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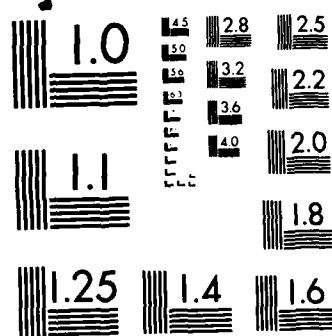
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THEORY AND APPLICATIONS OF RANDOM FIELDS

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October, 1985

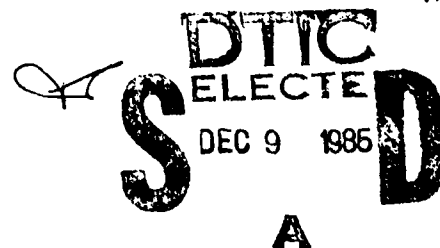
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19. ABSTRACT (Continue on reverse if necessary and identify by block number) The main results described in this report include: 1. A discussion of rough surfaces and their modelling by chi-squared processes and fields. Results which describe how the theory of the latter differs from the Gaussian theory; 2. A development of a new methodology for determining deviation from normality in process and field data; 3. New results on the distribution of the maxima of general Gaussian processes; and 4. Development of a new representation of generalized Gaussian fields and those fields subordinated to it via central limit theorems for additive functionals of Markov processes.					
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INTRODUCTION

In the original research proposal that is being reported on here there were essentially three distinct, albeit related, projects. This report, for ease of writing, reading, and evaluation is built around these three projects in the following fashion.

For each project I have included a brief recapitulation of what was presented in the original proposal. (A reader who is familiar with, and still remembers the details of, the original proposal can bypass the recapitulation.) Following this is, in each of the three cases, a report on the progress made towards realizing the goals of the proposal. The reports are generally rather brief, since they merely summarize results already presented in research papers to which the reader can turn for more details.

Following each report is a brief comment on further avenues of research (if any) opened up by the work done to date.

During the year I commenced work on a fourth project, that was not mentioned in the original proposal, but that grew out of it in a natural fashion. This work, on the theory of generalised processes, is described in a fourth section.

At the end of the reports is a list of research papers that resulted from the research described, as well as a list of conferences attended and visits made to American universities.

For the sake of completeness, we start by including some general background material on random fields.

SOME BACKGROUND ON RANDOM FIELDS

Random fields are simply stochastic processes, $X(t)$, whose "time" parameter, t , varies over some rather general space rather than over the more common real line. The simplest of these occur when the parameter space is some multi-dimensional Euclidean space, and it is these fields that will be at the centre of our study. Of these, the most basic arise when the parameter space is the two-dimensional plane, so that we are dealing with some kind of random surface. When the parameter space is three-dimensional then we have a field (such as ore concentration in a geological site) that varies over space, while when the dimension increases to four we are generally dealing with space-time problems.

More complicated examples of random fields arise as the parameter space becomes more esoteric. Typical examples are parameter spaces of classes of sets, such as arise in the statistical theory of multi-variate Kolmogorov-Smirnov tests and set indexed empirical processes. Another common example is provided by fields indexed by families of functions. Although these arise, once again, in the theory of empirical processes, they are much more famous for their appearance in Quantum Field Theory in Mathematical Physics. There they appear, among other guises, as continuum limits of such well known discrete parameter random fields as the Ising model of Statistical Mechanics.

In the introduction to the original research proposal, written some three years ago, as well as in the introduction to the proposal being reported on here, there was a disclaimer pointing out that fields of the type described in the previous paragraph lay outside the interests of our research program. This is no longer strictly true, as the reader will see for himself as he continues further. The reasons as to how and why such seemingly abstract objects entered a project that was essentially concerned with more practical things should also then become clear.

For now, however, let us return to the simple setting of continuous parameter random fields defined on a Euclidean space. (Note that discrete parameter fields, such as the Ising model, are still not within our domain of interest.) The theory of these fields is now quite substantial, with four separate monographs on various aspects of the subject having appeared in the past four years. (Adler (1981a), Rozanov (1982), Vanmarcke (1983) and Yadrenko (1983).) Roughly speaking, the theory breaks quite naturally into two quite distinct parts.

In the first case, we assume that the sample functions (realisations) of the random field satisfy certain basic regularity conditions, such as continuity, differentiability, etc.. It is then possible to study problems such as the

structure of the field in the neighbourhood of extrema, and the rate at which the field "crosses" (a term which requires careful definition) various levels. These problems turn out to be very important in the application of random fields to the study of rough surfaces, as discussed in the following section.

The second class of problems in the study of continuous parameter random fields arises when the regularity conditions mentioned above are not imposed. Conventionally, one then studies such sample path properties as the (Hausdorff) dimension of various random sets generated by the field. Although these fields, despite their somewhat esoteric properties, are both theoretically interesting and of applied importance, (as the current theory of fractal geometry due to Mandelbrot (1982) and his colleagues has shown beyond any shadow of doubt), they are only of peripheral concern to the main thrust of the current project.

Although, as just noted, the theory of continuous parameter random fields is well developed, it is important to note that in one respect at least it is still very restricted. This is a consequence of the fact that throughout the literature, both theoretical and applied, there is an almost universal assumption of normality. This is an assumption that has a substantial simplifying affect on the mathematics of random fields, but is undesirable for two quite distinct reasons. The first, which comes from purely practical considerations, is that real life fields to which one might like to apply the theory are very often non-Gaussian. For example, the rough metallic surfaces described in the following section are known to be highly non-Gaussian (Adler (1981b)). Assuming, incorrectly, that they are Gaussian leads to the development of a theory of surface structure that invariably fails to tie in with experiment. The second difficulty with the Gaussian assumption is that it hamstring the Mathematician by limiting the phenomena available for his investigation to that case only.

Of the four reports that follow, two are intimately concerned with non-Gaussian processes. The first involves the development and study of a model that can often be used in place of a Gaussian one without too great an increase in the level of difficulty of the mathematics. The second involves the development of a procedure for testing whether or not a particular set of data is consistent with an assumption of normality. The remaining two reports are concerned primarily with Gaussian processes, although the last, via its concern with Wick powers, also has a distinctly non-Gaussian side to it.

Overall, the common thread that runs through the project is the extension of both the theory and applications of random fields, with the aim of increasing our understanding of the Gaussian situation while at the same time attempting to extend our horizons beyond it.

ROUGH SURFACES AND CHI-SQUARED PROCESSES

It is now a well established fact that all surfaces used in engineering practice are rough when judged by the standards of molecular dimensions. This fact has played a major role in the development of Tribology, a science that, among other problems, is concerned with the nature of contact between two surfaces under load and its relationship to problems such as wear, friction, and the conduction of heat and electricity between two surfaces in contact.

Because of the difficulties inherent in observing what happens when two surfaces are in actual physical contact Tribology has made substantial use of mathematical models. The basic idea underlying this has been to develop models of surface structure (at the microscopic level) and then apply these together with, say, a theory of surface deformation, to predict observable (macroscopic) phenomena. Although there has been an enormous amount of activity in this area over the past twenty years (see Thomas (1982) for a recent exhaustive survey) there is still very often disconcerting disagreement between theory and practice. This is despite the fact that very sophisticated random field models have been used for the rough surfaces.

The reason for this is very simple. Almost without exception, rough surfaces have been modeled as Gaussian fields, when, in fact, they are highly non-Gaussian. This point was emphasised in Adler and Firman (1981), following an analysis of both old and new rough surface data. Consequently, irregardless of the sophistication of the model, it is not surprising that the current models fail to yield a theory that squares with practice.

It was precisely this problem that initiated the current study of chi-squared processes and fields. Chi-squared processes can be easily defined via a representation as a sum of squared Gaussian processes. This simple trick yields a family of fields that are at the same time substantially different to Gaussian fields in their sample path behaviour and yet mathematically close enough to their Gaussian parents to be analytically tractable. Furthermore, it yields a family of fields that turn out to model rough surfaces very closely, and to generate a theory that yields results akin to those observed in the laboratory (c.f. Adler (1981b)).

It was from this background that it was decided that a systematic study of chi-squared processes and fields be undertaken. This study has progressed very satisfactorily, and a reasonably complete picture of the sample function behaviour of chi-squared processes and fields is now available.

Our approach to this study has been two phased. Due to the extreme difficulty of the algebra involved in handling chi-squared fields, in the first phase we concentrated on a detailed study of chi-squared processes. Our intention, which has fortunately turned out to be realisable, was that the experience obtained, and intuition developed, in the case of processes would make the treatment of fields somewhat easier.

As far as processes go, a reasonably complete picture, is now available. For example, in the paper Aronowich and Adler (1985a) we obtained the distribution of various quantities associated with local extrema of chi-squared processes. Examples include the distribution of the height of local maxima and minima, as well as the curvature of the process at such extrema. All of these quantities are of substantial interest in the rough surface setting. These results were mentioned in the last annual report, where we also indicated that they seemed to imply that chi-squared extrema, while being almost as mathematically tractable as their Gaussian counterparts, behaved in a quite different fashion.

We have taken this point considerably further over the past year, in the development of (Slepian) model processes for chi-squared processes in the neighbourhood of local extrema and level crossings. These model processes show that while chi-squared maxima behave in a fashion not too different to their Gaussian counterparts, their minima show remarkably different behaviour. These are, in general, much flatter than in the Gaussian case. (This is, clearly, an "obvious" consequence of the fact that chi-squared processes are bounded from below.)

One of the important consequences of the development of these models is that they enable a comparatively easy study of what the high and low parts of the process look like, and this is precisely where the action is when one applies these models to study contact between surfaces.

Another result that we managed to obtain for chi-squared processes was that of the asymptotic Poisson nature of their point processes of minima. This result, which complements a similar result of Sharpe (1978) and Lindgren (1980) for the maxima of these processes, can be combined with the model processes to give a reasonably accurate picture of the low levels of a chi-squared process as being made up of a number of parabolic disks sited on the points of a Poisson process. This work has been written up in the paper Aronowich and Adler (1985b).

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All of the above work was carried out jointly by myself and Michael Aronowich, a Technion doctoral student who was supported by the Technion while working on this project. Following the completion of the first phase of our study of chi-squared processes, Aronowich continued, by himself, to investigate the field situation. This work has been written up in his thesis (Aronowich (1985a)) and in a paper (Aronowich (1985b)).

The algebra required for a full analysis of the random field case would seem to be of an order of difficulty that makes it basically unmanageable. However, it turned out that, while it was impossible to describe exactly the form of the Slepian model process for chi-squared fields, it was possible to obtain information on this at asymptotically high and low levels. Again, one sees that at extrema the chi-squared field adopts a parabolic form generally much flatter than that adopted by its Gaussian counterpart. This ties in very well with what is actually observed for rough surfaces.

We have thus more or less completed the proposed theoretical analysis of chi-squared processes and fields. The next class of problems to be tackled here is the application of these models to actual problems in the theory of rough surfaces. We hope to carry this out during the next year or so. Preliminary work in this area seems to indicate that the application should be both interesting and useful.

DIFFERENTIATING BETWEEN GAUSSIAN AND NON-GAUSSIAN PROCESSES AND FIELDS

A very simple problem that arises as soon as one starts to think about non-Gaussian processes and fields is how to determine if a particular set of data, either from a field or a process, is actually consistent with a Gaussian hypothesis. Despite the simplicity of this problem, and the ubiquity of Gaussian models, it is rather surprising that there is no truly satisfactory answer to this question in the statistical literature other than that based on poly-spectra.

The poly-spectra approach to this problem (see, for example, Rao and Gabr (1984) for a full account of this procedure) is based on testing for departure from normality via the cumulant structure of the process. My own feeling is that this is a particularly uninteresting and uninformative aspect of any process, since it is not at all clear how the cumulant structure of a process influences its sample function behaviour. It is this latter aspect of the process that is usually the most interesting in practical situations.

Another problem with the poly-spectra approach, that is perhaps more important from our point of view, is that it seems to be almost impossible to use it for random fields. For example, the bi-spectrum of a single parameter process is a two-parameter creature, that generally has to be viewed at some time during its estimation. The bi-spectrum of a two-parameter random field is a four-dimensional creature, and thus somewhat difficult to observe even with the most modern of computer graphics.

Consequently, we proposed a procedure for differentiating between Gaussian and non-Gaussian processes and fields via their level crossing rates. The basic idea was that since Gaussian processes and fields have characteristic level crossing rates, departures from these by data should provide a simple test of normality.

At the time of the proposal only preliminary work had been completed on this project. Since then, considerable progress has been made.

As with the previous project, our attack was in two distinct phases. Initially we considered only processes, leaving fields for later. The process problem has now been essentially solved, and the results have been written up in Adler and Feingold (1985). (Ms. Feingold was employed under the grant as a research associate/programmer, and did a fine piece of computational work.)

Basically, the results in the paper involve the development of two statistics, both based on level crossings, for testing for normality. Since in both cases it is quite clear that the distribution theory of these statistics is impossibly difficult, there are a substantial number of numerical calculations and simulations in order to establish the power of the procedure suggested.

The random field problem has not, as yet, been fully investigated. In principle, it works much like the single parameter situation. However, in order to develop it fully it will be necessary to carry out further numerical work as well as simulations to determine its efficiency. We propose to do this in the near future

MAXIMA OF GAUSSIAN FIELDS

Unlike the case for their Markov counterparts, very little is known about the precise distribution of the maxima of Gaussian processes. In fact, if attention is restricted to the stationary case, then there are only six Gaussian processes on the line for which this distribution is known exactly. A fortiori, the situation is worse for Gaussian fields, and in fact, there is not a single Gaussian field, neither stationary nor non-stationary, for which the precise distribution of the maximum is known.

Partly, or perhaps primarily, because of this dearth of results a large amount of effort has been devoted to studying the asymptotic distribution of these maxima. The basic result in this area is due to Fernique, Landau, Marcus and Shepp, and states that for any almost surely continuous, mean zero, Gaussian process, $X(t)$, where t varies over almost any parameter set, we have the following result for any $a > 0$ and any x large enough.

$$P\{ \sup[X(t)] > x \} < \exp\{ -x^2[1-a]/2m \},$$

where $m = \sup\{\text{var}[X(t)]\}$.

The suggestion made, implicitly, in the proposal was that, based on experience from a result in empirical processes, obtained the previous year with Larry Brown (Adler and Brown (1985)) it should be possible to remove the rather annoying factor of a from the right hand side of the above inequality.

That this is in fact the case has now been established for a wide variety of situations, and the results have been presented in Adler and Samorodnitsky (1985). (Samorodnitsky is an exceptionally gifted doctoral student supported by the Technion while working on this project.)

The examples that we have considered up until now include all Gaussian fields defined on finite dimensional Euclidean spaces, as well as a number of fields defined on classes of subsets of Euclidean space, such as the families of all half-spaces, all triangles, all quadrilaterals, etc. All of these examples are of particular interest in the study of multi-dimensional empirical processes.

This work is still in a very active phase, with the next step being the distribution of the maximum of Gaussian fields defined over families such as the family of convex sets in the plane. The results are both of pure theoretical interest and of substantial possible usefulness in the application of advanced multi-dimensional Kolmogorov-Smirnov tests.

GENERALISED GAUSSIAN FIELDS

The probabilist's appeal to the ubiquitous Central Limit Theorem is his standard procedure for justifying to the world his not inconsiderable fascination with, and dependence on, the normal distribution and the Gaussian process. In order to give a heuristic background to the so-called "free field" of Quantum Field Theory, Wolpert (1978) proved a delightful central limit theorem that showed how this generalised process (i.e. a process indexed by infinitely differentiable functions rather than time or some other simple parameter) could be realised as a normalised sum of the self-intersection local times of an infinite collection of Brownian motions. Although his method is too long to describe here, let it suffice to say that his motivation was, essentially, to allow one to think of the free field as a consequence of the activities of certain physical particles undergoing Brownian motion.

Wolpert actually went beyond this, for he also showed how the Wick powers of the free field (which are essentially the "generalised chi-squared process" generated by the field) are related to the elementary Brownian particles.

Wolpert's work related only to one generalised Gaussian process - the free field. From recent work of Dynkin (1980, 1981, 1983) it is reasonably clear that Wolpert's procedure should work in far greater generality, and that his central limit theorem should give an alternate representation of generalised processes and their functionals to that usually provided by the theory of multiple Wiener-Ito integrals.

I have been working on this idea together with a doctoral student, Ms. Raisa Epstein. At this stage we have, in fact, managed to extend Wolpert's result to arbitrary Gaussian fields, where the building blocks are now arbitrary Markov processes rather than Brownian motion.

Work on this project is continuing apace, and once we have a more complete theory, we hope to do a fair amount of writing up.

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PUBLICATIONS PREPARED UNDER THE GRANT

1. Tail behaviour for the suprema of empirical processes,
(R. J. Adler and L. D. Brown). To appear in the Annals
of Probability.
2. The supremum distribution of the Gaussian process,
(R. J. Adler and G. Samorodnitsky) Submitted.
3. Testing for the normality of a stochastic process,
(R. J. Adler and S. Feingold) Submitted.
4. Multivariate Kolmogorov-Smirnov statistics,
(L. D. Brown and R. J. Adler) In preparation.
5. Extrema and level crossings of chi-squared processes,
(M. Aronowich and R. J. Adler) Submitted.
6. Extrema and level crossings of chi-squared random fields,
(M. Aronowich) In preparation.

CONFERENCES ATTENDED AND VISITS

I attended the 14th Conference on Stochastic Processes and their Applications in Gotenborg, Sweden, from June 12-16, 1984, and presented an (invited) paper entitled "Tail behaviour for the suprema of empirical processes".

During the summer of 1984 I visited the following American institutions, and conferred there with the following colleagues:

1. University of Washington: Pyke, Alexander, Bass (4 weeks)
2. Colorado State University: Resnick, Davis, Tavaré (1 week)
3. Centre for Stochastic Processes, Chapel Hill: Cambanis, Leadbetter, Kallianpur (1 week)

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