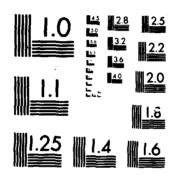
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## MARCHING DIFFERENCE SCHEMES FOR ILL-POSED INITIAL VALUE PROBLEMS

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## FINAL REPORT

by

Alfred S. Carasso, Principal Investigator

November 30, 1985

U.S. ARMY RESEARCH OFFICE

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Center for Applied Mathematics National Bureau of Standards Gaithersburg, Maryland 20899

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## Introduction

This final report summarizes work accomplished under Funding Document MIPR ARO 63-82 during the period July 16, 1982 through September 30, 1985. Six research manuscripts were completed and submitted for publication during this period, with copies of each manuscript transmitted to ARO at the time of submission. These six research papers are listed below under items one through six, together with a summary of the principal results obtained.

It is noteworthy that all of the research described in this report traces its mathematical ancestry to the problem of linear inverse heat transfer in gun barrels, first studied under previous ARO sponsorship (DAA629-78-G-0091). After successfully extending the computational algorithm to the nonlinear case of temperature-dependent thermal properties, (see item 1 below), it was quickly realized that the mathematical theory of the linear ill-posed heat transfer problem spilled over, and was germane to several important areas of nondestructive evaluation unrelated to heat conduction. Such problems include for example, the detection of imperfections in optical fiber waveguides, or the location of flaws in metallic plates by ultrasonic testing. Typical experimental work in these areas proceeds via the determination of the "impulse response" of the system. A pulsed signal is used to interrogate the "black box" and the output waveform is recorded. Careful deconvolution of the output waveform reveals singularities in the system's impulse response which can then be related to flaws. Such ill-posed reconstruction problems occur in many areas of measurement science, and are of potential significance in Army Electronics and Materials research. When the probe pulse has the shape of an "inverse Gaussian", the resulting deconvolution problem is mathematically equivalent to the linear inverse heat transfer problem mentioned above. Using ideas from advanced probability theory, a way was found to relate a very rich variety of probe waveforms to "generalized" inverse heat transfer problems. In this way, the research on the gun barrel problem evolved into a powerful systematic technique for pulse probing and characterization of linear time invariant systems. This theory is currently being successfully applied in basic experimental research work in Ultrasonics, at the National Bureau of Standards.

## Scientific Personnel Supported by this Project

Alfred S. Carasso, Principal Investigator. No advanced degrees awarded.

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 A. S. Carasso, "Nonlinear Inverse Heat Transfer in Gun Barrels," <u>Transactions of the First Army Conference on Applied Mathematics and</u> <u>Computing</u>, pp. 325-353, ARO Report 84-1 (1984), U.S. Army Research Office, P.O. Box 12211, Research Triangle Park, North Carolina 27709.

This paper documents an important computational experiment on a nonlinear inverse heat transfer problem whose exact solution is known. We consider the problem of determining the temperature and temperature gradient histories at the inside wall of a gun barrel, from embedded thermocouple measurements at various points in the annular region between the inner and outer radii of the cannon. A fictitious nonlinear heat conduction problem was created so as to have an exact solution simulating conditions presumed to exist in a 155 mm cannor. The relevant parameters were made available by the U.S. Army Ballistics Research Laboratory. In this simulation, the temperature at the wall rises from 300°K to almost 1000°K in the first 10 milliseconds, while the postulated temperature-dependent conduction coefficient undergoes a 280 percent change. A regularized explicit step by step marching procedure was devised to solve the nonlinear inverse problem, based on integrating the heat equation as an initial-value problem in the x-variable. Recorded temperature histories are assumed given at two thermocouple locations and are used to create the necessary initial data for the inverse calculation. Detailed comparisons are provided between the exact and estimated histories at the wall. These comparisons clearly indicate the above technique to be a reliable tool in such calculations. Similar marching procedures can be applied to other ill-posed initial value problems.

2. A. S. Carasso and N.N. Hsu - "The Inverse Gaussian Pulse in the Experimental Determination of Linear System Green's Functions," <u>Transactions of the Second Army Conference on Applied Mathematics and</u> <u>Computing</u>, pp. 389-404, ARO Report 85-1 (1985), U.S. Army Research <u>Office</u>, P.O. Box 12211, Research Triangle Park, North Carolina 27709.

This report considers the problem of determining the impulse response of a linear time-invariant system by probing the system with a specifically designed approximation to the Dirac  $\delta$ -function. The use of an "inverse Gaussian" probe pulse is advocated because of the connection with heat conduction. This connection can be exploited to approximately solve the resulting deconvolution problem, under weak a-priori assumptions on the data noise and the unknown system response. It is pointed out that such weak assumptions are necessary in many problems where the impulse response is neither positive nor differentiable. Computational reconstructions are presented in the context of elastic Green's functions, using a step by step marching procedure.

3. A. S. Carasso and N. N. Hsu - "Probe Waveforms and Deconvolution in the Experimental Determination of Elastic Green's Functions," <u>SIAM J. Appl.</u> Math., 45 (1985), pp. 369-383.

This paper contains a detailed discussion and elaboration of the ideas presented under item 2 above, as is appropriate for a paper in a refereed mathematical journal. The concepts of partial and continuous deconvolution are introduced and tied to the possibility of obtaining rigorous error bounds, in the L<sup>2</sup> norm, under weak a-priori assumptions. Such notions are only possible when the integral operator can be embedded in a semi-group. Several computational experiments are presented. The last section of the manuscript, "Concluding Remarks", emphasizes the advantages of using a pulse which is intimately related to heat conduction; in particular, the advantages of a marching calculation are stressed.

4. A. S. Carasso and N. N. Hsu - "L<sup>∞</sup> Error Bounds in Partial Deconvolution of the Inverse Gaussian Pulse," SIAM J. Appl. Math., 45 (1985). In Press.

This paper represents an important new contribution to the inverse Gaussian pulse theory. The concept of partial deconvolution introduced in items 2 and 3 above, is here shown to be especially significant from the standpoint of error bounds. Under weak  $L^2$  a-priori assumptions on the data noise and the unknown system response, error estimates in the L<sup>∞</sup> norm are obtained for the partial deconvolution and any of its derivatives. Such strong estimates are not generally known in the literature on ill-posed problems and/or deconvolution. However, error bounds in the L<sup>∞</sup> norm are especially valuable when the unknown response contains sharp features such as spikes, cusps, jumps and the like. The pointwise reliability of the partial deconvolution as the L<sup>2</sup> noise level tends to zero, assures the user of the genuineness of any developing sharp characteristics, provided the L<sup>2</sup> noise level is sufficiently low. Under the usual L<sup>2</sup> error bounds, pointwise reliability is open to question. In this paper, the roles of heat conduction and the associated analytic semi-group are apparent and are reflected in the above strong estimates.

5. A. S. Carasso - "Infinitely Divisible Pulses, Continuous Deconvolution, and the Characterization of Linear Time Invariant Systems," Submitted for publication.

This lengthy manuscript contains a far reaching generalization of the inverse Gaussian theory using tools from advanced probability theory. In addition, striking computational experiments are presented on reconstructing highly singular elastic Green's functions from smooth synthetic data in the presence of noise. The motivation for this extension stems from the need to provide a systematic theory which allows for perturbations of the inverse Gaussian. Such perturbations and distortions generally introduce wiggles and originate from interfacing devices such as power amplifiers, transducers, and transient recorders. The work of Paul Levy on stable laws and infinitely divisible probability density functions provides the framework for this extension. A rich variety of pulse shapes is isolated, the class D; this class includes quite complicated wiggly pulses which in addition to being one-sided probability densities, are C∞ functions on the whole line. It is shown that pulses in class D can be associated with an inverse heat flow problem in which the governing equation is a parabolic pseudo-differential equation in two independent variables. Fourier and Laplace transform techniques are then used to construct an efficient computational algorithm. Most important, the concepts of partial and continuous deconvolution remain valid for pulses in class D, along with L<sup> $\infty$ </sup> error estimates under L<sup>2</sup> a-priori constraints. The paper integrates diverse ideas from probability theory, integral equations and partial differential equations, and should prove to be a significant contribution to the literature on deconvolution. This manuscript is currently being evaluated for possible publication in the SIAM Journal on Applied Mathematics.

6. A. S. Carasso - "Deconvolution in Slow Motion and the Experimental Determination of the Impulse Response of a Linear Time Invariant System." To appear in the Proceedings of the IMA Conference on Mathematics in Digital Signal Processing, University of Bath, United Kingdom, September 1985; The Institute of Mathematics and Its Applications, Southend-on-Sea, Essex, United Kingdom, Oxford University Press.

The contents of this paper were presented at an international conference on digital signal processing, sponsored by the British Intitute of Mathematics and its Applications. The audience consisted mostly of electrical engineers, physicists and statisticians, from governmental and industrial research laboratories, representing almost all Western European countries as well as the U.S.A., Canada, and Australia. Accordingly, the presentation took the form of a survey of the main results contained in items 2-5, designed to give an overview of the theory and steer the reader through the detailed mathematical developments in those papers. The presentation was extremely well-received, and the concepts of partial and continuous deconvolution elicited a warm response. This manuscript is currently being evaluated for possible publication in the Proceedings of the above conference.

