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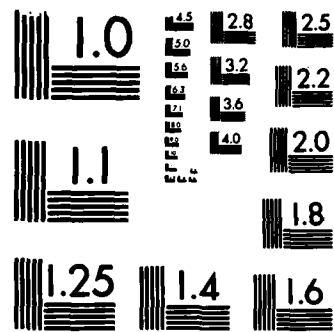NATIONAL BUREAU OF STANDARDS-1963-A

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CONCAVITY OF SOLUTIONS OF THE
POROUS MEDIUM EQUATION

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CONCAVITY OF SOLUTIONS OF THE POROUS MEDIUM EQUATION

Philippe Benilan[*],[1] and Juan Luis Vazquez[**],[1],[2]

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ABSTRACT

We consider the problem

$$(P) \quad \begin{cases} u_t = (u^m)_{xx} & \text{with } x \in R, t > 0 , \\ u(x,0) = u_0(x) & \text{for } x \in R , \end{cases}$$

where $m > 1$ and u_0 is a continuous, nonnegative function that vanishes outside an interval (a,b) and such that $(u_0^{m-1})'' < -C < 0$ in (a,b). Using a Trotter-Kato formula we show that the solution conserves the concavity in time: for every $t > 0$, $u(x,t)$ vanishes outside an interval $\Omega(t) = (\zeta_1(t), \zeta_2(t))$ and

$$(u^{m-1})_{xx} < - \frac{C}{1 + C \frac{m(m+1)}{m-1} t}$$

in $\Omega(t)$. Consequently the interfaces $x = \zeta_i(t)$, $i = 1,2$, are concave curves. These results also give precise information about the large time behaviour of solutions and interfaces.

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SIGNIFICANCE AND EXPLANATION

The flow of a gas through a porous medium is governed by a degenerate quasilinear parabolic equation. It is known that the nonnegative solutions to this equation possess a lower bound for the second derivative of the pressure in the spatial variables. This bound plays an important role in the mathematical treatment and is related to the entropy of the flow. Since the solutions exhibit interfaces across which v_x jumps positively, no upper bound is possible globally for v_{xx}. Nevertheless we prove it is proven that the concavity of $v(\cdot,t)$ in the region where v is positive is preserved in time. This is in itself an interesting geometric property of the solution. It also allows one to obtain precise information about the asymptotic behaviour of the flow.

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CONCAVITY OF SOLUTIONS OF THE POROUS MEDIUM EQUATION

Philippe Benilan[*],[(1)] and Juan Luis Vazquez[**],[(1),(2)]

Introduction

We consider the Cauchy problem

$$(P) \quad \begin{cases} u_t = (u^m)_{xx} & \text{in } Q = R \times (0,\infty) \\ u(x,0) = u_0(x) & \text{for } x \in R \end{cases}$$

where $m > 1$ and u_0 is a non-negative continuous function vanishing outside a bounded interval (a,b) with $u_0(x) > 0$ for $x \in (a,b)$.

It is well known (see for instance [V2] for general references on this problem) that there exists a unique generalized solution $u(x,t)$ of (P) that is non-negative bounded and continuous on $\overline{Q}$. Moreover there exist two continuous monotone curves $x = \zeta_1(t)$, $x = \zeta_2(t)$ called the interfaces such that $\zeta_1(0) = a$, $\zeta_2(0) = b$ and

$$\Omega \overset{\text{def}}{=} \{(x,t)\epsilon Q; u(x,t) > 0\} = \{(x,t)\epsilon Q; \zeta_1(t) < x < \zeta_2(t)\} \quad .$$

Also $u \in C^\infty(\Omega)$.

In the description of the flow of a gas through a porous medium, u represents the density of the gas and $v = \dfrac{m}{m-1} u^{m-1}$ the pressure. We shall retain this denomination in the sequel. Of course $v \in C^\infty(\Omega)$ and it satisfies on Ω the equation

$$(0.1) \qquad v_t = (m-1)vv_{xx} + v_x^2 \quad .$$

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We recall also the basic "convexity inequalities"

$$(0.2) \qquad v_{xx} \geqslant - \frac{1}{(m+1)t} \quad \text{in } D'(Q)$$

$$(0.3) \qquad (-1)^i (\zeta_i'' + \frac{m}{(m+1)t} \zeta_i') \geqslant 0 \quad \text{in } D'(\mathbb{R}^+)$$

Now we assume that the initial pressure $v_0 = \frac{m}{m-1} u_0^{m-1}$ is concave in (a,b); more precisely we assume

$$(0.4) \qquad v_{0,xx} \leqslant -C \quad \text{in } D'(a,b)$$

with some constant $C \geqslant 0$. The purpose of this paper is to study the concavity of the pressure v on Ω and of the interfaces ζ_i on $\mathbb{R}^+$, and their consequences. The main result is

THEOREM 1. <u>Under assumption</u> (0.4), <u>the following "concavity inequalities"</u> <u>hold</u>

$$(0.5) \qquad v_{xx} \leqslant - \frac{C}{1+(m+1)Ct} \quad \text{on } \Omega$$

$$(0.6) \qquad (-1)^i (\zeta_i'' + \frac{mC}{1+(m+1)Ct} \zeta_i') \leqslant 0 \quad \text{in } D'(\mathbb{R}^+) \quad \underline{\text{for}} \quad i = 1,2 \ .$$

In the case $C = 0$, the concavity (0.5) has already been proved by Graveleau and Jamet [GJ]. For proving (i) in the general case $C \geqslant 0$, we will follow their approach based on splitting (0.1) into the two equations

$$(0.7) \qquad v_t = (m-1)vv_{xx}$$

$$(0.8) \qquad v_t = v_x^2 \ .$$

We then recover (0.1) from (0.7), (0.8) via a Trotter-Kato formula.

In the case $C > 0$, the combination of (0.5) and (0.6) with (0.2) and (0.3) gives strong information about the asymptotic behaviour of the pressure and the interfaces. It was proved in [V1] that this asymptotic behaviour is determined in first approximation by the two invariants of the motion, namely the total mass

$$(0.9) \qquad M = \int u_0(x)dx = \int u(x,t)dx$$

and the center of mass

(0.10)
$$x_0 = M^{-1} \int x u_0(x) dx = M^{-1} \int x u_0(x) dx \quad .$$

In fact there is a selfsimilar (Barenblatt) solution given in terms of the pressure by

(0.11)
$$\bar{v}(x,t) = \frac{[r(t)^2 - (x-x_0)^2]_+}{2(m+1)t}$$

with

(0.12)
$$r(t) = c(M^{m-1}t) \quad , \quad c = c(m) > 0 \quad ,$$

such that as $t \to \infty$, $v-\bar{v} = o(t^{-(m+1)})$ uniformly in $x \in \mathbb{R}$, $v_x + x/(m+1)t = o(t^{-1})$ uniformly in Ω, $\zeta_i(t) = x_0 + (-1)^i r(t) + o(1)$ and $\zeta_i'(t) = (-1)^i r'(t) + o(t^{-1})$.

As a consequence of Theorem 1 these results can be improved as follows:

THEOREM 2. <u>Under the assumption</u> (0.4) <u>with</u> $C > 0$ <u>we have as</u> $t \to \infty$

(0.13)
$$v_{xx}(x,t) = -\frac{1}{(m+1)t} + O(\frac{1}{t^2})$$

<u>uniformly in</u> x <u>for</u> $(x,t) \in \Omega$ <u>and</u>

(0.14)
$$\zeta_i''(t) = (-1)^i r''(t) \; (1 + O(\frac{1}{t})) \quad .$$

These rates of convergence are optimal as it can be easily checked on the time-delayed Barenblatt solutions $v(x,t;\tau) = \bar{v}(x,t+\tau)$, where $\bar{v}$ is given by (0.11) and $\tau > 0$. From (0.13), (0.14) sharp estimates follow for v, v_x, ζ and ζ'.

The main problem in proving concavity results lies in the fact that across a moving interface of a solution to problem (P) v_x has a jump discontinuity, therefore v_{xx} is very singular and positive on the interfaces. Using equations (0.7), (0.8) allows to overcome this difficulty because the interfaces in (0.7) do not move while in (0.8) we are reduced to study the characteristic lines of a first-order equation.

On the contrary in proving lower bounds for v_{xx} no such difficulty arises since we can always approximate our solution by smooth positive solutions and apply the maximum principle to the equation satisfied by v_{xx} as in the proof of (0.2), cf. [AB]. Thus if v is a solution of (0.1) whose initial datum v_0 satisfies

(0.15) $$v_{0,xx} \geq -C \quad \text{in} \quad D'(\mathbb{R})$$

then

(0.16) $$v_{xx} \geq - \frac{C}{1+(m+1)Ct} \quad \text{in} \quad D'(Q) \quad .$$

It is worth noting that we can always use the maximum principle method of [AB] when dealing with problem (P) for $0 < m < 1$ (the so-called fast-diffusion case) or $m = 1$ (the heat equation) and the above results (0.5), (0.16) are true if we define $v = - \frac{m}{1-m} u^{-(1-m)}$ if $m < 1$, $v = \log(u)$ if $m = 1$. We shall leave the verification of these facts to the reader, but let us recall that $v \in C^{\infty}(Q)$ in both cases and $v < 0$ if $m < 1$.

The plan of the paper is as follows: in section 1 we study the Cauchy problems associated to (0.7) and (0.8) for v_0 satisfying (0.4). We prove the statement about v_{xx} in Theorem 1 via the Trotter-Kato formula in section 2. Finally, section 3 studies the interfaces and the asymptotic behavior.

1. PRELIMINARIES

In this section we study the Cauchy problems for equations $v_t = \lambda v v_{xx}$ and $v_t = v_x^2$. To make their application in Section 2 easier it is convenient to introduce the following notations. Let N be the set of continuous non-negative functions $f : R \to R$ satisfying for some bounded interval $I(f) = (a,b)$

$$f(x) = 0 \text{ if } x \notin (a,b) \quad , \quad f'' \leq 0 \text{ in } D'(a,b) \quad .$$

It is clear that if $f \in N$, then either $f \equiv 0$ or $I(f) = \{x \in R; f(x) > 0\}$. N is a closed set in the span $C_c(R)$ of continuous real functions on R with compact support with the usual topology. To measure the concavity of a function $f \in N$ we define

$$C(f) = \sup\{C \geq 0 \; ; \; f_{xx} \leq -C \text{ in } D'(I(f))\}$$

$$c(f) = \sup\{c \geq 0 \; ; \; cf_{xx} \geq -1 \text{ in } D'(R)\}$$

with the convention $C(f) = 0$ if $f \equiv 0$. Clearly $0 \leq C(f) \leq c(f)^{-1} \leq \infty$ and the maps $f \mapsto C(f), f \mapsto c(f)$ are upper semi continuous functionals on N (endowed with the topology of $C_c(R)$).

Let $N_1 = \{f \in N; c(f) > 0\}$. The following results can be proved by elementary calculus.

LEMMA 1.1. $\underline{\text{Let}}$ $f \in N_1$ $\underline{\text{with}}$ $f \not\equiv 0$ and $I(f) = (a,b)$.

i) $f \in W^{2,\infty}(a,b)$ and

$$C(f) = \inf_{(a,b)} \text{ess} - f'' \leq \sup_{(a,b)} \text{ess} - f'' = c(f)^{-1}$$

ii) $f \in W^{1,\infty}(R)$ and

$$f \leq \frac{(b-a)^2}{c(f)} \quad , \quad (f')^2 \leq \frac{2\|f\|_\infty}{c(f)}$$

iii) $f' \in BV(R)$, $f'(a_+) > 0 > f'(b-)$ and

$$f'' = f''\chi_{(a,b)} + f'(a_+)\delta_a - f'(b-)\delta_b \quad \text{in} \quad D'(\mathbb{R}).$$

We solve first the Cauchy problem

(1.4a) $\qquad v_t = \lambda v v_{xx} \quad \text{in} \quad Q = \mathbb{R} \times (0,\infty)$,

(1.4b) $\qquad v(x,0) = v_0(x) \quad \text{for} \quad x \in \mathbb{R}$,

where λ is a positive constant and $v_0 \in N$. We obtain

PROPOSITION 1.2. Let $v_0 \in N$ and $I(v_0) = (a,b)$. There exists a unique

function $v \in C(\overline{Q})$ that solves problem (1.4) in the sense that

(a) $v \geqslant 0$ in Q, v is positive precisely on the set $\Omega = (a,b) \times (0,\infty)$

and $\qquad v \in C^\infty(\Omega)$.

(b) $v_t = \lambda v v_{xx}$ in Ω.

(c) $v(x,0) = v_0(x)$ for every $x \in \mathbb{R}$.

Moreover, if $v(t) = v(\cdot,t)$ then for every $t > 0$, $v(t) \in N_1$ and

(1.5) $\qquad\qquad\qquad 0 \leqslant v(t) \leqslant v_0$ in $\mathbb{R}$,

(1.6) $\qquad -\dfrac{1}{c(v_0)+\lambda t} \leqslant v(t)_{xx} \leqslant -\dfrac{C(v_0)}{1+C(v_0)\lambda t}$ in $D'(a,b)$.

If $c(v_0) > 0$ then for every $(x,t) \in Q$

(1.7) $\qquad\qquad\qquad v_0(x) - v(x,t) \leqslant \dfrac{\lambda t v_0(x)}{c(v_0)}$.

Proof. Let us assume first that $c(v_0) > 0$. To avoid the degeneracy of the

equation (1.4a) at the level $v = 0$ we begin by considering the problems

$$(P_\varepsilon) \quad \begin{cases} v_t = \lambda v v_{xx} & \text{in } \Omega = \{a < x < b, \ t > 0\} \\[2mm] v(x,0) = v_{0\varepsilon}(x) & \text{for } a < x < b \\[2mm] v(a,t) = v(b,t) = \varepsilon(\lambda t C(v_0) + 1)^{-1} & \text{for } t > 0 \ , \end{cases}$$

where ε is positive and $\{v_{0\varepsilon}\}_\varepsilon$ is a sequence of C^∞, positive functions

in $[a,b]$ such that

    i) $(v_{0\varepsilon})_{xx} < C(v_0)$ and $(v_{0\varepsilon})_{xx} > 1/c(v_0)$,

    ii) $v_{0\varepsilon} \to v_0$ uniformly in $[a,b]$ as $\varepsilon \to 0$,

    iii) $v_{0\varepsilon}(x) = \varepsilon$ and $v_{0\varepsilon,xx} = -C(v_0)$ at $x = a,b$.

Since (P_ε) is a quasilinear parabolic problem and we have the a priori

estimate $\varepsilon < v(x,t) < \sup_x\{v_{0\varepsilon}(x)\}$ for the solutions, by the standard theory

(cf. [LSU]) there exists a solution $v_\varepsilon \in C^\infty(\overline{\Omega})$ of problem (P_ε).

    Consider now the function $p(x,t) = v_{\varepsilon xx}(x,t)$. It satisfies in Ω the

equation

$$p_t = \lambda v_\varepsilon p_{xx} + 2\lambda v_{\varepsilon x} p_x + \lambda p^2 \ ,$$

with initial condition $p(x,0) = v_{0\varepsilon,xx}(x)$ and boundary conditions

$$p(a,t) = p(b,t) = -\frac{C(v_0)}{\lambda t C(v_0)+1} \ .$$

(Note that $p = v_{\varepsilon t}/(\lambda v)$.) Since for every $k > 0$ the function

$$P(x,t;k) = -(\lambda t+k)^{-1}$$

is an explicit solution of the equation satisfied by p, it follows from the

maximum principle that

$$(1.8) \qquad -\frac{1}{\lambda t + c(v_0)} < p(x,t) < -\frac{c(v_0)}{\lambda t c(v_0) + 1}$$

in Ω. Since $p = v_{\epsilon,xx}$ and $v_\epsilon(a,t) = v_\epsilon(b,t) < \epsilon$ it follows from Lemma 1.1 and (1.8) that the following estimates

$$0 \qquad > v_{\epsilon t} > -\frac{\lambda v_\epsilon}{\lambda t + c(v_0)} \quad,$$

$$|v_\epsilon| \qquad < \epsilon + \frac{(b-a)^2}{2(\lambda t + c(v_0))} \quad,$$

$$|v_{\epsilon x}|^2 < \frac{2\|v_0\|_\infty}{c(v_0) + \lambda t} \quad \text{and} \quad |v_{\epsilon x}| < \frac{(b-a)}{2(c(v_0) + 2t)}$$

hold in Ω. Therefore if we pass to the limit $\epsilon \to 0$ there is a subsequence $\{v_{\epsilon_k}\}$ of $\{v_\epsilon\}$ that converges uniformly in $[a,b] \times [0,T]$ for every $T > 0$ to a function $\bar{v} \in C(\bar{\Omega})$ and the above estimates hold for $\bar{v}$ so that $\bar{v} \in W^{1,\infty}((a,b) \times (\tau,\infty))$ for every $\tau > 0$. Because of (1.8) in a neighborhood of every point $(x,t) \in \Omega$ the sequence $v_\epsilon(x,t)$ is uniformly bounded away from 0, hence by standard bootstrap arguments (cf. [LSU] or [A]) we conclude that $\bar{v} \in C^\infty(\Omega)$ and $\bar{v}_t = \lambda \bar{v} \bar{v}_{xx}$ is satisfied in the classical sense. It is also clear that $v \in C(\bar{\Omega})$ and both the initial and boundary conditions are satisfied.

To obtain a solution v in Q we extend $\bar{v}$ by $v(x,t) = 0$ if $x \notin (a,b)$ and $t > 0$.

The uniqueness of the solution is a consequence of the maximum principle (it follows in particular that all the sequence $\{v_\epsilon\}$ converges to $\bar{v}$). In fact if we have two solutions v,w of (1.4) corresponding to initial data $v_0, w_0 \in N_1$ and $v_0 > w_0$ then $v > w$ in Q.

In case $c(v_0) = 0$ we can yet use the above procedure to obtain a solution and only the convergence at $t \to 0$ poses a problem that can be

solved by approximating the initial data v_0 above and below with data in N_1 and using the above comparison results.   #

REMARK. Independently of the authors M. Ughi [U] has studied the equation $u_t = uu_{xx} - u(1-u)$ in $R \times (0,T)$, for initial data not necessarily concave. She obtains results about existence, uniqueness and nonuniqueness for suitable classes of weak solutions. The results apply to equation (1.4a) with obvious changes.

Consider now the initial-value problem

(1.9a) $v_t = |v_x|^2$ in Q ,

(1.9b) $v(x,0) = v_0(x)$ for $x \in R$.

Equation (1.9a) is one of the simplest and best-known examples of Hamilton-Jacobi equation. There is now a general theory of viscosity solutions for equations of the type $u_t + H(u_x) = 0$ where H is a continuous function. In particular we have the following result proved by Crandall and P. L. Lions.

PROPOSITION 1.3. [cf. [CL], Theorem VT.2). <u>Let</u> $v_0 \in$ BUC (R), <u>the set of</u> <u>bounded uniformly continuous functions in</u> R. <u>There exists a unique</u> $v \in C(\overline{Q})$, <u>which is a viscosity solution of</u> $u_t + H(u_x) = 0$ <u>in</u> Q <u>with</u> <u>initial data</u> $v(x,0) = v_0(x)$.

<u>Moreover the maps</u> $T(t)$: BUC$(R) \rightarrow$ BUC(R) <u>defined for</u> $t > 0$ <u>by</u> $T(t)v_0 = v(\cdot,t)$ <u>form a strongly continuous, order-preserving contraction</u> <u>semigroup in</u> BUC(R).

The fact that in problem (1.9) the nonlinearity $H(p) = p^2$ is convex considerably simplies the construction of the solutions of (1.9), especially if the initial data $v_0 \in N$. In this case we can use the classical method of characteristics (cf. [Bt], [L]) as follows. Assume for simplicity that $v_0 \in C^1([a,b])$, where (a,b) is the support of v_0. Through every point $(\xi,0)$ with $a < \xi < b$ we construct a characteristic line

(1.10)
$$x(\xi,t) = \xi - 2v_0'(\xi)t \quad .$$

Along this line the derivative $w = v_x$ of every classical solution of $v_t = |v_x|^2$ must be constant. Therefore we get

(1.11)
$$v(x,t) = v(\xi,0) + \int_0^t \frac{d}{d\tau} v(x(\tau),\tau)d\tau = v_0(\xi) - |v_0'(\xi)|^2 t \quad .$$

The concavity of v_0 in (a,b) implies that these characteristics do not cross each other. In that way a C^1 solution of (1.9a) can be constructed in the region where $v > 0$. It is easy to see that this region has the form

(1.12)
$$\Omega = \{(x,t) \in Q : s_1(t) < x < s_2(t)\}$$

where $x = s_1(t)$ and $x = s_2(t)$ are Lipschitz-continuous curves (the interfaces), $s_1(0) = a$, $s_2(0) = b$, and $(-1)^i s_i(t)$ is nondecreasing in time. Along the interfaces the derivative v_x is discontinuous, i.e. a chock propagates according to the well-known Rankine-Hugoniot conditions.

The following results about problem (1.9) will be needed in the proof of Theorem 1.

PROPOSITION 1.4. <u>Let</u> v <u>be the viscosity solution of</u> (1.9) <u>with</u> $v_0 \in$.
<u>Then</u> $v \in W^{1,\infty}(Q_\tau)$ <u>for every</u> $\tau > 0$, $v(t) = T(t)v_0 \in {}_1$ <u>for every</u> $t > 0$
<u>and</u>

(1.13a)
$$c(v(t)) \geqslant c(v_0) + 2t \quad ,$$

(1.13b)
$$C(v(t)) \geqslant \frac{C(v_0)}{1 + 2tC(v_0)} \quad ,$$

<u>Moreover if</u> $c(v_0) > 0$ <u>then</u>

(1.14)
$$v(t) - v_0 \leqslant \frac{2t\|v_0\|_\infty}{c(v_0)} \quad .$$

<u>Proof.</u> By the contracting property of $T(t)$ we may assume that $C(v_0)$, $c(v_0) > 0$ and $v_0 \in C^1([a,b])$. To prove (1.13) let $t > 0$ and let x_1 and x_2 be two points $s_1(t) < x_1 < x_2 < s_2(t)$. There are well-defined starting-

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points ξ_1 and ξ_2 in (a,b) for the characteristic line (1.10) through (x_1,t) and (x_2,t) respectively. Setting $w_1 = v_x(x_1,t)$, $w_2 = v_x(x_2,t)$ we have

$$x_1 - x_2 = \xi_1 - \xi_2 + 2t(w_2 - w_1) \quad .$$

But v_x is constant along the characteristics, hence $w_2 - w_1 = v_0'(x_2) - v_0'(x_1) > 0$ since v_0 is strictly concave. Therefore if we write

$$\frac{x_1 - x_2}{w_2 - w_1} = \frac{\xi_1 - \xi_2}{w_2 - w_1} + 2t$$

and let $x_1 - x_2 \to 0$ we obtain the relationship

(1.15)
$$\frac{1}{v_{xx}(x,t)} = \frac{1}{v_0''(\xi)} + 2t$$

valid in Ω. (1.13) follows immediately. From this we get

(1.16)
$$v_t = v_x^2 < \frac{2\|v_0\|_\infty}{2t + c(v_0)} \quad .$$

(1.14) is now immediate. #

2. THE TROTTER-KATO FORMULA

Let us now consider the problem

(2.1a) $$v_t = (m-1)vv_{xx} + v_x^2 \quad \text{in} \quad Q$$

(2.1b) $$v(\cdot,0) = v_0 \in N \quad .$$

To solve (3.1) we shall use a Trotter-Kato formula based on the results of the previous section (with $\lambda = m-1$). It is interesting in that respect to think of the solution $v(x,t)$ of problem (1.4) as a semigroup $S(t) : N \to N$, $t > 0$, defined by $S(t)v_0 = v(t)$ if $v(t) = v(\cdot,t)$, v being the solution of (1.4) with initial data v_0. In the same way the solution to problem (1.9) defines another semigroup $T(t) : N \to N$, $t > 0$. Both are order-preserving continuous semigroups in N. The concavity estimates (1.6), (1.13) can be reformulated as

(2.2a) $$c(S(t)v_0) \geq c(v_0) + (m-1)t \quad ,$$

(2.2b) $$c(T(t)v_0) \geq c(v_0) + 2t \quad ,$$

(2.2c) $$C(S(t)v_0) \geq \frac{C(v_0)}{1+(m-1)C(v_0)t} \quad ,$$

and

(2.2d) $$C(T(t)v_0) \geq \frac{C(v_0)}{1+2C(v_0)t} \quad .$$

Result of the type of Chernoff and Trotter-Kato formulas for pairs of continuous semigroups are known when both semigroups are contractive in some Banach space X, cf. [BCP]. Unfortunately in our case, while the semigroup T is contractive in $L^\infty(\mathbb{R})$ or $BUC(\mathbb{R})$, the semigroup S is not. In fact the only contraction properties known for the solutions of (1.4) apply to the function $u = \log(v)$, which satisfies the equation $u_t = \lambda(\exp(u))_{xx}$, cf. [BC].

The convergence of our Trotter-Kato formula relies on the compactness due to (2.2). We proceed as follows. For every $\epsilon > 0$ we define the approximate solution $v_\epsilon(t)$ to problem (2.1) by

$$(2.3) \qquad v_\epsilon(t) = \frac{t-(n-1)\epsilon}{\epsilon} \, v_\epsilon^n + \frac{n\epsilon - t}{\epsilon} \, v_\epsilon^{n-1}$$

if $(n-1)\epsilon < t < n\epsilon$, where for $n = 1,2,\ldots$

$$(2.4) \qquad v_\epsilon^n = (S(\epsilon)T(\epsilon))^n(v_0) \quad,$$

we obtain the following convergence result

PROPOSITION 2.1. <u>As</u> $\epsilon \to 0$, v_ϵ <u>converges to a function</u> $v \in (\overline{Q}) \cap W^{1,\infty}(Q_\tau)$ <u>for every</u> $\tau > 0$ <u>uniformly on compact subsets of</u> $\overline{Q} = R \times [0,\infty)$, v <u>satisfies</u>

$$(2.5) \qquad v_t = \frac{m-1}{2} \, (v^2)_{xx} - (m-2)v_x^2$$

<u>in</u> $D'(Q)$. <u>Moreover if</u> $v(t) : x \to v(x,t)$ <u>then for every</u> $t > 0$, $v(t) \in {}_1$

<u>and</u>

$$(2.6a) \qquad C(v(t)) > \frac{C(v_0)}{1+(m+1)t} \quad,$$

$$(2.6b) \qquad c(v(t)) > c(v_0) + (m+1)t \quad.$$

<u>Also</u>

$$(2.7) \qquad v_x^2 < \frac{2\|v_0\|_\infty}{c(v_0)+(m+1)t} \quad,$$

$$(2.8) \qquad \frac{(m-1)\|v_0\|_\infty}{c(v_0)+(m+1)t} < v_t < \frac{2\|v_0\|_\infty}{c(v_0)+(m+1)t} \quad.$$

<u>Proof.</u> By induction we have $v_\epsilon^n \in N_1$,

$$(2.9a) \qquad 0 < v_\epsilon^n < T(n\epsilon)v_0 < \|v_0\|_\infty \quad,$$

$$(2.9b) \qquad C(v_\epsilon^n) > \frac{C(v_0)}{1+(m+1)n\epsilon C(v_0)} \quad,$$

$$(2.9c) \qquad c(v_\epsilon^n) > c(v_0) + (m+1)n\epsilon \quad,$$

$$(2.9d) \qquad \text{supp}(v_\epsilon^n) \subset \text{supp}(T(n\epsilon)v_0) \quad.$$

Observe that (2.9a) follows from the fact that $S(\varepsilon)v \leqslant v$ and $T(\varepsilon)$ and $S(\varepsilon)$ are order-preserving maps. (2.9b) and (2.9c) follow from (2.2) and finally (2.9d) is a consequence of (2.9a).

Assume now that $c(v_0) > 0$ and fix $t_0 > 0$. We have

$$v_\varepsilon \in W^{1,\infty}(\mathbb{R} \times (0,t_0)) \quad ,$$

$$0 \leqslant v_\varepsilon \leqslant \|v_0\|_\infty \quad ,$$

$$\operatorname{supp}(v_\varepsilon(t)) \subset \operatorname{supp}(T(t_0+\varepsilon)v_0) \quad \text{if} \quad 0 < t < t_0 \quad ,$$

$$v_{\varepsilon,x}^2 \leqslant 2\|v_0\|_\infty/c(v_0) \quad .$$

Moreover

$$\frac{d^+ v_\varepsilon}{dt} = \frac{S(\varepsilon)w_\varepsilon - w_\varepsilon}{\varepsilon} + \frac{T(\varepsilon)\tilde{v}_\varepsilon - \tilde{v}_\varepsilon}{\varepsilon} \quad ,$$

where $\tilde{v}_\varepsilon(t) = v_\varepsilon^{n-1}$ if $(n-1)\varepsilon \leqslant t < n\varepsilon$ and $w_\varepsilon = T(\varepsilon)\tilde{v}_\varepsilon$. Since $c(w_\varepsilon(t)) \geqslant c(\tilde{v}_\varepsilon(t)) + 2\varepsilon \geqslant c(v_0)$ we have by (1.7) and (1.14)

$$-\frac{(m-1)\|v\|_\infty}{c(v_0)} \leqslant \frac{d^+ v_\varepsilon}{dt} \leqslant \frac{2\|v\|_\infty}{c(v_0)} \quad .$$

It follows in particular that v_ε is relatively compact in $C(\mathbb{R} \times [0,t_0])$. If v_ε converges uniformly to a function $v \in C(\mathbb{R} \times [0,t_0])$ along a sequence $\varepsilon = \varepsilon_k \to 0$, we have $v \in W^{1,\infty}(\mathbb{R} \times [0,t_0])$

$$0 \leqslant v \leqslant T(t)v_0 \quad , \quad |v_x^2| \leqslant \frac{2\|v_0\|_\infty}{c(v_0)}$$

and

$$-\frac{(m-1)\|v_0\|_\infty}{c(v_0)} \leqslant v_t \leqslant \frac{2\|v_0\|_\infty}{c(v_0)} \quad .$$

Furthermore the estimates (2.6a) and (2.6b) follow from (2.9b) and (2.9c) and the upper-semi-continuity of the functionals c and C.

To end the proof of the proposition we have yet to check that equation (2.5) holds. For that we write

$$\frac{d^+ v_\varepsilon}{dt} = \frac{1}{\varepsilon} \int_0^\varepsilon (S(\tau)w_\varepsilon(t))_\tau d\tau + \frac{1}{\varepsilon} \int_0^\varepsilon (T(\tau)\widetilde{v}_\varepsilon(t))_\tau d\tau =$$

$$(2.10) \qquad = \frac{m-1}{2\varepsilon} \int_0^\varepsilon \{(S(\tau)w_\varepsilon(t))\}_{xx} d\tau + \frac{1}{\varepsilon} \int_0^\varepsilon \{(T(\tau)\widetilde{v}_\varepsilon(t))_x\}^2 d\tau -$$

$$- \frac{m-1}{\varepsilon} \int_0^\varepsilon \{(S(\tau)w_\varepsilon(t))_x\}^2 d\tau \equiv I_1 + I_2 + I_3 \ .$$

We want to pass to the limit $\varepsilon \to 0$. Let us begin for instance with I_2.
Since $v_{\varepsilon_k} \to v$ uniformly in $\mathbb{R} \times [0,t_0]$, using also (1.14) it follows that

$$\sup_{\substack{0 < \tau < \varepsilon \\ 0 < t < t_0}} \| T(\tau)\widetilde{v}_\varepsilon(t) - v(t) \|_\infty \equiv 0 \quad \text{as} \quad \varepsilon_k \to 0 \ .$$

If we now observe that $c(T(\tau)\widetilde{v}_\varepsilon(t)) \geqslant c(v_0)$ and $\text{supp}(T(\tau)\widetilde{v}_\varepsilon(t)) \subset$

$\text{supp } T(((t_0+\varepsilon)v_0) \subset [-R,R]$ for some $R > 0$ the convergence

$$(2.11) \qquad |(T(\tau)\widetilde{v}_\varepsilon(t))_x|^2 \to |v(t)_x|^2 \quad \text{in} \quad C([0,t_0];L^1(\mathbb{R}))$$

will hold uniformly in $\tau \in (0,\varepsilon)$ as a consequence of the following compact-
ness result applied to the family $\{f_{t,\tau,\varepsilon}(x) = T(\tau)\widetilde{v}_\varepsilon(t)\}$, $0 < t < t_0$,
$\varepsilon > 0$, $0 < \tau < \varepsilon$.

LEMMA 3.2. **For every constant** c_0, $R > 0$ **the set**

$$N_{c_0,R} = \{f \in N : c(f) \geqslant c_0,\ \text{supp}(f) \subset [-R,R]\}$$

is a compact subset of $W^{1,p}(\mathbb{R})$ **for any** $p \in [1,\infty)$. **Moreover there exist**
constants $C = C(c_0,R) > 0$ **and** $\theta \in (0,1)$ **such that**

$$(2.12) \qquad \|f_1 - f_2\|_{W^{1,p}(\mathbb{R})} \leqslant C\|f_1 - f_2\|_{L^p(\mathbb{R})}^\theta$$

For any two functions $f_1 f_2 \in N_{c_0,R}$.

For a proof of this result see [L, Lemma 10.1]. It is based on proving
that $N_{c_0,R} \subset W^{s,p}(\mathbb{R})$ for some $s > 1$ and $p > 1$ and then using
interpolation.

Continuing with the proof of Proposition 3.1 it follows from (2.11) that

$$I_2 = \frac{1}{\varepsilon} \int_0^\varepsilon \{(T(\tau)\widetilde{v}_\varepsilon(t))_x\}^2 \to |v(t)_x|^2 \quad \text{in} \quad C([0,t_0] ; L^1(\mathbb{R})) \ .$$

In the same way we prove that

$$I_1 \equiv \frac{1}{\varepsilon} \int_0^\varepsilon \{(S(\tau)w_\varepsilon(t))_x\}^2 d\tau \to (v(t)_x)^2 \quad \text{in} \quad C([0,t_0] ; L^1(\mathbb{R})) \quad .$$

Finally since

$$F_\varepsilon(t) = \frac{1}{\varepsilon} \int_0^\varepsilon (S(\tau)w_\varepsilon(t))^2 d\tau \to v(t)^2 \quad \text{in} \quad C(\mathbb{R} \times [0,t_0])$$

it follows that $I_3 = F_\varepsilon(t)_{xx} \to (v(t)^2)_{xx}$ in $D'(\mathbb{R} \times (0,t_0))$. Therefore we may let $\varepsilon_k \to 0$ in (2.10) to obtain (2.5).

The case $c(v_0) = 0$ is easily dealt with by comparison since for every $\delta > 0$

    i) $S(\delta)v_0 \leqslant v_0 \leqslant T(\delta)v_0$ and $\|T(\delta)v_0 - S(\delta)v_0\|_\infty \to 0$ as $\delta \to 0$,

    ii) $c(S(\delta)v_0) > 0$, $c(T(\delta)v_0) > 0$.

To conclude the proof of the first part of Theorem 1 we have to identify the function v as the pressure of the solution to problem (0.1).

PROPOSITION 2.3. <u>Let</u> $v \in C(\overline{Q}) \cap W^{1,\infty}(Q_\tau)$ <u>for every</u> $\tau > 0$ <u>satisfy</u>

    i) <u>for every</u> $t > 0$, $v(\cdot,t) \in N$

    ii) $v_t = \frac{m-1}{2}(v^2)_{xx} + (2-m)|v_x|^2$ <u>in</u> $D'(Q)$

    iii) $v(\cdot,0) = \frac{m}{m-1} u_0^{m-1}$,

<u>where</u> u_0 <u>is nonnegative, continuous and bounded. Then</u> v <u>is the pressure associated to the solution of</u> (0.1).

<u>Proof.</u> Let $u = ((m-1)v/m)^{\frac{1}{m-1}}$ and let $\overline{u}$ be the usual solution of (0.1). By the preceding results if Ω is the subregion of Q where v is positive Ω has the form $\{(x,t) \in Q : s_1(t) < x < s_2(t)\}$ and we have $u \in C(\overline{\Omega})$. Also (by standard regularity theory) $u \in C^\infty(\Omega)$ and $u_t = (u^m)_{xx}$ is satisfies in Ω in the classical sense. Since $u \leqslant \overline{u}$ on the parabolic boundary of Ω by the maximum principle we get $u \leqslant \overline{u}$ in Ω.

Moreover since $(u^m)_x = uv_x$ and $v(\cdot,t) \in N$ it is easy to see that $(u^m)_x \to 0$ as (x,t) tends to the lateral boundary of Ω, hence for every

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$t > 0$

$$\frac{d}{dt} \int u(x,t)\,dx = \left[(u^m)_x \right]_{x=s_1(t)}^{x=s_2(t)} = 0$$

and we have

$$\int u(x,t)\,dx = \int u_0(x)\,dx = \int \bar{u}(x,t)\,dx \quad .$$

Therefore $u = \bar{u}$ in Q.   #

3. INTERFACES AND ASYMPTOTIC BEHAVIOUR

In this section we complete the proof of Theorem 1 by studying the concavity of the interfaces of the solutions to problem (P) and derive its consequences for the asymptotic behaviour of solutions and interfaces, Theorem 2.

Proof of the concavity of statement (0.6) of Theorem 1.

We may assume that v is a solution of (0.4) whose initial datum v_0 satisfies (0.4) with $C > 0$. Also we may consider only the right-hand interface and drop the subindex, $\zeta(t) = \zeta_2(t)$. Following the idea of the proof of the converse inequality (0.3) ([CF], [V1]) we compare our solution v with the Barenblatt solution w that has best contact with it, this time from above, at a given point of the interface, the main difference lying in the fact that we have to use a time-delayed Barenblatt. The outline of the proof is as follows: let $t_1 > 0$. Since $v(\cdot, t_1)$ is strictly concave in its support, cf. (0.5), $v_x(\zeta(t_1) - 0, t_1)$ is necessarily negative, therefore by the interface equation

$$(3.1) \qquad\qquad \zeta'(t) = -v_x(\zeta(t) - 0, t)$$

([A], [K]) we have $\zeta'(t)$. We consider the Barenblatt solution

$$(3.2) \qquad\qquad w(x,t) = \frac{(r(t+\tau)^2 - (x-x_1)^2)_+}{2(m+1)(t+\tau)}$$

where $r(t)$ is as in (0.12). The parameters M, $\tau > 0$ and $x_1 \in \mathbb{R}$ shall be adjusted so as to have

(i) $\quad v(\zeta(t_1), t_1) = w(\zeta(t_1), t_1)$

(ii) $\quad v_x(\zeta(t_1)-0, t_1) = w_x(\zeta(t_1)-0, t_1)$

(iii) $\quad v_{xx}(\xi, t_1) < w_{xx}(\xi, t_1) \quad$ whenever $\quad v(\xi, t_1) > 0$.

By (0.5) and (3.2) the last inequality is implied by

$$- \frac{C}{1+(m+1)Ct_1} < - \frac{1}{(m+1)(t_1+\tau)} \quad ,$$

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namely $\tau > 1/((m+1)C)$. The best choice is

$$\tau = 1/((m+1)C) \quad .$$

We can now choose M and x_1 so that (i) and (ii) are satisfied. It follows from (i) − (iii) that $v(\cdot,t_1) < w(\cdot,t_1)$. Since both v and w are solutions of (0.7) in $S = \mathbb{R} \times (t_1,\infty)$ it follows that $v(x,t) < w(x,t)$ in S. Therefore for every $t > t_1$ we have

$$\zeta(t) < \zeta_w(t) \equiv x_1 + r(t+\tau) \quad .$$

Since at $t = t_1$ we have $\zeta(t_1) = \zeta_w(t_1)$ and $\zeta'(t_1) = \zeta_w'(t_1)$ (by (i), (ii)) and

(3.3)
$$\zeta_w''(t) = - \frac{m}{(m+1)(t+\tau)} \zeta_w'(t)$$

we conclude as in [CF], [V1] that

(3.4)
$$\zeta''(t_1) < - \frac{m}{(m+1)(t_1+\tau)} \zeta'(t_1)$$

in the sense of measures in $(0,\infty)$, i.e. (0.6).   #

(0.3) and (0.6) together imply that ζ'' is a locally bounded function. (0.6) can be reformulated as saying that $\zeta'(t)/r'(t+\tau)$ is nonincreasing as a function of t.

We now study the asymptotic behaviour. We state in detail the results for the interface.

PROPOSITION 3.1. _Let_ v _be a solution of_ (0.4) _such that_ v_0 _satisfies_ (0.4) _with_ $c > 0$ _and let_ $\tau = 1/((m+1)C)$. _Then as_ $t \to \infty$ _we have_

(3.5)
$$r(t) + x_0 < \zeta(t) < r(t+\tau) + x_0$$

(3.6)
$$r'(t) > \zeta'(t) > r'(t+\tau) \quad .$$

Moreover $\zeta(t) - r(t+\tau) \uparrow x_0$ _and_ $\zeta'(t)/r'(t+\tau) \downarrow 1$.

Proof. Since $\zeta'(t)/r'(t+\tau)$ is nonincreasing and $\zeta(t)/r(t) \to 1$, cf. [V1], it follows that $\zeta'(t)/r'(t+\tau) \downarrow 1$. Therefore $\zeta(t) - r(t+\tau)$ is nondecreasing. Since $r(t+\tau) - r(t) \to 0$ and $\zeta(t) = r(t) + x_0$ as $t \to \infty$ ([V1]) we have $\zeta(t) - r(t+\tau) \uparrow x_0$. This proves the right-hand

inequalities. The left-hand inequalities were proved in [V1].

COROLLARY 3.2. <u>As</u> $t \to \infty$

$$(3.7) \qquad\qquad |\zeta(t) - (r(t) + x_0)| < C_1 \tau t^{-\frac{m}{m+1}}$$

$$(3.8) \qquad\qquad |\zeta'(t) - r'(t)| < C_2 \tau t^{-(\frac{m}{m+1} + 1)}$$

$$(3.9) \qquad\qquad |\zeta''(t) - r''(t)| < C_3 \tau t^{-(\frac{m}{m+1} + 2)}$$

<u>where</u> C_1, C_2, C_3 <u>depend only on</u> m.

<u>Proof.</u> (3.7) and (3.8) follow resp. from (3.5) and (3.6). Then the estimate
for ζ'' follows from this, (0.3) and (0.6).

To end the proof of Theorem 2 we remark that (0.13) comes from (0.2) and
(0.5). By integration we obtain estimates for v and v_x. It is to be
remarked that the estimates for v, v_x, ζ and ζ' are valid also in the case
of symmetric solutions without the assumption of concavity, cf. [V1, Theorem
B]. In that case the result extends to several space dimensions. Let us
finally remark another consequence the above results.

COROLLARY 3.3. <u>The interface of a concave solution to problem</u> (P) <u>has zero</u>
<u>waiting time. More precisely with the above notations we have</u>

$$(3.10) \qquad\qquad \zeta'(0) > r_M'(\tau) = (c/(m+1))(M^{m-1}\tau^{-m})^{1/(m+1)} \quad ,$$

$$(3.11) \qquad\qquad \zeta'(0) > C(b-a)/2 \quad .$$

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We consider the problem

$$(P) \quad \begin{cases} u_t = (u^m)_{xx} & \text{with } x \in R, t > 0, \\ u(x,0) = u_0(x) & \text{for } x \in \mathbb{R}, \end{cases}$$

where $m > 1$ and u_0 is a continuous, nonnegative function that vanishes

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ABSTRACT (continued)

outside an interval (a,b) and such that $(u_0^{m-1})'' \leq -C \leq 0$ in (a,b). Using a Trotter-Kato formula we show that the solution conserves the concavity in time: for every $t > 0$, $u(x,t)$ vanishes outside an interval $\Omega(t) = (\zeta_1(t), \zeta_2(t))$ and

$$(u^{m-1})_{xx} \leq - \frac{C}{1 + C \frac{m(m+1)}{m-1} t}$$

in $\Omega(t)$. Consequently the interfaces $x = \zeta_i(t)$, $i = 1,2$, are concave curves. These results also give precise information about the large time behaviour of solutions and interfaces.

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