Statistical Modeling of Quarterly Contractor Overhead Costs

by

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Presented at the
19th Annual Department of Defense
Cost Analysis Symposium
Xerox Training Center
Leesburg, Virginia
September 17 - 20 1985

Introduction

Overhead costs generally are estimated by using estimated overhead rates which are then applied to estimated labor hours or costs in each of several functional categories, such as engineering or manufacturing. Total overhead is then obtained by summing across all the functions. This approach is not entirely satisfactory since changes in operating rates cause changes in overhead rates which are reflected only after a significant lag. For firms in which output fluctuates significantly, this approach can result in poor estimates of overhead costs with corresponding difficulties for product pricing. In instances where the Federal government is the sole purchaser of the product, actual production costs (both direct and indirect) are important inputs into the price and quantity negotiation process. With aerospace contractor overhead comprising 30 to 50 percent of total costs to the Federal government, it is imperative that overhead costs be estimated with greater accuracy.

An alternative approach to estimating overhead costs is to estimate these costs directly and, hence, forego direct reliance upon overhead rates. Two examples of this are provided ~~by [7] and [4]~~. Martinson reclassified overhead costs from the usual functional categories into an input-oriented categorization and then regressed these new categories of overhead costs on various operating variables. Current conventional wisdom holds that the Martinson ~~approach~~ has been unsuccessful in al~~most all~~ of its subsequent trials.

Gross and Dienemann estimated various categories of overhead costs using direct labor and material costs on a pooled time-series, cross-section sample of aerospace firms. The categories which they used were similar to those used by Martinson. Unfortunately, there are major technical difficulties with the methodology of Gross and Dienemann. Almost all of their regression models used lagged values of the dependent variable as one of the explanatory variables, yet they reported only the Durbin-Watson statistic as the measure of the degree of autocorrelation present in their models. It is well-known (see [5] or [6], for example) that the use of lagged values of the dependent variable as an explanatory variable results in an upward bias of the Durbin-Watson statistic (that is, the statistic does not find autocorrelation when it is actually present). Since it is also well-known that the presence of positive autocorrelation in a regression model biases downward the standard errors and biases upward the R-squared statistic, most of the results of Gross and Dienemann have unknown reliability.

The procedures described below attempt to estimate total overhead costs from five aerospace contractors as functions of the number of direct manufacturing personnel. These procedures are extensions to both more data and more firms of the analyses performed in [1]. The focus here is on determining the effectiveness of a procedure which can be routinized and, hence, utilized by persons with relatively low degrees of statistical sophistication. Consequently, the number of explanatory variables is purposefully kept to a minimum.

Data Sources and Characteristics

Data were obtained for five major United States defense aircraft manufacturers. The data, however, are proprietary and are not releaseable. To preserve the anonymity of the data and results, any specific reference to individual manufacturers will be in the form of contractors A through E.

Prior to the obtaining of any data, a particular format for collection of overhead cost data was determined in order to assure uniformity of data categories across the different firms. The overhead cost data from the major manufacturing divisions of the contractors were collected within this defined format on a quarterly basis beginning with the first quarter 1979 through second quarter 1984. The format for overhead costs has five major categories with several subcategories within each category. The five major categories are (i) labor-related, (ii) facilities-related, (iii) operations, (iv) mixed labor and facilities, and (v) corporate transfers. Other additional data pertaining to production and operating characteristics of the divisions of the firms were also obtained.

The various categories of cost data were converted from current to constant fourth quarter 1982 dollars using indices from Bureau of Labor Statistics (BLS) and Bureau of Economic Analysis publications. The labor-related data were converted using the BLS SIC 3721 (aircraft industry) index and other categories were converted using GNP deflators for structures, services, etc. It is recognized that these indices, along with almost all others, are imperfect, but they were selected in an attempt to provide the best measures of inflation from among all readily-available indices relevant to these particular categories.

Modeling Quarterly Overhead Costs

Sequential cost and operating data, as with most other time series data resulting from firm operations, can be expected to exhibit some level of autocorrelation. This is because firm expenditures from period to period are not totally random but tend to change relatively smoothly. Consequently, the error process of a time series-based statistical model of the costs of a firm does not exhibit the desired (normal) random structure but, instead, exhibits a structure in which errors in one period tend to be related to errors in other periods. Although the presence of some form of autocorrelation in the residuals of a regression model does not create any problems in obtaining unbiased estimates of the regression coefficients themselves, it does result in biased estimates of the standard errors of the regression coefficients. Hence, any hypothesis tests which rely upon either the standard errors or functions of the standard errors may result in erroneous conclusions. This includes the standard t-tests for the statistical significance of the difference of the regression coefficient value from zero. Consequently, it is desirable to obtain not only unbiased estimates of the regression coefficients but also unbiased estimates of their standard errors.

First order autocorrelation occurs when the errors of the model are related to the errors in the adjacent, prior periods. The errors are said to follow a first order autoregressive, or AR(1), process. Yearly cost and operating data tend to have errors which follow an AR(1) process. The use of quarterly data, however, may cause the autocorrelation to take on a special form. Instead of, or perhaps in addition to, the standard, first order autocorrelation, one might encounter a special form of fourth order autocorrelation (see [8]). This special form corresponds to a seasonal pattern in the errors, since each error is correlated only with the error which occurred four quarters previously. Plots of the raw data confirmed that this form of autocorrelation is potentially present since, within each year, there was a clearly discernable tailing off of expenditures toward the final quarters. This pattern is a typical one for organizations which operate in an environment of known, binding budgets with all funds available at the beginning of the budget period.

The general model utilized in this analysis is of the form

$$y_t = X_t\beta + \epsilon_t \ , \tag{1}$$

$$\epsilon_t = \rho_t\epsilon_{t-i} + \eta_t \ , \ t=1,\ldots,T, \tag{2}$$

where X_t is, in general, a Txk matrix, β is a kx1 vector, and i is either 1 for an AR(1) process, 4 for an AR(4) process, or a mixture of these two processes. The y_t are total overhead costs, and the columns of X_t are direct personnel and a constant term. The error component of the model, ϵ_t, has the specific structure indicated by equation (2), where η_t has the zero-mean and constant-variance properties usually assumed for the error component of a regression model. Note that this model assumes a special form of the general fourth-order autoregressive (AR(4)) process. The general AR(4) process can be written as

$$\epsilon_t = \rho_1\epsilon_{t-1} + \rho_2\epsilon_{t-2} + \rho_3\epsilon_{t-3} + \rho_4\epsilon_{t-4} + \eta_t \tag{3}$$

For i=4 in equation (2), this assumes that the effects of the prior three quarters are negligible compared to the effect of the corresponding year-earlier quarter. This is the seasonal pattern discussed above.

After selection of the independent variable(s) for a particular model, the general procedure was to first perform an OLS regression on the untransformed data and then test for the presence of the above form of the AR(4) process, the AR(1) process, or some mixture of the two in the residuals. Following [8], the test statistic for the AR(4) process can be written as

$$d_4 = \frac{\sum_{t=5}^{T}(e_t - e_{t-4})}{\sum_{t=1}^{T}e_t^2} \ , \tag{4}$$

where

$$e_t = y_t - \hat{y}_t \ ,$$

$$\hat{y}_t = X_t\hat{\beta} \ , \text{ and}$$

$\hat{\beta}$ is the estimator of β obtained from the OLS regression indicated by equation (1). The null hypothesis is that of zero autocorrelation in the residuals, and the alternative hypothesis is that of the quarterly form of autocorrelation. This test and test statistic are exact analogs to the Durbin-Watson test and test statistic which are used for an AR(1) process. The reader should note well that both the venerable Durbin-Watson test and this Wallis test use null hypotheses of zero autocorrelation. If this is unacceptable, then other tests should be used. Tables of the upper and lower critical points of the distribution of d_4 are given by Wallis [8] for the above type of model.

If a test reveals the presence of the AR(1) or the AR(4) process in the residuals, then the model must be reestimated using a transformed version of the original data. Such transformations for an AR(1) process are discussed in [5] and [6]. For the AR(4) process, the dependent variable is transformed as

$$y_t^* = y_t (1 - \rho_4^2) , \qquad t=1,2,3,4, \text{ and} \tag{5}$$

$$y_t^* = y_t - \rho_4 y_{t-4} , \qquad t=5,\ldots,T. \tag{6}$$

Similarly, the independent variables are transformed as

$$X_t^* = X_t (1 - \rho_4^2) , \qquad t=1,2,3,4, \text{ and} \tag{7}$$

$$X_t^* = X_t - \rho_4 X_{t-4} , \qquad t=5,\ldots,T. \tag{8}$$

Each of these transformations requires an estimate of ρ_4. There are a number of ways to estimate this parameter, but only the most straightforward technique was selected here because of the potential requirement that this entire procedure be replicable by persons with relatively low levels of statistical sophistication. Although [1] evaluated three alternative estimators, the following one performed as well as more complex estimators. For the AR(4) process, the estimate of ρ_4 is given by

$$\hat{\rho}_4 = 1 - .5d_4 . \tag{9}$$

This estimator is derived from equation (4) via equation (2) and asymptotic arguments. Note that the value of this estimator is easily obtainable from the value of the test statistic calculated from equation (4).

After each model was reestimated using the transformed data of equations (5) through (8) and the estimator of ρ_4 given by equation

(9) (or equivalent transformations and estimators for an AR(1) process), the model was checked for the presence of first order autocorrelation using the Durbin-Watson statistic. Then each model was checked for the presence of seasonal autocorrelation using the statistic shown in equation (4). An examination of the residual plots provided further evidence of the presence or absence of any systematic effects.

Structural Analyses

The procedures outlined in the previous section will be illustrated in detail using total overhead costs from one of the contractors. Following this illustration, the results for each contractor will be presented and discussed. All statistical results will be reported to three significant digits.

Table 1 presents the results of these procedures applied to the regression of total overhead costs for contractor A (TOTOHA) upon total direct personnel for contractor A (DIRPERA). The results of the regression on the original, untransformed data can be seen as poor.

TABLE 1

Model: TOTOHA = a + b DIRPERA

Untransformed Data

Sum of Squared Residuals:	378. E7
Adjusted R-Squared:	.110
F-Statistic:	3.53
Durbin-Watson Statistic:	1.96
Estimate of a:	76800.
Standard Error:	70900.
Estimate of b:	11.3
Standard Error:	5.99
Estimate of d_4:	.468
Estimate of ρ_4:	.766

Transformed Data

Sum of Squared Residuals:	216. E7
Adjusted R-Squared:	.893
F-Statistic:	176.
Durbin-Watson Statistic:	1.06
Estimate of a:	4250.
Standard Error:	5150.
Estimate of b:	16.3
Standard Error:	1.23

Upon testing this model for the presence of the special form of fourth-order autocorrelation discussed above, the null hypothesis of no fourth-order autocorrelation is clearly rejected since the calculated value of d_4 is below the upper and lower five percent significance points (see [8]). The data were then transformed as described above and the model was reestimated. The regression results for the transformed data show that there is indeed a great deal of information contained in this model of overhead costs. As indicated above, the residuals of the transformed regression were carefully examined. Tests and plots indicate that the transformed model conforms to both the Gauss-Markov theorem and residual normality. Therefore, this model has all desirable statistical properties, with the possible exception that there may be some AR(1) process remaining in the residuals after adjustment for the presence of the AR(4) process; this is indicated by the value of the Durbin-Watson statistic after adjustment. Note, however, that the Durbin-Watson statistic prior to the adjustment indicated that there was no AR(1) process present. In summary, the regression model using transformed data yields excellent results, but the adjustment for this special form of autocorrelation clearly is necessary in order to obtain these results.

Table 2 presents the results of the modeling of total overhead costs for all contractors. Results for other major categories of overhead costs for contractors A and B are presented in [1].

Results similar to those for contractor A are obtained when the procedures described in the previous section are applied to the regression of total overhead costs for contractor B (TOTOHB) upon total direct personnel for contractor B (DIRPERB). Very poor results were obtained initially when using untransformed data, the presence of this special form of fourth-order autocorrelation was indicated clearly by the test, and excellent results were obtained using transformed data.

Since all costs are measured in thousands of dollars, the first model in Table 2 may be interpreted as indicating that there is a fixed component of total overhead costs (when a function of direct personnel) of approximately $4.25 million, with each additional direct person costing about $16,300 in total overhead costs.

These structural results may be used to compare overhead costs experienced by contractors A and B. The two models for total overhead costs are

TABLE 2

Summary Regressions

Model: TOTOHA = a + b DIRPERA

Sum of Squared Residuals:	216. E7
Adjusted R-Squared:	.893
F-Statistic:	176.
Durbin-Watson Statistic:	1.06
Estimate of a:	4250.
Standard Error:	5150.
Estimate of b:	16.3
Standard Error:	1.23

Model: TOTOHB = a + b DIRPERB

Sum of Squared Residuals:	116. E7
Adjusted R-Squared:	.933
F-Statistic:	295.
Durbin-Watson Statistic:	1.75
Estimate of a:	3460.
Standard Error:	5450.
Estimate of b:	15.8
Standard Error:	.922

Model: TOTOHC = a + b DIRPERC

Sum of Squared Residuals:	248. E7
Adjusted R-Squared:	.472
F-Statistic:	19.8
Durbin-Watson Statistic:	1.57
Estimate of a:	54500.
Standard Error:	10300.
Estimate of b:	5.30
Standard Error:	1.19

Model: TOTOHD = a + b DIRPERD

Sum of Squared Residuals:	240. E6
Adjusted R-Squared:	.821
F-Statistic:	97.4
Durbin-Watson Statistic:	.718
Estimate of a:	1510.
Standard Error:	2580.
Estimate of b:	12.9
Standard Error:	1.31

Model: TOTOHE = a + b DIRPERE

Sum of Squared Residuals:	155. E7
Adjusted R-Squared:	.589
F-Statistic:	31.1
Durbin-Watson Statistic:	1.96
Estimate of a:	-286.
Standard Error:	17200.
Estimate of b:	15.2
Standard Error:	2.73

$$TOTOHA = 4250 + 16.3\ DIRPERA \text{ and}$$
$$TOTOHB = 3460 + 15.8\ DIRPERB.$$

It may be seen that the regression for contractor B lies everywhere below the regression for contractor A; not only does contractor B have a lower fixed cost (no statistical difference) but also it has a lower variable cost (no statistical difference). Comparisons between other contractors may also be made using the results in Table 2.

The reader should be aware that these comparisons imply only that, with the same number of direct personnel, contractor B experiences lower total overhead costs than contractor A. These comparisons do not imply that contractor B has lower overhead costs than contractor A, regardless of the circumstances. This observed, but not statistical, difference is at least partially due to the different personnel classification systems used by the two contractors.

The results shown for contractors C and D were obtained using the same procedures as were used for contractors A and B. However, there were some additional complicating factors which were not present for contractors A and B. The Wallis test statistic (equation 4) for contractor C showed that there was no evidence of AR(4); however, the Durbin-Watson showed that there was evidence of AR(1). After adjustment for AR(1), there still appeared to be some AR(4) remaining in the residuals. If this is true, then it indicates that there is most likely a mixture of AR(1) and AR(4) which must be removed from the regression residuals.

After adjusting the model for contractor D for the presence of the AR(4) process, there was still strong evidence from the Durbin-Watson statistic that some AR(1) process remained in the residuals. Again, it appears that there is a mixture process in the residuals.

The results shown for contractor E were obtained with no adjustment of the variables. Both the Durbin-Watson and the Wallis statistics indicated that AR(1) and AR(4) were not present.

Predictive Analyses

Since the results shown in Table 2 using total overhead costs for most contractors were of such high quality, it was determined that predictive tests of these regressions for each contractor should be undertaken. The general procedure was to fit the regression model to a sample of only the first four and one-half years (eighteen observations), predict the last year (four observations), and compare the predicted to the actual values of overhead cost.

The regression model using transformed data was estimated exactly as described in the previous section except that only the first eighteen observations were used. Based upon these estimated results, the last four observations were predicted via the equation

$$\hat{y}_t = \hat{\rho}_i y_{t-i} + (X_t - \hat{\rho}_i X_{t-i})\hat{\beta}\ ,\ t=19,\ldots,22, \tag{10}$$

where y_t and X_t are defined below, i is either 1 or 4 (depending on whether the AR(1) or AR(4) process is adjusted for), and $\hat{\beta}$ and $\hat{\rho}_i$ are the values obtained from the estimation based on the first eighteen observations.

Regression models of the quality of those discussed in the previous section generally have good predictive capabilities. Standard approaches to measuring the predictive power of regression models assume knowledge of all right hand side (RHS) variables. This poses particular problems for the current approach since predictions at t=18 for t=19,. . .,22 require future values of variables on the RHS of equation (10). Note that this makes the predictions, $\hat{y}_t$, conditional on the values of the RHS variables.

To generate predictions which are unconditional on the values of the RHS variables requires an independent process to provide forecasts of RHS which are unknown at the time of the forecast. If the residual process is determined to be AR(4) or AR(1), then values of X_t are required for use in equation (10). For an AR(1) process, values of y_t also are required.

To obtain independently forecasted values of X_t for t=19,. . .,22, the first eighteen observations were modeled using a Box-Jenkins approach (see [2] and [3]). Since only eighteen observations were available, the underlying ARMA models were kept as parsimonious as possible. Application of this methodology resulted in the models shown in Table 3. Forecasts of direct personnel for each contractor which resulted from these models were then used in equation (10) to obtain forecasts of overhead costs.

Since the error process for contractor C is AR(1), lagged future values of y_t are required on the RHS of equation (10) in order to pre-

TABLE 3

Box-Jenkins Models of Direct Personnel

Contractor	Model
A	ARMA(4,2)
B	ARMA(1,0)
C	ARMA(1,0)
D	ARMA(1,0)
E	ARMA(1,0)

dict future values of y_t at time t=18. Unlike the AR(4) case in which unconditional forecasts are available from equation (10) immediately upon obtaining independent forecasts of X_t, the AR(1) case requires lagged values of y_t which fall within the prediction window, t=19,. . .,22. Since the y_t's are lagged and hence known, unconditional forecasts of y_t are obtained using the independently forecasted X_t and rolling the unconditional y_t's forward one period.

Conditional and unconditional predicted values of overhead costs were then compared to the observed values of overhead costs using (1) a Pearson correlation coefficient, (2) the root mean squared forecast error, and (3) the mean absolute percentage error. These results are shown in Table 4.

For conditional predictions of contractor A, the Pearson correlation coefficient value of .981 indicates that there is a strong tendency for the predicted values of total overhead costs to follow closely the actual values. A measure of the size of the forecast errors is given by the ratio of the root mean squared error to the mean of the four actual values to be predicted. For contractor A, the root mean squared error is 5.4 percent of this mean and shows that the forecast errors are small relative to the actual values. A second measure of the size of the forecast errors is given by the mean absolute percentage error. This measure for contractor A indicates that the forecast errors are approximately 5.3 percent of the actual, observed values.

Except for contractor C, conditional prediction results for the other four contractors were similar in quality to those of contractor A. Again, the difficulty with contractor C (as well as contractor D) is that a mixture process is present but has not been eliminated from the residuals. If this were done, the prediction results for contractors C and D would improve considerably.

TABLE 4

Prediction Results

	Conditional	Unconditional
Model: TOTOHA = a + b DIRPERA		
Correlation coefficient between actual and predicted values	.981	.512
Root mean squared error divided by the mean of the actual values	.0540	.0639
Mean absolute percentage error (in percent)	5.28	5.83
Model: TOTOHB = a + b DIRPERB		
Correlation coefficient between actual and predicted values	.883	.900
Root mean squared error divided by the mean of the actual values	.0436	.0448
Mean absolute percentage error (in percent)	3.65	3.95
Model: TOTOHC = a + b DIRPERC		
Correlation coefficient between actual and predicted values	.544	.745
Root mean squared error divided by the mean of the actual values	.199	.228
Mean absolute percentage error (in percent)	19.1	21.4
Model: TOTOHD = a + b DIRPERD		
Correlation coefficient between actual and predicted values	.899	.896
Root mean squared error divided by the mean of the actual values	.114	.122
Mean absolute percentage error (in percent)	8.87	9.92
Model: TOTOHE = a + b DIRPERE		
Correlation coefficient between actual and predicted values	.845	.743
Root mean squared error divided by the mean of the actual values	.0755	.0980
Mean absolute percentage error (in percent)	5.31	6.99

Unconditional predictions for contractors B and D were very similar to their conditional predictions The unconditional predictions for contractors A and E were slightly worse and for contractor C was better than the respective conditional prediction. These results are very similar to those for the conditional models and indicate that not much predictive power is lost in moving to the unconditional models. This conclusion itself is conditioned on the prediction period, the prediction window, and, just as for the conditional models, the underlying modeling processes. Again, extending the sample size will improve these modeling results, especially for the residual processes which may be mixtures of AR(1) and AR(4).

Another general approach to estimating direct personnel is to use some other even more readily-available variable to attempt to predict direct personnel. The most logical and most available is units of output. In the case of one of the contractors, the most straightforward approach of regressing direct personnel on units of output of type 1, units of output of type 2, etc., produced a surprisingly high R-squared statistic of .84. In general, however, some assumptions about the production technology will be necessary in order to utilize this approach. Also, this approach requires a larger sample size than that utilized in the above approach since it estimates a larger number of coefficients.

Summary

The statistical models for analyzing overhead costs which have been presented in this paper have yielded, in general, excellent structural results. Additionally, predictive analyses were undertaken of the best structural models. These predictive analyses showed that reasonable predictions are possible for all contractors and that excellent predictions are available for most. These results indicate that this entire procedure may yield fruitful results when applied to other contractors and over longer time series. The above results indicate that overhead, at least for this sample, tends to follow variations in output levels through the number of direct personnel. More research is necessary in this area to evaluate the advantages and disadvantages of the alternative approaches for estimating and predicting costs of these and similar firms.

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