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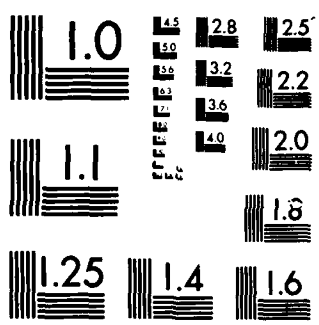

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ADAPTIVE CONTROL TECHNIQUES FOR LARGE SPACE STRUCTURES

AD-A164 240

Annual Technical Report

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This report summarizes the research performed during the period 1 June 1984 to 31 May 1985 on adaptive control techniques for Large Space Structures (LSS). The research effort concentrated on two areas: (1) a further development of an earlier idea on how to perform on-line robust design from identified models - what is referred to here as adaptive calibration; and (2) an analysis of slow-adaptation for adaptive control of LSS. The report summarizes the results obtained in these areas and also includes Appendices which contain detailed descriptions and technical articles.

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TABLE OF CONTENTS

SECTION 1

BACKGROUND AND PROBLEM STATEMENT

1.1 HISTORY OF CURRENT RESEARCH PROGRAM

The Large Space Structure (LSS) research program was originally formulated in late 1982 in response to the increasing concern that performance robustness of Air Force LSS type systems would be inadequate to meet mission objectives. In particular, uncertainties in both system dynamics and disturbance spectra characterizations (both time varying and stochastic uncertainty) significantly limit the performance attainable with fixed gain, fixed architecture controls. Therefore, the use of an adaptive system, where disturbances and/or plant models are identified prior to or during control, gives systems designers more options for minimizing the risk in achieving performance objectives.

The aim of adaptive control is to implement in real-time and on-line as many as possible of the design functions now performed off-line by the control engineer; to give the controller "intelligence". To realize this aim, both a theory of stability and performance of such inherently nonlinear controls is essential as well as a technology capable of achieving the implementation. As has been noted by Astrom (1981) recent advances in each of these fronts have brought us to the position where adaptive control has been applied to many processes and standard adaptive controllers can be purchased, e.g., Egardt (1984).

First Year Technical Results (1 June 1983 - 31 May 1984)

Motivated by the uncertain order of LSS models, the first year research was originally directed toward using variable order adaptive systems. Early in the research, however, lack of a well-developed robustness theory for adaptive mechanizations required a reexamination of the problem at a more fundamental level, i.e., development of model and disturbance uncertainty bounds for which adaptive algorithms would exhibit (stable) desired performance. Adaptive theory at that time only considered model and disturbance parameter uncertainty, and hence, did not provide the means to obtain the relation between achievable performance and unmodeled effects, e.g., unmodeled residual modes and small broadband disturbance. The theory required:

(1) a complete parametric model (no unmodeled dynamics/disturbance)

(2) known model order

(3) known high frequency gain

Under these conditions, it was possible to show that the adaptive system was globally stable, i.e., all the signals remained bounded for any initial set of parameter values and any bounded reference command. If, in addition, the reference input was "sufficiently rich", then the parameters converged exponentially fast to a unique tuned set of values (the paper by Goodwin, Ramadge, and Caines (1980) summarizes the above theory).

For the LSS problem, it is not possible to satisfy a single one of the above theoretical requirements. In the first place, the LSS is theoretically of infinite order and a complete parametric model would be a partial differential equation, not the ordinary differential equation required by the theory. Secondly, and most importantly, the theory could not tolerate even small deviations from the above assumptions. It was shown in simulations by Rohrs et al. (1981, 1982) that unmodeled high frequency dynamics, together with a small high frequency noise - both well outside the controller bandwidth - could cause a parameter drift which eventually destabilized the system. This drift phenomena was also reported to occur in on-site process control applications where it was necessary to re-set the parameters every so often (Wittenmark and Astrom, 1982). Simulation studies involving LSS exhibited similar problems where the parameters would not converge unless the initial parameter values were close to the unknown tuned values (Gupta, Lyons et al., 1981; Sundararajan and Montgomery, 1982).

Because of these difficulties, the first year effort focused on relaxing the requirements on adaptive theory by emphasizing local rather than global results. The distinction between global and local is that in local theory there are restrictions on the magnitudes and frequency content of the inputs as well as the initial parameter values. In global theory there are no such restrictions. Thus, the local theory is more practical and is able to use a priori information that is available.

To summarize, there were two major accomplishments in the first year effort:

(1) Development of a local theory of adaptive control with broad applicability. These results are reported in several joint papers by Kosut and co-workers from 1982-1984 (see References).

(2) Methodology development for LSS based on the above local theory. This result used key ideas from parameter estimation and robust control design under "slow" adaptation.

By "slow" we mean that there is sufficient time to run batch identification before the control system is modified. The use of slow adaptation is anticipated for a large class of LSS missions which have quiescient periods useful for "calibration". The methodology we have developed provides a guaranteed level of performance given an "identified" model of the system together with the model error between the system and the identified model. In fact, our methodology generates performance vs. model error tables (to be stored in the computer) from which the control design is determined strictly on the basis of model error and performance demand, rather than trial and error. These results are summarized in our annual report (ISI Report 43, 1984) and in Kosut and Lyons (1984).

The status of adaptive theory at the end of the first year effort is summarized in Table 1-1. The maturity of adaptive control theory is compared to that of linear control theory with respect to the impact of modeling assumptions and modeling errors. Table 1-1 is primarily applicable to linear finite dimensional systems. Adaptive theory is virtually undeveloped either for ∞ - dimensional linear systems or general nonlinear systems.

TABLE 1-1. STATUS OF THEORY#

	MODELING ASSUMPTIONS		MODELING ERRORS	
	CRITICAL PARAMETERS	EXTERNAL DISTURBANCES	UNMODELED DYNAMICS	UNMODELED DISTURBANCES
LINEAR CONTROL	SMALL, SLOWLY VARYING DEVIATIONS FROM NOMINAL	KNOWN TYPE; KNOWN PARAMETERS (MAG. AND FREQ.)	KNOWN UPPER BOUND OF MAGNITUDE vs. FREQUENCY	KNOWN UPPER BOUND OF MAG. vs. FREQ.
ADAPTIVE CONTROL	LARGE, POSSIBLY RAPID CHANGES	KNOWN TYPE; UNKNOWN PARAMETERS (MAG. AND FREQ).	EMERGING * THEORY	EMERGING * THEORY

Table is primarily applicable to linear finite dimensional systems. Theory is less developed for ∞ - dimensional systems and/or nonlinear systems in each case.

* Primary research areas covered by current contract.

Second Year Results (1 June 1984 - 31 May 1985)

The second year effort continued with theoretical work on further developing the local theory with particular regard to the fact that LSS modes are uncertain, densely packed and very lightly damped. These characteristics of LSS dynamics are among the primary difficulties faced by practical identification/adaptation algorithms. A major task in this effort was to investigate the properties of adaptive systems incorporating algorithms with multi-rate (two time scale) structures, and persistent excitation. These algorithms were studied under slow adaptation. This made it easier to assure achievable specified performance levels despite unmodeled dynamics and disturbances. The effect of speeding up the adaptation has been proposed for study in our current work under Contract F49620-85-C-0094.

New Results and Directions

An important series of developments took place during this second year which will have a positive effect for our present and future research. Our initial efforts at developing a local stability theory have just recently been further advanced by Riedle and Kokotovic (1984). Their results indicate that by combining the averaging theory of Hale (1969) with the local stability theory of Kosut and Anderson (1983), a sharp stability-instability boundary can be obtained for the case of slow adaptation/identification. Insofar as LSS technology is concerned this is a major breakthrough, because as mentioned before, slow adaptation/identification is sufficient in many cases. This new insight made it very important to more thoroughly develop the theory during this second year. A heuristic explanation of our local stability theory and the method of averaging is provided in Section 2.1. A more detailed overview is contained in the summary paper by Kosut in Appendix B.

Collaborative Research Effort

An important point to be made, regarding this research, is that there is a great deal of collaborative effort involved among several researchers who share a common interest in this field. In fact, quite soon after the initial work on input-output stability theory of adaptive systems was published by Kosut and Friedlander (1982, 1985)*, other pockets of research groups began extending these results in a variety of directions, e.g., Ortega, Praly, and Landau (1984), Riedle and Kokotovic (1984), Kosut, Johnson, and Anderson (1983). It became apparent that these researchers should get together for informal meetings. One was arranged by Landau at the University of Grenoble in July 1984 with Kosut, Ortega, and Praly attending. A second was arranged by Kokotovic at Montana State University in August 1984, with most of the above researchers attending.

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* Supported under AFOSR contract F4920-81-C-0051.

These small, rather intensely focused meetings, have proven to be a major catalyst for advancing the research efforts in this field. A third meeting, arranged by Kokotovic, occured in Nov. 1984, at the University of Illinois, the purpose of which is to organize efforts on the further development and use of the averaging theory of Hale (1969). A fourth meeting took place in the summer of 1985.

Also during this period, Dr. Kosut received AFOSR approval to accept an invitation from the Australian National University to work there as a Visiting Scholar. He spent a one month period and worked in close collaboration with Professor Brian Anderson and his collegues. The results are reported in Kosut, Anderson, and Mareels (1985) and included here as Appendix D.

The purpose of describing all those meetings here, is to emphasize the collaborative effort that has been involved in the development of this theory, and also to indicate the degree of interest and excitement in the adaptive control research community about this endeavor. Obviously, without the continuing support from the various government agencies, e.g., AFOSR, NSF, etc., none of this would be possible, at least not at this pace. We are also pleased to report that another result of these collaborations is a forthcoming book by Kosut, Anderson, Kokotovic, et. al., on stability theory for adaptive systems (MIT Press, Spring 1986).

Other Related Research

This program provides for the development of theory which will work in synergy with other related research activities both at Integrated Systems, Inc., and at Stanford University, where Dr. Kosut has a position as Consulting Processor in the Department of Electrical Engineering. At Integrated Systems, Inc., these related programs support development of hardware architectures and associated hardware to provide practical adaptive controls for real DOD/NASA missions. One activity, with the Army Munitions and Chemical Command (AMCCOM), requires development of programmable board level processors to implement adaptive gun turret pointing controls. Control synthesis and simulation is followed directly by programming of the real-time processors, thus sharply reducing real-time system development costs. Another activity is a more general treatment of the AMCCOM problem where robust adaptive mechanizations for complex systems such as LSS will be implementable in a real-time system level hardware architecture. This program was funded under the AFOSR/SBIR office.

At Stanford University, Dr. Kosut meets regularly with Professor G.F. Franklin, and several students, all of whom are working towards their Ph.D. in the area of adaptive control. The students interests vary from LSS to robotics to theory. Two of the students have been working on implementing algorithms for use on experimental flexible systems in the Guidance and Control Laboratory in the Department of Aeronautics and Astronautics under the direction of Professor R.H. Cannon. Their results, both positive and negative, have provided a strong impetus in the direction of our research program with AFOSR.

1.2 RESEARCH OBJECTIVES

The study being reported on here, which is part of our ongoing research, will extend adaptive theory and its application to LSS problems in several directions. These include the following:

(1) Theoretical development - The present emphasis is to merge our present local adaptive theory with the method of averaging of Hale (1969). First attempts involved slow adaptation, since this covers many LSS situations. Later on we will examine fast adaptation. The theory developed here provides for: (a) estimates of robustness, i.e., stability margins vs. performance bounds; (b) estimates of regions of attraction and rates of parameter convergence to these regions. Later on, extensions of the present linear finite dimensional adaptive theory will include nonlinear and infinite dimensional plants and controller structures; and (d) extensions to decentralized systems.

(2) Parameter adaptive algorithms - Assess the behavior of different algorithms, including: gradient, recursive least squares, normalized least mean squares, and nonlinear observer (e.g., Extended Kalman Filter).

(3) Parametric models - Assess the impact of model choices. In particular, the effect of explicit and implicit model choices. An explicit model, for example, is a transfer function whose coefficients are all unknown. In an implicit model transfer function, the coefficients would be functions of some other parameters. Implicit models usually arise from physical or experimental ground data, whereas explicit models are selected for analytical convenience.

(4) Adaptive nonlinear control - Although our early effort is to study adaptive linear control, there are many LSS situations where the control is nonlinear.

Consider, for example, Figure 1-1 which depicts two LSS control modes: (1) vibration suppression, and (2) tracking/slewing. In the vibration suppression situation a "slow" adaptive algorithm (sequential identification/adaptation) may be sufficient, whereas in the tracking/slewing case a "rapid" algorithm (parallel identification/adaptation) may be necessary. These two possibilities stretch adaptive theory at both ends, particularly with regard to convergence rate requirements.

Note also that the two adaptive controllers in Figure 1-1 involve not only different convergence rate requirements, but also involve different controller structures, i.e., linear for vibration suppression and nonlinear for tracking/slewing.

Clearly many combinations of models and control structures are possible, and at this point it is not possible to enumerate all that may be relevant. However, by examining a few sensible choices, such as those compatible with typical LSS objectives, the ensuing theoretical development will indicate the necessary modifications required in each case.

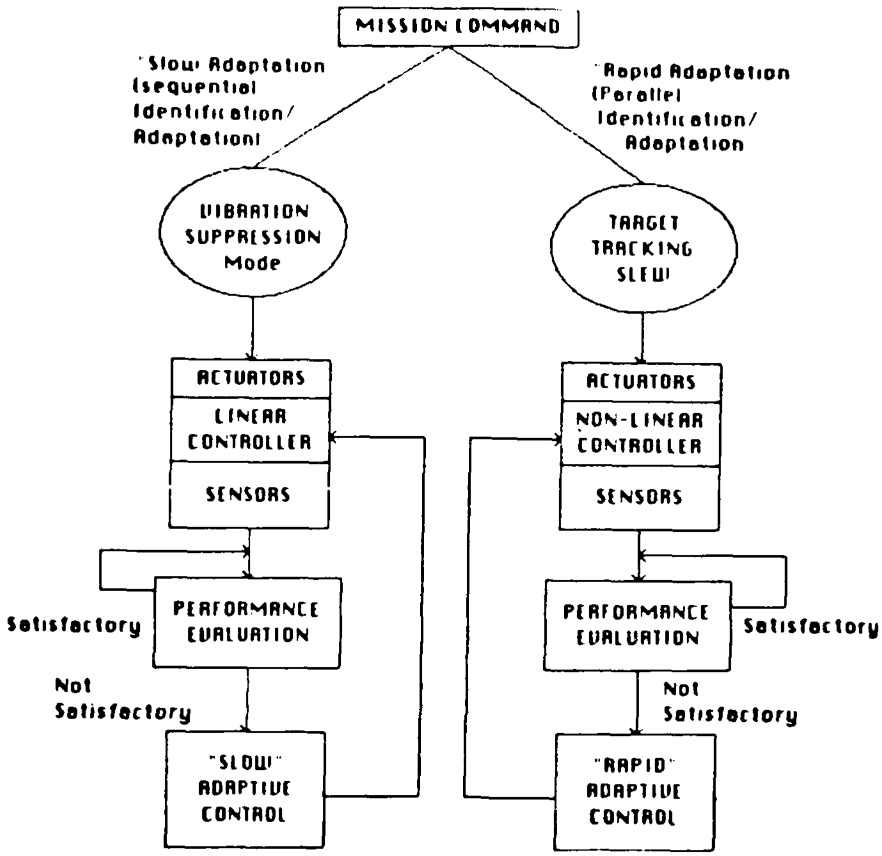

Figure 1-1. Adaptive Structures

1.3 MOTIVATION AND LONG-RANGE GOALS

The issues of performance sensitivity, robustness, and achievement of very high performance with low-order controllers can be effectively addressed using adaptive algorithms. The need to identify modal frequencies, for example, in high-performance disturbance rejection systems has been shown in ACOSS (1981) and VCOSS (1982). The deployment of high-performance optical or RF systems may require on-line identification of critical modal parameters before full control authority can be exercised. Parameter sensitivity, manifested by performance degradation or loss of stability (poor robustness) may be effectively reduced by adaptive feedback mechanizations. Reducing the effects of on-board disturbance sources on the system performance (disturbance rejection) is particularly important for planned Air Force missions. For these cases, adaptive control mechanizations are needed to produce the three-to-five orders-of-magnitude reductions in line-of-sight jitter required by the mission.

Research is essential to identify the performance limitations of adaptive strategies for LSS control both from theoretical and hardware mechanization viewpoints. The long range goal of this proposed research program is to establish guidelines for selecting the appropriate strategy, to evaluate performance improvements over fixed-gain mechanizations, and to examine the architecture necessary to produce a practical hardware realization. The initial thrust, however, is to continue to build a strong theoretical foundation without losing sight of the practical implementation issues.

Impact on Other Aerospace Applications

The adaptive theory being developed for LSS control will spillover to other aerospace applications. Table 1-2 shows a comparison of aerospace system adaptive control applications. The LSS control problems clearly involve aspects of the other disciplines.

1.4 REPORT OUTLINE

In the next section we summarize the technical issues involved in the adaptive control of large space structures. Various detailed technical papers and reports are included as supporting Appendices.

TABLE 1-2. A COMPARISON OF AEROSPACE SYSTEM ADAPTIVE CONTROL APPLICATIONS

Problem Characteristics	ROTORCRAFT VIBRATION SUPPRESSION	LARGE SPACE STRUCTURE POINTING CONTROL	AIRCRAFT WING/STORE FLUTTER SUPPRESSION
Nature of Adaptive Requirement	• Find narrowband model for different flight and operating conditions, for different flexible rotorcraft	• Ground Testing impossible; many mode model that changes with temperature, aging, and configuration	Level 1 • Adjust to differing flight conditions Level 2 • Adapt to sudden model instabilities from unanalyzed store combinations
Key apriori information	• Disturbance frequency	• Sufficient time to partition and identify model bandwidth	• Timing of when stores are dropped
Special Identification Aspects	Near singular disturbance that does not enter through the control	• Multiple narrow and wideband disturbances, large maneuvering loads. • Quiet periods available for long ID runs • Persistent excitation required	• Very short time to identify • Model mismatch and hence model form are very important • Physical Model tuning parameters are very helpful
Special Control Problem Aspects	• Inherently MIMO • Often non-minimum phase • Autotuning capability is essential	• Inherently MIMO • Need very high disturbance rejection gain • Controller bandwidth is packed with modes • Off-line robustness analysis and design are helpful	• Stability is the key problem • Some off-line analysis of a-priori models and stability boundaries essential to guarantee stability
Adaptive Mechanization Aspects	• Frequency-shaped time domain model concentrates controller energy • Notch filter reduces parameter estimation load • Hysteresis adaptive on/off logic bypasses persistent excitation problems	• Controller update rate can be slower than basic sampling and ID rate. • Decentralized control may be valuable due to model complexity, system reliability, computational requirements and step-wise deployment	• High computational load can benefit from parallel architectures • Persistent excitation is a problem - tuning on/off excitation, freezing adaptation, and co-variance modification are potential solution

1.5 SELECTED PUBLICATIONS TO DATE

Journals

1. R.L. Kosut and M.G. Lyons, "Issues in the Adaptive Control of Large Space Structures", AIAA J. Guid. Control, submitted.

2. #R.L. Kosut, B.D.O. Anderson, and I Mareels, "Stability Theory for Adaptive Systems: Methods of Averaging and Persistency of Excitation:, IEEE Trans. Auto. Control, to appear.

3. B.D.O. Anderson, R.R. Bitmead, C.R. Johnson, Jr. and R.L. Kosut, "Stability Theorems for the Relaxation of the SPR Condition in Hyperstable Adaptive Systems", IEEE Trans. Auto Control, submitted.

4. R.L. Kosut, and B.D.O. Anderson, "A Local Stability Analysis for a Class of Adaptive Systems", IEEE Trans Aut. Control, to appear.

5. R.L. Kosut, and C.R. Johnson, Jr., "An Input-Output View of Robustness in Adaptive Control", Automatica: Special Issue on Adaptive Control, Sept., 1984.

6. * R.L. Kosut and B. Friedlander, "Robust Adaptive Control: Conditions for Global Stability", IEEE Trans. Auto Control, Vol. Ac-30, No. 7, July 1985.

Books

R.L. Kosut, P. Kokotovic, B.D.O. Anderson, et. al., Stability of Adaptive Systems, in preparation, MIT Press, Spring, 1986.

R.L. Kosut, "Methods of Averaging for Adaptive Systems", Adaptive Systems: Theory and Applications" Editor: K.S. Narendra, Plenum Press, Fall, 1985.

R.L. Kosut, "Adaptive Calibration for the Identification and Control of Large Space Structures", Adaptive Systems: Theory and Applications, Editor, K.S. Narendra, Plenum Press, Fall, 1985.

Research performed while R. Kosut was a Visiting Fellow at the Australian National University.

* Started under contract F4920-81-C-0051, while R. Kosut was wit' SCT

SECTION 2

PROBLEM FORMULATION AND TECHNICAL DISCUSSION

The development of a design methodology for adaptive control of LSS involves many different issues. In this section we present a selective discussion of the theoretical and practical issues that seem most relevant. A more in depth discussion of the overall LSS control design problem is presented in Appendix C. The discussions there cover various types of control design approaches, including both robust (non-adaptive) as well as adaptive. The major points, however, are summarized in this section.

2.1 LSS CONTROL PROBLEM SETTING

Control Design Objectives

Problems associated with vibration control and accurate pointing of DEW/LSS systems typically involve a combination of the following control-performance objectives.

(1) modal damping augmentation to enhance transient settling or improve quasi-static vibration propagation behavior,

(2) stabilization of the attitude control system,

(3) eigenvector modification to reject narrow band steady-state disturbances, and

(4) maneuver load management to minimize structural loads or modal excitation (transient or steady-state).

Modeling

The basis for selecting a control strategy must include an adequate description of the relevant structural dynamics together with a description of how system performance is to be measured. Initially, continuum models were suggested as the basis for proper system design since discretization of the model could be postponed or eliminated, Hughs and Skelton, 1981. Unfortunately, practical spacecraft configurations do not present simple boundary conditions or simple shapes, hence, p.d.e., representations are nearly impossible to write. However, such continum models have provided useful insight into appropriate discrete representations. Finite element models can provide adequate fidelity, at least over the frequency range needed for the control design model, and are supported with sophisticated software tools easily adapted to the needs of control design, ACOSS (1981).

Two Level Control Architecture

The basic control architectures can readily be combined into a two-level control system architecture consisting of a wide-band, low-authority control (LAC) and a narrow-band, high-authority control (HAC), see Figure 2-1. The LAC introduces low damping (2-10%) in a wide range of modes for maximum robustness. HAC provides high damping and mode shape adjustment in selected modes to meet performance requirement.

LAC synthesis principally involves passivity methods and rate feedback mechanizations, usually with co-located actuators and sensors, Aubrun (1980), Iwens, et. al (1980).

HAC synthesis, in addressing performance goals associated with dynamic wavefront and line-of-sight error suppression, requires high modal damping and mode shape changes. Hence, the HAC is dependent on accurate narrow-band models. For such requirement, it is essential that control design techniques manage both dependence on model fidelity and system gain in regions where model fidelity is poor. This has generally been accomplished using fixed-gain robust control theory, e.g., Kosut, Salzwedel, and Emami (1983). With this architecture it is likely that only the HAC would be tuned by an adaptive system since the LAC is inherently robust.

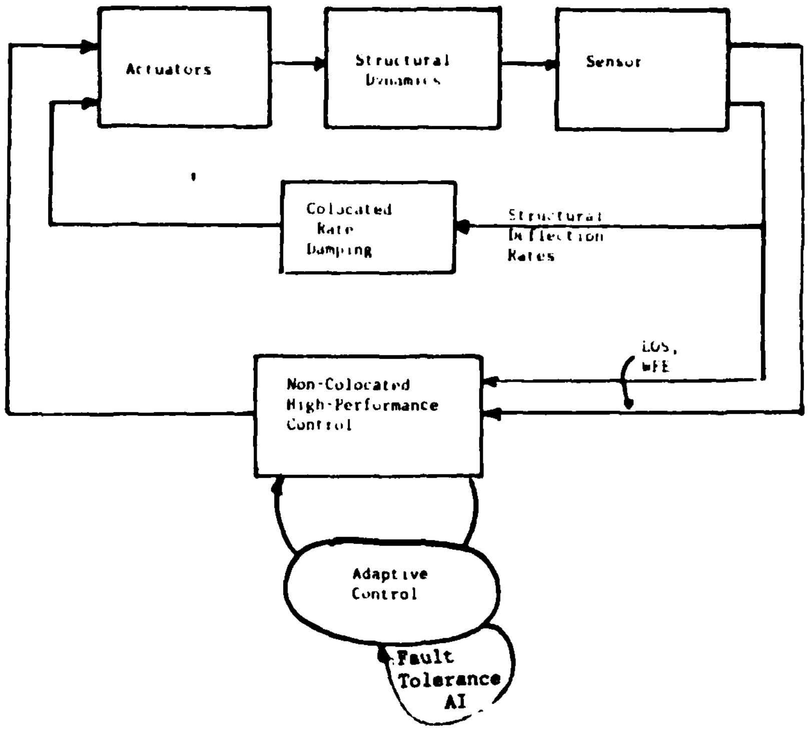

Figure 2-1. Features of Control Architecture

Adaptive Techniques

In general, uncertainties in both disturbance spectra and system dynamical characteristics limit the performance obtainable with fixed gain, fixed order controls, e.g., HAC system. The use of an adaptive control mechanization where disturbance and/or plant dynamics are identified prior to or during control, gives system designers more options for minimizing the risk in achieving performance benchmarks.

In the case of LSS/DEW systems, the performance levels are extremely high. Hence, it is necessary that disturbance and plant models are accurately known. Since model data obtained from ground testing is unlikely to sufficiently match the actual on-orbit system, it follows that on-line procedures are needed for identification and control.

The generic properties of closed-loop system performance vs. structural parameter variations are depicted in Figure 2-2.

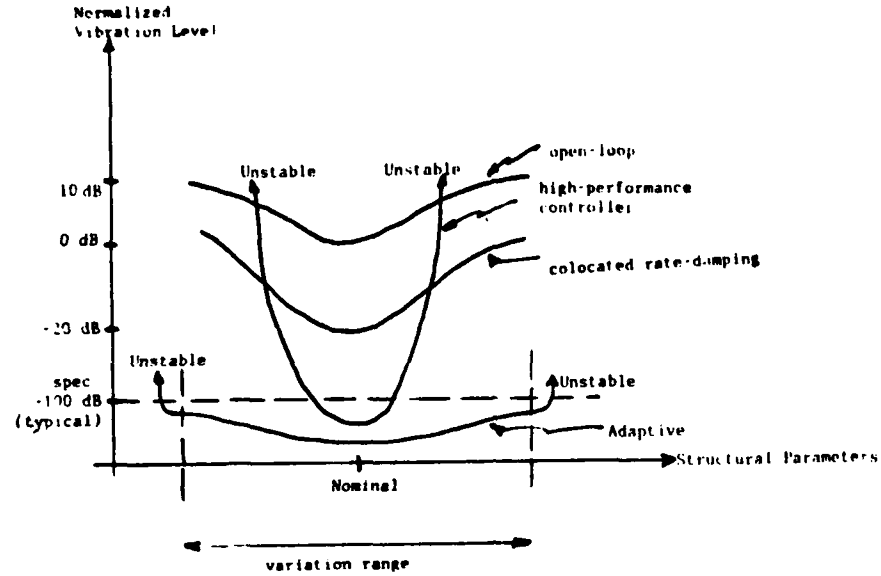

Figure 2-2. Performance Vs. Strucutral Parameter Variations
(Generic Properties)

Assessment of Adaptive Techniques

Available Algorithms

Many adaptive control and identification algorithms exist for lumped parameter, finite-dimensional linear systems, e.g., Goodwin and Sin (1984); Ljung and Soderstrom (1983). Most available algorithms can be cast into the form shown in Figure 2-3. For example, a user could select from the following catalog of model forms, control design procedures, and parameter adaptive mechanisms:

Model	Control Design	Adaptation
ARMAX	Model Reference	Gradient
State-Space	Self-Tuning Pole-Placement	Recursive Least Squares Recursive Max Likelihood Extended Kalman Filter

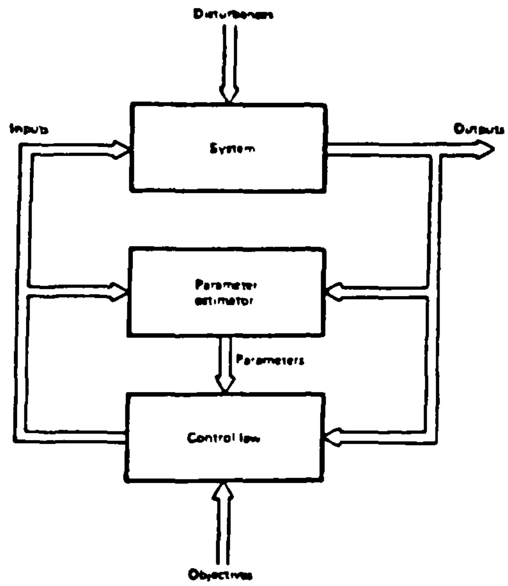

Figure 2-3. Adaptive Control System

These schemes also differ in terms of update rates. Typically the outer control loop is at a fast rate, whereas the parameters from identification are updated more slowly. Adaptive schemes are generally referred to as recursive if the identification rate is a fixed multiple of the controller rate. If identification is used for occasional tuning or calibration, the scheme is referred to here as adaptive calibration.

Consider, for example, Figure 1-1, which depicts two LSS/DEW control modes: (1) vibration suppression, and (2) tracking/slewing. In the vibration suppression situation a "slow" adaptive algorithm (sequential identification/adaptation) may be sufficient, whereas in the tracking/slewing case a "rapid" algorithm (parallel identification/adaptation) may be necessary. These two possibilities stretch adaptive theory at both ends, particularly with regard to convergence rate requirements.

Note also that the two adaptive controllers in Figure 1-1 involve not only different convergence rate requirements, but also involve different controller structures, i.e., linear for vibration suppression and non-linear for tracking/slewing.

Direct application of available algorithms to the LSS system is restricted because of a lack of theory regarding the system's robustness to model error. In addition, it is not known how limitations imposed by a decentralized for pre-selected control architecture will effect achievable performance. Those two issues will now be discussed.

Robustness to Model Error

The use of available identification and adaptive algorithms, which are based on finite-dimensional, linear models, on LSS systems introduces a troublesome source of model error, i.e., high frequency unmodeled structural modes, of which there are theoretically an infinite number. Other sources of model error include uncertain actuator/sensor dynamics and neglected nonlinearities in joints and damping mechanisms.

Although available theory can handle a finite number of parameter errors, it cannot deal effectively with other types of model error, specifically, the unmodeled high frequency structural modes and dynamics. It was shown in simulations by Rohrs et al. (1981, 1982) that unmodeled high frequency dynamics, together with a small high frequency noise - both well outside the controller bandwidth could cause a parameter drift which may eventually destabilize the system. This drift phenomena was also reported to occur in on-site process control applications where it was necessary to re-set the parameters every so often (Wittenmark and Astrom, 1982). Simulation studies involving LSS exhibited similar problems where the parameters would not converge unless the initial parameter values where close to the unknown tuned values (Sundararajan and Montgomery, 1982).

Robustness to model error is more well understood in the context of nonadaptive linear control theory, e.g., Doyle and Stein (1981); Zames and Francis (1983); Chen and Desoer (1982). The common theoretical basis for these robustness theories is the input-output view of feedback systems (Zames, 1966; Desoer and Vidyasagar, 1975, Safonov, 1980). Another viewpoint on robustness, which follows more along the lines of Liapunov stability theory, characterizes the solutions of perturbed nonlinear ordinary differential equations, e.g., LaSalle and Lefschetz (1961), Hale (1969).

A main intent of our research program has been to merge the input-output view and the Liapunov stability view with adaptive mechanizations to develop a theory of robust adaptive control. Some of the groundwork has already been accomplished (Kosut, et. al., 1982-1985) and as such, we are now in a good position to address those issues as they related to LSS systems.

Decentralized Adaptive Control

Limitations on control authority and the information pattern are the main features of the decentralized control problem. The general structure of such a decentralized control system is illustrated in Figure 2-4. The dashed lines indicate a partial information exchange, e.g., the local controller receives reference commands (or discretes) from a higher level control (the coordinator) and/or information from other local controllers in the form of an "aggregated" state.

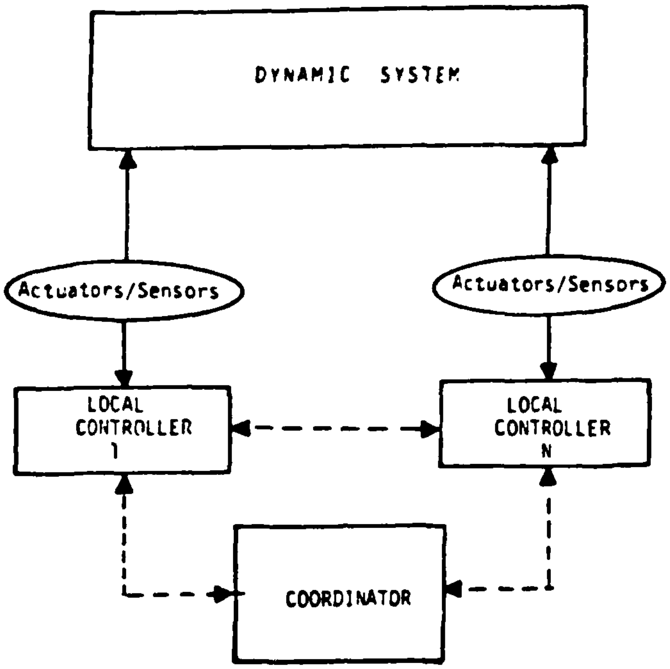

Figure 2-4. Decentralized LSS Control. Dashed Lines Indicate a Partion Exchange of Information

For LSS/DEW systems, decentralization normally results because a natural separation is physically or geographically present between functional components of the system. For example, some decompositions result from spatial differences: weak dynamic interaction effects can be easily identified. A decomposition also occurs from temporal differences; phenomena occurring at different time-scales, e.g., a separation between fast and slow modes or between low frequency and high frequency effects. For example, groups of the modes can be separately controlled by separate controllers which do not destabilize each other. Specific combinations of weak dynamic coupling and separation of slow and fast modes can often be identified, e.g., Figure 2-5.

A number of very useful results are available for non-adaptive decentralized systems, e.g., Vidyasagar (1981), Siljak (1978). Those results show how total system properties are dependent on subsystem and interconnection properties.

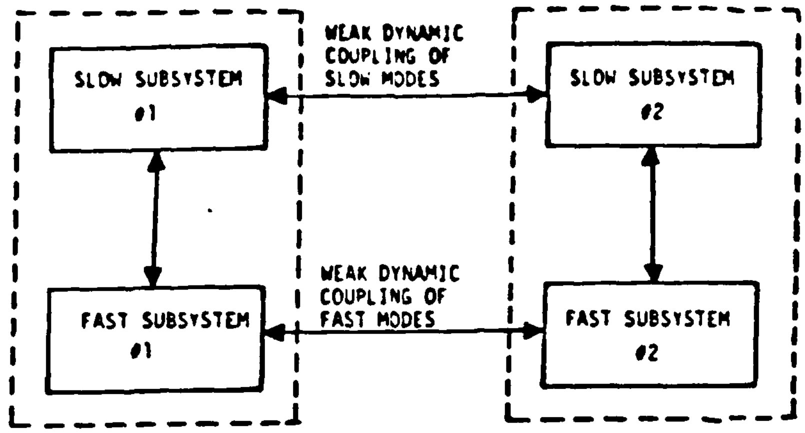

Figure 2-5. Weak Dynamic Coupling with Slow and Fast Modes

Much less is known about adaptive decentralized control, although there are some promising preliminary results available, e.g., Ioannou (1984). There is, however, a similarity in robustness theory and the theory of decentralized control which we hope to explore and exploit.

Research Objectives

The dream of adaptive control researchers has always been to develop a "global" stability theory, i.e., performance is guaranteed independent of initial conditions and disturbance spectra. Such results have been obtained, but as mentioned, the available theory requires that a finite dimensional parameterization exists such that an exact matching condition is obtained (see e.g. Ljung and Soderstrom, 1983; Goodwin and Sin, 1984). Such a stringent requirement is impossible to obtain in an LSS system, principally because there are many sources of model error which defy an exact parameterization (see Section 1).

Because of these difficulties, recent research efforts have focused on relaxing the requirements on adaptive theory by emphasizing local rather than global results. The distinction between global and local is that in local theory there are restrictions on the magnitudes and frequency content of the inputs as well as the initial parameter values. In global theory there are no such restrictions. Thus, the local theory is more practical and provides robustness to model error by utilizing a priori information that is available. In addition, restrictions imposed by a decentralized architecture can, in principal, also be accounted by the form of the theory.

The basic ideas for local stability rest on two fundamental stability theories. One is the small gain theory of Zames (1966), and the other is the method of averaging as described by Hale (1969). These theories have different origins and to some extent have application in different regimes for nonlinear systems. Small gain theory determines the stability of trajectories. With regard to LSS problems, small gain theory is generally applicable to "fast" recursive adaptive control; averaging theory is applicable to "slow" identification and then control, i.e., adaptive calibration. One can envision small gain theory as providing a macroscopic view while averaging provides a microscopic view. Our intention is to merge these as much as possible and thus, broaden the application of each approach, particularly for the LSS system. This is not without precedent.

Small gain theory was applied originally to continuous-time adaptive systems by Kosut and Friedlander (1982, 1985) and more recently extended by Kosut and Johnson (1984), and Kosut and Anderson (1984). The method of averaging was applied to continuous-time adaptive systems with almost periodic inputs for the first time by Riedle and Kokotovic (1984) and Astrom (1984). Extensions to more general input classes have been obtained by Kosut, Anderson, and Mareels (1985). The averaging theory has also been able to accurately predict the drift phenomena observed by Rohrs, et al. (1982).

Although small gain results apply equally to continuous or discrete-time systems, the averaging theory is underdeveloped for the discrete-time case. A major proposed early effort, therefore, is to develop a discrete-time averaging theory useful for sample-data adaptive systems. This will provide a foundation for later efforts in sampled-data adaptive systems with multi-rate and decentralized processors.

2.2 SUMMARY OF UNDERLYING THEORY

In this section we present a heuristic discussion which summarizes the underlying theory. As mentioned, the basic ideas rest on two fundamental theories: 1) is the small gain theory of Zames (1966), and 2) the method of averaging as described by Hale (1969).

To heuristically describe our approach, consider the feedback system in Figure 2-6, below where G_1 and G_2 represent operators.

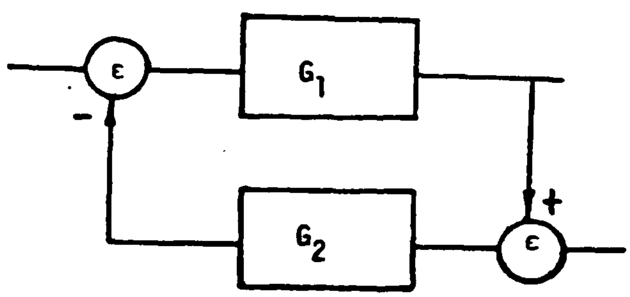

Figure 2-6. Feedback System

Small gain theory asserts that if

$$\text{Gain}(G_1) \cdot \text{Gain}(G_2) < 1 \qquad (1)$$

then the feedback system is stable. In robustness theory, the feedback loop is arranged so that G_2 represents all of the nominally zero uncertain elements of the system. The operator, G_1, referred to as a return-difference operator, is everything else. It is always possible to arrange a feedback loop this way (Safonov, 1978). Thus, G_1, is dependent on the presumed uncertainty location. Hence, stability margin is the maximum allowable uncertainty which guarantees stability, i.e.,

$$\text{Stability Margin} = 1/\text{Gain}(G_1) \qquad (2)$$

For linear-time-invariant systems it is not difficult to calculate Gain (G_1); in fact, this can be accomplished in the frequency-domain. These results parallel Bode analysis, and offer the engineer a very useful design tool. At the present time, however, no such tool exists for adaptive systems. The main difficulty is that in the adaptive case G_1 is nonlinear. Nonetheless, certain types of nonlinearities in G_1 can result in a frequency-domain test, e.g., Popov criterion, hyperstability concepts, positivity, etc. These latter results follow from passivity theorems, and interestingly enough, this is the main tool in current adaptive theory for proving stability. Unfortunately, however, by proceeding this way, it can be shown (Kosut, 1982a) that the resulting stability margins are very small and are easily violated in any realistic environment.

The approach we are proposing here is to by-pass the conservative passivity conditions and rearrange the adaptive system so that the two operators are intrinsically small. This requires introducing the notion of a tuned controller (Kosut, 1985) and a tuned set model representation of the plant, which accounts for uncertainty. As a result of this transformation we can introduce averaging. Hence, a less conservative calculation is

$$\text{Stability Margin} = 1/\text{Avg Gain } (G_1) \tag{3}$$

The averaged gain is significantly smaller than the usual gain calculation, and thus, produces a much larger stability margin, i.e., greater performance.

In Section 2.3 we outline this approach. Further details using small gain theory can be found in Kosut and Anderson (1984) with averaging results in Reidle and Kokotovic (1984) and in Kosut, Anderson, and Mareels (1985). Appendix B contains a detailed report summarizing the application of averaging methods to adaptive systems.

2.3 ADAPTIVE CONTROL THEORY

In this section we will discuss issues and new directions in adaptive control theory. To illustrate the ideas we will consider a continuous-time generic representation for control or identification of a scalar (single-input-single-output) plant. In all cases the comments apply equally to discrete-time, as well as multivariable systems. The generic adaptive system, shown in Figure 2-7, is described by,

$$e = e_* - c'x \qquad \dot{x} = Ax + bz'\Theta$$

$$z = z_* - Dx \qquad \dot{\Theta} = \gamma ze \tag{4}$$

where $\Theta(t) = \hat{\Theta}(t) - \Theta_* \in R^P$ is the parameter error vector; $\hat{\Theta}(t)$ is the current parameter vector estimate; Θ_* is a constant tuned parameter vector setting; $z(t) \in R^P$ is the regressor vector; $x(t) \in R^n$ is the system state consisting of plant and filter states; $e(t)$ is the error signal; and $\gamma > 0$ is the adaptive gain. The signals $e_*(t)$ and $z_*(t)$, referred to as the tuned error and tuned regressor, respectively, are outputs from the ideal tuned system where $\Theta(t) = 0$, i.e., $\hat{\Theta}(t) = \Theta_*$.

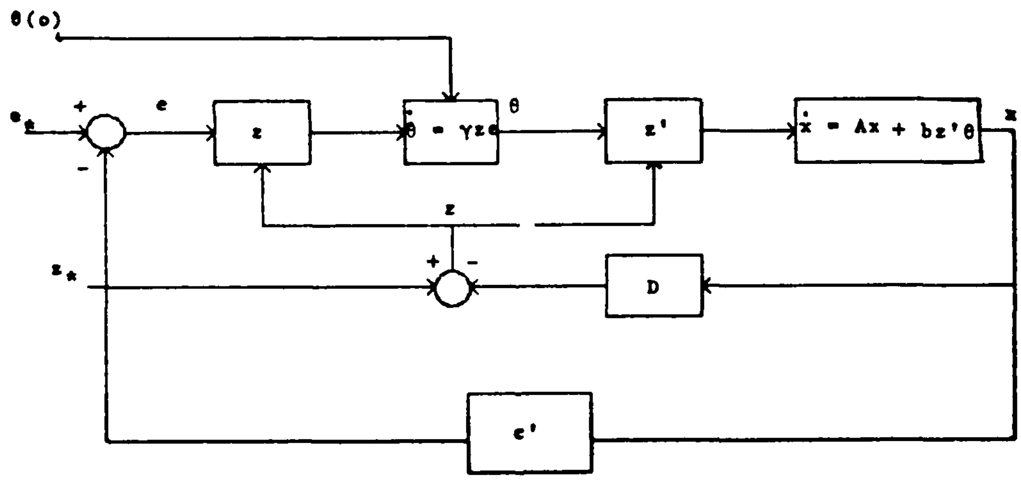

Figure 2-7. Generic Adaptive System

Appendix (A) shows how the nonlinear system (4) arises in the analysis of adaptive control or identification. More detail on the development of (2.1) can be found in Kosut and Friedlander (1982, 1985), and Kosut and Johnson (1984).

Global Stability Theory

The assumptions required to insure global stability and convergence of (2.1) are quite strict, e.g., Narendra, Lin, and Valavani (1980), Goodwin, Ramadje, and Caines (1980). These assumptions include:

(A1) $e_*(t) \rightarrow 0$ as $t \rightarrow \infty$

(A2) $z(t)$ persistently exciting (PE)

(A3) $c'(sI-A)^{-1}b$ is strictly positive real (SPR)

In all practical cases these assumptions are violated. Assumption (A1) implies that there are no continually acting unmodeled noises or disturbances. Assumption (A2) can not be guaranteed unless (A1) holds. The SPR assumption in (A3) is the most restrictive, since it requires a priori knowledge of model order, relative degree, and high frequency gain, all of which are not available or infeasible to obtain. Moreover, these requirements make no sense at all for an LSS whose order is theoretically infinite. Unfortunately, even small deviations from these assumptions can critically disrupt an adaptive system, e.g., Rohrs et al. (1981, 1982), Reidle, and Kokotovic (1984).

Example of Adaptive Calibration

The basic problem with control based on identified modeling is that without a measure of model error it is very easy to destabilize the system - particularly when the goal is high performance - as in an SBL. Adaptive calibration is an approach which incorporates a measure of model error with robust control design in an iterative way so that identification is performed only where it is needed. One proposed adaptive calibration system is shown in Figure 2-8, with test results, using the CSDL #2 model, shown in Figure 2-9. The adaptive calibration procedure involves the following steps:

Step 1: The model M(s) is a 10-mode model which has been obtained from I/O data.

Step 2: Estimate $\hat{\delta}(\omega)$ = model error vs. frequency using FFT. This is dashed curve in Figure 2-9(a).

Step 3: Using the identified model M(s) and the model error $\hat{\delta}$, synthesize a robust control (e.g., Appendix C).

Step 4-5: Calculate $\delta_{SM}(\omega)$ - stability margin (or $\delta_{pm}(\omega)$ - performance margin). This is dark curve in Figure 2-9(a). Compare to model error $\delta(\omega)$. Both plotted in Figure 2-9(a). If $\delta(\omega)<\delta_{pm}(\omega)$ go to Step 7 and implement controller. Otherwise, go to Step 6.

Step 6: Modify filter windows, number of parameters, or input spectrum and then repeat Step 1 to obtain new ID model. Figure 2-9(b) shows result of ID after one mode is added in frequency domain region where test fails.

Step 7: Implement controller.

These preliminary results are quite promising. A major portion of our initial research effort has been to understand the nature of such schemes at a fundamental level.

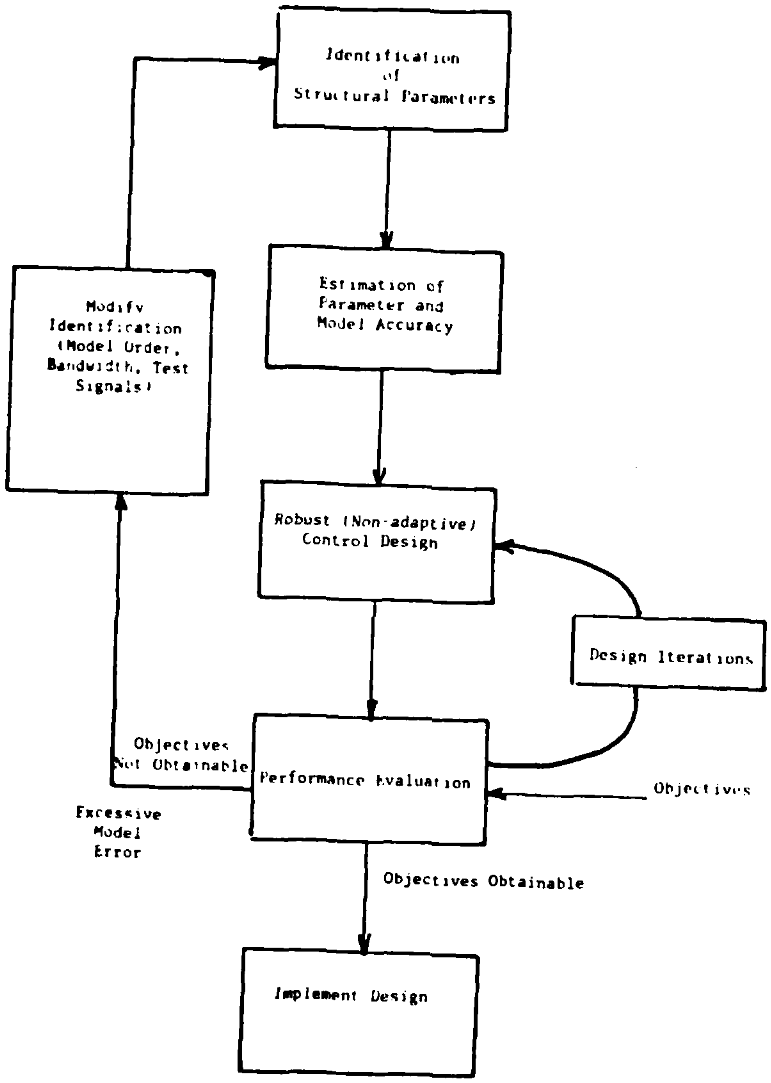

Figure 2-8. A Procedure for Adaptive Calibration of LSS

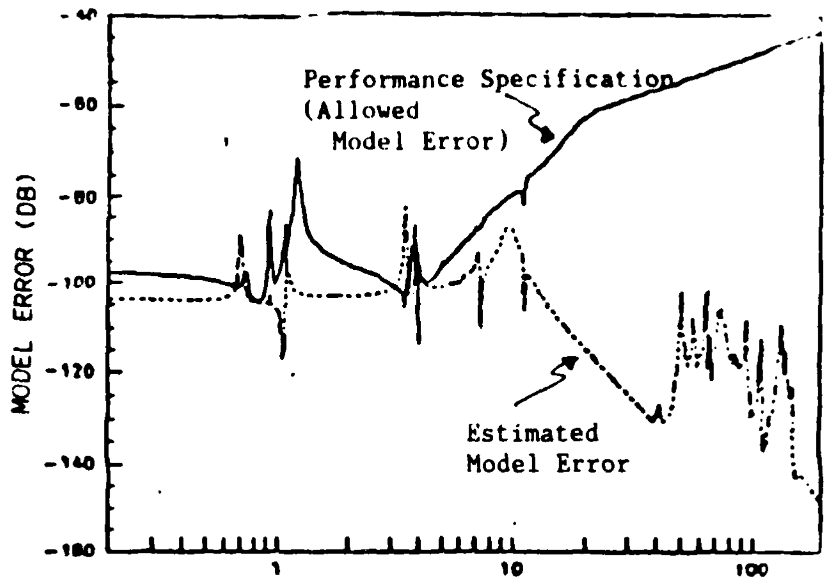

Figure 2-9(a). Comparison of Allowable and Estimated Model Errors

● PLOTS SHOW WHERE ERROR IS > ALLOWABLE

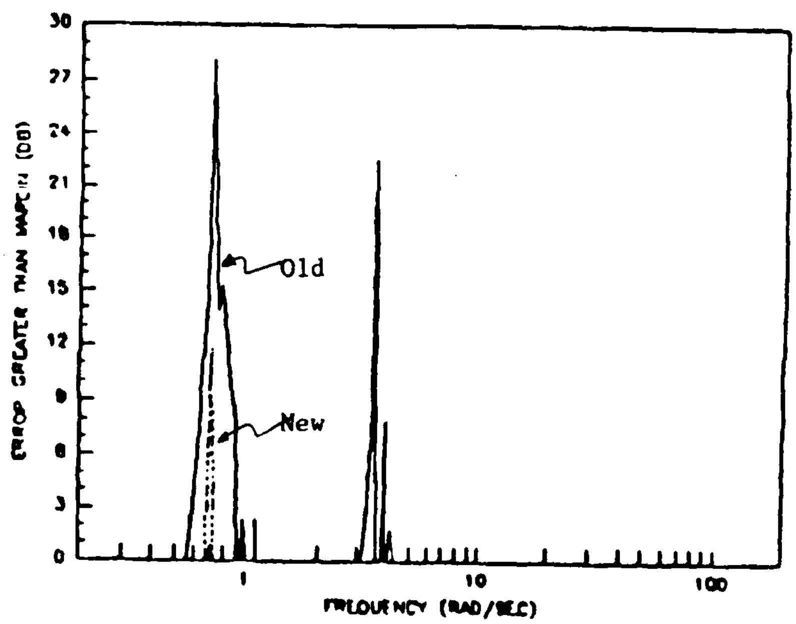

Figure 2-9(b). Data Before and After ID Cycle

Local Stability Theory

Boundedness Results (Fast Adaptation - High Gain)

With regard to practical adaptive design techniques, the need for the SPR condition - hence, the restrictive modeling assumptions--can be eliminated by considering conditions for local stability rather than global stability, see e.g., Kosut and Anderson (1984). The term 'local' refers to the use of known restrictions on the system external inputs, uncertain parameters, and unmodeled dynamics. For example, since persistent excitation induces exponential stability (Anderson, 1977), and since an exponentially stable system is inherently robust, it is logical to expect that unmodeled dynamics could be robustly tolerated. Other mechanisms--including persistent excitation--can ensure stability of the adaptive system, without SPR, provided certain other restrictions are enforced, e.g., slowly varying signals, approximate SPR, gain retardation, and restricted signal magnitudes and bandwidths (Ioannou and Kokotovic, 1982-1984; Kosut, 1984). It is our intention to utilize these theoretical results, whenever appropriate, in the adaptive LSS study, as this local theory emerges.

Stability Results (Slow Adaptation-Low Gain)

The above local theory, which proceeds from the Small Gain Theory of Zames (1966), and Desoer and Vidyasager (1975) can be considered as giving conditions for boundedness (in the input-output sense) rather than stability (in the Liapunov sense). The recent work of Reidle and Kokotovic (1984) utilizes the averaging theory of Hale (1969) to obtain a sharp stability - instability boundary for slow adaptation, i.e., when γ in (2.1) is sufficiently small. To apply the averaging theory requires transforming (2.1) to the following form:

$$\dot{\theta} = \gamma f(t,\theta,\xi,\gamma) \qquad (5)$$

$$\dot{\xi} = A\xi + \gamma g\,(t,\theta,\xi,\gamma)$$

where f and g are time-varying nonlinear functions. As $\gamma \to 0$ the stability of (2.2) is identical to the stability of the "averaged" system

$$\begin{aligned} \dot{\theta} &= \gamma f_o(\theta) \\ \dot{\xi} &= A\xi \end{aligned} \tag{6}$$

where

$$f_o(\Theta) = \lim_{T \to \infty} \frac{1}{T} \int_o^T f(t,\Theta,o,o)\,dt \tag{7}$$

Without going into the details here, it can be shown that there exists a sufficiently small positive γ such that (6) is stable -- and hence, (5) is stable - if and only if a persistent-excitation type of condition is satisfied. The important results here may be summarized as follows:

(1) For sufficiently slow adaptation, there exists a sharp stability - instability boundary.

(2) The conditions do not require SPR, but do require persistent excitation.

Much work has to be done to further develop this theory in general and to tie it together with the boundedness results by Kosut, et. al. For LSS in particular it is necessary to extend the theory to discrete-time, ∞ -dimensional and nonlinear systems. Note, however, that the form of the system (2.2) required by the averaging theory can be nonlinear.

Extensions to Discrete-Time

An important step in our research is to develop the averaging theory of Hale (1969) to discrete-time adaptive systems. There appears to be no theory available at the present time. The obvious extension is to consider the discrete-time system analagous to (5), i.e.,

$$\begin{aligned} \Delta\theta &= \gamma f(t,\theta,\xi,\gamma) \\ \Delta\xi &= A\xi + \gamma g(t,\theta,\xi,\gamma) \end{aligned} \tag{8}$$

where Δ is the finite-difference operator $(\Delta x)(t) = x(t+1)-x(t)$ and t takes on discrete-values $t = (1, 1, 2, \ldots)$. One can make the conjecture that the discrete-time averaging theory states that as $\gamma \to 0d$, the stability of (2.2)' is identical to the stability of the averaged system

$$\begin{aligned} \Delta\theta &= \gamma f_o(\theta) \\ \Delta\xi &= A\xi \end{aligned} \tag{9}$$

where

$$f_o(\theta) = \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T} f(t,\theta,0,0) \tag{10}$$

We will conduct research to determine if this conjecture is true, or what the necessary modifications involve.

2.4 ADAPTIVE CONTROL STRUCTURES

In this section we explore variations of (4) arising from different models, control structure definitions, and parameter update laws. These variations will be useful in different LSS control applications.

Structure of the Parameter Adaptive Algorithm

The adaptive parameter update algorithm in (4), $\dot{\Theta} = \gamma ze$, although useful for analysis, is too simple in practical cases. The more general form:

$$\dot{\Theta} = \gamma\, h(t,z,e) \tag{11}$$

can arise from normalization,

$$h(t,z,e) = ze/(1 + |z|^2) \tag{12}$$

least squares considerations,

$$\begin{aligned} h(t,z,e) &= R(t)^{-1}\, ze/(1 + z'\, R(t)^{-1}\, z) \\ \dot{R}(t) &= z(t)\, z(t)' \end{aligned} \tag{13}$$

dead-zones,

$$h(t,z,e) = \begin{cases} ze, & |e| > m \\ o, & |e| \leq m \end{cases} \qquad (14)$$

or combinations of all of the above. These all have their use and characteristics properties [see Goodwin and Sin (1984) or Ljung and Sodestrom (1983) for a survey]. Which combination of these or other structures is useful for LSS needs to be determined.

Multi-Rate Adaptive Control

Multi-rate adaptive schemes (or two-times scale schemes) refers to an adaptive structure where the adaptive control parameters are updated more slowly than the basic control sampling rate. This scheme allows a period of plant identification to be followed by a change in control parameters. Allowing more time in the identification phase also allows for more reliable pass-fail tests. For example, we can perform a fit-error test after parameter identification on the old data as well as estimating its information content. If either test fails then the control gains are not updated. If they both pass then the gains can be updated in the direction of maximum information. We can also include a robustness test based on a priori (or new) model error bounds. Ideas such as these have been considered for LSS applications by Sundararajan and Montgomery (1982).

Hybrid Adaptive Control

If, in the multi-rate adaptive structure, the basic control sampling rate becomes continuous, then the structure is referred to as a "hybrid" structure: the feedback is continuous but is updated at discrete times.

Such a scheme reduces intersample ripple and also tends to increase bandwidth and noise attenuation. These structures may prove quite practical for the LSS.

Linear Finite-Dimensional Models

The modeling assumptions influence the adaptive control structure. Assume that the plant to be controlled can be accurately represented by a finite-dimensional model of the form

$$y = -\theta' x_m$$
$$\dot{x}_m = A_m x_m + B_m (u, y)' \tag{15}$$

where u is the input, y is the output, A_m is a stable nxn matrix, and (A_m, B_m) is controllable. (see e.g. ch. 2, Kailath, 1980). Under these conditions, the transfer function $P_m(s,\theta)$ from u into y is:

$$P_m(s,\theta) = \frac{\theta_{n+1}s^{n-1} + \theta_{n+2}s^{n-2} + \ldots + \theta_{2n}}{s^n + \theta_1 s^{n-1} + \ldots + \theta_n} \tag{16}$$

This type of model is referred to as an <u>explicit</u> parametric model. The advantage of this representation is that the parameter vector θ appears linearly in (15). In Appendix A we discuss how this form is compatible with <u>equation error</u> and <u>output error</u> identification algorithms. Moreover, these algorithms can be transformed to the form of (4) and the theory discussed earlier can be applied. Also, in adaptive control, the linear relation between y and θ in (15) simplifies the transformation from model parameter estimates $\hat{\theta}$ to control parameters, i.e., $k = f(\hat{\theta})$.

There are many cases where the explicit model (15), although analytically tractable, is oblivious to useful information obtained from a physical analysis. Consider the implicit linear modal,

$$
\begin{aligned}
y &= c(\alpha)' x_m \\
\dot{x}_m &= A(\alpha) x_m + b(\alpha) u
\end{aligned}
\tag{17}
$$

where $A(\alpha)$, $b(\alpha)$, $c(\alpha)$ are nonlinearly dependent on the parameter vector $\alpha \in \Omega$, where Ω is the parameter space, a subset of R^k. This model (2.9) retains the physical meaning of the parameters, e.g., in an LSS the elements with α can consist of equivalent masses, spring constants, and dampers. There will be far fewer of these parameters than there are coefficients in the equivalent transfer function, i.e., the transfer function $P_m(s,\alpha)$ from u into y for (17) is:

$$
P_m(s,\alpha) = \frac{b_1(\alpha)s^{n-1} + \ldots + b_n(\alpha)}{s^n + a_1(\alpha)s^{n-1} + \ldots + a_n(\alpha)} \tag{18}
$$

It is clear that the parametric model of (18) can be made identical to (15) simply by equating each coefficient i.e., $\theta_1 = a_1(\alpha)$, $\theta_2 = a_2(\alpha)$, and so on. Since the model (16) is linear in Θ whereas (18) is nonlinear in α, there is a great difference in the identification schemes. For (18) there are fewer parameters to be identified, but several nonlinear relations. For (16), there are many more parameters to be identified but all enter linearly into the model. In LSS modeling it may be prudent to examine the effect of using implicit parametric models, because of the reduced number of parameters to be identified. This type of model, however, leads to more complicated forms than (4) and involves a nonlinear observer. One such scheme using an EKF is discussed next.

Identification using EKF

The implicit parametric linear model (18) is compatible with an identifier based on the extended Kalman filter (EKF). The EKF parameter estimator has the form:

$$\dot{\hat{\alpha}} = k_\alpha(\hat{\alpha})(y - c(\hat{\alpha})'\hat{x})$$
$$\dot{\hat{x}} = A(\hat{\alpha})\hat{x} + b(\hat{\alpha})u + k_x(\hat{\alpha})(y - c(\hat{\alpha})'\hat{x}) \tag{19}$$

where the gain $k_\alpha(\hat{\alpha})$ and $k_x(\hat{\alpha})$ are the Kalman filter gains obtained by linearizing (17) about the current estimate $\hat{x}$. Although EKF has been extensively studied (e.g., Ljung and Soderstrom, 1983) there still remains no concrete proof of guaranteed convergence for identification and no proof of stability in the adaptive case, i.e., when $\hat{\alpha}$ is used to generate a control law. In as much as explicit parametric models may better represent the LSS, we will develop the theory in this area. There have been some attempts in this direction by Safonov and Athans (1978) and Vidyasagar (1980). In this latter work it is assumed that certain signals are bounded in order to prove convergence. In the adaptive case this assumption needs to be proved first, because the signals that need to be bounded are inside the adaptive loop, and hence, dependent on the parameter estimator. The approach we take is to apply the averaging theory of Hale (1969) by transforming the EKF based adaptive system to the form of (5).

Infinite-Dimensional Systems

Systems of the form (4) not only presume a linear finite dimensional plant but also a linear finite-dimensional controller. Extending the theory to the use of infinite dimensional plant and finite-dimensional controller may not prove to be too difficult, e.g., Kosut and Friedlander (1982). However, the case for a distributed parameter controller (DPC), which is perhaps more realistic for LSS, is more difficult. What is involved here is adjusting a few parameters in the DPC from discrete spatial-temporal measurements. This is certainly an area for basic research.

Nonlinear Control Structures

During tracking and slewing the control can be nonlinear, involving switching curves (time-optimal) and actuator saturations. Adaptive schemes here must be rapid (high gain) and must account for nonlinearities. Dominant kinematic or structural nonlinearities will also engender nonlinear control structures.

Lattice Filters for Adaptive Control

The uncertainty in the number of modes to be selected for adequate control of LSS raises some special difficulties. Conventional adaptive control schemes involve controller structures of fixed order. In the LSS context it seems necessary to adapt the controller order as well as its parameters. The lattice structure is especially well suited for variable order modeling and control as was discussed in Friedlander (1982), Sundararajan and Montgomery (1982).

The theoretical development of lattice filters for control purposes is only at a preliminary stage of development. Its applications to practical control situations have been very limited. A number of issues need to be resolved before the potential of lattice filters can be fully realized, including:

(i) The development of order-recursive lattice models for plants with both poles and zeros. The work of Sundararajan and Montgomery (1982) was limited to the all pole case. It is expected that pole-zero models will provide better fit to the LSS problem. Some work has been done in this area (Friedlander, 1982) but questions remain regarding the simultaneous determination of the orders of the poles and the zeros.

(ii) The development of direct lattice controllers. The main application of lattice filters so far has been in the area of system identification. The identified model can then be used to design a conventional controller. It is possible, however, to use a controller implemented in lattice structure and adjust its parameters directly, based on the observed data. This leads to an adaptive controller structure in which both the gains and the order are being adjusted.

(iii) Development of a prediction error lattice filter. The current versions of the adaptive lattice are of the residual type (i.e., the predictor coefficients depend on current as well as past data. In the prediction error version they depend on past data only). The control problem is naturally related to the prediction error form. This development is a straightforward extension of available results.

Finally, it is necessary to test some candidate lattice algorithms on real and simulated data to gain better insight into their properties when applied to very high order plants.

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APPENDIX A

DERIVATION OF GENERIC ADAPTIVE SYSTEM

In this section we show how the nonlinear system (see Figure 2.1)

$$e = e_* - c'x \qquad \dot{x} = Ax + Bz'\theta \tag{A.1}$$
$$z = z_* - Dx \qquad \dot{\theta} = \gamma ze$$

is representative of a large class of adaptive control and identification systems. In (A.1), e(t) and $e_*(t)$ are scalars, x(t) is an n-vector, $\theta(t)$ and z(t) are p-vectors, A, b, c, and D are constant matrices or vectors of appropriate dimensions, and γ is a positive constant. A more thorough discussion on the derivation of (A.1) can be found in Kosut and Friedlander (1982, 1985), or Kosut and Johnson (1984).

Adaptive Control

Consider the model reference adaptive control (MRAC) system of Narendra, Lin, and Valavani (1978):

Plant: $\dot{x}_p = A_p x_p + b_p u_p,\ x_p(o) \in R^\ell$

$$y_p = d + c_p' x_p \tag{A.2a}$$

Reference model: $\dot{x}_m = A_m x_m + b_m r,\ x_m(o) = 0 \in R^m$

$$y_m = c_m'\, x_m \tag{A.2b}$$

Control: $u_p = -\hat{\theta}'z$ (A.2c)

Filter (Regressor): $\dot{z} = A_f z + b_{fy} y_p + b_{fu} u_p - b_{fr} r, \; z(o) = 0 \; \varepsilon \; R^p$ (A.2d)

Parameter Update: $\dot{\hat{\theta}} = \gamma z e, \; \hat{\theta}(o) \; \varepsilon \; R^p, \; \gamma > 0$ (A.2e)

$$e = y_p - y_m$$

The external signals are the reference command r(t) and the disturbance d(t). Let θ_* be a constant vector in a subset Ω_* of R^p such that when the fixed-gain (non-adaptive) control

$$u = -\theta_*' z \tag{A.3}$$

is applied to the plant the resulting system is stable and in addition exhibits acceptable performance characteristics. Any system corresponding to any $\theta_* \; \varepsilon \; \Omega_*$ is referred to as a <u>tuned system</u> and is described by:

$$\dot{x}_{p*} = A_p x_{p*} + b_p u_{p*} \; , \; y_{p*} = d + c_p' x_{p*}$$

$$\dot{z}_* = A_f z_* + b_{fy} y_{p*} + b_{fu} u_{p*} b_{fr} r$$

$$u_{p*} = -\theta_*' z_* \tag{A.4}$$

$$e_* = y_{p*} - y_m$$

Define the parameter error vector

$$\theta = \hat{\theta} - \theta_* \tag{A.5}$$

Combining (A.2), (A.4), and (A.5) and comparing the resulting system to (A.1) gives the error state as

$$x = \begin{pmatrix} x_p - x_{p*} \\ z - z_* \end{pmatrix} \tag{A.6}$$

and system matrices:

$$A = \begin{pmatrix} A_p & -b_p\theta_*' \\ b_{fy}c_p' & A_f - b_{fu}\theta' \end{pmatrix} , \quad b = \begin{pmatrix} b_p \\ b_{fu} \end{pmatrix} \tag{A.7a}$$

$$c' = \quad (c_p' \quad 0), \quad D = (0 \quad I_p) \tag{A.7b}$$

Since the tuned system is stable by definition, it follows that A is stable, i.e., all the eigenvalues of A have negative real parts.

Identification

The algorithms studied in Ljung and Sodestrom (1983) and Goodwin and Sin (1984) will be used as representatives. Consider the possibly unstable plant

$$\begin{aligned} y_p &= c_p' x_p + d \\ \dot{x}_p &= A_p x_p + b_p u_p \quad , x_p(o) \in R^{\ell} \end{aligned} \tag{A.8}$$

with (A_p, b_p, c_p) controllable and observable, to be identified by using the linear parametric model

$$y_m = \theta' x_m$$

$$\dot{x}_m = A_f x_m - b_{fy} y + b_{fu} u_p \quad , \quad x_m(o) \in R^p \tag{A.9}$$

where y is set equal to either y_p or y_m, A_f is stable, and $(A_f, [b_{fy}\ b_{fu}])$ is controllable. In the case when $\ell = p$ and $d(t) = 0$, a unique $\theta_* \in \mathbb{R}^p$ exists such that $y_p = y_{m*}$ where y_{m*} satisfies:

$$y_{m*} = \Theta_*' x_m$$

$$\dot{x}_{m*} = A_f x_{m*} - b_{fy} y + b_{fu} u_p \tag{A.10}$$

$$y = y_p = y_{m*}$$

In practice $\ell > p$ due to unmodeled dynamics and $d(t) \neq 0$ due to sensor noise and disturbances. In this case $\theta_* \in R^p$ can be chosen to minimize either the <u>equation error norm</u>

$$J_1(\theta) = ||y_p(.) - y_m(.)||_{y = y_p} \tag{A.11}$$

or the <u>output error norm,</u>

$$J_2(\theta) = ||y_p(.) - y_m(.)||_{y = y_m} \tag{A.12}$$

where the norm $||.||$ is defined as

$$||f(.)|| = (\lim_{T \to \infty} \frac{1}{T} \int_0^T |f(t)|^2 \, dt)^{1/2} \tag{A.13}$$

When $\ell = p$ and $||d(.)|| = 0$, the equation error solution is identical to the output error solution and is given by the unique Θ_* satisfying (A.10). Hence, in this case, $J_1(\theta_*) = J_2(\theta_*) = 0$. When $\ell > p$ or $||d(.)|| > 0$, the solutions are different. In general, output error provides some filtering of the disturbance.

The on-line or adaptive procedures for obtaining output error or equation error estimates have the following forms. For equation error identification:

$$\text{Error:} \quad e = y_p - y_m \quad , \; y_m = \hat{\theta}' x_m$$

$$\text{Model:} \quad \dot{x}_m = A_f x_m - b_{fy} y_p + b_{fu} u_p \tag{A.14}$$

$$\text{Update:} \quad \dot{\hat{\Theta}} = \Upsilon x_m e$$

For output error identification:

$$\text{Error:} \quad e = y_p - y_m \quad , \quad y_m = \hat{\theta}' x_m$$

$$\text{Model:} \quad \dot{x}_m = A_f x_m - b_{fy} y_m + b_{fu} u_p \tag{A.15}$$

$$\text{Update:} \quad \dot{\hat{\Theta}} = \Upsilon x_m e$$

Let θ_* denote a minimizing solution of either (A.11) or (A.12) and let (A.10) describe the resulting <u>tuned</u> model. Define the parameter error

$$\theta = \hat{\theta} - \theta_* \tag{A.16}$$

Combining (A.10), (A.14), and (A.16) gives the equivalent equation error system:

$$e = e_* - \theta' z \quad , \; e_* = y_p - \theta_*' z$$

$$z = x_m \tag{A.17}$$

$$\dot{\theta} = \gamma z e$$

which is a degenerate form of (A.1), i.e., the state is not driven by the non-linear term $z'\theta$ as it is in the adaptive case (A.7).

In the output error case, combining (A.10), (A.15), and (A.16) gives the equivalent system

$$e = e_* - \theta' z, \quad e_* = y_p - \theta_*' x_{m*}$$

$$\dot{z} = A_f z - b_{fu} u_p + b_{fy} z' \theta \tag{A.18}$$

$$\dot{\theta} = \gamma z e$$

which is also a degenerate form of (A.1). Note, however, that in the output error case, the state equation is driven by $z'\theta$ whereas in the equation error case it is not. Hence, the output error systems is much more like the adaptive system (A.7) and can exhibit the same type of instabilities and parameter drift that have been reported in several studies, e.g., Rohrs et al. (1982, 1983), Reidle and Kokotovic (1984).

APPENDIX B

METHODS OF AVERAGING FOR ADAPTIVE SYSTEMS*

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Abstract

A summary of methods of averaging analysis is presented for continuous-time adaptive systems. The averaging results of Riedle and Kokotovic (1984) and of Ljung (1977) are examined and are shown to be closely related. Both approaches result in a sharp stability-instability boundary which can be tested in the frequency domain and interpreted as a signal dependent positivity condition.

1. Introduction

The theory developed in Kosut and Anderson (1983, 1985) shows that the stability of adaptive systems in the neighborhood of the equilibrium trajectories is dependent on the stability of a system of linear time-varying equations. These essentially are a linearization of the adaptive system and are referred to here as the linearized adaptive system.

In a recent paper by Riedle and Kokotovic (1984) a classical method of averaging as described by Hale (1969) was applied to the linearized adaptive system. The result is a sharp stability-instability boundary determined by a signal dependent positivity condition. This result is significantly weaker than the SPR (strictly positive real) condition required in the proof of stability of adaptive systems, e.g., Narendra, Lin, and Valavanni (1980), Landau (1979). In order to apply the averaging theory to obtain this result, the linearized system has first to be decoupled into slow (parameter) states and fast states. It is this transformation which is essential to the averaging approach and is a major contribution in the Riedle-Kokotovic method.

Averaging has also been applied to the counter-example of Rohrs et al. (1982) by Astrom (1983, 1984). In this analysis, by "freezing" the parameters, the parameter and state equations are decoupled thereby obtaining the asymptotic trajectories. Both of these averaging analyses assume that the system is periodic or almost periodic, an assumption that can be dispensed with by introducing the notion of a local (moving) average, Kosut, Anderson, and Mareels (1985). In the same reference, the averaging approach is also shown to be applicable to discrete-time systems.

In Riedle and Kokotovic (1985), the averaging approach is extended to nonlinear systems - and generalizes Astrom's analysis by introducing the <u>integral manifold</u> which completely separates the parameter and state equations. This latter approach is valid for the nonlinear adaptive system, and not just the linearized part. Similar results can also be found in Fu, Bodson, and Sastry (1985).

An averaging method was also developed by Ljung (1977) for use in discrete-time recursive parameter estimation. The analysis shows that the convergence properties of the estimates can be determined from the stability properties of a related set of ordinary differential equations; the method usually referred to as the ODE analysis.

In this paper we summarize the results obtained by Riedle and Kokotovic (1984, 1985) and show (heuristically) how they are related to the local stability analysis of Kosut and Anderson (1983, 1984) and the ODE averaging approach of Ljung (1977).

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2. Adaptive Error System

Although it is unlikely that a truly generic adaptive error system can be formed to capture all the nuances of adaptive systems, the SISO adaptive system shown in Figure 1 is offered as a good representation for the purposes of analysis. The system equations are:

$$e = e_* - H_{ev} v \qquad (2.1a)$$

$$z = z_* - H_{zv} v \qquad (2.1b)$$

$$v = z'\Theta \qquad (2.1c)$$

$$\dot{\Theta} = e\Gamma z \qquad (2.1d)$$

The development of (2.1) can be found in Kosut and Friedlander (1982, 1985) and in Kosut and Johnson (1984). In (2.1), $e(t) \in R$ is a measured error signal which drives the parameter update (2.1d), $z(t) \in R^p$ is the regressor, and $\Theta(t) \in R^p$ is the parameter error between the current estimate at t and a tuned parameter setting $\Theta_* \in R^p$. The selection of Θ_* is based on complete knowledge of the actual plant and distrubances. The system corresponding to this setting is referred to as the tuned system. The signals $e_*(t) \in R$ and $z_*(t) \in R^p$ are outputs of the tuned system, and are referred to as the tuned error and tuned regressor, respectively. The signal $v(t) \in R$ can be regarded as the adaptive control error.

The operators H_{ev} and H_{zv} are dependent on Θ_* and describe how v effects the error and regressor signals. We assume here that H_{ev} and H_{zv} are linear-time-invariant (LTI) with stable proper transfer functions $H_{ev}(s)$ and $H_{zv}(s)$. This would arise, for example, when the plant to be controlled is LTI and the adaptive controller is linear in the adaptive parameters. The stability of H_{ev} and H_{zv} is a consequence of the definition of Θ_* as the tuned parameter setting.

The operator Γ depends on the choice of parameter update algorithm. We will restrict attention here to the following representatives:

Gradient

$$(\Gamma z)(t) = \varepsilon z(t) \qquad (2.2)$$

$$\varepsilon > 0$$

Recursive Least Squares

$$(\Gamma z)(t) = P(t)z(t)$$

$$\frac{d}{dt} P^{-1}(t) = z(t)\, z(t)' \qquad (2.3)$$

$$P(0) = P(0)' > 0$$

3. Global Stability and Passivity

It is of interest to determine under what conditions the adaptive error system (2.1,3) produces bounded outputs (Θ, e, v, z) for all bounded initial parameter errors $\Theta(o) \in R^p$. This is what is meant here by "global" stability. As it turns out, such a result is possible to prove provided that:

1) $H_{ev}(s) \in$ SPR with gradient (3.1)

2) $H_{ev}(s) - 1/2 \in$ SPR with least squares (3.2)

3) $z_*, \dot{z}_* \in L^p_\infty$ and either (3.3)

a) $e_*, \dot{e}_* \in L_2 \cap L_\infty$ (3.4)

b) $e_*, \dot{e}_* \in L_\infty$ and $z \in$ PE (persistently exciting) (3.5)

Parameter convergence to a constant in R^p or to a well defined subset in R^p, requires that (3.4) be strengthened to:

$$e_*, \dot{e}_* \in L_2 \cap L_\infty, \; z_* \in \text{PE} \qquad (3.6)$$

The above results can be found in Kosut and Friedlander (1982, 1985) and in Boyd and Sastry (1984). Although of theoretical significance, they are not feasible to obtain in practice. In the first place, due to unmodeled dynamics (Rohrs et al. 1982, 1984), $H_{ev}(s) \in$ SPR is pratically impossible to achieve in adaptive feedback and even in some output error identifica- tion. (This is not the case in equation error identification.) Secondly, when $e_* \in L_\infty$ as in (3.5), it is required that $z \in$ PE which can not be guaranteed in advance since z is inside the adaptive loop. Case (3.6) which requries $z_* \in$ PE which is feasible to establish - conflicts with $e_*, \dot{e}_* \in L_2 \cap L_\infty$. The latter implies $e_*(t) \to 0$ which can only occur for $z^* \in$ PE - and where there are no unmodeled dynamics which we argue is not possible.

With these impossible to satisfy theoretical requirements, it is doubtful that a global stability theory can be attained which relies on passivity, i.e., condition (3.1,2). On the practical side, however, there is substantial evidence of well engineered algorithms that work without SPR, e.g., Astrom (1983). These do not work for all $\Theta(o)$ and for all e_*, z_* in L_∞, but rather, for restricted magnitudes and signal spectrums. For example, if $H_{ev}(s)$ is SPR for $\omega \le \omega_{BW}$ then it is expected that the adaptive system will be well behaved provided there is insignificant excitation above ω_{BW}.

4. Local Stability: Small Gain Theory and Averaging

By restricting the magnitude of $\theta(o)$ and the magnitude and spectrum of $z_*(t)$ and $e_*(t)$, it is possible to obtain conditions to prove local stability, e.g., Kosut and Anderson (1983, 1985). The local stability property hinges on two premises: (1) the error system trajectories are in a (not necessary small) neighborhood of the tuned solution, and (2) the linear time varying system which maps $w \to \theta$ as given by

$$\dot{\theta} = -(\Gamma z_*)(t)\, H_{ev}(z_*(.)\theta(.)) + (\Gamma w)(t) \qquad (4.1)$$

is L_∞- stable, i.e., there exists constants k and b s.t. $||\theta||_\infty \le k||w||_\infty + b$. The choice of Γ comes from (2.2) or (2.3) and $z_*(t)$ is the tuned regressor. We can regard (4.1) as a linearization of the update algorithm. There are several ways to establish the L_∞ stability of (4.1).

Gradient Algorithm

We first consider the case when Γ represents the gradient algorithm, i.e., $(\Gamma z_*)(t) = \epsilon z_*(t)$ with $\epsilon > o$.

In Anderson (1977), it is shown that if $H_{ev}(s) \in$ SPR and $z_* \in$ PE, then for all $\epsilon > o$, $w \to \theta$ is exponentially stable, and hence, L_∞- stable. In Anderson et al. (1985), if $H_{ev}(s) = \bar{H}_{ev}(s) + s\Delta(s)$, $\bar{H}_{ev}(s) \in$ SPR, $\Delta(s)$ is stable, and $z_* \in$ PE then for sufficiently small ϵ and $||\dot{z}_*||_\infty$, $w \to \theta$ is still exponentially stable, and hence is L_∞-stable. This latter method relies on loop-transformations and applicaiton of small gain theory.

Another approach is to use averaging. In Riedle and Kokotovic (1984) it is shown that if $z_* \in$ PE with the Fourier series representation

$$z_*(t) \sim \sum_k \alpha(\omega_k) e^{j\omega_k t} \qquad (4.2)$$

and if the eigenvalues of the real matrix

$$B = \sum_k \alpha(\omega_k)\, \alpha(-\omega_k)'\, H_{ev}(-j\omega_k) \qquad (4.3)$$

all have positive real parts, then for all sufficiently small $\epsilon > o$, $w \to \theta$ is exponentially stable, and hence, L_∞-stable. Moreover, if any one eigenvalue of B has a negative real part, then $w \to \theta$ is exponentially unstable. Hence, there exists $w \in L_\infty$ s.t. $|\theta(t)| \to \infty$ as $t \to \infty$ exponentially fast. It is obvious then when $H_{ev}(s)$ is not SPR, but only approximately so, then the Riedle Kokotovic result provides a sharp stability-instability boundary. Note than when $H_{ev}(s)$ is SPR and $z_* \in$ PE we have from Anderson (1977) that $w \to \theta$ is exp. stable for all $\epsilon > o$. On the other hand, the result in Anderson et. al. (1985) remains valid for $H_{ev}(s) \in$ SPR ($\Delta(s) = o$) because then $\epsilon > o$ is bounded above by infinity.

B. Recursive Least Squares Algorithm

In this case we have from (2.3) that

$$(\Gamma z_*)(t) = P_*(t) z_*(t)$$

$$\frac{d}{dt} P_*(t)^{-1} = z_*(t) z_*(t)', \quad P_*(0) > 0.$$

When $z_* \in$ PE there exists $\alpha > 0$ such that

$$P_*(t)^{-1} = P_*(o)^{-1} + \int_o^t z_*(\tau) z_*(\tau)' d\tau \ge \alpha t \,.\, I$$

Thus, it is convenient to define

$$R(t) = \frac{1}{t} P_*(t)^{-1} \text{ for } t > o$$

Hence

$$R(t)^{-1} = t\, P_*(t) \le \frac{1}{\alpha} I$$

and we can write (4.1) and (2.3) as,

$$\dot{\theta} = \frac{1}{t} R^{-1}[w \quad z_* H_{ev}(z_*'\theta)] \qquad (4.4)$$

$$\dot{R} = \frac{1}{t}(z_* z_*' - R)$$

When $H_{ev}(s) - \frac{1}{2}$ is not SPR we can now follow Ljung (1977) and for $t \ge s$ and s sufficiently large, replace the right hand side by its average. Letting "overbar" denote average (assuming it exists) we have:

$$\theta(t) = \frac{1}{t} \bar{R}^{-1} (\bar{w} - (\overline{z_* H_{ev} z_*'})\bar{\theta}) \qquad (4.5)$$

$$\bar{R}(t) = \frac{1}{t}(\overline{z_* z_*'} - \bar{R})$$

Integrating from s to s + T, T > o, gives

$$[\theta(s+T) - \theta(s)]/\int_s^{s+T} dt/t = \bar{R}^{-1}(w - (\overline{z_* H_{ev} z_*'})\bar{\theta}) \qquad (4.6)$$

$$[R(s+T) - R(s)]/\int_s^{s+T} dt/t = \overline{\tau_* \tau_*'} - \bar{R} \qquad (4.7)$$

Now change time scales $s + T \to \tau + \Delta\tau$, $\Delta\tau = \int_s^{s+T} dt/t$ and letting $s \to \infty$ gives the differential equations:

$$\dot{\theta}_A(\tau) = R_A(\tau)^{-1}[\bar{w} - B\, \theta_A(\tau)] \qquad (4.8)$$

$$\dot{R}_A(\tau) = \overline{z_* z_*'} - R_A(\tau) \qquad (4.9)$$

with $B = \overline{z_* H_{ev} z_*'}$ given by (4.3). These equations describe the asymptotic behavior of (4.1) in just the same way as they do for discrete-time (Ljung 1977).

Observe that in (4.9) as $t \to \infty$, $R_A(t) \to \overline{z_* z_*'}$. Thus, when $H_{ev}(s) - \frac{1}{2}$ is not SPR and $z_* \epsilon$ PE with Fourier representation (4.2) the asymptotes are stable if

$$\operatorname{Re} \lambda (L) > 0 \qquad (4.10)$$

where

$$L = (\overline{z_* z_*'})^{-1} B$$
$$= \Big(\sum_k \alpha(\omega_k)\alpha(-\omega_k)'\Big)^{-1} \sum_k \alpha(\omega_k)\alpha(\omega_k)' H_{ev}(-j\omega_k) \qquad (4.11)$$

If $\operatorname{Re} H(j\omega_k) \geq \rho > 0$ at low frequencies, and if $|\alpha(\omega_k)|$ is small at frequencies where $\operatorname{Re} H(j\omega_k) \leq 0$, then $\operatorname{Re} \lambda(L) \approx \rho$. Thus, all parameter asymptotes have a uniform rate of convergence which is not the case for the gradient algorithm with a time-invariant gain.

5. Averaging: A More General Approach

In this section we will establish a general form of the adaptive error system (2.1,3) which is useful for application of averaging methods. The first step is to transform (2.1,3) into a set of nonlinear time-varying differential equations. To do this observe that if $H_{ev}(s)$ are strictly proper functions (a convenient illustrative, but not necessary, assumption) then we can write

$$H_{ev}(s) = c'(sI-A)^{-1}b$$
$$H_{zv}(s) = D(sI-A)^{-1}b \qquad (5.1)$$

where $A \in R^{n \times n}$, $b \in R^n$, $c \in R^n$, $D \in R^{p \times n}$, with $(A,b,[c\ D'])$ a minimal representation. Also, $\operatorname{Re} \lambda(A) < 0$ reflecting the fact that H_{ev} and $H_{zv}(s)$ are stable. The error system (2.1) is then equivalently expressed as

$$e = e_* - c'x$$
$$z = z_* - Dx$$
$$\dot{x} = Ax + bz'\theta \qquad (5.2)$$
$$\dot{\theta} = (\Gamma z)e$$

By eliminating the variables e and z we can reduce (5.2) to the coupled state-space description:

$$\dot{\phi} = \gamma(t)\, f(t,\phi,x) \qquad (5.3)$$

$$\dot{x} = Ax + g(t,\phi,x) \qquad (5.4)$$

with the gradient algorithm (2.2), let

$$\phi(t) = \theta(t)$$
$$\gamma(t) = \epsilon \qquad (5.5)$$

and

$$f(t,\theta,x) = z_*(t)e_*(t) - Q_*(t)\,x + c'x\,D\,x$$
$$Q_*(t) = z_*(t)c' + e_*(t)D \qquad (5.6)$$
$$g(t,\theta,x) = b(z_*(t) - Dx)'\theta$$

With the recursive least squares algorithm (2.3), define:

$$R(t) = \frac{1}{t} P(t)^{-1} \qquad (5.7)$$

and let

$$\phi(t) = \begin{pmatrix} \theta(t) \\ col\{R(t)\} \end{pmatrix}, \quad \gamma(t) = \frac{1}{t} \qquad (5.8)$$

where the opreralor col{R} stacks up the columns of the matrix R to form a vector. Thus,

$$f(t,\phi,x) = \begin{bmatrix} R^{-1}(z_*(t)e_*(t) - Q_*(t)\,x + c'xDx) \\ col\{z_*(t)z_*(t)' - z_*(t)(Dx)' - Dxz_*(t)' + Dx(Dx)' - R\} \end{bmatrix} \qquad (5.9)$$

$$g(t,\phi,x) = b(z_*(t) - Dx)'\theta \qquad (5.10)$$

The col{.} operator was used by Ljung (1978) to develop the discrete-time version of (5.3), (5.4).

Heuristics: The Integral Manifold

The basic idea in the application of averaging methods to (5.3,4) is to see what happens when $\gamma(t)$ is small. Essentiallyl, $\phi(t)$ slows down and we can repalce the right hand side of (5.3) with its average, i.e.,

$$\dot{\phi} = \gamma(t)\bar{f}(\phi) \qquad (5.11)$$

$$\text{where } \bar{f}(\phi) = \lim_{T\to\infty} \frac{1}{T}\int_0^T f(t,\phi,\bar{x}(t,\phi))dt \qquad (5.12)$$

assuming the limit exists. (Such is the case, for example, when $f(t,\phi,x)$ and $g(t,\phi,x)$ are periodic in t for all bounded ϕ and x). To arrive at (5.11) formally requires the introduction of the integral manifold as suggested by Reidle and Kokotovic (1985) [see Hale (1969) for discussion of the integral manifold]

The integral manifold M of (5.3,4) is the set,

$$M = \{t,\phi,x\ :\ x(t) = h(t,\phi(t))\} \qquad (5.13)$$

By substituting $x = h(t,\phi)$ into (5.3,4), the manifold function $h(t,\phi)$ is seen to satisfy the partial differential equation

$$\frac{\partial h}{\partial t} + \gamma(t)\,\frac{\partial h}{\partial \phi}\,f(t,\phi,h) = Ah + g(t,\phi,h) \qquad (5.14)$$

Whenever $\gamma(t)$ is sufficiently small, a reasonable approximation is to $h(t,\phi)$ is given by $h_o(t,\phi)$ which is the solution to

$$\begin{aligned}\frac{\partial h_o}{\partial t} &= Ah_o + g(t,\phi,h_o)\\ &= F(\theta)h_o + bz_*(t)'\theta \qquad (5.15)\end{aligned}$$

where the last line follows from (5.6) with

$$F(\theta) = A - b\theta' D \qquad (5.16)$$

In (5.14), θ and t are regarded as independent variables and, hence, we can define the stabilizing parameter set

$$D_s = \{\theta \in R^p : \text{Re}\,\lambda(F(\theta) < o\} \qquad (5.17)$$

Thus, for $\gamma(t)$ sufficiently small, we can refer to $h(t,\phi)$ with $\theta \in D_s$ as the <u>stable manifold</u>, which we will approximate by $h_o(t,\theta)$, $\theta \in D_s$.

The final transformation on (5.3,4) is obtained by examining the behavior of (ϕ,x) in the neighborhood of the stable manifold. Introduce the error state,

$$\xi = x - h(t,\phi) \qquad (5.18)$$

Using (5.18), and (5.3,4), with the approximation $h_o(t,\theta)$ for $h(t,\phi)$, we have

$$\dot{\phi} = \gamma(t)f(t,\phi,h_o(t,\theta)+\xi) \qquad (5.19)$$

$$\dot{\xi} = F(\theta)\xi \qquad (5.20)$$

If $\gamma(t)$ is sufficiently small and θ remains (moving slowly) in D_s then $\xi(t) \to o$ exp. fast. As a result, by the same reasoning as in Section 4, the stability of (5.19) is identical to the stability of the asymptotic system:

$$\dot{\phi}_A(\tau) = \bar{f}(\phi_A(\tau)) \qquad (5.21)$$

where

$$\bar{f}(\phi) = \lim_{T\to\infty} \frac{1}{T}\int_o^T f(t,\phi,h_o(t,\theta))dt \qquad (5.22)$$

assuming the limit exists. The stability of (5.21) is given as follows. The proof is in Hale (1969).

Theorem

Let ϕ^o denote a solution of

$$\bar{f}(\phi^o) = o$$

and define the matrix,

$$G = \frac{\partial \bar{f}}{\partial \phi}(\phi^o)$$

Provided Re $\lambda(G) \neq o$:

(i) If Re $\lambda(G) < o$ then ϕ^o is u.a.s.

(ii) If max Re $\lambda(G) > o$ then ϕ^o is unstable.

<u>Application to Gradient Algorithm</u>

Applying this result to (5.5,6) with the gradient algorithm and with $z_* \in$ PE and $\overline{z_* e_*} = o$, gives $G = -B$ with B from (4.3). Since $\gamma(t) = \varepsilon > o$, we can only conclude that if Re $\lambda(B) > o$ and ε is sufficiently small, then $\theta(t)$ approaches an ε-neighborhood of $\theta = o$ as $t \to \infty$. Provided of course that $\theta(t) \in D_s$ long enough for transients to die out, which is unprovable as yet in general.

<u>Application to Recursive Least Squares Algorithm</u>

Under ths same conditions and with the same provisions as above, $G = -L$ with L from (4.11). This time, since $\gamma(t) = 1/t \to o$ as $t \to \infty$, we can conclude that if Re $\lambda(L) > o$, then $\theta(t) \to o$ as $t \to \infty$ at a rate $1/t$.

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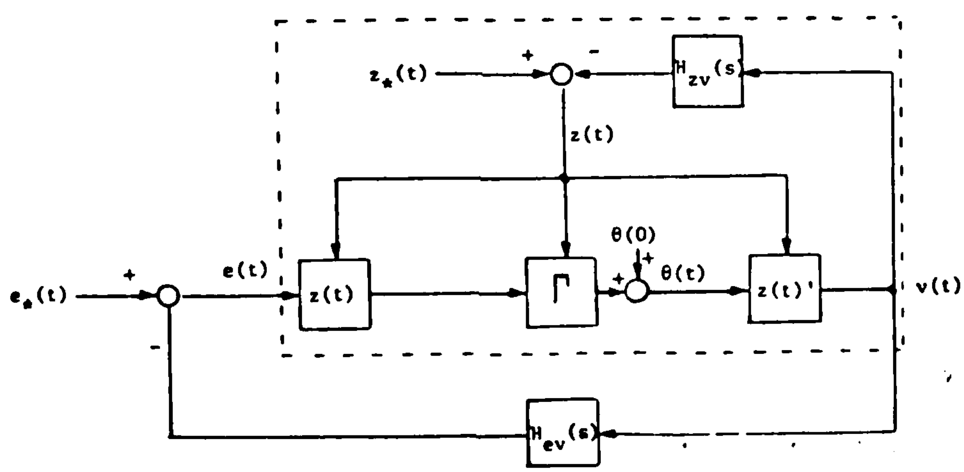


Figure 1. Adaptive Error System

APPENDIX C

Fundamentals of Control for Large Space Structures: Robust and Adaptive Design

Abstract

This report gives a summary overview of robust and adaptive control design for LSS. It is one section of a larger internal report in progress.

II-2.2.7.4.1 Control Design for HAC/LAC Architecture. In this section we will discuss the steps involved in control design for the HAC/LAC architecture. Although the architecture is specialized, the control design methodology is not and can be quite general. We will discuss three methodologies for design: (1) an LQG based methodology whose genesis is the ACOSS/VCOSS programs, and (2) a more recent approach involving what is known as "Q-parameterization" and "H∞-optimization". These latter methods are frequency-domain oriented rather than state-space oriented like the LQG approach. (3) We will also discuss an adaptive control strategy which can be utilized for online self-tuning. We refer to this approach as "adaptive calibration". This approach has been developed by ISI under an on-going research contract with the Directorate of Aerospace Sciences in AFOSR.

Limitations of Design. Independent of the design method, the defining characteristic of the vibration control problem is there are an infinite number (theoretically) of elastic modes, with low natural damping, and the controller bandwidth extends over a significant number of these modes (Fig. II.2-28). The low frequency modes interact not only with the attitude controller but contribute directly to the deformation geometry of the structure which itself may require accurate control. Proper control synthesis requires that performance criteria be precisely formulated or the control problem is ill-posed.

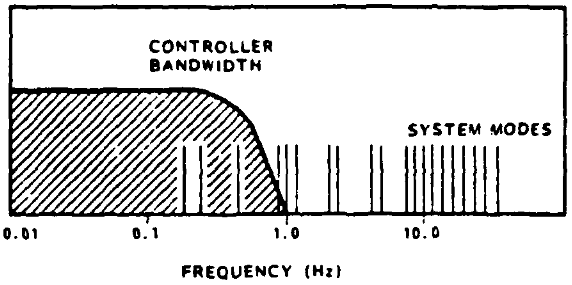


Fig. II.2-28 Flexible Structure Mode Location and Controller Bandwidth

The control design approach must properly handle the poorly known higher-frequency modes by not destabilizing them while controlling the low-frequency modes. Indeed, no matter where the controller roll-off frequency is situated, the infinite nature of the modal spectrum implies that there will be modes within and beyond the roll-off region, furthermore, destabilization is likely and almost certain to occur in the roll-off region, a situation which can only worsen for closely packed modes and low natural damping. This phenomenon sometimes referred to as "spillover" is one of the most crucial problems faced by the control designer. In more general terms, spillover can be viewed as an aspect of the problem of robust control design; this will be discussed more in a later section.

Modeling of Flexible Spacecraft. A central issue in the active control of space structures is the development of "correct" mathematical models for the open- and closed-loop dynamical plants. Programs such as NASTRAN and SPAR are the primary current tools for generating dynamical models of conceptual spacecraft whose structure cannot be idealized by simple models of beams, plates, and beams with lumped masses.

Finite-element structural programs generally provide the control designers with a set of modal frequencies and a set of mode shapes (eigenvectors) corresponding to appropriate boundary values (e.g., free-free modes). These eigenvectors are given in discretized form, i.e., a set of modal displacements in the x, y, and z directions at each nodal station. In some cases, modal rotations are also required. In addition, coordinates and a "map" of the structure's nodes must be provided to allow the reconstruction of physical displacements in terms of their modal expansions.

The important point here is that, for any nontrivial flexible satellite configuration, the volume of information is so large that the data handling must remain entirely within the computer and its mass-storage facilities. Development of this database, in a form usable by control synthesis software, is a fundamental necessity for the synthesis and evaluation of complex controls which require modal truncation, actuator/sensor location and type changes, and evaluation of system performance for parameter and system order changes. Preparation of a structure for controls is a major part of the overall effort required to develop structural control systems.

Nonlinear Models. For single-body monolithic structures, the fine-pointing attitude dynamics are subsumed in the rotational rigid body modes included in the modal matrix. When only "small" motions of a space structure are being considered, the conventional linear structural dynamics analyses (NASTRAN and SPAR) are adequate, and the rigid-body modes are formally handled together with the elastic modes, even though the actuators necessary to control them will be different, in general, from those used to control elastic vibrations. When larger attitude angles need to be considered, if the angular rates remain small, the linear equations are still applicable provided that the rigid body modes are now given in terms of three attitude angles which then constitute the first three modal coordinates. The displacements are then interpreted as the linear deformations of the structure with respect to the rotated frame. This procedure removes the kinematic nonlinearities resulting from the linear stretching of the structure under the classical rigid-body modes. However, for large angular rates, nonlinear dynamic effects have to be modeled, even though structural deformations can still be represented by linear equations.

Two-Level Control Design: The HAC/LAC Methodology. The two-level approach consists of a wide-band, low-authority control (LAC) and a narrow-band, high-authority control (HAC). HAC provides high damping or mode-shape adjustment in a selected number of modes to meet performance requirements. LAC, on the other hand, introduces low damping in a wide range of modes for maximum robustness. Figure II.2-29 shows the control-design procedure with integrated LAC and HAC designs.

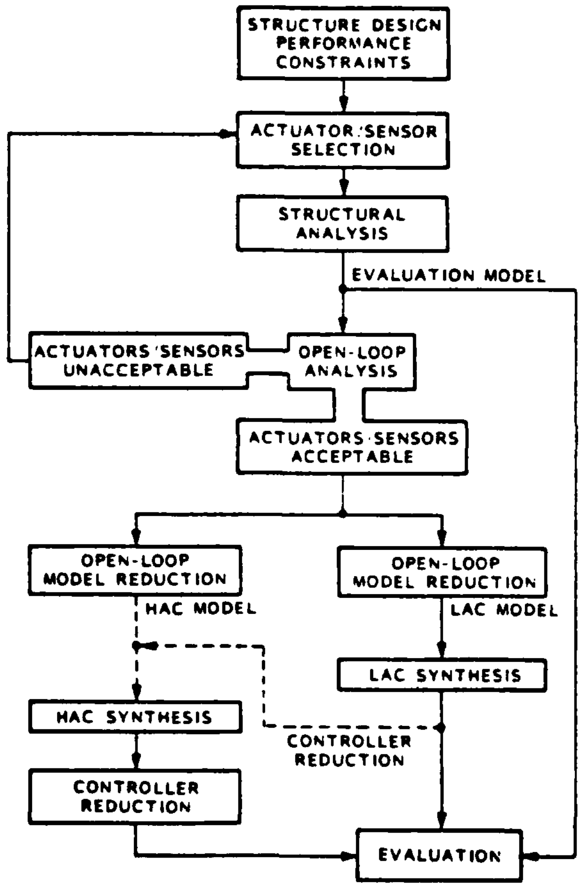


Fig. II.2-29 Analytical Control-Design Procedure

LAC is usually implemented with colocated sensors and actuators. However, the theory, based on the work of Aubrun, is applicable to multiple actuators/sensors with cross-feedback and possible filters [Aub 1].

HAC uses a collection of sensors and actuators not necessarily colocated. Selecting the increase in damping ratio is realized by any number of methods, including: LQG with frequency shaping, Q-parametization, or H∞-optimization. These methods provide roll-off over desired frequency regions. HAC may destabilize modes not used in the design. LAC is, therefore, necessary to "clean up" problems created by HAC.

The need to integrate HAC with LAC is shown in Fig. II.2-30. HAC is based on models valid over a limited frequency region. It produces large increases in damping ratio and disturbance rejection in the frequency range of interest. The effect of the HAC controller on modes not used in the control design and outside the controller bandwidth may be stabilizing or destabilizing. LAC is designed to provide protection such that adequate damping is provided in the mode most adversely perturbed by HAC. With reference to Fig. II.2-30, the LAC moves the entire uncertainty region above the zero level damping ratio.

In the next few sections, a more in-depth discussion of the blocks in Fig. II.2-29 will be presented, in particular: actuator/sensor location, model and controller reduction methods, and HAC/LAC synthesis. These methodologies rely on certain properties of feedback control; this raises the issue of robust control design which is fundamental to the whole design philosophy of feedback, especially for LSS, and this will be discussed first.

Robust Control Design. This section will describe how to evaluate the robustness of a control design. The evaluation is independent of the methodology used to achieve a particular design. To illustrate the technique we will consider the robust control problem of vibration suppression with unmodeled high-frequency dynamics. Figure II.2-31 shows the control system where P(s) is the plant transfer function matrix from actuator inputs to LOS sensor measurements, and where C(s) is the controller transfer

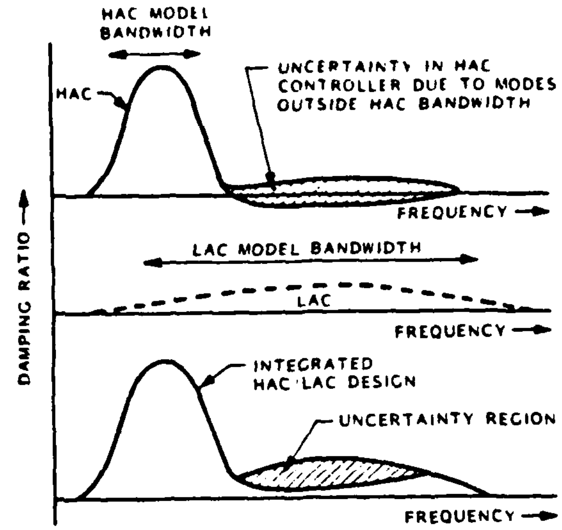


Fig. II.2-30 Need to Integrate High-Authority Controller (HAC) and Low-Authority Controller (LAC)

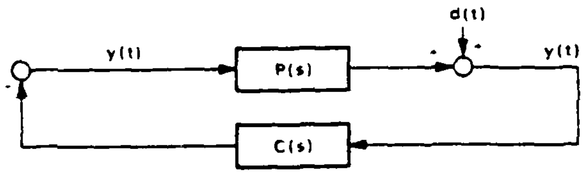


Fig. II.2-31 Vibration Suppression Control System

function matrix. Neglecting the rigid body modes in P(s) and assuming infinite bandwidth sensors and actuators,

$$P(s) = \sum_{k=1}^{\infty} C_k(s)$$

where

$$C_k(s) = \frac{1}{s^2 + 2\zeta_k\omega_k s + \omega_k^2} M_k$$

Suppose that n of the modes are known. Let $P_n(s)$ denote the known part of P(s).

For example, $P_n(s)$ can be obtained from P(s) by modal truncation, i.e., the first n-modes of P(s) are retained. One can ask the question: is this the best choice for a given model order n? In general, it depends on what is meant by "best." For closed-loop control it is usually better to retain those n-modes which most affect the closed-loop performance. How to select these modes will be discussed in the section on model reduction.

Assuming the modes have been selected, define model error as

$$\Delta(s) = P(s) - P_n(s) - \sum_{k \in \Omega_n} C_k(s)$$

observe that $\delta(s)$ is stable because both $P(s)$ and $P_n(s)$ are stable. Hence, it can be shown that the closed-loop system is stable if,

$$\bar{\sigma}\left[\Delta(j\omega)\right] < \delta_{sm}(\omega) = 1/\bar{\sigma}\left[Q_n(j\omega)\right]$$

where $Q_n(s)$ is given by $Q_n(s) = C(s) [I + P_n(s)C(s)]^{-1}$ and where $\bar{\sigma}(.)$ denotes the maximum singular value of the matrix argument. The quantity $\Delta_{sm}(\omega)$ is referred to as the "stability margin", hence, the subscripts "sm". (See [Doy. 1], [Kos 1]).

The stability robustness test depends on the location of uncertainty. Additive perturbations such as those just discussed result in the test as shown. The table in Fig. II.2-32 shows a variety of stability margins corresponding to generic forms of model error. In Fig. II.2-32, P = plant, C = control, M nominal model, and Δ = model error. The stability margin is expressed as a function of C and M which are known quantities. Examples of some model error tests are shown in Fig. II.2-33 for the CSDL #2 VCOSS model.

Performance Robustness. The stability robustness tests can be extended to evaluate performance robustness to model error. The evaluation is determined by how performance is measured. Consider the closed-loop system

$$y(t) = H(s)d(t)$$

with $H(s)$ the closed-loop transfer function. Although $d(t)$ is not precisely known, it can be considered as the output of a weighting filter $W(s)$ driven by "noise" $w(t)$, then $d(t) = W(s)\,w(t)$.

Output performance versus input is shown in Fig II.2-34. The table is derived by bounding the "noise" input $w(t)$ in terms of power, energy, and magnitude. Except for the mag in-mag-out bound, all other bounds depend directly on the frequency dependent quantity $[H(j\omega)W(j\omega)]$. A natural frequency domain performance criterion is then

$$\bar{\sigma}\left[H(j\omega)W(j\omega)\right] \le \rho(\omega)$$

where $\rho(\omega)$ is selected on the basis of power, energy, and magnitude specifications on the output signals. In terms of model error performance specification is satisfied if

$$\bar{\sigma}\left[\Delta(j\omega)\right] < \delta_{PM}(\omega)$$

with $\delta_{pm}(\omega)$ is the performance margin given by

$$\delta_{PM}(\omega) = \left[1 - \rho_n(\omega)/\rho(\omega)\right]\delta_{SM}(\omega)$$

and where $\delta_n(\omega)$ is the performance of the nominal closed-loop system $H_n(s)$ with no model error. Then,

$$\rho_n(\omega) = \bar{\sigma}\left[H_n(j\omega)\,W(j\omega)\right]$$

which must always be smaller than $\rho(\omega)$ in order for $\delta_{pm}(\omega)$ to be meaningful. Note that $\delta_{pm}(\omega) > \delta_{sm}(\omega)$, as would be expected since performance includes stability. As before, the location of uncertainty modifies the calculation of $\delta_{pm}(\omega)$.

Usefulness of Stability/Performance Robustness Tests

The stability/performance robustness tests are indispensible in obtaining a realistic preliminary design. They are used in a number of places in the design cycle to establish the HAC/LAC gains, effect of actuator/sensor dynamics, and the criteria for model and controller reduction, which will be discussed in the next section. The tests are also invaluable in establishing criteria for online system identification and control, which will be discussed later on in this section.

Model Reduction. In general, the requirements for model reduction for active control of large space structures must include the following:

1. The reduced model should be suitable for control design and synthesis. It should incorporate all features critical for the selection of a feedback

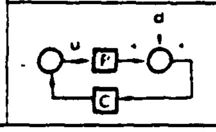
GENERIC FORM OF MODEL ERROR			STABILITY MARGIN	
				CONTROLLED SPACE
P : ACTUAL PLANT M : MODEL Δ : MODEL ERROR			GUARANTEED STABILITY IF $\bar{\sigma}(\Delta) < \delta_{SM}$	
ADDITIVE $P = M + \Delta$	• NEGLECTED RESIDUAL MODES, e.g., – SPILLOVER – REDUCED ORDER MODELING – UNCERTAIN INTERACTING STRUCTURAL MODES		$\delta_{SM} = 1/\bar{\sigma}\,[C(I + MC)^{-1}]$	
OUTPUT MULTIPLICATIVE $P = (I + \Delta)M$	• SENSOR ERRORS – MISALIGNMENTS – BANDWIDTH – SCALE FACTORS	• NEGLECTED HIGH FREQUENCY PHENOMENA, e.g., – MODEL APPROXIMATIONS – FRICTION – STICTION	$\delta_{SM} = 1/\bar{\sigma}\,[(I + MC)^{-1}MC]$	
INPUT MULTIPLICATIVE $P = M(I + \Delta)$	• ACTUATOR ERRORS – BANDWIDTH – NONLINEARITIES – PARASITICS – QUANTIZATION		$\delta_{SM} = 1/\bar{\sigma}\,[(I + CM)^{-1}CM]$	

Fig. II.2-32 Source of Model Error in Spacecraft System

structure and control gains.

2. The reduced model should accurately incorporate actuator effectiveness, sensor measurements and disturbance distribution [ACOSS, 1].
3. The dynamical characteristics of interest in the structure should be represented in the reduced model.

A basic methodology for model reduction which has been used successfully in ACOSS/VCOSS and a number of other programs, internal balancing, is now described. Other approaches also exist which will be examined in the program.

Internal Balancing

To determine the most important modes for control design, many criteria must be considered including controllability, disturbability, observability in performance, and observability in the measurements. Any mode which is highly controllable, observable, and disturbable must clearly be included in the design model, however highly controllable-but-unobservable modes, for example, are difficult to judge. Moore [Moo. 1] has developed an "internal balancing" approach whereby asymptotically stable linear models are transformed to an essentially unique coordinate representation for which controllability and observability rankings are identical. The definition of internally balanced coordinates follows:

<u>Def</u>. An asymptotically stable model

$$\left\{\begin{array}{lll} \dot{x} & = & Ax + Bu \\ y & = & Cx \end{array}\right\}$$ is <u>internally</u> <u>balanced</u> over $[0,\infty]$ IFF

$$\int_0^\infty e^{At}BB^Te^{A^Tt}dt = \int_0^\infty e^{A^Tt}C^TCe^{At}dt = \Sigma^2$$

where $\Sigma^2 = \text{diag}\,|\sigma_1^2\,\sigma_2^2 \ldots \sigma_n^2|$

$$i > j,\ \sigma_i^2 \geq \sigma_j^2$$

Notice that the balanced representation is such that the controllability Gramian and observability Gramian are equal and diagonal. The σ_i's are

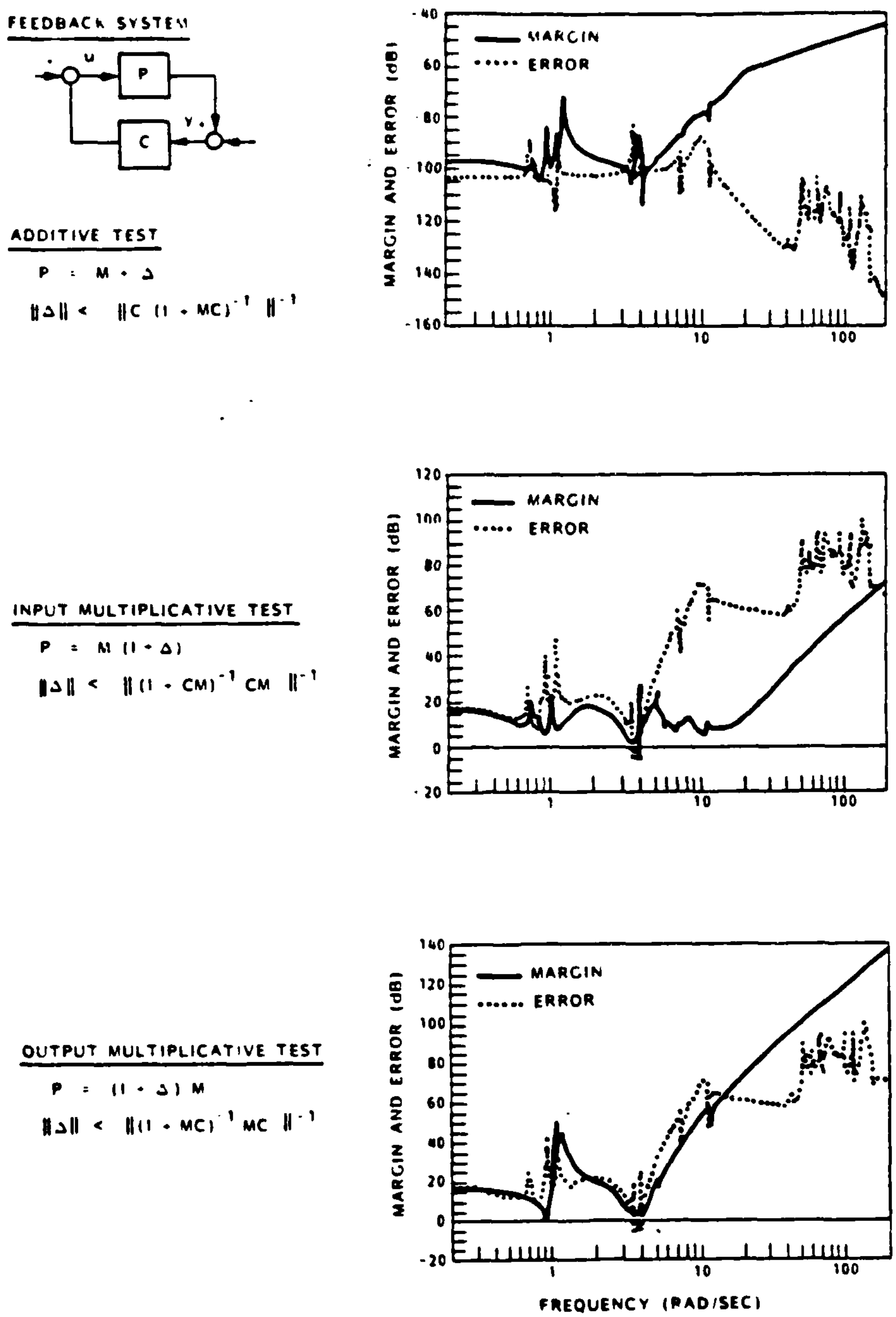


Fig. II.2-33 Results of Model Error Tests

OUT $Y(t)$ / IN $W(t)$	POWER	ENERGY	MAC
POWER	$\max_{\omega} \bar{\sigma}\left[H(j\omega)\ W(j\omega)\right]$	∞	∞
ENERGY	0	$\max_{\omega} \bar{\sigma}\left[H(j\omega)\ W(j\omega)\right]$	$\left(\int_{-\infty}^{\infty} \bar{\sigma}\left[H(j\omega)\ W(j\omega)\right]^2 d\omega\right)^{1/2}$
MAC	$\max_{\omega} \bar{\sigma}\left[H(j\omega)\ W(j\omega)\right]$	∞	$\int_0^{\infty} \bar{\sigma}\left[HW(t)\right] dt$

Fig. II.2-34 Performance Robustness Criteria

termed "second-order modes." In general, the required transformation "scrambles" the original coordinate system such that the physical meaning of the states is lost.

However, for lightly damped structural models with decoupled dynamics, the internally balanced coordinate representation is approximately equal to a scaled representation of the model states. Thus it is possible to write approximate formulae for the states in terms of the original model. Three modal rankings are considered:

- disturbance inputs to LOS
- actuator inputs to LOS
- actuator inputs to sensor outputs

These "second-order modes" rankings give important evaluations about which modes to retain and validity of actuator/sensor placement. These rankings are shown in Fig. II.2-35 along with LOS modal cost [Gre. 1] computed using the colored noise disturbance.

Here the absolute values of the modal costs (for the VCOSS 1 model) are used. The RMS second-order modes and modal costs are plotted versus mode number in Fig. II.2-35. Immediately ident is the clustering of these modal phenomena. The disturbance effect as seen through the line-of-sight is constrained to clusters of modes as is the ability to measure and control the model. The coincidence of the controllable clusters and disturbable clusters indicates a favorable actuator/sensor configuration for the problem.

Frequency Weighted Balanced Realizations. Balanced realization model reduction can be

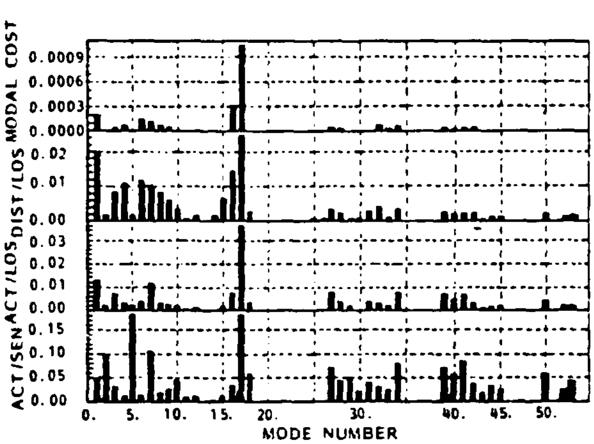


Fig. II.2-35 Open-Loop Model Analysis

extended to finding a reduced model $P_n(s)$ of a high order model $P(s)$ such that

$$\sup_{\omega} \bar{\sigma}\left\{ W_o(j\omega)\left[P(j\omega) - P_n(j\omega)\right] W_i(j\omega)\right\} < 1$$

where $W_o(s)$ and $W_i(s)$ are output and input frequency dependent weighting matrices. These can be chosen to reflect closed-loop requirements on model error, vis a vis, frequency-domain stability and performance margins. For example, stability of the closed-loop system with $C(s)$ designed from $P_n(s)$ is guaranteed if:

$W_o(s) = I$

$W_i(s) = W_i(s) = C_n(s)[I + P_n(s)C_n(s)]^{-1}$

The problem is that $W_i(s)$ is dependent on $P_n(s)$ which is unknown. The let out is that its shape is partially determined by the performance spec; thus, we can make an initial guess. This technique is referred to as "advanced loop shaping." This involves an iterative problem which is solvable via successive approximation.

Compensator Order-Reduction. An alternate to plant order reduction is to design a high order compensator and then reduce the compensator order. Let C(s) denote a high order compensator of order N designed to control P(s) of order N or larger. Let $C_n(s)$ denote a reduced version of C(s) to order $n<N$. Motivated by the stability robustness theory, view $C(s) - C_n(s)$ as a perturbation. Hence, the closed-loop system with P(s) and $C_n(s)$ is stable if:

$$\sup_{\omega} \bar{\sigma} \left| W(j\omega) \left[C(j\omega) - C_n(j\omega) \right] \right| < 1$$

where

$W(s) = (I + P(s)\,C(s))^{-1}\,P(s)$

The weight W(s) is stable because the high order control C(s) stabilizes the closed loop system. In this case W(s) is known and we can apply internal balancing to find $C_r(s)$. The disadvantage to this method is that it is necessary to find a high-order compensator. The advantage is that once it is found, internal balancing applies immediately since the weights are known. On the other hand, direct plant order reduction does not involve control design for the high order plant, but does involve an iterative process since the weights are functions of the (unknown) reduced model.

Low-Authority Control Design. Low-authority control (LAC) systems, when applied to structures, are vibration control systems consisting of distributed sensors and actuators with limited damping authority. The control system is allowed to modify only moderately the natural modes and frequencies of the structure. This basic assumption, combined with Jacobi's root perturbation formula, leads to a fundamental LAC formula for predicting algebraically the root shifts produced by introducing an LAC structural control system. Specifically, for an undamped, open-loop structure, the predicted root shift $(d\lambda_n)_p$ is given by

$$(d\lambda_n)_p \approx \frac{1}{2} \sum_{a,r} C_{ar}\, \phi_{an}\, \phi_{rn} \quad \text{(II-1)}$$

where the coefficient matrix C_{ar} is a matrix of (damping) gains, and ϕ_{an}, ϕ_{rn} denote, respectively, the values of the nth mode shape at actuator station a and sensor station r.

Equation (II-1) may also be used to compute the unknown gains C_{ar} if the $d\lambda_n$ are considered to be desired root shifts or, equivalently, desired modal dampings. While an exact "inversion" of Eq. (II-1) does not generally exist, weighted least-squares type solutions can be devised to determine the actuator control gains C_{ar} necessary to produce the required modal damping ratios. This determination of the gains is the synthesis of LAC systems.

For structures which already have some damping or control systems in which sensor, actuator, or filter dynamics can either be ignored or are already embedded in the plant dynamics, the root perturbation techniques and cost function minimization methods above can similarly be used to synthesize low-authority controls.

Robustness of LAC systems. When sensors and actuators are colocated (i.e., a = r), are complementary, and only rate feedback is used, formula (II-1) reduces to

$$d\lambda_n = -\zeta_n \omega_n \approx \frac{1}{2} \sum_{a} C_a\, \phi^2_{an}$$

which shows that the root shifts are always towards the left of the j-axis if all the gains are negative. This robustness result is obviously based on the assumption that both sensors and actuators have infinite bandwidth, and also that the structure was initially undamped. Several departures from this idealization occur in the actual practical implementation of the LAC systems. The most severe of these results from the finiteness of the actuators' bandwidths. More precisely, the second-order roll-off introduced by the actuator dynamics will always destabilize an undamped structure. However, when some natural damping is present in the structure, or when a passive damper is mounted in parallel with the actuator, additional active damping can be obtained without destabilizing the structure

High-Authority Control Design. The HAC control design procedure can be based on any number of multivariable design methods, e.g., LQG, Q-parameterization, H_∞-optimization, etc. Increased penalties in the LQG cost functional are placed at those frequencies where less response is desired. The concept of frequency-shaped cost functionals was introduced prior to ACOSS [Gup. 1].

The frequency shaping methods are useful in several areas of large space structures control. Three principal applications are important: (1) robustness (spillover avoidance), (2) disturbance rejection, and (3) state estimation.

Management of Spillover. Spillover in closed-loop control of space structures is managed by injecting minimum control power at the natural frequencies of the unmodeled modes. Procedures for controlling spillover at high-frequencies are usually discussed, although similar techniques are applicable for other regimes.

The high-frequency spillover may be controlled by modifying the state or the control weighting. Conversion to the frequency domain gives the following performance index:

$$R(j\omega) = \left(\frac{(\omega^2 + \omega_o^2)}{\omega_o^2}\right) R$$

The problem of robustness (spillover management is solved by making Q and R functions of frequency. Figure II.2-36 depicts the modification to the nominal LQG controller. Observe that frequency shaping adds filters whose inputs are the innovations outputs of the state-estimator in the LQG controller.

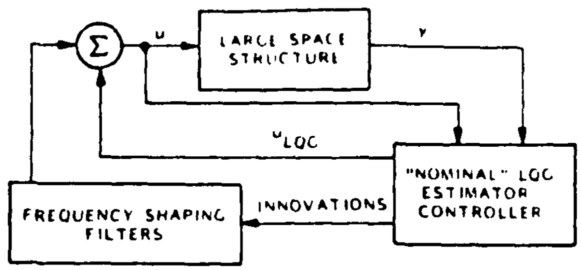


Fig. II.2-36 LQG Control With Frequency-Shaping Filters

Summary. The application of frequency-shaping methods to large space structures leads to a linear controller with memory. However, additional states are needed to represent frequency-dependent weights, hence, there is an increase in the controller order. The software needed for these controller designs is similar to that for standard LQG problems.

Controller Design Using Q-parametrization and H-Optimization. During the last decade, mathematical theories of servo design have been based mainly on quadratic minimization of the Weiner-Hopf-Kalman type, usually applied to state-space models, e.g., LQG controls. However, despite the academic success of these methods, classical frequency response techniques relying on "lead-lag compensators" to reduce sensitivity have continued to dominate industrial servo design. One reason is that quadratic design tends to have poor sensitivity. On the other hand, the frequency domain description has proven to be more suitable to characterize uncertainties which arise in the plant approximation/identification, and frequency domain technique usually results in more robust design, e.g., frequency-shaped LQG can be viewed as an indirect frequency-domain design approach.

Two direct multivariable frequency domain design techniques have become popular in recent years, the Q-parametrization technique and the H_∞-optimal sensitivity.

Q-Parametrization Design. Consider the linear unity-feedback systems shown in Fig II 2-37 where P(s) is the given linear time-invariant plant. C(s) is the linear compensator. u_1 is the reference input. u_2 and d_o are respectively the plant-input disturbance and plant-output disturbance. and y_2 is the plant output.

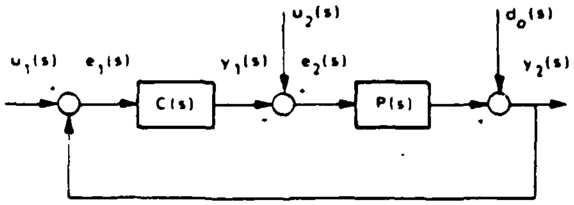


Fig. II.2-37 The Unity-feedback Systems

The closed-loop system input-output transfer function is given by

$$H_{yu} = \begin{bmatrix} C(I+PC)^{-1} & PC(I+PC)^{-1} & -C(I-PC)^{-1} \\ PC(I+PC)^{-1} & P(I+CP)^{-1} & (I+PC)^{-1} \end{bmatrix};$$

(For simplicity. we drop the argument s in P(s). C(s). etc. in this section.)

By introducing the parameter (transfer function)

$$H_{yu} = \begin{bmatrix} u_1 \\ u_2 \\ d_o \end{bmatrix} \rightarrow \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$Q = C(I+PC)^{-1}$$

H_{yu} can be rewritten as

$$H_{yu} = \begin{bmatrix} Q & -QP & -Q \\ PQ & (I-PQ)P & I-PQ \end{bmatrix}$$

Note that the closed-loop input-output transfer function. for the given plant P. is completely specified by the parameter Q in a very simple manner: it involves only sums and products of P and Q.

In a typical control system design problem. the two most important closed-loop transfer functions are $H_{y_2u_1}$ and $H_{y_2d_0}$: $H_{y_2u_1}$ is the transfer function from reference input u_1 to output y_2 and $H_{y_2d_0}$ is the transfer function from plant-output disturbance d_o and output y_2. They specify respectively the servo-performance and regulator performance of the feedback system S. The two transfer functions are given by

$$H_{y_2u_1} = PQ \text{ AND}$$

$$H_{y_2d_0} = I-PQ$$

Therefore. the control design problem reduces to choosing the parameter Q so that the closed-loop system S is stable and that $H_{y_2u_1}$ and $H_{y_2d_0}$ are "satisfactory". After the parameter Q is chosen. the corresponding compensator C can be obtained by the formula [Cal. 1. Chap. 8]

$$C = Q(I-PQ)^{-1}$$

Hence. there is a one-to-one correspondence between C and Q. Consequently. for each parameter Q chosen. there is a unique compensator C which achieves the specified Q.

The selection of the parameter Q in the design process raises several questions: What are the conditions on Q so that the resulting compensator C is realizable (e.g.. proper)? What are the class of all Q's which result in a stable feedback system? How is an "optimal" Q chosen?

Realizability. If the plant P is realizable. then the compensator C is realizable if and only if the parameter Q is realizable. Note that a physical plant is always realizable.

Global Parametization. If the open-loop plant P is stable. then the closed-loop system S is stable if and only if Q is stable since sums and products of stable transfer function matrices are stable. Consequently. the class of all stabilizing compensators is given by

$$\{Q(I-PQ)^{-1} \mid Q \text{ IS STABLE}\}$$

and the class of all achievable stable input-output transfer matrix $H_{y_2u_1}$ and the class of all achievable stable disturbance-to-output transfer matrix $H_{y_2d_0}$ are given respectively by

$$\{PQ \mid Q \text{ IS STABLE}\}$$

and

$$\{I-PQ \mid Q \text{ IS STABLE}\}$$

These sets give global parametrization of all stabilizating compensators, and all achievable I/O characteristics in terms of a stable proper transfer matrix Q. In other words, the class of all "feasible" designs are parametrized by Q.

If the open-loop plant P is not stable, additional constraints have to be added to the choice of Q, in addition to stability and realiability of Q. For example, Q must contain right half plane zeroes to cancel the unstable poles of P. Currently, there are three approaches to obtain global parametrization of a given unstable plant: (i) Factorization representation theory [Des. 1]; (ii) Direct approach [Zam. 1]; (iii) Two-step compensation [Zam. 1].

Optimality. The Q-parametrization alone does not quantatively address the issue of optimal design. The designer selects Q from the class of "feasible" designs, on the basis of the desired input-output response, a priori knowledge of external disturbances, bandwidth, dynamic range and uncertainty of the plant, etc.

Optimal design based on the Q-parametrization and fractional representation framework has become very popular in the research community. The H∞-optimal sensitivity design are among the results available.

H∞-Optimal Sensitivity Design. The H∞-optimal sensitivity design is an extension of the Q-parametrization technique to include a quantitative performance measure of the closed-loop system and achievable optimality based on the performance measure. Roughly speaking, the H∞ design problem is the following: given an open-loop plant P(s) and a low-pass weighting function W(s), find the compensator C(s) so that the H∞-norm of the weighted sensitivity $(I+PC)^{-1}W$ is minimized subject to the stability of the closed-loop system.

Using the Q-parametrization formulation, the problem is equivalent to the following: find a Q in H∞ such that the closed-loop system is stable and that $(I-PQ)W$ is minimized. Since the weighted sensitivity function is affine in Q, the equivalent problem is easier to solve than the original problem.

Solution to the H∞-optimal Sensitivity Problem. Based on the fractional representation (Coprime factorization) formulation, several solutions have been proposed and algorithms given. However, all the proposed algorithms are conceptual in nature, suitable only for simple text book example. More effort is needed toward a numerically robust synthesis procedure. Relevant research results available during the JOSE period of performance will be evaluated and used as appropriate.

II-2.2.7.4.2 Adaptive Control Techniques. Uncertainties in both disturbance spectra and system dynamical characteristics will limit the performance obtainable with fixed gain, fixed order controls. The use of adaptive type control, where disturbance and/or plant dynamics are identified prior to or during control, give system designers more options for minimizing the risk in achieving performance benchmarks. For the case of SBL spacecraft, where performance levels are extremely high, it is absolutely necessary that disturbance and plant models be equally accurate. Since data from ground tests do not usually represent the flight condition accurately, it follows that an on-line procedure for identification and control is necessary.

In this section a method for on-orbit identification and control of flexible spacecraft referred to as "adaptive calibration", is described. This method is being developed by ISI in an on-going basic research program in adaptive control supported by AFOSR Directorate of Aerospace Science. The basic objective of this research program is to establish the theoretical foundations and performance limitations for adaptive control applications to large space structures. An important element of the research is to examine implementation concepts which can lead to appropriate hardware development. A summary of recent results is in [Kos. 1].

The program was originally formulated in late 1982 in response to the increasing concern that performance robustness of Air Force LSS type system would be inadequate to meet mission objectives. The need to identify modal frequencies, for example, in high-performance disturbance rejection systems has been shown in ACOSS (1981) and VCOSS (1982). The deployment of high-performance optical or RF systems may require on-line identification of critical modal parameters before full control authority can be exercised. Parameter sensitivity manifested by performance degradation or loss of stability (poor robustness) may be effectively reduced by adaptive feedback mechanizations. Reducing the effects of on-board disturbance sources on the system performance (disturbance rejection) is particularly important for planned Air Force missions. For these cases, adaptive control mechanizations are needed to produce the three-to-five orders-of-magnitude reductions in line-of-sight jitter required by the mission.

AFOSR Research Program Summary. The research was originally directed toward real-time adaptation using standard estimator and controllers forms in the basic adaptive control structure, as shown in Fig. II.2-38. Most adaptive control algorithms can be described in this form. For example, one could select from the following catalogs of major areas:

Model	Control Design	Adaptation
ARMAX	Model Reference	Gradient
State-	Self-Tuning	Recursive Least Squares
Space	Pole-Placement	Recursive Max Likelihood
		Extended Kalman Filter

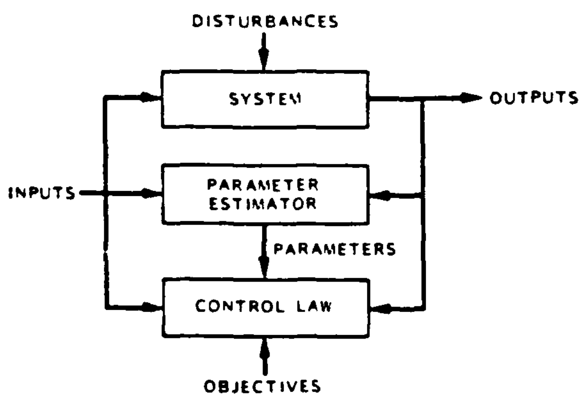


Fig. II.2-38 Adaptive Control System

The schemes also differ in terms of update rates. Typically the outer control loop is at a fast rate, whereas the parameters from identification are updated more slowly. Adaptive schemes are referred to as "recursive" if the identification rate is a fixed multiple of the controller rate. If identification is used when necessary for calibration the scheme is referred to as adaptive calibration.

Although a great deal of research results are available about adaptive control and identification, unmodeled dynamics and broadband disturbances will significantly upset most algorithms. Hence, the lack of a well-developed robustness theory for adaptive mechanizations required a reexamination of the problem at a more fundamental level, i.e., development of model and disturbance uncertainty bounds for which adaptive algorithms would exhibit (stable) desired performance. Toward this end there have been two major accomplishments:

(1) **Development of Theory**: In examining the possible use of recursive adaptive control it was necessary to generate new theory of use on large space structures [Kos. 3]. This theory accounts for the effect of unmodeled dynamics with distributed parameter systems, such as flexible space structures, and extends current adaptive theory in several directions.

In the first place, current adaptive theory provides conditions for "global" stability, i.e., bounded-input, bounded-output stability with no limitation on the size (or spectrum) of the bounded-inputs (e.g., disturbances and references). Secondly, the theory is limited to finite-dimensional linear systems. This latter condition cannot be satisfied by a flexible space structure, which is a distributed parameter system. Also, the disturbance and reference inputs effecting the spacecraft have limited magnitudes and spectrums and these limits are known, although not precisely. The developed theory circumvents those difficulties by providing conditions for "local" stability, i.e., limitations in input size and spectrum are accounted. The theory also allows for a distributed system as well as providing quantifiable bounds on permissible model error. These results extend the state-of-the-art in adaptive theory beyond the current limits.

(2) **Adaptive Calibration**: The use of "slow" adaptive control, which is more practical than recursive adaptive control in most space applications, necessitated a new methodology development

merging key ideas in parameter estimation, system identification, and robust control design The term "slow" means that there is sufficient time to run batch identification before the control system is modified The methodology developed resolves a long standing problem with adaptive systems of this type, namely, the means to provide a guaranteed level of performance given an "identified" model of the system together with the model error between the system and the identified model. In fact, the methodology generates performance versus model error tables (to be stored in the computer) from which the control design is immediately obtained. Moreover, the order of the control design is determined strictly on the basis of model error and performance demand, rather than trial and error as has been suggested in the past.

Application of Adaptive Calibration. The basic problem with control based on identified models is that without a measure of model error it is very easy to destabilize the system—particularly when the goal is high performance—as in SBL. Adaptive calibration is an approach which incorporates a measure of model error with robust control design in an iterative way so that identification is performed only where it is needed. A proposed adaptive calibration system is shown in Fig. II.2-39, with test results, using the CSDL #2 model, shown in Fig. II.2-40. The adaptive calibration procedure involves the following steps:

1. The model M(s) is a 10-mode model which has been obtained from I/O data.

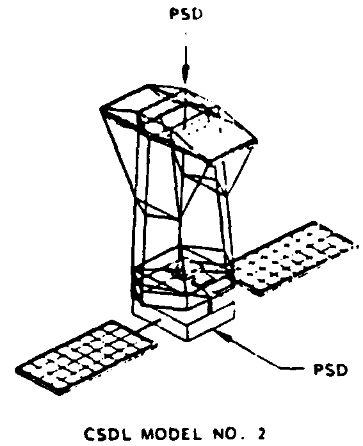


Fig. II-2-40 Draper Simulation System

2. Estimate $\delta(\omega)$ = model error versus frequency using FFT. This is dashed curve in Fig. II.2-41.
3. Using the identified model M(s) and the model error $\delta(\omega)$, synthesize a robust control (e.g., Section 2.2.7.5).
4. Calculate δ_{SM} — stability margin. This is the dashed curve in Fig.II.2-41. Compare to model error δ both plotted in Fig. II.2-42. If acceptable go to Step 7 and implement controller. Otherwise go to Step 6.
6. Modify filter windows, number of parameters (e.g., number of modes), or input spectrum and then repeat Step 1 to obtain new ID model. Fig. II.2-42 shows result of identification after one mode is added in the frequency domain region where the test fails.

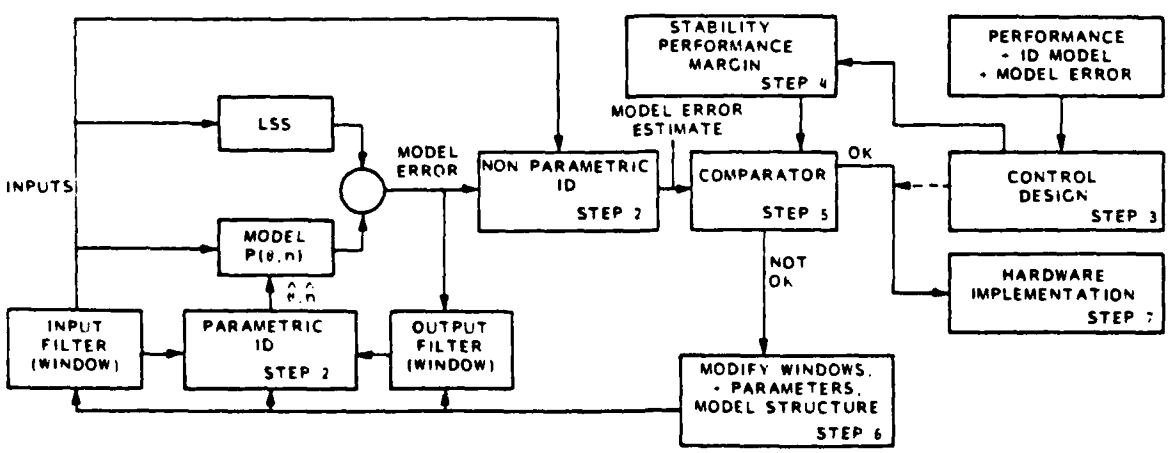


Fig. II.2-39 Adaptive Calibration for LSS

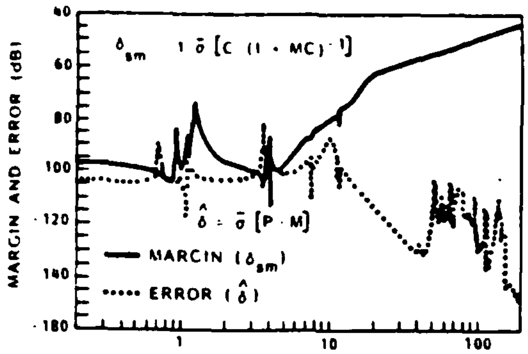


Fig. II.2-41 Comparison of Margin and Model Error

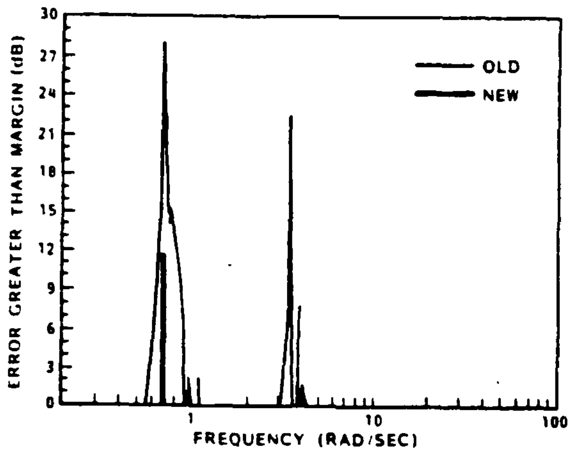


Fig. II.2-42 Data Before and After ID Cycle Plots Show Where Error is Margin

7. Implement controller.

This strategy has been successfully implemented in ground test hardware. Similar techniques are being used for adaptive (hardware) gun turret stabilization for an Army helicopter weapons platforms.

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APPENDIX D

STABILITY THEORY FOR ADAPTIVE SYSTEMS: METHODS OF AVERAGING AND PERSISTENCY OF EXCITATION**

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ABSTRACT

A method of averaging is developed for the stability analysis of linear differential equations with small time-varying coefficients which do not necessarily possess a (global) average. The technique is then applied to determine the stability of a linear equation which arises in the study of adaptive systems where the adaptive parameters are slowly varying. The stability conditions are stated in the frequency-domain which shows the relation between persistent excitation and unmodeled dynamics.

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1. INTRODUCTION

For a large class of adaptive feedback systems, as well as for some output error identification schemes, a stability analysis in the neighborhood of the desired behavior leads to investigating the stability of the following homogeneous linear system of differential - operator equations (see e.g. [1] - [3])

$$\dot{\theta} = -\varepsilon\, u(t)\, H(u(\cdot)'\theta(\cdot)), \qquad \forall t \in R_+ \tag{1.1}$$

where $\theta(0) \in R^p$, ε is a positive constant, $u(\cdot): R_+ \to R^p$ is regulated and bounded, and H is a linear time-invariant operator whose transfer function $\hat{H}(s)$ is proper, rational, and stable, i.e., all poles have negative real parts.

Linearization and Local Stability

In [2], for example, system (1.1) is obtained as a result of linearization of the adaptive system in the neighborhood of a "tuned" system, i.e., a system where the adaptive parameters are set to a constant value $\theta_* \in R^p$ and whose behavior is deemed acceptable. Hence, in (1.1), $\theta(t)$ is the vector of parameter errors between the parameter estimate at time t and the tuned value θ_*, $u(t)$ is the regressor vector from the tuned system (e.g., filtered revisions of measured signals), and the scalar ε is the magnitude of the adaptation gain which essentially controls the rate of adaptation. The operator H depends on the actual system being controlled or identified and also on the tuned parameter setting θ_*.

It is shown in [2,3] that if the zero solution of (1.1) is uniformly asymptotically stable (u.a.s), then the adaptive system is locally stable, i.e., the adaptive system behavior will remain in a neighborhood of the desired behavior provided the initial parameter error $\theta(0)$ and the effect of external disturbances are sufficiently small. Although the results in [2,3] were arrived at using input-output properties [16], the local stability property also follows from the results on "total" stability [4].

Unmodeled Dynamics and Slow Adaptation

In the ideal case there are a sufficient number of adaptive parameters (the number p) such that the tuned parameter setting results in $\hat{H}(s)$ being strictly positive real (SPR), i.e., Re $\hat{H}(j\omega) > 0$, $\forall\omega \in R_+$. Under these conditions, we have the following results (see e.g , [5]-[8], [1]): (1) the zero solution of (1.1) is stable, i.e., $\theta(t)$ is bounded but not necessarily constant; (2) if, in addition, u(t) is persistently exciting, then the zero solution is uniformly asymptotically stable (u.a.s.), thus, $\theta(t) \to 0$ exponentially fast as $t \to \infty$. The trouble starts when there are an insufficient number of parameters to obtain $\hat{H}(s) \in$ SPR, as is the case in adaptive control when the plant has unmodeled dynamics (see e.g. [2, 7], [12]).

In this paper we will examine the stability of (1.1) when ϵ is small, u(t) is persistently exciting, and $\hat{H}(s)$ is not necessarily SPR but only stable. Reidle and Kokotovic [9] refer to this case as "slow adaptation" and by using the methods of averaging described by Hale [10], they show that the stability of the zero solution of (1.1) is critically dependent on the spectrum of the excitation in relation to the frequency response $\hat{H}(j\omega)$. With the same assumptions, Astrom [11] uses averaging techniques to analyze the interaction between unmodeled dynamics and external inputs in the counter-example posed by Rohrs et al. [12]. Both these analyses require the assumption that u(t) is almost periodic. In this case Reidle and Kokotovic [9] show that the zero solution of (1.1) is u.a.s. if

$$\lambda\Big(\sum_{\omega\in\Omega} [\alpha(\omega)\alpha(\omega)^*] \text{ Re } \hat{H}(j\omega)\Big) > 0 \qquad (1.2)$$

where Ω and $\{\alpha(\omega), \omega \in \Omega\}$ are, respectively, the Fourier exponents and coefficients of u(t). Condition (1.2) can be considered as a positivity condition, but unlike the SPR condition Re $\hat{H}(j\omega)$ is not required to be positive at all frequencies.

Averaging: Uses and Limitations

The main contribution of this paper is to extend the theory of averaging to include the case when u(t) does not have a (generalized) Fourier series representation, but is only known to be regulated and bounded. Thus, u(t) need not be almost periodic nor even possess a (global) average value. We also state stability conditions in the frequency-domain in a form similar to (1.2). Analagous results are stated for the discrete-time system

$$\theta(t+1) = \theta(t) - \varepsilon\, u(t)\, H(u(\cdot)'\theta(\cdot)), \quad \forall t \in Z_+ \tag{1.3}$$

where we only require $u(\cdot) \in \ell_\infty^P$ and H to be linear-time-invariant and stable. Averaging results for (1.3) with H = 1 and with u(·) not almost periodic can also be found in [13]; and this suggests the possibility of being able to dispense with the almost periodicity assumption on u(·) and analyzing (1.1) with a non-SPR operator H.

The averaging theory developed here, as well as averaging theory in general, has its uses and limitations for adaptive system. In the first place, the theory requires slow adaptation which can be counter-productive because performance can be below par for the long period of time it takes for the parameters to readjust. Secondly, averaging theory is a form of linearization, thus, the (nonlinear) adaptive system must be initialized in a (not necessarily small) neighborhood of the tuned system. On the positive side, however, we do obtain frequency domain conditions which explain the system behavior near the tuned solutions. In this sense, we can consider the results of averaging theory to be necessary conditions for good performance of adaptive systems.

To obtain the heralded goal of frequency domain stability conditions, it may be inevitable to encounter linearization. Somewhat less intuitively appealing results can be obtained without resorting to direct linearization or averaging, e.g., in [2,3], [14] and [15] the results arise from a combination of small gain theory and perturbation theory.

Organization of Paper

The paper is organized as follows: Section 2 develops methods of averaging for general systems described by linear differential equations -- both homogenous and inhomogeneous systems are considered. The reader can regard this section independently from the rest of the paper, because the systems of linear equations considered are the most general and need not arise from adaptive systems. In Section 3 we apply the general results of Section 2 to (1.1) and obtain frequency domain stability conditions. In Section 4 we analyze the effect of unmodeled dynamics. In Section 5 we state the discrete-time versions of the results obtained in Section 1, and, as in Section 1, these results are of general interest.

2. METHOD OF AVERAGING

2.1 LINEAR HOMOGENEOUS SYSTEMS

In this section we will consider the homogeneous linear system

$$\dot{x} = \varepsilon A(t)x \tag{2.1}$$

Lemma (2.2)

Suppose in (2.1) that $A(\cdot): R_+ \to R^{nxn}$ is regulated and bounded. Then $\forall s,\tau \in R_+$, the transition matrix $\phi(s+\tau,s)$ of (2.1) is given by

$$\phi(s+\tau,s) = \exp[\varepsilon\tau \bar{A}_\tau(s)] + R(s,\varepsilon\tau) \tag{2.3}$$

where

$$\bar{A}_\tau(s) = \frac{1}{\tau}\int_s^{s+\tau} A(t)\,dt \tag{2.4}$$

is the local average value of $A(t)$ on the interval $s \leq t \leq s + \tau$, and

$$||R(\cdot,\ \varepsilon\tau)||_\infty \leq r(\varepsilon\tau||A||_\infty) := (\varepsilon\tau||A||_\infty)^2 \exp(\varepsilon\tau||A||_\infty) \tag{2.5}$$

See Appendix for proof.

Remarks

(1) Assuming that $A(t)$ is regulated and bounded is sufficient for the existence and uniqueness of solutions [17].

(2) Observe that Lemma (2.2) is valid $\forall s,\tau \in R_+$ and $\forall \varepsilon \in R$. In the sequel we use Lemma (2.2) only for the case when $\varepsilon > 0$ and $\varepsilon\tau$ is small.

The stability properties of (2.1) can be established by application of Lemma (2.2) as stated in Theorem (2.9) below. We first require

Definition [16] (2.6)

The function $\mu(\cdot)$: $C^{nxn} \to R$, defined by

$$\mu(M) = \lim_{\alpha \downarrow 0} (|I + \alpha M|_i - 1)/\alpha \qquad (2.7)$$

is called the measure of the matrix M, where $|\cdot|_i$ is the matrix norm on C^{nxn} induced by the vector norm $|\cdot|$ on C^n. For example, if $|\cdot|$ is the Euclidean norm then $\mu(M) = \max \lambda[(M+M^*)/2]$. For any norm on C^n we have the relation,

$$-\mu(-M) \leq \mathrm{Re}\,\lambda(M) \leq \mu(M), \quad \forall M \in C^{nxn} \qquad (2.8)$$

Theorem (2.9)

Suppose $A(t)$ in (2.1) is regulated and bounded with the sequence of local average values $\{\bar{A}_T(kT),\ \forall k \in Z_+\}$. Then:

(i) If $\exists\ T > 0$ and $\alpha > 0$ such that

$$\mu[\bar{A}_T(kT)] \leq -\alpha, \qquad \forall k \in Z_+ \tag{2.10}$$

then $\exists\ \eta > 0$ such that $\forall \varepsilon T \in (0, \eta)$ the zero solution of (2.1) is u.a.s.

(ii) If $\exists\ T > 0$ and $\alpha > 0$ such that

$$\mu[-\bar{A}_T(kT)] \leq -\alpha, \qquad \forall k \in Z_+ \tag{2.11}$$

then $\exists\ \eta > 0$ such that $\forall \varepsilon T \in (0, \eta)$ the zero solution of (2.1) is completely unstable.

Remarks

(1) The proof (see Appendix) is based on Lemma (2.2) and the inequality [16]:

$$\exp\{-\varepsilon T\mu[-\bar{A}_T(kT)]\} \leq |\phi((k+1)T,kT) - R(kT,\varepsilon T)|$$

$$\leq \exp\{\varepsilon T\ \mu[\bar{A}_T(kT)]\}, \qquad \forall k \in Z_+ \tag{2.12}$$

where $\phi(\cdot,\cdot)$ and $R(\cdot,\cdot)$ are as defined in Lemma (2.2).

(2) Whenever (2.10) holds we have $|\exp\{\varepsilon T\bar{A}_T(kT)\}| < 1$, $\forall k \in Z_+$ which insures a contraction (for small εT) on the interval $s \leq t \leq s + T$. It is possible to weaken condition (2.10) and still have a contraction by just enforcing $|\exp\{\varepsilon T\ \bar{A}_T(kT)\}| < 1$ directly as is done by Coppel [18].

(3) Note that Thereom (2.9) can be stated in terms of stronger conditions on $\mu[\bar{A}_T(s)]$, $\forall s \in R_+$.

Using the same technique, but allowing $A(\cdot)$ (equivalently $\bar{A}_T(\cdot)$) to possess a global average, we obtain the following sharper result.

<u>Theorem</u> (2.13)

Suppose $A(t)$ in (2.1) is regulated, bounded, and has a (global) average $\bar{A} \in R^{nxn}$, i.e.,

$$\lim_{T\to\infty} \bar{A}_T(s) = \bar{A} \tag{2.14}$$

uniformly $\forall s \in R_+$ with $\mathrm{Re}\ \lambda(\bar{A}) \neq 0$. Under these conditions:

(i) If $\exists\ \alpha > 0$ such that

$$\mathrm{Re}\ \lambda(\bar{A}) \leq -\alpha \tag{2.15}$$

then $\exists\ \varepsilon_o > 0$ such that $\forall \varepsilon \in (0, \varepsilon_o)$, the zero solution of (2.1) is u.a.s.

(ii) If $\exists\ \alpha > 0$ such that

$$\max \mathrm{Re}\, \lambda(\bar{A}) \geq \alpha \tag{2.16}$$

then $\exists \epsilon_o > 0$ such that $\forall \epsilon \in (0, \epsilon_o)$, the zero solution of (2.1) is unstable.

Discussion

The results in Theorem (2.9) and Theorem (2.13) generalize some results obtained by averaging methods such as those described by Hale [10], or as obtained by Coppel [18] using the notion of integral smallness. Theorem (2.13) is a classical result of averaging theory, except that as stated allows for functions which are not necessarily almost periodic. The class of functions allowed in theorem (2.13) - regulated, bounded, with an average - is not precisely characterized. Obviously it includes the class of asymptotically almost periodic functions of the form [19]

$$A(t) = A_p(t) + A_1(t) \tag{2.17}$$

where $A_p(t)$ is almost periodic and $A_1(\cdot) \in L_1^{nxn}$.

Theorem (2.9) considers a larger class of functions -- those without an average -- at the expense of a weaker result: the stability -- instability boundary is not as sharp as in theorem (2.13).

An example of a function which satisfies the conditions of theorem (2.9), but not of (2.13) is:

$$A(t) = A_o + (1/\sqrt{2})\, A_1 \,(\sin \log t + \cos \log t) \tag{2.18}$$

where $A_i = A_i' > 0$, $i=1, 2$ such that $A_o - A_1 > 0$. This function does not have a global average, as can be seen from

$$\frac{1}{T}\int_s^{s+T} A(t)dt = A_o + A_1[\sin\log(s+T) + \frac{s}{T}\sin\log(1+\frac{T}{s})] \quad (2.19)$$

However, it satisfies the conditions of Theorem (2.9) because from (2.18),

$$\frac{1}{T}\int_s^{s+T} A(t)\,dt \geq A_o - A_1 > 0, \quad \forall s \in R_+, \forall T > 0. \qquad (2.20).$$

From the proof of Theorem (2.9) we can extract a value for η and also state bounds on the exponential rates of growth or decay of the transition matrix $\phi(t, \tau)$ for all $t \geq \tau$. Specifically, we have:

Corollary (2.21)

If $A(t)$ is regulated and bounded with $||A(\cdot)||_\infty \leq m$, then:

(i) Whenever (2.10) holds for some $T > 0$, the zero solution of (2.1) is u.a.s. $\forall \varepsilon T \in (0, \eta)$ where:

(a) $\eta > 0$ satisfies $\exp(-\eta\alpha) + r(\eta m) = 1$ (2.22)

(b) $\forall t,\tau \in R_+$ with $t \geq \tau$, (2.23)

$$|\phi(t,\tau)| \leq K \exp(-\varepsilon(t-\tau)\beta)$$

where

$$K = \exp(\varepsilon T(m+\beta)) > 1$$

$$\exp(-\varepsilon T\beta) = \exp(-\varepsilon T\alpha) + r(\varepsilon Tm) < 1$$

(ii) Whenever (2.11) holds for some $T > 0$, the zero solution of (2.1) is unstable $\forall \epsilon T \in (0,\eta)$ where:

$$\text{(a)} \quad \eta > 0 \text{ satisfies } \exp(\eta\alpha) - r(\eta m) = 1 \tag{2.24}$$

$$\text{(b)} \quad \forall t,\tau \in R_+ \text{ with } t \geq \tau, \tag{2.25}$$

$$|\phi(t,\tau)| \geq K \exp(\epsilon(t-\tau)\beta)$$

where

$$K = \exp(-\epsilon T(m+\beta)) < 1$$

$$\exp(\epsilon T\beta) = \exp(\epsilon T\alpha) - r(\epsilon Tm) > 1$$

2.2 LINEAR INHOMOGENEOUS SYSTEMS

In this section we extend the results of Section 2 to the inhomogeneous system

$$\dot{x} = \epsilon(p(t) + A(t)x) \tag{2.26}$$

<u>Theorem</u> (2.27)

Suppose in (2.26) that $A(\cdot): R_+ \to R^{nxn}$ and $p(\cdot): R_+ \to R^n$ are regulated and bounded. Let $\bar{A}_\tau(s)$, $\forall s,\tau \in R_+$ denote the local average value of $A(t)$ as defined by (2.4) and let $\bar{p}_\tau(s)$ denote the local average value of $p(t)$ defined by

$$\bar{p}_\tau(s) = \frac{1}{\tau}\int_s^{s+\tau} p(t)\, dt. \qquad (2.28)$$

Under these conditions, $\forall s,\tau \in R_+$,

$$|x(s+\tau)-\phi(s+\tau,s)[x(s) + \varepsilon\tau\bar{p}_\tau(s)]| \le q(\varepsilon\tau,||A||_\infty)||p||_\infty \qquad (2.29)$$

where

$$q(\varepsilon\tau,||A||_\infty) = (\varepsilon\tau)^2||A||_\infty \exp(\varepsilon\tau||A||_\infty) \qquad (2.30)$$

Combining this result with Theorem (2.9) and Corollary (2.21) gives

Corollary (2.31)

Under the conditions of Theorem (2.27), if $\exists\ \alpha > 0$ and $T > 0$ such that

$$\mu[\bar{A}_T(kT)] \le -\alpha, \qquad \forall k \in Z_+ \qquad (2.32)$$

then $\forall \varepsilon T \in (0, \eta)$,

$$||x||_\infty \le q_1 \sup_{k\in Z_+} |\bar{p}_T(kT)| + q_2||p||_\infty \qquad (2.33)$$

with

$$q_1 = \varepsilon T \exp(\varepsilon T(m-\beta))/(1 \quad \exp(-\varepsilon T\beta)) \qquad (2.34)$$

$$q_2 = \varepsilon T \exp(\varepsilon Tm)[1 + \varepsilon Tm \exp(\varepsilon Tm)/(1 \ \exp(-\varepsilon T\beta))]$$

where $m = ||A||_\alpha$ and where $\eta > 0$ and $\beta > 0$ satisfy (2.22) and (2.23), respectively, i.e.,

$$\exp(-\eta\alpha) + (\eta m)^2 \exp(\eta m) = 1$$
$$\exp(-\varepsilon T\beta) = \exp(-\varepsilon T\alpha) + (\varepsilon Tm)^2 \exp(\varepsilon Tm) \tag{2.35}$$

Remarks

Observe that as $\varepsilon T \to 0$, $q_1 \to 1/\beta$ and $q_2/\varepsilon T \to 1$. Hence, as $\varepsilon T \to 0$ we see that $||x||_\infty$ is overbounded by the largest value of the sequence of local averages $\{|\bar{p}_T(k\tau)|, k \in Z_+\}$ and not $||p||_\infty$. For example, if $p(t) = \sin \frac{2\pi t}{T}$ then $||p||_\infty = 1$ whereas $|\bar{p}_T(kT)| = 0$, $\forall k \in Z_+$.

3. FREQUENCY-DOMAIN STABILITY CONDITIONS

In this section we apply the results of Section 2 to the homogeneous linear system (1.1), i.e.,

$$\dot{\theta} = -\varepsilon\, u(t)\, H(u(\cdot)'\theta(\cdot)), \quad \theta(0) \in R^p \tag{3.1}$$

where H is a linear time invariant operator with transfer function $\hat{H}(s)$. We first show that for sufficiently small $\varepsilon > 0$, the stability analysis of (3.1) can be determined from the stability of an "averaged" system

$$\dot{\theta} = \varepsilon\, \mathrm{avg}\{u(t)(Hu)'(t)\}\theta$$

where avg $\{\cdot\}$ has yet to be precisely defined. Using this result we then establish stability conditions in the frequency-domain involving the Fourier transform $\hat{H}(j\omega)$ and the "spectral" content of $u(t)$, where this notion has also to be defined. Finally, we show that the appropriately defined spectral content of $u(t)$ necessarily requires that $u(t)$ have a persistency of excitation property, and that the dominant excitation should be at those frequencies where Re $\hat{H}(j\omega) > 0$.

The first step in the analysis is to transform (3.1) into a form suitable for application of Theorem (2.9).

Lemma (3.2)

Assume in (3.1) that u(t) is regulated and bounded, and H is a causal linear time-invariant operator where transfer function $\hat{H}(s)$ is proper, rational, and stable. Under these conditions, $\exists\ \varepsilon_o > 0$ such that $\forall\varepsilon \in (0,\varepsilon_o)$, (3.1) is equivalent to,

$$\dot{\theta} = -\varepsilon\,[\beta(t) + \varepsilon\, u(t)v(t)']\theta \tag{3.3a}$$

where

$$B(t) = u(t)(Hu(\cdot)')(t) \tag{3.3b}$$

and where the function $v(\cdot) : R_+ \to R^p$ is regulated and bounded.

Remarks

(1) Since v(t) is bounded, it follows that we can approximate (3.3) to first order in ε by $\dot{\theta} = -\varepsilon\,\beta(t)\theta$ which is the form required by Theorem (2.9). Precise conditions on this approximation are stated in Theorem (3.11) below.

(2) Estimates for ε_o and $||v||_\infty$ are obtained from the proof of Lemma (3.2). Specifically, let $\hat{H}(s) = d + c'(sI-A)^{-1}b$ where, with no loss in generality, (A,b,c,d) is minimal, $|c| = 1$, and Re $\lambda(A) < 0$. Then:

$$\varepsilon_o = a(\ell - \ell_o)/[K||u||_\infty(d||u||_\infty + \ell)\ell] \tag{3.4}$$

$$||v||_\infty \leq (1 - \ell_o/\ell)/\varepsilon_o \tag{3.5}$$

where $K \geq 1$ and $a > 0$ are found from

$$|\exp(tA)| \leq K \exp(-ta), \quad \forall t \in R_+ \tag{3.6}$$

with

$$\ell = \ell_o[1 + (1 + d||u||_\infty/\ell_o)^{1/2}] \tag{3.7}$$

$$\ell_o = ||L_o||_\infty \tag{3.8}$$

where $L_o(t)$ satisfies,

$$\dot{L}_o = AL_o + bu(t)', \quad L_o(0) = 0 \in R^{nxp} \tag{3.9}$$

We now use Lemma (3.2) to establish frequency domain stability conditions for (3.1). This requires that u(t) be restricted to those functions which have a Fourier series representation on any finite interval. A known class of such functions is defined as follows.

<u>Definition</u> [20]

A function $f(\cdot)$: $R_+ \rightarrow R^n$ is a C_δ^n function if it is regulated, bounded and $\exists$ a constant $\delta > 0$ such that any two points t_1, $t_2 \in R_+$ where $f(\cdot)$ is discontinuous are separated by at least an interval δ, i.e., $|t_1 - t_2| \geq \delta$.

Frequency-domain stability conditions for (3.1) can now be stated.

Theorem (3.11)

Assume in (3.1) that:

(A1) H is linear with stable proper rational transfer function $\hat{H}(s)$ and impulse response h(t). Thus, $\exists$ a > 0 and b > 0 such that

$$|h(t) - h(0)\delta(t)| \leq a \exp(-bt), \quad \forall t \in R_+ \tag{3.12}$$

(A2) $u(\cdot) \in C^p_\delta$ with piece-wise Fourier series representation,

$$u(t) \sim \sum_{m \in Z} \alpha_k(\omega_m) e^{j\omega_m t}, \quad \forall t \in [kT,(k+1)T], \ \forall k \in Z_+ \tag{3.13}$$

for any $T \geq \delta$ where $\omega_m = 2\pi m/T$.

Define the matrix sequence in R^{pxp} by

$$R_T(k) = \sum_{m \in Z} \alpha_k(\omega_m)\alpha_k(\omega_m)^* \hat{H}(-j\omega_m), \quad \forall k \in Z_+ \tag{3.14}$$

Under these conditions:

(i) If $\exists$ $T \geq \delta$ and $\alpha > 0$ such that

$$\mu[-R_T(k)] \leq -(\alpha + 2(a/b^2)||u||_\infty/T), \quad \forall k \in Z_+ \tag{3.15}$$

then $\exists$ $\varepsilon_o > 0$ such that $\forall \varepsilon \in (0, \varepsilon_o)$ the zero solution of (3.1) is u.a.s.

(ii) If $\exists$ $T \geq \delta$ and $\alpha > 0$ such that

$$\mu[R_T(kT)] \leq -(\alpha + 2(a/b)||u||_\infty^2/T), \qquad \forall k \in Z_+ \tag{3.16}$$

then $\exists\ \varepsilon_0 > 0$ such that $\forall \varepsilon \in (0, \varepsilon_0)$ the zero solution of (3.1) is unstable.

<u>Remarks</u>

(1) The existence of the piece-wise Fourier series representation (3.13) for $u(t)$ is guaranteed by $u(\cdot) \in C_\delta^p$ [17]. The Fourier coefficients $\alpha_k(\omega_m)$ are the coefficients of the T-periodic function

$$u_k(t) = \sum_{m \in Z} \alpha_k(\omega_m) e^{j\omega_m t}, \ \forall t \in R_+ \tag{3.17}$$

which is equal to $u(t)$ for $kT \leq t \leq (k+1)T$ and, in general, not equal to $u(t)$ on any other interval. Thus, $u_k(t)$ is just $u(t)$, $\forall t \in [kT, (k+1)T]$, repeated with period T. Observe that the spectrum of $u_k(t)$ is what determines the stability-instability boundary and not the spectrum of $u(t)$. These will merge only when $u(\cdot)$ has a (global) average as assumed in Theorem (3.23) below.

(2) The matrix $R_T(k)$ can be equivalently expressed as the local average value of $u_k(t)(Hu_k)_\infty'(t)$, i.e.,

$$R_T(k) = \frac{1}{T} \int_{kT}^{(k+1)T} u_k(t)(Hu_k)_\infty'(t)dt \tag{3.18}$$

where $(Hu_k)_\infty(t)$ is the "steady-state" part of $(Hu_k)(t)$, i.e.,

$$(Hu_k)_\infty(t) = \sum_{m \in Z} \hat{H}(j\omega_m)\alpha_k(\omega_m)e^{j\omega_m t}, \ \forall t \in R_+ \tag{3.19}$$

(3) If we use the measure $\mu(M) = \max \lambda(M+M^*)/2$, then (3.15) and (3.16) become, respectively,

$$\lambda[Q_T(k)] \geq \alpha + 2(a/b^2)||u||_\infty^2/T, \qquad \forall k \in Z_+ \qquad (3.20)$$

and

$$\lambda[Q_T(k)] \leq -(\alpha + 2(a/b^2)||u||_\infty^2/T), \qquad \forall k \in Z_+ \qquad (3.21)$$

where $Q_T(k)$ is the Hermitian part of $R_T(k)$, i.e.,

$$Q_T(k) = \sum_{m \in Z_+} \text{Re}[\alpha_k(\omega_m)\alpha_k(\omega_m)^*] \; \text{Re}[\hat{H}(j\omega_m)] \qquad (3.22)$$

(5) The "initial conditions" at $t = kT$ contribute to the term $2(a/b^2)||u||_\infty^2/T$ in (3.15)-(3.16) or (3.20)-(3.21). Hence, the average energy in $u_k(t)(Hu_k)_\infty'(t)$ must dominate long enough (T sufficiently large) to overcome these (possibly) negative effects.

As before, if $u(t)$ is further restricted such that $R_T(\cdot)$ has a global average, then we can sharpen the stability-instability boundary. For example, if $u(t)$ is almost periodic then a Fourier series representation exists $\forall t \in R_+$ and $R_T(\cdot)$ has an average [10]. The stability conditions for this case are stated as follows.

Theorem (3.23)

Suppose in (3.1) that u(t) is almost periodic with Fourier series

$$u(t) \sim \sum_{\omega \in \Omega} \alpha(\omega)\, e^{j\omega t}, \quad \forall t \in R_+ \tag{3.24}$$

where $\Omega \in R$ are the distinct Fourier exponents and $\{\alpha(\omega),\ \omega \in \Omega\}$ are the Fourier coefficients. Define the matrix $R \in R^{nxn}$ by

$$R = \sum_{\omega \in \Omega} \alpha(\omega)\, \alpha(\omega)^*\, \hat{H}(-j\omega) \tag{3.25}$$

If $\text{Re}\ \lambda(R) \neq 0$ then $\exists\ \varepsilon_o > 0$ such that $\forall \varepsilon \in (0, \varepsilon_o)$ the zero solution of (3.1) is:

(i) u.a.s if $\text{Re}\ \lambda(R) > 0$ (3.26)

(ii) unstable if $\max \text{Re}\ \lambda(R) < 0$ (3.27)

Discussion

The proof of Theorem (3.23) is entirely analogous to that of Theorem (2.13). Theorem (3.23) is the result obtained in [9] when u(t) is almost periodic. Theorem (3.11) is a generalization in that $u(\cdot) \in C_\delta^p$.

Observe that the stability-instability boundary determined by (3.20) - (3.22) exists if and only if

$$\lambda[Q_T(k)] \neq 0, \quad \forall k \in Z_+ \tag{3.28}$$

By (3.22), this will hold if and only if for some finite integer $q \geq p$,

$$\text{rank}\,[\alpha_k(0),\ \alpha_k(\omega_1),\ .\ .\ .\ .\ ,\ \alpha_k(\omega_q)] = p,\ \forall k \in Z_+. \tag{3.29}$$

Hence, Theorem (3.11) implicitly restricts $u(\cdot) \in C_\delta^p$ to those functions whose (time-varying) Fourier coefficients satisfy the rank condition above. This class of functions, however, are precisely those which can be categorized as persistently exciting:

<u>Definition</u> [1] (3.30)

A function $f(\cdot): R_+ \to R^n$ is <u>persistently exciting</u> over an interval h if it is regulated, bounded, and $\exists$ constants $h > 0$ and $\beta > 0$ such that

$$\min \lambda\left(\frac{1}{h}\int_s^{s+h} f(t)f(t)'dt\right) \geq \beta, \qquad \forall s \in R_+ \tag{3.36}$$

Denote such functions by $u(\cdot) \in PE^n(h,\beta)$.

Hence, we immediately see that if u(t) in (3.1) is in $PE^p(h,\beta)$ and C_δ^p then (3.29) will hold for $\forall T \geq h \geq \delta$. It is important to emphasize, however, that even if u(t) is PE, u.a.s. of the zero solution of (3.1) is guaranteed if (3.20) holds. The implication then is that u(t) must have a dominant spectrum at those frequencies where $Re[\hat{H}(j\omega_m)] > 0$. Thus, we can view (3.20) as a <u>generalized positivity condition</u> on the operator H.

Condition (3.20) is significantly weaker than the usual positivity conditions on H. For example, a strictly proper transfer function $\hat{H}(s)$ is strictly positive real (SPR) if it is exponentially stable and $\exists$ constant $\rho > 0$ such that [16]:

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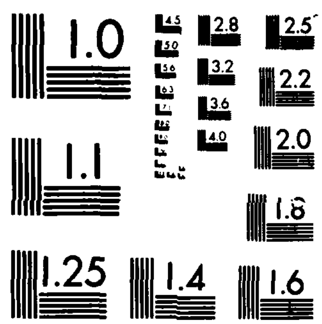

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$$\mathrm{Re}\,[\hat{H}(j\omega)] \geq \rho|\hat{H}(j\omega)|^2, \qquad \forall\omega \in R_+ \tag{3.32}$$

This condition must hold at every frequency, whereas (3.20) requires $\mathrm{Re}\,[\hat{H}(j\omega_m)] > 0$ at those discrete frequencies in R_+ where the magnitude of the input spectrum is large. Conversely, at those frequencies in R_+ where $\mathrm{Re}[\hat{H}(j\omega_m)] < 0$, the magnitude of the input spectrum should be small. Since (3.25) will fail if $\mathrm{Re}\,\hat{H}(j\omega) < 0$, $\forall\omega \in R_+$, it follows that $\mathrm{Re}\,\hat{H}(j\omega) > 0$ at some frequencies, hence, the motivation to refer to (3.20) as a positivity condition.

Although condition (3.20) is weaker than condition (3.32), we do pay the Piper. Suppose $\hat{H}(s)$ is SPR and (3.32) holds. If $u(t)$ is persistently exciting than Theorem (3.11) states that the zero solution of (3.1) is u.a.s. for sufficiently small $\varepsilon > 0$. However, from other arguments (see e.g. [1]) we know that under these same conditions the zero solution of (3.1) is u.a.s. for all $\varepsilon > 0$. Thus, Theorem (3.11) is conservative in this case. However, when $\hat{H}(s)$ is not SPR but (3.32) holds at some frequencies, Theorem (3.11) is now applicable whereas the results in [1] do not apply. In fact in this latter case when ε gets too large the zero solution of (3.1) can be unstable, even if (3.20) holds. For example, if in (3.1) $u(t) = \sin(0.35t)$ and $\hat{H}(s) = 1/(s^2 + 2s + 2)$ then condition (3.20) is satisfied. The simulations in Figure 3.1 with $\theta(0) = 1$ show that the zero solution is u.a.s. for $\varepsilon = 4$ but is completely unstable for $\varepsilon = 8$.

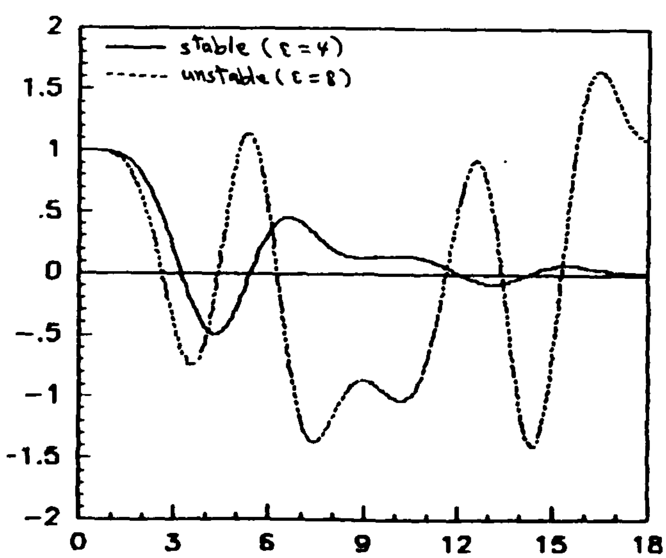

Fig. 3.1

4. EFFECT OF UNMODELED DYNAMICS

In this section we will consider the system

$$\dot{\theta} = -\varepsilon\, u(t)\, H(u(\cdot)'\theta(\cdot)), \qquad \theta(0) \in R^p \tag{4.1}$$

where H as before is linear with stable transfer function $\hat{H}(s)$. In addition we assume that $\hat{H}(s)$ has the decomposition

$$\hat{H}(s) = \hat{H}_o(s) + \hat{\Delta}(s) \tag{4.2}$$

where $\hat{H}_o(s)$ is SPR, i.e., $\exists\ \rho > 0$ such that

$$\mathrm{Re}\ \hat{H}_o(j\omega) \geq \rho |\hat{H}_o(j\omega)|^2, \qquad \forall \omega \in R_+ \tag{4.3}$$

and where $\hat{\Delta}(s)$ represents unmodeled dynamics such that

$$|\hat{\Delta}(j\omega)| \leq \delta_H(\omega), \qquad \forall \omega \in R_+ \tag{4.4}$$

We also assume that u(t) satisfies the conditions of Theorem (3.11) in that $\exists\ T > 0$ such that u(t) has the piece-wise Fourier series representation

$$u(t) \sim \sum_{m \in R} \alpha_k(\omega_m) e^{j\omega_m t}, \qquad kT \leq t \leq (k+1)T \tag{4.5}$$

where $\omega_m = 2\pi m/T$ and $k \in Z_+$. We also decompose

$$\alpha_{kT}(\omega_m) = \overset{\circ}{\alpha}_{kT}(\omega_m) + \beta_{kT}(\omega_m), \qquad \forall m,k \in Z \tag{4.6}$$

where $\overset{\circ}{\alpha}_{kT}(\omega_m)$ is due to predetermined inputs and $\beta_{kT}(\omega_m)$ is due to disturbances bounded by

$$|\beta_{kT}(\omega_m)| \leq \delta_u(\omega_m), \qquad \forall m \in Z_+ \tag{4.7}$$

Hence, the functions $\omega \to \delta_H(\omega)$ and $m \to \delta_u(\omega_m)$ represent, respectively, bounds on the effect of unmodeled dynamics in $\hat{H}(s)$ and unknown elements of $u(t)$ as a function of frequency. Combining the above assumptions with Theorem (3.11) and using (3.25) gives:

<u>Lemma</u> (4.8)

The zero solution of (4.1) is u.a.s. if $\epsilon > 0$ is sufficiently small and if $\exists\ \alpha > 0$ and $T > 0$ such that $\forall k \in Z_+$,

$$\lambda\{\sum_{m\epsilon Z_+} \rho|\hat{H}_\circ(j\omega_m)|^2 \operatorname{Re}[X^\circ_{kT}(\omega_m)]\} \geq$$

$$q_{kT} + \alpha + 2(a/b^2)||u||_\infty/T \tag{4.9}$$

where

$$q_{kT} = \sum_{m \in Z_+} (\delta_H(\omega_m) [\, |X^{\circ}_{kT}(\omega_m)| + \mu_{kT}(\omega_m) |\hat{H}_o(j\omega_m)| \tag{4.10}$$

with

$$\begin{aligned} X^{\circ}_{kT}(\omega_m) &= \alpha_{kT}(\omega_m)\, \alpha_{kT}(\omega_m)^* \\ \mu_{kT}(\omega_m) &= \delta_u(\omega_m)[\delta_u(\omega_m) + |\alpha^{\circ}_{kT}(\omega_m)|] \end{aligned} \tag{4.11}$$

Discussion

Condition (4.8) shows that the dominant excitation must act in the frequency range where $\hat{H}(s) \approx \hat{H}_o(s) \in$ SPR. Moreover, there must be enough excitation and positivity in Re $\hat{H}_o(j\omega)$ in this range to overcome initial conditions (the 1/T term) and the effect of unmodeled dynamics and unknown disturbances (the q_{kT} term). Typically, the disturbances and unmodeled effects occur at high frequencies and the known efforts in $\hat{H}_o(s)$ and $\alpha^{\circ}_{kT}(\omega_m)$ at low frequencies. For example, if there is a frequency ω_c such that

$$\begin{aligned} &\alpha^{\circ}_{kT}(\omega),\ \hat{H}_o(j\omega) \text{ small for } \omega >> \omega_c \\ &\delta_H(\omega),\ \delta_u(\omega) \text{ small for } \omega << \omega_c \end{aligned} \tag{4.12}$$

then condition (4.8) holds if $\forall k \in Z_+$,

$$\lambda\{ \sum_{\omega_m \le \omega_c} \rho |\hat{H}_o(j\omega_m)|^2 \ \mathrm{Re}[X^{\circ}_{kT}(\omega_m)]\} >> \sum_{\omega_m \ge \omega_c} \delta_H(\omega_m)\delta^2(\omega_m) \tag{4.13}$$

Observe that robustness conditions (4.8) or (4.13) are dependent on the input signal spectrum as as well as the unmodeled dynamics. In non-adaptive linear systems the robustness conditions only involve system dynamics.

5. DISCRETE-TIME SYSTEMS

In this section we state the discrete-time versions of Lemma (2.2), Theorem (5.2) and Theorem (2.13) for the linear homogeneous difference equation

$$x(t+1) = (I + \varepsilon A(t))\, x(t), \qquad t \in Z_+ \tag{5.1}$$

The results are identical to the continuous-time results, and the same comments apply mutatis mutandis. The proof of the following Lemma is in the appendix.

Lemma (5.2)

Suppose in (5.1) that $A(\cdot) \in \ell_\infty^{nxn}$. Then $\forall s,\tau \in Z_+$, the transition matrix $\phi(s+\tau,s)$ of (5.1) is given by

$$\phi(s+\tau,s) = I + \varepsilon\tau\, \bar{A}_\tau(s) + R(s,\varepsilon\tau) \tag{5.3}$$

where

$$\bar{A}_\tau(s) = \frac{1}{\tau} \sum_{t=s}^{s+\tau-1} A(t) \tag{5.4}$$

is the local average value of $A(t)$ on the interval $s \leq t \leq s+\tau$, and

$$\|R(\cdot,\ \varepsilon\tau)\|_\infty \leq \begin{cases} (1+\varepsilon\|A\|_\infty)^{\tau} - (1+\varepsilon\tau\|A\|_\infty), & \forall\varepsilon > 0 \\ \frac{1}{2}(\varepsilon\tau\|A\|_\infty)^2, & \forall\varepsilon < 1/\|A\|_\infty \end{cases} \tag{5.5}$$

Remarks

(1) A similar result can be found in [13].

(2) The conditions on $A(\cdot) : Z_+ \to R^{n \times n}$ are weaker than those imposed in the continuous-time version in Lemma (2.2). In the discrete-time case we only need $A(\cdot) \in \ell_\infty^{n \times n}$ whereas in continuous-time $A(\cdot)$ is regulated and bounded.

The following stability result follows immediately from Lemma (5.2).

Theorem (5.6)

Suppose in (5.1) that $A(\cdot) \in \ell_\infty^{n \times n}$ with the sequence of local average values $\{\bar{A}_T(kT),\ k \in Z_+\}$. Then:

(i) If $\exists\ T \in Z_+$ and $\alpha > 0$ such that

$$\mu[\bar{A}_T(kT)] \leq -\alpha, \qquad \forall k \in Z_+ \tag{5.7}$$

then $\exists\ \eta > 0$ such that $\forall \varepsilon T \in (0,\ \eta)$ the zero solution of (5.1) is u.a.s.

(ii) If $\exists\ T \in Z_+$ and $\alpha > 0$ such that

$$\mu[-\bar{A}_T(kT)] \leq -\alpha, \qquad \forall k \in Z_+ \tag{5.8}$$

then $\exists\ \eta > 0$ such that $\forall\ T \in (0, \eta)$ the zero solution of (5.1) is unstable.

If we let $A(\cdot)$ have a (global) average than by applying the same argument in the proof of Theorem (2.13) we obtain the following analogous result.

<u>Theorem</u> (5.9)

Suppose in (5.1) that $A(\cdot) \in \ell_\infty^{nxn}$ with (global) average $\bar{A} \in R^{nxn}$, i.e.,

$$\lim_{T \to \infty} \bar{A}_T(s) = \bar{A} \tag{5.10}$$

uniformly $\forall s \in Z_+$ where $\mathrm{Re}\ \lambda(\bar{A}) \neq 0$. Under these conditions:

(i) If $\exists\ \alpha > 0$ such that

$$\mathrm{Re}\ \lambda(\bar{A}) \leq -\alpha \tag{5.11}$$

then $\exists\ \epsilon_o > 0$ such that $\forall \epsilon \in (0, \epsilon_o)$, the zero solution of (5.1) is u.a.s.

(ii) If $\exists\ \alpha > 0$ such that

$$\max\ \mathrm{Re}\ \lambda(\bar{A}) \geq \alpha \tag{5.12}$$

then $\exists\ \epsilon_o > 0$ such that $\forall \epsilon \in (0, \epsilon_o)$, the zero solution of (5.1) is unstable.

APPENDIX

A. Proof of Lemma (2.2)

Using the Peano-Baker series representation for the transition matrix of (2.1) gives:

$$\phi(s+\tau,s) = I + \varepsilon \int_s^{s+\tau} A(t)dt$$

$$+ \sum_{k=2}^{\infty} \varepsilon^k \int_s^{s+\tau} A(t_1) \int_s^{t_1} A(t_2) \ldots \int_s^{t_{k-1}} A(t_k)dt_1 \ldots dt_k \quad \text{(A.1)}$$

Using definitions (2.3) - (2.4) for $R(s,\varepsilon\tau)$ and $\bar{A}_\tau(s)$, respectively, together with the series expansion for $\exp(\varepsilon\tau\,\bar{A}_\tau(s))$ results in,

$$R(s,\varepsilon\tau) = \sum_{k=2}^{\infty} [-(\varepsilon\tau\,\bar{A}_\tau(s))^k/k!$$

$$+ \varepsilon^k \int_s^{s+\tau} A(t_1) \int_s^{t_1} A(t_2) \ldots \int_s^{t_{k-1}} A(t_k)dt_1 \ldots dt_k] \quad \text{(A.2)}$$

$$\leq 2\sum_{h=2}^{\infty} (\varepsilon\tau||A||_\infty)^k/k!, \quad \forall s \in R_+ \quad \text{(A.3)}$$

$$= (\varepsilon\tau||A||_\infty)^2 \exp(\varepsilon\tau||A||_\infty) \quad \text{(A.4)}$$

since $||\bar{A}_\tau(\cdot)||_\infty = ||A(\cdot)||_\infty$. This proves (2.5).

B. Proof of Theorem (2.9) and Corollary (2.21)

The following inequality holds $\forall M \in R^{nxn}$ and $\forall t \in R_+$ [16]:

$$\exp[-t\mu(-M)] \leq |\exp(tM)| \leq \exp[t\mu(M)]. \tag{B.1}$$

Combining this inequality with (2.3) gives.

$$\exp[-\varepsilon\tau\mu(-\bar{A}_\tau(s)] \leq |\phi(s+\tau,s) - R(s,\varepsilon\tau)|$$

$$\leq \exp[\varepsilon\tau\mu(\bar{A}_\tau(s))], \ \forall s,\tau \in R_+ \tag{B.2}$$

which implies,

$$|\phi(s+\tau,s)| \leq \exp[\varepsilon\tau\mu(\bar{A}_\tau(s))] + r(\varepsilon\tau m) \tag{B.3}$$

$$|\phi(s+\tau,s)| \geq \exp[-\varepsilon\tau\mu(\bar{A}_\tau(s))] - r(\varepsilon\tau m) \tag{B.4}$$

where we have used (2.5) with $||A||_\infty = m$.

We first prove part (i) by using condition (2.10) and inequality (B.3) with $\tau = T$ and $s = kT$. This gives,

$$|\phi((k+1)T,\ kT)| \leq \exp(-\varepsilon T\alpha) + r(\varepsilon Tm), \ \forall k \in Z_+ \tag{B.5}$$

We now need to show that $\theta(kT) \rightarrow \theta((k+1)T)$ is a contraction mapping, i.e., the right hand side of (B-5) is less than one for sufficiently small εT, i.e., $\exists\ \eta > 0$ and $\beta > 0$ such that $\forall\ \varepsilon T \in (0,\ \eta)$,

$$\exp(-\epsilon T\alpha) + r(\epsilon Tm) = \exp(-\epsilon T\beta)$$
$$\exp(-\eta\alpha) + r(\eta m) = 1 \tag{B.6}$$

From the definition of $r(\cdot)$ in (2.5) it is obvious that (B.6) holds. Observe that the expressions in (B.6) appear also in conditions (2.22), (2.23) of Corollary (2.21).

For any t, $s \in R_+$ with $t \geq s$, there exists an integer $k \geq 0$ such that $s + kT \leq t \leq s + (k+1)T$. Thus,

$$\begin{aligned}
|\phi(t,s)| &= |\phi(t,s+kT)\phi(s+kT,s+(k-1)T)\ .\ .\ .\ \phi(s+T,\ s)| \\
&\leq |\phi(t,s+kT)|\exp\ (-\epsilon kT\beta), && \text{by (B.6)} \\
&\leq |\phi(t,s+kT)|\exp\ (-\epsilon(t-s-T)\beta), && \text{by } kT \geq t-s-T \\
&\leq \exp(\epsilon T(m+\beta))\exp\ (-\epsilon(t-s)\beta) && \text{(B.7)}
\end{aligned}$$

by the inequality [16]:

$$|\phi(t,s+kT)| \leq \exp(\int_{s+kT}^{t} \mu[\epsilon A(\tau)]d\tau) \tag{B.8}$$

$$\leq \exp(\epsilon m(t-s-kT)), \qquad \text{by } \mu[A(\tau)] \leq |A(\tau)| \leq m$$

$$\leq \exp(\epsilon mT) \tag{B.9}$$

by $t-s-kT \in (0,T)$. This proves part (i) of Theorem (2.9). Note that (B.7) is the same as (2.23) and, hence, we have also established part (i) of Corollary (2.21).

Using the same techniques, but starting with (B.4), we can also prove parts (ii) of Theorem (2.9) and Corollary (2.21).

C. Proof of Theorem (2.13)

Assumption (2.14) means that $\forall\delta > 0$, $\exists\, T_o(\delta) > 0$ such that $\forall T \geq T_o$,

$$|\bar{A}_T(s) - \bar{A}| \leq \delta, \quad \forall s \in R_+ \tag{C.1}$$

From (B.3), with $||A(\cdot)||_\infty = m$ and $T \geq T_o(\delta)$,

$$|\Phi(s+T,s)| \leq \exp[\varepsilon T\mu(\bar{A} + \bar{A}_T(s) - \bar{A})] + (\varepsilon Tm)^2 \exp(\varepsilon Tm) \tag{C.2}$$

$$\leq \exp[\varepsilon T(\mu(\bar{A}) + \delta)] + (\varepsilon Tm)^2 \exp(\varepsilon Tm) \tag{C.3}$$

Now, choose as a norm on R^n,

$$|x| = (x'Px)^{1/2} \tag{C.4}$$

where $P = P' > 0$ is the solution of the Lyapunov equation,

$$\bar{A}'P + P\bar{A} + 2I = 0 \tag{C.5}$$

Under this norm, the measure of a matrix $M \in R^{nxn}$ becomes,

$$\mu(M) = \max \lambda(M^T P + PM)/2 \qquad (C.6)$$

Hence, (C.3) becomes,

$$|\phi(s+\tau,s)| \leq \exp[-\varepsilon T(1-\delta)] + (\varepsilon TM)^2 \exp(\varepsilon TM) \qquad (C.7)$$

By assumption (2.14) it is always possible to select $T_o(\delta)$ in (C.1) such that $\delta < 1$. By inspection of (C.7), these exists $\varepsilon_o > 0$ such that $\forall \varepsilon \in (0, \varepsilon_o)$, $|\phi(s+T,s)| < 1$, $\forall s \in \mathbb{R}_+$, which completes the proof of part (i). Part (ii) can be proven in an analagous manner starting with (B.4) and using (C.5) with $\bar{A}$ replaced by $-\bar{A}$.

D. Proof of Theorem (2.27)

By the variation of constants formula any solution $x(s+\tau)$, $\forall s,\tau \in R_+$, of (2.26) satisfies,

$$x(s+\tau) = \phi(s+\tau,s)x(s) + \varepsilon \int_s^{s+\tau} \phi(s+\tau,u)\, p(u)\, du \qquad (D.1)$$

where $\phi(s+\tau,s)$ is the transition matrix for $\dot{x} = \varepsilon A(t)x$.

Set

$$f(u) = \int_u^{s+\tau} p(t)dt \qquad (D.2)$$

Hence,

$$f(s) = \tau\, \bar{p}_\tau(s) \qquad (D.3)$$

$$f(s+\tau) = 0$$

where $\tilde{p}_\tau(s)$ is defined in (2.28). Using (D.2) and integrating (D.1) by parts gives,

$$x(s+\tau) = \phi(s+\tau,s)[x(s) + \varepsilon\tau\tilde{p}_\tau(s)] + \varepsilon^2 \int_s^{s+\tau} \phi(s+\tau,u)\, A(u)\, f(u)\, du \qquad (D.4)$$

Using,

$$|\phi(s+\tau,u)| \le \exp\,(\varepsilon(s+\tau-u)||A||_\infty) \qquad (D.5)$$

$$|f(u)| \le \tau||p||_\infty$$

in (D.4) gives,

$$|x(s+\tau) - \phi(s+\tau,s)[x(s) + \varepsilon\tau\tilde{p}_\tau(s)]|$$

$$\le \varepsilon^2\ \tau||A||_\infty||p||_\infty \int_s^{s+\tau} \exp[\varepsilon(s+\tau-u)||A||_\infty]du \qquad (D.6)$$

$$= \varepsilon\tau||p||_\infty\ (\exp(\varepsilon\tau||A||_\infty) - 1)$$

$$\le (\varepsilon\ \tau)^2\ ||A||_\infty\ \exp(\varepsilon\tau||A||_\infty)||p||_\infty,\ \text{by}\ e^\sigma - 1 \le \sigma e^\sigma,\ \sigma \in R_+$$

$$= q(\varepsilon\tau,||A||_\infty)||p||_\infty \qquad (D.7)$$

which establishes (2.28)-(2.30).

E. Proof of Corollary (2.31)

For any $t \in R_+$, $\exists$ integer $k \geq 0$ such that $kT \leq t \leq (k+1)T$.

Hence,

$$|x(t)| = |\phi(t, kT)\, x(kT) + \varepsilon \int_{kT}^{t} \phi(t,\tau)\, p(\tau)\, d\tau| \tag{E.1}$$

$$\leq \exp(\varepsilon Tm)(|x(kT)| + \varepsilon\tau\, ||p||_\infty) \tag{E.2}$$

where $m = ||A||_\infty$. Since (2.32) holds by assumption, it follows that $|\phi(kT,(k-1)T)| \leq \exp(-\varepsilon T\beta)$ where β satisfies (2.35). Combining this with (D.7) for $t=(k+1)T$ gives,

$$|x(kT)| \leq \exp(-\varepsilon T\beta)[|x(k-1)T)| + \varepsilon T||\bar{p}_T||_\infty] + q(\varepsilon T,m)||p||_\infty \tag{E.3}$$

Using (E.3) recursively together with (E.2) and the assumption $x(0) = 0$ gives,

$$|x(kT)| \leq (\varepsilon T \exp(-\varepsilon T\beta)||\bar{p}_T||_\infty + q(\varepsilon T,m)||p||_\infty$$

$$\cdot (1 + \exp(-\varepsilon T\beta) + \dots + \exp(-(k-1)\varepsilon T\beta))$$

$$\leq (\varepsilon T \exp(-\varepsilon T\beta)||\bar{p}_T||_\infty + q(\varepsilon T,m)||p||_\infty)/(1-\exp(-\varepsilon T\beta))$$

Combining this with (E.2) gives the results (2.33), (2.34).

F. Proof of Lemma (3.2)

Using $\hat{H}(s) = d + c'(sI-A)^{-1}b$ we can write (3.1) as

$$\begin{aligned} \dot{\theta} &= -\varepsilon(duu'\theta + uc'x), \quad \theta(0) \in R^p \\ \dot{x} &= Ax + bu'\theta, \quad x(0) = 0 \in R^n \end{aligned} \tag{F.1}$$

We now use the Lyapunov transformation developed in [9] where

$$x = L\theta \tag{F.2}$$

and L satisfies,

$$\begin{aligned} \dot{L} &= AL + bu' + \varepsilon(dLuu' + Luc'L) \\ L(0) &= 0 \in R^{nxp} \end{aligned} \tag{F.3}$$

We will show subsequently that $|L(t)|$ is bounded if ε is sufficiently small. In fact, if ε is not small enough it is possible that $|L(t)| \to \infty$ in finite time. Assume for the moment that $\exists\ \varepsilon_o > 0$ such $\forall \varepsilon \in (0, \varepsilon_o)$, $|L(t)|$ is bounded. Combining (F.1) - (F.3) gives,

$$\dot{\theta} = -\varepsilon\ (duu' + uc'L)\theta \tag{F.4}$$

We can decompose L as follows:

$$L = L_o + \varepsilon L_1 \tag{F.5}$$

where

$$\dot{L}_o = AL_o + bu' \tag{F.6}$$

$$\dot{L}_1 = AL_1 + d(L_o + \varepsilon L_1)uu' + (L_o + \varepsilon L_1)uc'(L_o + \varepsilon L_1) \tag{F.7}$$

Thus, (F.4) becomes

$$\dot{\theta} = -\varepsilon[duu' + uc'L_o + \varepsilon uc'L_1\theta$$

$$= -\varepsilon[u(Hu)' + \varepsilon uc'L_1]\theta \tag{F.8}$$

which is precisely the form in (3.3) with $v = c'L_1$. It remains to prove that $|L(t)|$ is bounded.

By the variation of constants formula, any solution L of (F.3) satisfies,

$$L(t) = L_o(t) + \varepsilon \int_o^t \exp[(t-\tau)A]\; G[\tau,\, L(\tau)]\; d\tau \tag{F.9}$$

where $G(t,L)$: $R_+ \times R^{nxp} \to R^{nxp}$ is defined by,

$$G(t,L) = dLu(t)u(t)' + Lu(t)c'L \tag{F.10}$$

Without loss of generality choose $c \in R^n$ such that $|c| = 1$. Then, $\forall\ \ell > 0$,

$$|L| < \ell \Rightarrow |G(t,L)| < g(\ell)|L| \tag{F.11}$$

where

$$g(\ell) = ||u||_\infty (d||u||_\infty + \ell) \tag{F.12}$$

Since Re $\lambda(A) < 0$ in (F.1), there exists positive constants K and a such that $|\exp(tA)| \leq K \exp(-at)$, $\forall t \in R_+$. For those values of $t \geq 0$ for which $|L(t)| \leq \ell$ we have from (F.11),

$$|L(t)| \leq \ell_o + \varepsilon\, k\, g(\ell) \int_o^t \exp(-a(t-\tau))|L(\tau)|d\tau \tag{F.13}$$

where $\ell_o = ||L_o||_\infty < \ell$. By the Bellman-Gronwall Lemma [16],

$$|L(t)| \leq \frac{\ell_o}{a-\varepsilon K\ g(\ell)} [a - \varepsilon K\ g(\ell) \exp(-(a - \varepsilon K\ g(\ell))t)] \tag{F.14}$$

for all values of $t \geq 0$ for which $|L(t)| \leq \ell$, provided that $a - \varepsilon k\ g(\ell) > 0$. Hence, $|L(t)| \leq \ell$ for all t if

$$\ell_o\, a/[a - \varepsilon k\, g(\ell)] \leq \ell \qquad (F.15)$$

Combining (F.14) and (F.15) gives

$$\varepsilon \leq a\, (\ell - \ell_o)/[K\, \ell ||u||_\infty (d||u||_\infty + \ell)] \qquad (F.16)$$

Choosing $\ell = \ell_*$ where

$$\ell_* = \ell_o\, [1 + (1 + d||u||_\infty/\ell_o)^{1/2}] \qquad (F.17)$$

maximizes the right hand side of (F.16). The results in (3.4) - (3.9) follow. This proves Lemma (3.2).

G. Proof of Theorem (3.11)

Using Lemma (3.2) we have that $\forall \varepsilon \in (0, \varepsilon_o)$, (3.1) is equivalent to,

$$\dot{\theta} = - \varepsilon\, F(t)\theta \qquad (G.1)$$

$$F(t) = u(t)(Hu)(t)' + \varepsilon\, u(t)v(t)$$

with ε_o and $||v||_\infty$ are given in (3.4)-(3.9). Thus, we have the local average,

$$\bar{F}_T(k) = \frac{1}{T} \int_{kT}^{(k+1)T} F(t)dt \tag{G.2}$$

$$= \frac{1}{T} \int_{kT}^{(k+1)T} u(t)(Hu)(t)'dt + \frac{\varepsilon}{T} \int_{kT}^{(k+1)T} u(t)v(t)'dt \tag{G.3}$$

$$= R_T(k) + \frac{1}{T} \int_{kT}^{(k+1)T} u(t)[(Hu)(t)-(Hu_k)_\infty(t)]'dt$$

$$+ \frac{\varepsilon}{T} \int_{kT}^{(k+1)T} u(t)v(t)'dt \tag{G.4}$$

where we have used the expression for $R_T(k)$ in (3.18) and the definition of $(Hu_k)_\infty(t)$ in (3.19). Using assumption (3.12) we will show below that

$$\left|\int_{kT}^{(k+1)T} u(t)[(Hu)(t) - (Hu)_\infty(t)]'dt\right| \le 2(a/b^2)||u||^2_\infty \tag{G.5}$$

Hence, from (G.4),

$$\mu[-\bar{F}_T(k)] \le \mu[-R_T(k)] + 2(a/b^2)||u||^2_\infty/T + \varepsilon\ ||u||_\infty||v||_\infty \tag{G.6}$$

$$\le -\alpha + \varepsilon\ ||u||_\infty||v||_\infty \tag{G.7}$$

by (3.15). Using Theorem (2.9) with $\varepsilon < \alpha/(||u||_\infty||v||_\infty)$ establishes part (i). Part (ii) follows by using (3.17) to overbound $\mu[\bar{F}_T(k)]$.

We now establish the bound in (G.5). For $kT \le t \le (k+1)T$,

$$(Hu)(t) = \int_0^t h(t-\tau)u(\tau)d\tau$$

$$= \int_0^t h(t-\tau)u_k(\tau)d\tau + \sum_{r=0}^{k-1} \int_{rT}^{(r+1)T} h(t-\tau)u_r(\tau)d\tau$$

(by definition (3.17) for $u_k(t)$)

$$= \int_0^{t-kT} h(\tau)u_k(t-\tau)d\tau + \sum_{r=0}^{k-1} \int_{t-(r+1)T}^{t-rT} h(\tau)u_r(t-\tau)d\tau$$

$$= (Hu)_\infty(t) - \int_{t-kT}^{\infty} h(\tau)u_k(t-\tau)d\tau + \sum_{r=0}^{k-1} \int_{t-(r+1)T}^{t-rt} h(\tau)u_r(t-\tau)d\tau$$

by definition (3.19). Thus, from (3.12) we have

$$|(Hu)(t) - (Hu)_\infty(t)| \leq ||u||_\infty \, (a/b) \, [e^{-b(t-kT)} + \sum_{r=0}^{k-1} (e^{-b(t-(r+1)T)} - e^{-b(t-rt)})]$$

Using this expression in the left hand side of (G.5) gives the upper bound,

$$\int_{kT}^{(k+1)} |u(t)|\cdot|(Hu)(t) - (Hu)_\infty(t)|dt \leq ||u||_\infty^2 (a/b^2)(1-e^{-bT})$$

$$\cdot[1 + (e^{bT}-1) \sum_{r=0}^{k-1} e^{-(k-r)bT}]$$

$$= ||u||_\infty^2 (a/b^2)(1-e^{-bT})\{1+(e^{bT}-1)[\ 1 + \frac{1 - e^{-(k+1)bT}}{1 - e^{-bT}}\ ]\}$$

(by the geometric series formula)

$$= 2\ ||u||_\infty^2(a/b^2)(1-e^{-bT})(1-e^{-kbT})$$

$$\leq 2||u||_\infty^2(a/b^2)$$

which establishes (G.5) and completes the proof.

H. Proof of Lemma (5.2)

The transition matrix for (5.1) is given by

$$\phi(s+\tau,\ s) = \prod_{t=s}^{s+\tau-1} (I + \varepsilon\ A(t)) \tag{H.1}$$

$$= I + \varepsilon \sum_{t=s}^{s+\tau-1} A(t) + \varepsilon^2 \sum_{t_1=s}^{s+\tau-1} A(t_1) \sum_{t_2=s}^{t_1} A(t_2) + \ .\ .\ .$$

$$+ \varepsilon^\tau A(s+\tau-1)\ .\ .\ .\ A(s)$$

$$= I + \varepsilon\tau\bar{A}_\tau(s) + R(s,\ \varepsilon\tau) \tag{H.2}$$

by definitions (5.3) and (5.4). Hence, using the binomial series formulae we have,

$$|R(s,\ \varepsilon\tau)| \leq \frac{\tau(\tau-1)}{2!}(\varepsilon||A||_\infty)^2 + \frac{\tau(\tau-1)(\tau-2)}{3!} + (\varepsilon||A||_\infty)^3$$

$$+ \ldots + (\varepsilon ||A||_{\infty})^{\tau} \tag{H.3}$$

$$= (1 + \varepsilon||A||_{\infty})^{\tau} - (1 + \varepsilon\tau||A||_{\infty}) \tag{H.4}$$

$$\leq \frac{1}{2}(\varepsilon\tau||A||_{\infty})^2 \ , \ \forall\varepsilon < 1/||A||_{\infty}. \tag{H.5}$$

The last inequality follows by Lagrange's remainder theorem. This proves the Lemma.

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