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ESTIMATION OF POWER SPECTRA FROM DATA CONTAINING
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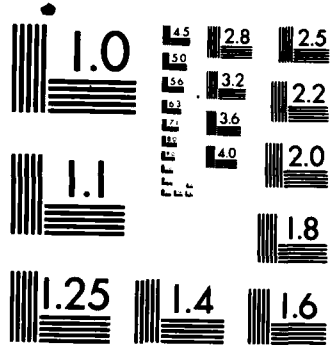
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Estimation of Power Spectra From Data Containing Recurring Pulses

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Preface

This reprint report was prepared under NUSC Project No. A75031, "Data Adaptive Detection and Estimation," Principal Investigator Dr. Roger F. Dwyer (Code 3314), Program Manager Dr. N. Gerr, ONR (Code 411 (SP)).

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ESTIMATION OF POWER SPECTRA FROM DATA CONTAINING RECURRING PULSES

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There are many physical sources that generate recurring pulses. Rotating fans and propellers are two examples. The objective of this paper is to present a signal processing methodology to extract recurring pulses from data. The method relies on obtaining discrete frequency components over a band or discrete temporal components over a time interval. It is shown that the results for the time and frequency domains are duals of each other. These ideas are then generalized to the spatial domain by including an array of sensors. It is shown that the time-spatial domain and the frequency-wavenumber domain are also duals of each other.

INTRODUCTION

The physical principle of extracting recurring pulses from data relies on obtaining discrete discernable components either in the frequency or time domain. Data from rotating machinery usually produce discrete harmonically related frequency components. On the other hand, discrete recurring temporal pulses are produced by other sources. For example, helicopter-radiated noise [1].

We shall first consider a signal processing methodology for a single sensor, to extract recurring pulses or harmonically related frequency components. The results for these two cases are related through a time-frequency duality principle. In addition, the extracted components are equivalent to the maximum likelihood estimate. On the other hand, for an array of sensors the signal processing method for these two cases are related through a time-spatial-frequency-wavenumber duality principle.

In the paper the data are assumed to be filtered to the desired passband and appropriately A/D converted.

The discrete Fourier transform is defined as follows,

$$X(K) = (1/\sqrt{N}) \sum_{i=0}^{N-1} x(i) \exp(-j2\pi Ki/N)$$

and its inverse

$$x(i) = (1/\sqrt{N}) \sum_{K=0}^{N-1} X(K) \exp(j2\pi Ki/N)$$

For an interval of time consisting of N samples let the data be represented by a succession of pulses recurring every τ seconds. This can be written as follows,

$$x(i) = \sum_{k=0}^{w-1} \delta[i - (i_g + k\tau)]$$

Where $\delta(\)$ is the Kronecker delta function. Here, i_g represents the sample on which the first of w pulses starts.

Notice that the spectrum of the above process is given by

$$X(K) = (w/\sqrt{N}) \exp[-j2\pi(i_g + \frac{w-1}{2}\tau) K/N] \cdot \left[\frac{\sin(\pi w K \tau / N)}{w \sin(\pi K \tau / N)} \right]$$

The location of the peaks of this spectrum are found from the solutions to the equation

$$\sin(\pi K \tau / N) = 0$$

which are given by, $K = (N/\tau)l_0$, for $0 \leq K \leq N-1$, and $l_0 = 0, 1, 2, \dots$

Therefore, r controls both the number and locations of the discrete components. As $w \rightarrow \infty$, the spectrum of the process will approach a purely line spectrum.

This example reveals the principle on which the signal processing method is based. Namely, all the information associated with recurring pulses is contained at discrete frequencies. The dual of this result is that all the information associated with the recurring pulses is contained at discrete time locations.

MATRIX METHOD

The discrete Fourier transform and its inverse can be conveniently discussed in terms of matrices and vectors.

Let

$$\underline{x} = (x(0), x(1), \dots, x(N-1))^T$$

be a vector of time domain data samples. Here T represents transpose.

Similarly, the frequency domain data is given by the vector

$$\underline{X} = (X(0), X(1), \dots, X(N-1))^T$$

where the two vectors are related through the transformation

$$\underline{X} = F \underline{x}$$

and F is a $(N \times N)$ transform matrix representing the DFT operation as follows:

$$F = (1/N)^{1/2} \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 \exp(-j2\pi/N) & \exp(-j2\pi 2/N) & \dots & \exp(-j2\pi(N-1)/N) \\ 1 \exp(-j2\pi 2/N) & \exp(-j2\pi 4/N) & \dots & \exp(-j2\pi 2(N-1)/N) \\ \vdots & \vdots & \vdots & \vdots \\ 1 \exp(-j2\pi(N-2)/N) & \exp(-j2\pi 2(N-2)/N) & \dots & \exp(-j2\pi(N-2)(N-1)/N) \\ 1 \exp(-j2\pi(N-1)/N) & \exp(-j2\pi 2(N-1)/N) & \dots & \exp(-j2\pi(N-1)(N-1)/N) \end{bmatrix}$$

and its inverse exists and is denoted by

$$F^{-1} = (1/N)^{1/2} \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 \exp(j2\pi/N) & \exp(j2\pi 2/N) & \dots & \exp(j2\pi(N-1)/N) \\ 1 \exp(j2\pi 2/N) & \exp(j2\pi 4/N) & \dots & \exp(j2\pi 2(N-1)/N) \\ \vdots & \vdots & \vdots & \vdots \\ 1 \exp(j2\pi(N-2)/N) & \exp(j2\pi 2(N-2)/N) & \dots & \exp(j2\pi(N-2)(N-1)/N) \\ 1 \exp(j2\pi(N-1)/N) & \exp(j2\pi 2(N-1)/N) & \dots & \exp(j2\pi(N-1)(N-1)/N) \end{bmatrix}$$

We will now define a diagonal $(N \times N)$ operator matrix Q which is applied in the frequency domain.

The extracted time domain data vector \underline{x}_F , after the matrix Q is applied in the frequency domain is given by the expression

$$\underline{x}_F = F^{-1} Q F \underline{x}$$

Notice that the time domain transformation $F^{-1} Q F$ is real. If we subtract \underline{x}_F from \underline{x} we obtain the resultant data vector, \underline{x}_R .

$$\underline{x}_R = \underline{x} - F^{-1} Q F \underline{x}$$

If $Q=I$ (identity matrix), then, $\underline{x}_R = \underline{0}$ (zero vector).

Whereas, if $Q = \tilde{0}$ (zero matrix), then $\underline{x}_R = \underline{x}$.

Therefore we can choose Q to obtain the desired result.

In applications the elements of the diagonal matrix Q are chosen to represent the locations where the discrete components occur in the frequency domain.

For example, let $N=1024$, and let there be four exact solutions located at $K=0, 256, 512$, and 768 .

$$\text{Then } Q_{ij} = \text{diag} [q_{ij}; i, j=0, 1, \dots, 1023]$$

where,

$$q_{ij} = \begin{cases} 1, & \text{at } i=j = 0, 256, 512, \text{ and } 768 \\ 0, & \text{otherwise} \end{cases}$$

Therefore,

$$F^{-1}QF = (4/N)E$$

where,

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & \dots & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & \dots & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & \dots & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & \dots & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & & & & & & & & & & & & & & & & & \\ 0 & & & & & & & & & & & & & & & & & \\ 0 & & & & & & & & & & & & & & & & & \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & \dots & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & \dots & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & \dots & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

(1024 x 1024)

The extracted data vector for this example is found from the expression

$$\underline{x}_E = (4/N) E \underline{x} = \underline{x}$$

So that, the extracted data vector is equal to the original data vector. The reason for this is that, in this example, all the information was located at discrete frequencies. Now, if we add noise of the form \underline{n} then

$$\underline{z} = \underline{x} + \underline{n}$$

and

$$\begin{aligned} \underline{z}_E &= (4/N) E \underline{x} + (4/N) E \underline{n} \\ &= \underline{x} + (4/N) E \underline{n} \end{aligned}$$

which reduces to, $\underline{z}_E = \underline{x}$. Since the noise is random, the noise vector will not match the transformation E and consequently the noise will be effectively averaged out.

The resultant data vector is obtained from the expression,

$$\underline{x}_R = \underline{x} - (4/N) E \underline{x} = 0.$$

Adding noise again we obtain,

$$\underline{z}_R = \underline{n} - (4/N) E \underline{n} = \underline{n}.$$

Therefore, for this example, the extraction method is nearly perfect even with additive noise. In practice the matrix method will be limited by the ability to choose the appropriate operator matrix Q .

We shall now compare this method with the maximum likelihood (ML) estimate [2]. Let,

$$\underline{z} = \underline{x} + \underline{n}$$

be the data. Here \underline{n} is i.i.d. Gaussian noise. We want to estimate the deterministic waveform \underline{x} at each sample point. These sample points will be denoted by \underline{a} . The ML estimate of \underline{a} is given by the procedure. Choose the value of \underline{a} , say $\hat{\underline{a}}$, that maximizes $f(\underline{z}; \underline{a})$, where, here, $f(\underline{z}; \underline{a})$ is a multidimensional Gaussian probability density function. Since, the waveform is a recurring pulse of duration h , the estimate reduces to the expression

$$\hat{a}_i = (1/w) \sum_{k=0}^{w-1} x(i_g + i + hk)$$

for $i=0,1,2,\dots,h-1$. So, each sample of the pulse is averaged over all respective samples of the successive pulses. This is exactly what the transformation $F^{-1}QF$ performs.

For a sinusoid, which is periodic, the same type of transformation, but dependent upon frequency, will evolve from the extraction procedure. Here, however, two components are associated with each sinusoid [3].

For example, let the frequency of a sinusoid be $k=256$. Then the operator matrix Q will have two nonzero components located at q_{ij} ($i=j=256$ and 768). The transformation will reduce to the following,

$$F^{-1}QF = (2/N) \begin{bmatrix} 1 & 0 & -1 & 0 & 1 & 0 & -1 & \dots & 0 & 1 \\ 0 & 1 & 0 & -1 & 0 & 1 & & & & \\ -1 & & & & & & & & & \\ 0 & & & & & & & & & \\ \vdots & & & & & & & & & \\ 0 & & & & & & & & & \\ 1 & & & & & & & & & 01 \end{bmatrix}$$

(1024 x 1024)

It is clear from this result that the exact sinusoid, including phase, will be extracted.

Now consider a frequency domain transformation. This procedure is a dual of the time domain transformation.

The extracted frequency domain data vector, \underline{X}_E is given by the expression

$$\underline{X}_E = FQ_t F^{-1} \underline{X}$$

Where Q_t is a diagonal operator matrix applied in the time domain. Now, $FQ_t F^{-1}$ will be complex in general. But the transformation performs a similar operation as its time domain dual. Namely, it averages the appropriate frequencies to produce the extracted data vector. This is also equivalent, under the stated conditions, to the ML estimate.

The resultant frequency domain data vector, \underline{X}_R , is given by the expression,

$$\underline{X}_R = \underline{X} - FQ_t F^{-1} \underline{X}$$

Transforming this result into the time domain we obtain the result,

$$\underline{x}_R = [I - Q_t] \underline{x}$$

The transformation $[I - Q_t]$ is equivalent to the ideal non-linearity discussed in [3,1]. This means that if we can identify the recurring pulses, or if we have a priori knowledge of them, in the time domain and apply the transformation $[I - Q_t]$ we will have subtracted the frequency domain ML estimate from \underline{x} .

Suppose the nonzero components of Q_t are located at the sample points, 0, 256, 512 and 768. Then the transformation will also reduce to $FQ_t F^{-1} = (4/N)E$.

An important application of this method is obtaining the power spectrum of a signal when recurring impulsive interference is present. The resultant data vector will demonstrate the ability to remove this interference.

From the above result we obtain the power spectrum from the the expression

$$\underline{x}_R^* \underline{x}_R = \underline{x}^* \underline{x} - \underline{x}^* FQ_t F^{-1} \underline{x}$$

Therefore, the resultant power spectrum is obtained by subtracting the power spectrum of \underline{x}_E from the power spectrum of the original data. An error analysis of this procedure was discussed in [4].

SPATIAL DOMAIN

These results can be generalized for an array of sensors. The two-dimensional discrete Fourier transform is given by the expression

$$X(L,K) = (1/\sqrt{M})(1/\sqrt{N}) \sum_{J=0}^{M-1} \sum_{i=0}^{N-1} x(J,i) \cdot \exp[-j2\pi(JL/M + iK/N)]$$

where, $J=0,1,\dots,M-1$ represent the spatial sensors, i.e., $x(0,i)$ is the first sensor at the i -th time sample. If, $N=1$, $x(J,0)$ and $X(L)$ will have the same relationship as $x(i)$ and $X(K)$ did before. But, now, J represents spatial position and L spatial frequency.

It is convenient to write this equation in matrix form.

$$\underline{X}_{FW} = F_{FW} S_{FW} \underline{x}_{TS}$$

where, \underline{X}_{FW} is a $(MN \times 1)$ vector and F_{FW} is a block diagonal $(MN \times MN)$ matrix. Each block is the $(N \times N)$ matrix F . The other matrix S_{FW} is a $(M \times MN)$ matrix of the form,

$$S_{FW} = (1/\sqrt{M}) \begin{bmatrix} I & I & & & I \\ I & Ie^{-j2\pi/M} & & & Ie^{-j2\pi(M-1)/M} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ I & Ie^{-j2\pi(M-1)/M} & \cdot & \cdot & Ie^{-j2\pi(M-1)(M-1)/M} \end{bmatrix}$$

where I is a $(N \times N)$ identity matrix, and \underline{x}_{TS} is a $(MN \times 1)$ vector.

We now define a $(MN \times MN)$ diagonal operator matrix Q_{FW} , so that

$$\underline{x}_{TS}^E = S_{FW}^{-1} F_{FW}^{-1} Q_{FW} F_{FW} S_{FW} \underline{x}_{TS}$$

is the extracted data vector of the time-space domain.

The dual of this result is the following,

$$\underline{x}_{FW}^E = F_{FW} S_{FW} Q_{TS} S_{FW}^{-1} F_{FW}^{-1} \underline{x}_{FW}$$

where Q_{TS} is a $(MN \times MN)$ diagonal operator matrix applied in the time-space domain.

SUMMARY

Since there are many physical sources that produce discrete spectra or discrete temporal components, a signal processing methodology was developed to take advantage of these phenomena. If a source produced discrete frequencies then all of the information associated with the source is contained at those discrete frequencies. By extracting only those frequencies a time domain enhancement was possible. By considering the transformation of the method it was revealed that the extracted data is equivalent to a maximum likelihood estimate under the assumption of i.i.d. Gaussian noise. A dual relationship was shown to hold for discrete temporal components. These results were then generalized for an array of sensors. In this case the results revealed a time-space-frequency-wavenumber duality.

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