7T3.DOC        17 Feb.,1985
10 DIM A(6,5), B(2,3), C(8), D(3), D(5), T(5)
20 DATA -3, -2, -1, 0, 1, 2, 3
30 FOR i=0 TO 6
40 READ A(i,0)
50 NEXT i
55 REM STUDENT T
60 DATA 99.99, 6.314, 2.920, 2.353, 2.132, 2.105
70 FOR j=0 TO 5
80 READ T(j)
90 NEXT j
100 C(3) = 7
105 REM ENTER SMOOTHING TIME
110 C(0) = 2160
115 REM ENTER QUESTIONABLE POINTS (QP's)
120 DATA -10, 0, 0, -2, 0, 0, 0
130 FOR i=0 TO 6
140 READ A(i,1)
150 NEXT i
160 C(1) = A(3,1)
165 REM ENTER OBSERVED XO
170 DATA 33238.6,  33267.7,  33313.5,  33374.2, 33434.8, 33510.5,
33592.5
180 FOR i=0 TO 6
190 READ A(i,2)
200 NEXT i
210 C(2) = A(3,2)
215 REM DETERMINE NS
220 FOR i=0 TO 6
230 IF A(i,1)<0 THEN C(3)=C(3)-1 ELSE C(3)=C(3)
240 NEXT i
245 REM CHECK IF SMOOTHING POSSIBLE
250 IF C(3)<3 THEN GO TO 260 ELSE GO TO 290
260 C(4)=0 : C(5)=0  C(6)=C(2)
270 C(7)=0 : C(8)=T(0)
280 GO TO 1050
290 C(4)=0
295 REM START SMOOTHING
300 C(4)=C(4)+1                                         'ITERATION
305 REM ZERO S AND D CELLS
310 FOR j=0 TO 5
320 S(j)=0
330 NEXT j
340 FOR j=0 TO 3
350 D(j)=0
360 NEXT j
363 REM PRELIMINARY CALCULATIONS
366 REM CALCULATE XAV AND XM's
370 FOR i=0 TO 6
```

```
380 S(0)=S(0)+A(i,2)
390 NEXT i
400 S(0)=S(0)/7                                  'XAV
410 FOR i=0 TO 6
420 A(i,3)=A(i,2)-S(0)                           'XM
430 NEXT i
435 REM CALCULATE SUMS OF PRODUCTS
440 FOR i=0 TO 6
450 S(1)=S(1)+A(i,3)*A(i,0)                      'SSX
460 S(2)=S(2)+A(i,3)*A(i,0)                      'S1X
470 S(3)=S(3)+A(i,3)*(A(i,0)^2)                  'S2X
480 S(4)=S(4)+A(i,3)*(A(i,0)^3)                  'S3X
490 NEXT i
500 S(5)=(28*S(4)-196*S(2))/28                   'S3X1
505 REM CALCULATE FM1
510 B(0,1)=C(3)-2                                'DF1
520 SSR1=(28*S(1)-S(2)^2)/28
530 B(0,2)=SQR(SSR1/B(0,1))                      'SDR1
540 B(0,3)=B(0,2)*T(B(0,1))/SQR(C(3))            'FM1
545 REM CHECK QUADRATIC FIT
550 B(1,1)=C(3)-3                                'DF2
560 IF B(1,1)<1 THEN GO TO 690 ELSE GO TO 570
565 REM CALCULATE FM2
570 SSR2=(84*SSR1-S(3)^2)/84
580 B(1,2)=SQR(SSR2/B(1,1))                      'SDR2
590 B(1,3)=B(1,2)*T(B(1,1))/SQR(C(3)             'FM2
595 REM CHECK CUBIC FIT
600 B(2,1)=C(3)-4                                'DF3
610 IF B(2,1)<1 THEN GO TO 620 ELSE GO TO 640
620 B(2,2)=0 : B(2,3)=T(0)
630 GO TO 670
635 REM CALCULATE FM3
640 SSR3=(216*SSR2-S(5)^2)/216
650 B(2,2)=SQR(SSR3/B(2,1))                      'SDR3
660 B(2,3)=B(2,2)*T(B(2,1))/SQR(C(3))            'FM3
665 REM SELECT POLYNOMIAL
670 IF B(2,3)<B(1,3) AND B(2,3)<B(0,3) THEN GO TO 850
    ELSE GO TO 680
680 IF B(1,3)<B(0,3) THEN GO TO 770 ELSE GO TO 690
685 REM FIT LINEAR
690 C(5)=1 : C(8)=B(0,3)                         'k=1 FM=FM1
695 REM ESTABLISH COEFFICIENTS
700 D(1)=S(2)/28                                 'b21
705 REM DETERMINE XE, RES
710 FOR i=0 TO 6
720 A(i,4)=S(0)+D(1)*A(i,0)                      'xe
730 A(i,5)=A(i,2)-A(i,4)                         'res
740 NEXT i
750 C(6)=A(3,4) : C(7)=C(2)-C(6)
760 GO TO 930
765 REM FIT QUADRATIC
770 C(5)=2 : C(8)=B(1,3)                         'k=2 FM=FM2
```

```
775 REM ESTABLISH COEFFICIENTS
780 D(2)=S(3)/84 : D(1)= S(2)/28 : D(0)=-4*D(2)
785 REM DETERMINE XE, RES
790 FOR i=0 TO 6
800 A(i,4)=S(0)+D(0)+D(1)*A(i,0)+D(2)*(A(i,0)^2)      'xe
810 A(i,5)=A(i,2)-A(i,4)                               'res
820 NEXT i
830 C(6)=A(3,4) : C(7)=C(2)-C(6)
840 GO TO 930
845 REM FIT CUBIC
850 C(5)=3 : C(8)=B(2,3)                              'k=3 FM=FM3
855 REM ESTABLISH COEFFICIENTS
860 D(3)=S(5)/216 : D(2)=S(3)/84
870 D(1)=(S(2)-196*D(3))/28 : D(0)=-4*D(2)
875 REM DETERMINE XE RES
880 FOR i=0 TO 6
890  A(i,4)=S(0)+D(0)+D(1)*A(i,0)+D(2)*(A(i,0)^2)+D(3)*(A(i,0)^3)
900 A(i,5)=A(i,2)-A(i,4)                             'res
910 NEXT i
920 C(6)=A(3,4) : C(7)=C(2)-C(6)
925 REM TERMINATE IF NOT QP
930 IF A(3,1)=0 THEN GO TO 1050 ELSE GO TO 940
935 REM CHECK RESIDUALS
940 FOR i=0 TO 6
950 IF A(1,i)<0 AND ABS(A(i,5))>1 THEN GO TO 980 ELSE GO TO 960
960 NEXT i
970 GO TO 1050
980 IF C(4)>9 THEN GO TO 1050 ELSE GO TO 990
990 FOR i=0 TO 6
1000 IF A(i,1)<0 THEN A(i,2)=A(i,4) ELSE A(i,2)=A(i,2)
1010 NEXT i
1020 GO TO 300
1045 REM TERMINATE
1050 FOR j=0 TO 8
1060 LPRINT C(j);
1070 NEXT j
1080 LPRINT
1090 STOP
```

C. DISCUSSION OF THE PROGRAM

The smoothing program can now be discussed in some detail.

STEP 1. Enter the segment data.

Before starting the smoothing the pertinent information on the data segment must be entered. This includes the time (program line 110), the QP's (line 120), and the observed xo's (line 170).

STEP 2. Determine NS.

This is the number of legitimate observations in the segment. (lines220-240)

STEP 3. Check if smoothing possible.

If NS<3, then even linear polynomial cannot be fitted to the data segment by the Least-Squares Method. (lines 250-280) If no fit is possible, then terminate (STEP 15).

STEP 4. Zero iteration counter and cells in Arrays S and D.

S cells cumulate data products and D cells contain parameter coefficients. (lines 290-360)

STEP 5. Calculate xav (S(0)) and xm's (A(i,3).

xav is the average of the xo's and the xm's are the differences xm(j)=xo(j)-xav. This reduces computational round-off error. (lines 370-430)

STEP 6. Calculate other S(j)'s.

These are sums of products of data values. (lines 440-500)

STEP 7. Determine capability of linear fit.

Establish DF1 (B(0,1)), SDR1 (B(0,2)), and FM1 (B(0,3)). (lines 510-540)

STEP 8. Check quadratic fit.

If NS < 4 then DF2 < 1 and no quadratic fit is possible. Otherwise establish SDR2 and FM2. (lines 550-590)

STEP 9. Check cubic fit.

If NS < 5 then DF3 < 1 and no cubic fit is possible. Otherwise establish SDR3 and FM3. (lines 600-660)

STEP 10. Select polynomial for fitting data segment.

If FM3 < FM2 and FM3 < FM1 then select cubic, set k = C(5) = 3 and fit cubic in STEP 13. Otherwise, if FM2 < FM1 then select quadratic, set k = C(5) = 2 and fit quadratic in STEP 12. Finally, if neither of the above then select linear, set k = C(5) = 1 and fit linear in STEP 11. (lines 670-680).

STEP 11. Fit linear polynomial.

Establish linear coefficients (D(i)'s) and calculate xe(i)'s and r(i)'s, also xe(0) in cell C(6) and res in cell C(7). (lines 690-750)

STEP 12. Fit cubic polynomial.

Establish quadratic coefficients and calculate estimates and residuals. (lines 770-830)

STEP 13. Fit cubic polynomial.

Establish cubic coefficients and calculate estimates and residuals. (lines 850-920)

STEP 14. Check QP's.

If xo(0) is legitimate observation (A(3,1) = 0) then terminate (STEP 15). Otherwise check number of iterations (C(4)). If C(4) = 10 then terminate. Finally, check residuals at times where A(i,1)<0 (at missing points and outliers). If any of these are greater than unity in magnitude then repeat the smoothing (STEP 4). If none of them are then terminate. (lines 930-1020)

STEP 15. Terminate.

Print Array C and STOP. (lines 1050-1090)

IV. APPLICATION

The example used to illustrate this smoothing program is the sample selected consists of 40 points from times t=2121 to t=2160. Additional points at both ends of the sample are required in order to smooth at these times. Appropriate inputs for this sample are given in the first four columns of Table 1. It is convenient to have the contents of Figure 1 on the monitor when performing the data smoothing. Inputs can then be changed easily by making the appropriate changes in lines 110, 120, and 170. This is particularly convenient when smoothing values other than outliers or missing values. The computer printout of the results of smoothing this sample are shown in Figure 2 and summarized in the last three columns of Table 1.

It is of some interest to compare the results in Table 1, established using the BASIC Program LS7T3.BAS on an IBM PC, with the results given in Table 1 of Reference 7 which were established using a Texas Instruments calculator (TI-59). The differences can be attributed to the differences in the number of places used in the computations.

Operation of the smoothing program can be examined in more detail by revising the program to version LS7T4.BAS. Application of this revised program to the treatment of the missing point at t=2160 is shown in Figure 3a. This figure presents the elements of the other arrays as well as Array C. Several aspects of the smoothing of this missing point are discussed below.

1. The initial value value for xo(0) at time t=2160 which is entered in A(3,2) is the temporary value xo(0)=(xo(-1)+xo(+1))/2=33374.2. This value also appears in C(2) of the output. The value shown in A(3,2) in the printout of Array A is the value of xe(0) in A(3,4) from the next to last iteration.

2. The initial value of xo(-3) entered in cell A(0,2) is the smoothed value for the outlier at t=2157 which was established when that outlier was smoothed. Again, the value in A(0,2) in the printout of Array A is the value of xe(-3) in A(0,4) from the next to last iteration.

3. The residual error r(0)=0.184 in A(3,5) is the difference between the values of xo(0) in A(3,2) and xe(0) in A(3,4) whereas the residual error in C(7) is the difference between the temporary value of xo(0) initially entered in A(3,2) and in C(2) and the smoothed value xe(0) in A(3,4) and in C(6).

4. In each iteration of the smoothing process the value of xo(0) in A(3,2) was replaced by the estimate xe(0) in A(3,4). The difference r(0) shown in A(3,5) is r(0)=xo(0)-xe(0) which is displayed in Array A as A(3,5)=A(3,2)-A(3,4)=0.2. The residual error shown in C(7) is the difference between the origninal value in A(3,2) which is saved in C(2) and the final estimate xe(0) in C(6) and in A(3,4) so that it is the difference between the initial temporary value and the final smoothed value,

i.e.,

C(7)=C(2)-C(6)=4.8.

5. In the same way, the residual error in A(-3,5) is the difference

A(-3,5)=A(3,2)-A(3,4)=0.1

whereas the fitting error at t=2157 is the difference between the initial value in A(-3,2) which is the previously smoothed value (and also the input value) and the smoothed value of xe(-3) in A(-3,2) so that

RES(-3)=33238.6-33234.5=4.1.

The final estimate xe(-3) in A(-3,4) is consistent with the 7-point data segment centered at t=2160 whereas the initial value xo(-3) is consistent with the 7-point data segment centered at t=2157. This brings up the question of whether previously smoothed values should be iteratively smoothed when smoothing other points. This warrants further investigation.

TREATMENT OF MISSING POINT AT T = 2160

INPUT OP'S

-10 0 0 -2 0 0 0

INPUT XO'S

33238.6 33267.7 33313.5 33374.2 33434.8 33510.5 33592.5

ARRAY C (OUTPUT)

2160 -2 33374.2 5 3 3 33369.4 4.804688 1.941253

ARRAY A

-3	-10	33234.56	-154.457	33234.5	.0625
-2	0	33267.7	-121.3203	33267.94	-.2421875
-1	0	33313.5	-75.51953	33313.25	.25
0	-2	33369.58	-19.44141	33369.4	.1835938
1	0	33434.8	45.78125	33435.35	-.5507813
2	0	33510.5	121.4805	33510.1	.4023438
3	0	33592.5	203.4805	33592.6	-.1015625

ARRAY B

0	3	26.00435	27.36421
0	2	1.84462	2.408822
0	1	.687484	1.941253

ARRAY S

33389.02 102914.5 1680.715 412.1133 11728.02 -36.98438

ARRAY D

-19.62444 61.2241 4.906111 -.171224

TABLE 1
Smoothing Sample NWS2AX1

T	COLL	XO	QP	XE	RES	FM
2117	3	33533.2	0	-	-	-
2118	3	33567.7	0	-	-	-
2119	3	33603.6	0	-	-	-
2120	m	(33634.1)	-2	33634.9	(-0.8)	1.51
2121	3	33664.4	0	33665.3	-0.7	1.53
2122	3	33695.0	0	33693.5	1.5	1.41
2123	3	33718.5	0	33720.9	-2.4	2.41
2124	3	33745.8	0	33744.6	1.2	1.64
2125	3	33767.1	0	33764.6	2.6	3.02
2126	3	33780.7	0	33783.7	-3.0	3.13
2127	3	33798.0	0	33799.7	-1.7	3.47
2128	m	(33810.9)	-2	33816.3	(-5.4)	5.17
2129	3	33823.7	0	33828.1	-4.4	4.73
2130	3	33827.3	0	33821.6	5.7	8.35
2131	3	33794.2	0	33786.6	7.6	11.23
2132	3	33726.1	0	33724.1	2.1	2.46
2133	3	33637.7	0	33640.1	-2.4	2.62
2134	12	33556.5	0	33556.9	-0.4	5.07
2135	12	33486.6	0	33490.0	-3.4	3.45
2136	m	(33466.5)	-12	33452.3	(14.2)	4.61
2137	3	33446.5	0	33451.2	-4.7	5.16
2138	3	33485.1	0	33489.9	-4.8	8.82
2139	3	33559.3	0	33560.9	-1.6	3.44
2140	3	33650.2	0	33649.8	0.4	1.67
2141	3	33738.5	0	33734.5	4.0	5.50
2142	3	33799.0	0	33795.6	3.4	7.38
2143	3	33825.8	0	33823.1	2.7	2.48
2144	m	(33798.7)	-12	33813.0	(-14.3)	2.93
2145	3	33771.6	0	33767.4	4.2	6.62
2146	3	33698.3	0	33696.5	1.8	5.11
2147	3	33607.7	0	33614.6	-6.9	9.35
2148	12	33528.5	0	33530.0	-1.3	3.59
2149	12	33455.2	0	33451.7	3.5	3.84
2150	3	33381.2	0	33383.4	-2.2	2.74
2151	3	33323.5	0	33325.2	-1.7	3.27
2152	m	(33285.1)	-2	33277.9	(7.2)	2.36
2153	3	33246.6	0	33243.5	3.1	3.02
2154	3	33219.5	0	33222.1	-2.6	4.60
2155	3	33212.5	0	33214.0	-1.5	4.64
2156	3	33221.0	0	33218.8	2.2	2.63
2157	3	33273.5	-10	33238.6	34.9	2.54
2158	3	33267.7	0	33269.7	-2.0	2.51
2159	3	33313.5	0	33313.1	0.4	3.27
2160	m	(33374.2)	-2	33369.4	(4.8)	1.94
2161	3	33434.8	0	-	-	-
2162	3	33510.5	0	-	-	-
2163	3	33592.5	0	-	-	-

FIGURE 1
BASIC SCREEN FOR SMOOTHING DATA

```
LPRINT                          "SAMPLE NWS2AX1"

LPRINT                          "OUTLIERS"

LIST 110
110 C(0)=2160
0

LIST 120
120 DATA -10, 0, 0, -2, 0, 0, 0
0
LIST 170
170 DATA 33238.6, 33267.7, 33313.5, 33374.2, 33434.8, 33510.5, 33592.5
0

RUN

LPRINT  "MISSING POINTS"

LPRINT  "OTHER POINTS"
```

TREATMENT OF SAMPLE NWS2A1X USING LS7T3.BAS

OUTLIERS								
2136	-12	33466.5	6	4	3	33452.28	14.22266	4.610405
2144	-12	33789.7	6	4	3	33812.96	-14.25781	2.9266.79
2157	-10	33273.5	5	4	2	33238.63	34.875	2.541182
MISSING POINTS								
2120	-2	33634.1	6	1	2	33634.88	-.7734375	1.475461
2128	-2	33810.0	6	3	3	33816.34	-5.4375	5.16895
2152	-2	33285.1	6	3	2	33277.93	7.175782	2.356863
2160	-2	33374.2	5	3	3	33369.4	4.804688	1.941253
OTHER POINTS								
2121	0	33664.6	6	1	2	33665.28	-.6757813	1.526676
2122	0	33695	6	1	2	33693.54	1.460938	1.408332
2123	0	33718.5	6	1	2	33720.87	-2.371094	2.412839
2124	0	33745.8	7	1	2	33744.58	1.226563	1.636923
2125	0	33767.1	6	1	2	33764.55	2.550781	3.020508
2126	0	33780.7	6	1	2	33783.69	-2.988281	3.133061
2127	0	33798	6	1	2	33799.72	-1.722656	3.467283
2129	0	33823.7	6	1	3	33828.12	-4.421875	3.467283
2130	0	33827.3	6	1	3	33821.64	5.664063	8.346241
2131	0	33794.2	6	1	2	33786.6	7.605469	11.22599
2132	0	33726.1	7	1	3	33724.05	2.050781	2.459811
2133	0	33637.7	6	1	3	33640.13	-2.433594	2.61768
2134	0	33556.5	6	1	3	33556.91	-.40625	5.074126
2135	0	33486.6	6	1	3	33489.96	-3.359375	3.445967
2137	0	33446.5	6	1	2	33451.15	-4.652344	5.162057
2138	0	33485.1	6	1	3	33489.95	-4.792969	8.815254
2139	0	33559.3	6	1	3	33560.92	-1.617188	3.443657
2140	0	33650.2	7	1	3	33649.8	.3984375	1.671507
2141	0	33738.5	6	1	3	33734.47	4.0312575	5.498794
2142	0	33799	6	1	3	33795.56	3.445313	7.382694
2143	0	33825.8	6	1	2	33823.13	2.671875	2.47908
2145	0	33771.6	6	1	3	33767.36	4.242188	6.616293
2146	0	33698.3	6	1	3	33696.5	1.800781	5.109362
2147	0	33607.7	6	1	3	33614.61	-6.910157	9.347524
2148	0	33528.5	7	1	3	33529.77	-1.269531	3.590484
2149	0	33455.2	6	1	2	33451.71	3.492188	3.838284
2150	0	33381.2	6	1	3	33383.39	-2.1875	2.737984
2151	0	33323.5	6	1	2	33325.17	-1.664063	3.271418
2153	0	33246.6	6	1	2	33243.54	3.066406	3.016105
2154	0	33219.5	5	1	2	33222.07	-2.570313	4.597027
2155	0	33212.5	5	1	2	33214.04	-1.542969	4.636977
2156	0	33221	6	1	2	33218.8	2.199219	2.634624
2158	0	33267.7	5	1	2	33269.67	-1.964844	2.507949
2159	0	33313.5	5	1	3	33313.05	.4492188	3.274325

V. CONCLUSIONS AND RECOMMENDATIONS

The BASIC program LS7T3.BAS presented in this report uses the Least-Squares Method to select a polynomial of order 1, 2, or 3 to fit 7-point segments of data. It establishes an estimate for the actual value at the midpoint of the data segment and a Figure of Merit (FM) to provide an indication of the quality of that estimate.

A modified version of this program (LS7T4.BAS) is available for a more detailed examination of the application of the program to any 7-point data segment.

It is recommended that this program be used for smoothing NUWES 3-D data. It is also recommended that this program be transferred to a spreadsheet format for use with LOTUS-123 with the intent of using macros to automate its application to the extent possible.

Possible modification of this program to change the treatment of multiple questionable points in a data segment needs further examination as suggested in Section IV.

APPENDIX A

FLOW CHART: LS7T3 BASIC PROGRAM

FIGURE 1

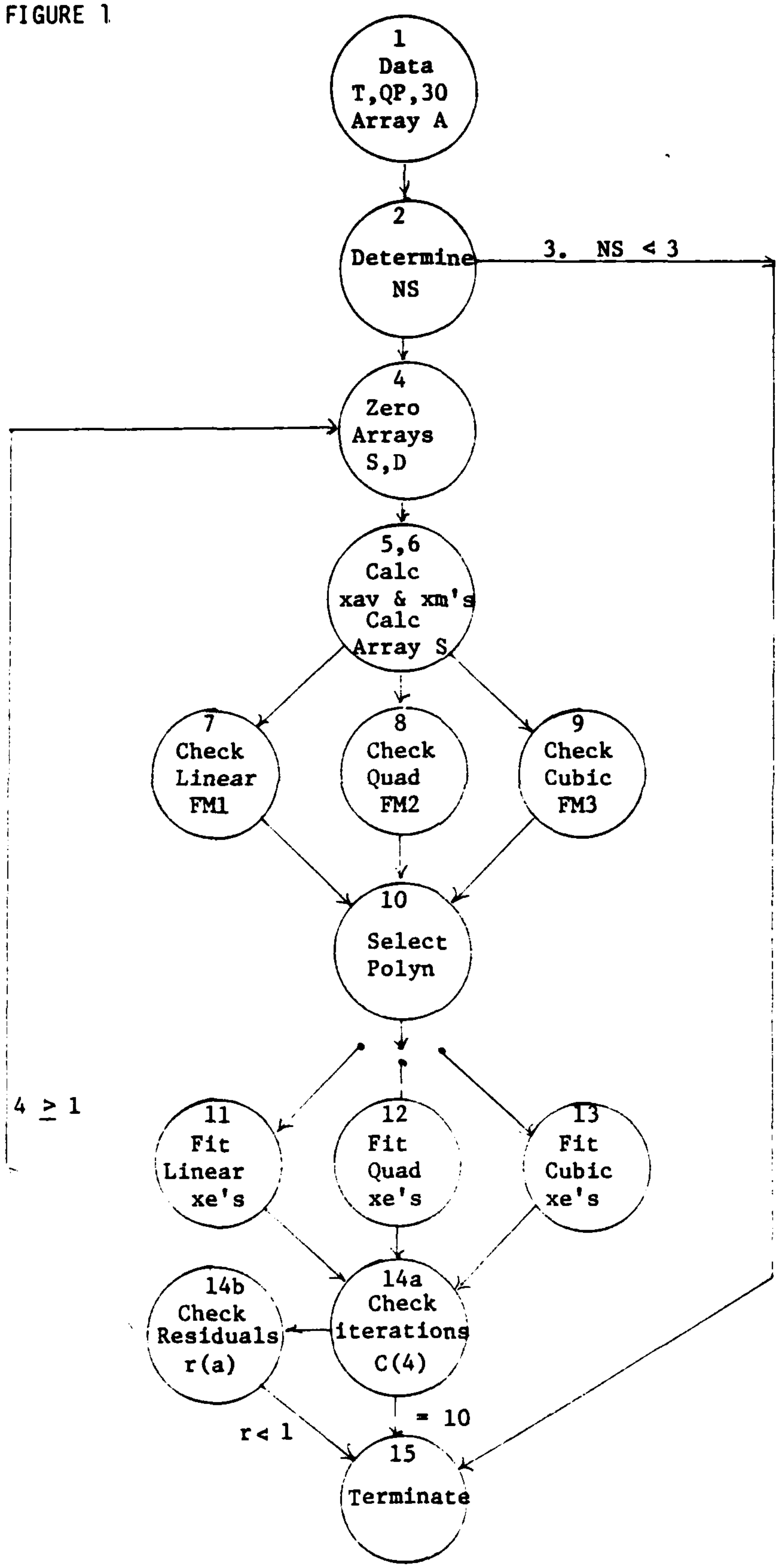
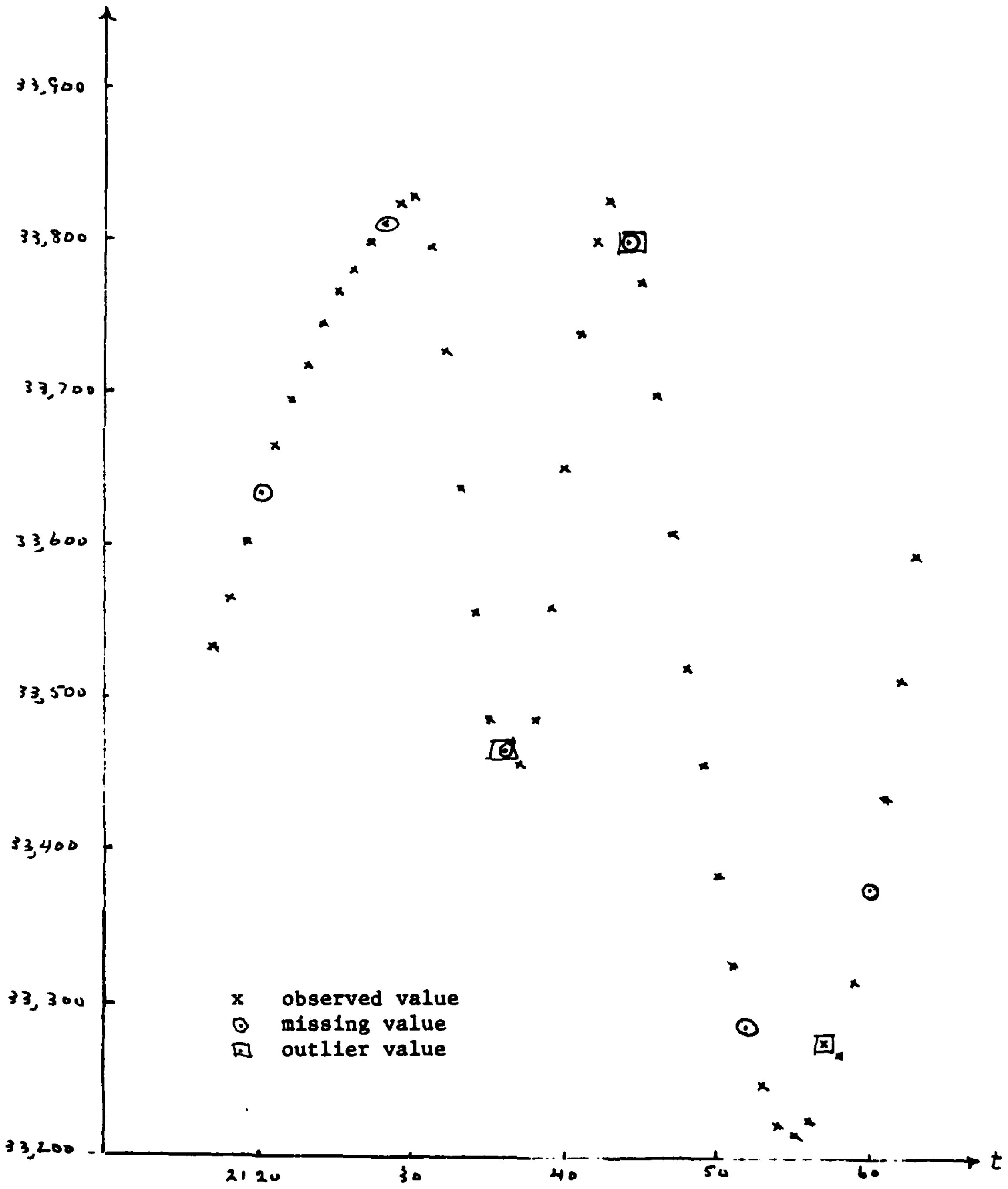


FIGURE 2

SAMPLE NWS2AX1



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