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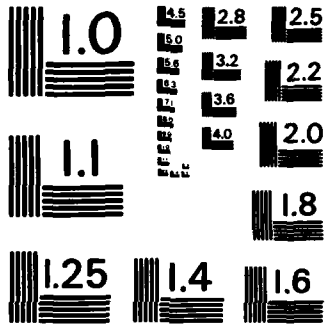


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This research was concerned with a number of topics related to error control for noisy communications channels. Specific topics included: probability of undetected error for error detection codes, coding for discrete-time continuous-amplitude signalling, tail-biting for convolutional codes, combined modulation and coding using amplitude, phase and frequency modulation, and coding for fault tolerant systems and VLSI.

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AD-A162 196

Final Scientific Report

AFOSR-82-0061

"A Study of Error Detection and Correction Codes"

for the period

February 1, 1982 to July 31, 1985

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## ABSTRACT

This research was concerned with a number of topics related to error control for noisy communications channels. Specific topics included: probability of undetected error for error detection codes, coding for discrete-time continuous-amplitude signalling, tail-biting for convolutional codes, combined modulation and coding using amplitude, phase and frequency modulation, and coding for fault tolerant systems and VLSI.



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Final Report on AFOSR-82-0061

"A Study of Error Detection and Correction Codes"

## I. Introduction

This three and one-half year effort was concerned with a number of topics concerned with error detection and error correction codes. The objective was to find and analyze good codes to achieve efficient and reliable transmission of data over noisy communications channels.

In the first year, the effort was specifically concerned with error detection codes and their performance. Since such codes are often used to insure the integrity of information and control messages, it is imperative that we know the probability that an error detection code fails to detect errors which actually are present in a received message. This probability, called the probability of undetected error, is the single-most important design parameter of an error detection code. An often used bound for binary codes is that the probability of undetected error for an error detecting code is less than or equal to "two" raised to an exponent equal to the negative of the number of parity bits in the code. This bound is not always correct and its use can be very misleading in instances where the bound does not hold. The first year's effort was concerned primarily with a study of the probability of undetected errors for error detecting codes, and, in particular, establishing general rules for which this bound can be used correctly. Details of the results are given below.

A subject which was initiated during the first year and carried over to the second year was coding for discrete-time amplitude-continuous signalling. Here we are concerned with adding redundancy to a sequence of real or complex numbers so as the resultant redundant sequence has immunity to impulse type errors. The codes discovered were closely related to Reed-Solomon codes. A number of topics were studied related to these codes. In particular, we considered decoding algorithms which allowed for small errrors on each of the components in addition to the large impulse-type errors.

In the second and third year of this research, two problems related to convolutional codes were investigated. The first concerns a technique called tail-biting whereby no rate loss occurs when a convolutional code is truncated to form a block code. This is achieved by having the initial state of the

convolutional code carry information and then force the encoder to end in that state. (In a generalization of this scheme, called generalized tail-biting, only part of the initial state carries information.)

In its broadest context, this technique allowed for a general theory to be developed which describes both convolutional codes and quasi-cyclic block codes. In particular, good convolutional codes were used to yield good quasi-cyclic block codes and (perhaps more importantly) vice versa. A number of related topics were also considered, one being the design of a new class of unequal error protection codes.

A study of another application of convolutional codes was also initiated during the second year. Here we began with the work of Ungerboeck (G. Ungerboeck, "Channel Coding with Multilevel/Phase Signals," IEEE Trans. on Information Theory, vol. IT-28, pp. 55-67, Jan. 1982) who used amplitude and phase modulation to improve the performance of signal designs without sacrificing data rate or increasing bandwidth. We generalized this approach to the case where we use amplitude, phase and frequency modulation. We found that under certain definitions of bandwidth (e.g. the so-called "99% bandwidth") the new codes had better Euclidean distance than the Ungerboeck codes for the same bandwidth and power.

During the last year's effort, work was initiated on finding codes which were particularly suited to fault tolerant decoders, especially decoders implemented in VLSI. Several new codes were found, one being a class of majority logic decodable burst error correcting codes and the second being a generalization of Gilbert burst error correcting codes. A new fault tolerant implementation of a bank of modulo 2 adders was found. This circuit is particularly useful for syndrome calculation in VLSI. Second, the notion of a redundant syndrome was investigated whereby redundant parity equations were used to construct a syndrome which possessed error detection and correction capabilities. Finally, a fault tolerant majority decoder was designed.

Finally, work was begun on utilizing the notions of error control to design high density mangetic recording systems. The basic idea followed to date was to assume that the recording system was equalized to that of a known partial response channel (e.g., the Class IV partial response channel) and then to design modulation and coding systems which work well with such a channel. This work was in its preliminary stage when the grant expired.

Two M.S. dissertations and two Ph.D. dissertations were completed by students supported under this grant. Two other Ph.D. students supported by this grant, are still carrying out their research. Both of these theses are expected to be completed within the next year.

Since most of the details of this research is contained in a series of publications authored by the principal investigator and his graduate students, the abstracts of these publications are given in the next section. The last section contains the abstracts of the two Ph.D. dissertations completed under this grant.

## II. Publications and Abstracts

1. J.K. Wolf, "Redundancy and the Discrete Fourier Transforms," Proceedings of the Sixteenth Annual Conference on Systems Science and Systems, pp 156-158, Princeton, New Jersey, March 1982

Abstract: The relationship between the discrete Fourier transform and error control codes is examined. Under certain conditions we show that discrete time sequences carry redundant information which then allow for the detection and correction of transmission errors at the receiver. An application of this technique to impulse noise cancellation is described.

2. R. Padovani and J. K. Wolf, "Data Transmission Using Error Detection Codes," Conference Record, IEEE Global Telecommunications Conference, vol 2, pp 626-631, November 1982

Abstract: The problem considered is the calculation of the probability of undetected error for linear binary block codes used solely for error detection. Certain codes are shown to be optimal for error detection in that their probability of undetected error is as small as could possibly be obtained for codes with the same block length and same redundancy. We find that certain well known codes, namely Reed-Muller codes and certain product codes have poor performance as error detection codes.

3. J. K. Wolf, "Redundancy, the Discrete Fourier Transform, and Impulse Noise Cancellation, IEEE Trans. on Communications, vol COM-31, pp 458-461, no 3, March 1983

Abstract: The relationship between the discrete Fourier transform and error-control codes is examined. Under certain conditions we show that discrete-time sequences carry redundant information which then allow for the detection and correction of errors. An application of this technique to impulse noise cancellation for pulse amplitude modulation transmission is described.

4. J. K. Wolf and T. K. Philips, "The Detection and Estimation of a Single Impulse Using the Discrete Fourier Transform," Neuvieme Colloque sur le Traitement du Signal et ses Applications, pp 131-134, May 1983

Abstract: Given a vector $\underline{x}$ = (x[0],x[1],...,x[N-1]) whose components x[i] are complex numbers. Assume that the Discrete Fourier Transform (D.F.T.) of the vector $\underline{x}$ has the property that two consecutive components are identically zero.

First we show that if $\underline{y} = \underline{x} + I\underline{\delta}_i$ where $\underline{\delta}_i$ = <----i----> (0,0,...,0,1,0,...,0) and I is an arbitrary complex number, then the original vector $\underline{x}$ can be uniquely determined from the observed vector $\underline{y}$ (without error). Then we investigate the case where $\underline{y} = \underline{x} + I\underline{\delta}_i + \underline{n}$ where $\underline{n}$ is a vector whose components are i.i.d. Gaussian random variables. We find the optimal decision rule for determining the location of the impulse I (that is, for determining the value of i). Finally we determine the performance of this optimal system.

5. R. Padovani and J. K. Wolf, "Poor Error Correction Codes are Poor Error Detection Codes," IEEE Trans. on Information Theory, vol IT-30, pp 110-111, no 1, January 1984

Abstract: We show that binary group codes that do not satisfy the asymptotic Varshamov-Gilbert bound have an undesirable characteristic when used as error detection codes for transmission over the binary symmetric channel.

6. H. H. Ma and J. K. Wolf, "On Tail Biting Convolutional Codes," accepted for publication in IEEE Trans. on Communications

Abstract: In this paper we introduce generalized tail biting encoding as a means to ameliorate the rate deficiency caused by zero-tail convolutional encoding. This technique provides an important link between quasi-cyclic block and convolutional codes. Optimum and sub-optimum decoding algorithms for these codes are described and their performance determined by analytical and simulation techniques.

7. H. H. Ma and J. K. Wolf, "Binary Unequal Error-Protection Block Codes Formed from Convolutional Codes by Generalized Tail Biting," submitted for publication, IEEE Trans. on Information Theory. Also presented at IEEE International Symposium on Information Theory, Brighton, England, 1985

Abstract: A design procedure is presented for synthesizing threshold decodable, rate 1/n and (n-1)/n, self-orthogonal block codes obtained from truncating convolutional codes by GTB. When GTB is applied to an equal error protection threshold decodable convolutional code, the result is a class of unequal error protection threshold decodable block codes. These codes are related to unequal error protection codes previously described by Mandelbaum. Furthermore, the codes of Mandelbaum are generalized. Finally, a relationship between quasi-cyclic self-orthogonal block codes and tail biting self-orthogonal convolutional codes is discussed.

8. R. Padovani and J. K. Wolf, "Coded Phase/Frequency Modulation," presented at IEEE International Symposium on Information Theory, Brighton, England, June 1985. Also submitted to IEEE Trans. on Information Theory

Abstract: Channel coding combined with expanded signal sets has been shown to improve error performance over uncoded modulation without expanding the bandwidth of the transmitted signals. In this paper new coded modulation formats defined over an expanded set of signals varying both in phase and frequency are presented. The new schemes combine FSK and PSK modulation and make use of trellis coding and Viterbi decoding to improve error performance over uncoded modulation.

The free Euclidean distance is calculated for several classes of codes and upper bounds and simulation results are also presented for some simple codes. The spectral characteristics of the proposed coded modulation formats are evaluated and compared to conventional two-dimensional modulation formats.

Differential encoding and various extensions of the basic scheme are also discussed.

9. R. Padovani and J. K. Wolf, "Spectral Analyses of Phase/Frequency Modulation," submitted for publication to the IEEE Trans. on Communications

Abstract: In this paper, the power spectral density of a form of digital signalling is derived. The modulation format combines PSK and FSK modulation for transmissions of m bits of information per modulation interval T.

## III. Ph.D. Dissertations and Abstracts

1. R. Padovani, "Signal Space Channel Coding: Codes for Multilevel/Phase/Frequency Signals," University of Massachusetts, Amherst, MA, February 1985

Abstract: Channel coding combined with expanded signal sets has been shown to improve error performance over uncoded modulation without expanding the bandwidth of the transmitted signals. Remarkable coding gains are obtainable by combining rate $R = m/(m+1)$ convolutional encoders with $2^{m+1}$-point signal constellations for transmission of m bits of information per modulation interval.

The redundancy introduced in the signal space can be effectively exploited by a maximum-likelihood decoder (Viterbi decoder) to improve error performance as compared with uncoded transmission of the same information.

The same approach is applied to new modulation formats that combine amplitude, phase, and frequency modulation. Several new codes are presented and their performance analyzed on both the additive white Gaussian noise channel and a bandlimited channel.

The spectral characteristics of the proposed codes are derived and compared with those of conventional two-dimensional modulation formats. Finally, phase recovery and differential encoding to resolve phase ambiguities are discussed.

2. H. H. Ma, "Generalized Tail-Biting Convolutional Codes," University of Massachusetts, Amherst, MA, February 1985

Abstract: Generalized tail-biting (GTB) convolutional encoding has been presented as a means to ameliorate the rate deficiency caused by zero-tail convolutional encoding. This GTB encoding provides a block structure for convolutional codes without sacrificing any rate loss. Two special schemes of GTB are discussed. The first scheme uses the full constraint lengths (m) of information blocks as tails and is called full tail-biting (FTB) encoding. The second scheme uses $m'(0<m'<m)$ information blocks as tails and is called partial tail-biting (PTB) encoding. It is shown that the FTB encoding increases the code complexity by a factor of $2^m$ and the PTB encoding increases the code complexity by a factor of $2^{m'}$ when compared to an (n,k,m) zero-tail convolutional code.

Using the generator matrix characterization, an equivalence relationship is developed between binary rate k/n convolutional codes and quasi-cyclic block codes of period n. This relationship is utilized to design both good systematic convolutional codes and block codes. These codes are then compared with existing good codes. It is concluded that some of the systematic convolutional codes found from the best quasi-cyclic block codes achieve larger minimum distances than those of previously known convolutional codes. The

block codes obtained from truncating the best convolutional codes by FTB are also found to possess good minimum distance property.

Maximum likelihood (optimum) Viterbi-like decoding is developed for the classes of FTB and PTB codes. This scheme basically performs exhaustive searches (by Viterbi decoding trials) for the codeword that satisfies the FTB or PTB encoding constraint. Therefore, two suboptimum decoding schemes are proposed to reduce the decoding complexity of the FTB codes. They both focus upon the important task of estimating the starting state of the transmitted sequence.

The first suboptimum scheme performs several Viterbi decoding trials until the starting state is equal to the ending state for the decoded sequence. Meanwhile, it always uses the ending state for the previous decoded sequence as the starting state for the next Viterbi decoding trial. This decoding scheme is most efficient when the equivalent block length (L+m) is about five constraint lengths long.

The second suboptimum scheme preprocesses the received polynomial algebraically using the continued fraction technique to obtain an ordered list of the $2^m$ starting states. Then, usual Viterbi trials are performed using each entry on the list as their starting state. Finally, decoding terminates when the codeword that satisfies the encoding constraint is found.

The decoding performance of both optimum and suboptimum decoding schemes for the binary symmetric channel are then evaluated using a transfer function technique, a Viterbi FTB decoding analysis method, and simulations.

A design procedure is presented for synthesizing threshold decodable, rate 1/n and (n-1)/n, self-orthogonal block codes obtained from truncating convolutional codes by GTB. When GTB is applied to an equal error protection threshold decodable convolutional code, the result is a class of unequal error protection threshold decodable block codes. These codes are related to a class of unequal error protection codes previously described by Mandelbaum. Furthermore, the codes of Mandelbaum are generalized. Finally, a relationship between quasi-cyclic self-orthogonal block codes and tail-biting self-orthogonal convolutional codes is discussed.

In addition, the following Ph.D. dissertations are presently under preparation:

W.-L. Zhang, "Coding for Fault Tolerant Systems"

E. Zehavi, "Modulation and Coding for Magnetic Recording Systems"

Finally, two M.S. dissertations were completed under this grant. They are:

R. Padovani, "The Probability of Undetected Error for Linear Block Codes," M.S. Thesis, University of Massachusetts, Department of Electrical and Computer Engineering, January 1983

T. Philips, "Error Correcting Codes Based on the Discrete Fourier Transform," M.S. Thesis, University of Massachusetts, Department of Electrical and Computer Engineering, April 1983

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