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ERROR BOUNDS FOR NEWTON'S ITERATES
DERIVED FROM THE KANTOROVICH THEOREM

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July 1985

(Received July 15, 1985)

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ERROR BOUNDS FOR NEWTON'S ITERATES DERIVED FROM THE KANTOROVICH THEOREM

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Technical Summary Report #2843
July 1985

ABSTRACT

In this paper, it is shown that the upper and lower bounds of the errors in the Newton iterates recently obtained by Potra-Pták [11] and Miel [7], with the use of nondiscrete induction and majorizing sequence, respectively, follow immediately from the Kantorovich theorem and the Kantorovich recurrence relations. It is also shown that the upper and lower bounds of Miel are sharper than those of Potra-Pták. *Keywords; numerical analysis;*

AMS (MOS) Subject Classifications: 65G99, 65J15

Key Words: Error bounds, Newton's method, the Kantorovich theorem, Potra-Pták's bounds; Miel's bounds.

Work Unit Number 3 (Numerical Analysis and Scientific Computing)

SIGNIFICANCE AND EXPLANATION

To find precise error bounds for iterative solutions of equations is one of the important subjects in numerical analysis. This paper shows that the upper and lower bounds of the errors in the Newton iterates recently obtained by Potra-Pták [11] and Miel [7] follow from the Kantorovich theorem, and that the bounds of Miel are sharper than those of Potra-Pták.

ERROR BOUNDS FOR NEWTON'S ITERATES DERIVED FROM THE KANTOROVICH THEOREM

Tetsuro Yamamoto[*]

1. Introduction

The Kantorovich theorem for the Newton method is of fundamental importance in the study of nonlinear equations in Euclidean and Banach spaces. Let X and Y be Banach spaces, D be an open convex subset of X and $F : D \subseteq X \to Y$ be a Fréchet differentiable operator which satisfies a Lipschitz condition in D. Then, the theorem guarantees the existence and uniqueness of a solution of the equation $F(x) = 0$ and the convergence of the Newton process to the solution. By replacing the original assumption that F belongs to C^2-class in D by a weaker one of the Lipschitz continuity of F' in D, an affine invariant version of the theorem is stated as follows:

Theorem 1 (Kantorovich-Akilov [4; Theorem 6 (1.XVIII)]). Let $F : D \subseteq X \to Y$ be Fréchet differentiable. Assume that, at some $x_0 \in D$, $F'(x_0)$ is invertible and that

$$\|F'(x_0)^{-1}(F'(x) - F'(y))\| \leq K\|x - y\|, \qquad x,y \in D \tag{1}$$

$$\|F'(x_0)^{-1}F(x_0)\| \leq \eta, \qquad h = K\eta \leq 1/2 , \tag{2}$$

$$\bar{S}(x_0,t^*) \subseteq D, \qquad t^* = 2\eta/(1 + \sqrt{1 - 2h}) . \tag{3}$$

Then:

i) The Newton iterates $x_{n+1} = x_n - F'(x_n)^{-1}F(x_n)$, $n \geq 0$, are well-defined, lie in $\bar{S}(x_0,t^*)$ and converge to a solution x^* of $F(x) = 0$.

ii) The solution x^* is unique in $S(x_0,t^{**}) \cap D$, $t^{**} = (1 + \sqrt{1 - 2h})/K$ if $2h < 1$, and in $\bar{S}(x_0,t^{**})$ if $2h = 1$.

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iii) Error estimates

$$\|x^* - x_n\| \leq \begin{cases} t^* & (n = 0) \\ 2^{1-n}(2h)^{2^n-1}\eta & (n \geq 1) , \end{cases} \tag{4}$$

are valid.

<u>Remark 1.</u> The importance of such an affine invariant formulation is stressed in Deuflhard-Heindl [2].

<u>Remark 2.</u> The condition (3) may be replaced by the weaker conditions

$$x_0 \in D, \quad \bar{S}(x_1, t^* - \eta) \subseteq D , \tag{5}$$

which are due to Schmidt [14]. In fact, by induction on n and the well-known majorant principle for the Newton iterates, we can prove that, under the assumptions (1), (2) and (5), the Newton iterates are well-defined and $x_n \in \bar{S}(x_1, t^* - \eta)$, $n \geq 1$.

There are many literatures ([1],[3],[5]-[7],[9]-[11],[16]) on the improvements of the estimates (4). For example, under the assumptions of Theorem 1 (or, by replacing (3) by (5)), the following results hold.

Theorem 2 (Gragg-Tapia [3]).

$$\|x^* - x_n\| \leq \begin{cases} \dfrac{2}{h}\sqrt{1 - 2h}\,\dfrac{\theta^{2^n}}{1 - \theta^{2^n}}\,\|x_1 - x_0\| & \text{if } 2h < 1 , \\[4mm] 2^{1-n}\|x_1 - x_0\| & \text{if } 2h = 1 , \end{cases} \tag{6}$$

and

$$\dfrac{2\|x_{n+1} - x_n\|}{1 + \sqrt{1 + \dfrac{4\theta^{2^n}}{(1 + \theta^{2^n})^2}}} \leq \|x^* - x_n\| \leq \theta^{2^{n-1}}\|x_n - x_{n-1}\|, \quad n \geq 1 , \tag{7}$$

where $\theta = t^*/t^{**} = (1 - \sqrt{1 - 2h})/(1 + \sqrt{1 - 2h})$.

Theorem 3 (Potra-Pták [11]). Let $\alpha = \sqrt{1 - 2h}/K$ and

$$\gamma(t) = \sqrt{\alpha^2 + 4t^2 + 4t\sqrt{\alpha^2 + t^2}} - (t + \sqrt{\alpha^2 + t^2}) .$$

Then

$$\gamma(|x_{n+1} - x_n|) \leq |x^* - x_n| \leq \sqrt{\alpha^2 + |x_n - x_{n-1}|^2} - \alpha . \tag{8}$$

Theorem 4 (Miel [7]). Let $\Delta = t^{**} - t^*$. Then

$$\frac{2|x_{n+1} - x_n|}{1 + \sqrt{1 + \frac{4}{\Delta} \cdot \frac{1 - \theta^{2^n}}{1 + \theta^{2^n}} |x_{n+1} - x_n|}} \leq |x^* - x_n| \leq \frac{1 - \theta^{2^n}}{\Delta} |x_n - x_{n-1}|^2 \tag{9}$$

if $2h < 1$, and

$$2^{1-n}\eta\left(\sqrt{1 + \frac{2^n}{\eta} |x_{n+1} - x_n|} - 1\right) \leq |x^* - x_n| \leq \frac{2^{n-1}}{\eta} |x_n - x_{n-1}|^2 \tag{10}$$

if $2h = 1$.

<u>Remark 3.</u> Let $f(t) = \frac{1}{2} Kt^2 - t + \eta$ and define the sequence $\{t_n\}$ by

$$t_0 = 0, \quad t_{n+1} = t_n - f(t_n)/f'(t_n), \quad n \geq 0 .$$

Then the well-known majorant principle due to Kantorovich asserts that

$$|x^* - x_n| \leq t^* - t_n, \quad n \geq 0 . \tag{11}$$

The more general arguments are developed in Ortega-Rheinboldt [9] and Schmidt [14], [15]. On the basis of Ostrowski's results [10; Appendix F], however, we can show that the bounds (11) are the same as (6), provided that $\eta = |x_1 - x_0|$. Furthermore, we note that the upper bounds in Theorems 2-4 coincide for $n = 1$, and are equal to $t^* - t_1 = (1 - h - \sqrt{1 - 2h})/K$ with $\eta = |x_1 - x_0|$.

Theorem 2 was derived with the use of the Kantorovich recurrence relations. Theorems 3 and 4, improved versions of Theorem 2, were obtained recently by nondiscrete induction and the majorizing sequence, respectively. In [7], Miel

has mentioned that it turns out that the upper bounds in (9) are sharper than those in (8), and that numerical experiments also indicate that the lower bounds in (9) are finer than those in (8). In this paper, with the use of the same technique as in the previous paper [17], we shall show that Theorems 3 and 4 follow immediately from Theorem 1 and that Theorem 4 improves Theorem 3.

2. Basic Lemmas

To derive Theorems 3 and 4 from Theorem 1, we need the following basic lemmas and give their proofs for the sake of completeness.

Lemma 1. Under the assumptions of Theorem 1, define three sequences $\{B_n\}$, $\{\eta_n\}$, and $\{h_n\}$ by

$$B_0 = 1, \quad B_n = \frac{B_{n-1}}{1 - h_{n-1}},$$

$$\eta_0 = |x_1 - x_0|, \quad \eta_n = \frac{h_{n-1}\eta_{n-1}}{2(1 - h_{n-1})},$$

$$h_0 = h = \kappa\eta, \quad h_n = KB_n\eta_n = \frac{h_{n-1}^2}{2(1 - h_{n-1})^2}, \quad n = 1, 2, \ldots,$$

respectively. Then we have

$$|F'(x_n)^{-1}F'(x_0)| \leq B_n \quad \text{and} \quad |F'(x_n)^{-1}F(x_n)| \leq \eta_n.$$

In particular, if $2h = 1$, then $2h_n = 1$ and $\eta_n = 2^{-1}\eta_{n-1} = 2^{-n}\eta$.

<u>Proof.</u> This is a direct application of the original recurrence relations to $F'(x_0)^{-1}F$ (cf. Rall [13]). Q.E.D.

Lemma 2. The speed of convergence of the iterates is estimated by

$$|x^* - x_n| \leq \frac{KB_{n-1}}{2}|x^* - x_{n-1}|^2.$$ (12)

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Similarly we have

$$\|x_{n+1} - x_n\| \leq \frac{KB_n}{2} \|x_n - x_{n-1}\|^2 . \qquad (13)$$

Proof. The estimates (12) follow from (1), Lemma 1 and the relations

$$x^* - x_n = -F'(x_{n-1})^{-1}\{F(x^*) - F(x_{n-1}) - F'(x_{n-1})(x^* - x_{n-1})\}$$

$$= -F'(x_{n-1})^{-1}F'(x_0) \int_0^1 F'(x_0)^{-1} \cdot$$

$$\cdot \{F'(x_{n-1} + t(x^* - x_{n-1})) - F'(x_{n-1})\}(x^* - x_{n-1})dt .$$

Similarly, the estimates (13) follow from the relations

$$x_{n+1} - x_n = F'(x_n)^{-1}F(x_n)$$

$$= F'(x_n)^{-1}[F(x_{n-1}) + F'(x_{n-1})(x_n - x_{n-1})$$

$$+ \int_0^1 \{F'(x_{n-1} + t(x_n - x_{n-1})) - F'(x_{n-1})\}(x_n - x_{n-1})dt]$$

$$= F'(x_n)^{-1}F'(x_0) [\int_0^1 F'(x_0)^{-1} \cdot$$

$$\cdot \{F'(x_{n-1} + t(x_n - x_{n-1})) - F'(x_{n-1})\}(x_n - x_{n-1})dt] . \qquad Q.E.D.$$

Lemma 3 (Basic Error Estimates). Under the assumptions of Theorem 1, we have

$$\frac{2\|x_{n+1} - x_n\|}{1 + \sqrt{1 + 2KB_n\|x_{n+1} - x_n\|}} \leq \|x^* - x_n\| \leq \frac{2\|x_{n+1} - x_n\|}{1 + \sqrt{1 - 2KB_n\|x_{n+1} - x_n\|}} . \qquad (14)$$

Proof. Replace x_0 and η in Theorem 1 by x_n and $\|x_{n+1} - x_n\|$, respectively. Then (1) is replaced by

$$\|F'(x_n)^{-1}(F'(x) - F'(y))\| \leq \|F'(x_n)^{-1}F'(x_0)\| \cdot \|F'(x_0)^{-1}(F'(x) - F'(y))\|$$

$$\leq KB_n\|x - y\|, \qquad x,y \in D .$$

Therefore, the upper bounds in (14) follow from Theorem 1. Furthermore, we have

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from Lemma 2

$$|x_{n+1} - x_n| \leq |x_{n+1} - x^*| + |x^* - x_n| \leq \frac{KB_n}{2} |x^* - x_n|^2 + |x^* - x_n| \, ,$$

or

$$\frac{KB_n}{2} |x^* - x_n|^2 + |x^* - x_n| - |x_{n+1} - x_n| \geq 0 \, .$$

Solving this yields the lower bounds in (14).   Q.E.D.

Lemma 4. We have

$$B_n\sqrt{1 - 2h} = \sqrt{1 - 2h_n}, \qquad n \geq 0 \, , \qquad\qquad (15)$$

and

$$B_n\sqrt{1 - 2h + (K\eta_{n-1})^2} = 1, \qquad n \geq 1 \, . \qquad\qquad (16)$$

<u>Proof</u>. The equalities (15) and (16) are trivial for $n = 0$ and $n = 1$, respectively. If $n \geq 1$, then we obtain

$$1 - 2h_n = 1 - \frac{h_{n-1}^2}{(1 - h_{n-1})^2} = \frac{1 - 2h_{n-1}}{(1 - h_{n-1})^2} = \frac{1 - 2h_0}{(1 - h_{n-1})^2 \cdots (1 - h_0)^2}$$

$$= (1 - 2h)B_n^2 \, ,$$

which proves (15). Furthermore, if $n \geq 2$, then we have

$$B_n^{-2} = \prod_{i=0}^{n-1} (1 - h_i)^2$$

$$= (1 - 2h_{n-1}) \prod_{i=0}^{n-2} (1 - h_i)^2 + h_{n-1}^2 \prod_{i=0}^{n-2} (1 - h_i)^2$$

$$= 1 - 2h + \{KB_{n-1}\eta_{n-1} \prod_{i=0}^{n-2} (1 - h_i)\}^2$$

$$= 1 - 2h + (K\eta_{n-1})^2$$

since $B_{n-1} \prod_{i=0}^{n-2} (1 - h_i) = 1$. This proves (16).   Q.E.D.

3. Proof of Theorem 3

Let us now prove Theorem 3. We put $e_n = \|x_{n+1} - x_n\|$. Then, by Lemma 4, we have

$$B_n^{-1} = \sqrt{1 - 2h + (K\eta_{n-1})^2} \geq \sqrt{1 - 2h + (Ke_{n-1})^2} \;.$$

Hence we obtain from Lemmas 2-4

$$\|x^* - x_n\| \leq \frac{2e_n}{1 + \sqrt{1 - 2KB_n e_n}} \leq \frac{2e_n}{1 + \sqrt{1 - 2KB_n \eta_n}}$$

$$\leq \frac{KB_n e_{n-1}^2}{1 + \sqrt{1 - 2h_n}} = \frac{Ke_{n-1}^2}{B_n^{-1} + \sqrt{1 - 2h}}$$

$$\leq \frac{Ke_{n-1}^2}{\sqrt{1 - 2h + (Ke_{n-1})^2} + \sqrt{1 - 2h}} = \sqrt{\alpha^2 + e_{n-1}^2} - \alpha$$

where $\alpha = \sqrt{1 - 2h}/K$. Next, to derive the lower bounds in (8), we observe that by Lemma 4

$$\sqrt{\alpha^2 + \eta_{n-1}^2} = \frac{1}{K}\sqrt{1 - 2h + (K\eta_{n-1})^2} = \frac{1}{KB_n}$$

so that

$$e_n \leq \frac{1}{2} KB_n e_{n-1}^2 = \frac{e_{n-1}^2}{2\sqrt{\alpha^2 + \eta_{n-1}^2}} \leq \frac{e_{n-1}^2}{2\sqrt{\alpha^2 + e_{n-1}^2}}$$

or

$$e_{n-1}^2 \geq 2e_n\sqrt{\alpha^2 + e_{n-1}^2}$$

which are equivalent to

$$\left(\sqrt{\alpha^2 + e_{n-1}^2} - e_n\right)^2 \geq \alpha^2 + e_n^2 \;. \tag{17}$$

Moreover we have

$$\sqrt{\alpha^2 + e_{n-1}^2} \geq e_n, \quad n = 1, 2, \ldots \;. \tag{18}$$

In fact, the inequalities (18) follow from the inequalities

$$e_n \leq \frac{KB_n}{2} e_{n-1}^2 \leq \frac{h_{n-1}}{2(1 - h_{n-1})} e_{n-1} \leq \frac{1}{2} e_{n-1} \leq e_{n-1} .$$

Hence we have from (17)

$$\sqrt{\alpha^2 + e_{n-1}^2} \geq e_n + \sqrt{\alpha^2 + e_n^2} .$$

Consequently we obtain from Lemma 3

$$|x^* - x_n| \geq \frac{2e_n}{1 + \sqrt{1 + 2KB_n e_n}}$$

$$= \frac{2e_n}{1 + \sqrt{1 + 2e_n/\sqrt{\alpha^2 + \eta_{n-1}^2}}}$$

$$\geq \frac{2e_n}{1 + \sqrt{1 + 2e_n/\sqrt{\alpha^2 + e_{n-1}^2}}}$$

$$\geq \frac{2e_n}{1 + \sqrt{1 + 2e_n/(e_n + \sqrt{\alpha^2 + e_n^2})}}$$

$$= \frac{2e_n(e_n + \sqrt{\alpha^2 + e_n^2})}{e_n + \sqrt{\alpha^2 + e_n^2} + \sqrt{(e_n + \sqrt{\alpha^2 + e_n^2})(3e_n + \sqrt{\alpha^2 + e_n^2})}}$$

$$= \gamma(e_n) .$$

This completes the proof of Theorem 3.

4. Proof of Theorem 4

To prove Theorem 4, we use Gragg-Tapia's result

$$\theta^{2^n} = \frac{1 - \sqrt{1 - 2h_n}}{1 + \sqrt{1 - 2h_n}} , \qquad n = 0, 1, 2, \ldots . \tag{19}$$

If $2h < 1$, then we have from (19)

$$B_n = \frac{\sqrt{1 - 2h_n}}{\sqrt{1 - 2h}} = \frac{1}{\sqrt{1 - 2h}} \cdot \frac{1 - \theta^{2^n}}{1 + \theta^{2^n}} = \frac{2}{K\Delta} \cdot \frac{1 - \theta^{2^n}}{1 + \theta^{2^n}} \tag{20}$$

where $\Delta = t^{**} - t^*$. If $2h = 1$, then obviously we have $B_n = 2^n$ and

$$\frac{2e_n}{1 + \sqrt{1 + 2KB_n e_n}} = \frac{2e_n}{1 + \sqrt{1 + \frac{2^n}{\eta} e_n}} = \frac{\eta}{2^{n-1}} \left(\sqrt{1 + \frac{2^n}{\eta} e_n} - 1 \right) .$$

Therefore the lower bounds in Theorem 4 coincide with those in Lemma 3. Furthermore, we obtain from (20) and (19)

$$\frac{1 - \theta^{2^n}}{\Delta} = \frac{KB_n}{2} (1 + \theta^{2^n}) = \frac{KB_n}{2} \frac{2}{1 + \sqrt{1 - 2h_n}} = \frac{KB_n}{1 + \sqrt{1 - 2h_n}} ,$$

if $2h < 1$. Hence the upper bounds in (9) reduces to

$$\| x^* - x_n \| \le \frac{KB_n e_{n-1}^2}{1 + \sqrt{1 - 2h_n}} = \frac{Ke_{n-1}^2}{B_n^{-1} + \sqrt{1 - 2h}} , \tag{21}$$

which follow immediately from Lemmas 2 and 3. The upper bounds in (10) also reduce to (21) because $B_n = 2^n$ and $K = 1/2\eta$ if $2h = 1$. The proof is completed.

5. Observations

By our proofs of Theorems 3 and 4, we see that Potra-Pták's upper bounds are obtained by replacing B_n^{-1} in the last expression of (21) by the smaller quantities $\sqrt{1 - 2h + (Ke_{n-1})^2}$. Similarly the lower bounds $\gamma(e_n)$ in (8) are obtained if we replace KB_n in the lower bounds in (14) by the larger

quantities $(e_n + \sqrt{\alpha^2 + e_n^2})^{-1}$. Therefore we can conclude that Theorem 4 is finer than Theorem 3. We remark also that Gragg-Tapia's upper bounds in (6) and (7) are equal to

$$\tau_n = \frac{2\eta_n}{1 + \sqrt{1 - 2h_n}} \quad \text{and} \quad \frac{\tau_n}{\eta_{n-1}} e_{n-1} , \tag{22}$$

respectively (cf. Yamamoto [17]). As was shown in [11], the upper bounds of Potra-Pták are sharper than (22). Furthermore, it is easy to see that the lower bounds of Potra-Pták improve those of Gragg-Tapia in (7). (This fact remains unproved in [11].) To prove this, we note that the lower bounds in (7) may be written

$$\frac{2e_n}{1 + \sqrt{1 + 2h_n}}$$

and that

$$\frac{e_n}{e_n + \sqrt{\alpha^2 + e_n^2}} \leq \frac{\eta_n}{\eta_n + \sqrt{\alpha^2 + \eta_n^2}} = \frac{\eta_n}{\eta_n + (KB_{n+1})^{-1}} = h_n ,$$

where we have used the fact that the function $g(t) = t/(t + \sqrt{\alpha^2 + t^2})$ is monotonically increasing with respect to t. Hence we obtain that

$$\gamma(e_n) = \frac{2e_n}{1 + \sqrt{1 + 2e_n(e_n + \sqrt{\alpha^2 + e_n^2})^{-1}}} \geq \frac{2e_n}{1 + \sqrt{1 + 2h_n}} .$$

REFERENCES

1. Dennis, J.E.: On the Kantorovich hypotheses for Newton's method. SIAM J. Numer. Anal. **6**, 493-507 (1969)

2. Deuflhard, P., Heindl, G.: Affine invariant convergence theorems for Newton's method and extensions to related methods. SIAM J. Numer. Anal. **16**, 1-10 (1979)

3. Gragg, W.B., Tapia, R.A.: Optimal error bounds for the Newton-Kantorovich theorem. SIAM J. Numer. Anal. **11**, 10-13 (1974)

4. Kantorovich, L.V., Akilov, G.P.: Functional analysis in normed spaces. Oxford, Pergamon, 1964

5. Miel, G.J.: The Kantorovich theorem with optimal error bounds. Amer. Math. Monthly **86**, 212-215 (1979)

6. Miel, G.J.: Majorizing sequences and error bounds for iterative methods. Math. Comp. **34**, 185-202 (1980)

7. Miel, G.J.: An updated version of the Kantorovich theorem for Newton's method. Computing **27**, 237-244 (1981)

8. Ortega, J.M.: The Newton-Kantorovich theorem. Amer. Math. Monthly **75**, 658-660 (1968)

9. Ortega, J.M., Rheinboldt, W.C.: Iterative solution of nonlinear equations in several variables. New York: Academic Press, 1970

10. Ostrowski, A.M.: Solution of equations in Euclidean and Banach spaces. New York: Academic Press, 1973

11. Potra, F.A., Pták, V.: Sharp error bounds for Newton's process. Numer. Math. **34**, 63-72 (1980)

12. Pták, V.: The rate of convergence of Newton's process. Numer. Math. **25**, 279-285 (1976)

13. Rall, L.B.: Computational solution of nonlinear operator equations. New York: John Wiley, 1969

14. Schmidt, J.W.: Regular-falsi-Verfahren mit konsistenter Steigung und Majorantenprinzip. Periodica Math. Hungarica 5, 187-193 (1974)

15. Schmidt, J.W.: Untere Fehlerschranken für Regula-falsi-Verfahren. Periodica Math. Hungarica 9, 241-247 (1978)

16. Tapia, R.A.: The Kantorovich theorem for Newton's method. Amer. Math. Monthly 78, 389-392 (1971)

17. Yamamoto, T.: Error bounds for Newton's process derived from the Kantorovich theorem. To appear in Japan J. Appl. Math.

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1. REPORT NUMBER 2843	2. GOVT ACCESSION NO. AD - A160959	3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle) ERROR BOUNDS FOR NEWTON'S ITERATES DERIVED FROM THE KANTOROVICH THEOREM	5. TYPE OF REPORT & PERIOD COVERED Summary Report - no specific reporting period
	6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s) Tetsuro Yamamoto	8. CONTRACT OR GRANT NUMBER(s) DAAG29-80-C-0041

9. PERFORMING ORGANIZATION NAME AND ADDRESS Mathematics Research Center, University of 610 Walnut Street Wisconsin Madison, Wisconsin 53705	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Work Unit Number 3 - Numerical Analysis and Scientific Computing

11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office P. O. Box 12211 Research Triangle Park, North Carolina 27709	12. REPORT DATE July 1985
	13. NUMBER OF PAGES 12

14. MONITORING AGENCY NAME & ADDRESS(If different from Controlling Office)	15. SECURITY CLASS. (of this report) UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
Error bounds
Newton's method
the Kantorovich theorem
Potra-Pták's bounds
Miel's bounds

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this paper, it is shown that the upper and lower bounds of the errors in the Newton iterates recently obtained by Potra-Pták [11] and Miel [7], with the use of nondiscrete induction and majorizing sequence, respectively, follow immediately from the Kantorovich theorem and the Kantorovich recurrence relations. It is also shown that the upper and lower bounds of Miel are sharper than those of Potra-Pták.

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