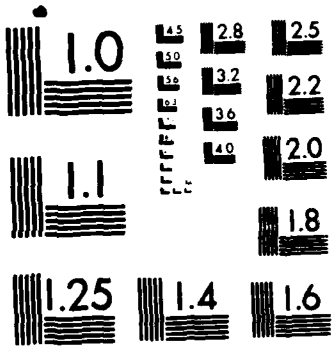
AD-A162 029 RELIABILITY ASSESSMENT FOR SYSTEMS SUBJECT TO MAINTENANCE AND REPAIR(U) CITY COLL NEW YORK DEPT OF MATHEMATICS M BROWN AUG 85 AFOSR-TR-85-1084 1/1

UNCLASSIFIED AFOSR-84-0095 F/G 14/4 NL



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963 A

8a. NAME OF FUNDING/SPONSORING ORGANIZATION	8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
AFOSR	NM	AFOSR-84-0095

8c. ADDRESS (City, State and ZIP Code)
Bolling AFB, D.C. 20332-6448

10. SOURCE OF FUNDING NOS.

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT NO.
61102F	2304	A5	

11. TITLE (Include Security Classification) Reliability Assessment for Systems Subject to Maintenance and Repair

12. PERSONAL AUTHOR(S)
Mark Brown

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
Annual	FROM 1 Apr 84 TO 30 Jun 85	August 1985	7

16. SUPPLEMENTARY NOTATION

17. COSATI CODES		
FIELD	GROUP	SUB. GR.

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
Reliability modeling, age-weighted distributions, repairable systems.

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The main thrust of the research under this grant is to develop the methodology to deal with more realistic models for the quantitative reliability assessment and analysis of complex systems. Inequalities for increasing failure rate distributions have been derived. A new theory of age-weighted distributions is being developed. Work is beginning on exploiting properties of a pratial ordering of distributions.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT	21. ABSTRACT SECURITY CLASSIFICATION
UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☒ DTIC USERS ☐	Unclassified

AFOSR-TR- 1084

Interim Scientific Report to the Air Force
Office of Scientific Research

Grant AFOSR-84-0095

Reliability Assessment for Systems Subject to Maintenance and Repair

Department of Mathematics
The City College, CUNY
New York, NY 10031

Mark Brown, Principal Investigator

August, 1985

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Grant AFOSR-84-0095 was awarded for the period 4/1/84 - 3/31/85. It was subsequently extended to 6/30/85 without further funding. A renewal proposal covering 7/1/85 - 6/30/86 was submitted and approved. This report will discuss the research accomplishments so far achieved during the grant period.

The main thrust of the research performed under the grant is the development of methodology to deal with more realistic models for the quantitative reliability assessment and analysis of complex systems. This methodology is potentially applicable to the programs of the U.S. Air Force in particular and the Department of Defense in general. In addition the results obtained are of value in a variety of other applied areas, and in probability and statistical theory.

1) <u>Inequalities for IFR Distributions</u>.

A technical report "Inequalities for distributions with increasing failure rate" was issued in December 1984. The paper will appear in a special volume in celebration of the 65th birthday of Herbert Solomon.

Several new inequalities for IFR (distributions) are derived. Included is a two sided bound on the renewal function as well as the exact sup norm distance between the renewal function and its asymptotic linear approximation.

This is one of several papers the author has written on the subject of exponential approximation. The importance of this study is that quite often the first two moments of a distribution are known along with the reliability class. The above approximations give uniform two sided bounds for the cdf which are excellent when the parameter ρ is small. As the distributions of interest are often intractable, the ability to do such approximation is invaluable. In particular it allows one to closely approximate the distribution of the time to first failure for repairable systems.

2) Age-weighted distributions.

Under the current grant I have been developing a new theory which looks interesting and potentially useful. I plan to write a technical report in the near future summarizing the current state of development.

Given a distribuiton F on $[0,\infty)$ with finite mean μ define AF, the age-weighted distribution of F by:

$$d(AF)(t) = \frac{t}{\mu}\, dF(t) .$$

Age-weighted distributions arise in many different areas. Two prominent examples are:

(i) If F is the interarrival time distribution for a renewal process, then AF is the distribution of the length of the interval covering the point 0 in a stationary renewal process on the whole real line with interarrival distribution F. The fact that F and AF have different distributions, with AF being larger, is the content of the waiting time paradox discussed by Feller (1966).

(ii) Suppose that $X_1 \sim F_1$ and $X_2 \sim F_2$ with F_1 and F_2 mutually absolutely continuous. Let f_1 and f_2 be the densities of F_1 and F_2 with respect to λ (a measure dominating both F_1 and F_2). Then the Neyman-Pearson test of F_1 vs F_2 is based on the ratio f_2/f_1. Define $Y_1 = \frac{f_2}{f_1}(X_1)$ and $Y_2 = \frac{f_2}{f_1}(X_2)$, the Neyman-Pearson statistic when F_1 and F_2, respectively, are true. Then $Y_2 = AY_1$, i.e. the distribution of the Neyman-Pearson likelihood ratio under F_2 is the age-weighted distribution of its distribution under F_1.

Below are some of the results I have obtained during the current grant period for age-weighted distributions.

Let $X \sim F$ and $T \sim AF$. Then:

(1) $$ET = (EX)^{-1} .$$

More generally, let $X_1,\ldots,X_n$ be i.i.d. as X and independent of T. Define $S_n = \Sigma_{i=1}^{n} X_i$. Then:

(2) $$E(S_n+T)^{-1} = [(n+1)EX]^{-1} .$$

The relationship between X and T^{-1} is symmetric, i.e.:

(3) $$T^{-1} = (AX)^{-1} \quad \text{and} \quad X = (AT^{-1})^{-1} .$$

Defining $c_X^2 = 1-(EXEX^{-1})^{-1}$ and $c_T^2 = 1-(ETET^{-1})^{-1}$ it follows from (1)-(3) that:

(4) $$c_X^2 = \mathrm{Var}\, T/ET^2$$

(5) $$c_T^2 = \mathrm{Var}\, X/EX^2 \; .$$

Finally as a consequence of (1)-(5), let g be a convex function with $w_g = \sup(x^3 g''(x)) < \infty$. Then:

(6) $$0 \le Eg(X)-g(EX) \le \frac{1}{2}\,(EX^{-1}-(EX)^{-1})w_g \; .$$

The import of (6) is that X may have a distribution with large or infinite variance. As a result the standard delta method for approximating $Eg(X)$ may not be applicable or useful. However, if g is rapidly decreasing, the mean of $g(X)$ should not be extremely sensitive to the tail of the distribution of X. Relationship (6) gives an alternative to the delta method in which the variance (which measures the departure of EX^2 from $(EX)^2$), is replaced by the departure of EX^{-1} from $(EX)^{-1}$. This appears to be a fundamental idea, well worth deeper study.

3) Partial ordering of distributions.

In previous papers I obtained some interesting results for IMRL and DFR distributions by exploiting a partial ordering between F (IMRL or DFR) and its stationary renewal distribution G.

I am currently abstracting this partial ordering and developing its properties. It looks like a promising new tool. Below are some of my results.

Let F_1, F_2 be probability distributions on $[0,\infty)$. Define $F_1 \geq F_2$ if $\overline{F}_1(t)/\overline{F}_2(t)$ is increasing in t for $\overline{F}_2(t) > 0$. It turns out that this partial ordering is equivalent to each of:

(i) $X_1 = \min(X_2, Z)$ where $X_1 \sim F_1$, $X_2 \sim F_2$ and Z is a (possibly improper) random variable, independent of X_2.

(ii) $G_1 \geq G_2$ is the sense of monotone likelihood ratio, where G_i is the stationary renewal distribution corresponding to F_i, $i = 1,2$.

(iii) Let $X_1 \sim F_1$, $X_2 \sim F_2$. Then for all $t \geq 0$, the conditional distribution of $X_1 - t$ given $X_1 > t$ is stochastically larger than the conditional distribution of $X_2 - t$ given $X_2 > t$.

The main results I have derived for this partial ordering are:

(a) Let $F_1 \geq F_2$, with F_1 continuous and $H_1(t) = -\ln(\overline{F}_1(t))$.

Then:

$$\sup_{B \in \mathcal{B}} |F_1(B) - F_2(B)| \leq 1 - \int_0^\infty \overline{F}_2(t)\, dH_1(t)$$

where $\mathcal{B}$ is the collection of Borel subsets of $[0,\infty)$.

(b) Let X_1, X_2 be independent with $X_1 \sim F_1$, $X_2 \sim F_2$ and $F_1 \leq F_2$. Let $h(x,y)$ be a function of two variables with $h(x,y) - h(y,x)$ increasing in $x \geq y$. Then:

$$Eh(X_1, X_2) \geq Eh(X_2, X_1) \quad (7)$$

The inequality (7) leads to several interesting results obtained by proper choice of h. I am now exploring their implications.

Bibliography

I - Technical report issued under Grant 84-0095 "Inequalities for distributions with increasing failure rate," City College CUNY Report No. MB84-01, AFOSR Technical Report No. 84-01.

II - Previous AFOSR papers of Mark Brown published during the grant period, or in press.

1) Brown, M. and Ge, G. (1984). "Exponential approximations for two classes of aging distributions." Annals of Probability, 12, 869-875.

2) Brown, M. (1984). "On the reliability of repairable systems." Operations Research, 32, 607-615.

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6) "A measure of variability based on the harmonic mean and its use in approximations." To appear in Annals of Statistics, September, 1985.

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