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MICROCOPY RESOLUTION TEST CHART

NATIONAL BUREAU OF STANDARDS-1963-A

(2)

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION	1b. RESTRICTIVE MARKINGS
UNCLASSIFIED	

2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT
	Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	

4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S)
	AFOSR-TR- 85-1205
	AFOSR-83-0343

6a. NAME OF PERFORMING ORGANIZATION	6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION
Purdue University		AFOSR/NE

6c. ADDRESS (City, State and ZIP Code)	7b. ADDRESS (City, State and ZIP Code)
Hovde Hall Lafayette, IN 47907	Building 410 Bolling AFB Washington, DC 20332-6448

8a. NAME OF FUNDING/SPONSORING ORGANIZATION	8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
AFOSR	NE	AFOSR-83-0343

8c. ADDRESS (City, State and ZIP Code)	10. SOURCE OF FUNDING NOS.			
	PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT NO.
Same as 7b	61102F	2305	B2	

11. TITLE (Include Security Classification)
Nonlinear Nonreciprocity:"Theory, And Applications to the Enhancement of the

12. PERSONAL AUTHOR(S)
Sagnac Effect, Optical Bistability"   Kaplan

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
FINAL	FROM 01-Sep- TO 31-Oct 84	26 Nov 85	

16. SUPPLEMENTARY NOTATION

7.	COSATI CODES		18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR.	
			Nonlinear. Sagnac Effect. Optical Bistability

19. ABSTRACT (Continue on reverse if necessary and identify by block number)
Several novel implementations of optical bistability have been proposed and analyzed. Novel applications of optical nonlinearities in a ring have been proposed; particularly interesting is the use of nonlinear nonreciprocity of counterpropagating waves in a ring to enhance the Sagnac Effect, and, thereby, increase the sensitivity of a passive ring optical gyro. The use of moderate energy electron beams traversing superlattices to generate coherent x-rays by the coherent addition of transition radiation has also been proposed and analyzed.

DTIC FILE COPY

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT	21. ABSTRACT SECURITY CLASSIFICATION
UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☒ DTIC USERS ☐	UNCLASSIFIED

22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE NUMBER (Include Area Code)	22c. OFFICE SYMBOL
Dr Howard Schlossberg	202-767-4931	NE

DD FORM 1473, 83 APR   EDITION OF 1 JAN 73 IS OBSOLETE.

AD-A164 374

SELECTED FEB 14 1986 D

Bistable Solitons

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Abstract

It is demonstrated that a nonlinear Schrödinger equation with certain nonlinearities allows for an existence of multi-stable singular solitons (i.e., singular solitons with the same carried power but different propagation parameters). In nonlinear optics, these solitons may exist either in the form of short bistable pulses, or bistable self-trapping (both two- and three-dimensional).

PACS numbers: 42.65.Bp, 03.40.Kf.

In this letter we demonstrate that for a certain class of nonlinearities, the soliton solution of the (generalized) nonlinear Schrödinger equation becomes multi-stable. This implies that more than one amplitude profile and speed of propagation of a singular soliton may exist for the same amount of the total power carried by the soliton. The existence of multi-stable solitons is related to the properties of nonlinear susceptibility as a function of the intensity of light. For example, the multistable soliton waves can not be observed in a Kerr-like nonlinear medium; they may exist only if the nonlinear component of the susceptibility as function of intensity is either changing its sign or its derivative has a sufficiently sharp peak (e.g. is a step-like function).

The soliton bistability can result in such effects as bistable (or multistable, in general) self-trapping of light in media with nonlinear refractive index as well as bistable propagation of short soliton pulses in nonlinear optical fibers since both of them may be described by the same nonlinear equation. Both of these effects may be viewed as an ultimate manifestation of multistable wave propagation since they are based on the simplest possible propagation configuration. They may also provide new opportunities in the field of optical bistability[1]. Indeed, for example, a bistable self-trapping of light provides a potential for optical bistable device entirely free either from any cavity or Fabry-Perot resonators[1], single nonlinear interfaces[2] or nonlinear waveguides formed by the nonlinear interfaces[3], retroreflection self-action effects[4], four-wave mixing[5], etc. On the other hand, since the propagation of singular pulses in a homogeneous nonlinear medium[6,7] and in nonlinear fiber waveguides[8] is also governed by a nonlinear Schrödinger equation, these soliton pulses in the system with an appropriate nonlinearity may provide the first (to the best of our knowledge) known opportunity to attain a temporal (or dynamic) bistability as opposed to all known kinds of optical bistability which were so far formulated in terms of steady-state regimes. The very notion of steady-state optical bistability comes into the inevitable contradiction with the applications most of which assume fast pulse regime of operations. When exploited in a dynamic regime, such effects still demonstrate hysteretic behavior which, however, can hardly be identified with the original "adiabatic", steady-state hysteresis. The dynamic hysteresis is more strongly affected by the relaxation processes rather than by steady-state bistable states especially when the total switching cycle has the duration time of the same order as relaxation times. The truly dynamic (or temporal) bistability discussed in this paper is based on bistable pulse shapes (as well as on bistable duration of the pulses) and offers a way to resolve this contradiction.

We consider the generalized nonlinear Schrödinger equation for the complex amplitude of field E in the form

$$2i\,\partial E/\partial z + \partial^2 E/\partial x^2 + Ef(|E|^2) = 0 \qquad (1)$$

where $f(|E|^2)$ is an arbitrary function of the intensity $|E|^2$ with $f(0) = 0$. When $f(|E|^2) = \alpha|E|^2 (\alpha = \text{const})$, (1) is so called cubic nonlinear Schrödinger equation[7,9-11]

(which corresponds to Kerr-nonlinearity in optical propagation). In the case of two-dimensional self–trapping[11], z is a normalized axis of the soliton propagation and x is a normalized transversal axis (both of them are dimensionless and correspond to the real coordinate $\tilde{z}$ and $\tilde{x}$ multiplied by the wave number $k = \omega n/c$), whereas in the case of pulse propagation (along the axis $\bar{z}$), $\tilde{x}$ is interpreted as $t - \tilde{z}/v$, $v = d\omega/dk$ is a group velocity in the linear case, with the initial equation of propagation being[6-8]

$$2ik\partial E/\partial\tilde{z} + (dv/d\omega)v^{-2}\partial^2 E/\partial\tilde{x}^2 + k^2 Ef(|E|^2) = 0 , \qquad (1')$$

and can readily be transformed into Eq (1) by proper scaling. In both of the cases f is proportional to the normalized nonlinear (i.e. intensity dependent) component $\Delta\epsilon^{NL}$ of the dielectric constant ϵ of medium. The nonlinear Schrödinger equation is obtained from the Maxwell equations in a conventional approximation of slowly varying envelop (i.e. $\dfrac{\partial^2 E}{\partial z^2} \ll \dfrac{\partial^2 E}{\partial x^2}$) which implies either small (quasi-optical) diffraction [Eq (1)] or relatively small dispersion [Eq (1')].

The stationary solutions (in particular, singular solitons) of Eq (1) have nonvarying intensity profile, $\partial|E|^2/\partial z = 0$, i.e. such solutions are written as $E(x,z) = u(x)\exp(\delta 2/2 + i\phi)$, where $\phi = $ const and δ is (unknown) real speed (or propagational constant) of the soliton. Thus, the equation for the real amplitude $u(x)$ is

$$d^2u/dx^2 + u[f(u^2) - \delta] = 0 \qquad (2)$$

whose soliton solution must satisfy the condition $u \to 0$ as $|x| \to \infty$ in order for the total power $P = \int_{-\infty}^{\infty} u^2 dx$ to be limited. This provides for the first integral of Eq (2) in the form

$$(du/dx)^2 = 2 \int_0^u u[\delta - f(u^2)]du \qquad (3)$$

integration of which gives the soliton amplitude profile $u(x)$ for each particular δ and $f(u^2)$. In order to evaluate a total power P, however, one needs not to know an explicit form $u(x)$. Indeed, by making use of Eq. (3) and introducing $I = u^2 = |E|^2$, one shows that

$$P(\delta) = \int_0^{I_m(\delta)} dI/\sqrt{\delta - F(I)} , \qquad (4)$$

where

$$F(I) = I^{-1}\int_0^I f(I)dI , \qquad (F(0)=0) \qquad (5)$$

i.e. P is determined immediately by f(I) and δ. In Eq (4), $I_m(\delta)$ is a peak intensity of

the soliton; it is defined as a minimal positive root of the equation $F(I) = \delta$. The multistability of a singular soliton is realized when the function $\delta(P)$ implicitly determined by Eq (4) becomes multivalued.

It is readily shown that a Kerr-like nonlinearity ($f \propto I$) results only in a one-valued singular soliton (with $\delta \propto P^2$), see Fig. 1, curve 5; the same is valid also for any other nonlinearity with $f \propto I^\mu$ where $\mu > 0$ (but $\mu \neq 2$). The nonlinearity $f \propto I^2$ plays a special role in two-dimensional propagation in the sense that in this case the total energy carried by any singular soliton is the same regardless of its spatial profile and preparation constant. Indeed, for $f = I^2/I_0^2$ where $I_0 = $ const, the intensity profile $I(x)$ and propagation constant δ are defined[12] from Eq (3):

$$I(x) = I_m/\cosh(2I_m x/I_0\sqrt{3}) ; \qquad \delta = I_m^2/3I_0^2 \qquad (6)$$

where maximal intensity of soliton I_m is an arbitrary constant; the total power is $P = \pi/2I_0$. One may note from Eq (4) that in the general case of arbitrary $f(I)$, a constant δ may be viewed as a first integral ("energy") of some system with a potential $F(I)$, Eq (5). The motion of this system in some p domain can then be described by the equation

$$d^2I/dp^2 + 8d[F(I)]/dI = 0 \qquad (7)$$

where if p is interpreted as a "time", $P(\delta)$ is a total "period" of oscillation of the system for any given "energy" of excitation δ. Particularly, one may see that the case $f \propto I^2$ (and therefore, $F \propto I^3$) corresponds to a "linear oscillator", with the "period" of oscillation P independent on its "energy" δ, i.e. $dP/d\delta = 0$ as suggested above.

In order to demonstrate the existence of the countable set of states of the singular soliton (with more than one state) we consider first the step-nonlinearity:

$$f(I) = 0, \text{ if } I < I_0 , \text{ and } f = \Delta , \text{ if } I > I_0 \qquad (8)$$

where I_0 and Δ are some positive constants. Substituting (8) into (4) one gets

$$P(\delta) = \frac{I_0}{\sqrt{\Delta}} \frac{1}{1-\beta} \left(\frac{1}{\sqrt{\beta}} + \frac{\arcsin\sqrt{\beta}}{\sqrt{1-\beta}} \right) ; \qquad \beta \equiv \frac{\delta}{\Delta} . \qquad (9)$$

The function β vs P determined by (9) is a two-valued function (Fig. 1, curve 1) for any $P > P_{cr1} \approx 3.44I_0/\sqrt{\Delta}$ with $\beta(P_{cr}) \approx 0.21$. The further example is given by the nonlinearity

$$f(I) = 0 , \text{ if } I < I_0, \text{ and } f(I) = \Delta(1 - I_0^2/I^2) , \text{ if } I > I_0 . \qquad (10)$$

In contrast with (8), $f(I)$ is now a continuous function, whereas its derivative df/dI is still discontinuous. The total power (4) now is

$$P = \frac{I_0}{\sqrt{\Delta}} \frac{1}{1-\beta}(\frac{1}{\sqrt{\beta}} + \frac{\text{arc cos }\sqrt{\beta}}{\sqrt{1-\beta}}) ; \quad \beta = \frac{\delta}{\Delta} \tag{11}$$

which essentially represents the same kind of behavior as (9), i.e. provides two-valued solution $\beta(P)$ for any $P > P_{cr2} \approx 4.28 \, I_0/\sqrt{\Delta}$ with $\beta(P_{cr}) \approx 0.26$. In these cases, the nontrivial branches of the function $P(\delta)$ tend to infinity as $\delta \to 0$ and $\delta \to \Delta$ (note that the third, "trivial", branch with $\delta \equiv 0$, P-arbitrary, corresponds to a nontrapped beam with $I_m < I_0$). This suggests a bistability without hysteresis and is due to the fact that nonlinearity $f(I)$ differs from zero only for some finite $I > I_0$. The same kind of soliton bistability is exhibited by the system, if either (i) $f'(0) < 0$ but $f(I)$ becomes positive at some I, e.g. when $f = -a_1 I + a_2 I^2 - a_3 I^3$ where $a_1, a_2, a_3 > 0$ and $9 a_1 a_3 < 2 a_2^2$, or (ii) $f(I) > 0$ in the vicinity of $I = 0$ but $f(I) = o(I^2)$; e.g. $f(I) = a_1 I^3 - a_2 I^4 (a_1, a_2 > 0)$ or $f(I) = a_1 I^3/(1 + I^3/I_o^3)$, $(a_1, I_o > 0)$. The latter nonlinearity may result from the three-photon resonant absorption of light by two-level systems with saturation.

In order to attain truly hysteretic bistable behavior [i.e. that characterized by the S-shape steady-state curves (see e.g. curves 2 and 3 at Fig. 1) which causes both "on" and "off" jumps between different branches of the curve], the function $f(I)$ must be positive at least in some range $0 < I < I_1$ and have a distinct peak of its first derivative $f'(I)$ in this range. The existence of hysteretic jumps is secured if $d\delta/dP = \infty$ (or $dP/d\delta = 0$) for two (or more) discrete values of P (or δ), where $dP/d\delta$ is found from (4) as

$$\frac{dP}{d\delta} = \frac{1}{2\delta} \int_0^{I_m} [1 - 2 \frac{F(d^2F/dI^2)}{(dF/dI)^2}] \frac{dI}{\sqrt{\delta - F(I)}} \tag{12}$$

A derivative $\frac{dP}{d\delta}(\delta)$ is strongly affected by $F''(I)$ and therefore by $f'(I)$; bistability may exist if $f'(0) > 0$, and if at some point $I = \tilde{I}$, there is $f''(\tilde{I}) = 0$ and $f'(\tilde{I}) > f'(0)$. As an example of such a function, consider

$$f = a_1 I + a_2 I^3 - a_3 I^5 \tag{13}$$

where $a_1, a_2, a_3 > 0$. S-shape behavior of $P(\delta)$ (Fig.1, curve 3) is possible if the condition is satisfied

$$a_1 a_3/a_2^2 < S_{cr} = O(1) \tag{14}$$

where S_{cr} is some critical quantity; the rough estimate gives $S_{cr} \sim 0.1-0.2$. In general case, the critical situation [when the curve $P(\delta)$ at some point $\delta = \delta_{cr}$ has $\frac{dP}{d\delta} = \frac{d^2P}{d\delta^2} = 0$, see, e.g. Fig.1, curve 4] corresponds to the conditions

$$dP/d\delta_{cr} = 0 \quad \text{and} \quad 2 \, (d^2F/dI_{cr}^2) \, F = (dF/dI_{cr})^2 \tag{15}$$

[where I_{cr} is the minimal solution of the equation $\delta_{cr} = F(I_{cr})$], which determines both

δ_{cr} and the required parameters of the function F(I) [and therefore, f(I)]. In the case when $f(I) = O(I^2)$ at I=0, the function $\delta(P)$ forms a hysteresis if $d^2f/dI^2 > 0$; $d^3f/dI^3 > 0$, and $d^4f/dI^4 < 0$ at I=0, e.g., $f = a_2I^2 + a_3I^3 - a_4I^4$ ($a_2, a_3, a_4 > 0$). In such a case, the lower (stable) branch of $\delta(P)$ corresponds to non-trapped beam ($\delta=0$), see Fig. 1, curve 2.

A stability of each of possible solitons which correspond to the same total power P is an important issue. The detailed small perturbation analysis [13] of the spatial stability of multistable solitons in the case of step-nonlinearity (8) shows that the lower branch of curve 1, Fig. 1 corresponds to the unstable solitons and the upper- to the stable ones; the trivial solution ($\delta = 0$) is stable for any P. This suggests a general criterion for an arbitrary f(I), and therefore $\delta(P)$: the stable solitons are those for which $d\delta/dP > 0$ and vice versa (see Fig. 1, curves 1-3). In a further study, it is of considerable interest to study a "collision" of two solitons that belong to upper and lower branches of the curve $\delta(P)$.

The bistable solitons may exist also in the case of three-dimensional propagation. Stationary self-trapping of a cylindrical beam, for instance, is governed by the "nonlinear Bessel" equation [instead of (2)]:

$$d^2u/dr^2 + (1/r)(du/dr) + u[f(u^2) - \delta] = 0 \qquad (16)$$

where r is a radial coordinate in the plane normal to z axis. For cylindrical beams, a Kerr-nonlinearity, $f \propto I$, plays the same role as $f \propto I^2$ in two dimensional case: for such a nonlinearity, the total power of the beam does not depend of its size or its peak intensity [7]. Therefore, in order to attain a nonhysteretic bistable soliton propagation of the kind depicted by curve 1, Fig. 1, the lower required degree of nonlinearity at $I \to 0$ is $f \propto I^2$ [with f attaining some maximum or saturation when I increases, e.g. $f = \alpha I^2/(1 + I^2/I_0^2)$]. Such a nonlinearity can be originated e.g. by the two-photon resonant absorption [13]. Hysteretic characteristic curve $\delta(P)$ similar to curve 2, Fig. 1, can be provided now by the nonlinearity $f(I) = a_1I + a_2I^2 - a_3I^3$ ($a_1, a_2, a_3 > 0$), with the critical condition in the same form as Eq (14) [but with different $S_{cr} = O(1)$].

In conclusion, we demonstrated an existence of multi-stable solitons of generalized nonlinear Schrödinger equation. In order for those solutions to exist, the nonlinearity must satisfy some special conditions, e.g. its dependence on the light intensity must have a range where it increases sufficiently sharply. In nonlinear optics, these solitons may manifest themselves either as singular pulses (e.g. in nonlinear fibers) or self-trapped channels (both in two- and three-dimensional cases). Bistable solitons present the ultimate case of multistable wave propagation and may find an application to the dynamic (temporal) optical bistability and bistable resonator-free self-trapping of light.

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12. Note that the solution (6) is <u>not</u> the same as for well known singular solitons of the cubic nonlinear Schrödinger equation with $f(I) \propto I$; in the latter case $I(x) \propto 1/\cosh^2(Ax)$ where A is some constant.

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Captions to Figure

Fig. 1. A propagation constant δ vs the total power P carried by the soliton. Curves 1-5 corresponds to various functions of nonlinearity: 1 - step-function Eq (8); 2 — $f = a_2 I^2 + a_3 I^3 - a_4 I^4 (a_2, a_3, a_4 > 0)$; 3 - Eq (13) with $a_1 a_3 < a_2^2 S_{cr}$; 4 - Eq (13) with $a_1 a_3 = a_2^2 S_{cr}$; 5 - Kerr-nonlinearity, $f \propto I$. The broken lines at curves 1-3 correspond to the unstable solitons. In the insertion, the intensity profiles I(x) are depicted of solitons that carry the same power but correspond to different branches of function $\delta(P)$ - upper branch (U) and lower branch (L).
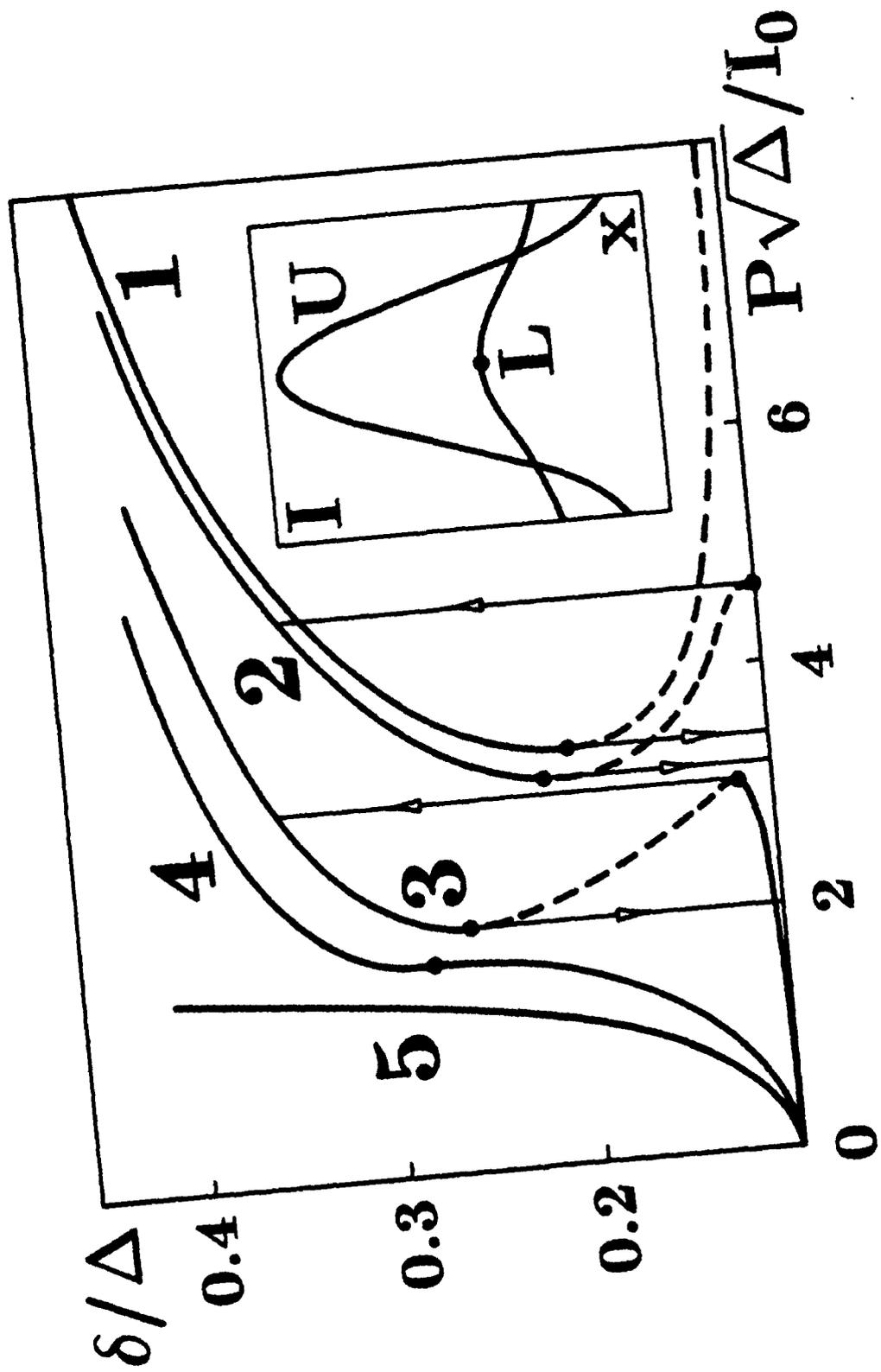
Multistable Self-trapping of Light and Multistable
Soliton Pulse Propagation

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Abstract

It is demonstrated that a nonlinear Schrödinger equation with certain nonlinearities allows for an existence of multi-stable single solitons (i.e., single solitons with the same carried power but different propagation parameters). In nonlinear optics, these solitons may exist either in the form of short bistable pulses, or bistable self-trapping (both two- and three-dimensional).

1. Introduction

The most known optical bistability principles and devices[1] are based on Fabry-Perot resonators filled by a nonlinear material. The entirely different principle of switching effect has first been proposed[2] and subsequently observed in the experiment[3] which does not exploit any resonant effect and is based on the reflection of light at the single nonlinear interface formed by semi-infinite linear and nonlinear media. In the recent work[4] (with the earlier proposals[5]) this principle has been extended to the so-called nonlinear waveguides formed by two nonlinear interfaces (i.e. sandwich structure with a linear layer between two semi-infinite nonlinear layers[4,5] or vice versa[5]). Although the nonlinear waveguides suggest new interesting opportunities, they are still substantially based on the nonlinear interface principle.

In this paper, the feasibility of a fundamentally novel bistable and switching optical effect is demonstrated which consists in a multivalued self-trapping of light in an *interface-free* nonlinear medium. For the certain class of nonlinearities the self-trapped beam of light occurs to have more than one possible propagation constant (and respectively more than one possible intensity profile in its cross-section) for the same total power carried by the beam.

Self-trapping and self-focusing of light in nonlinear media may be described by a parabolic nonlinear partial differential equation[6-8] (the so-called nonlinear Schrödinger equation) whose single solutions (solitons[9]) correspond to self-trapped channels[8-10]. We show that the existence of multi-soliton solutions of the (generalized) Schrödinger equation is related to the properties of nonlinear susceptibility as a function of the intensity of light. For example, the multistable soliton waves can not be observed in a Kerr-like nonlinear medium; they may exist only if the nonlinear component of the susceptibility as function of intensity is either changing its sign or its derivative has a sufficiently sharp peak (e.g. is a step-like function). This may be e.g. due to such

mechanisms as either light-induced phase transition in a material or multi-photon saturation of resonant levels. This special requirement may probably be a reason why the multi-stable soliton effects has not been discussed earlier.

Furthermore, since the propagation of single pulses in a homogeneous nonlinear medium[8,11] and in nonlinear fiber waveguides[12] is also governed by a nonlinear Schrödinger equation, these soliton pulses in the system with an appropriate nonlinearity may provide the first (to the best of our knowledge) known opportunity to attain a temporal (or dynamic) bistability as opposed to all known kinds of optical bistability which were so far formulated in terms of steady-state regimes. The very notion of steady-state optical bistability comes into the inevitable contradiction with the applications most of which assume fast pulse regime of operations. When exploited in a dynamic regime, such effects still demonstrate hysteretic behavior which, however, can hardly be identified with the original "adiabatic", steady-state hysteresis. The dynamic hysteresis is more strongly affected by the relaxation processes rather than by steady-state bistable states especially when the total switching cycle has the duration time of the same order as relaxation times. The truly dynamic (or temporal) bistability discussed in this paper is based on bistable pulse shapes (as well as on bistable duration of the pulses) and offers a way to resolve this contradiction.

Both of these effects (multistable self-trapping of light and multistable soliton pulses) may be viewed as an ultimate manifestation of multistable wave propagation since they are based on the simplest possible propagation configuration. They may also provide new opportunities in the field of optical bistability.

2. Wave Equation.

We assume that the EM field $\vec{E}(\vec{r})e^{i(\vec{k}\vec{r}-\omega t)}$ propagates in a lossless medium having intensity-dependent susceptibility ϵ, such that

$$\epsilon = \epsilon^L + \Delta\epsilon^{NL}(|E|^2) \tag{1}$$

and introduce $f(|E|^2) = \Delta\epsilon^{NL}/\epsilon^L$ where f can be an arbitrary function of the intensity $|E|^2$ with $f(0) = 0$. The case of $f \propto |E|^2$ corresponds to the Kerr-nonlinearity. We assume also that the axis of the propagation is $\bar{z}$ (i.e. $\vec{k} = k\hat{e}_z$, where $k^2 = \omega^2\epsilon^L/c^2$) and introduce dimensionless coordinates $z = k\bar{z}$, $x = k\bar{x}$, $y = k\bar{y}$. Then from the Maxwell equation, in the conventional approximation of a slowly varying envelope[7,8] ($\partial^2 E/\partial z^2 << \Delta_\perp E$, where $E_\perp$ is a Laplacian operator in a plane normal to the z axis), one readily gets the (generalized) nonlinear Schrödinger equation governing the nonlinear wave propagation:

$$2i\partial E/\partial z + \Delta_\perp E + E f(|E|^2) = 0 \tag{2}$$

In the two-dimensional case (e.g. with $\partial/\partial y = 0$), this equation is reduced to the form

$$2i\partial E/\partial z + \partial^2 E/\partial x^2 + E f(|E|^2) = 0 \tag{3}$$

In the case of the one-dimensional pulse propagation along the z_1 axis in the slightly dispersive medium with a nonlinearity $f_1(|E|^2)$, the equation of the propagation is [8,11,12]

$$2i\partial E/\partial z_1 + (dv/d\omega)v^{-2}\partial^2 E/\partial\xi^2 + k E f_1(|E|^2) = 0 \tag{4}$$

where $\xi = t - z_1/v$; $v = d\omega/dk$ is the group velocity of linear propagation. Eq. (4) can readily be transformed into Eq. (3) by proper scaling, e.g. by assuming

$$z_1 = z/k^2(dv/d\omega); \quad \xi = x/kv; \quad f_1 = fk(dv/d\omega). \tag{5}$$

The slow varying envelope approximation implies either small diffraction [Eqs. (2), (3)] or small dispersion [Eq. (4)].

The stationary solutions (in particular single solitons) of Eqs. (2-4) have nonvarying intensity profile, $\partial |E|^2/\partial z = 0$, i.e. such solutions are written in the form

$$E(x,y,z) = u(x,y)x\,p(i\delta z/2 + i\phi) \tag{6}$$

where $u = |E|$ is a real amplitude of the field, $\phi = $ count is a real phase and δ is (unknown) real speed (or propagation constant) of the soliton.

3. Bistable Soliton Pulses and Two-Dimensional Self-Trapping

Both two-dimensional self-trapping and one-dimensional soliton pulses (e.g. in fibers [12]) are governed by the same Eq. (3). Substituting Eq. (6) into Eq. (3) and bearing in mind that in this case $u = u(x)$, one gets the equation for the real amplitude u:

$$d^2u/dx^2 + u[f(u^2) - \delta] = 0 \tag{7}$$

whose soliton solution must satisfy the condition $u \to 0$ as $|x| \to \infty$ in order for the total power $P = \int_{-\infty}^{\infty} u^2 dx$ to be limited. This provides for the first integral of Eq (7) in the form

$$(du/dx)^2 = 2 \int_0^u u[\delta - f(u^2)]du \tag{8}$$

integration of which gives the soliton amplitude profile u(x) for each particular δ and $f(u^2)$. In order to evaluate a total power P, however, one needs not to know an explicit form u(x). Indeed, by making use of Eq. (8) and introducing $I = u^2 = |E|^2$, one shows that

$$P(\delta) = \int_0^{I_m(\delta)} dI/\sqrt{\delta - F(I)}\,, \tag{9}$$

where

$$F(I) = \Gamma^{-1} \int_0^I f(I)dI \ , \quad (F(0)=0) \tag{10}$$

i.e. P is determined immediately by $f(I)$ and δ. In Eq (9), $I_m(\delta)$ is a peak intensity of the soliton; it is defined as a minimal positive root of the equation $F(I) = \delta$. The multistability of a single soliton is realized when the function $\delta(P)$ implicitly determined by Eq (9) becomes multivalued.

It is readily shown that a Kerr-like nonlinearity ($f \propto I$), Fig. 2, curve 1, results only in a one-valued single soliton (with $\delta \propto P^2$), see Fig. 1, curve 5; the same is valid also for any other nonlinearity with $f \propto I^\mu$ where $\mu > 0$ (but $\mu \neq 2$). The nonlinearity $f \propto I^2$ Fig. 2, curve 2, plays a special role in the two-dimensional propagation in the sense that in this case the total energy carried by any single soliton is the same regardless of its spatial profile and propagation constant. Indeed, for $f = I^2/I_0^2$ where $I_0 = $ const, the intensity profile $I(x)$ and propagation constant δ are defined[13] from Eq (8):

$$I(x) = I_m/\cosh(2I_m x/I_0 \sqrt{3}) \ ; \quad \delta = I_m^2/3I_0^2 \tag{11}$$

where maximal intensity of soliton I_m is an arbitrary constant; the total power is $P = \pi/2I_0$. One may note from Eq (9) that in the general case of arbitrary $f(I)$, a constant δ may be viewed as a first integral ("energy") of some system with a potential $F(I)$, Eq (10). The motion of this system in some p domain can then be described by the equation

$$d^2I/dp^2 + 8d[F(I)]/dI = 0 \tag{12}$$

where if p is interpreted as a "time", $P(\delta)$ is a total "period" of oscillation of the system for any given "energy" of excitation δ. Particularly, one may see that the case $f \propto I^2$ (and therefore, $F \propto I^3$) corresponds to a "linear oscillator", with the "period" of oscillation P independent on its "energy" δ, i.e. $dP/d\delta = 0$ as suggested above.

In order to demonstrate the existence of the countable set of states of the single soliton (with more than one state) we consider first the step-nonlinearity (this problem has first been considered in [14]):

$$f(I) = 0, \text{ if } I < I_0, \text{ and } f = \Delta, \text{ if } I > I_0, \tag{13}$$

Fig. 2, curve 3, where I_0 and Δ are some positive constants. Substituting (13) into (9) one gets

$$P(\delta) = \frac{I_0}{\sqrt{\Delta}} \frac{1}{1-\beta} \left(\frac{1}{\sqrt{\beta}} + \frac{\arcsin\sqrt{\beta}}{\sqrt{1-\beta}} \right); \quad \beta \equiv \frac{\delta}{\Delta}. \tag{14}$$

The function β vs P determined by (14) is a two-valued function (Fig. 1, curve 1) for any $P > P_{cr1} \approx 3.44 I_0/\sqrt{\Delta}$ with $\beta(P_{cr}) \approx 0.21$. The further example is given by the nonlinearity (Fig. 2, curve 4)

$$f(I) = 0, \text{ if } I < I_0, \text{ and } f(I) = \Delta(1 - I_0^2/I^2), \text{ if } I > I_0. \tag{15}$$

In contrast with (13), $f(I)$ is now a continuous function, whereas its derivative df/dI is still discontinuous. The total power (9) now is

$$P = \frac{I_0}{\sqrt{\Delta}} \frac{1}{1-\beta} \left(\frac{1}{\sqrt{\beta}} + \frac{\arccos\sqrt{\beta}}{\sqrt{1-\beta}} \right); \quad \beta = \frac{\delta}{\Delta} \tag{16}$$

which essentially represents the same kind of behavior as (14), i.e. provides two-valued solution $\beta(P)$ for any $P > P_{cr2} \approx 4.28\, I_0/\sqrt{\Delta}$ with $\beta(P_{cr}) \approx 0.26$. In these cases, the nontrivial branches of the function $P(\delta)$ tend to infinity as $\delta \to 0$ and $\delta \to \Delta$ (note that the third, "trivial", branch with $\delta \equiv 0$, P - arbitrary, corresponds to a nontrapped beam with $I_m < I_0$). This suggests a bistability without hysteresis and is due to the fact that nonlinearity $f(I)$ differs from zero only for some finite $I > I_0$. The same kind of soliton bistability is exhibited by the system, if either (i) $df(0)/dI < 0$ but $f(I)$ becomes positive at some I, e.g. when $f = -a_1 I + a_2 I^2 - a_3 I^3$ where $a_1, a_2, a_3 > 0$ (Fig. 2, curve 5) and $9a_1 a_3 < 2a_2^2$, or (ii) $f(I) > 0$ in the vicinity of $I = 0$ but $f(I) = o(I^2)$; e.g.

$f(I) = a_1I^3 - a_2I^4(a_1,a_2 > 0)$ or $f(I) = a_1I^3/(1 + I^3/I_o^3)$, $(a_1,I_o > 0)$, Fig. 2, curve 6. The latter nonlinearity may result from the three-photon resonant absorption of light by two-level systems with saturation.

In order to attain truly hysteretic bistable behavior [i.e. that characterized by the S-shape steady-state curves (see e.g. curves 2 and 3 at Fig. 1) which causes both "on" and "off" jumps between different branches of the curve], the function $f(I)$ must be positive at least in some range $0 < I < I_1$ and have a distinct peak of its first derivative df/dI in this range. The existence of hysteretic jumps is secured if $d\delta/dP = \infty$ (or $dP/d\delta = 0$) for two (or more) discrete values of P (or δ), where $dP/d\delta$ is found from (9) as

$$\frac{dP}{d\delta} = \frac{1}{2\delta} \int\limits_0^{I_m} [1 - 2\frac{F(d^2F/dI^2)}{(dF/dI)^2}]\frac{dI}{\sqrt{\delta - F(I)}} \tag{17}$$

A derivative $\frac{dP}{d\delta}(\delta)$ is strongly affected by d^2F/dI^2 and therefore by df/dI; bistability may exist if $df/dI > 0$, and if at some point $I = \tilde{I}$, there is $d^2f/d\tilde{I}^2 = 0$ and $df/d\tilde{I} > df(0)/dI$. As an example of such a function, consider (Fig. 2, curve 7)

$$f = a_1I + a_2I^3 - a_3I^5 \tag{18}$$

where $a_1,a_2,a_3 > 0$. S-shape behavior of $\delta(P)$ (Fig.1, curve 3) is possible if the condition is satisfied

$$a_1a_3/a_2^2 < S_{cr} = O(1) \tag{19}$$

where S_{cr} is some critical quantity; the rough estimate gives $S_{cr} \sim 0.1-0.2$. In general case, the critical situation [when the curve $P(\delta)$ at some point $\delta = \delta_{cr}$ has $\frac{dP}{d\delta} = \frac{d^2P}{d\delta^2} = 0$, see, e.g. Fig.1, curve 4] corresponds to the conditions

$$dP/d\delta_{cr} = 0 \quad \text{and} \quad 2\ (d^2F/dI_{cr}^2)\ F = (dF/dI_{cr})^2 \tag{20}$$

[where I_{cr} is the minimal solution of the equation $\delta_{cr} = F(I_{cr})$], which determines both

δ_{cr} and the required parameters of the function $F(I)$ [and therefore, $f(I)$]. In the case when $f(I) = O(I^2)$ at $I=0$, the function $\delta(P)$ forms a hysteresis if $d^2f/dI^2 > 0$, $d^3f/dI^3 > 0$, and $d^4f/dI^4 < 0$ at $I=0$, e.g., $f = a_2I^2 + a_3I^3 - a_4I^4$, $(a_2, a_3, a_4 > 0)$, see Fig. 1, curve 2. In such a case, the lower (stable) branch of $\delta(P)$ corresponds to non-trapped beam $(\delta=0)$.

To analyze the conditions (20) on the general case of an arbitrary function $f(I)$ is not an easy task since the dependence $P(\delta)$ is implicitly determined by the integral (9) which is not evaluated in the analytic form for an arbitrary $f(I)$ [and therefore $F(I)$, Eq. (10)]. Therefore, for the practical purpose, it is important to have a good analytic approximation of the function $P(\delta)$ at least in the range $0 < I < I_B$ where I_B is the point at which $d^2F(I_B)/dI^2 = 0$ [one may see from Eq. (20) that the critical point $I = I_{cr}$ is located in this range, i.e. $I_{cr} < I_B$]. For such a purpose, it is more convenient to operate with I_m rather than δ [remember that $\delta = F(I_m)$]. Then, if $F(I_m)$ is positive and monotonically increasing in $(0, I_B)$, a good approximation is given by the formula

$$P(I_m) \approx 2C[\sqrt{F_m} - \sqrt{(F_m' - F_0')(F_m - F_0' I_m)/F_m'}]/F_o' , \qquad (21)$$

where $F_m = F(I_m) = \delta$; $F_m' = dF(I_m)/dI$; $F_0' = dF(0)/dI$, and $C > 0$ is some constant of the order of unity which is determined by the type of the function $F(I)$. For the calculations related to the conditions (20), this constant is not important, since the first of Eqs. (20) is to be replaced now by the condition $dP/dI_{cr} = 0$ which makes insignificant any scaling of $P(I_m)$.

4. Stability of Two-Dimensional Bistable Solitons.

A stability of each of possible solitons which correspond to the same total power P is an important issue. Assuming that the amplitude profile $u_s(x)$ of the particular soliton [determined by Eq (8)] is known, we represent a solution of Eq (2) in the form of perturbed soliton solution (6):

$$E(x,z) = [u_s(x) + \Delta u(x,z)]exp[i\delta z/2 + i\phi(x,z)] \qquad (22)$$

where Δu, $\partial\phi/\partial z$ and $\partial\phi/\partial x$ are small real perturbations and assume the factorization of the perturbation in the form

$$\Delta u = w(z)[u_s^{'}(x)p(x)]; \quad \phi = \psi(z)\xi(x), \qquad (23)$$

where w, p, ψ, and ξ are some unknown real functions of a single variable. Substituting Eqs. (22) and (23) into Eq. (3), linearizing Eq. (3) with respect to the small perturbations, equalizing the real and imaginary parts of the obtained equation separately to zero, separating the terms that depend only on x or z, and making use of Eq. (7), one finally gets the equations for the unknown functions w, p, ψ, and ξ:

$$dw/dz = \nu\psi; \quad d\psi/dz = \lambda w \qquad (24)$$

$$(p^{'}u_s^{'2})^{'} = \lambda(u_s^2)^{'}\xi; \qquad (25)$$

$$(\xi^{'}u_s^2)^{'} = \nu(u_s^2)^{'}p, \qquad (26)$$

where ν and λ are some unknown real constants, and prime denotes now a derivative with respect to x. Eqs. (25) and (26) can be also reduced to a single equation of the fourth order for one variable, e.g. p:

$$\left\{ u_s^2[(p^{'}u_s^{'2})^{'}/(u_s^2)^{'}]^{'} \right\}^{'} = \gamma(u_s^2)^{'}p \qquad (27)$$

where $\gamma = \nu\lambda$. The eigenmodes of Eqs. (25-27) must satisfy a condition

$$pu_s^{'} \to 0, \quad \xi^{'} \to 0, \quad \text{as} \quad |x| \to \infty. \qquad (28)$$

If these eigenmodes as well as their respective eigenvalues ν and λ (and therefore $\gamma = \nu\lambda$) are found, the sufficient requirement for the stability of the respective soliton $E_s = u_s(x)exp(i\delta z/2)$ is that all γ must be negative. The soliton is unstable if any of γ is positive (the sufficient condition); the case with $\gamma = 0$ has to be further investigated

although normally it corresponds to a stable soliton. The lower order eigenmodes of Eqs. (25-27) with $\gamma = 0$ are readily found. These are:

(i) $\lambda = \nu = 0$, $p = \text{const} \neq 0$ (i.e. $\Delta u = C u_s'(x)$, $C \ll 1$), $\phi = 0$. This mode corresponds to the shift of the spatial position of the soliton peak at the x axis (with all other characteristics of the soliton being intact).

(ii) $\lambda = \nu = 0$, $\xi = \text{const} \neq 0$, $p \simeq 0$, which corresponds to the shift of the soliton phase ϕ.

(iii) $\lambda = 0$; $\nu \neq 0$; $p = \text{const} \neq 0$; $\xi = (\nu p)x$. This corresponds to the shift of the "angle" of the soliton propagation.

(iv) $\lambda \neq 0$; $\nu = 0$; $\xi = \text{const} \neq 0$; and

$$p = \lambda \xi \int_0 \frac{u_s^2}{u_s'^2}\, dx = \lambda \xi \int \frac{dx}{\delta - F(u_s^2)} \tag{29}$$

This eigenmode corresponds to the small change of the propagation constant of the soliton δ (and therefore, the total power carried by the soliton), with conservation of the single soliton character of the entire propagation. Analytical solution for the higher modes in the case of arbitrary f(I) is still to be found. The detailed analysis in the case of step-nonlinearity (13) shows that the lower branch of curve 1, Fig. 1 corresponds to the unstable solitons and the upper-one to the stable ones; [the trivial solution ($\delta = 0$) is stable for any P]. This suggests a general criterion for an arbitrary f(I), and therefore $\delta(P)$: the stable solitons are those for which $d\delta/dP > 0$ and vice versa (see Fig. 1, curves 1-3). Although this statement seems to be intuitively almost obvious, it is still to be proven. It is also of considerable interest to study a "collision" of two solitons that belong to upper and lower branches of the curve $\delta(P)$.

5. Bistable Three-Dimensional Self-Trapping

The bistable solitons may exist also in the case of three-dimensional propagation. Stationary self-trapping of a cylindrical beam, for instance, is governed by the "nonlinear Bessel" equation which follows from Eqs. (2) and (6) when $u(x,y)$ has a polar symmetry:

$$d^2u/dr^2 + (1/r)(du/dr) + u[f(u^2) - \delta] = 0 \tag{30}$$

where $r = \sqrt{x^2+y^2}$ is a radial coordinate in the plane normal to z axis. With the step-nonlinearity (13) the solution of Eq (30) is

$$u(r) = \begin{cases} A\ J_0(kr\sqrt{\Delta-\delta})\ , & \text{when } u > u_0 \equiv \sqrt{I_0}\ ,\ r < r_0 \\ B\ K_0(kr\sqrt{\delta})\ , & \text{when } u < u_0\ ,\ r > r_0 \end{cases} \tag{31}$$

where J_ν and K_ν are ν-order ordinary and modified Bessel functions respectively; A, B, and r_0 are some constants. By requiring continuity of $u(r)$ and du/dr at $u = u_0$, one gets an equation determining the radius of the beam r_0 at the level $u = u_0$:

$$\sqrt{\frac{\Delta}{\delta}-1}\ \frac{J_1(kr_0\sqrt{\Delta-r})}{J_0(kr_0\sqrt{\Delta-r})} = \frac{K_1(kr_0\sqrt{\delta})}{K_0(kr_0\sqrt{\delta})}\ . \tag{32}$$

With r_0 known, A and B are determined as $A = u_0/J_0(kr_0\sqrt{\Delta-\delta})$; $B = u_0/K_0(kr_0\sqrt{\delta})$. For cylindrical beams, a Kerr-nonlinearity, $f \propto I$, plays the same role as $f \propto I^2$ in the two dimensional case: for such a nonlinearity, the total power of the beam does not depend of its size or its peak intensity[8]. Therefore, in order to attain a nonhysteretic bistable soliton propagation of the kind depicted by curve 1, Fig. 1, the lower required degree of nonlinearity at $I \to 0$ is $f \propto I^2$ [with f attaining some maximum or saturation when I increases, eg. $f = \alpha I^2/(1+I^2/I_0^2)$] which resembles curve 6, Fig. 2. Such a nonlinearity can be originated e.g. by the two-photon resonant absorption[15]. S-shape hysteretic characteristic curve $\delta(P)$ can be provided now by the nonlinearity $f(I) = a_1I + a_2I^2 - a_3I^3$ $(a_1,a_2,a_3 > 0)$ which resembles curve 7, Fig. 2, with the critical

condition in the same form as Eq (19) [but with different $S_{cr} = 0(1)$].

6. Conclusion.

In conclusion, we demonstrated an existence of multi-stable solitons of generalized nonlinear Schrödinger equation. In order for those solutions to exist, the nonlinearity must satisfy some special conditions, e.g. its dependence on the light intensity must have a range where it increases sufficiently sharply. In nonlinear optics, these solitons may manifest themselves either as single pulses (e.g. in nonlinear fibers) or self-trapped channels (both in two- and three-dimensional cases). Bistable solitons present the ultimate case of multistable wave propagation and may find an application to the dynamic (temporal) optical bistability and bistable resonator-free self-trapping of light.

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Captions to Figures

Fig. 1. A propagation constant δ vs the total power P carried by the soliton. Curves 1-5 corresponds to various functions of nonlinearity: 1 - step-function Eq (13); 2 - $f = a_2 I^2 + a_3 I^3 - a_4 I^4$, $(a_2, a_3, a_4 > 0)$; 3 - Eq (18) with $a_1 a_3 < a_2^2 S_{cr}$; 4 - Eq (18) with $a_1 a_3 = a_2^2 S_{cr}$; 5 - Kerr-nonlinearity, $f \propto I$. The broken lines at curves 1-3 correspond to the unstable solitons. In the insertion, the intensity profiles I(x) are depicted of solitons that carry the same power but correspond to different branches of function $\delta(P)$ - upper branch (U) and lower branch (L).

Fig. 2. Various functions of nonlinearity f vs. the field intensity I (see details in the text).

f(I)

ARBITRARY UNITS

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I

0

1 2 3 4 5 6 7
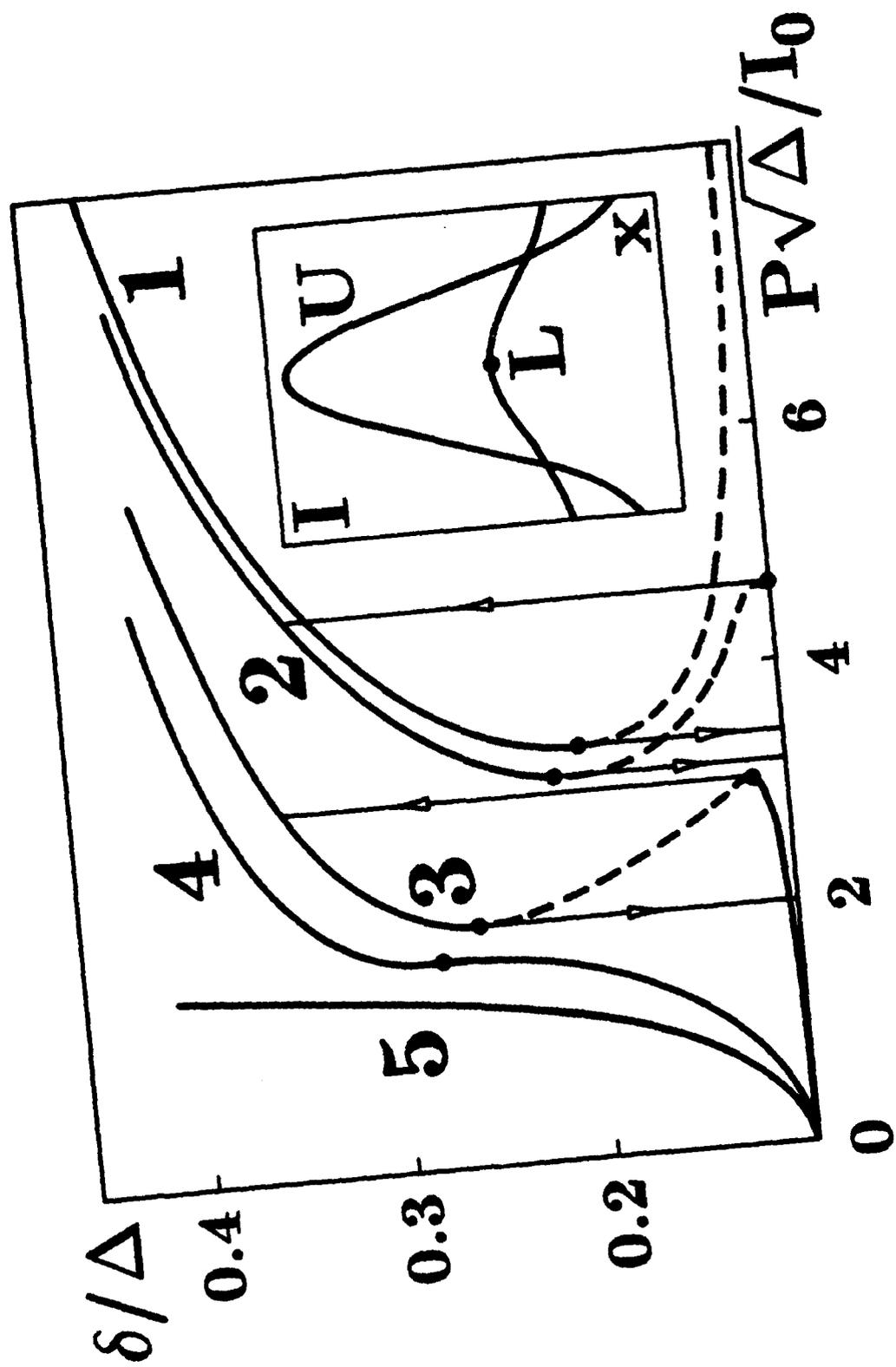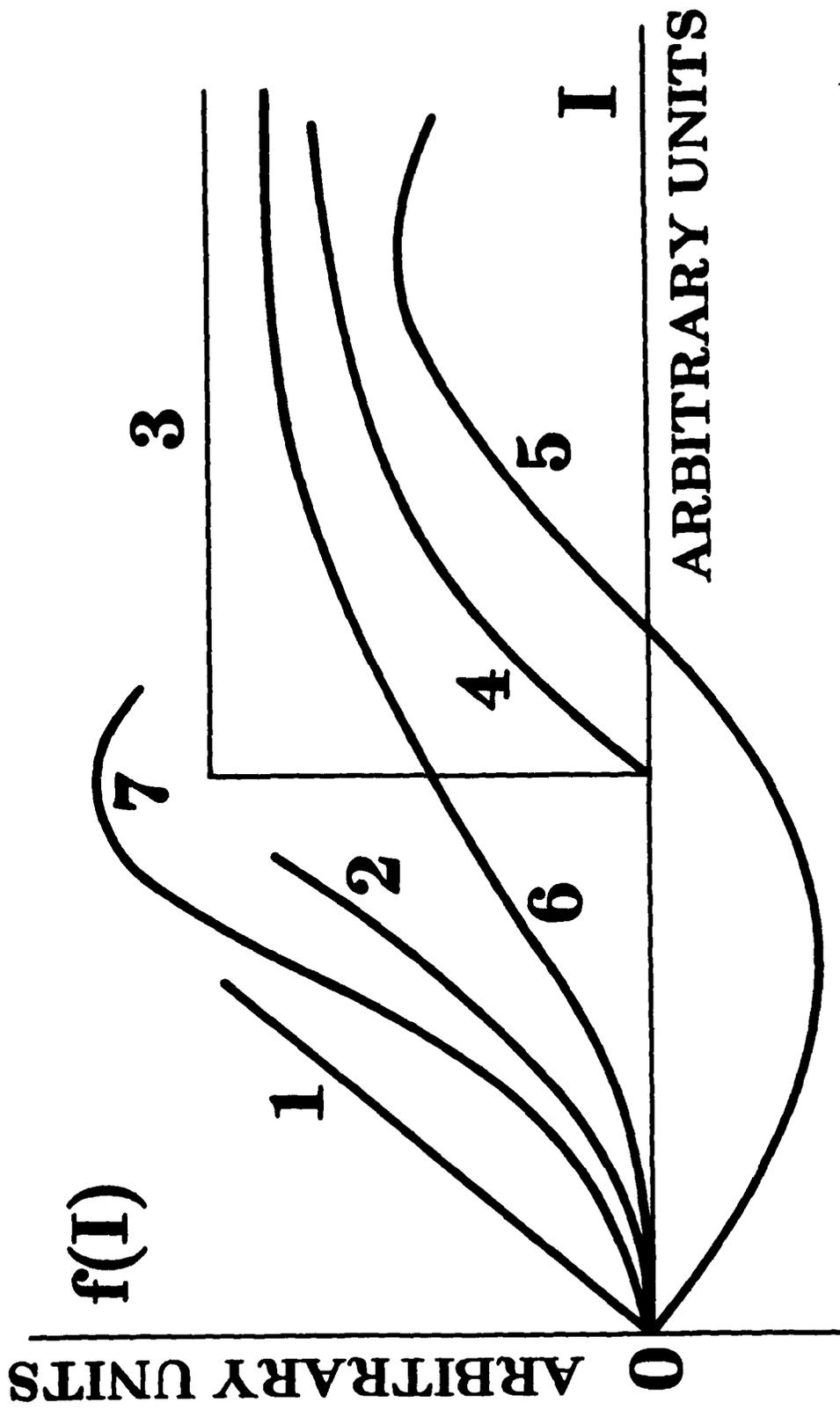
Ultimate Bistability: Hysteretic Resonance of a Slightly - Relativistic Electron

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ABSTRACT

It was recently predicted by us that cyclotron resonance of free electrons in vacuum and conduction electrons in semiconductors may exhibit bistable and hysteretic behavior which is due to relativistic mass-effect (or pseudo relativistic - in semiconductors). Based on this prediction, the hysteretic cyclotron resonance of a trapped single electron in vacuum has recently been experimentally observed by G. Gabrielse et al. In this paper we consider this phenomenon as an ultimate bistability since it is based on the most fundamental mechanism of nonlinearity (relativistic mass-effect), involves the interaction of EM wave with the simplest single elementary particle, and demonstrate the first known intrinsic bistability with no macroscopic optical feedback. We also show that a hysteretic resonance of a single electron based on relativistic effects is feasible also in one-dimensional parabolic potential (with no magnetic field required to attain a resonance).

Introduction

Optical bistability[1] is a rapidly expanding and promising field in nonlinear optics which offers both new insights in nonlinear interactions of light with matter and potentials for superfast switching devices for optical computers and optical signal processing. Therefore, the fundamental physical problems related to that phenomenon have become important as well. One of the most intriguing questions is: what is the ultimate physical level of bistable interaction of light with matter? It is feasible to realize (and possibly, to exploit) the bistable interaction of the microscopic level?

In our recent work[2] it was predicted that even a slight relativistic mass-effect of a single free electron may result in large nonlinear effects such as hysteresis and bistability in cyclotron resonance under action of EM wave. The predicted effect may be regarded as the ultimate and fundamental one in many respects:

(i)   it suggests the bistable interaction of EM wave with the single simplest microscopic physical object - an electron

(ii)   the nonlinearity that makes the bistable interaction possible is based on one of the most fundamental physical effects - relativistic change of electron mass

(iii)   it was the first proposed effect that offered bistability based on the intrinsic property of an microscopic object rather than on macroscopic optical feedback in a nonlinear medium.

Very recently, based on the prediction[2], the hysteretic (bistable) cyclotron resonance of a free electron has been discovered by G. Gabrielse et al.[3] in experiment in which a simple electron has been trapped in a Penning trapp for the period of time as long as 10 months[4].

In Ref [2] it was also suggested that the analogous effect (i.e. hysteretic cyclotron resonance) can be expected also in semiconductors with narrow energy gap between conduction and valence bands. This problem was later addressed in Ref [5]. In semiconductors, the mass-effect required for the hysteretic resonance, is provided by the

nonparabolicity of the semiconductor conduction band which causes a pseudo-relativistic dependence[6,7] of the effective mass of conduction electrons on their momentum or energy.

In the both cases (i.e. free-space electrons and conduction electrons), the hysteretic resonance is attributed to the dependence of the cyclotron frequency of forced oscillations on the relativistic mass of the electron, and hence on its momentum (or kinetic energy). The critical condition for bistability to occur is that the nonlinear (relativistic) shift of the cyclotron frequency must be sufficiently greater than the linewidth of the resonance. To some extent, this effect resembles hysteresis in a classic nonlinear oscillator[8] or in nonlinear parametric systems[9].

From the electrodynamic viewpoint, one of the most important features of the new effect is that it is based on the intrinsic properties of the microscopic components, not on the macroscopic optical feedback. This differs fundamentally from all conventional mechanisms of optical bistability[1] which so far have always been based on macroscopic nonlinear properties of the media. Nonlinear change in macroscopic susceptibility under action of the strong EM wave provides dramatic change in the optical condition of propagation of this wave under various special circumstances which, in turn, leads again to the change in the susceptibility. This so-called optical feedback in nonlinear macro-systems results in the existence of multistable (in particular, bistable) steady states. No such optical feedback exists in the hysteretic electron resonances. One of the consequences of this fact is that those effects exibit also so called cavityless (or resonator-free) bistability[10]. Recently, some new mechanisms of optical bistability based on the intrinsic bistability analogous to bistable cyclotron resonance (in the sense that they are attributed to the nonlinear shift of resonant frequency of material) have been proposed[11,12] and experimentally observed[11].

In this paper we review the theory of hysteretic (bistable) cyclotron resonance of a single free electron (Section 1) and conduction electrons in semiconductors (Section 2).

We show also that in fact the presence of a magnetic field (which is required to attain a cyclotron resonance) is not a necessary condition for a hysteretic effect; one may attain a bistability based on relativistic mass-effect even in any one-dimensional oscillator having parabolic potential well (Section 3).

1. Bistable (hysteretic) cyclotron resonance of a free electron

We shall demonstrate[2] this effect for the simplest case of a single electron immersed in a strong constant magnetic field H_0 and interacting with an electromagnetic field E_{in} of amplitude E (E$\ll$H$_0$). The plane EM wave propagates along the axis z parallel to H_0. Field H_0 provides a cyclotron resonance with the unperturbed frequency

$$\omega_0 = eH_0/m_0c \qquad (1.1)$$

where e is an electrical charge of the electron, m_0 is its rest mass, and c is the speed of light. We also assume that a small constant electric field $\vec{g}(z)$ is applied along axis z; this field is provided by some potential to arrange a trapping of the electron[13] and to compensate a radiation force caused by the EM wave.

We treat this problem classically. The equation of motion for the electron moving with arbitrary velocity v is [14]

$$d(m\vec{v})/dt = (e/c)\vec{v} \times \vec{H}_\Sigma + e\vec{E}_\Sigma + \vec{F}_f , \qquad (1.2)$$

$$m = m_0(1 - |\vec{v}|^2/c^2)^{-1/2} , \qquad (1.3)$$

where $\vec{H}_\Sigma$ is the total magnetic field (including the EM wave component $\vec{H}_{EM}$, i.e. $\vec{H}_\Sigma = \vec{H}_0 + \vec{H}_{EM}$), $\vec{E}_\Sigma = \vec{E}_{in} + g(z)\hat{e}_x$ is the total electric field, and the term $\vec{F}_f$ represents energy losses of the electron. In the ultimate case in which the losses are caused by EM radiation of the rotating electron (and $|v| \ll c$) this term can be written as [14]

$$\vec{F}_l = (2e^2/3c^2)d^2\vec{v}/dt^2 \ . \tag{1.4}$$

In the general case the losses may be much larger and caused by various factors, basically by collisions, so that the radiation losses could be neglected. The force is then proportional to the velocity of the electron, e.g., $\vec{F}_l = -\gamma m_0\omega_0\vec{v}$, where γ is the dimensionless width of cyclotron resonance. The radiation losses can also be represented by this formula, since one can assume[14] that $d^2\vec{v}/dt^2 \approx -\omega_0^2\vec{v}$, which yields

$$\gamma_{rad} = \frac{2e^2\omega_0}{3m_0c^3} = \frac{2}{3}\, r_e k_0 \ll 1 \ , \tag{1.5}$$

where $r_e = e^2/m_0c^2 = 2.8 \times 10^{13}$ cm is an electron radius and $k_0 = \omega_0/c$ is a resonance wave number. Since for the plane wave

$$\vec{H}_{EM} = \vec{k} \times \vec{E}_{in}/\omega \tag{1.6}$$

(where ω is a frequency of EM field, and $k = \omega/c$), Eq (1.2) governing the interaction of the "magnetized" electron with an arbitrary propagating plane wave $\vec{E}_{in}$ can be written in the form:

$$d(m\vec{v})/dt + \gamma m_0\omega_0\vec{v} = e\left[\vec{E}_{in} + \frac{1}{c}\,\vec{v}\times\vec{H}_o\right] + e\left[\frac{\vec{v}}{\omega} \times [\vec{k}\times\vec{E}_{in}]-\vec{g}\right] \tag{1.7}$$

where the term $e\vec{v}\times[\vec{k}\times\vec{E}_{in}]/\omega$ is a radiation force applied to the electron. Usually this term is neglected (see, e.g. Ref. 15), except in Ref. 16 in which, however, losses and other possible forces like counter-potential $\vec{g}$ were not considered. However, all of these interactions become important[2] when considering excited steady states (and multi-stability) of the electron under the action of the sufficiently intense EM wave.

We introduce dimensionless notations:

$$\vec{\beta} = \frac{\vec{v}}{c} \ ; \quad \vec{\mu} = \frac{e\vec{E}_{in}}{m_0c^2}\,\frac{1}{k_0} \ ; \quad \rho(z) = \frac{eg(z)}{m_0c^2} \cdot \frac{1}{k_0} \ . \tag{1.8}$$

All these variables (as well as γ) are supposed to be very small compared to unity:

$$|\vec{\beta}| , |\vec{\mu}| , |\rho| , \gamma \ll 1 \qquad (1.9)$$

Using these conditions, and making use of definitions (1.8) and of the fact that under the assumed geometry the vectors $\vec{H}_0$, $\vec{k}$, $\vec{g}$, and $\vec{e}_z$ are parallel to each other, one can rewrite Eq (1.7) in the form:

$$\frac{1}{\omega_0}(1 + \frac{|\vec{\beta}|^2}{2})\frac{d\vec{\beta}}{dt} + \gamma\vec{\beta} = \vec{\mu} + \vec{\beta} \times \hat{e}_z + \hat{e}_z[\vec{\beta} \cdot \vec{\mu} - \rho(z)] \qquad (1.10)$$

where the term $|\beta|^2/2$ represents a small nonlinearity which is due to weak relativistic mass effect. We also assume that the EM wave $\vec{E}_{in}$ is circularly polarized (which maximizes the expected effect) and rotates in the same direction as the electron, i.e.

$$\vec{\mu} = \mu[\hat{e}_x\sin(\omega t - kz) + \hat{e}_y\cos(\omega t - kz)] . \qquad (1.11)$$

The required solution to Eq (1.10) can then be written in the form:

$$\vec{\beta}(t,z) = \beta[\hat{e}_x\sin(\omega t - kz + \phi) + \hat{e}_y\cos(\omega t - kz + \phi)] + \beta_z\hat{e}_z . \qquad (1.12)$$

By virtue of conditions (1.9), the unknown variables β, β_z, and ϕ vary little in the time ω^{-1}, which allows us to use slow-varying envelope approximation, i.e. to neglect their second-order time derivatives and higher harmonics, and to write down the set of truncated first-order equations:

$$\dot{\beta}/\omega_0 = -\gamma\beta + \mu \cos\phi \qquad (1.13)$$

$$\dot{\phi}/\omega_0 = \beta_z - (\Delta + \beta^2/2 + \mu \sin\phi/\beta) \qquad (1.14)$$

$$\dot{\beta}_z/\omega_0 = -\gamma\beta_z - \rho(z) + \mu\beta\cos\phi ; \qquad (1.15)$$

$$\dot{z} = c\beta_z \qquad (1.16)$$

Here $\Delta = (\omega - \omega_0)/\omega_0 \ll 1$; Δ is a dimensionless resonant detuning. The steady-state solution $(d/dt = 0)$ is thus determined by the relationships

$$\mu^2 = \beta_s^2[\gamma^2 + (\Delta + \beta_s^2/2)^2] \qquad (1.17)$$

$$(\beta_z)_s = 0 \;; \quad \rho(z_s) = \gamma\beta_s^2 \qquad (1.18)$$

$$\tan\phi_s = -(\Delta + \beta_s^2/2)/\gamma \qquad (1.19)$$

where the subscript "s" labels characteristics of the steady-state regime. It is seen from Eqs (1.17) and (1.19) that the steady-state kinetic energy $\beta_s^2/2$ does not depend on parameters of the force $\rho(z)$. This force cancels the radiation force of the incident wave and is small compared to all the other forces in the problem; e.g., in the region of interest $\rho \sim \gamma^2 \sim \beta_s^4$. It is obvious, though, that the characteristics of the transient regime, described by Eqs (1.13-1.16) depend on the spatial behavior of the force $\rho(z)$. For example, the frequency of transient oscillations along the z axis should depend upon gradient $d\rho/dz$ in the vicinity of $z = z_s$, Eq. (1.18). However, the magnitude of these oscillations at the onset of the hysteretic jumps can be assumed to be arbitrarily small, provided that the amplitude (or the frequency) of the incident wave is swept sufficiently slow to prevent the transient regime from masking the hysteretic behavior of the steady-state regime. Under the threshold conditions

$$\mu^2 > \mu_{th}^2 \equiv (16/3\sqrt{3})\gamma^3 \;, \quad \Delta < \Delta_{th} \equiv -\gamma\sqrt{3} \;, \qquad (1.20)$$

Eq. (1.17) yields a three-valued solution for β_s. The plot of steady-state kinetic energy $\beta_s^2/2$ as a function of resonant detuning Δ and incident intensity μ^2 is shown in Fig. 1. At the threshold point this value is $\beta_{th}^2 = 2\gamma/\sqrt{3}$ (curve 2 in Fig. 1), and the radius of orbit is $r = \beta_{th}/k_0 \ll \lambda_0$. In the case of the multivalued solution the examination of Eqs. (1.13-1.16), linearized in the close vicinity of the steady-state solutions (1.17-1.19), shows that only those states are stable which satisfy the energy criterion

$$d(\beta_s^2)/d(\mu^2) > 0 \qquad (1.21)$$

(solid branches of the curves in Fig. 1); otherwise, they become unstable (dashed

branches in Fig. 1). This meets the physical expectations, and leads to hysteretic behavior of the electron under conditions (1.20).

Let us make some quantitative estimates. A magnetic field of strength $H_0 = 100$ kG produces the cyclotron frequency $\omega_0 = 1.7 \times 10^{12}$ sec^{-1} ($\lambda_0 \sim 1.07$ mm). Then in the ultimate case of radiation losses, the resonance width is $\gamma \sim 10^{-11}$, which yields the threshold field amplitude to be as small as $E_{th} = 1.7 \times 10^{-1}$ V/cm, and the kinetic energy as small as $\beta_s^2/2 \simeq 1.2 \times 10^{-11}$. This is, in fact, even near α^{-1} times smaller than a quantum limit of the energy of excitation which is $2\hbar\omega_0/mc^2$ (here $\alpha = e^2/\hbar c = 1/137$ is the fine-structure constant). Therefore, in the close vicinity of the threshold (1.20), only the quantum approach[17] can give an adequate description of the phenomenon, whereas for sufficiently strong driving field ($\mu \gg \mu_{th}$) the classical results (and, in particular, hysteretic jumps) remain valid.

Very recently, the hysteretic cyclotron resonance of a free electron in vacuum has been first experimentally observed by G. Gabrielse et. al[3] using a single electron kept continuously in a Penning trap for more than 10 months. The hysteresis was so pronounced that it provided the best signal to noise ratio ever observed with a single particle in a trap. The cyclotron energy was measured via a second manifestation of the relativistic mass increase. The electron was weakly confined in a Penning trap and oscillated in a direction which is orthogonal to the cyclotron motion with a frequency that was measureably shifted in proportion to the electron's kinetic energy. The typical hysteresis observed in the experiment[3] is depicted at Fig. 2.

2. Bistable cyclotron resonance in semiconductors

Here we shall consider a bistable (hysteretic) cyclotron resonance in semiconductors[5]. This effect is feasible due to the nonparabolicity of the semiconductor conduction band which causes a pseudo-relativistic behavior[6,7] of the effective mass of conduction electrons in the narrow-gap semiconductors such as e.g. InSb.

The main features of this effect as compared with the free-electron case, are as follows:

i) The nonlinearity of conduction electrons in semiconductors is many orders of magnitude larger than the relativistic nonlinearity of the free electron. This allows one to attain a fairly low critical pumping intensity (although still much greater than in vacuum) for observation of the effect even taking into consideration the much faster relaxation in semiconductors.

ii) The effective mass of the electron in some semiconductors (e.g. InSb) is very small which results in a considerable increase of the cyclotron frequency (up to 70-80 times) as compared to the free electron for the same magnetic field (or allows one to correspondingly decrease the required magnetic field). In the case of InSb, this allows one to use the 10.6 μm radiation of a CO_2 laser with a magnetic field $H_o \sim$ 140 kG.

iii) In semiconductors, the nonlinear cyclotron resonance in the strong optic resonant field is accompanied by some small electrostatic potential between the faces of the semiconductor layer. This effect is due to the radiation force of the driving EM field and to the redistribution of electric charges in semiconductor. This potential exhibits a hysteretic behavior as well. Therefore, this effect could be the first proposed all-optical nonlinear phenomenon which results simultaneously in optic as well as opto-electronic instability. This property provides for an immediate electronic indication of the effect.

We shall demonstrate the feasibility of this effect using a classical model for the interaction of an optical wave with a single electron in the conduction band of a semiconductor. The thin semiconductor film is immersed in the homogeneous magnetic field H_o which is perpendicular to the layer. The semiconductor is also subject to the action of an EM wave $\vec{E}_{in}$ of amplitude E which propagates along the same direction as

the magnetic field (axis z), the same as in the free-electron case. However, now the small static electric field $\vec{g}$ which is directed along the z axis and compensates the radiation force, is not provided anymore by the external potential, but is caused by the background charge. This is due to a redistribution of the electron density between the faces of the semiconductor film, i.e. the problem becomes self-consistent as far as this potential is concerned.

In narrow-gap semiconductors which can be described by the Kane two-band model[6] with isotropic nonparabolic bands, the conduction band energy W (which is an analog of kinetic energy) can be written as

$$W(p) = \sqrt{m_o^{*2}v_o^4 + p^2v_o^2} \qquad (2.1)$$

where $\vec{p}$ is the momentum of the conduction electron, m_o^* is its effective mass at the bottom of the conduction band, $v_o = \sqrt{W_G/2m_o^*}$ is some characteristic speed, and W_G is the band gap (the energy W in Eq. (2.1) is measured with respect to the middle of the gap). The velocity $\vec{v}$ of the conduction electron is given by[18] $\vec{v}(\vec{p}) = \partial W(p)/\partial \vec{p}$. By virtue of Eq. (2.1), this yields

$$\vec{p} = m_o^*\vec{v}/\sqrt{1 - v^2/v_o^2} \; ; \; \vec{v} = \vec{p}/m_o^*\sqrt{1+p^2/p_o^2} \, , \qquad (2.2)$$

where $p_o \equiv m_o^*v_o = \sqrt{W_G m_o^*/2}$ is some characteristic momentum. One can see from Eqs. (2.1) and (2.2) that relations among W, v and p are completely relativistic, with v_o playing a role as an "effective speed of light," and $W_G/2$ as an "effective rest energy" of the electron. For InSb, $W_G = 0.24$ eV, $m_o^* = 0.014 \, m_o$ (where m_o is the rest mass of an electron in vacuum), so that $v_o \sim 1.15 \times 10^8$ cm/sec. Therefore, it is natural to look for applications of this property of conduction electrons in solid state analogous to those of free electrons. For example, since the relativistic mass-effect of the free electron in vacuum is a basic phenomenon for the cyclotron maser ("gyrotron")[19,15], it was recently proposed to develop a solid-state cyclotron maser[20] by exploiting the pseudo-

relativistic behavior of conduction electrons in semiconductors.

From Eqs. (2.1) and (2.2) one readily gets a formula for the effective mass of conduction electron $m^*(v)$:

$$m^* = m_o^*(1 - |v|^2/v_o^2)^{-1/2} \qquad (2.3)$$

which again has a relativistic form, see Eq. (1.3). Therefore, one can readily arrive at the equation of the electron motion, Eq. (1.10), where now $\vec{\beta}$ is interpreted as $\vec{\beta} = \vec{v}/v_o$, $\omega_o = eH_o/m_o^*c$, $k_o = \omega_o\sqrt{\epsilon(\omega_o)}/c$, and all the rest of variables in Eq. (1.8) are defined through m_o^* instead of m_o. This immediately leads to the truncated dynamic equations (1.13)-(1.16), and therefore - to the equations (1.17)-(1.19) for the steady-state regime and the threshold conditions (1.20). The only new feature is that now, since the quasi-static electric field g (or variable $\rho=eg/m_o^*ck_o$) is developed by the redistribution of electrons inside the film, these equations are to be supplemented by the equation for the field g. Strictly speaking, the single-electron model cannot provide the required equation. The complete theory of the dynamics of the system can be developed only in a framework of a kinetic approach which takes into consideration the distribution function of electrons in the system. However, in order to get results for the steady-state regime as well as equations for the small perturbations of that regime, one can use these arguments. By the analogy with other effects in which the redistribution of charges is caused by an external force (e.g. as in Hall effect, or in the case when an external electrostatic field is applied to the film), it is easy to understand that the distribution of electrons remains almost homogeneous inside the film such that no free bulk charges occur while the induced charges are localized at the surfaces of the film. The thickness of the charged surface layers is of the order of the Debye's radius r_D, which is fairly small (e.g. for $\epsilon \sim 5$ and $N_o \sim 0.5 \cdot 10^{15}$ cm^{-3} where N_o is the concentration of electrons, one has[18] $r_D \sim 10^{-5}$ cm at room temperature). The entire film then can be considered as a capacitor with capacitance $C = \dfrac{\epsilon}{4\pi d}$ per unit area (where d is its

thickness) whose potential U determines the field g: $g = U/d = q/Cd$. Here, the electrical charge per unit area q is related with the current J per unit area by the relationship $dq/dt = J = \sigma E_z$, where σ is a conductivity of the material, and $E_z = \sqrt{\epsilon}E(v/c)\cos\phi - g$.

From these relationships one gets $q = Cdg = \epsilon g/4\pi$, or, after taking time derivative, $\epsilon \dot{g}/4\pi = \dot{q} = J = \sigma E_z$. Making use of Eq. (1.15), one finally gets the required equation for g:

$$\tau_\sigma e\, dg/dt = m_o^*(<\dot{v}_z> + \omega_o<v_z>) = e < E_z > \tag{2.4}$$

where $\tau_\sigma = \epsilon/4\pi\sigma$ is a relaxation time for the redistribution of the charge, and angle brackets denote an averaging over ensemble of electrons. The steady-state magnitude of g (or ρ), however, is still determined by Eq. (1.18), i.e. $\rho_s = \gamma\beta_s^2$, or

$$eg_s = m_o^* c\, k_o \gamma\beta_s^2. \tag{2.5}$$

The threshold conditions (1.20) with application to the amplitude of the required critical field amplitude E_{th} gives

$$\left| eE \right|^2 > \left| eE_{th} \right|^2 = \frac{8}{3\sqrt{3}} W_G\, m_o^* \gamma^3 \omega_o^2; \quad \omega_o - \omega > \gamma\sqrt{3} \tag{2.6}$$

Let us make some qualitative estimates for the threshold intensity E_{th}^2 of the incident wave. In the case of InSb, $m_o^* = 0.014\, m_o$, $W_G = 0.24$ eV. The damping rate γ depends primarily on the concentration of impurities. We assume it to be $\gamma \sim 10^{-2}$ for $\lambda_0 \sim 10.6\ \mu$m, if a CO_2 laser is used as a source of the driving radiation. The magnetic field required to get a cyclotron resonance in the neighborhood of $\lambda \sim 10.6\ \mu$m is $H_o \sim 140$ kG, and Eq. (2.6) provides for a critical field required to observe hysteresis: $E_{th} \sim 300$ V/cm. This corresponds to the incident power of about 240 W/cm² which can be readily attained.

Let us also estimate the quasi-static potential $U = dg$ (d is the thickness of the layer) which accompanies the phenomenon. From Eq. (2.5), one can see that the field g

must experience hysteresis as long as the velocity β_s does. At the point of higher excitation where $\beta_{max} \sim \mu/\gamma$ [see Eq. (1.13)], this gives

$$g_{max} \simeq eE^2\omega_o\sqrt{\epsilon}/\gamma m_o^* c \qquad (2.7)$$

In the above instance, if the pumping intensity is slightly above the threshold E_{th}^2 (for instance, $E = 500$ V/cm), one gets $g_{max} \sim 1$ V/cm. For a thickness of the layer $d \sim 0.1$ cm this gives $U = dg \sim 0.1$ V which can be readily measured. The relaxation time for the potential U is of order of $\tau_\sigma = \epsilon/4\pi\sigma$.

The presence of two different relaxation times (τ_σ and $\gamma^{-1} \cdot \omega_o^{-1}$) in the system can cause even more complicated behavior which depends on the ratio of these times i.e. quantity $\tau_\sigma\gamma\ \omega_0$. In the limit $\tau_\sigma \to 0$, i.e. if $\tau_\sigma\gamma\ \omega_0 << 1$, it can be shown that two states out of the three possible steady-states (in the case of a multi-valued solution) are stable with regard to small perturbations. These two stable steady-states must satisfy the same condition (1.21) as for free electrons, i.e. those are the states which belong to upper and lower branches of hysteresis. However, if $\tau_\sigma\gamma\ \omega_0 \sim 1$, there could be some domains on the upper branch (which are close to the onset of the reverse jump from the upper branch to the lower one) where these steady states becomes unstable. This may result in excitation of self-oscillations and possibly chaotic motion. Using the usual formula for conductivity [18] $\sigma = eNM$ where N is the concentration of conduction electrons and $M = e\omega_o/m_o^*\gamma$ is their mobility, one gets the expression for parameter $\tau_\sigma\omega_o\gamma$:

$$\tau_\sigma\gamma\ \omega_o = \frac{\epsilon m_o^*\omega_o^2}{4\pi e^2 N} = \left(\frac{\omega_o}{\omega_p}\right)^2; \quad \omega_p = \left(\frac{4\pi e^2 N}{\epsilon m_o^*}\right)^{1/2} \qquad (2.8)$$

where ω_p is the plasma frequency [18]. In the above mentioned instance of InSb with $\gamma \sim 10^{-2}$ this yields $\tau_\sigma\omega_o\ \gamma \sim 10^{18}/N$ where N is expressed in cm^{-3}. One must note, though, that a more comprehensive theory of stability (as well as of the entire phenomenon) has to involve consideration of cooperative motion of electrons, i.e. the kinetics of distribution functions rather than the motion of a single electron.

3. Bistable noncyclotron resonance of a slightly - relativistic electron

In the above sections, the bistable effect was discussed which is based on a rotation of a slightly - relativistic electron in a magnetic field (i.e. on a nonlinear cyclotron effect). However, it is obvious that the presence of a magnetic field is not a necessary condition; the only factors substantial for the bistable resonant excitation to exist are presence of nonlinearity (in our case - relativistic mass-effect) and sufficiently sharp resonance. The latter one can be provided by any potential well. In the simplest and probably most characteristical case it is one-dimensional parabolic potential well (which would correspond to a conventional linear oscillator has the electron mass not varied due to relativistic effect). We assume that the electron oscillates along the x axis, and is excited by the driving periodic electric field $E_{in} = E \cos \omega t$, which is directed along the same axis (for the sake of simplicity, we shall neglect here the action of the radiation force, since it does not affect directly the steady-state regime, see Section 1), and that the motion is only slightly relativistic (i.e. $\dot{x}^2 \ll c^2$). In such a case, the motion of the electron is governed by the equation

$$\ddot{x} + 2\gamma\omega_o\dot{x} + x\omega_o^2(1 - \dot{x}^2/2c^2) = \frac{eE}{m_o} \sin \omega t \qquad (3.1)$$

where ω_o is an eigenfrequency of a respective linear oscillator in the potential well. We use the same approximation as in Section 1 and look for the forced solution of Eq (3.1) in the form

$$x(t) \approx v_m(t) \, \omega^{-1} \sin(\omega t + \phi) \qquad (3.2)$$

where the maximal speed of electron v_m and its phase ϕ are slow varying functions of time. We introduce again a dimensionless variables

$$\beta_m = \frac{v_m}{c} ; \quad \Delta = \frac{\omega - \omega_o}{\omega_o} , \quad \mu = \frac{eE}{m_oc^2} \cdot \frac{1}{k_o} \qquad (3.3)$$

and assume that all of them are much smaller than unity. Then, in the same fashion as

in Section 1, one gets a truncated equation for their dynamics:

$$\dot{\beta}_m/\omega_o = -\gamma\beta_m + \mu \cos \phi/2 \tag{3.4}$$

$$\dot{\phi}/\omega_o = -(\Delta + \beta_m^2/32 + \mu \sin \phi/2\beta_m) \tag{3.5}$$

which (with modified coefficients and $\beta_z = 0$) reproduce respectively Eqs (1.13) and (1.14). The steady-state regimes ($d/dt = 0$) follows immediately:

$$\mu^2 = 4\beta_m^2[\gamma^2 + (\Delta + \beta_m^2/32)^2] \tag{3.6}$$

which again reproduces Eq (1.17) with modified coefficients. By introduction $\tilde{\mu} = \mu/8$, and $\tilde{\beta}_s = \beta_m/4$, Eq (3.5) is reduced exactly to Eq (1.17) for $\tilde{\mu}$ and $\tilde{\beta}_s$. Therefore, taking into consideration Eqs (1.20) and (3.3), the threshold conditions required in order to observe bistability of exitation of slightly - relativistic oscillator (3.1) are

$$\left| eE \right| > 32 \ k_o m_o c^2 (\gamma/\sqrt{3})^{3/2} \ ; \quad \omega - \omega_o > \omega_o \gamma \sqrt{3} \ . \tag{3.7}$$

If the damping rate γ is again due to radiation of EM wave by the electron, the required threshold intensity of the driving field is of the same order of magnitude as that one discussed in Section 1 for the bistable cyclotron resonance, in vaccuum.

Conclusion

In conclusion, we demonstrated the feasibility of hysteretic behavior and bistability of cyclotron resonance of free electrons in vacuum and conduction electrons in semiconductors under the action of sufficiently strong quasiresonant driving radiation. We showed also that the same effect must be peculiar for a conventional (i.e. noncyclotron) resonance of a slightly - relativistic electron in a one-dimensional potential well. Future research should involve quantum as well as kinetic theory of the phenomenon. Even far from the onset of the hysteresis, the action of the strong pumping should cause a dramatic change in the location and the shape of the resonant curve of cyclotron

resonance; in particular, the dependence on the frequency (or magnetic field) should become drastically asymmetrical. This effect may provide a new experimental method to measure the nonparabolicity of the conduction band in semiconductors, effective mass, nonlinear relaxation, etc. Probably, the most attractive and fundamental feature of all these effects is that for the first time they provide an unique opportunity to experimentally (and theoretically) study hysteretic and bistable phenomenon at a quantum level.

Acknowledgement

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Captions to figures

Fig. 1.   The plots of normalized kinetic energy of the electron $\beta^2/2$ (a) vs normalized resonant detuning Δ/γ for various intensities of incident EM field, and (b) vs normalized incident intensity $\mu^2/2\gamma^3$ for various detunings. Curves: (a) 1, $\mu^2/2\gamma^3 = 1$; $\mu^2/2\gamma^3 = (2\sqrt{3})^3$; 3, $\mu^2/2\gamma^3 = 3$; (b) 1, $\Delta/\gamma = 0$; 2, $\Delta/\gamma = -\sqrt{3}$; 3, $\Delta/\gamma = -3$.

Fig. 2.   Experimentally observed[3] axial frequency shift (left vertical scale) as a function of the frequency of the driving force when this frequency is swept downward (2a) and upward (2b) through resonance. The right vertical scale is the kinetic energy in the cyclotron motion in eV.

United States Patent [19]

Kaplan et al.

[11] **4,410,239**

[45] **Oct. 18, 1983**
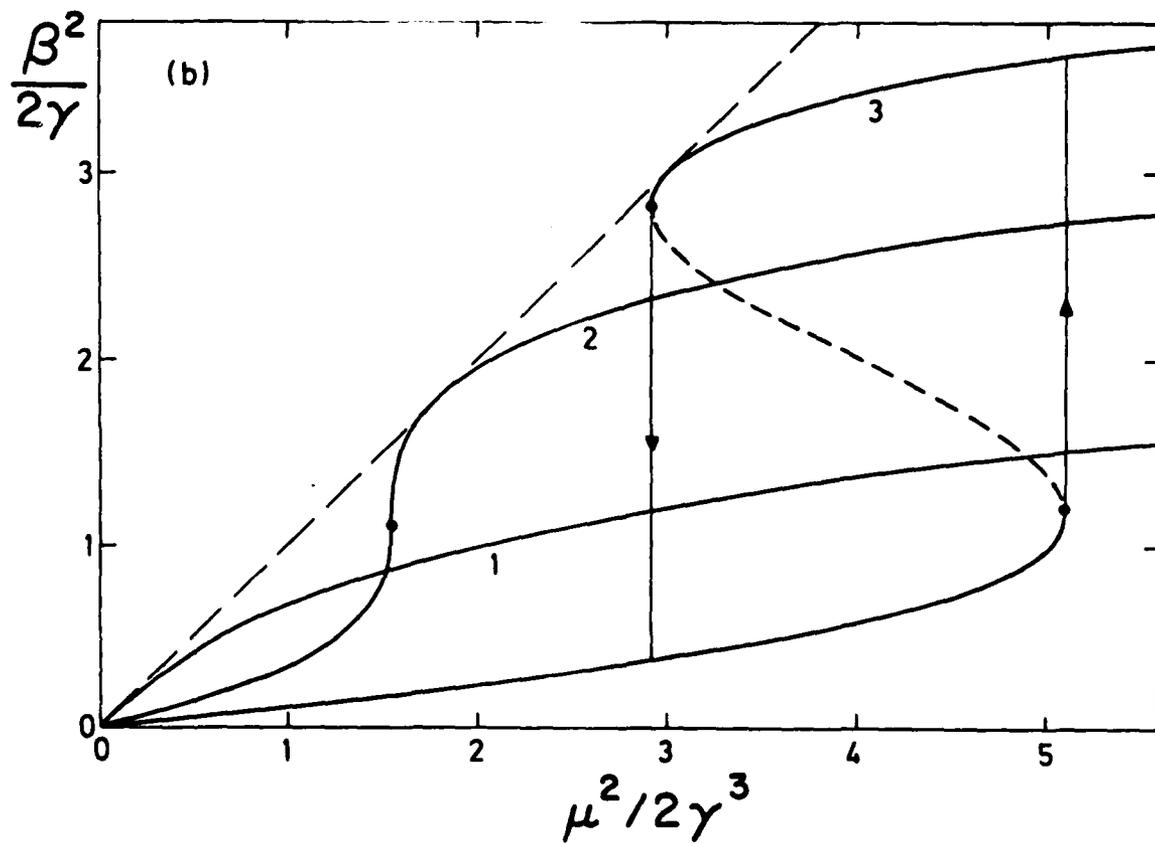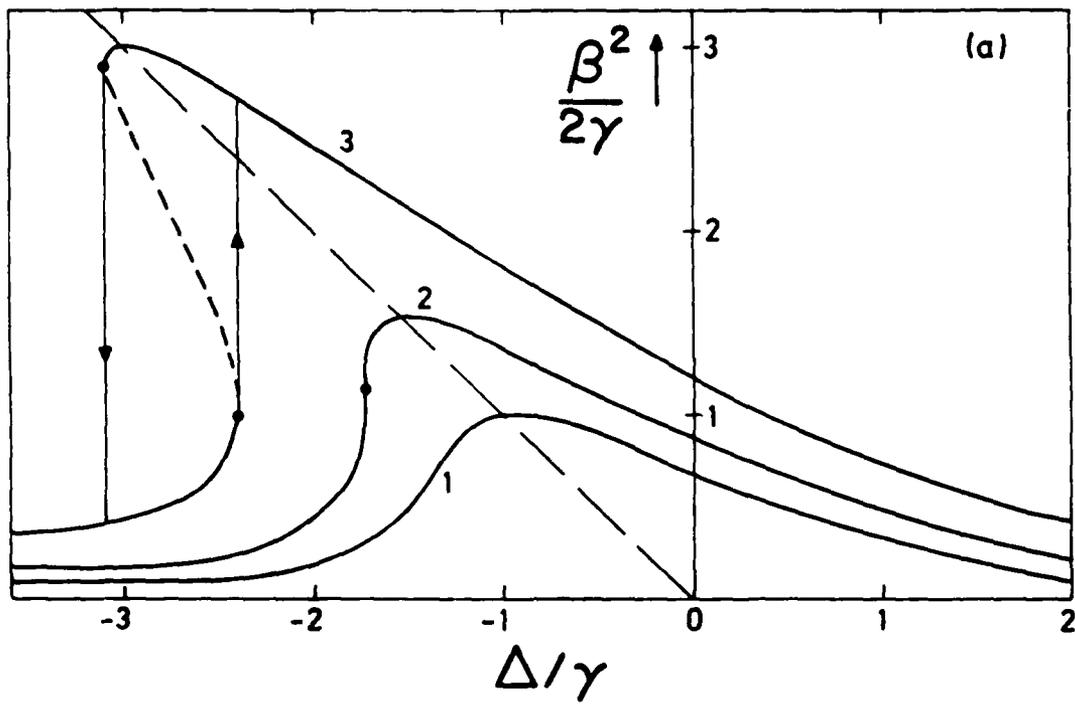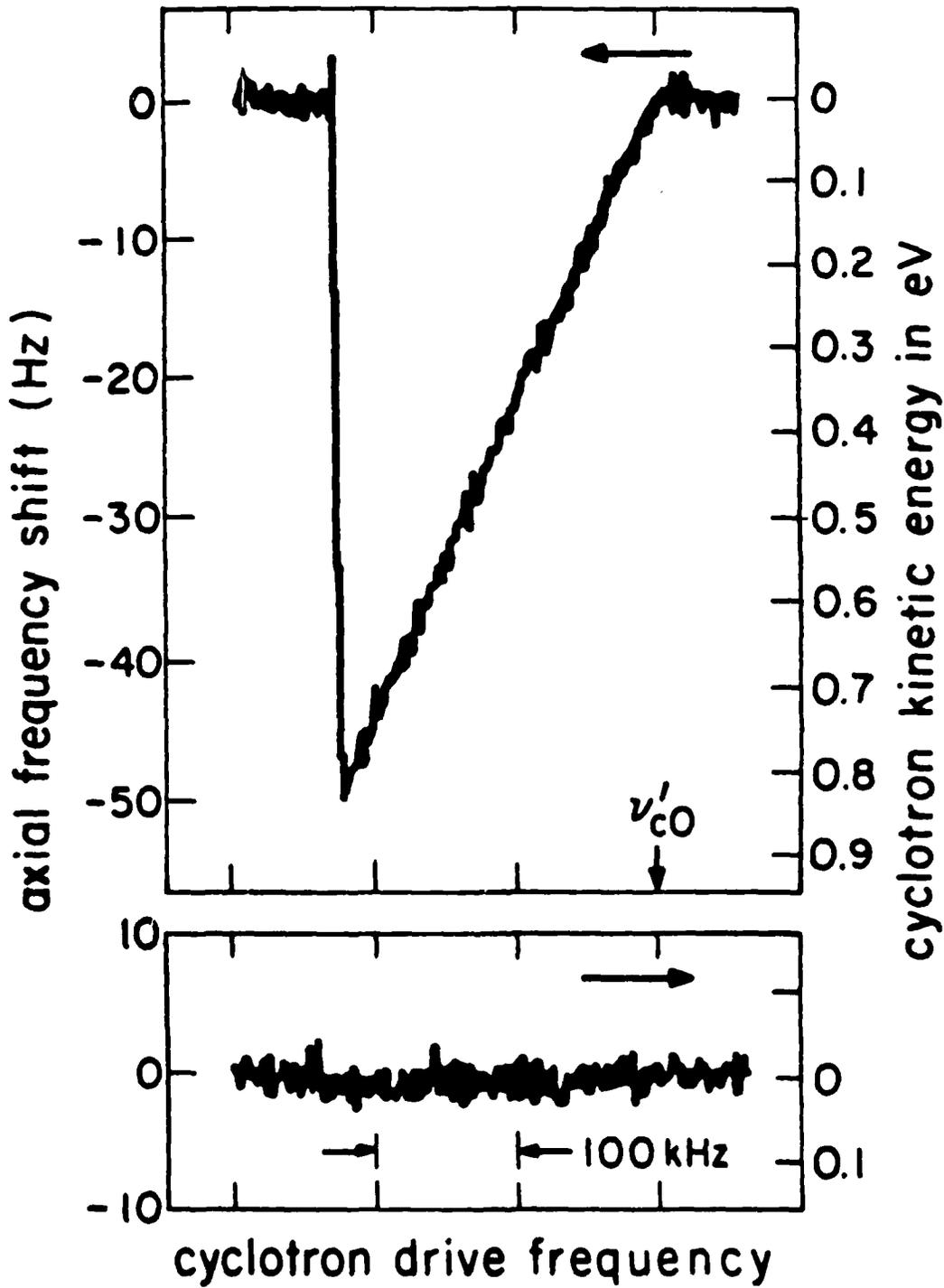
[54] **NONLINEAR OPTICAL DEVICE USING SELF-TRAPPING OF LIGHT**

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[21] Appl. No.: **255,291**

[22] Filed: **Apr. 17, 1981**

[51] Int. Cl.³ G02F 1/35
[52] U.S. Cl. 350/354; 350/385; 350/390; 350/393; 372/99
[58] Field of Search 350/379, 385–386, 350/375–376, 390–391, 393, 354, 356, 162.17; 372/103, 1, 96, 99

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Attorney, Agent, or Firm—Daniel D. Dubosky; Erwin W. Pfeifle

[57] **ABSTRACT**

Many prior art bistable optical devices require resonant optical cavities and are therefore limited in their opera-tion due to the long lifetimes associated with their high-finesse cavities. A bistable optical device that does not use a resonant cavity is disclosed wherein a nonlinear medium whose index of refraction increases with in-creased light intensity is arranged to have input and output faces into which and out of which a laser beam having a nonuniform spatial profile can be propagated. A mirror having a predetermined area of reflectivity is positioned with respect to the output face of a nonlinear medium so as to reflect only the light energy that propa-gates in an area at the output face that is approximately equal to the area which the beam presents at this face when the beam is propagating at a critical power level, that is, when the beam is self-trapped.

5 Claims, 2 Drawing Figures

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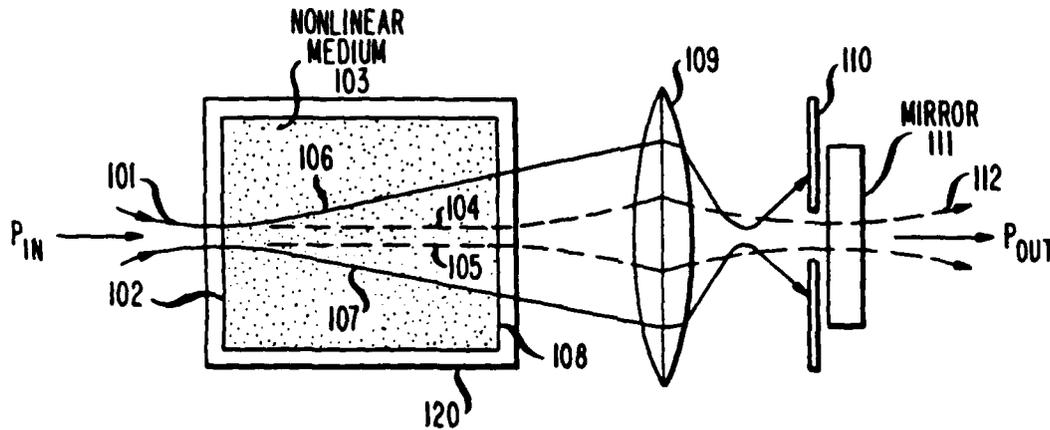
NONLINEAR OPTICAL DEVICE USING SELF-TRAPPING OF LIGHT

BACKGROUND OF THE INVENTION

The study of optically bistable devices and their generic characteristics has received increased attention in the scientific community. See, for example, the special issue on optical bistability *IEEE Journal of Quantum Electronics*, QE-17, March 1981. Bistable optical devices are often classified into one of two categories. In the first category are intrinsic devices which are those devices in which the feedback required for bistability is optical. In the second category are hybrid devices in which devices some form of electrical feedback, sometimes in conjunc n with optical feedback, is used. Intrinsic devices are of particular interest because of their potential for ultra-fast switching.

Many of the intrinsic devices known in the prior art require resonant optical cavities. As a result of this characteristic, these prior art intrinsic devices require that the input light to the device be tuned to a special frequency. In addition, these devices are frequently sluggish in their operation due to the long lifetimes associated with their high-finesse cavities. It is expected that the operation of an intrinsic optically bistable device could be improved if the device did not require a resonant cavity.

SUMMARY OF THE INVENTION

The present invention is based upon a fundamentally new type of intrinsic optical bistability. The operation of devices using the present invention is based on self-focusing of light. Self-focusing occurs when a light beam having a nonuniform spatial profile, such as a Gaussian laser beam, progagates through a nonlinear medium having an index of refraction that increases with increased light intensity. When the light intensity increases to a critical power level, P_{cr}, the input laser beam passes through the medium with no change in spot size, and this situation is referred to as self-trapping.

In accordance with the present invention a nonlinear medium whose index of refraction increases with increased light intensity is arranged to have input and output faces into which and out of which a light beam having a nonuniform spatial profile can be propagated. A mirror is combined with the output face of the nonlinear medium so as to reflect only the light energy that propagates in an area at the output face that is approximately equal to the area which the beam presents at this face when the beam is propagating at the critical power level, that is, when the beam is self-trapped. When beam power has been increased to the critical power level, a substantial fraction of the beam will be reflected back into the nonlinear medium. The beam intensity can then be decreased to a lower power level and the beam will remain self-trapped because of the light energy that is reflected from the mirror, and the device will therefore exhibit a bistable optical characteristic.

In the embodiment which was constructed, a lens is positioned behind the output face of the nonlinear medium in order to image the light at the output face of the medium onto a light absorbent disk having an aperture approximately equal in size to the spot size that the beam would have at the disk when the beam is self-trapped. A mirror is positioned on the side of the optically absorbent disk that is opposite to the propagating

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beam. The mirror is selected to be partially transmitting in order to provide an output optical beam.

BRIEF DESCRIPTION OF THE DRAWING

The invention will be more readily understood after reading the following detailed description in conjunction with the drawing wherein:

FIG. 1 is a schematic block diagram of an apparatus constructed in accordance with the present invention, and

FIG. 2 is a graph of optical output power versus input power for the device shown in FIG. 1.

DETAILED DESCRIPTION

The basic principles upon which the new class of device operates can be understood from a discussion of the apparatus shown in FIG. 1. A laser beam 101 of power, P_{in}, having a Gaussian intensity profile is focused onto the input face 102 of a nonlinear medium 103 having an intensity dependent index of refraction. For self-focusing to be possible the medium must be chosen such that its refractive index increases with increasing light intensity. A measure of the strength of this nonlinearity is called the "critical power," P_{cr}. When $P_{in} = P_{cr}$, the input laser beam passes through the medium with no change in spot size, as shown by dashed lines 104 and 105; this situation is referred to as "self-trapping." For $P_{in} < P_{cr}$ the laser beam diverges less rapidly than it would in the absence of the nonlinearity and for $P_1 > P_{cr}$ the beam converges. For $P_1 < < P_{cr}$ there is essentially no self-focusing; the input focal spot size is chosen such that, for these conditions, the laser beam diverges appreciably in passing through the nonlinear medium, as shown by the solid lines 106 and 107.

The optical field at output face 108 of the nonlinear medium 103 is imaged by a lens 109 onto the partially transmitting mirror 111 which is aligned normal to the laser beam. Immediately in front of the mirror is an appropriately-sized optically absorbent disk 110 having an aperture positioned to permit the self-trapped beam to pass through to the mirror 111. The aperture size is adjusted to satisfy two criteria. First, it must be small enough that in the absence of self-focusing the fraction of the incident light feedback into the nonlinear medium by the mirror is small. Secondly, it is large enough that under self-trapping conditions essentially all the light passes through the aperture and is reflected back upon itself by the mirror. This strong feedback reinforces the self-focusing in the nonlinear medium and it allows self-trapping to be maintained even when the input power is reduced below P_{cr}. This is the mechanism which gives rise to the optical bistability and hysteresis. The aperture and mirror could be combined and deposited on the exit face of the nonlinear medium to minimize device length and optical transit times. Clearly other arrangements can be devised by those skilled in the art which lead to optical bistability using the same basic principles.

In the embodiment which was constructed, atomic sodium vapor was utilized as the nonlinear medium 103. Sodium vapor has the virtue of having a large nonlinear index for frequencies near its resonance transitions. In addition, steady-state self-focusing and self-trapping have been observed in it using cw dye lasers. See the article entitled "cw Self-Focusing and Self-Trapping of Light in Sodium Vapor," by J. E. Bjorkholm and A. Ashkin, *Physical Review Letters*, Vol. 32, pages 129-132,

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Jan. 28, 1974. The sodium vapor was contained in a 20-cm-long heated cell 120 constructed of pyrex. The input laser beam 101 was obtained from a single-mode cw ring dye laser (Spectra-Physics Model 380A). The transverse mode of the laser was TEM$_{00}$ (Gaussian mode) and its focal spot size on the input face 102 of the medium 103 was approximately 80 μm; the corresponding confocal parameter of the laser beam was about 6.8 cm so that in the absence of self-focusing the spot size on the output face 108 of the medium was about 480 μm. A linear polarizer and a quarter wave plate situated between the laser and the vapor cell was used as an isolator; thus circularly polarized light was incident onto the cell. Lens 109 was a 16-cm focal length lens positioned to image the optical field at the exit face 108 of the medium, with unity magnification, onto flat mirror 111 having a reflectance of 94 percent. A disk 110 having an aperture with a diameter of approximately 150 μm was placed several mm in front of the mirror.

Without feedback from the mirror strong self-focusing and self-trapping were readily observed with approximately 150 mW of light tuned roughly 1 GHz above the resonant frequency of the $3S_1(F=2) \rightarrow 3P_1$ transition at 5896 Å. The sodium density was nominally 2×10^{12} cm^{-3}. To observe bistability the mirror was aligned normal to the laser beam and the input light was amplitude modulated at about 50 Hz with a spinning transmission grating composed of closely spaced fine wires. The input power (P_{in}) of beam 101 and the power of beam 112 passing through the mirror 111 (P_{out}) were monitored as functions of time using photodiodes and a dual-channel digital oscilloscope (Nicolet Explorer II); the signals were also displayed and recorded as P_{out} vs. P_{in}.

FIG. 2 presents data obtained when the parameters of the device were adjusted to exhibit bistable behavior; the laser was tuned roughly 1 GHz above the $3S_1(F=2) \rightarrow 3P_1$ transition. As the input power was increased upward, switching from a low-transmission state to a high-transmission state occurred at an input power of about 130 mW. At the switching point, the transmission abruptly increased by a factor of 4.1; the rise-time of the switch was about 20 μsec. With input power decreasing, a downward switch of the same speed occurred at about 95 mW of input power. Optical pumping of sodium undoubtedly plays a role in the nonlinearity of the medium and the observed switching time may be more characteristic of optical pumping than of the switching process itself. The dashed lines show the calculated low and high transmission limits, based on the measurement of 45 mW incident onto the 0.15 mm aperture for an input power of 140 mW and no feedback. The high transmission limit assumes 100 percent transmission by the aperture. The upward switch occurs at a power level roughly equal to P_{cr}. Visual observation of the resonance fluorescence induced by the laser beam showed dramatically that, at switching, the laser beam abruptly changes from diverging propagation to what appeared to be self-trapping. Under different conditions several switching levels and several hysteresis loops were obtained, perhaps corresponding to oscillations of the spot size as the laser beam propagated through the self-focusing medium.

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As will be readily apparent to those skilled in the art, a device that exhibits the bistable characteristic of the type shown in FIG. 2 can function as a memory element. For example, the device can be biased at an input power level of P_b at a point substantially midway between the two step changes shown in FIG. 2. At this bias level of P_b, the output power level is dependent on the previous history of input power levels. If the input power level is increased to a value in excess of 130 mW and then returned to the bias level of P_b the device will operate at the point designated as 202 in FIG. 2 and an output power level of about 1.9 mW will be present. If on the other hand the input power level is decreased to a value below 95 mW and then returned to the bias level of P_b the device will operate at a point corresponding to 201 in FIG. 2 and an output power level of about 0.5 mW will be present. In this way, the device remembers what the previous input power level has been and can therefore function as an optical memory.

What has been described hereinabove is an illustrative embodiment of the present invention. Numerous departures may be made by those skilled in the art without departing from the spirit and scope of the present invention. For example, if the medium 103 has a sufficiently strong nonlinearity, the entire device may be scalable to short lengths giving the potential for fast response times. By focusing the input laser beam in such a device to a spot size on the order of λ, the wavelength of the light, device lengths on the order of 50λ with corresponding transit times for the light of about 0.2 psec, should be possible.

What is claimed is:

1. A nonlinear bistable optical device for use with an input light beam having a nonuniform spatial profile comprising a nonlinear medium (103) having an input face (102) and an output face (108), said nonlinear medium having an index of refraction that increases with increasing light intensity, and means (109, 110 and 111) including a reflective central aperture positioned at the output face of said nonlinear medium for reflecting light back into the medium only from a limited central area of said output face.

2. A nonlinear bistable optical device as defined in claim 1 wherein said limited central area has a diameter approximately equal to the diameter of an optical beam propagating within said nonlinear medium at the self-trapping power level.

3. A nonlinear bistable optical device as defined in claim 1 wherein the means for reflecting light comprises an optically absorbent disk (110) having an aperture approximately equal to the diameter of an optical beam propagating within said nonlinear medium at the self-trapping power level, and a partially transmitting mirror (111) positioned adjacent to said disk with the disk between said medium and said mirror.

4. A nonlinear bistable optical device as defined in claim 3 wherein said means for reflecting light further includes a lens (109) positioned between said disk and said medium.

5. A nonlinear bistable optical device as defined in claim 4 wherein said nonlinear medium is an atomic sodium vapor contained in a heated cell (120) constructed of pyrex.
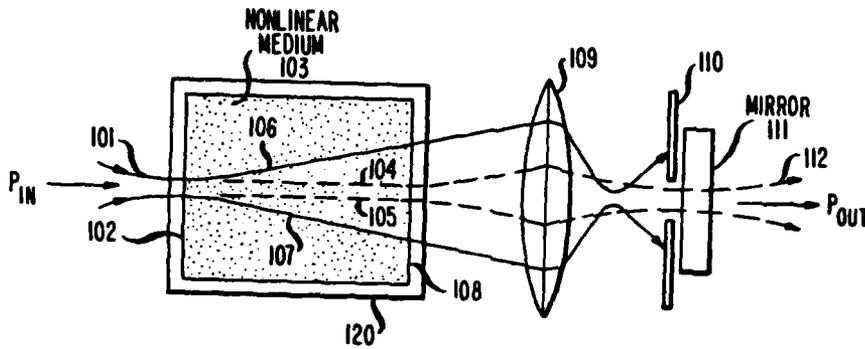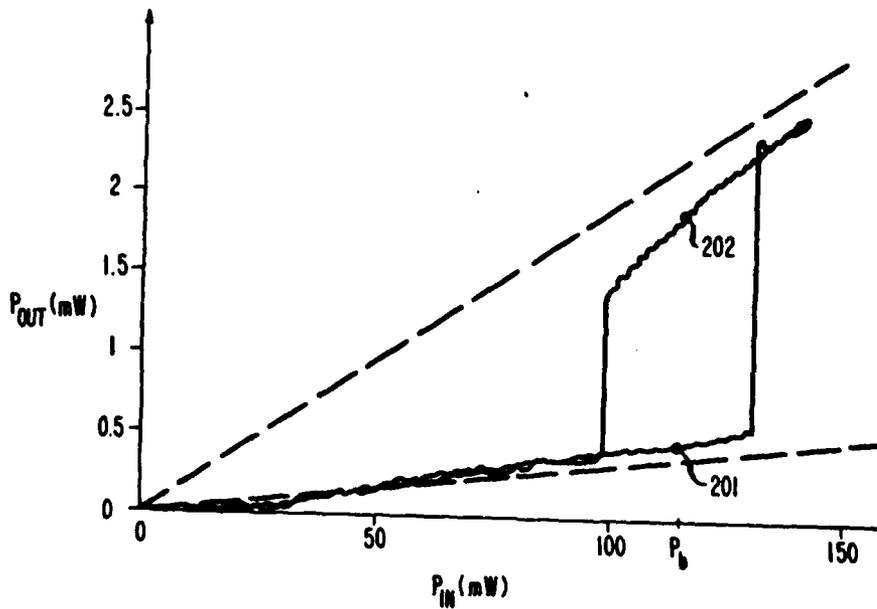
* * * * *

FIG. 1

FIG. 2

Extreme-Ultraviolet and X-ray Emission
and Amplification by Non-relativistic Electron Beams
Traversing a Superlattice

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Abstract

High-energy electrons emit resonant electromagnetic radiation when passing through a spatially periodic medium. It is conventionally assumed that ultra-relativistic electron beams are required to obtain significant emission. We demonstrate theoretically the feasibility of exploiting solid-state superlattices with short periods to obtain both spontaneous and stimulated emission in the far-ultraviolet and soft X-ray range using non-relativistic beams.

Introduction

Fast-moving electrons emit electromagnetic waves when moving from one medium into another with a different dielectric constant [1-3]. This is known as transition radiation and was predicted by Ginzburg and Frank [1]. In a spatially periodic medium the waves emitted at different interfaces interfere so that a resonant emission is obtained when the following condition is satisfied [2-4]:

$$\sqrt{\bar{\epsilon}} \cos\theta = c/v - n\lambda/\ell \tag{1}$$

where λ is the wavelength of radiation, ℓ is the period of the spatially varying dielectric constant ϵ of medium (it is conventionally assumed that the variations are very small), v is velocity of the electrons (assumed normal to the interfaces), θ is the angle between the direction of wave propagation and electron motion, n is an integer, and $\bar{\epsilon}$ is a "mean" ϵ. This condition is readily derived by requiring that the waves emitted at different interfaces interfere constructively at a distant point. Usually the period $\ell >> \lambda$, so that ultra-relativistic beams ($v/c \simeq 1$) are required in order to satisfy Eq(1) for real θ. Recently the possibility of stimulated resonance radiation of ultra-relativistic (~ 50 Gev) electrons traveling through a stack of metal foils was considered [4], with $\ell \sim 7$ cm.

In this paper, we demonstrate the feasibility of using non-relativistic electron beams in order to attain both spontaneous and stimulated emission in the ultraviolet and X-ray range using solid-state superlattices with $\ell \sim 100$ Å so that $\ell/n\lambda \sim 1$. We show that the wavelength of resonant radiation and the required energy of electrons are determined by the parameter $Q = n\lambda_p/\ell$, where λ_p is a "mean" plasma wavelength of the medium. If $n\lambda_p << \ell$ (i.e. $Q << 1$), as is assumed in all previous work [2-4], then the wavelength of the resonant radiation has an order of magnitude of $\lambda \sim \lambda_p Q$, and the kinetic (dimensionless) energy of the electrons eU/mc^2 must exceed the critical amount $\sim 1/\sqrt{Q^2-\theta^2} >> 1$ ($\theta < Q$) which constitutes the use of ultra-relativistic beams. On the contrary, if the period of the spatially periodic medium is chosen so that $Q >> 1$ (i.e. period ℓ is much shorter than the plasma wavelength, e.g. $\ell \sim 50\text{-}200$ Å), the wavelength of resonant radiation becomes of the order of $\sim \lambda_p/Q = \ell/n$, and the critical kinetic energy of the beam turns

out to be extremely small: $eU/mc^2 \sim 1/2Q^2$, which may be less than a few kilo-volts even for very short wavelength radiation. The advantages of the proposed method are: (1) the frequency of radiation can be easily tuned in a very wide range by simply varying accelerating potential of beam (which is very hard to do with ultra-relativistic beams) (2) the range of the possible angles of the wave propagation is almost unlimited, and (3) the cost of equipment and energy required for experiments with non-relativistic beams is insignificant compared to large accelerating machines. This last consideration is perhaps the most important.

The main requirements for non-relativistic short-wavelength resonant radiation is a periodic medium with a very short spatial period. Fortunately, the development of molecular beam epitaxy (MBE) and other techniques in recent years has made it possible to grow very thin films (~ 100 Å and less) with precise boundaries. Periodic structures composed of thin films of different materials, in particular superlattices, have also been fabricated [5]. Using these structures, non-relativistic electron beams with energies 20-200 KeV can be used to generate radiation of wavelength 100-200 Å and less. The concept of EM radiation of electron beam in spatially periodic structures in general is known since microwave traveling wave tube amplifiers (with nonrelativistic beams) and their recent optical modification - free-electron lasers (with relativistic beams). An important feature of transition radiation discussed in this paper is that the electron beam travels *through* the material structure rather than in vacuum *above* the structure (as in traveling wave tubes), or through a spatially modulated magnetic field (as in many free-electron lasers). This makes it possible to use very short spatial periods. The problem is the energy transfer from the electron beam to the material structure causing heating and possible damages. This problem will be briefly addressed at the end of this paper.

In this paper, we show that resonant spontaneous emission with a total power of 0.1 mW (around a wavelength ~ 200 Å) can be obtained with a 75 KeV electron beam carrying a current of only 1 mA. The spontaneous emission can be used as a narrowband source by selecting the radiation in a narrow angular range. To get stimulated emission with a gain of 5% per pass, however, requires a significantly larger current (5×10^{10} A/cm^2); to avoid sample burnout it will be necessary to use pulsed operation with pulse lengths ~ 0.1 ps, which is a difficult task. However, it should be noted that coherent sources are not available at these short wavelengths and getting significant stimulated emission is a difficult problem in general.

Spontaneous Transition Emission

We will first obtain the resonant wavelength of radiation from Eq(1) noting that [3] $\ell\sqrt{\bar{\epsilon}} = \ell_1\sqrt{\epsilon_1} + \ell_2\sqrt{\epsilon_2}$, where $\ell_{1,2}$ and $\epsilon_{1,2}$ are respectively the thicknesses and dielectric constants of alternating layers forming the superlattice, $\ell = \ell_1 + \ell_2$ is its spatial period. For short wavelengths ($\lambda^2 << \lambda_{1,2}^2$), $\epsilon_{1,2} = 1 - \lambda^2/\lambda_{1,2}^2$ where $\lambda_{1,2}$ are the plasma wavelengths of the two materials forming the superlattice, ($\lambda_{1,2}^2 = 4\pi mc^2/e^2 N_{1,2}^e$ where N^e is the density of electrons). Thus, the mean dielectric constant may be written as $\bar{\epsilon} \simeq 1 - \lambda^2/\lambda_p^2$, where $\lambda_p^{-2} = \ell^{-1}(\ell_1\lambda_1^{-2} + \ell_2\lambda_2^{-2})]$. Substituting this into (1) and solving it for λ, one gets the resonant wavelengths:

$$\lambda = \lambda_p \frac{Q}{Q^2 + \cos^2\theta} \left[\frac{1}{\beta} \pm \frac{\cos\theta}{Q} \sqrt{(\gamma_{cr}^2 - 1)^{-1} - (\gamma^2 - 1)^{-1}} \right], \qquad (2)$$

where $Q = n\lambda_p/\ell$; $\beta = v/c$; $\gamma = (1 - \beta^2)^{-1}$, and $\gamma_{cr} = [1 + (Q^2 - \sin^2\theta)^{-1}]^{1/2}$ is the

critical energy required for the excitation of resonant radiation. For $Q << 1$, $\lambda \simeq \lambda_p(Q \pm \sqrt{\gamma_{cr}^{-2}-\gamma^{-2}})$. Here $\gamma_{cr} \simeq (Q^2-\theta^2)^{-1/2} >> 1$, such that only an ultra-relativistic beam can excite radiation. On the other hand, when $Q >> 1$, the critical kinetic energy turns out to be extremely low, $(eU/mc^2)_{cr} = \gamma_{cr}-1 \simeq 1/2Q^2$, which is less than 10 KeV for all conventional materials if $\ell \sim 100$ Å. For sufficiently higher (but still non-relativistic) energies eU, Eq(2) gives simply

$$\lambda \simeq \frac{\ell}{n}(\frac{1}{\beta}-\cos\theta) \simeq \frac{\ell}{n}(\sqrt{mc^2/2eU}-\cos\theta); \quad eU << mc^2 = 0.51 \text{ MeV.} \quad (3)$$

For instance, if $\ell=100$ Å, $n = 1$, $eU = 75$ KeV, and $\theta = 45°$, one has $\lambda = 113.6$ Å; for $n = 10$, $\lambda = 11.3$ Å.

The resonant radiation (i.e. spontaneous emission in the system) can provide a narrowband source of radiation. A single electron traversing multilayer structure with N layers radiates energy I in a solid angle $d\Omega$ in the frequency interval between ω and $\omega+d\omega$, given by [1-4]

$$d^2I/d\omega d\Omega = (d^2I_o/d\omega d\Omega) \cdot 4\sin^2(\xi\ell_1/\ell)\sin^2(\xi N/2)/\sin^2\xi \quad (4)$$

where $\xi = (\frac{1}{\beta}-\sqrt{\epsilon}\cos\theta)\pi\ell/\lambda$, and $d^2I_o/d\Omega d\omega$ is a radiation produced by a single interface. According to Ginsburg-Frank theory [1-3], for non-relativistic electrons ($\beta^2 << 1$) and small variations of ϵ ($|\Delta\epsilon| = |\epsilon_1-\epsilon_2| << \bar\epsilon$) the distribution of single-interface radiation is given by

$$d^2I_o/d\Omega d\omega = e^2\beta^2[\Delta\epsilon(\omega)]^2\sin^2\theta/4\pi^2c \quad (5)$$

If the number N of layers is sufficiently large, ($N >> |\bar\epsilon/\Delta\epsilon|$), Eq(4) provides for very narrow spectral peaks of radiation for each particular angle θ (with central wavelength determined by Eq(3)), which also implies that any frequency is radiated in a very narrow intervals of angles. Noting that $\Delta\epsilon = \lambda^2(\lambda_1^{-2}-\lambda_2^{-2})$, and integrating Eq(4) over ω and Ω (with $d\Omega = 2\pi\sin\theta d\theta$), one gets the total radiation in each order n

$$I \simeq 16e^2L\ell^2(\lambda_1^{-2}-\lambda_2^{-2})^2\sin^2(n\pi\ell_1/\ell)/3\beta n^4\pi , \quad (6)$$

(where $L = N\ell/2$ is the total thickness of the structure) with the wavelengths of radiation being in the range $\frac{\ell}{n}(\frac{1}{\beta}-1) < \lambda < \frac{\ell}{n}(\frac{1}{\beta}+1)$. The total energy of radiation increases as speed β decreases. In order to calculate the power of resonant radiation emitted by an electron beam with an electrical current J, one must multiply Eqs(4)-(6) by J/e. If $\ell = 100$ Å, $\ell_1 = \ell_2 = \ell/2$, $L = 1$ μm, $eU = 75$KeV, $J = 1$mA, and $\lambda_1 \simeq 400$ Å (e.g. Zn, Cu, Ag or Au), $\lambda_2 \simeq 800$ Å (e.g. Si or Ge, see [6]), the system can provide a radiation of first harmonic (n = 1) with a total power ~ 0.1 mW and a mean wavelength ~ 200 Å.

Stimulated Emission (Amplification)

We will derive now an amplification caused by the stimulated emission. This effect may be viewed in the following way. An EM wave having a wave vector component $k_s = k_o\cos\theta$ along the axis z (which coincides with the electron trajectory) produces the higher order spatial harmonics with $k_s = k_s \pm 2\pi n/\ell$ which is due to the periodicity of medium. The phase velocities of these harmonics along the axis x are, therefore, $v_n = c/\sqrt{\epsilon}(\cos\theta\pm2\pi n/\ell k_o)$. If the resonant condition (1) for λ is fulfilled, one of these phase velocities coincides with the speed of the electron that results in an exchange of energy between the EM wave and the electron. For some

frequencies in the neighborhood of resonance, the electron loses energy to the EM wave; this results in a coherent gain of the wave, or stimulated emission.

Essentially, this resembles a common mechanism of amplification for many kinds of microwave devices based on the interaction of electrons with "slow" EM wave. The important point is to find the intensities of the resonant spatial harmonics of the field. In all the previous work on resonant radiation [2-4] it is assumed that $\lambda << \ell$ (which is always valid in the ultra-relativistic case, see the introductory section). This allows one to use the WKB approximation. This approximation is not valid in our case since λ may be of order or longer that ℓ. Instead we will find a solution of the exact wave equation (with periodic parameters) based on the assumption of smallness of variations of susceptibility (i.e., $|\Delta\epsilon/\bar{\epsilon}| << 1$, which is always true for short wavelengths); no assumption is made regarding the ratio λ/ℓ. Furthermore, the spatial variation of $\epsilon(z)$ is usually approximated by a cosine function [2-4]. In this approach, the relative amplitude ρ_n of nth harmonic of the EM field is $\rho_n \sim (\Delta\epsilon/\bar{\epsilon})^n$, so that for small $\Delta\epsilon$ ρ_n is negligible for all but the smallest n $(= 1)$. In our approach, we can treat any arbitrary function of $\epsilon(z)$, in particular the true rectangular function. We show that ρ_n falls off algebraically like the Fourier coefficients of $\epsilon(z)$. Significant radiation is expected even for large n provided the interfaces are sharp enough. In this paper, we approach the problem using a single-particle picture which provides direct insight into the mechanism of the electron-EM wave interaction. The problem can also be treated using either the Boltzmann equation [4] or a quantum mechanical formalism; we plan to address these aspects in a subsequent publication [7].

We consider the exact Maxwell equation for the EM field [9] with $\epsilon(z)$ being an arbitrary periodic function in z. We assume a plane wave; it can be shown that only the EM wave with its electric field $\vec{E}$ polarized in the plane of incidence (i.e. plane x,z) may be amplified by the beam [8]. By virtue of the Floquet's theorem for wave equations with periodic coefficients [9], any component of the EM field can be written as a sum of spatial harmonics:

$$u = u_o \exp(j\vec{k}\cdot\vec{r}-j\omega t)[1 + \sum_{n\neq 0}\rho_n \exp(2jn\pi z/\ell + j\phi_n)]; \qquad (7)$$

where $\vec{k}\cdot\vec{r} = k_o x\sin\theta + k_o z\cos\theta$, and ρ_n is the amplitude of the nth spatial harmonics. We make the conventional assumption that there is no retroreflection, which is valid if $N|\Delta\epsilon/\bar{\epsilon}|^2 << 1$, and $|\Delta\epsilon/\epsilon_o| << \cos\theta$. This assumption is strictly true in the vicinity of $\theta = 45°$ (see [8]). Substituting the EM field in the form (7) into the Maxwell equations, collecting the terms with $\exp(j\vec{k}\cdot\vec{r}+2jn\pi z/\ell)$ for each particular n and retaining only terms that are first order in $a_n (<<\cos\theta)$, where the a_n's are the Fourie coefficients of $\epsilon(z)$:

$\epsilon(z) = \bar{\epsilon} + \sum_{n=1}^{\infty} a_n\cos(2n\pi z/\ell + \psi_n)$, one gets the amplitudes u_o, ρ_n of the spatial harmonics of nonvanishing components of electric and magnetic fields (E_x, H_y, E_z): $E_{x_0} = E_o\cos\theta$; $H_{y_0} = E_o\sqrt{\bar{\epsilon}}$; $E_{z_0} = -E_o\sin\theta$, and

$$\begin{pmatrix} \rho_{x_n} \\ \rho_{y_n} \\ \rho_{z_n} \end{pmatrix} = \frac{a_n/2}{q(2\cos\theta + q)} \begin{pmatrix} 1 + q\sin^2\theta/\cos\theta \\ 1 + q\cos\theta \\ 1 - q\cos\theta - q^2 \end{pmatrix}; \qquad (8)$$

where E_o is the amplitude of the principal harmonic of total electric field, and $q = 2\pi n/\ell k_o = \lambda n/\ell\sqrt{\bar{\epsilon}}$. Further calculations are based on the conventional

model of energy exchange between the EM field and an electron which is used, e.g. in the theory of free-electron lasers (see e.g. [10]). From the Lorentz equation $mc\,d(\vec{\beta}\gamma)/dt = e(\vec{E}+[\vec{\beta}\times\vec{H}])$. one gets the equation for the energy $\ell = \gamma mc^2$ of electron

$$d\ell/dt = ec(\vec{\beta}\vec{E}) = ec(\vec{\beta}\vec{E}+\vec{E}\Delta\vec{\beta}+\vec{\beta}\Delta\vec{E}); \qquad (9)$$

where $\vec{E} = \vec{E}[\vec{r}(t),t]$ is the field at the instantaneous location of the electron, $\vec{\beta}$ and $\vec{E}$ are unperturbed vectors, $\Delta\vec{\beta}$ is a small perturbation of electron velocity due to interaction, and $\Delta\vec{E}$ is a small perturbation of the field seen by the electron due its spatial displacement in respect to the unperturbed trajectory, i.e.

$$\Delta\vec{E} = \frac{\partial\vec{E}}{\partial z}\,\Delta z(t); \Delta z = c\int_0^t \Delta\beta_z dt \qquad (10)$$

For the assumed polarization of the field, it follows from the Lorentz equation that

$$\Delta\beta_x = \frac{e}{\gamma mc}\int_0^t (E_x-\beta H_y)dt; \quad \Delta\beta_z = \frac{e}{\gamma^3 mc}\int_0^t E_z dt \qquad (11)$$

In Eqs (9)-(11) one has to take into account only that particular n^{th} component of the wave which is "resonant" to the speed of electron, i.e. that one with $|\xi-n\pi| << 1$. After substituting $z = c\beta t$ and the amplitudes (8) of the proper resonant harmonic of E_x, H_y, and E_z, into (9)-(11), integrating over the temporal interval $[0, \tau = L/\beta c]$, where τ is a time for an electron to pass through the superlattice, one has to average the result over all the possible phases ϕ_n of the relevant field harmonic. We denote this operation by angle brackets. Note that the term $<\vec{\beta}\vec{E}>$ in (9) vanishes, i.e. the stimulated emission is only due to changes in the electron motion caused by the field. Finally, one gets the total averaged change of the electron energy per pass:

$$<\Delta\ell_n> = \pi\frac{e^2 E_o^2 L^3}{m\beta^3 c^2\gamma\lambda}\cdot\frac{1}{(\nu_n\tau)^3}\,[\rho_{x_n}\cos\theta(\rho_{x_n}\cos\theta-\beta\sqrt{\epsilon}\rho_{y_n})\frac{\nu_n}{\omega}\times$$
$$(1-\cos\nu_n\tau)+\gamma^{-2}\rho_{z_n}^2\sin^2\theta(-2+2\cos\nu_n\tau+\nu_n\tau\sin\nu_n\tau)]; \qquad (12)$$

where $\nu_n = \omega[\beta\sqrt{\epsilon}(\cos\theta+q)-1] = \beta c(n\pi-\xi)/\ell$ is a resonant factor. For some ν_n, the change of energy $<\Delta\ell>$ becomes negative which constitutes the gain of EM field, $<\Delta\ell_{EM}> = -<\Delta\ell>$. In the non-relativistic case, the main contribution to the change of energy is due to z-components of $\Delta\vec{\beta}$ and $\Delta\vec{E}$ i.e., in (12) $\frac{\nu}{\omega}$ $(\rho_x^2, \rho_x\rho_y) << \rho_z^2 \simeq (a_n/2)^2(1-\beta\cos\theta)^2$. Replacing the term $(-2+2\cos\nu\tau+\nu\tau\sin\nu\tau)/\nu^3\tau^3$ by its negative extremum $-4/\pi^3$ ($\nu\tau \simeq \pi$), one gets the maximal EM-wave gain per electron per pass:

$$<\Delta\ell_{EM}> = a_n^2 e^2 E_o^2 L^3 \sin^2\theta(1/\beta-\cos\theta)^2/\pi^2 mc^2\beta\lambda \qquad (13)$$

In order to obtain an amplification Γ per pass in the system bombarded by an electron beam with the density of electric current $i(A/cm^2)$, one has to multiply $<\Delta\ell_{EM}>$ by i/e and divide by the energy flux of incident EM wave per unity area of the interface $E_o^2\cos\theta/2R$, where $R = 377\Omega$ is the vacuum impedance. One has also to take into account that for rectangular form of $\epsilon(z)$, $a_n/2 = (\Delta\epsilon/n\pi)\sin(n\pi\ell_1/\ell)$ with $\Delta\epsilon = \lambda^2(\lambda_1^{-2}-\lambda_2^{-2})$. Bearing in mind a resonant condition (3), one finally gets the maximal EM wave amplification per pass:

$$\Gamma = 8\mu ieRL^3\sin^2(\pi n\ell_1/\ell)\sin^2\theta/mc^2\ell\,\pi^4\cos\theta, \qquad (14)$$

where

$$\mu = \beta^{-1}\ell^4(\lambda_1^{-2}-\lambda_2^{-2})^2(1/\beta-\cos\theta)^5 n^{-5} = \lambda^5\ell^{-1}(\lambda_1^{-2}-\lambda_2^{-2})^2(\cos\theta+n\lambda/\ell).$$

If $\lambda = 140\text{Å}$, $\ell = 100\text{Å}$, $\ell_1 = \ell_2 = 50\text{Å}$, $n = 1$, $L = 1\mu m$, $\lambda_1 \simeq 400$ Å, $\lambda_2 \simeq 800$ Å, $\theta = 45°$, and $i = 5\times10^{10}$ A/cm^2 [4] (i.g. beam of 2 μm diameter with a current $\sim 1.5\times10^3$ A), one gets an amplification $\Gamma \simeq 5\%$ per pass. The required speed of electrons is $\beta \simeq 0.474$ which corresponds to energy eU = 69 KeV. For larger λ the amplification increases drastically. With the mirrors situated outside the superlattice to form a Fabry-Perot resonator to provide feedback, the system becomes a short-wave laser which may transform significant portion of energy of electron beam into coherent radiation. It is obvious that the amplifiers and lasers based on the proposed principle should work in the short pulse regime of operation, with the duration of current pulse being determined by the heating, ionization, diffusion of absorbed electrons, etc. As a rough approximation, the per atom heating rate caused by the energy losses of the electron beam [11], is

$$(n^e/N^a)d\mathcal{E}/dt = 4\pi i Z(mv^2)^{-1}\ln(\gamma^2 mv^3/e^2\omega) \tag{15}$$

where $n^e = i/ev$ is the electron beam density, N^a is the atomic density of the material, Z is the atomic number. For the parameters mentioned above, the duration of the current pulse must be shorter than $\sim10^{-13}$ sec in order for the energy transfer per atom to be of order of ~ 1 eV or less. One may note though that in the case of ultra-relativistic beams [4] with energy ~ 50 GeV for the same current, the losses (15) are even greater, such that one needs even shorter pulses. It will probably be hard to obtain such short and powerful pulses, but it may prove worthwhile. However, the first step is to attain spontaneous resonant emission as described in the beginning of this letter. The spontaneous emission can be used as a very narrowband source of radiation by selecting narrow range of angles [Eq. (4)]. It should also be noted that the spontaneous radiaton intensity depends on the total current in the electron beam unlike the stimulated emission gain which depends on the current density. Since there is no constraint on the current density, the conditions to obtan spontaneous emission are much more relaxed. According to Eqs. (4-6), the electrical current required to observe spontaneous emission with the same spotsize of the beam is 10^6–10^9 less than that required for stimulated emission.

Conclusion

In conclusion, we have demonstrated the feasibility of generating far-ultraviolet and soft X-ray radiation by electron beams with relatively low, non-relativistic energies, traversing the solid-state superlattice composed of very thin periodic layers. The use of low energies is a desirable feature as compared with ultra-relativistic beams. The proposed system can be used as a very efficient noncoherent source of narrowband radiation, and, under special conditions, as an amplifier and laser.

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Extreme-ultraviolet and x-ray emission and amplification by nonrelativistic electron beams traversing a superlattice

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High-energy electrons emit resonant electromagnetic radiation when passing through a spatially periodic medium. It is conventionally assumed that ultrarelativistic electron beams are required to obtain significant emission. We demonstrate theoretically the feasibility of exploiting solid-state superlattices with short spatial periods to obtain both spontaneous and stimulated emission in the extreme-ultraviolet and soft x-ray range using nonrelativistic beams.

Fast-moving electrons emit electromagnetic waves when moving from one medium into another with a different dielectric constant.[1-3] This is known as transition radiation and was predicted by Ginzburg and Frank.[1] In a spatially periodic medium the waves emitted at different interfaces interfere so that a resonant emission is obtained when the following condition is satisfied[2-4]:

$$\sqrt{\bar{\epsilon}} \cos \theta = c/v - n\lambda/l, \tag{1}$$

where λ is the wavelength of radiation, l the period of the spatially varying dielectric constant ϵ of medium (it is conventionally assumed that the variations are very small), v velocity of the electrons (assumed normal to the interfaces), θ the angle between the direction of wave propagation and electron motion, n an integer, and $\bar{\epsilon}$ a "mean" ϵ. Usually the period $l \gg \lambda$, so that ultrarelativistic beams ($v/c \simeq 1$) are required in order to satisfy Eq. (1) for real θ. Recently, the possibility of stimulated resonance radiation of ultrarelativistic (~ 50 GeV) electrons traveling through a stack of metal foils was considered,[4] with $l \sim 7$ cm.

In this letter we demonstrate the feasibility of using nonrelativistic electron beams in order to attain both spontaneous and stimulated emission in the extreme ultraviolet and x-ray range using solid-state superlattices with $l \sim 100$ Å so that $l/n\lambda \sim 1$. We show that the wavelength of resonant radiation and the required energy of electrons are determined by the parameter $Q = n\lambda_p/l$, where λ_p is a "mean" plasma wavelength of the medium. If $Q \ll 1$, as is actually assumed in all previous work,[2-4] then the wavelength of the resonant radiation has an order of magnitude of $\lambda \sim \lambda_p Q$, and the kinetic energy of the electrons eU/mc^2 must exceed the critical amount $\sim 1/\sqrt{Q^2 - \theta^2} \gg 1 (\theta < Q)$ which constitutes the use of ultrarelativistic beams. On the contrary, if the period of the spatially periodic medium is chosen so that $Q \gg 1$, the wavelength of resonant radiation becomes of the order $\sim \lambda_p/Q = l/n$, and the critical kinetic energy of the beam turns out to be extremely small: $eU/mc^2 \sim 1/2Q^2$. The advantages of the proposed method are (1) the frequency of radiation can be easily tuned in a very wide range by simply varying accelerating potential of beam (which is very hard to do with ultrarelativistic beams), (2) the range of the angles of the radiated emission is almost unlimited, and (3) the cost of equipment and energy required for experiments with nonrelativistic beams is insignificant compared to large accelerat-

ing machines. This last consideration is perhaps the most important.

Fortunately, the development of molecular beam epitaxy (MBE) and other techniques in recent years has made it possible to grow very thin films (~ 100 Å and less) with precise boundaries. Periodic structures composed of thin films of different materials (in particular, superlattices) have also been fabricated.[5] Using these structures, nonrelativistic electron beams with energies 70–200 keV can be used to generate radiation of wavelength 100–200 Å and less. An important feature of transition radiation is that the electron beam travels *through* the material structure rather than in vacuum *above* the structure (as in traveling wave tubes[6]), or through a spatially modulated magnetic field (as in many free-electron lasers[7]). This makes it possible to use very short spatial periods.

We will first obtain the resonant wavelength of radiation from Eq. (1) noting that[3] $l\sqrt{\bar{\epsilon}} = l_1 \sqrt{\epsilon_1} + l_2 \sqrt{\epsilon_2}$, where $l_{1,2}$ and $\epsilon_{1,2}$ are, respectively, the thicknesses and dielectric constants of alternating layers forming the superlattice; $l = l_1 + l_2$ is its spatial period. For short wavelengths ($\lambda^2 \ll \lambda_{1,2}^2$), $\epsilon_{1,2} = 1 - \lambda^2/\lambda_{1,2}^2$, where $\lambda_{1,2}$ are the plasma wavelengths of the two materials forming the superlattice ($\lambda_{1,2}^2 = 4\pi mc^2/e^2 N_{1,2}^e$, where N^e is the density of electrons). Thus, the mean dielectric constant may be written as $\bar{\epsilon} \simeq 1 - \lambda^2/\lambda_p^2$, where $\lambda_p^{-2} = l^{-1}(l_1 \lambda_1^{-2} + l_2 \lambda_2^{-2})$. Substituting this into Eq. (1) and solving it for λ, one gets the resonant wavelengths:

$$\lambda = \lambda_p \frac{Q}{Q^2 + \cos^2 \theta} \left[\frac{1}{\beta} \pm \frac{\cos \theta}{Q} \right.$$

$$\left. \times \sqrt{(\gamma_{cr}^2 - 1)^{-1} - (\gamma^2 - 1)^{-1}} \right], \tag{2}$$

where $\beta = v/c$, $\gamma = (1 - \beta^2)^{-1}$, and $\gamma_{cr} = [1 + (Q^2 - \sin^2 \theta)^{-1}]^{1/2}$ is the critical energy required for the excitation of resonant radiation. For $Q \ll 1$, $\lambda \simeq \lambda_p (Q \pm \sqrt{\gamma_{cr}^{-2} - \gamma^{-2}})$. Here, $\gamma_{cr} \gg 1$, such that only an ultrarelativistic beam can excite radiation. On the other hand, when $Q \gg 1$, the critical kinetic energy turns out to be extremely low, $\gamma_{cr} - 1 \simeq 1/2Q^2$, which gives less than 10 keV for all conventional materials if $l \sim 100$ Å. For sufficiently higher (but still nonrelativistic) energies eU, Eq. (2) gives simply

Quantum Theory of Spontaneous and Stimulated

Resonant Transition Radiation

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ABSTRACT

Resonant transition radiation, generated by high energy electron beams traversing a periodic medium, has been considered by many researchers as a potential source of both spontaneous and stimulated emission at short wavelengths. To our knowledge, this problem has only been treated classically. This paper presents a quantum mechanical theory that leads to a unified description of both spontaneous and stimulated emission and establishes a simple relation between them.

I. Introduction:

Electrons traveling at high speed emit electromagnetic waves when they move from one medium to another with a different dielectric constant. This is known as transition radiation[1]. In a spatially periodic medium there is an interference of the waves emitted at different interfaces producing a resonant transition radiation when the following condition is satisfied (Fig. 1)[2-4].

$$\sqrt{\bar{\epsilon}} \cos\theta = \frac{c}{v} - \frac{n\lambda}{\ell} \tag{1}$$

where

λ is the wavelength of radiation in free space

ℓ is the period of the spatially varying dielectric constant $E(z)$ [the variations are usually assumed to be small],

$\bar{\epsilon}$ is the 'mean' dielectric constant,

v   is the velocity of the electrons in the z-direction perpendicular to the interfaces,

θ is the angle between the direction of wave propagation and the z-axis,

and n is an integer

Resonant transition radiation has been considered by a number of workers as a possible source of short wavelength radiation. Usually the period ℓ is much greater than λ so that ultra-relativistic electron beams with $v/c \simeq 1$ are required to satisfy Eq. 1[4]. The possibility of using non-relativistic electron beams with short period solid state superlattices has also been considered recently[5].

To our knowledge the problem of resonant transition radiation has only been treated classically. In classical theory spontaneous and stimulated emission are two different problems. Spontaneous emission is obtained from Maxwell's equations, treating the electron beam as a fixed current source. To obtain stimulated emission, however, we must consider the effect of the electromagnetic wave on the electron beam.

This can be done using either a collective approach[4] (Boltzmann equation) or a single particle approach[5]. There is some discrepancy in the two results.

In this paper we will present a quantum mechanical treatment of resonant transition radiation. In the quantum mechanical approach, the electrons and the electromagnetic wave are treated simultaneously and both spontaneous and stimulated emission come out from the same formalism. A simple fundamental relation is established between the spontaneous emission rate and the stimulated emission gain. The spontaneous emission intensity obtained quantum mechanically agrees exactly with the classical results[3]. However, the stimulated emission gain obtained quantum mechanically differs from both Refs. 4 and 5.

In the quantum mechanical theory we consider the interaction between the electron beam and the electromagnetic normal modes of the periodic medium. These normal modes consist of an infinite number of spatial harmonics with wavenumbers $k_0 + \frac{2n\pi}{\ell}$ (n = integer, $k_0 = 2\pi/\lambda'$ $\lambda' = \lambda/\sqrt{\epsilon}$ whose amplitudes depend on the Fourier components of E(z). The spatial harmonics have phase velocities less than that of the fundamental (n = 0) by the factor $(1 + n\lambda'/\ell)$. Consequently it is possible for the electron beam to Cerenkov radiate into the higher spatial harmonics even if $v < c$. Resonant transition radiation can equivalently be viewed as a process of Cerenkov emission into the higher spatial harmonics. The results for both spontaneous and stimulated resonant transition radiation are the same as those for ordinary Cerenkov radiation multiplied by the squared amplitude of the nth spatial harmonic of an appropriate field component.

Section II gives a brief summary of the classical results for spontaneous resonant transition radiation. In Section III we describe the quantum theory of ordinary Cerenkov radiation; both the spontaneous emission rate and the stimulated emission gain are discussed and a simple fundamental relationship is established between them. These

results are modified in Section V for resonant transition radiation using the spatial harmonic amplitudes obtained in Section IV.

II. Classical Theory:

The energy U_0 radiated in a frequency range $d\omega$ in a solid angle $d\Omega$ by an electron traversing an interface between two media with dielectric constants ϵ_1 and ϵ_2 is given by[6]

$$\frac{d^2U_0}{d\omega d\Omega} = \frac{e^2}{4\pi\epsilon_0 c\sqrt{\bar{\epsilon}}} \frac{\sin^2\theta}{4\pi^2} \left|\frac{\Delta\epsilon}{\bar{\epsilon}}\right|^2 \beta^2 \left|\frac{1-\beta'\cos\theta-\beta'^2}{(1-\beta'\cos\theta)(1-\beta'^2\cos^2\theta)}\right|^2$$

where

$$\Delta\epsilon = |\epsilon_1 - \epsilon_2|/\epsilon_0$$

$$\bar{\epsilon} = (\epsilon_1 + \epsilon_2/2\epsilon_0$$

$$\epsilon_0 = \text{permittivity of vacuum}$$

$$\beta = v/c$$

$$\beta' = \beta\sqrt{\bar{\epsilon}}$$

Using Equation (1), we can rewrite Equation (2) as,

$$\frac{d^2U_0}{d\omega d\Omega} = \frac{e^2}{4\pi\epsilon_0 c\sqrt{\bar{\epsilon}}} \frac{\sin^2\theta}{4\pi^2} \left|\frac{\Delta\epsilon}{\bar{\epsilon}}\right|^2 \left|\frac{(nq-\beta')}{n^2q^2[2-nq\beta']}\right|^2 \qquad (2)$$

where $q = \lambda/\beta'\sqrt{\bar{\epsilon}}$

In a periodic medium with multiple interfaces the total radiated energy U is given by

$$\frac{d^2U}{d\omega d\Omega} = \frac{d^2U_0}{d\omega d\Omega} \cdot \frac{4 \sin^2(\xi \ell_1/2\ell)\sin^2(\xi N/2)}{\sin^2(\xi/2)} \tag{3}$$

where ℓ_1 is the length of one of the layers, 2N is the number of layers and $\xi = \frac{2\pi\ell}{\lambda}(\frac{1}{\beta} - \sqrt{\in}\cos\theta)$. If N is sufficiently large, the radiation shows very narrow spectral peaks around values of θ for which ξ is an integer multiple of 2π. The location of the peaks is given by

$$\xi = 2\pi \cdot n$$

or,

$$\frac{1}{\beta} - \sqrt{\in}\cos\theta = \frac{n\lambda}{\ell}$$

which is identical to Equation (1). The narrow spectral peaks can be approximated by delta functions to give for a particular value of n.

$$\frac{d^2U}{d\omega d\Omega} = \frac{d^2U_0}{d\omega d\Omega} \, 8\pi N\sin^2\left(\frac{n\pi\ell_1}{\ell}\right)\delta(\xi - 2n\pi)$$

$$= \frac{d^2U_0}{d\omega d\Omega} \, 4Nq \, \sin^2\frac{n\pi\ell_1}{\ell} \, \delta\left[\cos\theta + nq - \frac{1}{\beta}\right] \tag{4}$$

Integrating over the solid angle we get the energy U radiated into the frequency interval $d\omega$ for a particular value of n.

$$\frac{dU}{d\omega} = \frac{e^2}{4\pi\in_0 c\sqrt{\in}}\sin^2\theta\left|\frac{\Delta\in}{\in}\right|^2 \frac{2Nq}{\pi} \sin^2\frac{n\pi\ell_1}{\ell} \left|\frac{nq-\beta}{n^2q^2[2-nq\beta]}\right|^2 \tag{5}$$

where $\cos\theta = \frac{1}{\beta} - nq$

Here we have used Equation (3) for $d^2U_0/d\omega d\Omega$ and assumed that it varies slowly compared to the delta function. Equation (5) gives the total energy radiated by one electron; multiplying by J/e (J = current density) we get the radiated power per I unit

area.

$$\frac{dI}{d\omega} = \frac{eJ\sin^2\theta}{4\pi\epsilon_0 c\sqrt{\epsilon}} \frac{2Nq}{\pi} \left|\frac{\Delta\epsilon}{\bar{\epsilon}}\right|^2 \sin^2\frac{n\pi\ell_1}{\ell} \left|\frac{nq - \beta'}{n^2q^2(2 - nq\beta')}\right|^2 \tag{6}$$

III. Quantum Theory of Cerenkov Radiation:

In this section we will describe spontaneous and stimulated Cerenkov radiation using a quantum mechanical formalism. As we have mentioned in the introduction, the results for Cerenkov radiation are readily adapted to resonant transition radiation simply by multiplying with the squared amplitude of the spatial harmonic of an appropriate field component; this is done in Sections IV and V.

Consider a free non-relativistic electron (relativistic effects are incorporated later with a simple modification) interacting with a radiation field.

$$H_0 = \frac{p^2}{2m} + \sum_{\vec{k},\nu} \hbar\omega_k \, a_{\vec{k},\nu}^* a_{\vec{k},\nu} \tag{7a}$$

$$H_{int} = \sum_{\vec{k},\nu} K_{\vec{k},\nu} a_{\vec{k},\nu}^* + K_{\vec{k},\nu}^* a_{\vec{k},\nu} \tag{7b}$$

$$K_{\vec{k},\nu} = -\sqrt{\frac{\hbar}{2\epsilon\epsilon_0 V\omega_k}} \frac{e}{2m} \left[p_\nu e^{-i\vec{k}\cdot\vec{R}} + e^{-i\vec{k}\cdot\vec{R}}p_\nu\right] \tag{7c}$$

where

$\vec{k},\nu$ represent the wavevector and polarization of the photon mode,

$\omega_k = ck/\sqrt{\epsilon}$

V is the volume of normalization,

p is the momentum operator for the electron,

m is the electron mass,

$\vec{R}$ is the electronic position operator.

The initial state $|I>$ has the photon modes in harmonic oscillator states $|n_{\vec{k},\nu}>$ and the electron in the state $|\vec{\beta}_i>$ $\left[\equiv (V)^{-1/2}\exp i(\vec{\beta}_i\cdot\vec{R} - E_it/\hbar),\ E_i = \hbar^2\beta_i^2/2m\right]$.

$$|I> = |\vec{\beta}_i> \prod_{\vec{k},\nu} |n_{\vec{k},\nu}> \tag{8}$$

We consider transitions to a final state $|F>$ with the electron in state $|\vec{\beta}_f>$ and one more (or one less) photon in mode $\vec{k},\nu$ corresponding to photon emission (or absorption). The first-order matrix element M for this transition is given by

$$M = \sqrt{n_{\vec{k},\nu} + 1} \quad <\vec{\beta}_f|K_{\vec{k},\nu}|\vec{\beta}_i> \quad \text{(emission)} \tag{9a}$$

$$= \sqrt{n_{\vec{k},\nu}} \quad <\vec{\beta}_f|K_{\vec{k},\nu}^*|\vec{\beta}_i> \quad \text{(absorption)} \tag{9b}$$

Assuming that the length of the interaction region is L_x,L_y,L_z in the x,y,z directions and that the time of interaction is T, we get from Equation (9),

$$|M| = \sqrt{n_{\vec{k},\nu} + \frac{1}{2} \pm \frac{1}{2}} \sqrt{\frac{1}{2\varepsilon\epsilon_0 V\hbar\omega_k}} \frac{e\hbar}{2m} (\vec{\beta}_i + \vec{\beta}_f)_\nu \frac{L_xL_yL_z \cdot T}{V}$$

$$\frac{\sin(\Omega_xL_x/2)}{(\Omega_xL_x/2)} \cdot \frac{\sin(\Omega_yL_y/2)}{(\Omega_yL_y/2)} \cdot \frac{\sin(\Omega_zL_z/2)}{(\Omega_zL_z/2)} \frac{\sin(\Omega T/2)}{(\Omega T/2)} \tag{10}$$

where

$$\Omega_{x,y,z} = (\vec{\beta}_i - \vec{\beta}_f \mp \vec{k})_{x,y,z}$$

$$\Omega = (E_i - E_f \mp \hbar\omega_k)$$

and the upper (lower) sign is for emission (absorption).

The total transition probability is given by $|M|^2$ integrated over the possible final electronic states $d\vec{\beta}_f$. If L_x,L_y,L_z are sufficiently large we can treat the $\sin^2 x/x^2$ factors as delta functions in this integration.

$$\pm \Delta n_{\vec{k}} = (n_{\vec{k}} + \frac{1}{2} \pm \frac{1}{2}) \frac{L_x L_y L_z}{V} \frac{e^2 \hbar^2 \beta_i^2 \sin^2\theta}{2 \mathcal{E} \epsilon_0 V m^2 \hbar \omega_k} T^2 \left| \frac{\sin \Omega T/2}{\Omega T/2} \right|^2 \qquad (11)$$

$$\vec{\beta}_f = \vec{\beta}_i \mp \vec{k}$$

The electron couples only to the photons which are polarized in the plane containing $\vec{\beta}_i$ and $\vec{k}$ (Fig. 2). To account for relativistic effects we should use the Dirac equation rather than the Schrodinger equation for the electrons. However, since electron spin does not play any role in Cerenkov emission, we can use the Klein-Gordon equation[7,8]. It is shown in Ref. 7 that using the Klein-Gordon equation we get $m(\gamma_i + \gamma_f)/2$ in place of m in the expression for $|M|$ (Equation 10) where

$$E = \gamma m c^2$$
$$\gamma = 1/\sqrt{1-\beta^2} \qquad (12)$$

Consequently Equation (11) is modified to

$$\pm \Delta n_{\vec{k}} = (n_{\vec{k}} + \frac{1}{2} \pm \frac{1}{2}) \frac{L_x L_y L_z}{V} \frac{e^2 v^2 \sin^2\theta}{2 \mathcal{E} \epsilon_0 V \hbar \omega_k} \left| \frac{2\gamma_i}{\gamma_i + \gamma_f} \right|^2 T^2 \left| \frac{\sin \Omega T/2}{\Omega T/2} \right|^2 \qquad (13)$$

where we have used the relation $v = \hbar \beta_i / m \gamma_i$. Using the momentum conservation condition ($\Omega_{x,y,x} = 0$) and the relativistic energy-momentum relation ($E^2 = p^2 c^2 + m^2 c^4$) we can write,

$$\Omega = \mp k v \cos\theta \frac{2\gamma_i}{\gamma_i + \gamma_f} \mp \omega_k + \frac{\hbar k^2}{m(\gamma_i + \gamma_f)}$$

$$= \mp k v \frac{2\gamma_i}{\gamma_i + \gamma_f} \left[\cos\theta - \frac{1}{\beta\sqrt{\mathcal{E}}} \frac{\gamma_f + \gamma_i}{2\gamma_i} \mp \frac{k}{2\beta_i} \right] \qquad (14)$$

Usually the energy $\hbar \omega_k$ of the emitted photon is a very small fraction of the electron energy so that $E_f \simeq E_i$ and $\gamma_f \simeq \gamma_i = \gamma$. Also we can assume $L_x L_y L_z = V$ so that the wavefunctions are normalized to the interaction volume. With these assumptions we can simplify Equations (13) and (14),

$$\pm\Delta n_{\vec{k}} = \left[n_{\vec{k}} + \frac{1}{2} \pm \frac{1}{2}\right]\frac{e^2 v^2 \sin^2\theta}{2\mathcal{E}\epsilon_0 V\hbar\omega_k} \ T^2\left[\frac{\sin\,\Omega T/2}{\Omega T/2}\right]^2 \tag{15a}$$

where $\Omega = \mp kv\cos\theta \pm \omega_k + \hbar k^2/2\gamma m$

$$= \mp kv\left[\cos\theta - \frac{1}{\beta} \mp \frac{k}{2\beta_i}\right] \tag{15b}$$

The total spontaneous emission U, by an electron is obtained by integrating Equation (15a) (taking the upper sign with $n_{\vec{k}} = 0$) over all photon modes $\vec{k}$. If the time of interaction T is long enough we can treat the $\sin^2 x/x^2$ as a delta function in this integration to get

$$\frac{dU}{d\omega} = T \ \frac{e^2 \omega v}{4\pi\epsilon_0 c^2} \ \sin^2\theta \tag{16}$$

To obtain the gain per pass we need the difference between stimulated emission and absorption which arises from the small difference in Ω for the two cases (Equation 15b).

$$\frac{\Delta n_{\vec{k}}}{n_{\vec{k}}} = \frac{e^2 \omega v^2 \sin^2\theta}{4\epsilon_0 \gamma m c^2 V} \cdot T^2 \cdot F \tag{17}$$

where

$$F = \frac{d}{d\phi}\left[\frac{\sin^2\phi}{\phi^2}\right] = \frac{16}{\pi^3} \ (\text{maximum})$$

$$\phi = \Omega T/2$$

Equations (16) and (17) give the spontaneously radiated energy in a frequency interval ω to $\omega+d\omega$ and the gain per pass at a frequency ω due to a single electron traveling at a velocity v. For a beam of electrons with a current density $J(\equiv env$, $n =$ electron density) we should multiply Equation (16) by J/e to get the

intensity, $I(W/m^2)$ and Equation (17) by nV to get the total gain, Γ.

$$\frac{dI}{d\omega} = T \cdot \frac{eJ\omega v \sin^2\theta}{4\pi \epsilon_0 c^2} \tag{18a}$$

$$\Gamma = T^3 \cdot \frac{eJ\omega v \sin^2\theta}{4\epsilon_0 \gamma m c^2} \cdot F \tag{18b}$$

Equations (18a) and (18b) apply to ordinary Cerenkov radiation; it is shown in Sections IV and V that the results for resonant transition radiation are identical except for a multiplying factor depending on the relative amplitude of the appropriate spatial harmonic. From Equations (18a) and (18b) we can derive a simple relationship between the spontaneous emission intensity and the stimulated emission gain which will hold true even for resonant transition radiation.

$$\Gamma = \left[\frac{dI}{d\omega}\right] \cdot (F\pi) \cdot \frac{T^2}{\gamma} m \tag{19}$$

IV. Electromagnetic Waves in a Periodic Medium:

Consider an electromagnetic wave propagating at an angle θ to the z-direction which is perpendicular to the plane of the layers (Fig. 2). The coupling of the wave to the electrons is proportional to the component of the electron momentum along the direction of the electric field (Equation 10); consequently the electrons couple only to the waves polarized in the plane containing $\vec{\beta}_i$ and $\vec{k}$. For this polarization the field components are E_x, E_z and H_y; the component that determines the coupling strength is E_z since the electron current is along z. By Floquet's Theorem E_z can be written as

$$E_z = E_{z0} \exp[i(\vec{k} \cdot \vec{r} - \omega t)] \left[1 + \sum_{n \neq 0} \rho_n \exp\left[i\left(\frac{2n\pi z}{\ell} + \phi_n\right) \right] \right] \tag{20}$$

Starting from Maxwell's equations, assuming an arbitrary periodic dielectric constant $\epsilon(z)$, we can show that for small $\Delta\epsilon/\bar{\epsilon}$,

$$\rho_n = \frac{a_n(nq-\beta')}{2nq}[2 - nq\beta'] \tag{21}$$

where

$$\epsilon(z) = \bar{\epsilon} + \sum_{n=1}^{\infty} a_n \cos(\frac{2n\pi z}{\ell} + \psi_n).$$

V. Quantum Theory of Resonant Transition Radiation:

The results (Equations 18) in Section III have to be multiplied by $|\rho_n|^2$ in order to get the results for resonant transition radiation.

$$\frac{dI}{d\omega} = T\frac{eJ\omega v\sin^2\theta}{4\pi\epsilon_0 c^2}\frac{|a_n|^2}{4}\left|\frac{nq-\beta'}{nq(2-nq\beta')}\right|^2 \tag{22}$$

Assuming $\epsilon(z)$ has a rectangular form we can readily calculate its Fourier coefficients a_n.

$$a_n = \frac{2}{n\pi}\frac{\Delta\epsilon}{\bar{\epsilon}}\sin\frac{n\pi\ell_1}{\ell} \tag{23}$$

Using Equation (23) in Equation (22) we get

$$\frac{dI}{d\omega} = T\frac{eJ\omega v\sin^2\theta}{4\pi\epsilon_0 c^2}\left|\frac{\Delta\epsilon}{\bar{\epsilon}}\right|^2\sin^2\frac{r\pi\ell_1}{\ell}\frac{q^2}{\pi^2}\left|\frac{nq-\beta'}{n^2q^2(2-nq\beta')}\right|^2 \tag{24}$$

$$= \frac{eJ\sin^2\theta}{4\pi\epsilon_0 c\sqrt{\bar{\epsilon}}}\frac{2Nq}{\pi}\left|\frac{\Delta\epsilon}{\bar{\epsilon}}\right|^2\sin^2\frac{n\pi\ell_1}{\ell}\left|\frac{nq-\beta'}{n^2q^2(2-nq\beta')}\right|^2$$

using the relation $VT = N\ell$. This is in agreement with the classical result quoted earlier (Eq. 6). The stimulated emission gain Γ is readily obtained using the relationship between Γ and $\frac{dI}{d\omega}$ derived earlier (Eq. 19) for Cerenkov radiation. Since both

quantities are multiplied by $|\rho_n|^2$ (for resonant transition radiation) the relationship is still valid.

IV. Conclusions

Acknowledgements

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Far-Ultraviolet and X-ray Emission
and Amplification by Non-relativistic Electron Beams
Traversing a Superlattice

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Abstract

High-energy electrons emit resonant electromagnetic radiation when passing through a spatially periodic medium. It is conventionally assumed that ultra-relativistic electron beams are required to obtain significant emission. We demonstrate theoretically the feasibility of exploiting solid-state superlattices with short periods to obtain both spontaneous and stimulated emission in the far-ultraviolet and soft X-ray range using non-relativistic beams.

Fast-moving electrons emit electromagnetic waves when moving from one medium into another with a different dielectric constant [1-3]. This is known as transition radiation and was predicted by Ginzburg and Frank [1]. In a spatially periodic medium the waves emitted at different interfaces interfere so that a resonant emission is obtained when the following condition is satisfied [2-4]:

$$\sqrt{\bar{\epsilon}} \cos\theta = c/v - n\lambda/\ell \qquad (1)$$

where λ is the wavelength of radiation, ℓ is the period of the spatially varying dielectric constant ϵ of medium (it is conventionally assumed that the variations are very small), v is velocity of the electrons (assumed normal to the interfaces), θ is the angle between the direction of wave propagation and electron motion, n is an integer, and $\bar{\epsilon}$ is a "mean" ϵ. This condition is readily derived by requiring that the waves emitted at different interfaces interfere constructively at a distant point. Usually the period $\ell >> \lambda$, so that ultra-relativistic beams ($v/c \simeq 1$) are required in order to satisfy Eq(1) for real θ. Recently the possibility of stimulated resonance radiation of ultra-relativistic (~ 50 Gev) electrons traveling through a stack of metal foils was considered [4], with $\ell \sim 7$ cm.

In this paper, we demonstrate the feasibility of using non-relativistic electron beams in order to attain both spontaneous and stimulated emission in the ultraviolet and X-ray range using solid-state superlattices with $\ell \sim 100$ Å so that $\ell/n\lambda \sim 1$. We show that the wavelength of resonant radiation and the required energy of electrons are determined by the parameter $Q = n\lambda_p/\ell$, where λ_p is a "mean" plasma wavelength of the medium. If $n\lambda_p << \ell$ (i.e. $Q << 1$), as is assumed in all previous work [2-4], then the wavelength of the resonant radiation has an order of magnitude of $\lambda \sim \lambda_p Q$, and the kinetic (dimensionless) energy of the electrons eU/mc^2 must exceed the critical amount $\sim 1/\sqrt{Q^2-\theta^2} >> 1$ ($\theta < Q$) which constitutes the use of ultra-relativistic beams. On the contrary, if the period of the spatially periodic medium is chosen so that $Q >> 1$ (i.e. period ℓ is much shorter than the plasma wavelength, e.g. $\ell \sim 50$-200 Å), the wavelength of resonant radiation becomes of the order of $\sim \lambda_p/Q = \ell/n$, and the critical kinetic energy of the beam turns out to be extremely small: $eU/mc^2 \sim 1/2Q^2$, which may be less than a kilovolt even for very short wavelength radiation. The advantages of the proposed method are: (1) the frequency of radiation can be easily tuned in a very wide range by simply varying accelerating potential of beam (which is very hard to do with ultra-relativistic beams) (2) the range of the possible angles of the wave propagation is almost unlimited, and (3) the cost of equipment and energy required for experiments with non-relativistic beams is insignificant compared to large accelerating machines. This last consideration is perhaps the most important.

The main requirements for non-relativistic short-wavelength resonant radiation is a periodic medium with a very short spatial period. Fortunately, the development of molecular beam epitaxy (MBE) and other techniques in recent years has made it possible to grow very thin films (~ 100 Å and less) with precise boundaries. Periodic structures composed of thin films of different materials, called superlattices, have also been fabricated [5]. Using these superlattices, non-relativistic electron beams with energies 20-200 KeV can be used to generate radiation of wavelength 100-200 Å and less.

We will first obtain the resonant wavelength of radiation from Eq(1) noting that [3] $\ell\sqrt{\bar{\epsilon}} = \ell_1\sqrt{\epsilon_1} + \ell_2\sqrt{\epsilon_2}$, where $\ell_{1,2}$ and $\epsilon_{1,2}$ are respectively the thicknesses and dielectric constants of alternating layers forming the

superlattice, $\ell = \ell_1 + \ell_2$ is its spatial period. For short wavelengths ($\lambda^2 << \lambda_{1,2}^2$), $\epsilon_{1,2} = 1 - \lambda^2/\lambda_{1,2}^2$ where $\lambda_{1,2}$ are the plasma wavelengths of the two materials forming the superlattice, ($\lambda_{1,2}^{-2} = 4\pi mc^2/e^2 N_{1,2}^e$ where N^e is the density of electrons). Thus, the mean dielectric constant may be written as $\bar\epsilon \simeq 1 - \lambda^2/\lambda_p^2$, where $\lambda_p^{-2} = \ell^{-1}(\ell_1\lambda_1^{-2} + \ell_2\lambda_2^{-2})]$. Substituting this into (1) and solving it for λ, one gets the resonant wavelengths:

$$\lambda = \lambda_p \frac{Q}{Q^2 + \cos^2\theta}\left[\frac{1}{\beta} \pm \frac{\cos\theta}{Q}\sqrt{(\gamma_{cr}^2-1)^{-1}-(\gamma^2-1)^{-1}}\right], \tag{2}$$

where $Q = n\lambda_p/\ell$; $\beta = v/c$; $\gamma = (1-\beta^2)^{-1}$, and $\gamma_{cr} = [1+(Q^2-\sin^2\theta)^{-1}]^{1/2}$ is the critical energy required for the excitation of resonant radiation. For $Q << 1$, $\lambda \simeq \lambda_p(Q \pm \sqrt{\gamma_{cr}^{-2}-\gamma^{-2}})$. Here $\gamma_{cr} \simeq (Q^2-\theta^2)^{-1/2} >> 1$, such that only an ultra-relativistic beam can excite radiation. On the other hand, when $Q >> 1$, the critical kinetic energy turns out to be extremely low, $(eU/mc^2)_{cr} = \gamma_{cr}-1 \simeq 1/2Q^2$, which is less than 500 eV for all conventional materials if $\ell \sim 100$ Å. For sufficiently higher (but still non-relativistic) energies eU, Eq(2) gives simply

$$\lambda \simeq \frac{\ell}{n}(\frac{1}{\beta}-\cos\theta) \simeq \frac{\ell}{n}(\sqrt{mc^2/2eU}-\cos\theta); \quad eU << mc^2 = 0.51 \text{ MeV.} \tag{3}$$

For instance, if $\ell = 100$ Å, n = 1, eU = 75 KeV, and $\theta = 45°$, one has $\lambda = 113.6$ Å; for n = 10, $\lambda = 11.3$ Å.

The resonant radiation (which may be regarded as spontaneous emission in the system) can provide a narrowband source of radiation. A single electron traversing multilayer structure with N layers radiates energy I in a solid angle $d\Omega$ in the frequency interval between ω and $\omega + d\omega$, given by [1-4]

$$d^2I/d\omega d\Omega = (d^2I_o/d\omega d\Omega) \cdot 4\sin^2(\xi\ell_1/\ell)\sin^2(\xi N/2)/\sin^2\xi \tag{4}$$

where $\xi = (\frac{1}{\beta}-\sqrt{\bar\epsilon}\cos\theta)\pi\ell/\lambda$, and $d^2I_o/d\Omega d\omega$ is a radiation produced by a single interface. According to Ginsburg-Frank theory [1-3], for non-relativistic electrons ($\beta^2 << 1$) and small variations of ϵ ($|\Delta\epsilon| = |\epsilon_1-\epsilon_2| << \bar\epsilon$) the distribution of single-interface radiation is given by

$$d^2I_o/d\Omega d\omega = e^2\beta^2[\Delta\epsilon(\omega)]^2\sin^2\theta/4\pi^2c \tag{5}$$

If the number N of layers is sufficiently large, ($N >> |\bar\epsilon/\Delta\epsilon|$), Eq(4) provides for very narrow spectral peaks of radiation for each particular angle θ (with central wavelength determined by Eq(3)), which also implies that any frequency is radiated in a very narrow intervals of angles. Noting that $\Delta\epsilon = \lambda^2(\lambda_1^{-2}-\lambda_2^{-2})$, and integrating Eq(4) over ω and Ω (with $d\Omega = 2\pi\sin\theta d\theta$), one gets the total radiation in each order n

$$I \simeq 16e^2L\ell^2(\lambda_1^{-2}-\lambda_2^{-2})^2\sin^2(n\pi\ell_1/\ell)/3\beta n^4, \tag{6}$$

(where $L = N\ell/2$ is the total thickness of the structure) with the wavelengths of radiation being in the range $\frac{\ell}{n}(\frac{1}{\beta}-1) < \lambda < \frac{\ell}{n}(\frac{1}{\beta}+1)$. The total energy of radiation increases as speed β decreases. In order to calculate the power of resonant radiation emitted by an electron beam with an electrical current J, one must multiply Eqs(4)-(6) by J/e. If $\ell = 100$ Å, $\ell_1 = \ell_2 = \ell/2$, L = 1 μm, eU = 75KeV, J = 1mA, and $\lambda_1 \simeq 400$ Å (e.g. Zn, Cu, Ag or Au), $\lambda_2 \simeq 800$ Å (e.g. Si or Ge, see [6]), the system can provide a radiation of first harmonic (n = 1) with a total power ~ 0.33 mW and a mean wavelength ~ 200 Å.

We will derive now an amplification caused by the stimulated emission. This effect may be viewed in the following way. An EM wave having a wave

vector component $k_z = k_0 \cos\theta$ along the axis z (which coincides with the electron trajectory) produces the higher order spatial harmonics with $k_{z_n} = k_z \pm 2\pi n/\ell$ which is due to the periodicity of medium. The phase velocities of these harmonics along the axis x are, therefore, $v_n = c/\sqrt{\bar{\epsilon}}(\cos\theta \pm 2\pi n/\ell k_0)$. If the resonant condition (1) for λ is fulfilled, one of these phase velocities coincides with the speed of the electron that results in an exchange of energy between the EM wave and the electron. For some frequencies in the neighborhood of resonance, the electron loses energy to the EM wave; this results in a coherent gain of the wave, or stimulated emission.

Essentially, this resembles a common mechanism of amplification for many kinds of microwave devices based on the interaction of electrons with "slow" EM wave. The important point is to find the intensities of the resonant spatial harmonics of the field. In all the previous work on resonant radiation [2-4] it is assumed that $\lambda << \ell$ (which is always valid in the ultra-relativistic case, see the introductory section). This allows one to use the WKB approximation. This approximation is not valid in our case since λ may be of order or longer that ℓ. Instead we will find a solution of the exact wave equation (with periodic parameters) based on the assumption of smallness of variations of susceptibility (i.e., $|\Delta\epsilon/\bar{\epsilon}| << 1$, which is always true for short wavelengths); no assumption is made regarding the ratio λ/ℓ. Furthermore, the spatial variation of $\epsilon(z)$ is usually approximated by a cosine function [2-4]. In this approach, the relative amplitude ρ_n of nth harmonic of the EM field is $\rho_n \sim (\Delta\epsilon/\bar{\epsilon})^n$, so that for small $\Delta\epsilon$ ρ_n is negligible for all but the smallest n (= 1). In our approach, we can treat any arbitrary function of $\epsilon(z)$, in particular the true rectangular function. We show that ρ_n falls off algebraically like the Fourier coefficients of $\epsilon(z)$. Significant radiation is expected even for large n provided the interfaces are sharp enough. In this letter, we approach the problem using a single-particle picture which provides direct insight into the mechanism of the electron-EM wave interaction. The problem can also be treated using either the Boltzmann equation [4] or a quantum mechanical formalism; we plan to address these aspects in a subsequent publication [7].

We consider the exact Maxwell equation for the EM field with $\epsilon(z)$ being an arbitrary periodic function in z. We assume a plane wave; it can be shown that only the EM wave with its electric field $\vec{E}$ polarized in the plane of incidence (i.e. plane x,z) may be amplified by the beam [8]. By virtue of the Floquet's theorem for wave equations with periodic coefficients [9], any component of the EM field can be written as a sum of spatial harmonics:

$$u = u_0\exp(j\vec{k}\cdot\vec{r}-j\omega t)[1 + \sum_{n \neq 0}\rho_n\exp(2jn\pi z/\ell + j\phi_n)]; \qquad (7)$$

where $\vec{k}\cdot\vec{r} = k_0 x\sin\theta + k_0 z\cos\theta$, and ρ_n is the amplitude of the nth spatial harmonics. We make the conventional assumption that there is no retroreflection, which is valid if $N|\Delta\epsilon/\bar{\epsilon}|^2 << 1$, and $|\Delta\epsilon/\epsilon_0| << \cos\theta$. This assumption is strictly true in the vicinity of $\theta = 45°$ (see [8]). Substituting the EM field in the form (7) into the Maxwell equations, collecting the terms with $\exp(j\vec{k}\cdot\vec{r}+2jn\pi z/\ell)$ for each particular n and retaining only terms that are first order in $a_n(<<\cos\theta)$, where the a_n's are the Fourie coefficients of $\epsilon(z)$:

$\epsilon(z) = \bar{\epsilon} + \sum_{n=1}^{\infty} a_n\cos(2n\pi z/\ell + \psi_n)$, one gets the amplitudes u_0, ρ_n of the spatial harmonics of nonvanishing components of electric and magnetic fields (E_x, H_y, E_z): $E_{x_0} = E_0\cos\theta$; $H_{y_0} = E_0\sqrt{\bar{\epsilon}}$; $E_{z_0} = -E_0\sin\theta$, and

$$
\begin{Bmatrix} \rho_{x_n} \\ \rho_{y_n} \\ \rho_{z_n} \end{Bmatrix} = \frac{a_n/2}{q(2\cos\theta+q)} \begin{Bmatrix} 1+q\sin^2\theta/\cos\theta \\ 1+q\cos\theta \\ 1-q\cos\theta-q^2 \end{Bmatrix} ; \tag{8}
$$

where E_o is the amplitude of the principal harmonic of total electric field, and $q = 2\pi n/\ell k_o = \lambda n/\ell\sqrt{\epsilon}$. Further calculations are based on the conventional model of energy exchange between the EM field and an electron which is used, e.g. in the theory of free-electron lasers (see e.g. [10]). From the Lorentz equation $mc\, d(\vec{\beta}\gamma)/dt = e(\vec{E}+[\vec{\beta}x\vec{H}])$. one gets the equation for the energy $\mathcal{E} = \gamma mc^2$ of electron

$$
d\mathcal{E}/dt = ec(\vec{\beta}\vec{E}) = ec(\vec{\tilde{\beta}}\vec{\tilde{E}}+\vec{\tilde{E}}\Delta\vec{\beta}+\vec{\tilde{\beta}}\Delta\vec{E}); \tag{9}
$$

where $\vec{E} = \vec{E}[\vec{r}(t),t]$ is the field at the instantaneous location of the electron, $\vec{\tilde{\beta}}$ and $\vec{\tilde{E}}$ are unperturbed vectors, $\Delta\vec{\beta}$ is a small perturbation of electron velocity due to interaction, and $\Delta\vec{E}$ is a small perturbation of the field seen by the electron due its spatial displacement in respect to the unperturbed trajectory, i.e.

$$
\Delta\vec{E} = \frac{\partial\vec{\tilde{E}}}{\partial z}\,\Delta z(t); \Delta z = c\int_0^t \Delta\beta_z dt \tag{10}
$$

For the assumed polarization of the field, it follows from the Lorentz equation that

$$
\Delta\beta_x = \frac{e}{\gamma mc}\int_0^t (E_x-\beta H_y)dt; \ \Delta\beta_z = \frac{e}{\gamma^3 mc}\int_0^t E_z dt \tag{11}
$$

In Eqs (9)-(11) one has to take into account only that particular n^{th} component of the wave which is "resonant" to the speed of electron, i.e. that one with $|\xi-n\pi| << 1$. After substituting $z = c\beta t$ and the amplitudes (8) of the proper resonant harmonic of E_x, H_y, and E_z, into (9)-(11), integrating over the temporal interval $[0,\tau = L/\beta c]$, where τ is a time for an electron to pass through the superlattice, one has to average the result over all the possible phases ϕ_n of the relevant field harmonic. We denote this operation by angle brackets. Note that the term $<\vec{\tilde{\beta}}\vec{\tilde{E}}>$ in (9) vanishes, i.e. the stimulated emission is only due to changes in the electron motion caused by the field. Finally, one gets the total averaged change of the electron energy per pass:

$$
<\Delta\mathcal{E}_n> = \pi\frac{e^2 E_o^2 L^3}{m\beta^3 c^2\gamma}\cdot\frac{1}{(\nu_n\tau)^3}\,[\rho_{x_n}\cos\theta(\rho_{x_n}\cos\theta-\beta\sqrt{\epsilon}\rho_{y_n})\frac{\nu_n}{\omega}\times
$$
$$
(1-\cos\nu_n\tau)+\gamma^{-2}\rho_{z_n}^2\sin^2\theta(-2+2\cos\nu_n\tau+\nu_n\tau\sin\nu_n\tau)]; \tag{12}
$$

where $\nu_n = \omega[\beta\sqrt{\epsilon}(\cos\theta+q)-1] = \beta c(n\pi-\xi)/\ell$ is a resonant factor. For some ν_n, the change of energy $<\Delta\mathcal{E}>$ becomes negative which constitutes the gain of EM field, $<\Delta\mathcal{E}_{EM}> = -<\Delta\mathcal{E}>$. In the non-relativistic case, the main contribution to the change of energy is due to z-components of $\Delta\vec{\beta}$ and $\Delta\vec{E}$ i.e., in (12) $\frac{\nu}{\omega}$ $(\rho_x^2, \rho_x\rho_y) << \rho_z^2 \simeq (a_n/2)^2(1-\beta\cos\theta)^2$. Replacing the term $(-2+2\cos\nu\tau+\nu\tau\sin\nu\tau)/\nu^3\tau^3$ by its negative extremum $-4/\pi^3$ ($\nu\tau \simeq \pi$), one gets the maximal EM-wave gain per electron per pass:

$$
<\Delta\mathcal{E}_{EM}> = a_n^2 e^2 E_o^2 L^3 \sin^2\theta(1/\beta-\cos\theta)^2/\pi^2 mc^2\beta\lambda \tag{13}
$$

In order to obtain an amplification Γ per pass in the system bombarded by an electron beam with the density of electric current $i(A/cm^2)$, one has to multiply

- 6 -

$<\Delta \ell_{EM}>$ by i/e and divide by the energy flux of incident EM wave per unity area of the interface $E_o^2 \cos\theta/2R$, where $R = 377\Omega$ is the vacuum impedance. One has also to take into account that for rectangular form of $\epsilon(z)$, $a_n/2 = (\Delta\epsilon/n\pi) \sin(n\pi\ell_1/\ell)$ with $\Delta\epsilon = \lambda^2(\lambda_1^{-2}-\lambda_2^{-2})$. Bearing in mind a resonant condition (3), one finally gets the maximal EM wave amplification per pass:

$$\Gamma = 8\mu ieRL^3 \sin^2(\pi n\ell_1/\ell)\sin^2\theta/mc^2\ell\,\pi^4\cos\theta, \tag{14}$$

where

$$\mu = \beta^{-1}\ell^4(\lambda_1^{-2}-\lambda_2^{-2})^2(1/\beta-\cos\theta)^5 n^{-5} = \lambda^5\ell^{-1}(\lambda_1^{-2}-\lambda_2^{-2})^2(\cos\theta+n\lambda/\ell).$$

If $\lambda = 140\text{Å}$, $\ell = 100\text{Å}$, $\ell_1 = \ell_2 = 50\text{Å}$, $n = 1$, $L = 1\mu m$, $\lambda_1 \simeq 400$ Å, $\lambda_2 \simeq 800$ Å, $\theta = 45°$, and $i = 5\times10^{10}$ A/cm^2 [4] (i.g. beam of 2 μm diameter with a current $\sim 1.5\times10^3$ A), one gets an amplification $\Gamma \simeq 5\%$ per pass. The required speed of electrons is $\beta \simeq 0.474$ which corresponds to energy $eU = 69$ KeV. For larger λ the amplification increases drastically. With the mirrors situated outside the superlattice to form a Fabry-Perot resonator to provide feedback, the system becomes a short-wave laser which may transform significant portion of energy of electron beam into coherent radiation. It is obvious that the amplifiers and lasers based on the proposed principle should work in the short pulse regime of operation, with the duration of current pulse being determined by the heating, ionization, diffusion of absorbed electrons, etc.

In conclusion, we have demonstrated the feasibility of generating far-ultraviolet and soft X-ray radiation by electron beams with relatively low, non-relativistic energies, traversing the solid-state superlattice composed of very thin periodic layers. The use of low energies is a desirable feature as compared with ultra-relativistic beams. The proposed system can be used as a very efficient noncoherent source of narrowband radiation, and, under special conditions, as an amplifier and laser.

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BISTABLE CYCLOTRON RESONANCE IN SEMICONDUCTORS

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We consider the cyclotron resonance in a semiconductor which has a band with a relativistic energy—momentum disper-
sion and show that, when subject to quasi-resonant radiation, the energy of a free carrier in this band and the relative po-
tential drop between the two surfaces of the semiconductor facing the radiation exhibit bistability under classical single
electron approximation.

In a previous work by one of the authors [1], it
was shown that cyclotron resonance of relativistic
electrons exhibits bistability due to energy dependence
of the relativistic cyclotron frequency. In the present
letter, we consider the possibility of bistable cyclotron
resonance in a semiconductor which has a nonparabolic
band with a relativistic momentum dispersion in at
least part of the first Brillouin zone [2–4]. By means
of a classical analysis, we demonstrate that the cyclo-
tron of free carriers in the semiconductor exhibits
bistability if the semiconductor is immersed in a
homogeneous magnetic field B_0 and irradiated by a
circularly polarized light propagating along B_0.

The present analysis is similar to that of ref. [1].
However, there are a number of novel features in the
present problem. First, the nonlinearity of free carriers
in semiconductors are several orders of magnitude larger
than the nonlinearity of relativistic electrons in vac-
uum. This lowers the threshold intensities for the
onset of hysteretic jumps in semiconductors, despite
rapid thermal relaxation which broadens the cyclotron
resonance. In contrast to ref. [1], the strength of the
nonlinearity does not allow one to neglect the veloc-
ity terms higher than the fourth order and the har-
monics higher than the first. We give an exact result
in the steady state. Second, effective masses can be

quite small in semiconductors. For instance, in narrow
gap semiconductors they are nearly two orders of
magnitude smaller than the bare electron mass in the
vicinity of the band edge [5]. This increases the cy-
clotron frequency, making it possible to pump carriers
with optical or near optical frequencies. For example,
if the semiconductor is n-type InSb and $B_0 \sim 140$ kG,
one can use CO_2-laser at 10.6 μ. Third, the nonlinear
cyclotron resonance in semiconductors is accompanied
by the appearance of a voltage drop between the two
surfaces of the sample facing the radiation. The voltage
drop arises from the redistribution of rotating carriers
under the influence of the radiation pressure and ex-
hibits bistability. Therefore, the proposed effect could
be the first known all-optical nonlinear phenomenon
which yields an opto-electronic bistability.

Consider a semiconductor with a conduction band [1]
which has a relativistic momentum dispersion as in
Kane's isotropic two-band model [2] [2],

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[1] The argument is easily generalized to holes of a valence
band with a relativistic momentum dispersion.

[2] This type of semiconductors have been discussed exten-
sively for the purpose of obtaining a cyclotron maser. See
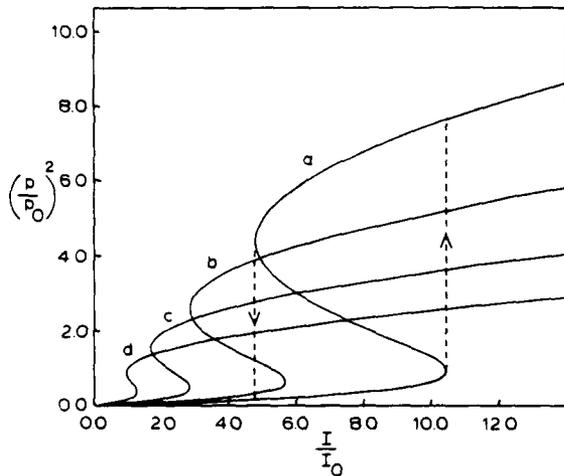refs. [3, 4].

Fig. 1. $(p/p_0)^2$ versus I/I_0 for $\omega_c^* = 10\Gamma$: (a) $\omega = 4\Gamma$; (b) $\omega = 5\Gamma$; (c) $\omega = 6\Gamma$; (d) $\omega = 7\Gamma$.

$$[(\omega_c^*/\omega)^{4/3} + (\omega_c^*/\omega)^{2/3} + 1] \, [(\omega_c^*/\omega)^{2/3} - 1]^{-2}$$

$$\lesssim (\omega/\Gamma)^2 < (\omega_c^*/\Gamma)^2 \,. \tag{6}$$

Of course, to observe the cyclotron resonance, one must have $\Gamma \ll \omega_c^*, \omega$.

It is seen from fig. 1 that, as I is increased from zero, the system moves along the lower branch of a curve corresponding to ω until I reaches I_2. At $I = I_2$, the system makes a hysteretic jump to the stable
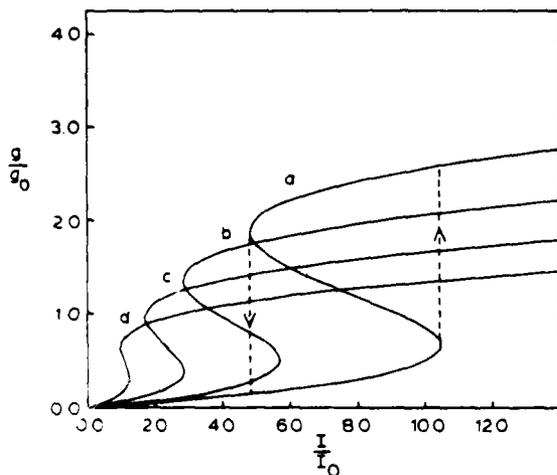

Fig. 2. g/g_0 versus I/I_0 for $\omega_c^* = 10\Gamma$. a, b, c and d are the same as in fig. 1.

upper branch and moves along the upper branch with increasing I. Prior to the jump at I_2, the momentum is approximately given by

$$p_2 \approx p_0 [(\omega_c^*/\omega)^{2/3} - 1]^{1/2} \,.$$

If I is decreased now, the system moves down the upper branch past I_2 to I_1. At $I = I_1$, the system jumps to the lower stable branch. Prior to the jump, the momentum is

$$p_1 \approx p_0 [\omega_c^*/\omega)^2 - 1]^{1/2} \,.$$

After the jump, the momentum is

$$p_3 \approx (p_0 \Gamma/\omega)(\omega_c^* + \omega)^{1/2}(\omega_c^* - \omega)^{-1/2} \,.$$

Note that the hysteretic jumps at I_1 and I_2 are contingent upon the charge distribution being in steady state, at least relative to cyclotron transitions. In other words, the charge relaxation must be fast such that $\tau_0 \Gamma \ll 1$. Furthermore, variations in I must be slow such that $\tau_0 dI/dt \ll I$. Using the expression for mobility $\mu = e/m^* \Gamma$ and the conductivity $\sigma = eN\mu$, where N is the free carrier density, one can write

$$\tau_0 \Gamma = \epsilon_0 m^* \Gamma^2 (4\pi e^2 N)^{-1} = (\epsilon_0/\epsilon_\infty)(\Gamma/\omega_p)^2 \ll 1 \,.$$

ω_p is the plasma frequency. This condition puts a lower limit on the carrier density such that [3]

$$N \gg N_1 = \epsilon_0 m^* \Gamma^2 (4\pi e^2)^{-1} \,.$$

If $N \lesssim N_1$, which corresponds to $\tau_0 \Gamma \gtrsim 1$, then charge oscillations may be excited and the system may not reach a stable steady state.

There is also an upper limit on the free carrier density N. This comes about from the fact that at low temperatures, band states are filled up to a Fermi momentum p_F and cyclotron transitions should excite the electron to states for which $p > p_F$. One can see from the curves in fig. 1 that the transition from the higher branch to the lower cannot occur if $p_3 < p_F$. Therefore, in order to observe the bistable behavior in full, one must have $p_3 > p_F$, and since $p_F = \hbar(3\pi^2 N)^{1/3}$ as temperature goes to zero,

$$\epsilon_0 m^* \Gamma (4\pi e^2)^{-1} = N_1 \ll N$$

$$< N_2 = (3\pi^2 \hbar^3)^{-1} p_0^3 [(\omega_c^*/\omega)^{2/3} - 1]^{3/2} \,. \tag{7}$$

[3] m^* here is the appropriate effective mass near the Fermi surface. Typically $m^* \sim 4m_0^*$.

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