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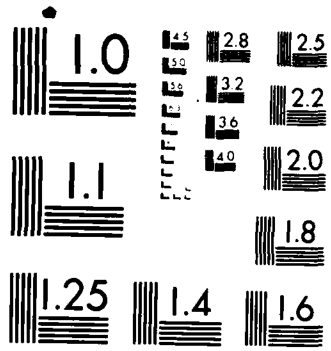

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NAVAL UNDERWATER SYSTEMS CENTER
NEW LONDON LABORATORY
NEW LONDON, CONNECTICUT 06320

Technical Memorandum

EXTRACTING SIGNAL INFORMATION FROM CONTAMINATED DATA WITH A TWO-DIMENSIONAL ARRAY

Date: 5 April 1985

Prepared by: Roger F. Dwyer
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Surface Ship
Sonar Department

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ABSTRACT

The use of sensors configured into various geometrical shapes called arrays are important in extracting signal information in many applications. This paper explores the effectiveness of a two-dimensional or planar array in extracting signal information from digitized data when additive Gaussian noise and additive impulsive interference are present. Based on the likelihood ratio approach the data output of each sensor is first passed through a zero-memory device. The zero-memory device is either a nonlinearity or linearity depending on whether the interference is present or not present, respectively. Further processing utilizes a receiver which converts the data at the output of each zero-memory device into frequency components using a discrete-Fourier transform. The frequency components are then weighted and summed over all spatial sensors. The weights are chosen by maximizing a defined performance measure. It is shown that this optimization procedure does not take into account the debilitating effects of impulsive interference. In order to treat theoretically the inclusion of a nonlinearity, to combat impulsive interference, the concept of an ideal nonlinearity is introduced.

ADMINISTRATIVE INFORMATION

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I. INTRODUCTION

In many applications an array of sensors is employed to extract useful information from a signal propagating in a medium. The information to be extracted may be the temporal, spatial, or spectral properties of the signal. Recently, the probability distribution information of received data were also shown to be of interest [1]. It seems natural to assume that the optimum processor would be configured based on the desired information to be extracted. One approach to design an optimum processor would be to specify, a priori , the information to be extracted from an expected signal and configure a receiver accordingly. If the signal is embedded in noise this complexity must also be taken into account. In this paper an array processor is defined and then optimized by choosing certain unspecified parameters by maximizing a performance measure.

The array of interest is configured as a rectangle. Although the initial mathematical details do not require an array configuration to be specified, the performance results do. Therefore, the array configuration is specified at the outset. As shown in figure (1), there are M sensors in each horizontal row and N sensors in each vertical column. In the paper, all mathematical relationships will be with respect to the top left corner sensor and cartesian coordinates will be utilized. Examples will be given for a plane wave arriving in the plane of the array.

In environments where the non-Gaussian noise, limiting signal detection, is impulsive theroetical predictions suggest that detection performance can be significantly improved by incorporating nonlinear zero-memory devices as preprocessors to the standard receiver optimum for Gaussian processes. However, in order to derive optimum tractable results from a likelihood ratio formulation somewhat restrictive assumptions are needed. Nevertheless, the usefulness of nonlinear preprocessors are clearly indicated in these analyses [2, 3, 4, 5, 6, 7]. A processor was discussed in references [8] which utilized a nonlinear preprocessor based on the estimation of quantiles [2]. This paper discusses the performance of a two-dimensional array processor which incorporates either a linearity or an ideal nonlinearity [9] at each sensor as a preprocessor.

The concept of an ideal nonlinearity is introduced in the time-domain to facilitate deriving the theoretical performance of the resultant receiver in the frequency-domain. A similar problem was discussed by Martin and Thompson [10]. They introduced a technique to improve spectrum estimation of a signal embedded in impulsive noise. But they did not theoretically evaluate its performance. The advantage of introducing an ideal nonlinearity is that this concept allows a theoretical performance prediction to be derived.

From its nature impulsive interference tends to produce a broad spectrum. For this type of process classical filtering methods are ineffective if the signal is contained in the same band as the impulsive interference. However, based on the principle of the likelihood ratio, under the assumption of independent and identically distributed samples, the optimum processor to combat impulsive noise is one which utilizes nonlinearities as preprocessors [2, 3, 4, 5, 6, 7]. In this paper several examples are presented which clearly show the benefit of using nonlinearities as preprocessors over classical filtering when impulsive interference limits signal detection. The impulsive interference is modeled based on measurements from a certain physical phenomenon which is shown later.

Before presenting the processing required for a two-dimensional array, frequency-wavenumber spectrum estimation for a line array will be discussed. This will reveal some advantages of a two-dimensional array over a line array and also justify the array processor to be defined in the next section.

Consider the coordinate system shown in figure (2). Distributed in the plane are several sensors. Their positions are given by the vectors $\vec{r}_J$, J = 1, 2, ..., N. In addition a plane wave is propagating in the plane at a velocity and direction described by the vector $\vec{\alpha}_0$. This vector is called the slowness vector [11, 12, 13, 14]. It points in the direction of propagation and has a magnitude equal to the reciprocal of the velocity of propagation, ie, $|\vec{\alpha}_0| = 1/v_0$, where v_0 is the velocity of propagation of the plane wave. The wavenumber is related to $\vec{\alpha}_0$ through the formula, $\vec{K}_0 = 2\pi f\vec{\alpha}_0$, where f is frequency.

The data received at the J-th sensor is given by

$$x(t, J) = s(t + \vec{\alpha}_0 \cdot \vec{r}_J)$$

where t represents time and $\vec{\alpha}_0 \cdot \vec{r}_J$ is the scalar product [15].

The information needed in this pap-- is contained in the cross-correlation function

$$E[x(t_1, J_1)\, x(t_2, J_2)] = E[s(t_1 + \vec{\alpha}_0 \cdot \vec{r}_{J_1})\, s(t_2 + \vec{\alpha}_0 \cdot \vec{r}_{J_2})]$$

$$= R_s[t_1 - t_2 + \vec{\alpha}_0 \cdot (\vec{r}_{J_1} - \vec{r}_{J_2})]$$

where it is assumed that the sensor output is space stationary [12, 16], E represents expectation, and $R_s[\ ]$ is the autocorrelation function of the signal.

If the sensors are located only in the x-direction then,

$$E[x(t_1, J_1)\, x(t_2, J_2)] = R_s[t_1 - t_2 - d \sin\theta / v_0]$$

where d is the separation of the sensors and θ is the arrival angle of the plane wave with respect to the array.

In this paper it is assumed that the output of each sensor is available in sampled form and the uniform sampling interval is one. In addition all time delays will be integral multiples of the sampling interval.

Given a line array, i.e., sensors located in one-dimension only, the frequency-wavenumber spectrum estimate for the discrete case can be defined as a two-dimensional discrete Fourier transform (DFT) as follows [12, 17, 18, 19, 20],

$$A(\lambda, K) = (1/\sqrt{N_o}) \sum_{J=o}^{N-1} \sum_{i=o}^{N_o-1} X(i, J)\ e^{-ji\lambda}\ e^{-jJK} \tag{1}$$

where, $j = \sqrt{-1}$, $\lambda = 2\pi r/N_o$ is the normalized frequency [11], $r = 0, 1, 2, \ldots, N_o-1$, and K is the normalized spatial frequency. Here, the coordinate system is defined with the line array along the x-axis and the first sensor at the origin.

The normalized spatial frequency is represented as

$$K = 2\pi\ K'/N,$$

where K' is defined as the discrete wavenumber component with respect to the array, and

$$\begin{aligned} K'/N &= (r/N_o)\ (d/c)\ \sin \theta_{K'} \\ &= d_r\ \sin\theta_{K'} \end{aligned}$$

where d_r is the sensor separation measured in wavelengths and c is the speed of propagation of the wavefront. Therefore the arrival angle is given by

$$\theta_{K'} = \sin^{-1}\left(\frac{K'}{d_r N}\right)$$

and the following relationship must hold

$$-1 \leq \frac{K'}{d_r N} \leq 1 \tag{2}$$

From the property of the DFT both positive and negative components, ie, frequencies or spatial frequencies, can be defined. But negative frequencies are redundant. However, both positive and negative spatial frequencies, associated with positive frequencies are meaningful. Therefore, K' will take on both positive and negative values according to equation (2).

An example of the frequency-wavenumber spectrum is shown in figure (3). This type of presentation of the frequency-wavenumber spectrum has been considered in reference [21]. A two-dimensional fast Fourier transform (FFT) was employed to produce the plot in the figure. Specifically a 128 point FFT and 64 point FFT were used to generate the frequencies and wavenumbers, respectively.

One disadvantage of the line array is that its beamwidth at end fire ($\Theta = \pm 90°$) is broader than its beamwidth at broadside ($\Theta = 0°$) [22]. This can be seen in figure (3) by counting the number of wavenumber bins for each 30° increment starting at broadside. There are 16 bins from broadside to 30°, 12 bins from 30° to 60°, and only 4 bins from 60° to end fire. The same relationship holds for the negative wavenumbers. In addition, as frequency is reduced the number of bins between the two end fire positions also reduces according to equation (2).

In the next section the array processor for a two-dimensional array will be defined. It will be a generalization of the discrete frequency-wavenumber spectrum estimate discussed above. However, the defined processor is also based on the work developed in the reference [23, 24, 25, 26].

II. ARRAY PROCESSOR

A two-dimensional array of omnidirectional sensors is shown in figure 1. The elements are designated as X_{LJ}, $L = 1, 2, \ldots, N$, $J = 1, 2, \ldots, M$ and the spacing between elements is denoted by d in the horizontal direction and denoted by λ is the vertical direction.

As shown in figure 1, a plane wave propagating in the plane of the array, is coincident with the first sensor and its wavefront makes an angle of Θ degrees with respect to the horizontal direction. The time it takes the wavefront to reach sensor X_{LJ} with respect to X_{11} can be computed from the equation

$$\xi_{LJ} = \lambda(L - 1)\cos\Theta/c \quad + d(J - 1)\sin\Theta/c \tag{3}$$

One advantage of a two-dimensional array over the line array is immediate. For example, let $L = 1$, then

$$\xi_{1J} = d(J - 1)\sin\Theta/c$$

If $\Theta = 0°$ or $180°$ the same value is obtained namely, $\xi_{1J} = 0$. In general there is an ambiguity associated with predicting the direction of arrival, Θ, of a plane wave with a line array. For this case, $\xi_{1J} = d(J - 1)\sin\Theta/c$, the ambiguous angles are denoted by the set $(\Theta, 180° - \Theta)$. For the other case, ξ_{L1}, the ambiguous angles are denoted by the set $(\Theta, -\Theta)$.

These ambiguous angles are completely resolved using equation (3) because an unique set of time delays are obtained for all arrival angles within the plane of the array. However, an ambiguity will again arise in predicting the direction of arrival if the plane wave is propagating out of the plane, say at some angle ϕ. Where, when $\phi = 90°$ the plane wave is propagating within the plane of the array. Then the right side of equation (3) will be multiplied by $\sin\phi$. As is well known, this amibiguity can also be resolved by utilizing a 3-dimensional array. But this generalization will not be needed in this paper. However, this ambiguity may also be resolved by utilizing additional information concerning the possible propagating direction [27].

As stated in the last section the data are assumed in digital form. Let the signal and noise be wide sense stationary, mutually independent and zero-mean processes. The interference will be a zero-mean deterministic process. The data at the output of each sensor are modeled in the form

$$x(i, L, J) = s(i, L, J) + n(i, L, J) + I(i, L, J)$$

$$i = 0, 1, \ldots, N_o-1$$
$$L = 1, 2, \ldots, N$$
$$J = 1, 2, \ldots, M$$

where $n(i, L, J)$ is spatially independent Gaussian noise, $I(i, L, J)$ is impulsive interference propagating from a specific direction which may coincide with the arrival direction of the signal. The interference is modeled as an impulse function of the form

$$I(i,\ L,\ J) = \sum_{k=o}^{w-1} A_k \left\{ \delta[i - (i_g + k\tau + \gamma_{LJ})] - \delta[i - (i_g + 1 + k\tau + \gamma_{LJ})] \right\} \quad (4)$$

where w is the number of impulse functions occurring in the data interval, is the separation of impulse functions, and γ_{LJ} represents the appropriate time delays for the interference. The symbol δ[ ] is the Kronecker delta function, and ig is the temporal location of the first impulse.

The information needed is contained in the cross-correlation function,

$$E[x(i_1,\ L_1,\ J_1)\ x(i_2,\ L_2,\ J_2)]$$

$$= R_s[i_1 - i_2 - (\xi_{L_1 J_1} - \xi_{L_2 J_2})]$$

$$+ R_n(i_1 - i_2)\ \ \delta[L_1 J_1 - L_2\ J_2]^{+}$$

$$+ \sum_{k_1=0}^{w-1} \sum_{k_2=0}^{w-1} A_{k_1}\ A_{k_2} \left\{ \left\{ \delta[i_1 - (i_g + k_1\tau + \gamma_{L_1 J_1})] - \delta[i_1 - (i_g + 1 + k_1\tau + \gamma_{L_1 J_1})] \right. \right.$$

$$\left. \left. X \left\{ \delta[i_2 - (i_g + k_2\tau + \gamma_{L_2 J_2})] - \delta[i_g + 1 + k_2\tau + \gamma_{L_2 J_2})] \right\} \right\} \quad (5)$$

The array processor is defined in the following way [23, 24, 25, 26],

$$\Lambda(\lambda) = \left| \sum_{L=1}^{N} \sum_{J=1}^{M} A_{LJ}(\lambda)\ (1/\sqrt{N_o}) \sum_{i=0}^{N_o-1} X(i,\ L,\ J)\ e^{-j\lambda i} \right|^2 \quad (6)$$

+ The symbol means that the cross terms are zero.

where the symbols are as defined previously. Here we utilize complex weights $A_{LJ}(\lambda)$ which differ from the frequency-wavenumber spectrum estimate discussed in the introduction. The weights will be chosen to maximize a performance measure. But, clearly they should include the proper phase shifts needed to steer the array toward a signal source. If the signal is not a plane wave equation (3) will not give the proper time delays. In this case a search for the appropriate time delays can be made by choosing the weights which maximize a performance measure. This procedure is sometimes called focusing the array [22].

Later we will include a nonlinearity in equation (5) to combate impulsive interference.

For this paper only the expected value of equation (5) will be considered. The expectation of equation (5) reduces to

$$E[\Lambda(\lambda)] = \sum_{L_1} \sum_{J_1} \sum_{L_2} \sum_{J_2} A_{L_1 J_1}(\lambda) \; A^*_{L_2 J_2}(\lambda) \Big\{ (1/N_0) \sum_{i_1} \sum_{i_2} E[X(i_1, L_1, J_1)\, X(i_2, L_2, J_2)] \; e^{-j\lambda i_1} \, e^{j\lambda i_2} \Big\} \tag{7}$$

where * represents complex conjugate.

The term in the bracket represents the cross-power spectral density. This will be derived first for the three components of equation (5). They are as follows:

1) Signal,

$$(1/N_o) \sum_{i_1} \sum_{i_2} R_s[i_1 - i_2 - (\xi_{L_1J_1} - \xi_{L_2J_2})]\, e^{-j\lambda i_1}\, e^{j\lambda i_2}$$

From the transformation of variables, $i_2 = i_2$, $v = i_1 - i_2$ and $D = \xi_{L_1J_1} - \xi_{L_2J_2}$ we obtain,

$$\sum_{v=-(N_o-1)}^{N_o-1} (1 - |v|/N_o)\, R_s(v - D)\; e^{-j\lambda D}$$

$$= e^{-j\lambda D} \sum_{v=-(N_o-1)-D}^{N_o-1-D} (1 - |v + D|/N_o)\, R_s(v)\, e^{-j\lambda v}$$

$$= e^{-j\lambda D}\; \hat{S}(\lambda) \xrightarrow[N_o \to \infty]{} S(\lambda)\, e^{-j\lambda\, \xi_{L_1J_1}}\, e^{-j\lambda\, \xi_{L_2J_2}} \quad (8)$$

where $S(\lambda)$ is an estimate of the spectrum of the signal and converges to the true spectrum as $N_o \to \infty$ [28, 29]. Therefore, the frequency properties have been separated from the spatial properties in the limit as $N_o \to \infty$.

2) Noise,

$$(1/N_0) \sum_{i_1} \sum_{i_2} E[n(i_1, L_1, J_1)\ n(i_2, L_2, J_2]\ e^{-j\lambda i_1} e^{j\lambda i_2}$$

$$\xrightarrow[N_0 \to \infty]{} N(\lambda) \quad \delta[L_1J_1 - L_2J_2] \tag{9}$$

where $N(\lambda)$ is the spectrum of the noise.

3) Interference,

$$(1/N_0) \sum_{i_1} \sum_{i_2} E[I(i_1, L_1, J_1)\ I(1_2, L_2, J_2)]\ e^{-j\lambda i_1} e^{j\lambda i_2}$$

$$= I(\lambda) \quad e^{-j\lambda\gamma_{L_1J_1}} \quad e^{j\lambda\gamma_{L_2J_2}} \tag{10}$$

where

$$I(\lambda) = (2\ A^2\ w^2/N_0) \left[\frac{\sin(w\lambda\ \tau/2)}{w\ \sin(\lambda\ \tau/2)}\right]^2 \quad [1 - \cos(\lambda)]$$

and we let $A_k = A$. $I(\lambda)$ is the power spectrum of impulsive interference. It depends on the amplitude A and the number of impulse functions occurring over the interval. Since the impulse function was defined to have zero mean value, the term $[1 - \cos(\lambda)]$ is present. At $\lambda = 0$, $I(\lambda) = 0$ due to this term.

The other term

$$\left[\frac{\sin(w\lambda\tau/2)}{w\sin(\lambda\tau/2)}\right]^2 \tag{11}$$

has maxima and minima occurring over the frequency band. The maxima can be found from the equation

$\sin(\lambda\tau/2) = 0$, which has solutions at

$\lambda = 2\pi I_0/\tau \quad , \qquad I_0 = 1, \pm 1, \pm 2,$ etc.

When the denominator is zero in equation (11) so is the numerator. The value of equation (11) at the solutions can be found using L' Hospital's rule. At the solutions equation (11) is equal to one. An example of this function will be given later in the paper. Since a model for the impulsive interference is available, it may be possible to estimate the parameters A, W, and τ and substitute them into $I(\lambda)$ to obtain an estimate of the spectrum of the impulsive interference directly. But it will be clear from the examples that this can't be done without error. However, this paper considers the performance of using nonlinearities only.

Using the above results equation (7) reduces to

$$E[\Lambda(\lambda)] = S(\lambda)\sum_{L_1}\sum_{J_1}\sum_{L_2}\sum_{J_2} A_{L_1J_1}(\lambda)\, A^*_{L_2\,J_2}(\lambda)\, e^{-j\lambda\xi_{L_1J_1}}\quad e^{j\lambda\xi_{L_2J_2}}$$

$$+ N(\lambda)\sum_{L}\sum_{J}\left|A_{LJ}(\lambda)\right|^2$$

$$+ I(\lambda)\sum_{L_1}\sum_{J_1}\sum_{L_2}\sum_{J_2} A_{L_1J_1}(\lambda)\, A^*_{L_2J_2}(\lambda)\quad e^{-j\lambda\gamma_{L_1J_1}}\, e^{j\lambda\gamma_{L_2J_2}} \qquad (12)$$

The weights can now be chosen to maximize a performance measure. But before this is done let,

$$A_{LJ}(\lambda) = e^{j\lambda\,\xi_{LJ}}$$

Then,

$$E[\Lambda(\lambda)] = S(\lambda)\ (NM)^2 + N(\lambda)\ NM$$

$$+ I(\lambda)\sum_{L_1}\sum_{J_1}\sum_{L_2}\sum_{J_2} e^{-j\lambda(\gamma_{L_1J_1}-\xi_{L_1J_1})}\, e^{j\lambda(\gamma_{L_2J_2}-\xi_{L_2J_2})}$$

In the above equation the signal is amplified by $(NM)^2$ and the noise is amplified by NM. Whereas, the amplification of $I(\lambda)$ depends on its direction of arrival. If the interference is coming from the same direction as the signal, then

$$\gamma_{L_1 J_1} = \xi_{L_1 J_1} \ , \ \gamma_{L_2 J_2} = \xi_{L_2 J_2}$$

and

$$E[\Lambda(\lambda)] = [S(\lambda) + I(\lambda)]\ (NM)^2 + N(\lambda)\ NM$$

Therefore, when the interference is coming from the same direction as the signal the array passes both equally.

These ideas can be stated more precisely if we define a performance measure.

Let the performance measure (PM) be defined as follows:

$$PM = \frac{E[\Lambda(\lambda) | H_1] - E[\Lambda(\lambda) | H_0]}{E[\Lambda(\lambda) | H_0]} \tag{13}$$

where,

$$H_1: \ x(i, L, J) = s(i, L, J) = n(i, L, J) + I(i, L, J)$$
$$H_0: \ x(i, L, J) = n(i, L, J) + I(i, L, J)$$

Rewriting equation (13) in a form convenient for optimization [25, 26] we obtain

$$PM = \frac{S(\lambda)}{N(\lambda)} \left\{ \frac{\underline{A}^{*T}\ \underline{V}\ \underline{V}^{*T}\ \underline{A}}{\underline{A}^{*T}\ Q\ \underline{A}} \right\}$$

where the term in the bracket is the gain of the array and its components are defined as follows,

$$\underline{A}^{*T} = (A_{11}(\lambda), A_{12}(\lambda), \ldots, A_{NM}(\lambda))$$

$$\underline{V}^{*T} = (e^{j\lambda\xi_{11}}, e^{j\lambda\xi_{12}}, \ldots, e^{j\lambda\xi_{NM}})$$

$$Q = I + \frac{I(\lambda)}{N(\lambda)} \underline{u}\,\underline{u}^{*T}$$

$$\underline{u}^{*T} = (e^{j\lambda\gamma_{11}}, e^{j\lambda\gamma_{12}}, \ldots, e^{j\lambda\gamma_{NM}})$$

where T represents transpose and I is an (NM x NM) identity matrix.

As is well known [25, 26, 30] the optimum gain is given by

$$G_o = \underline{V}^{*T} Q^{-1} \underline{V}$$

and the optimum weight vector is

$$\underline{A} = Q^{-1} \underline{V} = \left[I + \frac{I(\lambda)}{N(\lambda)} \underline{u}\,\underline{u}^{*T} \right]^{-1} \underline{V}$$

Now,

$$Q^{-1} = I - \frac{I(\lambda)}{N(\lambda) + I(\lambda)\ NM} \quad \underline{u}\ \underline{u}^{*T}$$

Therefore, the optimum weight vector reduces to

$$\underline{A} = \underline{V} - \frac{I(\lambda)}{N(\lambda) + I(\lambda)\ NM} \quad \underline{u}\ \underline{u}^{*T}\ \underline{V}$$

and

$$G_o = NM - \frac{I(\lambda)}{N(\lambda) + I(\lambda)\ NM} \quad \underline{V}^{*T}\ \underline{u}\ \underline{u}^{*T}\ \underline{V}$$

Before interpreting the function $\underline{V}^{*T}\ \underline{u}\ \underline{u}^{*T}\ \underline{V}$, let it equal NM. Then if $I(\lambda)\ NM >> N(\lambda)$, $G_o \sim NM - 1$. On the other hand if $V^{*T}\ \underline{u}\ \underline{u}^{*T}\ \underline{V} \sim (NM)^2$, then $G_o \sim 0$.

Therefore, the optimum gain depends on the value of the function $\underline{V}^{*T}\ \underline{u}\ \underline{u}\ \underline{V}$. This function can be interpreted as the array beam pattern (BP), since

$$BP = \underline{V}^{*T}\ \underline{u}\ \underline{u}^{*T}\ \underline{V} = \left| (1/NM) \sum_{L=1}^{N} \sum_{J=1}^{M} e^{-j\lambda\ (B-D)} \right|^2 \tag{14}$$

where,

$$B = \bar{\lambda}(L-1)\cos\theta_B/c + d(J-1)\sin\theta_B/c$$

$$D = \bar{\lambda}(L-1)\cos\theta/c + d(J-1)\sin\theta/c$$

and,

$$B-D = \bar{\lambda}(L-1)\;(\cos\theta_B - \cos\theta)/c + d(J-1)\;(\sin\theta_B - \sin\theta)/c$$

In this formulation the array is steered in the direction θ_B but the signal arrives from the direction θ.

From these results equation 14 simplifies to

$$BP = \left[\frac{\sin[N\lambda\bar{\lambda}a/2c]}{N\sin[\bar{\lambda}\lambda a/2c]}\right]^2 \left[\frac{\sin[M\lambda db/2c]}{M\sin[\lambda db/2c]}\right]^2 \tag{15}$$

where,

$$a = \cos\theta_B - \cos\theta$$

$$b = \cos\theta_B - \sin\theta$$

and the parameters a, b are bounded by

$$-2 \leq a \leq 2 \qquad \text{and} \qquad -2 \leq b \leq 2.$$

The spatial location of a signal coincides with the maximum value of BP. Therefore, for a specific signal direction θ, the function should have only one maximum value.

The possible maxima of BP can be found from the solutions of the equations

$$\sin(\lambda \ell a/2c) = 0 \qquad \text{and} \qquad \sin(\lambda d b/2c) = 0$$

which reduce to, respectively,

$$a = I_o \lambda_w/\ell \qquad \text{and} \qquad b = I_o \lambda_w/d$$

where $I_o = 0, \pm 1, \pm 2$, etc, *and* λ_w is the wavelength.

suppose, $\theta = 0$ and $\theta_B = \pi$

Then, $a = -2$ and $b = 0$. Therefore, if the array is not designed properly BP can have a maximum at $\theta = 0$ and $\theta_B = \pi$. This phenomenon is called a grating lobe [22]. It can be prevented if $\ell < \lambda_w/2$.

Similarly at $\theta = \pi/2$ and $\theta_B = 3/2\pi$, then $a = 0$ and $b = -2$. So if $d < \lambda_w/2$ a grating lob is also prevented from occurring.

These design parameters $\ell < \lambda_w/2$ and $d < \lambda_w/2$ also prevent the optimum weight approach from cancelling the signal due to grating lobes.

III. IDEAL NONLINEARITY

From the previous section the complex weights $A_{LJ}(\lambda)$ can be chosen to maximize the gain of the array processor. But for interference arriving from the same direction as the signal all that can be done is to allow both signal and interference to pass. Filtering is also not effective against impulsive interference. This can be seen from the relationship

$$E[|X_0(\lambda)|^2] = |H(\lambda)|^2 \; E[|X(\lambda)|^2]$$

where, $E[|X(\lambda)|^2]$ represents the spectrum of the data. Since the interference is impulsive, $E[|X(\lambda)|^2]$, has a broad spectrum and a filter $H(\lambda)$ would operate on the signal as well as the noise. However, from the likelihood ratio approach for i.i.d. data the correct procedure is to preprocess the data through a nonlinearity [2, 3, 4, 5, 6, 7]. We will employ a nonlinearity even though the data is not independent. Related work in this area can be found in references [31, 32, 33, 34, 35].

In order to derive a theoretical relationship for the array processor we define an ideal nonlinearity as our preprocessor.

Definition: Ideal Nonlinearity

$$y(i,L,J) = 0 \quad , \quad \text{at} \quad i = i_g + K\tau + \gamma_{LJ}$$

$$y(i,L,J) = 0 \quad , \quad \text{at} \quad i = i_g + 1 + k\tau + \gamma_{LJ}$$

$$y(i,L,J) = x(i,L,J) \quad , \quad \text{otherwise}$$

This definition can be equivalently expressed in the form

$$y(i,L,J) = z(i,L,J) - \sum_{k=0}^{W-1} z(i,L,J)\left\{\delta[i - (i_g + k\tau + \gamma_{LJ})]\right.$$

$$\left. + \delta[i - i_g + 1 + k\tau + \gamma_{LJ})]\right\}$$

where,

$$z(i,L,J) = s(i,L,J) + n(i,L,J)$$

As in the previous sections, the information needed is contained in the cross-correlation function

$$E[y(i_1,L_1,J_1)\ y(i_2,L_2,J_2)]$$

$$= E[z(i_1,L_1,J_1)\ z(i_2,L_2,J_2)]$$

$$-E[z(i_1,L_1,J_1)\ z(i_2,L_2,\ J_2)] \sum_{k=0}^{W-1}\left\{\delta[i_1 - (i_g + k\tau + \gamma_{L_1J_1})]\right.$$

$$\left. + \delta[i_1 - (i_g + 1 + k\tau + \gamma_{L_1J_1})]\right\}$$

$$-E[z(i_1,L_1,J_1)\ z(i_2,L_2,J_2)] \sum_{k=0}^{W-1}\left\{\delta[i_2 - (i_g + k\tau + \gamma_{L_2J_2})]\right.$$

$$\left. + \delta[i_2 - (i_g + 1 + k\tau + \gamma_{L_2J_2})]\right\}$$

$$+ E[z(i_1,L_1,J_1)\, Z(i_2,L_2,J_2)]$$

$$\sum_{k_1=0}^{W-1} \sum_{k_2=0}^{W-1} \left\{\delta[i_1 - (i_g + k_1\tau + \gamma_{L_1J_1})] + \delta[i_1 - (i_g + 1 + k\tau + \gamma_{L_1J_1})]\right\}$$

$$\times \left\{\delta[i_2 - (i_g + k_2\tau + \gamma_{L_2J_2})] + \delta[i_2 - (i_g + k_2\tau + \gamma_{L_2J_2})]\right\}$$

For simplicity we shall let $\gamma_{LJ} = \xi_{LJ}$, i.e., the interference is coming from the same direction as the signal, in the following analysis.

The cross-power spectral density estimate follows as

$$(1/N_0) \sum_{i_1} \sum_{i_2} E[y(i_1, L_1, J_1)\, y(i_2, L_2, J_2)]\, e^{-j\lambda i_1} e^{j\lambda i_2}$$

$$= \left\{S(\lambda) - S_{2,3}(\lambda,w) + S_4(\lambda,w)\right\} e^{-j\lambda \xi_{L_1J_1}} e^{j\lambda \xi_{L_2J_2}}$$

$$+ \left\{ N(\lambda) - N_{2,3}(\lambda,w) + N_4(\lambda,w) \right\} \delta[L_1J_1 - L_2J_2] \tag{16}$$

where

$$S_{2,3}(\lambda,w) = (1/N_o) \sum_{k=0}^{W-1} \sum_{i_3 = -\zeta_{L_2 J_2}}^{N_o - 1 - \xi_{L_2 J_2}} [G_s(i_3, k, \lambda) + G_s(i_3, k, 1, \lambda)]$$

$$+ \sum_{i_3 = -\zeta_{L_1 J_1}}^{N_o - 1 - \xi_{L_1 J_1}} [G_s(i_3, k, \lambda) + G_s(i_3, k, 1, \lambda)] ,$$

$$G_s(i_3, k, \lambda) = R_s[i_3 - (i_g + k\tau)]\, e^{j\lambda[i_3 - (i_g + k\tau)]}$$

$$G_s(i_3, k, 1, \lambda) = R_s[i - (i_g + 1 + k\tau)]\, e^{j\lambda[i_3 - (i_g + 1 + k\tau)]}$$

$$S_4(\lambda, w) = (1/N_o) \sum_{k_1} \sum_{k_2} \Big\{ 2\, R_s[(k_1 - k_2)\tau]\, e^{-j\lambda(k_1 - k_2)\tau}$$

$$+ R_s[(k_1 - k_2)\tau + 1]\; e^{-j\lambda(k_1 - k_2)\tau} e^{-j\lambda}$$

$$+ R_s[(k_1 - k_2)\tau - 1]\, e^{j\lambda(k - k_2)\tau} e^{j\lambda} \Big\}$$

$N_{2,3}(\lambda,w)$ and $N_4(\lambda,w)$ are of the same form as $S_{2,3}(\lambda,w)$ and $S_4(\lambda,w)$, respectively, but with $R_s(\ )$ replaced by $R_N(\ )$.

The terms $S_{2,3}(\lambda,w)$ and $N_{2,3}(\lambda,w)$ are functions of L_1J_1 and L_2J_2. But as $N_0 \to \infty$, their effect can be neglected just as was done for the cross-spectrum of the signal.

With this assumption the power spectrum can be separated from the spatial properties of the array. Therefore, the expected value of the array processor with the ideal nonlinearities included can be expressed as follows.

$$E[\Lambda_{NL}(\lambda)] = \sum_{L_1} \sum_{J_1} \sum_{L_2} \sum_{J_2} A_{L_1J_1}(\lambda) A^*_{L_2J_2}(\lambda) \Big\{ (1/N_0) \sum_{i_1} \sum_{i_2} E[y(i_1,L_1,J_1)\, y(i_2,L_2,J_2)]\, e^{-j\lambda i_1}\, e^{j\lambda i_2} \Big\}$$

$$= [S(\lambda) - S_{2,3}(\lambda,w) + S_4(\lambda,w)] \sum_{L_1} \sum_{J_1} \sum_{L_2} \sum_{J_2} A_{L_1J_1}(\lambda)\, A^*_{L_2J_2}\, e^{-j\lambda \xi_{L_1J_1}}\, e^{j\lambda \xi_{L_2J_2}}$$

$$+ [N(\lambda) - N_{2,3}(\lambda,w) + N_4(\lambda,w)] \sum_{L} \sum_{J} |A_{LJ}(\lambda)|^2 \qquad (17)$$

Several examples will now be given for the power spectrum of a signal in additive Gaussian noise and additive impulsive interference without and with the ideal nonlinearity included.

IV. ILLUSTRATIVE EXAMPLES

Figure 4 shows two impulse functions occurring in a sample of data. The first burst in the figure is due to a physical phenomenon but the second burst appears to be a reflection since it is 180 degrees out of phase with the first burst.

Our model for impulse functions, equation (4), can clearly represent the impulses of figure 4. However, the model is more general since other functions can also be represented. For the following examples a special case will be considered. Figure 5 shows the time history for our first example. The data consist of two additive sinusoids in addivitve non-white Gaussian noise and two additive impulse functions. One sinusoid has a level of 20 dB below the other. The impulse functions are separated by 25.0 msec and have a level of 4 volts.

The corresponsing spectra are shown in figure 6. The top plot represents the spectrum of signal and noise before the impulses are added. Since the noise is non-white the lower level sinusoid is detectable at a frequency where the noise level has diminished. The next plot in the middle of the figure is the spectrum with the additive impulses included. These results are predicted from equation (10). After the impulses are extracted by employing the ideal nonlinearity the resultant data have the spectrum shown in the bottom plot of figure 6. These results can be predicted from the theoretical results of the previous section.

Even though there is a significant improvement from employing the ideal nonlinearity in the spectrum there's still a residual degradation. With additional processing it may be possible to reduce the residual degradation still further. However, since our results are theoretical they can be used to compare other methods.

The next example is shown in figures 7 and 8. The time history of the data is identical except that the impulse functions are separated by 50.0 msec. Since the separation of impulse functions is doubled there are twice as many maxima and minima occurring over the band as shown in the middle plot of figure 8.

SUMMARY

The ability of a two-dimensional array with ideal nonlinear elements to extract spatial and frequency information from a signal which may be interferred with by impulsive interference was discussed. After some preliminary remarks on discrete frequency-wavenumber estimation the array processor for a two-dimensional array was defined. It consisted of a set of frequency dependent complex weights. It was shown that the weights could be chosen by maximizing a performance measure. This procedure would be advantageous when the wave could not be assumed to be a plane wave and when interference was present but propagating in a direction other than the signal direction.

For the impulsive interference considered it was shown that filtering was ineffective. However, from the results based on the principle of the likelihood ratio, nonlinearities were effective against impulsive interference. In order to discuss the effect of nonlinearities on the spectrum the concept of the ideal nonlinearity was introduced. This allowed the spectrum to be theoretically derived when ideal nonlinearities were included. Two examples were given to illustrate the theoretical results.

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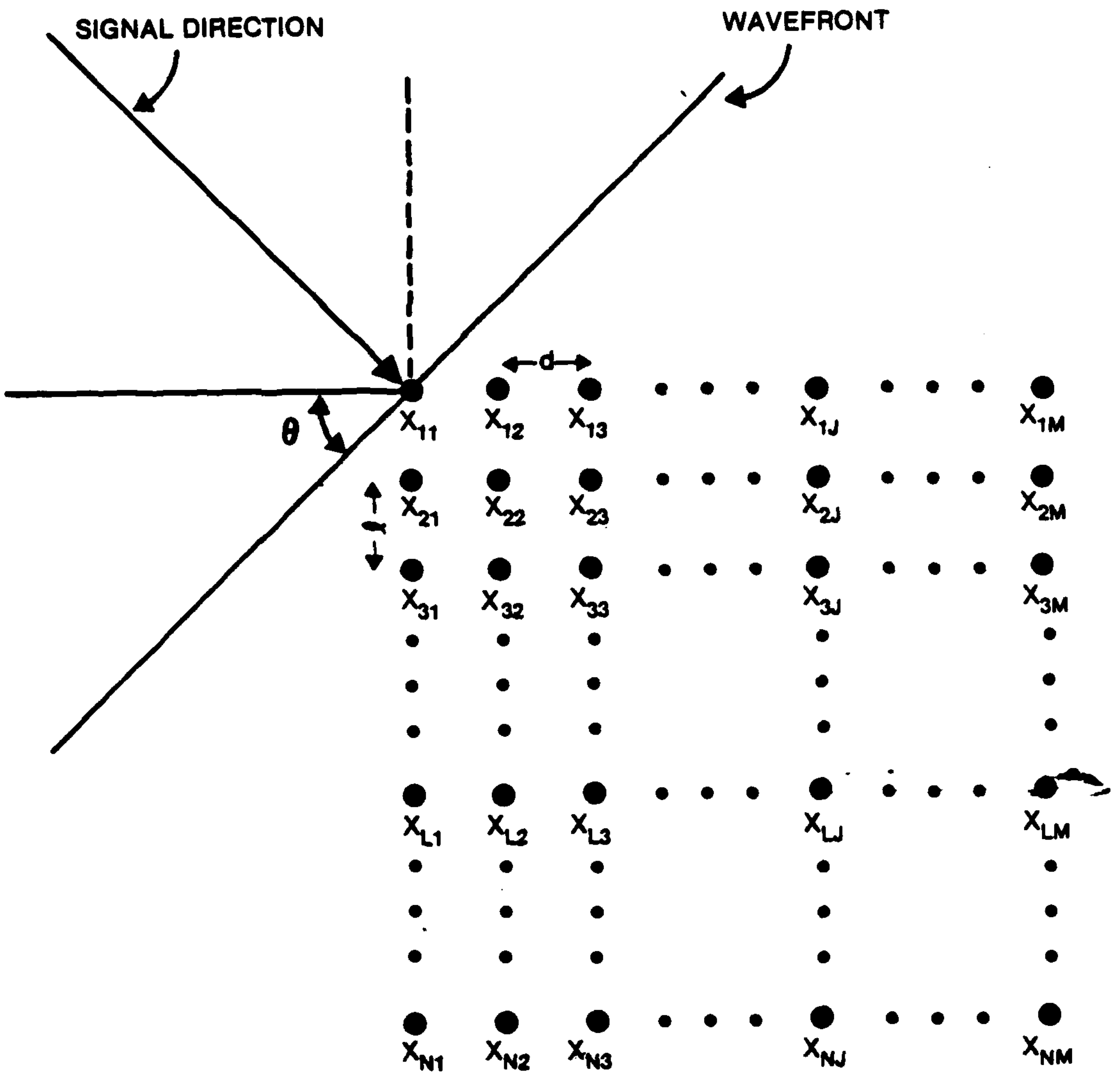

Figure 1. Two-Dimensional Array

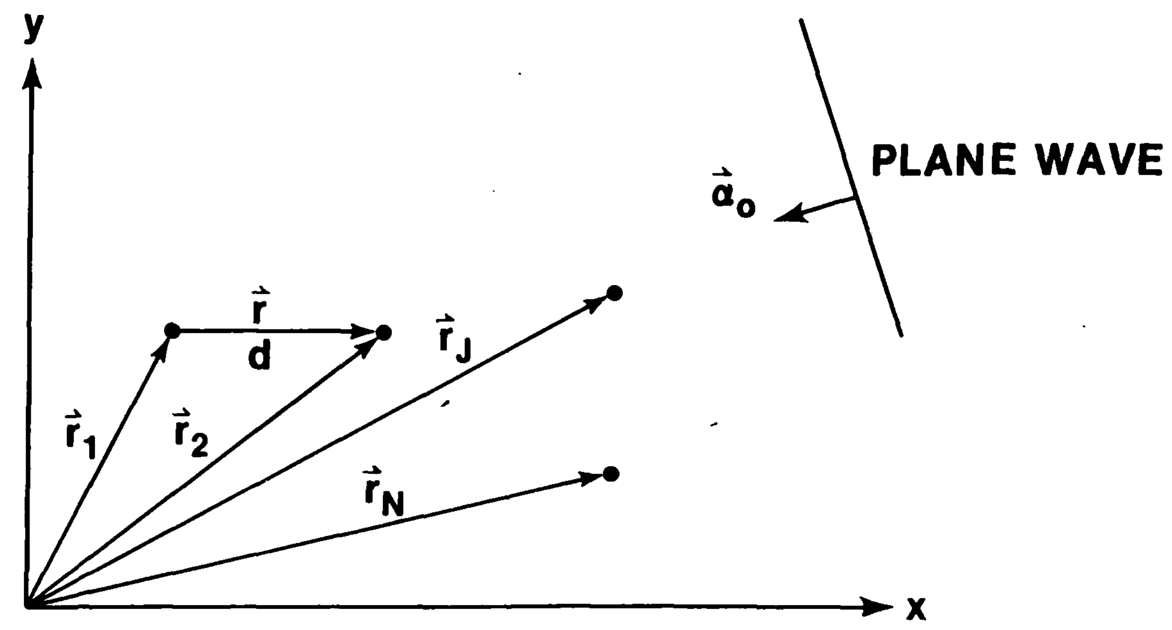

Figure 2. Two-Dimensional Coordinate System for Sensor Locations

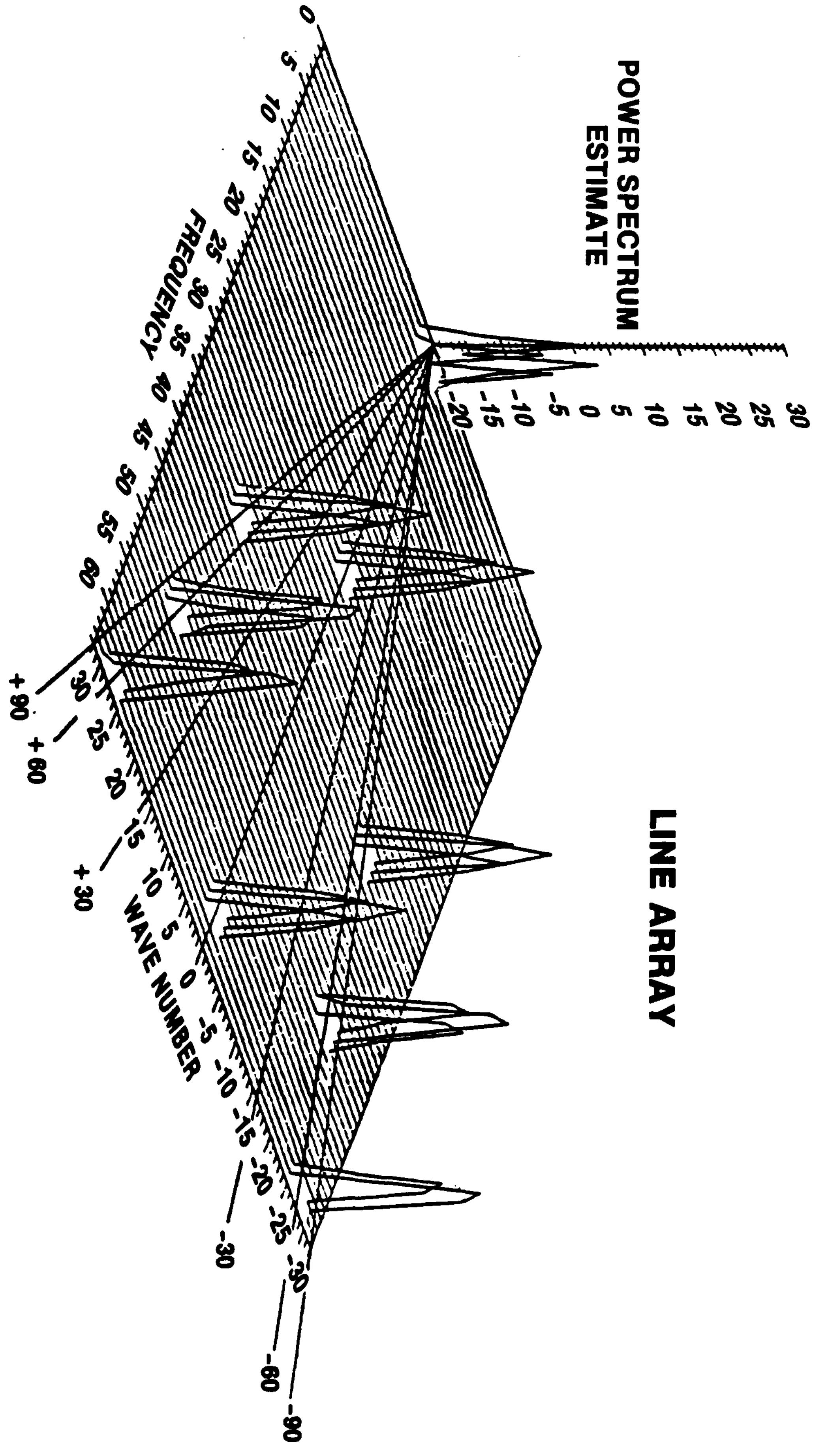

Figure 3. Frequency-Wavenumber Spectrum

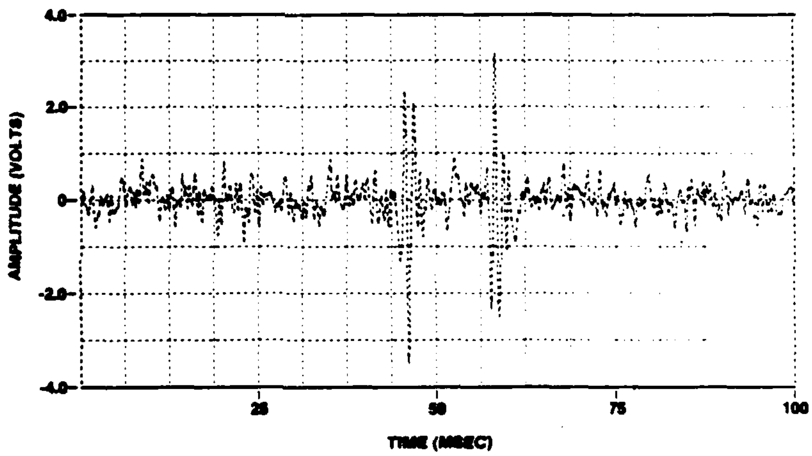

Figure 4. Impulse Function Interference

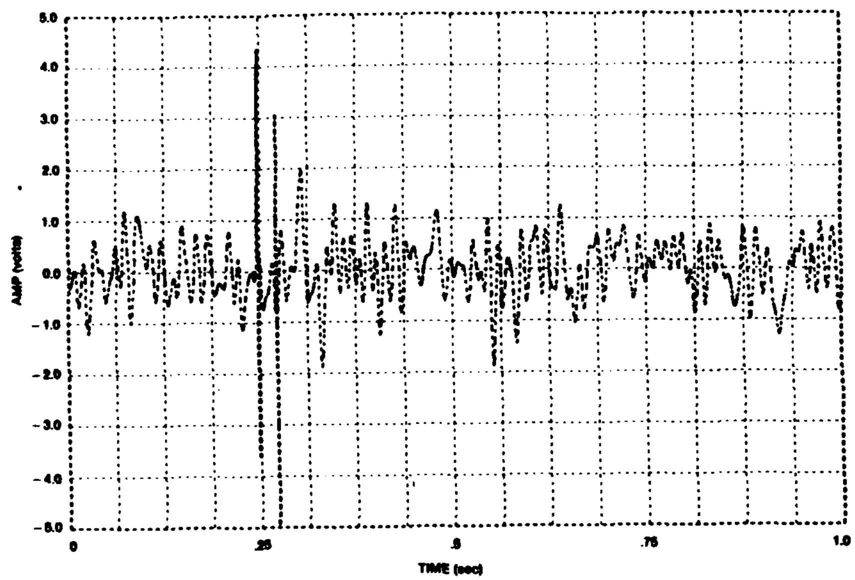

Figure 5. Model Impulse Function: Separation - 2 5 msec

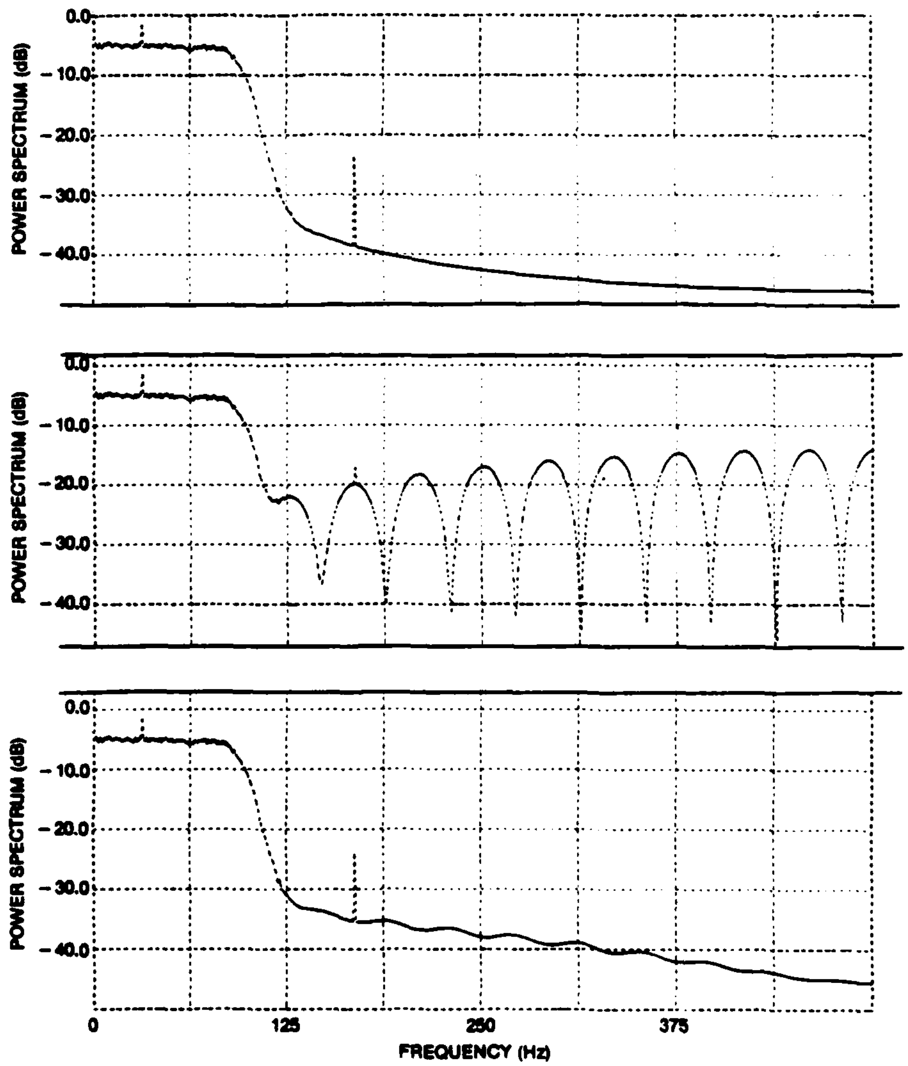

Figure 6. Spectrum Estimate: Top, Signal and Noise. Middle, with Impulse Functions. Bottom, Impulse Functions Removed by Ideal Nonlinearity

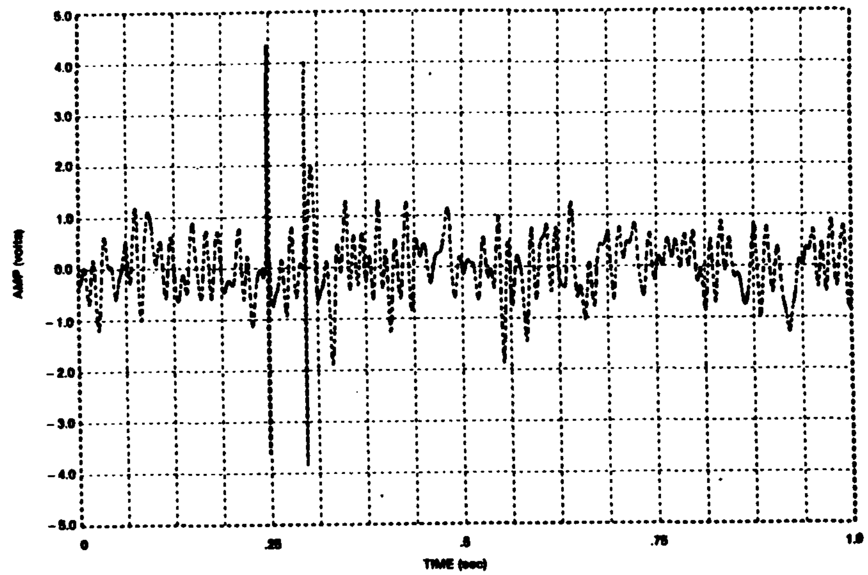

Figure 7. Model Impulse Function: Separation - 5 0.msec

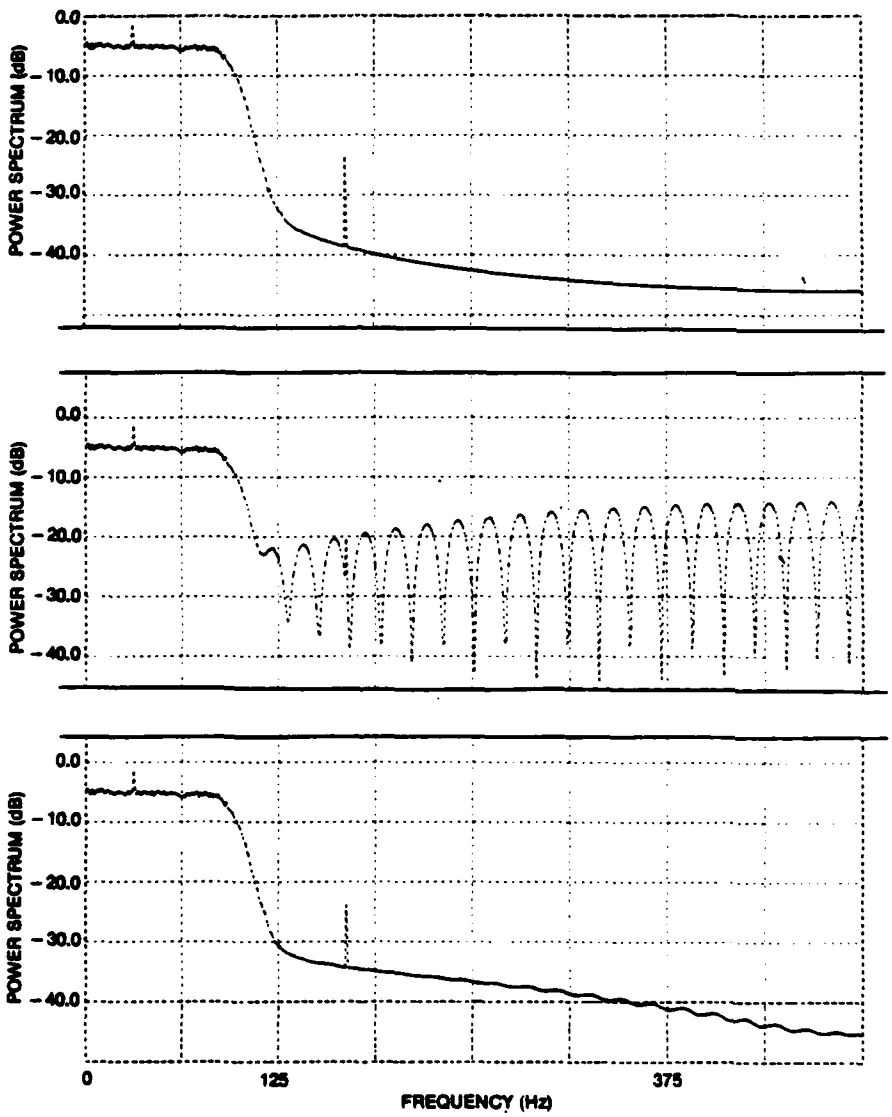

Figure 8. Spectrum Estimate: Top, Signal and Noise. Middle, with Impulses. Bottom, Impulse Functions Removed by Ideal Nonlinearity.

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DATA WITH A TWO-DIMENSIONAL ARRAY
Roger F. Dwyer
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