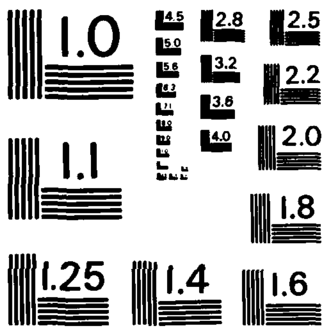
AD-A162 288 THE APPLICATION OF GENERALIZED GEOMETRIC PROGRAMMING (CONJUGATE DUALITY) T..(U) PITTSBURGH UNIV PA DEPT OF INDUSTRIAL ENGINEERING T R JEFFERSON 15 JUL 85 1/1

UNCLASSIFIED AFOSR-TR-85-1055 AFOSR-83-0234 F/G 9/2 NL


1.0
4.5
5.0
5.6
2.8
3.2
3.6
4.0
2.5
2.2
2.0
1.1
1.8
1.25
1.4
1.6

Final Technical Report

for

Air Force Grant #AFOSR 83-0234

The Application of Generalized Geometric Programming (Conjugate Duality) to the Analysis and Solution of Convex Programs

July 16, 1983 - July 15, 1985

Principal Investigator

Thomas R. Jefferson

Department of Industrial Engineering
University of Pittsburgh

SUMMARY

The research in this grant involves the application of generalized geometric programming (conjugate duality) to a variety of problems. The duality theory constructs a dual program which can provide insight into the problem and assist in solution. Composite geometric programming was developed as an important new class of mathematical programs. Applications studied included machining economics, resource allocation, assignment, nonlinear multicommodity network flow problems, mineral processing, statistical analysis of ordinal categorical data, and estimation.

Geometric programming was extended from functions of posynomial form to functions which include exponential, logarithmic and other factors by the development of composite geometric programming. This class retains the power of geometric programming while addressing new problems. Certain machining economics problems and chemical equilibrium problems fall into this new class of mathematical programs.

Research on the machining economics problem resulted in the problem being reduced from a nonlinear program to a one-dimensional search. In addition, duality theory provided easy parametric analysis.

In an unrelated problem of resource allocation a class of problems with linear return functions, the variables of which are restricted to intervals and must satisfy an aggregate convex constraint, was transformed through conjugate duality into a one-dimensional search.

In the control mineral processing circuits flow rates and species concentrations must be estimated from a partial set of data. Geometric programming provided a means for formulation and solution of these problems.

In the analysis of ordinal categorical data researchers have assumed equal spacing to the categories. A new method is developed which questions this assumption and provides a means to scale ordinal categorical data.

The results for the five topics described have been written up and submitted to journals. Three papers have been accepted for publication. Work continues on four more topics: Monotone convex assignment; convex multicommodity network flow; the relationships among entropy, estimation and duality; the relationship between certainity equivalents and stochastic constraints.

The objective to this research is to develop generalized geometric programming (conjugate duality) as a viable tool for the study of optimization problems. The potential of this analysis has been known for some twenty years. It pairs a second (dual) convex program with an original (primal) convex program. Optimality conditions tie the optimal solution for one to the optimal solution to the other. Thus, only one program need be solved to solve both. For real problems typically one of these two problems is easier to solve; sometimes the solution is surprisingly simple.

The calculation of the dual is not necessarily straightforward. Indeed through reformulation and flexibility in the application of generalized geometric programming many dual formulations become possible.

Which duals are best? How are they calculated?

The research goal, ultimately, is to answer these questions by the application of generalized geometric programming to a variety of problems so as to identify rules for its successful application.

A major result of this research is the development of an extension of geometric programming, called composite geometric programming. Geometric programming studies mathematical programs, the constraints and functions of which are posynomial. The terms of the posynomials are log-linear in the logarithms of the variables; thus, such functions are the sum of exponential functions compared with linear functions. Composite geometric programming uses the composition of exponential functions with convex function for building blocks. This generalization leads a class of programs which can be used to model problems not captured by geometric programming while retaining the power of geometric programming.

The first application comes from industrial engineering. In machining economics problems, the industrial engineer must make a trade-off between the cost of machine tools and the time it takes to machine a part. Such a problem can be formulated as a mathematical program and has been solved by conventional nonlinear programming codes in the literature. The application of generalized geometric programming to this problem produces a dual program, the solution of which is found by a one-dimensional search. Further insight into the machining economics problem comes easily from the dual program as does parametric analysis. The study of this problem leads to the development of quadratic geometric programming; a class of mathematical programs having many of the attractive properties of geometric programming.

The second completed application comes from resource allocation. The allocation of a resource over a number of competing projects is a common problem. Here a linear return function is assumed. The level of each project is limited by lower and upper bounds. Further, the cost function is a convex separable function of the allocation levels. Maximization of benefits subject to costs can take this form. Application of generalized geometric programming in a straightforward manner leads to a dual problem of high dimension. However, if the bounds are incorporated into the cost functions, the dual becomes a one-dimensional search regardless of the number of projects.

Mineral economics is the source of the next problem. In processing ore a variety of operations are used to separate the metal bearing species from the rock. To achieve high recovery rates material must be recycled through the operations. This leads to mineral processing circuits with few inputs and outputs and many streams, some of which feed partially processed ore back through various operations. Controlling the circuit is made difficult by variations in the concentrations of species in the ore and that mineral processing is affected by the quality of the ore. To control processing, assays of each stream are taken. These data are combined with the observed input and output flows to provide a picture of what is going on in the circuit.

Previously the method for combining the data involved satisfying flow conservation and alternately modifying flow rates and species concentrations until flows into each node of each species balanced flows out. An entropy model was developed which finds the most probable consistent estimates of species concentrations and flow rates. This model was used to analyze data and the results compared favorably with other methods.

Another estimation problem comes from the study of ordinal categorical data. Examples of such data arise in surveys, grades and other situations in which qualitative assessments must be made. At issue is the assumption of uniform weights or scales associated with each category when the data are analyzed and combined. If two or more observations of the same factor are made on a set of subjects, it is possible to estimate scales for categories rather than assuming them to be uniform. The new method, based on minimum discrimination information (related to entropy), was applied to four different data sets. Scales were generated for each category and found not to be uniform. A statistic for the measurement of consistency was also developed.

The previous five studies mentioned have been written up and submitted for publication. Three have been accepted for publication. The four research topics to be discussed are in various stages of completion. It is hoped they will be completed in a year.

The weapons allocation problem is one of a number of problems which falls in the class of monotone convex assignment problems. The problem is to allocate a limited store of munitions of different types to targets so as to maximize their effect. Such problems have n m variables for n targets and m weapon types. The dual has only $n + m$ variables. Monotonicity leads to further simplification. More work is required on the computational aspect of this problem.

In transportation planning one wishes to design or expand a transporation network so that needs are effectively met. In order to determine the impact a particular transportation network will have on demand, a nonlinear multicommodity flow problem must be solved. Since there are many alternatives to be considered and the cost of each is measured in millions, it is imperative that the problem be solved accurately and efficiently.

An analysis of the convex multicommodity flow problems was done using generalized geometric programming, and the algorithms appearing in the literature were studied and compared. By understanding the strengths and weaknesses of the various solution procedures a new method was proposed. It presently awaits final analysis.

The remaining two research topics are still in the formative stage. In financial planning one wishes to maximize return subject to cash flow restraints over time. The behavior of return and other factors are stochastic over time. The stochastic constraints are related to certainty equivalents over time. Duality provides new insight.

Finally there is a close relationship between entropy and maximum likelihood. This relationship is being explored using duality.

Publications

1. "Quadratic Geometric Programming Applied to Machining Economics" by T.R. Jefferson and C.H. Scott, Mathematical Programming 31 (1985) 137-152.

2. "Bounded Linear Allocation with a Non-linear Cost Constraint" by C.H. Scott and T.R. Jefferson, accepted for publication by INFOR.

3. "The Entropy Approach to Material Balancing in Mineral Processing Circuits," by T.R. Jefferson and C.H. Scott, accepted for publication by International Journal for Mineral Processing.

4. "An Entropy Approach to the Scaling of Ordinal Categorical Data" by T.R. Jefferson, J.H. May and N. Ravi, submitted to Psychometrika.

5. "Composite Geometric Programming" by T.R. Jefferson, Y.-P. Wang and C.H. Scott, submitted to Mathematical Programming.

6. "The Monotone Convex Assignment Problem" by T.R. Jefferson and C.H. Scott in preparation.

7. "Algorithms for Convex Multicommodity Flow" by T.R. Jefferson and C.H. Scott, in preparation.

8. "Dynamic Financial Planning: Certainity Equivalents and Stochastic Constraints" by T.R. Jeffereson and C.H. Scott, in preparation.

9. "Entropy, Maximum Likelihood, and Duality" by T.R. Jefferson and C. H. Scott, in preparation.

Personnel

Principal Investigator	Thomas R. Jefferson, Associate Professor Full Time - July 16 - August 31, 1983 May 1 - June 15, 1984 August 1 - August 31, 1984 May 1 - June 30, 1985
Research Assistant	Ya-Ping Wang, Graduate Student Full Time - Sept. 1, 1983 - April 30, 1984 Sept. 1, 1984 - Dec. 31, 1984
Research Assistant	Jon C. Yingling Full Time - Jan. 1, 1985 - April 30, 1985

These individuals worked part-time on the project at other times. Ya-Ping Wang presently at candidature stage of his doctorate on " Composite Geometric Programming: Theory and Applications" is expected to complete in a year. In addition, Jarai Komutdaeng, a graduate student, worked on the project part time as part of his master's degree..

Conferences

November 7-9, 1983 — ORSA/TIMS Conference in Orlando.
Delivered paper entitled "Quadratic Geometric Programming Applied to Machining Economics."

May 14-18, 1984 — TIMS/ORSA Conference in San Francisco.
Delivered paper entitled "The Monotone Convex Assignment Problem."

Nov. 26-28, 1984 — ORSA/TIMS Conference in Dallas.
Chaired session on Convex Programming. Delivered paper entitled "The Entropy Approach to the Material Balancing Problem."

April 29-May 1, 1985 — TIMS/ORSA Conference in Boston.
Chaired session on Mathematical Programming.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFOSR-TR- 85-1055	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER A162288
4. The Application of Generalized Geometric Programming (Conjugate Duality) to the Analysis and Solution of Convex Programs		5. TYPE OF REPORT & PERIOD COVERED Final Report July 1983 - July 1985
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) T.R. Jefferson		8. CONTRACT OR GRANT NUMBER(s) AFOSR - 83-0234
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Industrial Engineering University of Pittsburgh Pittsburgh, PA 15261		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 2304 A6
11. CONTROLLING OFFICE NAME AND ADDRESS AFOSR/NM, Building 410 Bolling AFB DC 20332		12. REPORT DATE 1985
		13. NUMBER OF PAGES 6
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Generalized Geometric Programming, Conjugate Duality

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This final report covers work done over the length of the project. The development of composite geometric programming is discussed. Results of applications in machining economics, resource allocation, mineral processing, and scaling of ordinal categorical data are presented.

END

FILMED

1-86

DTIC