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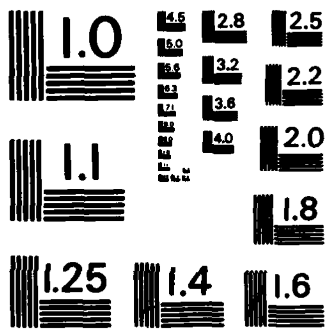

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An Efficient Random Access Algorithm for Packet Broadcast Channels with Long Propagation Delays

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Department of
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AN EFFICIENT RANDOM ACCESS ALGORITHM FOR PACKET BROADCAST CHANNELS WITH LONG PROPAGATION DELAYS

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Abstract

This paper introduces and analyzes an efficient algorithm for the random accessing of a broadcast channel by a large number of packet-transmitting, bursty users. The algorithm uses a mini-slot overhead, per packet, to extract detailed information regarding possible packet collisions. In the event of a collision, this information is used by the algorithm for accelerating the collision resolution process. The maximum stable throughput and the mean packet delay induced by the algorithm are evaluated via a systematic analysis method. The packet delay characteristics indicate that the proposed algorithm is well suited for operation on satellite channels, over a wide range of input traffic rates. Keywords: feedback; throughput.

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1. INTRODUCTION

We consider the random-accessing of a broadcast communication channel, (such as a satellite link, or a ground radio channel), by a large number of independent, packet-transmitting, bursty users, whose cumulative packet generating process is Poisson. Overlapping transmissions result in a collision, and all the packets involved must be retransmitted. Users monitor the channel activity and acquire some form of feedback information associated with packet transmissions. The feedback information, whose specific form depends on the precise model at hand, is then used by the users for scheduling packet transmissions and retransmissions in accordance with a distributed control scheme, referred to as random-access algorithm (RAA).

Given a specific model for the available feedback information, the performance of a random-access algorithm is commonly measured in terms of the packet delay characteristics that it induces, and the maximum throughput that it attains. For the Poisson infinite-population user model, the _maximum throughput_ is defined as the maximum input rate that the algorithm maintains with finite mean packet delay. A random-access algorithm is called _stable_ if its maximum throughput is greater than zero.

The design and performance of a random-access algorithm depend, critically, on the availability, quality, and timely acquisition of the feedback information, which in turn depend on the channel and user characteristics of the specific application at hand. Among the several aspects that characterize the feedback information structure we distinguish the following: 1) _feedback level_, 2) _feedback delay_. The feedback level specifies what kind of information, regarding the transmission activity on the channel, can be extracted by the users who monitor the channel. The feedback delay specifies when this information becomes available to the users, and is defined as the time it takes for a user to determine the current transmission activity on the channel. To be specific, let us assume that packets are of equal length, the channel is slotted with slot size equal to the packet transmission time, and that the users initiate transmission only at slot boundaries. In this case, the feedback information is commonly modeled as providing the "outcome" of each slot, at the end of each slot. (i.e., the feedback delay is assumed equal to one slot), where various such outcomes may be distinguished, depending on the feedback level. The most thoroughly studied feedback level is the <u>ternary</u>, which distinguishes among empty, busy with one packet (or successful), and busy with at least two packets (collision) slots. It has been shown that, under ternary feedback, the achievable maximum throughput cannot exceed 0.587 packets per slot [1]; the most efficient algorithm known to date attains a maximum throughput of 0.487 packets per slot [2]. In some applications, due to special modulation, coding and encryption techniques, or to channel noise conditions, ternary feedback might not be easily available. In such applications a binary feedback model is more realistic, and includes the following cases [3]: Distinguishing between either collision versus noncollision slots, (C-NC binary feedback) or empty versus nonempty slots, (E-NE binary feedback) or successful versus nonsuccessful slots, (S-NS binary feedback). Multiary feedback models have also been considered [4], where it is assumed that the existence of energy detectors enables users to determine the number of collided packets within each collision slot, whenever this number is below a certain limit; however, even with this more informative level of feedback, the maximum throughput of the existing algorithms does not exceed 0.532 packets per slot.

Given a feedback level, the performance of RAAs can be substantially improved if the feedback delay is small compared to the packet transmission time. This is, for example, the case in the multiple access channels used in some local area networks, where simple RAA's based on carrier sensing [5,6], attain maximum throughputs close to unity with uniformly low packet delays.

For channels with feedback delay equal to or larger than the packet transmission time, improvements in the maximum throughput are hard to obtain, as long as the algorithms rely on just the passive observation of channel history. For such channels, the maximum throughput can be improved by the use of reservation-based algorithms [7,8]. Reservation algorithms restrict the multiple access contention problem to a reservation subchannel. The reservation subchannel uses a fraction of the total channel capacity for the transmission of explicit reservation requests (mini-packets), and packets are transmitted collision-free following successful reservation requests. If the reservation overhead per packet is sufficiently small, reservation algorithms can attain maximum throughput close to unity, independently of how large the feedback delay is. In channels where the ratio between feedback delay and packet transmission time is large, however, this throughput improvement is obtained at the expense of higher mean packet delay at low input rates, as compared to the mean packet delay induced by stable RAA's. The reason for this delay increase is that packet transmissions are invariably associated with a minimum delay equal to the feedback delay of a reservation request. Considering, for example, a satellite channel where the round-trip propagation delay is equal to 270 msec, the use of reservations implies that packets suffer a delay of at least 540 msec before they reach their destination, even for near zero input rates. Note that the additional delay introduced by the required transit time of a reservation request may prove intolerable in application involving time critical interactive traffic, such as a remote computing, data base query, etc..

Hybrid schemes that combine the better features of random access and reservation access have also been proposed [9,10]. These schemes use TDMA or some other fixed assignment technique for the transmission of reservation packets, and for small user populations, they have excellent throughput-delay characteristics, However, their performance deteriorates rapidly as the number of users increases, since the reservation overhead is proportional to the user population size.

In this paper, we present and analyze a RAA that utilizes more detailed feedback information than the commonly used ternary feedback to accelerate the collision resolution process, and, therefore, to attain substantially higher maximum throughput. As explained below, the more detailed feedback information is created using a *partial reservation* strategy, which is based on the exchange of imprecise reservation requests conveying only a few "bits" of information; these partial reservation requests require little bandwidth on the channel, and their size is independent of the number of the channel users. In contrast to reservation algorithms, for channels with long propagation delay, the proposed algorithm exhibits good delay performance in both the low-throughput and the mid-throughput range.

To create the more detailed feedback the algorithm divides time into contiguous slots of equal length, taken as the unit of time. Each slot consists of a *control slot* (CS) and a *data slot* (DS). The CS is divided into m *control mini-slots* (CMS), each of which is of one bit (or a few bits) duration; the DS is long enough to accomodate a standard data packet, (see figure 1). Each time a user decides to transmit a packet, it transmits it in the DS and simultaneously it injects a pulse in one of the m CMS's chosen with equal probability. It is assumed that at the end of slot i+P all users can determine without error whether the transmissions in the i-th slot resulted in a packet collision in the DS (i.e., *C-NC binary feedback*), and whether a CMS of the i-th slot is empty (i.e., *E-NE binary feedback*); P denotes the channel's propagation delay in slots. The additional feedback information provided by the CS is used by the users to prevent idle slots and further collisions during the collision resolution phase, and is the principal mechanism for improving performance . Since the feedback from each CMS provides information only on the presence of absence of a pulse, the overhead needed for the GS is small. We shall see that this small investment of channel capacity results in substantial improvement in the overall system performance.

The control mini-slot concept, in conjuction with collision resolution algorithms, has been used by Huang and Xu [11], where ternary feedback is assumed available from each mini-slot, and Merakos and Kazakos [12], where the mini-slot feedback is of the binary type considered here. The algorithm considered in this paper is a generalized version of the second algorithm in [12], and is very similar to the one proposed and analyzed recently by Huang and Berger [13], [14], after an independent and concurrent investigation. An important contribution of the present paper is a novel delay analysis method, which is based on the asymptotic properties of regenerative processes. The proposed analysis method is of independent interest, and can be used for evaluating the performance of a large class of RAA's.

2. ALGORITHM DESCRIPTION AND ANALYSIS

In this section we describe and analyze an algorithm that operates on the basis of the feedback described in the introduction, when the feedback delay is unity, (i.e., when the propagation delay P is zero); this algorithm will be referred to as the zero propagation delay algorithm (ZPDA). In section 3, we show how the ZPDA can be modified to operate in a satellite channel, where $P \gg 1$, and we evaluate the performance attained by such modifications.

2.1 Description of the ZPDA

The ZPDA is defined by the following two rules:

Rule 1 (First Time Transmission)

All users observe the feedback at the end of each slot and utilize a counter, called global counter, to determine when to transmit a packet for the first time. A user initializes and updates his global counter according to the following rule.

> The counter is set to 1 at the beginning of the first slot, then incremented by (j-1) at the end of each collision slot with j nonempty mini-slots, and decremented by one at the beginning of each collision-free slot. When the counter reaches 0 it is immediately reset to 1 and the above described operation is repeated indefinitely.

Let G_i denote global counter indication at the beginning of slot i, $i>0$, and define the sequence $\{I_n\}_{n\geq 0}$ on the channel axis, and the sequence $\{t_n\}_{n\geq 0}$ on the arrival axis as follows:

$$I_0=1,\ I_{n+i}=\min\{i : G_i=0,\ i>I_n\}\ ,\ n=0,1,2,\ldots$$

$$t_0=0,\ t_{n+1}=t_n + \min\,(I_n-t_n,\Delta)$$

where Δ is a positive algorithmic parameter.

Using the above sequences, the first time transmission rule can be expressed as follows:

A packet arrived at time instant t is transmitted for the first time in slot I_n if $t \in [t_n, t_{n+1})$.

Rule 2 (Retransmission)

Each user with a packet involved in a collision, determines when to retransmit his packet using a second counter, called local counter, in accordance with the following rule.

Following a collision in which he is involved the user sets his local counter to (i-1), if he injected his pulse in the i-th nonempty mini-slot. Then, he increments it by (j-1) for each subsequent collision slot with j nonempty mini-slots, and decrements it by one for each subsequent collision-free slot. When the local counter reaches 0, the user retransmits in the next slot.

Remarks

1) As it can be seen from Rule 1, the ZPDA divides the arrival axis into contiguous time intervals $[t_n, t_{n+1})$, $n \geq 0$. The packets (if any) arrived in the interval $[t_n, t_{n+1})$ are first transmitted in slot I_n. If there is a collision in I_n, the collision is resolved, (i.e., the collided packets are eventually successfully transmitted), during slots I_n+1, $I_n+2, \ldots, I_{n+1}-1$, in accordance to Rule 2. This is illustrated in figure 2, where a collision of three packets is resolved.

The interval $[t_n, t_{n+1})$ will be referred to as the n-th enabled arrival interval (EAI); the interval $[I_n, I_{n+1})$ will be referred to as the n-th collision resolution interval (CRI), and the instants $I_n, n \geq 0$, will be referred to as collision resolution instants.

2) Rule 2 defines a collision resolution algorithm (CRA), which is a simple modification of the well-known Tree CRA [18]. The modification consists in utilizing the feedback information provided by the m CMS's to skip the empty

branches in an m-ary tree search, and, therefore, accelerate the collision resolution process.

3) The algorithm utilizes an elementary window flow control mechanism on the arrival axis by not allowing the length of an EAI to exceed Δ. The window size Δ will be optimized for throughput maximization.

2.2 Delay Analysis of the ZPDA

Consider the random access system operating with the ZPDA over the time interval $[0, +\infty)$. Let the arriving packets be labelled n=1,2,..., according to the order of arrival. We define the delay, $\mathcal{D}_n$, experienced by the nth packet as the time from its arrival at the transmitter until the completion of its successful reception by the receiver, (so that $\mathcal{D}_n = P+1$, when the nth packet is successfully transmitted beginning at the same moment it arrives at the transmitter - for the ZPDA, P=0). We are interested in evaluating the mean packet delay induced by the ZPDA.

From the rules of the algorithm it can be readily seen that, at each collision resolution instant, all packets arrived before time t_n have been successfully transmitted, and the packets arrived in the interval $[t_n, I_n)$ have not accessed the channel yet. Let $d_n \triangleq I_n - t_n, n \geq 0$; d_n will be referred to as the lag at I_n. Since $t_0=0$, $I_0=1$, we have $d_0=1$. Let $T_0 \triangleq I_0$ and define T_{i+1} as the first collision resolution instant after T_i, i=0,1,2,..., at which $d_{T_{i+1}}=1$.

From the operation of the algorithm it can be seen, after a little thought, that the delay process $\{\mathcal{D}_n\}_{n\geq 1}$ "probabilistically restarts itself" at each instant T_i, $i\geq 0$. The interval $[T_i, T_{i+1})$, $i \geq 0$, will be referred to as the ith session; since the packet arrival process (Poisson) is memoryless and has independent increments, the session lengths, $T_{i+1}-T_i, i\geq 0$, are independent, identically distributed (i.i.d.) random variables.

Let R_i, $i\geq 0$, denote the number of packets successfully transmitted in the interval $(T_0, T_i]$. (Note that R_i denotes also the number of packets arrived during $[0, T_i-1))$. Accordingly, $C_i = R_{i+1} - R_i$, $i\geq 0$, is the number of packets successfully transmitted during the ith session. The sequence $\{R_i\}_{i\geq 0}$ is a renewal process, since $\{C_i\}_{i\geq 0}$ is a sequence of nonnegative i.i.d. random variables. Furthermore, the delay process $\{\mathcal{D}_n\}_{n\geq 1}$ is regenerative with respect to the renewal process $\{R_i\}_{i\geq 0}$, with regeneration cycle C_0; that is, the process $\{\mathcal{D}_{R_i+n}\}_{n\geq 1}$, for every $i\geq 0$, is a probabilistic replica of the process $\{\mathcal{D}_n\}_{n\geq 1}$.

The regenerative probabilistic structure of the delay process makes it possible to express its asymptotic behavior in terms of quantities that refer only to one regeneration cycle of the process. This is made precise by the following elegant and powerful result from the theory of regenerative processes [15].

Theorem 1

If $C \triangleq E(C_0) < \infty$ and $S \triangleq E(\sum_{i=1}^{C_0} \mathcal{D}_i) < \infty$, then there exists a real number D such that

$$D = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \mathcal{D}_i = \lim_{n\to\infty} \frac{1}{n} E\{\sum_{i=1}^{n} \mathcal{D}_i\} = \frac{S}{C} \qquad \text{w.p.1}$$

Furthermore, if, in addition to the finiteness of C and S, the distribution of C_0 is not periodic, then $\{\mathcal{D}_n\}_{n\geq 1}$ converges in distribution to a random variable $\mathcal{D}_\infty$, and

$$E(\mathcal{D}_\infty) = \frac{S}{C}$$

In our case, the distribution of C_0 is clearly not periodic; thus, provided that both C and S are finite, the limiting average, the limiting expected average, and the mean of the limiting distribution of $\{\mathcal{D}_n\}_{n\geq 1}$, exist, coincide, and are finite; their common value D will be referred to as the mean packet delay. Note that D can be computed using only the per cycle quantities C and S.

Next we develop two systems of equations, whose solution is then used to compute the mean cycle length C, and the mean cumulative delay over a cycle S .

2.2.1 Computation of the quantities C and S.

Consider first the mean cycle length C. If the mean session length $H \triangleq E\{T_{i+1} - T_i\}$, $i \geq 0$, is finite, then by Wald's theorem we have that

$$C = \lambda H \tag{1}$$

To determine H we proceed as follows.

Let I(d) be a collision resolution instant at which the lag is equal to d; let T(1) be the first collision resolution instant after I(d) at which the lag is equal to one. The interval [I(d), T(1)) is called a d-session. Next we define the following random variables:

h_d : the length of a d-session

ℓ_d : the length of the first CRI of a d-session

Note that, by definition, $H = E(h_1)$,

From the description of the algorithm we have that the lags at two successive collision resolution instants satisfy the following relation.

$$d_{i+1} = \begin{cases} \ell_{d_i} & \text{if} \quad d_i \leq \Delta \\ d_i - \Delta + \ell_{d_i} & \text{if} \quad d_i > \Delta \end{cases} \tag{2}$$

From (2) and since $d_0 = 1$, we deduce that $d_i \in A$, for every $i \geq 0$, where A is a denumerable subset of the set of positive real numbers; specifically

$$A = \{d : d = k - i\Delta \geq 1;\ k,i \text{ integers}\}$$

From the operation of the algorithm and in view of (2), it can be readily deduced that the h_d, $d \in A$, satisfies the following relations:

$$h_d = \begin{cases} \ell_d & \text{if} \quad \ell_d = 1 \\ \ell_d + h_{\ell_d} & \text{if} \quad \ell_d > 1 \end{cases} \quad ,1 \leq d \leq \Delta \tag{3.a}$$

$$h_d = \ell_d + h_{d-\Delta+\ell_d} \quad , \; d>\Delta \tag{3.b}$$

If we let $H_d = E\,(h_d)$, then taking expectations in (3) yields

$$H_d = E\,(\ell_d) + \sum_{i=2}^{\infty} p_d(i)\, H_i \quad , \; 1 \le d \le \Delta \tag{4.a}$$

$$H_d = E\,(\ell_d) + \sum_{i=1}^{\infty} p_d(i)\, H_{d-\Delta+i} \quad , \; d > \Delta \tag{4.b}$$

where $p_d(i)$ is the probability distribution of $\ell_d, d \varepsilon A$. Note that for $d > \Delta$, $E\,(\ell_d) = E\,(\ell_\Delta)$, and $p_d(i) = p_\Delta(i)$, since when the lag is greater than Δ the enabled interval is of length Δ. Thus, the sequence $\{H_d\}_{d \varepsilon A}$ satisfies the following infinite dimensional system of linear equations

$$x_d = b_d + \sum_{t \varepsilon A} c_{dt}\, x_t \quad , \; d \;\varepsilon\; A \tag{5}$$

where $b_d = E\,(\ell_d)$, $1 \le d \le \Delta$, $b_d = E\,(\ell_\Delta)$, $d > \Delta$, and where c_{dt}, $d,t \;\varepsilon\; A$, are nonnegative coefficients that can be appropriately identified from (4).

Since $H = H_1$, we have from (1) that $C = \lambda\, H_1$. Thus, to compute C we need to compute H_1. This will be done by solving system (5). Before examining the solution of system (5), however, we first develop a similar system of equations that can be used to compute the mean cumulative delay S. For the development of such a system we need the following definitions:

w_d : the sum of the delays experienced by all the packets that were successfully transmitted during a d-session.

ω_d : the sum of the delays experienced by all the packets that were successfully transmitted during the first CRI of a d-session.

The operation of the algorithm yields the following relations for the w_d, $d \;\varepsilon\; A$.

$$w_d = \begin{cases} \omega_d & \text{if } \ell_d = 1 \\ \omega_d + w_{\ell_d} & \text{if } \ell_d > 1 \end{cases} \quad , \; 1 \leq d \leq \Delta \qquad (6.a)$$

$$w_d = \omega_d + w_{d-\Delta+\ell_d} \quad , \; d > \Delta \qquad (6.b)$$

If we let $W_d = E\,(w_d)$, then taking expectations in (6) yields

$$W_d = E\,(\omega_d) + \sum_{i=2}^{\infty} p_d(i)\, W_i \quad , \; 1 \leq d \leq \Delta \qquad (7.a)$$

$$W_d = E\,(\omega_d) + \sum_{i=1}^{\infty} p_d(i)\, W_{d-\Delta+i} \, , \quad d > \Delta \qquad (7.b)$$

From (7) we have that the sequence $\{W_d\}, d \in A$, satisfies the following infinite dimensional system of linear equations

$$x_d = b'_d + \sum_{t \in A} c_{dt}\, x_t \quad , \; d \in A \qquad (8)$$

where $b'_d = E\,(\omega_d)$, $d \in A$, and the nonnegative coefficients c_{dt}, $d, t \in A$, are as defined in system (5).

The mean cumulative delay S can be computed from the solution of system (8), since, by definition,

$$S = W_1 \qquad (9)$$

Before proceeding to the solution of systems (5), and (8) we elaborate on $E\,(\omega_d)$. With reference to figure 3 we define the following

n_d : the number of packets successfully transmitted during the first CRI of a d-session (Note that n_d is also the number of the Poisson arrivals in the enabled arrival interval (t_n, t_{n+1})).

u_d : the sum of the delays of the packets that were successfully transmitted during the first CRI of a d-session, measured at time t_{n+1}.

v_d : the sum of the delays of the packets that were successfully transmitted during the first CRI of a d-session after the first CRI begins.

Using the above defined random variables we can express ω_d as follows

$$\omega_d = u_d + v_d \qquad \text{if} \quad 1 \leq d \leq \Delta \tag{10.a}$$

$$\omega_d = u_\Delta + v_\Delta + n_\Delta\ (d-\Delta) \quad \text{if} \quad d > \Delta \tag{10.b}$$

Taking expectations in (10), and using known properties of the Poisson process we obtain

$$E\ (\omega_d) = E\ (v_d) + \frac{\lambda d^2}{2} \qquad \text{if } 1 \leq d \leq \Delta \tag{11.a}$$

$$E\ (\omega_d) = E\ (v_\Delta) + \frac{\lambda \Delta^2}{2} + \lambda\ \Delta\ (d-\Delta) \quad \text{if} \quad d > \Delta \tag{11.b}$$

Solving systems (5) and (8)

Formally, both system (5) and system (8) always have an infinite solution $x_d = +\infty$, $d \in A$. The following theorem specifies a sufficient condition under which both systems have nonnegative finite solutions ($0 \leq x_d < \infty$, $d \in A$, $d < +\infty$) that coincide with the corresponding algorithmic sequences induced by the ZPDA, and gives upper and lower bounds on such solutions.

Theorem 2

If

$$\Delta > E\ (\ell_\Delta) \tag{12}$$

then (i) system (5) has a solution $\{y_d\}$ $d \in A$, and system (8) has a solution $\{z_d\}$ $d \in A$, such that

$$\alpha_u\, d + \beta_\ell \leq y_d \leq \alpha_u\, d + \beta_u \qquad , d \in A \tag{13}$$

and

$$\gamma_u\, d^2 + \delta_u d + \zeta_\ell \leq z_d \leq \gamma_u\, d^2 + \delta_u d + \zeta_u \ , \ d \in A \tag{14}$$

where α_u, β_u, β_ℓ, γ_u, δ_u, ζ_u, ζ_ℓ are real coefficients whose expressions are given in Appendix A.

(ii) the algorithmic sequences $\{H_d\}$ $d \in A$ and $\{W_d\}$ $d \in A$ coincide with the solutions $\{y_d\}$ $d \in A$ and $\{z_d\}$ $d \in A$, respectively.

The proof of theorem 2 is given in Appendix A.

Under (12), theorem 2 gives the following bounds on H_d, W_d, $d \in A$.

$$\alpha_u d + \beta_\ell \triangleq {}^{(0)}H_d \leq H_d \leq H_d^{(0)} \triangleq \alpha_u d + \beta_u \ , \ d \in A \tag{15}$$

$$\gamma_u d^2 + \delta_u d + \zeta_\ell \triangleq {}^{(0)}W_d \leq W_d \leq W_d^{(0)} \triangleq \gamma_u d^2 + \delta_u d + \zeta_u \ , \ d \in A \tag{16}$$

The above initial bounds can be improved using the method described below. Consider the case where we want to improve the upper bound on H_d. To this end, define

$$H_d^{(1)} \triangleq E(\ell_d) + \sum_{i=2}^{\infty} p_d(i)\ H_i^{(0)} = H_d^{(0)} + E(\ell_d) + \alpha_u(E(\ell_d)-d) - (1+\lambda d)\exp(-\lambda d)(\alpha_u+\beta_u), \quad \underline{1 \leq d \leq \Delta}, \tag{17.a}$$

$$H_d^{(1)} \triangleq E(\ell_\Delta) + \sum_{i=1}^{\infty} p_\Delta^{(i)}\ H_{d-\Delta+i}^{(0)} = H_d^{(0)} + E(\ell_\Delta) - \alpha_u(\Delta - E(\ell_\Delta)), \ d > \Delta, \tag{17.b}$$

Under (12), the coefficients α_u, β_u have been chosen so that

$$H_d^{(1)} \leq H_d^{(0)} \ , \quad d \in A \tag{18}$$

Given a natural number $N > 2$, define, for $n > 1$,

$$H_d^{(n)} \triangleq E(\ell_d) + \sum_{i=2}^{N-1} p_d^{(i)} \, H_i^{(n-1)} + \sum_{i=N}^{\infty} p_d(i) \, H_i^{(0)} \quad , \quad 1 \leq d \leq \Delta \tag{19.a}$$

$$H_d^{(n)} \triangleq E(\ell_\Delta) + \sum_{i=1}^{N_0} p_\Delta^{(i)} \, H_{d-\Delta+i}^{(n-1)} + \sum_{i=N_0}^{\infty} p_\Delta^{(i)} H_{d-\Delta+i}^{(0)} \quad , \quad \Delta < d < N \tag{19.b}$$

where N_0 is the maximum integer satisfying $d-\Delta+N_0 < N$

For $n > 1$, combining (17) and (19) yields

$$H_d^{(n)} = H_d^{(1)} + \sum_{i=2}^{N} p_d^{(i)} \, (H_i^{(n-1)} - H_i^{(0)}) \qquad 1 \leq d \leq \Delta \tag{20.a}$$

$$H_d^{(n)} = H_d^{(1)} + \sum_{i=1}^{N_0} p_\Delta(i) \, (H_{d-\Delta+i}^{(n-1)} - H_{d-\Delta+i}^{(0)}) \quad , \quad \Delta < d < N \tag{20.b}$$

From (17), (18), and (19) we have that, for every $d \in [1,N)$ the sequence $\{H_d^{(n)}\}_{n \geq 0}$ is a monotonically decreasing sequence of upper bounds on H_d. The upper bound $H_d^{(n)}$, $n > 1$, $d \in [1,N)$ can be computed using (20). In (20), Δ is taken to be rational so that d assumes finitely many values in the interval $[1,N)$. The probabilities $p_d(i)$, $1 \leq i < N$, $1 \leq d \leq \Delta$, are obtained, in a recursive form using the rules of the algorithm in conjuction with straight forward combinatorial arguments; their expressions can be found in [16].

To improve the lower bound ${}^{(0)}H_d$ on H_d, we follow the same procedure except that now ${}^{(0)}H_d$ is used instead of $H_d^{(0)}$; this results in a monotonically increasing sequence, $\{{}^{(n)}H_d\}_{n \geq 0}$, of lower bounds on H_d, for $d \in [1,N)$. The same method can be used to obtain a monotone decreasing sequence, $\{W_d^{(n)}\}_{n \geq 0}$, of upper bounds, and a monotone increasing sequence $\{{}^{(n)}W_d\}_{n \geq 0}$, of lower bounds on W_d, for $d \in [1,N)$.

2.2.2 Mean packet delay bounds and maximum stable throughput.

From theorems 1, 2 and (1), (9) we have that, under (12),

$$D_0 \leq D \leq D^0 \tag{21}$$

where

$$D_0 \triangleq \frac{1}{\lambda} \frac{{}^{(0)}W_1}{H_1^{(0)}} = \frac{\gamma_u + \delta_u + \zeta_\ell}{\lambda(\alpha_u + \beta_u)}, \quad D^0 \triangleq \frac{1}{\lambda} \frac{W_1^{(0)}}{{}^{(0)}H_1} = \frac{\gamma_u + \delta_u + \zeta_u}{\lambda(\alpha_u + \beta_\ell)}$$

The bounds given by (17) can be improved using the method described at the end of the previous section. The improved bounds are

$$D_n \leq D \leq D^n \tag{22}$$

where

$$D_n = \frac{1}{\lambda} \frac{{}^{(n)}W_1}{H_1^{(n)}}, \quad D^n = \frac{1}{\lambda} \frac{W_1^{(n)}}{{}^{(n)}H_1}, \quad n = 1,2,\ldots.$$

The bounds given by (21) or (22) are valid for all λ's for which inequality (12) is satisfied. In Appendix B, it is shown that inequality (12) holds if

$$\lambda \leq \bar{\lambda}(m,\Delta)$$

where, for given m, $\bar{\lambda}(m,\Delta)$ is maximized for $\Delta = \Delta_m^*$. The values of $\bar{\lambda}(m,\Delta_m^*)$ and Δ_m^* are given in table 1, for different values of the parameter m. Table 2 gives the lower bound D_5, and the upper bound D^5 on the mean packet delay D for different values of m and for $\Delta = \bar{\Delta}_m$, where $\bar{\Delta}_m$ is a rational number close to Δ_m^*, (selected for convenience in the computation of the bounds).

If we define the maximum stable throughput, , of the ZPDA as

$$\eta \overset{\Delta}{=} \sup \{\lambda_0 : D < \infty \text{ for all } \lambda \in [0,\lambda_0)\}$$

then, for given m, $\overline{\lambda}$ (m,Δ_m^*) is a lower bound on η, since (12) is only sufficient for D to be finite. It can be easily shown, however, that the lower bound in (13), increases to infinity as $E(\ell_\Delta)$ increases to Δ, or equivalently as λ increases to $\overline{\lambda}(m,\Delta_m^*)$; thus, $\eta = \overline{\lambda}(m,\Delta_m^*)$. A more intuitive argument can also be used to show that (12) is a necessary condition for finite mean packet delay. Since the algorithm spends on the average $E(\Delta)$ units of time to successfully transmit the packets arrived during Δ units of time, inequality (12) is necessary for the ZPDA to keep up with the arrivals.

Finally, note that to account for the overhead created by the m control bits per slot the maximum stable thoughput η should be normalized to

$$\eta' = \frac{\eta}{1 + m\ r/L}$$

where L is the length of a standard data packet in bits, and r is the length of a CMS in bits. However, since the feedback from a CMS is of the E-NE type, r can be made very small--theoretically, 1 bit. On the other hand, L is of the order of 1000 bits. Thus, the reduction in throughput due to the overhead is small. The same normalization should be applied to the results listed in table 2; the values of λ should be divided by $(1 + m\ r/L)$, and the values of the delay bounds should be multiplied by $(1 + m\ r/L)$.

3. ADAPTATION OF THE ZPDA TO THE SATELLITE CHANNEL

Consider now a channel where the propagation delay, P, is much larger than the packet transmission time. A typical example is a satellite channel where the round-trip propagation is equal to 0.27 sec. If packets of 1125 bits are transmitted with a rate of 50 kbits/sec, then $P \simeq 12$ slots. Thus, the users of a satellite channel learn the outcome of a transmission $P(>>1)$ slots after the end of the transmission, (i.e, the feedback delay is equal to P+1 slots, where P>>1).

The simplest way to deal with the problem of non-zero propagation delay is to treat the random-access channel as $\overline{P} = P+1$ <u>interleaved</u> zero-propagation delay channels. In this scenario, slots i, $i+\overline{P}$, $i+2\overline{P}$,... of the actual channel form slots 1,2,3,... of the i-th interleaved channel, $i=1,2,\ldots,\overline{P}$. The Poisson arrival stream is subdivided into $\overline{P}$ interleaved substreams each one of which is served by a different interleaved channel in a <u>time division</u> (TD) manner. Specifically, a new packet that arrives during slot j-1, $j \geq 1$, is assigned to the interleaved channel corresponding to slots $j, j+\overline{P}, j+2\overline{P},\ldots$

Suppose now that the ZPDA is executed independently on each of the $\overline{P}$ interleaved channels. Then, clearly, the maximum stable throughput of the channel coincides with that induced by the ZPDA, i.e, $\overline{\lambda}(m,\Delta^*_m)$. The induced mean packet delay, D_{TD}, is given by (23),

$$D_{TD} = \overline{P}(D - \frac{1}{2}) + \frac{1}{2} \tag{23}$$

where D is the mean packet delay induced by the ZPDA. (23) follows from the fact that on the average a new packet waits half a slot before the next channel slot of the interleaved channel it is assigned to begins, and $\overline{P}\,(D - \frac{1}{2})$ channel slots until it is successfully received at its destination.

Bounds on D_{TD} are obtained by substituting the bounds on D, found in section 2.2.2, in (23). Figure 4 shows these bounds for different values of m, and for $P = 12$ slots.

Statistical time division (STD) is an alternative rule for assigning packets to the $\overline{P}$ interleaved channels. This rule assigns waiting packets to the first available interleaved channel. Specifically, let I_s denote the instant on the channel axis at which a CRI ends in any of the $\overline{P}$ interleaved channels; let also s be the time on the arrival axis for which it is known that all packets generated in the interval $[s, I_s)$ have not accessed any of the $\overline{P}$ channels yet. Then, all packets arrived in the interval $[s, s+\min(I_s - s, \Delta))$ are transmitted in slot I_s, i.e., they are assigned to the channel to which slot I_s belongs. The point s is updated as in the ZPDA, i.e., at each I_s, $s \rightarrow s+\min(I_s - s, \Delta)$; initially $s=\overline{P}$, since at the beginning of the operation of the system and for the first $\overline{P}$ slots packets are assigned to the $\overline{P}$ channels according to the TD rule. It is expected that the STD assignment rule induces improved delay characteristics as compared to those induced by the TD assignment rule, especially when the traffic is light to moderate. There are two reasons that support this claim, First, in contrast to TD, with STD no channel is idle while there are packets waiting for their first transmission. Second, since the arrival axis is not subdivided a priori, as in the TD rule, the CRI's start resolving enabled intervals of length closer to the optimal window size Δ. However, due to the interdependence in the operation of the interleaved channels, at this point we have not been able to analyze the delay characteristics induced by the interleaved system operating under the STD rule. The simulated mean packet delay performance is shown in figure 5 for $m=16$. For comparison purposes, in the same figure we have included the mean packet delay curve for $m=16$ induced by the TD assignment rule. It is clear that the STD scheme offers considerable delay performance improvement as compared to the TD scheme. In the same figure, the straight line at 25 slots (two times the round trip propagation delay plus one) corresponds to the minimum mean packet delay induced by any reservation scheme. As it can be seen from the figure, in the satellite environment the proposed schemes outperform any reservation scheme, at least for low to moderate input traffic values.

Appendix A

Proof of Theorem 2

Part (i)

Let E be the space of sequences $X = \{x_d\}: A \to R$. Given $Y^{(0)} \varepsilon E$, define

$$y_d^{(j+1)} \triangleq b_d + \sum_{t \varepsilon A} c_{dt}\, y_t^{(j)},\ d\varepsilon A,\ j=0,1,2,\ldots \tag{A-1}$$

where b_d, c_{dt}, $d,t\varepsilon A$, are defined in system (5). The sequence $\{Y^{(j)}\}$, $j=0,1,2,\ldots$, generated by (A-1) will be called the power sequence of system (5) with initial point $Y^{(0)}$.

Let $y_d^{(0)} = \alpha_u d + \beta_u$, $d\varepsilon A$. To establish the existence of a solution $Y=\{y_d\}$, such that $0 \leq y_d \leq \alpha_u d + \beta_u$, $d\varepsilon A$ it suffices to choose α_u, β_u so that

$$0 \leq y_d^{(1)} \leq y_d^{(0)} \text{ for every } d\varepsilon A \tag{A-2}$$

Under (A-2) and since $b_d \geq 0$, $c_{dt} \geq 0$, d, $t\varepsilon A$, we have, that

$$0 \leq y_d^{(j+1)} \leq y_d^{(j)},\ d\varepsilon A,\ j=0,1,2,\ldots \tag{A-3}$$

Thus, under (A-3), the following limit exists

$$\lim_{j\to\infty} y_d^{(j)} = y_d \quad ,\ d\varepsilon A \tag{A-4}$$

Passing to the limit as $j\to\infty$ in (A-1) shows that the sequence Y solves system (5).

From (A-3), (A-4) we have that

$$0 \leq y_d \leq \alpha_u d + \beta_u,\ d\varepsilon A$$

Next we choose α_u , β_u so that (A-2) is satisfied.

After straightforward manipulations, by (A-1), we obtain

$$y_d^{(1)} = y_d^{(0)} + E(\ell_d) + \alpha_u \, (E(\ell_d) - d - p_d(1)) - \beta_u \, p_d(1) \quad , \; 1 \le d \le \Delta \qquad \text{(A-5.a)}$$

$$y_d^{(1)} = y_d^{(0)} + E(\ell_\Delta) - \alpha_u \, (\Delta - E(\ell_\Delta)) \quad , \qquad d \ge \Delta \qquad \text{(A-5.b)}$$

From (A-5.b) we see that (A-2) is satisfied only if condition (12) in the theorem holds. Under (12), it can be readily seen from (A-5.a) that (A-2) is satisfied if we choose α_u, β_u as follows

$$\alpha_u = \frac{E(\ell_\Delta)}{\Delta - E(\ell_\Delta)} \quad , \; \beta_u = \sup_{1 \le d \le \Delta} (\psi(d))$$

where $\psi(d) = \dfrac{E(\ell_d) + \alpha_u \, (E(\ell_d) - d - p_d(1))}{p_d(1)}$, and where $p_d(1) = (1+\lambda d) \exp(-\lambda d)$.

To construct a lower bound on y_d we let $y_d^{(0)} = \alpha_\ell d + \beta_\ell$, $d \epsilon A$, and we choose α_ℓ and β_ℓ such that

$$y_d^{(0)} \le y_d^{(1)} \quad , \; d \epsilon A \qquad \text{(A-7)}$$

From (A-5) it can be readily seen that (A-7) holds if

$$\alpha_\ell = \alpha_u \quad , \; \beta_\ell = \inf_{1 \le d \le \Delta} (\psi(d))$$

where α_u, $\psi(d)$ are as given above.

The expectations $E(\ell_d)$, $1 \le d \le \Delta$, needed for the computation of the coefficients of the linear bounds can be computed as shown in Appendix B.

<u>System (8)</u>: Similarly to system (5), the solution $Z = \{z_d\} d \epsilon A$ to system (8) may be obtained as the pointwise limit of its power sequence $\{Z^{(j)}\}$, $j=0,1,2,\ldots$, defined by

$$z_d^{(j+1)} = b_d' + \sum_{t\varepsilon A} c_{dt} \; z_d^{(j)} \quad , \; d\varepsilon A \tag{A-8}$$

provided that $z_d^{(0)}$ is such that

$$z_d^{(0)} \geq 0, \; z_d^{(1)} \leq z_d^{(0)} \quad , \text{ for every } d\varepsilon A \tag{A-9}$$

Under (12), it can be shown by direct substitution in (A-8), that (A-9) holds if we choose $z_d^{(0)} = \gamma_u d^2 + \delta_u d + \zeta_u$, where

$$\gamma_u = \frac{\lambda\Delta}{2(\Delta - E(\ell|\Delta))}$$

$$\delta_u = \frac{E(\omega_\Delta) - \lambda\Delta^2 + \gamma_u \; (\Delta^2 + E(\ell_\Delta^2) - 2\;\Delta\; E(\ell|\Delta))}{\Delta - E(\ell|\Delta)}$$

$$\zeta_u = \sup_{1 \leq d \leq \Delta} (\phi(d))$$

$$\phi(d) = \frac{E(\omega_d) + \gamma_u(E(\ell_d^2) - d^2) + \delta_u(E(\ell_d) - d)}{(1+\lambda d)\; \exp(-\lambda d)} - (\gamma_u + \delta_u)$$

Under (12), and for every $d\varepsilon A$, the sequence $\{z_d^{(j)}\}_{j \geq 0}$ is non-increasing; thus $z_d \leq \gamma_u d^2 + \delta_u d + \zeta_u$.

The construction of a lower bound $\gamma_\ell d^2 + \delta_\ell d + \zeta_\ell$ on z_d is similar to that of the upper bound; the coefficients γ_ℓ, δ_ℓ, and ζ_ℓ are given below

$$\gamma_\ell = \gamma_u, \; \delta_\ell = \delta_u, \; \zeta_\ell = \inf_{1 \leq d \leq \Delta} (\phi(d))$$

The expectations in the expressions of the coefficients γ_u, δ_u, ζ_u, ζ_ℓ can be computed as shown in Appendix B.

Part (ii)

The proof involves two steps. In step 1 we show that the solutions found in part (i) are unique in the class of sequences

$$E_2 = \{X : \sup_{d\epsilon A} \frac{|x_d|}{d^2} < \infty\}$$

In step 2, we show that the algorithmic sequences $\{H_d\}$ $d\epsilon A$, and $\{W_d\}$ $d\epsilon A$ belong to E_2, and, therefore, coincide with the corresponding unique solutions in E_2.

Step 1: As in[17], we shall call the system

$$x_d = B_d + \sum_{t\epsilon A} C_{dt} x_t, \; d\epsilon A \tag{A-10}$$

majorant of the system

$$x_d = b_d + \sum_{t\epsilon A} c_{dt} \, x_t, \; d\epsilon A \tag{A-11}$$

if the following inequalities hold:

$$|c_{dt}| \leq C_{dt}, \quad b_d \leq B_d \;, \; d,t\epsilon A$$

A solution of a system of type (A-11) that is a pointwise limit of its power sequence with initial point $X^{(0)} = 0$, is called its principal solution.

With the above definitions, we state Lemmas A.1, A.2 below, which are essentially theorems I, II, & 2 of [17].

Lemma A.1 If the majorant system (A-10) has a nonnegative solution $\bar{S}$, then both system (A-10) and (A-11) have principal solutions $\bar{S}*$, $S*$, respectively. Moreover, $0 \leq |s_d^*| \leq \bar{s}_d^* \leq \bar{s}_d$, $d\epsilon A$.

Lemma A.2 If the majorant system (A-10) has a nonnegative solution $\bar{S}$, then the principal solution $S*$ of system (A-11) is unique in the class $E(\bar{S}*) \subset E$, defined as follows

$$E(\overline{S}^*) \triangleq \{ X \in E : \sup_{d\epsilon A} \frac{|x_d|}{\overline{s}^*_d} <\infty \}$$

Furthermore, S* is the pointwise limit of any power sequence of (A-11), with initial point any point in $E(\overline{S}^*)$

Using the above lemmas, we shall show that both the solution $Y = \{y_d\}_{d\epsilon A}$ of system (5), and the solution $Z = \{z_d\}_{d\epsilon A}$ of system (8) are unique in E_2. We start with system (8). Since system (8) has a nonnegative solution, Z, and is majorant of itself, from lemma A.1, it has a principal solution $Z^* = \{z^*_d\}_{d\epsilon A}$. According to lemma A.2, the solution Z* is unique in the class

$$E(Z^*) = \{X\epsilon E : \sup_{d\epsilon A} \frac{|x_d|}{z^*_d} <\infty\}$$

provided that $Z^{(0)} \epsilon E(Z^*)$. Since by definition (see part (i)), $Z^{(0)} \epsilon E_2$, the solution Z will be unique in E_2, if $E(Z^*) \equiv E_2$. To show that the latter holds, we use the following lemma whose easy proof is omitted.

<u>Lemma A.3</u> Let $F = \{f_d\}_{d\epsilon A}$, $G = \{g_d\}_{d\epsilon A}$ be two sequences in E. If

(a) $f_d \geq 0, g_d \geq 0$, $d\epsilon A$

(b) $\sup_{d\epsilon A} \frac{f_d}{g_d} <\infty$

(c) $\inf_{d\epsilon A} \frac{f_d}{g_d} >0$

then

$$\sup_{d\epsilon A} \frac{|x_d|}{g_d} <\infty \qquad \text{iff} \sup_{d\epsilon A} \frac{|x_d|}{f_d}, \quad X \in E;$$

i.e., the classes $E_F = \{X \in E: \sup_{d\epsilon A} \frac{|x_d|}{f_d} <\infty\}$ and $E_G = \{X \in E: \sup_{d\epsilon A} \frac{|x_d|}{g_d} <\infty\}$

Let $\{\tilde{Z}^{(n)}\}$, n=0,1,2,..., be the power sequence of system (8), with $\tilde{z}_d^{(0)}=0$, $d\epsilon A$. Clearly

$$\tilde{z}_d^{(1)} = b_d' \geq \varepsilon > 0, \text{ for every } d\epsilon A. \tag{A-12}$$

Also, it can be shown by induction that,

$$\tilde{z}_d^{(n)} = n\, E(\omega_d) - \frac{1}{2}\, n\, (n-1)\, \lambda\, \Delta(\Delta - E(\ell_\Delta)) \tag{A-13}$$

for every $d\epsilon A$, $n \geq 1$, such that $d > n\,\Delta$. For $d > 2\,\Delta$, let $n = \lfloor \frac{d}{\Delta} \rfloor - 1$ in (A-13), where $\lfloor x \rfloor$ denotes the maximum integer not exceeding x. Then, using the inequalities $E(\omega_d) > \lambda\,\Delta\,(d-\Delta) + \frac{\lambda\Delta^2}{2}$, (see (11.b)), $\lfloor \frac{d}{\Delta} \rfloor > \frac{d}{\Delta} - 1$, and $\Delta \lfloor \frac{d}{\Delta} \rfloor \leq d$ in (A-13) yields

$$\tilde{z}_d^{(n)} > \frac{1}{2}\,\lambda\, d^2 - \lambda\,\Delta\, d\ ,\ d > 2\,\Delta \tag{A-14}$$

From Lemma A.1 and the fact that

$b_d' \geq 0$, $c_{dt} \geq 0$, d, $t\epsilon A$, we have that

$$z_d^* \geq \tilde{z}_d^{(n)} \geq 0\ ,\ d\epsilon A,\ n \geq 1 \tag{A-15}$$

From (A-12) and (A-15) we obtain

$$z_d^* \geq \varepsilon > 0\ ,\ d\epsilon A \tag{A-16}$$

From (A-14), (A-15), and (A-16) we conclude that,

$$\inf_{d\epsilon A} \frac{z_d{}^*}{d^2} > 0 \tag{A-17}$$

From (14), and the inequality $z_d^* \leq z_d$, (lemmaA.1), we have

$$\sup_{d\epsilon A} \frac{z_d{}^*}{d^2} < \infty \tag{A-18}$$

Finally, from (A-17), (A-18), and lemma A.3 we have that $E(Z^*) \equiv E_2$.

To establish the uniqueness in E_2 of the solution $Y = \{y_d\}_{d\epsilon A}$ of system (5) we proceed as follows. Comparing the constant terms b_d of system (5) to the constant terms b_d' of system (8) we can readily show that there exists a positive number $M<\infty$, such that

$$b_d \leq Mb_d' \text{ , for every } d\epsilon A \tag{A-19}$$

From (A-19) we have that system (A-20)

$$X_d = Mb_d' + \sum_{t\epsilon A} c_{dt} X_t \quad , d\epsilon A \tag{A-20}$$

is majorant of system (5). Also, note that the sequence $\overline{Z} = \{\overline{z}_d\}_{d\epsilon A}$, $\overline{z}_d = Mz_d$, $d\epsilon A$, solves system (A-20), since $Z = \{z_d\}_{d\epsilon A}$ is a solution of system (8). Furthermore, system (A-20) has a principal solution $\overline{Z} = \{\overline{z}_d^*\}_{d\epsilon A}$ with

$$\overline{Z}_d^* = Mz_d^* \text{ , } d\epsilon A \tag{A-21}$$

where $Z^* = \{z_d\}$ is the principal solution of system (8)

Since system (A-20) is majorant of system (5), from lemma A.2, we have that the solution $Y = \{y_d\}_{d\epsilon A}$ is unique in the class

$$E(\overline{Z}^*) = \{X\epsilon E : \sup_{d\epsilon A} \frac{|x_d|}{\overline{x}_d^*} > \infty \}$$

provided that $Y^{(0)} \in E(\overline{Z}^*)$. From (A-21) we have that $E(\overline{Z}^*) \equiv E(Z^*)$. However, it has already been shown that $E(Z^*) = E_2$; thus, $E(\overline{Z}^*) \equiv E_2$. Since, by definition (see part (i)), $Y^{(0)} \epsilon E_2$, we conclude that Y is the unique solution of system (5) in E_2.

Step 2

Here, we show that the mean session length sequence $\{H_d\}$ $d\epsilon A$ belongs to the class E_2. Define the random variables $h_d^n \triangleq \min(h_d, n)$, $n=1,2,3,\ldots$, $d\epsilon A$, where h_d is the length of a d-session. Clearly,

$$0 \leq h_d^n \nearrow h_d \text{ , a.e., } d\epsilon A \tag{A-22}$$

If we let $H_d^n = E\{h_d^n\}$, then from the definition of h_d^n and the operation of the algorithm we have that

$$H_d^n \leq b_d + \sum_{t\varepsilon A} c_{dt} H_t^n , \ d\varepsilon A \tag{A-23}$$

Also, since $h_d^n \leq n$ a.e., $d\varepsilon A$, the sequence $\{H_d^h\}_{d\varepsilon A}$ belongs to E_2, for every n. Using (A-23) and the fact that $Y = \{y_d\}_{d\varepsilon A}$ is a solution of system (5) that belongs to E_2 it can easily be shown that

$$H_d^n \leq y_d , \ d\varepsilon A, \ n=0,1,2,... \tag{A-24}$$

From (A-22) and the monotone convergence theorem we have

$$H_d^n \nearrow H_d , \ d\varepsilon A \tag{A-25}$$

From A-24) and (A-25) we conclude that $H_d \leq y_d$; thus $\{H_d\}_{d\varepsilon A} \varepsilon E_2$. Since $\{H_d\}_{d\varepsilon A}$ solves system (5) it coincides with its unique solution Y in E_2.

The proof for the mean delay sequence $\{W_d\}$ $d\varepsilon A$ is similar.

Appendix B

Computation of $E(\ell_d)$, $E(\ell_d^2)$ and $E(v_d)$

Let X_d denote any of the random variables ℓ_d, ℓ_d^2, or v_d, and define

$E(X_d|k)$: The conditional expectation of the random variable X_d, given that the enabled arrival interval contains k packets, $k \geq 0$.

The conditional expectation $E(X_d|k)$ does not depend on d; therefore, we may write

$$E(X_d) = \sum_{k=0}^{\infty} E(X_1|k)\; e^{-\lambda d} \frac{(\lambda d)^k}{k!} \tag{B.1}$$

Note that, since the length of an enabled interval is at most Δ, we have that

$$E(X_d) = E(X_\Delta) \;\;, \text{ for } d > \Delta$$

Define $L_k \triangleq E(\ell_1|k)$, $M_k \triangleq E(\ell_1^2|k)$, and $V_k \triangleq E(v_1|k)$, $k \geq 0$. Using the rules of the algorithm, the quantities L_k, M_k, and V_k, $k \geq 0$, can be computed recursively, as follows.

$$L_0 = L_1 = 1 \;;\; L_k = 1 + m \sum_{j=1}^{k} b_j\,(k,1/m)\; L_j \;\;,\; k \geq 2,$$

where

$$b_j(k,1/m) = \binom{k}{j} \left(\frac{1}{m}\right)^j \left(1-\frac{1}{m}\right)^{k-j}$$

$M_0 = M_1 = 1$

$$M_k = 1+m \sum_{j=1}^{k} b_j(k,1/m) M_j + 2(L_k-1) + m(m-1) \sum_{k_1=1}^{k-1} \sum_{k_2=1}^{k-k_1} q_k(k_1,k_2,)\; L_{k_1} L_{k_2},\; k \geq 2,$$

where

$$q_k(k_1,k_2) = \frac{k!}{k_1!\,k_2!\,(k-k_1-k_2)!} \left(\frac{1}{m}\right)^{k_1+k_2} \left(1-\frac{2}{m}\right)^{k-k_1}$$

$V_0 = 0$, $V_1 = 1$

$$V_k = k+m \sum_{j=1}^{k} b_j(k,1/m)\; V_j + \sum_{\xi=0}^{m-2} \sum_{j=0}^{k} \sum_{i=1}^{k-j} b_j\left(k,\frac{\xi}{m}\right) b_i\left(k-j,\frac{1}{m-\xi}\right) (k-j-i)\; L_i \;\;,\; k \geq 2,$$

Using the above recursions, a finite number, M, of terms from the infinite series (B.1) can be easily computed. Also, for large K values, and based on the recursive expressions, simple upper and lower bounds on $E\{X_1|k\}$ can be developed. Those bounds can be used to tightly bound the sum

$$\sum_{k=M+1}^{\infty} E(X_1|k)\; e^{-\lambda d}\, \frac{(\lambda d)^k}{k!}$$

<u>The condition $\Delta > E(\ell_\Delta)$.</u>

Given the algorithmic parameters Δ and m, and in view of (B.1), the condition $\Delta > E(\ell_\Delta)$ in theorem 2 can be equivalently expressed as $\lambda < \bar{\lambda}(m,\Delta)$, where

$$\bar{\lambda}(m,\Delta) = \sup \{\lambda \colon \Delta > \sum_{k=0}^{\infty} L_k\, e^{-\lambda\Delta} \frac{(\lambda\Delta)^k}{k!} \}$$

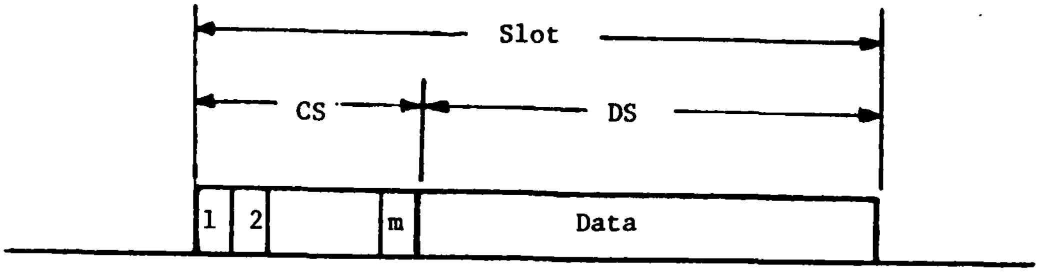

Figure 1

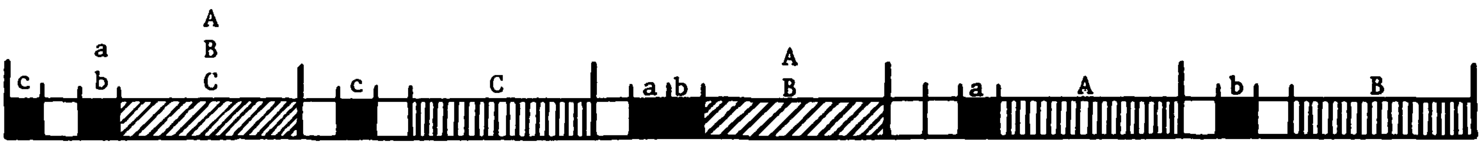

Figure 2. Illustration of the resolution of a collision involving the packets A,B,and C,when m=3.

Feedback : Empty, Non-empty, Collision, Non-collision.

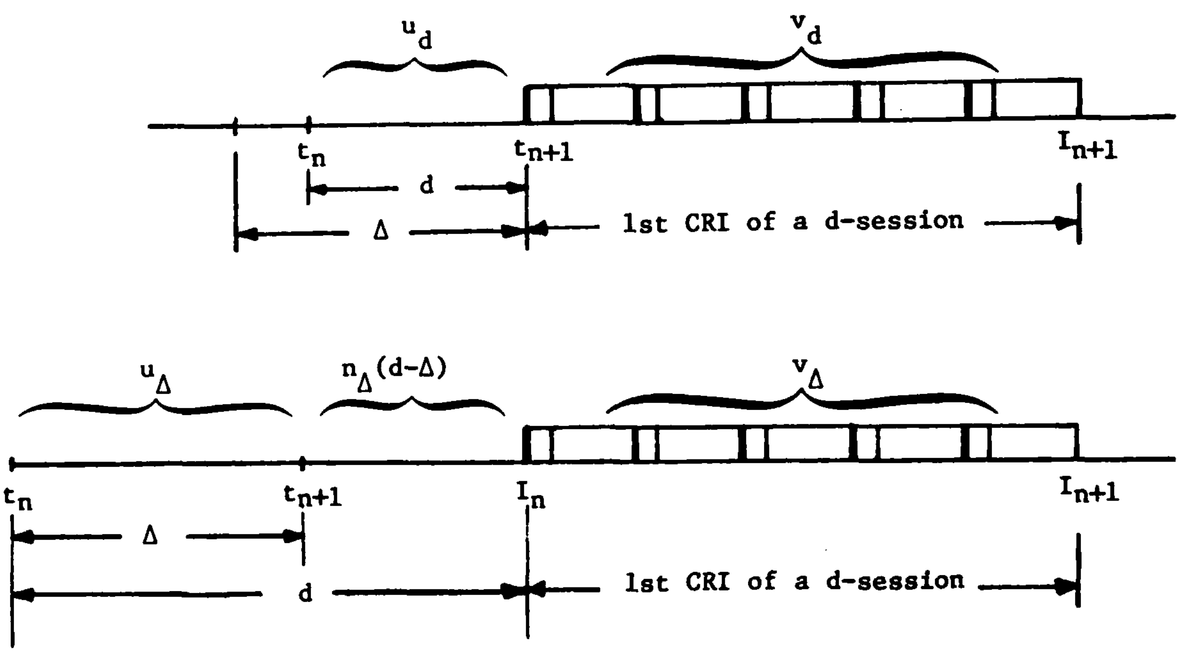

Figure 3.

m	$\bar{\lambda}(m,\Delta_m^*)$	Δ_m^*
2	0.5021	2.739
3	0.5782	3.182
4	0.6168	3.665
5	0.6424	4.347
6	0.6621	5.422
7	0.6785	6.476
8	0.6924	7.179
10	0.7145	8.051
12	0.7317	8.636
14	0.7456	9.091
16	0.7572	9.492
18	0.7672	9.847
20	0.7758	10.174
22	0.7834	10.479
24	0.7902	10.766
26	0.7963	11.025
28	0.8019	11.303
30	0.8070	11.555
32	0.8116	11.801

Table 1 The maximum stable throughput $\bar{\lambda}(m,\Delta_m^*)$, and the optimal window length Δ_m^*, for the ZPDA.

λ	$m = 4$		$m = 8$		$m = 16$	
	D_5	D^5	D_5	D^5	D_5	D^5
0.1	1.7068	1.7068	1.6820	1.6820	1.6724	1.6724
0.2	1.9945	1.9945	1.9233	1.9233	1.8958	1.8958
0.3	2.4494	2.4494	2.2749	2.2749	2.2088	2.2088
0.4	3.3041	3.3041	2.8426	2.8426	2.6795	2.6795
0.5	5.5664	5.5665	3.9299	3.9299	3.4589	3.4589
0.6	33.8550	34.1730	7.1034	7.1040	5.0305	5.0305
0.65	-	-	-	-	6.7409	6.7414
0.68	-	-	44.1240	44.6640	-	-
0.7	-	-	-	-	11.1590	11.1870
0.74	-	-	-	-	32.1660	33.0430

Table 2 The lower bound D_5 and the upper bound D^5 on D, for the ZPDA.

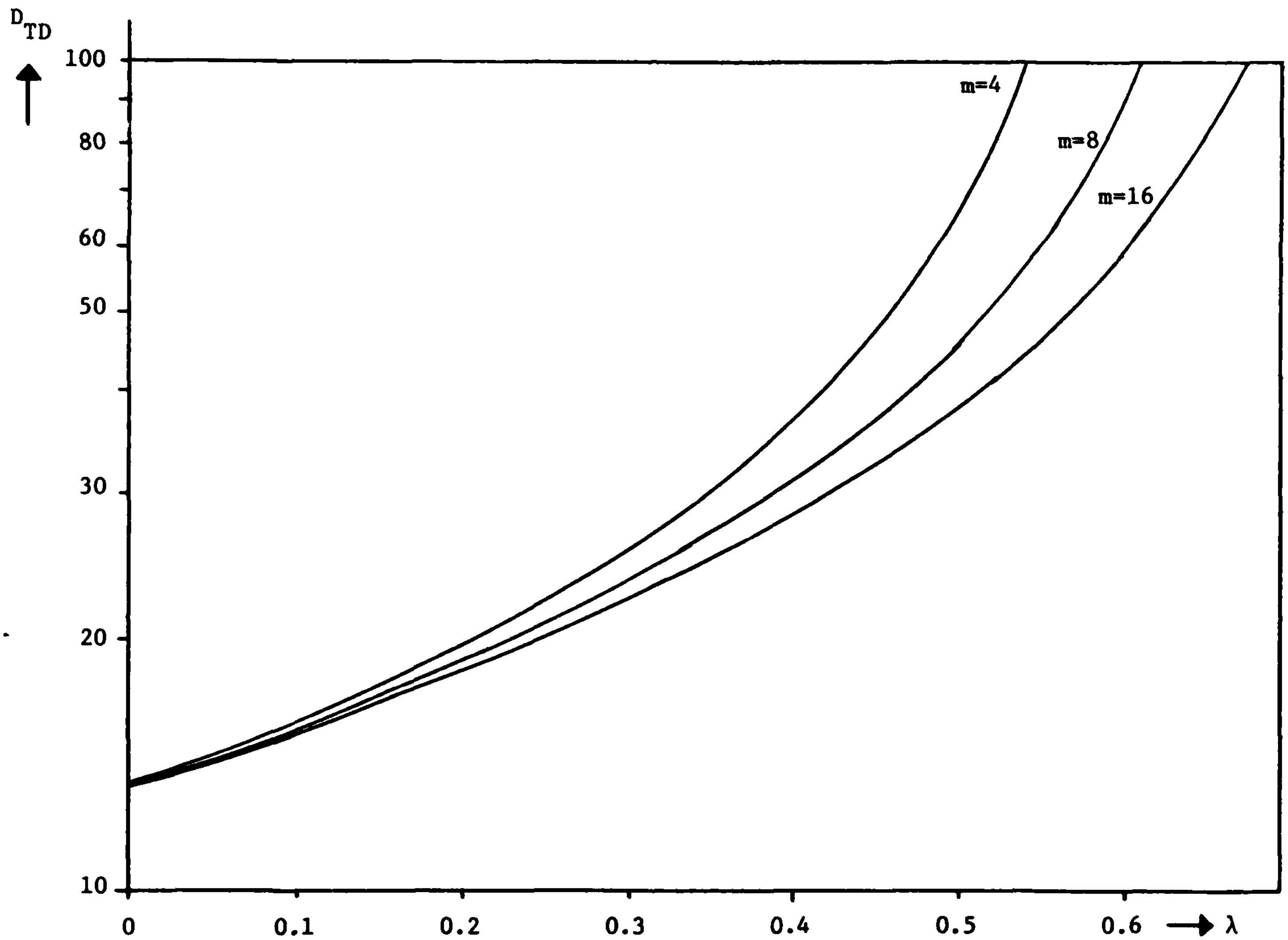

Figure 4

The mean packet delay D_{TD}, for P = 12

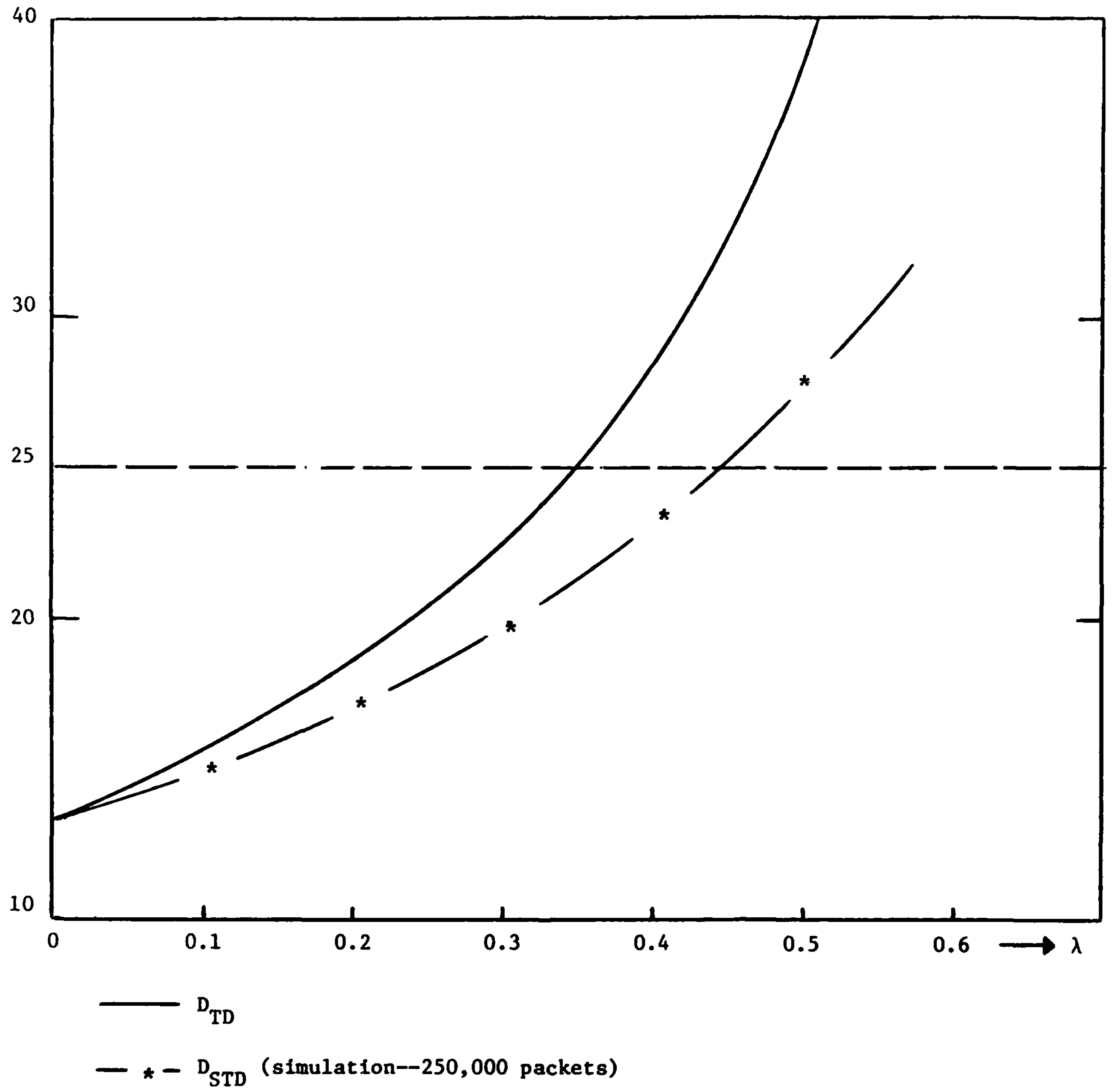

Figure 5

The mean packet delays D_{TD} and D_{STD}, for $m = 16$ and $P = 12$

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