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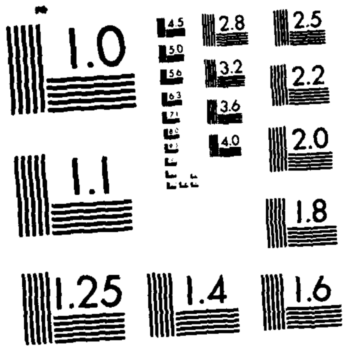

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During the period covered by this grant the principal investigator wrote 17 papers Titles include: "Scanning Control of a Vibrating String", "Dynamic Phase Transitions in a Van Der Waals Fluid", "The Viscosity-capillarity Admissibility for Shocks and Phase Transitions", "Lax-Friedrichs and the Viscosity-capilarity Criteria" and "Temporal and Spatial Chaos in a Van Der Waals Fluid Due to Periodic Thermal Fluctuations."

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FINAL PROGRESS REPORT AFOSR-81-0172

Non-linear linear systems in infinite dimensional state spaces

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In his research under AFOSR-81-0172 M. Slemrod has been involved in two main avenues of research. This first has been in nonlinear control problems for distributed parameter systems. ~~The second has been in~~ nonlinear continuum mechanics and related partial differential equations.

1. <u>Nonlinear control problems</u>

In joint work with John Ball and Jerald Marsden [1] I discussed the problem of bilinear control for a distributed parameter system. We formulated the problem as follows. Find p(t) a real valued scalar control which will drive the system

$$\frac{du}{dt} = Au + p(t)\ Bu$$

from $u(0)=u_0$ to $u(T)=u_1$. This a problem in <u>controllability</u>. We found that while in an infinite dimensional Hilbert space one cannot in general find such p(t) one can find p(t) if u_1 is restricted to a dense subspace of H. As an example illustrating our theory we showed that the vibrating beam equation

$$w_{tt} + w_{xxxx} + p(t)w_{xx} = 0$$

$$w = w_{xx} = 0 \text{ at } x = 0,1$$

$$w(x,0) = f(x), \quad w_t(x,0) = g(x)$$

(w,w_t) can be steered to a dense set of the Hilbert space

$H^2(0,1) \cap H_0^1(0,1) \times L^2(0,1)$ in finite time.

I continued this work on my own in [2] where I weakened some of the assumptions originally made in [1].

In his Ph.D thesis (completed June, 1985) E.L. Rogers studied feedback control of other bilinear systems. In this work our infinite dimensional partial differential equations was coupled to an ordinary differential equation making the system hybrid. He showed the stabilizabilty of such systems. This work will appear as a joint publication in the Quarterly of Applied Mathematics [3].

In another paper [4] I considered boundary feedback stabilization for the quasi-linear wave equation. In this problem (which models one dimensional elastic motion) one tries to find a feedback which yields the rest state of

$$w_{tt} = \sigma(w_x)_x$$
$$w(0,t) = 0$$
$$w_x(L,t) = f(t)$$

asymptotically stable. Here f(t) is a real valued control. The interest here of course is the fact that the system is highly nonlinear and special methods must be used.

In collaboration with J.R. McLaughlin of R.P.I. I wrote a paper on scanning controls for distributed systems. In particular we considered the problem of finding a controls which stabilize the wave equation

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2} + Ry + \sum_{i=1}^{N} \phi\left[x-\gamma_i(t)\right]y(x,t) ,$$

$$y = 0 \quad \text{at } x = 0,1 \quad .$$

Here $\gamma_i(t)$ are N real valued controls. We gave some interesting conditions relating ϕ and R which guarantee such stabilizing controls. This work will appear in [5].

Finally after considering some work of J. Hubbard of M.I.T. I decided to consider the problem of feedback stabilization of

$$\frac{du}{dt} = Au + Bf$$

in an infinite dimensional Hilbert space under the restriction $||f(t)|| \leq 1$. I found the theory of nonlinear semigroups of contractions applied nicely and one could find the desired control. I applied the theory to Hubbard's beam problem giving a correct analysis of a problem which he analyzed incorrectly. This work will appear in [6].

2. Nonlinear Continuum Dynamics

In this work I have basically considered the dynamics of the equations of gas dynamic under the assumption that the constitive equation for stress is given by the van der Waals equation of state

$$p(w,T) = \frac{RT}{w-a} - \frac{b}{w^2} \quad , \qquad a,b>0 \text{ constants} .$$

Here T is the absolute temperature, w = specific volume = $(\text{density})^{-1}$ and the stress = $-p(w,T)$. This work also can be applied to elastic solid when one takes w=strain. For example the isothermal inviscid balance of linear momentum yields the partial differential equations

$$\begin{aligned} v_t + p(w,T)_x &= 0 \\ w_t - v_x &= 0 \end{aligned} \qquad (2.1)$$

where we keep T fixed for the simple isothermal case. Since the above choice of p has both $p'<0$ and $p'>0$ for T sufficiently small these partial differential equations yield a mixed hyperbolic-elliptic initial value problem.

I have attempted to understand this initial value problem in several papers. My main contribution so far has been to put forward a new admissibility criterion which hopefully picks out the physically relevant solutions for the above system. This is important since weak solutions of quasi-linear equations are well known not to possess unique solutions. The main ideas is that the "good" solutions of (2.1) should be limits of a more "exact" system which contains both viscous and capillarity terms. This work has appeared in [7], [8], [9], and R. Hagan's Ph.D. thesis which appeared as [10].

In related work I have used the above ideas to show the Lax-Friedrichs finite difference scheme to be a reasonable method to solve (2.1) numerically [11],[12]. Also I have shown how chaos may occur in (2.1) under the assumption of T is spatially or temporally periodic (with J.E. Marsden [13]).

With M.E. Gurtin and J. Carr [14],[15] I considered a related equilibrium problem for solving the minimization problem

$$\begin{aligned} &\int_0^L \epsilon w''(x)^2 + W\left[w'(x)\right]dx \\ &\int_0^L w(x)dx = M \end{aligned} \qquad (2.2)$$

$w_x=0$ at $x=0,L$. Here W is the primitive of $-p$ and the ϵ term denotes the inclusion of the above mentioned capillarity term. We gave information on the nature of solutions of (2.2) and in fact an elegant rigorous estimate of the total mechanical energy as an asymptotic expansion in ϵ.

Finally with V. Roytburd, I am considering the existence of solutions to the initial value problem for (2.1). We are trying to apply Murat-Tartar's method of compensated compactness. We have put the results obtained so far in papers [16], [17]. We are still working on the problem.

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