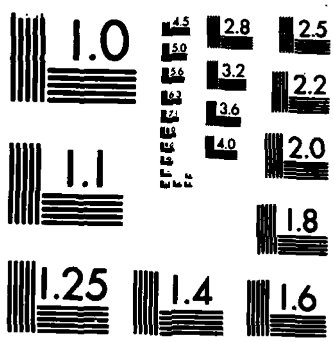
AD-A160 982 SINGULAR SOLUTIONS FOR SOME SEMILINEAR ELLIPTIC EQUATIONS(U) WISCONSIN UNIV-MADISON MATHEMATICS RESEARCH CENTER H BREZIS ET AL AUG 85 MRC-TSR-2858 1/1

UNCLASSIFIED DAAG29-80-C-0041 F/G 12/1 NL

END

FILMED

DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

(2)

MRC Technical Summary Report #2858

SINGULAR SOLUTIONS FOR SOME SEMILINEAR ELLIPTIC EQUATIONS

Haïm Brezis and Luc Oswald

Mathematics Research Center
University of Wisconsin—Madison
610 Walnut Street
Madison, Wisconsin 53705

August 1985

(Received August 7, 1985)

Approved for public release
Distribution unlimited

Sponsored by

U. S. Army Research Office
P. O. Box 12211
Research Triangle Park
North Carolina 27709

(A)

UNIVERSITY OF WISCONSIN-MADISON
MATHEMATICS RESEARCH CENTER

SINGULAR SOLUTIONS FOR SOME SEMILINEAR ELLIPTIC EQUATIONS

Haïm Brezis and Luc Oswald

Technical Summary Report #2858
August 1985

ABSTRACT

This document

~~We~~ give a new proof of Véron's result concerning the classification of

delta sub p The authors

isolated singularities for the equation $-\Delta u + u^p = 0$. ~~We~~ also establish that the singular behavior at a point can be prescribed and determines uniquely the solution (under fixed boundary conditions). Keywords: nonlinear elliptic equations, Scaling; applied mathematics; mathematical analysis.

AMS (MOS) Subject Classifications: 35J60

Key Words: Isolated singularities

Work Unit Number 1 (Applied Analysis)

Sponsored by the United States Army under Contract No. DAAG29-80-C-0041.

SIGNIFICANCE AND EXPLANATION

Nonlinear elliptic equations with isolated singularities occur in physical problems with point sources. A good example is the Thomas-Fermi theory of atoms and molecules which leads to the equation $-\Delta u + u^{3/2} = 0$ in $\mathbb{R}^3 \setminus \bigcup_{i=1}^{k} \{a_i\}$.

The points $\{a_i\}$ correspond to the location of positive nuclei of charge m_i. Near a_i the solution u has a singular behavior equivalent to $m_i E(x - a_i)$ where E is the fundamental solution of $-\Delta$, i.e. $E(x) = (4\pi|x|)^{-1}$. A striking result of L. Véron provides a complete classification of all singular solutions, and shows that isolated singularities of nonlinear problems are quite rigid. In this paper we present a new proof of Véron's result based on a simple scaling argument. We also establish that the singular behavior at a point can be prescribed very much like a boundary condition and determines uniquely the solution.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the authors of this report.

SINGULAR SOLUTIONS FOR SOME SEMILINEAR ELLIPTIC EQUATIONS

Haïm Brezis and Luc Oswald

Dedicated to Jim Serrin on his sixtieth birthday

1. Introduction

Let $B_R = \{x \in R^N;\ |x| < R\}$ with $N > 2$. Consider a function u which satisfies

$$(1) \quad \begin{cases} u \in C^2(B_R\setminus\{0\}), \quad u > 0 \quad \text{on} \quad B_R\setminus\{0\}, \\ -\Delta u + u^p = 0 \quad \text{on} \quad B_R\setminus\{0\}. \end{cases}$$

We are concerned with the behavior of u near $x = 0$. There are two distinct cases:

1) When $p > N/(N-2)$ and $(N > 3)$ it has been shown by Brezis - Véron [9] that u must be smooth at 0 (See also Baras-Pierre [1] for a different proof). In other words, isolated singularities are removable.

2) When $1 < p < N/(N-2)$ there are solutions of (1) with a singularity at $x = 0$. Moreover all singular solutions have been classified by Véron [22]. We recall his result:

Theorem 1 Assume $1 < p < N/(N-2)$ and u satisfies (1). Then one of the following holds:

(i) either u is smooth at 0,

(ii) or $\lim_{x\to 0} u(x)/E(x) = c$ where c is a constant which can take any value in the interval $(0,\infty)$,

(iii) or $\lim_{x\to 0} |u(x) - \ell(p,N)|x|^{-2/(p-1)}| = 0$.

Here $E(x)$ denotes the fundamental solution of $-\Delta$ and $\ell = \ell(p,N)$ is the (unique) positive constant C such that $C|x|^{-2/(p-1)}$ satisfies (1) - more precisely

Sponsored by the United States Army under Contract No. DAAG29-80-C-0041.

$$\ell = \ell(p,N) = \left[\frac{2}{(p-1)}\left(\frac{2p}{p-1} - N\right)\right]^{1/(p-1)} .$$

We shall first present a proof of Theorem 1 which is simpler than the original proof of Véron. In particular, it does not make use of Fowler's results [10] for the Emden differential equation. Instead, it relies on some simple scaling argument (see the proof of Lemma 5) which is similar to the one used by Kamin-Peletier [12] for parabolic equations.

Next, we emphasize that a singular behavior such as (ii) or (iii) can be prescribed together with a boundary condition, and these determine uniquely the solution.

More precisely, let Ω be a smooth bounded domain in $\mathbb{R}^N$ with $0 \in \Omega$ and let $\varphi > 0$ be a smooth function defined on $\partial\Omega$. We consider the problem

$$(2) \qquad \begin{cases} u \in C^2(\bar{\Omega}\backslash\{0\}), \quad u > 0 \quad \text{on} \quad \Omega \backslash \{0\} , \\ -\Delta u + u^p = 0 \qquad \text{on} \quad \Omega \\ \qquad u = \varphi \qquad \text{on} \quad \partial\Omega . \end{cases}$$

Theorem 2 Assume $1 < p < N/(N-2)$. Then:

(i) There is a unique solution u_0 of (2) which belongs to $C^2(\bar{\Omega})$.

(ii) Given any constant $c \in (0, +\infty)$ there is a unique solution u_c of (2) which satisfies

$$\lim_{x \to 0} u(x)/E(x) = c .$$

(iii) There is a unique solution u_∞ of (2) which satisfies

$$\lim_{x \to 0} |x|^{2/(p-1)} u(x) = \ell(p,N)$$

In addition, $\lim\limits_{c \downarrow 0} u_c = u_0$ and $\lim\limits_{c \uparrow \infty} = u_\infty$.

Singular solutions of (1) occur in the Thomas-Fermi theory with $N = 3$ and $p = 3/2$ (see e.g. [13] for a detailed exposition). Other results dealing with singular solutions of nonlinear elliptic equations have been obtained by a number of authors: J. Serrin [20], [21], Véron and Vazquez (See the exposition in [23]), P. L. Lions [14], W. M. Ni-J. Serrin [16]. Semilinear parabolic equations with isolated singularities have been considered by Brezis - Friedman [5], Brezis - Peletier - Terman [8], Kamin - Peletier [12], Oswald [18].

2. Some preliminary facts

We recall some known results dealing with functions u satisfying (1).

Set $\alpha = 2/(p-1)$ (for $1 < p < \infty$).

Lemma 1 Assume $u \in C^2(B_R)$ satisfies (1).

Then

$$u(0) < C(p,N)/R^{\alpha}$$

where $C(p,N)$ is defined by $C(p,N) = \text{Max}\left\{2\alpha N,\ 4\alpha(\alpha+1)\right\}^{1/(p-1)}$.

The proof of Lemma 1 uses a comparision function U of the same type as in Osserman [17] (or Loewner - Nirenberg [15]), namely set

$$U(x) = \frac{C(p,n)\ R^{\alpha}}{(R^2 - |x|^2)^{\alpha}} \quad \text{on } B_R .$$

A direct computation shows that

$$-\Delta U + U^p > 0 \qquad \text{on } B_R .$$

By the maximum principle we see that

$$u < U \quad \text{on } B_R$$

and in particular $u(0) < U(0)$.

Lemma 2 Assume u satisfies (1) with $1 < p < N/(N-2)$. Then, for $0 < |x| < R/2$, we have

$$u(x) < \frac{\ell(p,N)}{|x|^{\alpha}} \left(1 + \frac{C(p,N)}{\ell(p,N)} \left(\frac{|x|}{R}\right)^{\beta}\right)$$

where $\beta = 2\alpha + 2 - N > \alpha$.

Lemma 2 is established in Brezis - Lieb [6] (proposition A.4) for the special case where $N = 3$ and $p = 3/2$. The proof in the general case is just the same.

Lemma 3 Assume $1 < p < N/(N-2)$ and let $c > 0$ be a constant. Then, there is a unique function u satisfying

$$
(3)\qquad \begin{cases} u \in L^p(\mathbb{R}^N) \cap C^2(\mathbb{R}^N\setminus\{0\}) , \\ u > 0 \quad \text{on} \quad \mathbb{R}^N\setminus\{0\} , \\ -\Delta u + u^p = c\delta \quad \text{on} \quad \mathbb{R}^N \end{cases}
$$

We set $u = W_c$.

Lemma 3, as well as Lemma 4 below, are due to Benilan - Brezis (unpublished); the ingredients for the proofs may be found in [2], [3], [4] (and also [1] and [11]).

Finally, we assume that Ω is a smooth bounded domain in $\mathbb{R}^N$ with $0 \in \Omega$ and that $\varphi > 0$ is a smooth function defined as $\partial\Omega$.

Lemma 4 Assume $1 < p < N/(N-2)$ and let $c > 0$ be a constant.
Then, there is a unique function u satisfying

$$
(4)\qquad \begin{cases} u \in L^p(\Omega) \cap C^2(\bar{\Omega}\setminus\{0\}) \\ u > 0 \quad \text{on} \quad \Omega\setminus\{0\} \\ -\Delta u + u^p = c\delta \quad \text{on} \quad \Omega \\ \quad u = \varphi \quad \text{on} \quad \partial\Omega . \end{cases}
$$

3. A Scaling Argument

An important step in the proof of Theorem 1 is the following

Lemma 5 Assume $1 < p < N/(N-2)$. Then we have

$$\lim_{c\uparrow\infty} W_c(x) = \ell|x|^{-\alpha} \equiv W_\infty(x) .$$

Proof It is clear (by comparison) that $W_c(x)$ is a nondecreasing function of c. Moreover we have

$$W_c(x) \le \ell|x|^{-\alpha}$$

(by letting $R \to \infty$ in Lemma 2). Therefore $\lim_{c\uparrow\infty} W_c(x) = W_\infty(x)$ exists pointwise (for $x \neq 0$) and $W_\infty(x) \le \ell|x|^{-\alpha}$. The uniqueness of the solution of (3) implies that $W_c(x)$ is radial and so is $W_\infty(x)$. Next, we observe that the function

$$u(x) = k^\alpha W_c(kx) \qquad (k > 0)$$

satisfies

$$-\Delta u(x) + u^p(x) = k^{\alpha p} c\delta(kx) = k^{\alpha p-N} c\delta(x) .$$

It follows, again by uniqueness, that

$$k^\alpha W_c(kx) = W_{ck^{\alpha p-N}}(x) .$$

As $c\uparrow\infty$ we see that

$$k^\alpha W_\infty(kx) = W_\infty(x) .$$

Choosing $k = 1/|x|$ we obtain

$$W_\infty(x) = W_\infty\left(\frac{x}{|x|}\right)|x|^{-\alpha} = C|x|^{-\alpha}$$

where $C > 0$ is some constant.

Finally we note that since

$$-\Delta W_c + W_c^p = 0 \quad \text{in} \quad \mathcal{D}'(\mathbb{R}^N\backslash\{0\})$$

and

$$W_c \to W_\infty \quad \text{in} \quad L^p_{\ell oc}(\mathbb{R}^N\backslash\{0\}),$$

it follows that

$$-\Delta W_\infty + W_\infty^p = 0 \quad \text{in} \quad \mathcal{D}'(\mathbb{R}^N\setminus\{0\}).$$

This determines the value of the constant C to be $C = \ell$.

There is a similar result in balls: Set $u = V_c$ to be the unique solution of problem (4) with $\Omega = B_R$.

Lemma 6 Assume $1 < p < N/(N-2)$. Then we have $V_\infty(x) \equiv \lim_{c\uparrow\infty} V_c(x)$ exists pointwise on $B_R\setminus\{0\}$ and moreover

$$W_\infty(x) - \ell R^{-\alpha} \leq V_\infty(x) \leq W_\infty(x) \quad \text{on} \quad B_R .$$

Proof It is again clear (by comparison) that $V_c(x)$ is a nondecreasing function of c. Also we have

$$0 \leq V_c(x) \leq W_c(x) . \tag{5}$$

It follows from (4) and (5) that

$$-\Delta(W_c - V_c) \leq 0 \quad \text{on} \quad B_R ,$$

and consequently $\sup\limits_{B_R} (W_c - V_c) \leq \sup\limits_{\partial B_R} (W_c - V_c) \leq \sup\limits_{\partial B_R} W_\infty = \ell R^{-\alpha}$.

The conclusion follows by letting $c \to \infty$.

4. Proof of Theorem 1

Throughout this section we suppose $1 < p < N/(N-2)$. Assume u satisfies (1) and set

$$c = \limsup_{x \to 0} u(x)/E(x) .$$

We distinguish three cases:

Case (i) $c = 0$

Case (ii) $0 < c < \infty$

Case (iii) $c = \infty$.

Cases (i) and (ii).

Here, the main ingredient is the following:

Lemma 7 In cases (i) and (ii) the function u belongs to $L^p_{\ell oc}(B_R)$ and satisfies

$$-\Delta u + u^p = c_o \delta \quad \text{in} \quad \mathcal{D}'(B_R)$$

for some constant c_0.

Proof It is clear that $u \in L^p_{\ell oc}(B_R)$ since $E \in L^p_{\ell oc}(B_R)$ and $c < \infty$. We now use the same argument as in [7]: set

$$T = -\Delta u + u^p \in \mathcal{D}'(B_R) .$$

Since the support of T is contained in $\{0\}$, it follows from a classical result about distributions (see [19]) that

$$T = \sum_{0 \le |\alpha| \le m} c_\alpha D^\alpha(\delta) . \tag{6}$$

We claim that $c_\alpha = 0$ when $|\alpha| \geq 1$. Indeed let $\zeta \in \mathcal{D}(B_R)$ be any fixed function such that $(-1)^{|\alpha|} D^\alpha \zeta(0) = c_\alpha$ for every α with $|\alpha| \le m$. Multiplying (6) through by $\zeta_\varepsilon(x) = \zeta(x/\varepsilon)$ we obtain

$$- \int u \Delta \zeta_\varepsilon + \int u^p \zeta_\varepsilon = \sum_{0 \le |\alpha| \le m} c_\alpha^2 \varepsilon^{-|\alpha|} .$$

An easy computation - using the estimate $u \le CE$ - shows that

$$\begin{cases} \left|\int u \, \Delta\zeta_\varepsilon\right| \le C & \text{when } N \ge 3 \\ \left|\int u \, \Delta\zeta_\varepsilon\right| \le C|\log \varepsilon| + C & \text{when } N = 2 . \end{cases}$$

Since $\int u^p \zeta_\varepsilon \to 0$ as $\varepsilon \to 0$, we conclude that $c_\alpha = 0$ for $|\alpha| \ge 1$. Therefore we obtain

$$-\Delta u + u^p = c_0 \delta \quad \text{in} \quad \mathcal{D}'(B_R)$$

We conclude the proof of Theorem 1 in cases (i) and (ii) with the help of the following:

<u>Lemma 8</u> Assume $u \in C^2(B_R \setminus \{0\}) \cap L^p_{\ell oc}(B_R)$ satisfies

$$\begin{cases} u \ge 0 & \text{on } B_R , \\ -\Delta u + u^p = c_0 \delta & \text{in } \mathcal{D}'(B_R) \end{cases}$$

for some constant c_0.

We have

(i) if $c_0 = 0$, then u is smooth on B_R ,

(ii) if $c_0 \neq 0$, then $\lim_{x \to 0} u(x)/E(x) = c_0$.

<u>Proof</u>

(i) Assume $c_0 = 0$. Since u is subharmonic it follows that $u \in L^\infty_{\ell oc}(B_R)$ and thus $\Delta u \in L^\infty_{\ell oc}(B_R)$. We deduce that $u \in C^1(B_R)$ and then $u \in C^2(B_R)$. In fact $u \in C^\infty(B_R)$ since, by the strong maximum principle, we have either $u \equiv 0$ or $u > 0$ or B_R.

(ii) Assume $c_0 \neq 0$. By the maximum principle we have

$$u \le c_0 E + C \quad \text{on} \quad B_{R/2}$$

and therefore

$$\begin{aligned} -\Delta u &\ge c_0 \delta - (c_0 E + C)^p \\ &\ge c_0 \delta - C(E^p + 1) \quad \text{on} \quad B_{R/2} \end{aligned}$$

An elementary computation leads to

$$u(x) \geq c_0 E - o(E) \quad \text{as} \quad x \to 0 \ .$$

and we conclude that $\lim_{x \to 0} u(x)/E(x) = c_0$.

Remark 1 Assume $c_0 \neq 0$. The argument above provides in fact an estimate for $|u - c_0 E|$ as $x \to 0$. More precisely we have

a) If $N = 2$ and $1 < p < \infty$ or $N = 3$ and $1 < p < 2$, then

$$|u - c_0 E| \leq C \quad \text{on} \quad B_{R/2}$$

b) If $N = 3$ and $p = 2$, then

$$|u(x) - c_0 E(x)| \leq C(|\ell og|x|| + 1) \quad \text{on} \quad B_{R/2}$$

c) If $N = 3$ and $2 < p < 3$ or $N \geq 4$ and $1 < p < N/(N-2)$ then

$$|u(x) - c_0 E(x)| \leq C|x|^{2-(N-2)p} \quad \text{on} \quad B_{R/2}$$

and consequently

$$\left|\frac{u(x)}{E(x)} - c_0\right| \leq C\ |x|^{\nu} \quad \text{on} \quad B_{R/2}$$

with $\nu = N - (N-2)p > 0$.

Proof of Theorem 1 in the case (iii)

We first recall a result of Véron [22] (Lemma 1.5):

Lemma 9 Assume u satisfies (1). Then, there is a constant C (depending only as p and N) such that

$$\sup_{|x|=r} u(x) \leq C \inf_{|x|=r} u(x) \quad \text{for} \quad 0 < r < R/2.$$

The conclusion of Lemma 9 is a simple consequence of Harnack's inequality and the estimate of Lemma 1 - see [22] for the details.

We may now complete the proof of Theorem 1 with the help of the following:

Lemma 10 Assume u satisfies (1) and $\limsup_{x \to 0} u(x)/E(x) = \infty$. Then

$$|u(x) - \ell|x|^{-\alpha}| \leq C|x|^{\gamma} \quad \text{on} \quad B_{R/2}$$

for some constants $C = C(p,N,R)$ and $\gamma = \gamma(p,N) > 0$.

Proof By Lemma 2 we already have the estimate

$$u(x) \leq \ell|x|^{-\alpha} + C|x|^{\gamma} \quad \text{on} \quad B_{R/2}$$

with

$$\gamma = \beta - \alpha = \alpha + 2 - N > 0.$$

We now establish an estimate from below. Let $x_n \to 0$ be such that $\lim u(x_n)/E(x_n) = \infty$. Set $r_n = |x_n|$, so that we obtain from Lemma 9

$$\text{(7)} \qquad \inf_{|x|=r_n} u(x)/E(x) \underset{n\to\infty}{\to} \infty .$$

We recall that V_c is the unique solution of (4) when $\Omega = B_R$, so that $V_c \leq cE$ on B_R.

Given any constant $c > 0$, we see (by (7)) that

$$u(x) \geq cE(x) \quad \text{for} \quad |x| = r_n \quad \text{and} \quad n \quad \text{large enough} .$$

Therefore we obtain

$$u(x) \geq V_c(x) \quad \text{for} \quad |x| = r_n \quad \text{and} \quad n \quad \text{large enough} .$$

Applying the maximum principle in the domain $\{x \in \mathbb{R}^N; \ r_n < |x| < R\}$ we find that

$$u(x) \geq V_c(x) \quad \text{for} \quad r_n < |x| < R \quad \text{and} \quad n \quad \text{large enough} .$$

As $n \to \infty$ we conclude that

$$u(x) \geq V_c(x) \quad \text{on} \quad B_R\setminus\{0\}$$

and as $c \to \infty$ we see that

$$u(x) \geq V_\infty(x) \quad \text{on} \quad B_R\setminus\{0\} .$$

In Lemma 6 we had the estimate

$$V_\infty(x) \geq \ell(|x|^{-\alpha} - R^{-\alpha}) .$$

However it is not good enough to deduce conclusion (iii) of Theorem 1. We need a better estimate from below for $V_\infty(x)$; we claim that

$$\text{(8)} \qquad V_\infty(x) \geq \ell|x|^{-\alpha}\left(1 - \left(\frac{|x|}{R}\right)^{\beta}\right) \quad \text{on} \quad B_R ,$$

where β is defined in Lemma 2.

Clearly, it suffices to establish (8) for $R = 1$. The function V_∞ is radial and so we write $V_\infty(r)$. We define the function v or $(0,1)$ by the relation

$$v(r^\beta) = \ell^{-1} r^\alpha V_\infty(r)$$

so that $0 < v < 1$ on $(0,1)$, $v(1) = 0$ and $v(0) = 1$. Using the relation $-\Delta V_\infty + V_\infty^p = 0$ it is easy to deduce (as in the proof of Proposition A.4 [6]) that

$$-\beta^2 t^2 v''(t) + \ell^{p-1} v(t)(v^{p-1}(t) - 1) = 0 \quad \text{for} \quad t \in (0,1).$$

Consequently v is concave and thus we have

$$v(t) \geq 1 - t \qquad \forall t \in (0,1) ,$$

that is (8).

Remark 2 Véron [22] obtains in case (iii) an estimate of the form $|u(x) - \ell|x|^{-\alpha}| \leq C|x|^\delta$ with an exponent δ which is better than $\gamma = \beta - \alpha$.

5. Proof of Theorem 2.

Case (i) is classical.

Case (ii) The existence of a solution follows from Lemma 4 and 8.

Suppose now u satisfies (2) and $\lim_{x\to 0} u(x)/E(x) = c$. We deduce from Lemma 7 and 8 that $-\Delta u + u^p = c\delta$; uniqueness follows from Lemma 4.

Case (iii) We denote by u_c the unique solution of (4) given by Lemma 4. We claim that $u_\infty = \lim_{c\uparrow\infty} u_c$ has all the required properties.

Indeed $u_c(x)$ is a nondecreasing function of c. Fix $R > 0$ such that $2R < \mathrm{dist}(0,\partial\Omega)$. By Lemma 1 we have

$$u_c(x) \le C(p,N)R^{-\alpha} \quad \text{for} \quad |x| = R.$$

The maximum principle applied in the region

$$\Omega_R = \{x \in \Omega; \quad |x| > R\}$$

shows that, in Ω_R,

$$u_c(x) \le \mathrm{Max}\, \{\sup_{\partial\Omega} \varphi, \quad C(p,N)R^{-\alpha}\} .$$

Therefore $u_\infty(x) = \lim_{c\uparrow\infty} u_c(x)$ exists and u_∞ satisfies (2). By comparison on B_R we have

$$V_c \le u_c \quad \text{on} \quad B_R$$

and as $c \to \infty$ we obtain $V_\infty \le u_\infty$ on B_R.

It follows that $\lim_{x\to 0}|u_\infty(x) - \ell|x|^{-\alpha}| = 0$ (by Lemma 6 and Theorem 1).

We turn now the question of uniqueness. Suppose u_1 and u_2 satisfy (2) and $\lim_{x\to 0}|x|^\alpha u_i(x) = \ell$ for $i = 1, 2$. Lemma 10 implies that

$$|u_1(x) - u_2(x)| \le C|x|^\gamma \quad \text{on} \quad B_R$$

On the other hand we have

$$-\Delta(u_1 - u_2) + u_1^p - u_2^p = 0 \quad \text{on} \quad \Omega\backslash\{0\}$$

Applying the maximum principle in Ω_R we

$$\max_{\Omega_R}|u_1 - u_2| \leq \max_{\partial B_R}|u_1 - u_2| \leq CR^{\gamma}$$

and then we let $R \to 0$ to conclude that $u_1 \equiv u_2$.

References

[1] P. Baras - M. Pierre, Singularités éliminables pour des équations semi-linéaires, Ann. Inst. Fourier 34 (1984) p. 185-206.

[2] Ph. Benilan - H. Brezis - M. Crandall, A semilinear elliptic equation in $L^1(\mathbb{R}^N)$, Ann. Sc. Norm. Sup. Pisa 2 (1975) p. 523-555.

[3] H. Brezis, Some variational problems of the Thomas-Fermi type, in Variational inequalities, Cottle, Gianessi, Lions ed. Wiley (1980) p. 53-73.

[4] H. Brezis, Semilinear equations in $\mathbb{R}^N$ without condition at infinity, Applied Math. and Opt. 12 (1984) p. 271-282.

[5] H. Brezis - A. Friedman, Nonlinear parabolic equations involving measures as initial conditions, J. Math Pures et Appl. 62 (1983) p. 73-97.

[6] H. Brezis - E. Lieb, Long range potentials in Thomas-Fermi theory, Comm. Math. Phys. 65 (1979) p. 231-246.

[7] H. Brezis - P. L. Lions, A note on isolated singularities for linear elliptic equations, in Mathematical Analysis and Applications, Part A, a volume dedicated to L. Schwartz, L. Nachbin ed. Acad. Press (1981) p. 263-266.

[8] H. Brezis - L. Peletier - D. Terman, A very singular solution of the heat equation with absorption, Archive Rat. Mech. Anal. (to appear).

[9] H. Brezis - L. Véron, Removable singularities of some nonlinear elliptic equations, Archive Rat. Mech. Anal. 75 (1980) p. 1-6.

[10] R. H. Fowler, Further studies on Emden's and similar differential equations, Quarterly J. Math 2 (1931) p. 259-288.

[11] Th. Gallouet - [illegible]orel, Resolution of a semilinear equation in L^1, Proc. Roy. Soc. Edinburgh, 96A (1984) p. 275-288.

[12] S. Kamin - L. Peletier, Singular solutions of the heat equation with absorption (to appear).

[13] E. Lieb, Thomas-Fermi and related theories of atoms and molecules, Reviews of Modern Phys. 53 (1981) p. 603-641.

[14] P. L. Lions, Isolated singularities in semilinear problems, J. Diff. Eq. 38 (1980) p. 441-450.

[15] C. Loewner - L. Nirenberg, Partial differential equations invariant under conformal or projective transformations in Contributions to Analysis, Acad. Press (1974) p. 245-272.

[16] W. M. Ni - J. Serrin, Non-existence theorems for singular solutions of quasilinear partial differential equations (to appear).

[17] R. Osserman, On the inequality $\Delta u > f(u)$, Pacific J. Math. 7 (1957) p. 1641-1647.

[18] L. Oswald, Isolated singularities for a nonlinear heat equation, C. R. Acad. Sc. and detailed paper to appear.

[19] L. Schwartz, Théorie des distributions, Hermann (1966).

[20] J. Serrin, Local behavior of solutions of quasilinear equations, Acta Math. 111 (1964), p. 247-302.

[21] J. Serrin, Isolated singularities of solutions of quasilinear equations, Acta Math. 113 (1965), p. 219-240.

[22] L. Véron, Singular solutions of some nonlinear elliptic equations, Nonlinear Anal. 5 (1981), p. 225-242.

[23] L. Véron, Weak and strong singularities of nonlinear elliptic equations, AMS Symp. Pure Math. (to appear).

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 2858	2. GOVT ACCESSION NO. AD-A160 982	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) SINGULAR SOLUTIONS FOR SOME SEMILINEAR ELLIPTIC EQUATIONS		5. TYPE OF REPORT & PERIOD COVERED Summary Report - no specific reporting period
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Haïm Brezis and Luc Oswald		8. CONTRACT OR GRANT NUMBER(s) DAAG29-80-C-0041
9. PERFORMING ORGANIZATION NAME AND ADDRESS Mathematics Research Center, University of Wisconsin 610 Walnut Street Madison, Wisconsin 53706		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Work Unit Number 1 - (Applied Analysis)
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office P.O. Box 12211 Research Triangle Park, North Carolina 27709		12. REPORT DATE August 1985
		13. NUMBER OF PAGES 16
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Isolated singularities

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

We give a new proof of Véron's result concerning the classification of isolated singularities for the equation $-\Delta u + u^p = 0$. We also establish that the singular behavior at a point can be prescribed and determines uniquely the solution (under fixed boundary conditions).

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

END

FILMED

12-85

DTIC