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ANGULAR VELOCITY ESTIMATION USING ATTITUDE MEASUREMENTS WITH APPLICATION TO THE VSS/MARS SYSTEM

24 NOVEMBER 1982

Systems Analysis & Control

in Analysis & Control of Dynamic System

ANGULAR VELOCITY ESTIMATION USING ATTITUDE MEASUREMENTS WITH APPLICATION TO THE VSS/MARS SYSTEM

24 NOVEMBER 1982

PREPARED UNDER:

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TR-82-3

TABLE OF CONTENTS

SUMMARY

The existing Vertical Seeking Seat (VSS) control system uses angular velocity and attitude measurements provided by rate gyroscopes and the Micrad Attitude Reference System (MARS), respectively. A study has been conducted to determine the feasibility of estimating angular velocity from the MARS attitude measurements, and using these estimates in place of the measurements currently provided by rate gyros. This estimation is to be performed in the on-board microprocessor.

The design and performance of an angular velocity estimator will depend upon the sample or iteration rate of the microprocessors, the allowable errors and bandwidth requirements of the velocity estimate, and the errors and bandwidth characteristics of the attitude measurement, all of which are dependent upon the required bandwidth, stability, and response characteristics of the complete VSS system. Therefore, the first phase of this study included a literature search and technical discussion with appropriate personnel at the Naval Weapons Center to determine the design goals and preliminary specifications for the control system and sensor bandwidths. It was learned that the existing system was designed to optimize performance using the available sensing and actuating hardware, and not to meet quantitative bandwidth requirements. Since the control system and sensor bandwidth requirements are not known, but are necessary for determining the feasibility of using an angular velocity estimator, they are evaluated.

Several important assumptions are made in order to facilitate the analyses and evaluations performed in this study. One assumption is that analyses and evaluations of the roll axis control system are applicable to the pitch axis control system, as these systems are independent and very similar. A second assumption is that the MARS provides measurements of roll (and pitch) attitude relative to the vertical. Another assumption is that the control gains, actuator, and actuator compensation are those of the existing system model, since the scope of this study does not include redesigning the control system. The analyses and evaluation results obtained using all these assumptions are now summarized. Methods of determining the effects of these assumptions are presented later, along with other recommendations.

Since the MARS outputs are sampled and the estimator is to be implemented in the microprocessor, the control system is digital and specifications should be evaluated using a discrete time analysis. However, preliminary evaluations are performed using a continuous time analysis rather than a discrete time analysis, as the former is generally less complicated and provides more physical insight than the latter. Here it is assumed that the angular velocity and attitude measurements are outputs of continuous sensors. These analyses indicate that system performance requirements can be met with an angular velocity feedback loop bandwidth of 4 Hertz and an attitude feedback loop bandwidth of 1.4 Hertz. The feedback signal bandwidth should be about three times that of the corresponding loop, so that the angular velocity and attitude measurement bandwidths should be at least 12 Hertz and 4.2 Hertz, respectively. The effects of measurement bias and time delay are also evaluated. The effects of measurement bias are similar to those of aerodynamic moments in that they both contribute to the steady state attitude error of the seat. Since the aerodynamic moments may be large, resulting in large attitude errors, the errors due to bias should be a small fraction of the allowable 10 degrees of attitude error. If the errors resulting from angular velocity and attitude biases are each to be less than 2 degrees, then the maximum biases are 11.3 degrees/second and 2 degrees, respectively. The effect of a time delay between the taking of a measurement and the use of that measurement for application of control is to reduce the system stability margins. This reduction is minimal if the time delay is very small relative to the characteristic time (dominant time constant) of the closed loop system. If the angular velocity and attitude loop bandwidths are 4 Hertz and 1.4 Hertz, respectively, then associated measurement time delays less than 0.004 seconds and 0.01 second, respectively, should have minimal effect on the system stability and response characteristics.

Preliminary insight into the effects of sampling on the bandwidth requirements is obtained using a model with continuous angular velocity and attitude sensors whose outputs are sampled. A discrete time analysis of this model indicates that for sample rates below 100 samples/second, the angular velocity signal bandwidth must be at least 40 Hertz, which is approximately 3 times that required for the completely continuous model. However, the actual bandwidth requirements are evaluated later using a discrete time model of the control system and angular velocity estimator.

Three methods of estimating angular velocity are presented and analyzed. The first method is a kinematic estimator which is based only on kinematics. The second method is a dynamic estimator which is based on the system kinematics and dynamics. This estimator provides filtered estimates of angular velocity and attitude. The third method is a reduced order dynamic estimator which provides only angular velocity estimates. Analysis of these methods indicates that the dynamic estimator has several advantages in that it attenuates high frequency measurement noise, provides good estimates at high frequencies, and also provides a filtered estimate of attitude which can be used for feedback in place of the actual (noisy) measurement. As a result, this dynamic estimator is used in the discrete time analysis.

The dynamic estimator is based on a second order planar model of the seat rotational dynamics. If this model is "essentially" exact the estimator is ideal and the separation property allows the estimator and control system to be designed separately, but used together. However, the model used in this study is inexact as it neglects aerodynamic moments and cross coupling terms. As a result, the estimator is non-ideal and, therefore, the estimator and control system should not be designed separately. Since the scope of this study does not include redesign of the control system, an estimator is designed for use with the existing control system.

Estimator design and the evaluation of attitude measurement specifications are performed using a discrete time model of the combined estimator and control system. Design of the estimator consists of selecting the sample rate and gains. Sample rate selection results from a trade-off between obtaining required bandwidth and response characteristics, implying a high sample rate, and having time for processing the MARS measurements, implying a low sample rate. The sample rate should be at least ten times the required closed loop bandwidth. Since the required bandwidth of the rate loop is 4 Hertz, the sample rate should be at least 40 samples/second. However, the bandwidth of the existing rate loop is 6 Hertz, so the associated sample rate should be at least 60 samples/second. Since the MARS measurements are updated 80 times/second, a sample rate of 80 samples/second is selected. Estimator gain selection results from a trade-off between obtaining fast estimate convergence, implying high gains and bandwidths, and having adequate attentuation of measurement noise, implying low gains and bandwidths. The estimator bandwidth should be about four times the required closed loop bandwidth. Since the required rate loop bandwidth is 4 Hertz, the estimator should have a 16 Hertz bandwidth. However, the existing rate loop bandwidth is 6 Hertz, so an associated estimator should have a 24 Hertz bandwidth. The gains are selected to provide this bandwidth. A digital computer simulation of the resulting closed loop model indicates that the response is satisfactory and is similar to that of the continuous model of the existing system. A detailed description of the estimator is given in Appendix A.

The attitude measurement specifications are re-evaluated using the discrete time model of the combined estimator and control system. The minimum bandwidth is the same as established using a continuous model, that is, 4.2 Hertz. However, a maximum bandwidth must now be specified in order to minimize the effects of aliasing in the discrete system. Typically, aliasing effects are minimized by passing the continuous measurement through a low pass analog prefilter prior to sampling. The design of a prefilter, which determines the maximum bandwidth, is dependent upon the noise characteristics of the continuous measurements and additional filtering and processing performed prior to sampling. Since these noise characteristics are not known and the scope of this study does not include analysis of the measurement filtering and processing, a specific prefilter is not designed, but a design approach is discussed. The steady state effects of a constant bias are the same as for the continuous model and, therefore, the bias should still be kept below 2 degrees for attitudes near vertical. Since measurement noise may cause sustained oscillations of the actuator (TVC) and seat attitude, it is important that the noise RMS be specified so the RMS actuator deflection and attitude are within allowable limits. A complete evaluation of the system response to noise should include effects of measurement filtering and processing. Since evaluation of these effects is beyond the scope of this study, the system response to measurement noise is not determined. The allowable time delay is one sample period (0.0125 seconds) minus the time required for calculating the estimates. Delays longer than this will degrade the stability and response characteristics of the system.

NOMENCLATURE

B	constant bias
C	feedback gain matrix (continuous)
e	base of natural logarithms
F	system matrix (continuous)
g	acceleration due to gravity
G	control matrix (continuous)
H	measurement matrix
I	moment of inertia
K	estimator gain matrix
ℓ	distance from motor pivot to seat center of mass
m	total mass of system
n	sample index
p	roll angular velocity
R	measurement noise power spectral density (PSD)
s	LaPlace operator
t	sample period
T	torque
u	system input
U	control covariance matrix
x	state vector
X	state covariance matrix
y_i	relative horizontal displacement from launch point
z	measurement vector
z_i	relative vertical displacement from launch point
Γ	control matrix (discrete)
δ	thrust vector control deflection
η	dummy variable of integration
ϕ	roll attitude
Φ	system matrix (discrete)
ζ	damping ratio
τ	time constant; correlation time
χ	measurement noise variance
ω	natural frequency

Subscripts and Superscripts

a	augmented
c	command; control system
cl	closed loop
e	estimated
f	compensation filter
i	inertial
$(\tilde{\ })$	error in estimate of ( )
$(\hat{\ })$	estimated ( )
$(\ )^T$	transpose of ( ) matrix
$(\dot{\ })$	derivative of ( ) variable

INTRODUCTION

The China Lake Naval Weapons Center (NWC) is involved in developing a Vertical Seeking (ejection) Seat (VSS). After ejection, the VSS sensors provide measurements of seat attitude and angular velocity to the control system, which deflects a gimbaled rocket motor so that the seat aligns itself with the vertical. Currently, attitude relative to the vertical is measured using the Microwave Radiometric (MICRAD) Attitude Reference System (MARS) and angular velocity is measured using a triad of rate gyroscopes.

The MARS appears well suited for this application in that it is autonomous and can provide precise measurements immediately after ejection. Rate gyros are not as well suited, as there is a delay between the activation time and that at which adequate measurements are available. Also, since rate gyros are mechanical devices, questions of reliability arise. An alternative to using rate gyros or other rate sensors is to estimate seat angular velocity, in the onboard microprocessor, from attitude measurements provided by the MARS. The design and performance of such an estimator will depend on the sample or iteration rate of the microprocessor, the allowable errors and the bandwidth requirements of the velocity estimate, and the errors and bandwidth characteristics of the attitude measurement, all of which are dependent upon the required bandwidth, stability, and response characteristics of the complete VSS system.

This report describes a study to determine the feasibility of estimating angular velocity from the MARS attitude measurements, and using these estimates in place of measurements currently provided by rate gyros. The phases of this study were performed, and are presented, as follows. First, all available documentation of the current VSS-MARS system was reviewed, and technical discussions were held with NWC personnel who were involved in the development of the current system. Second, mathematical models used in this study are presented. Third, a simplified analysis is performed to evaluate specifications which should be placed on angular velocity and attitude measurements obtained from continuous sensors. Next, methods of estimating velocity are reviewed, and a candidate estimator selected. A roll control system model employing this estimator is then used to evaluate specifications which should be placed on angular velocity estimates, attitude measurements, and sample or iteration rates. Since the scope of this study does not include a redesign of the VSS control system, these specifications are evaluated assuming that the gains, actuator, and actuator compensation are those of the original system model. Finally, the results of this study are summarized and recommendations for future efforts are presented.

LITERATURE SEARCH AND REVIEW

A literature search was performed to obtain all documentation relating to the current design and development of the MARS-VSS system. Specifically, descriptions of design goals and preliminary specifications for the control system and sensor bandwidths and the means used to establish them were sought. Three reports were obtained and are listed as References 1, 2, and 3. Reference 1, which is a three volume final report published by NWC, describes the design and testing of the current system. This report documents the existing control system and sensors, but does not define goals or specifications for the bandwidths of interest. Reference 2 is a report generated by Grumman Aerospace Corporation for the Naval Air Development Center (NADC). This report describes an evaluation of various sensors which may be used in a similar VSS system, but again does not address bandwidth specifications. Reference 3 is a Systems Analysis and Control report describing an analysis of MARS control laws. The literature search did not produce information regarding design goals or preliminary requirements for the control system and sensor bandwidths.

After the literature search, discussions were held with NWC personnel who contributed to the VSS control system design. These discussions indicated that the current system was designed to optimize performance using the available sensing and actuating hardware, and not to meet quantitative bandwidth requirements. The control system and sensor bandwidth requirements are not known.

Since these bandwidth requirements are necessary for determining the feasibility of using an angular velocity estimator, they are evaluated in this study.

MATHEMATICAL MODELS

Three mathematical models are used and referred to in this study. These include a linear model of the roll axis control system, a model of the seat rotational dynamics, and a three degree of freedom (DOF) simulation model. Each of these models is described.

CONTROL SYSTEM

The VSS control system consists of independent pitch and roll axis controllers. Since these controllers are very similar, only the roll axis controller is modeled and analyzed. A block diagram of the roll controller linear model is shown in Figure 1. This model is obtained by eliminating the notch and low pass filters and linearizing the "hardware" model shown in Figure 3.5-1 of Reference 1. These filters are neglected in this analysis since their function is to attenuate body bending effects and noise, and not to provide compensation. Since the rate gyro bandwidth requirements are to be determined, the bandwidth is indicated as the variable ω in Figure 1. Also, the model is linearized by using small angle approximations for the actuator deflection, neglecting the electrical and mechanical limits of the actuator, and assuming that the output of the "MARS control law" is proportional to roll attitude relative to vertical.

The resulting model can be expressed in state variable form as

$$\dot{x} = Fx + Gu \tag{1}$$

$$u = -Cx \tag{2}$$

where

$$x = [\phi \; p \; \hat{p} \; \dot{\hat{p}} \; \ddot{\hat{p}} \; \delta \; \dot{\delta} \; \ddot{\delta} \; \delta_f]^T \tag{3}$$

Equation 1 represents the open-loop model. A closed-loop model is obtained by substituting Equation 2 into 1,

$$\dot{x} = [F - GC]x \tag{4}$$

Through proper specification of the feedback gain matrix C, Equation 4 may

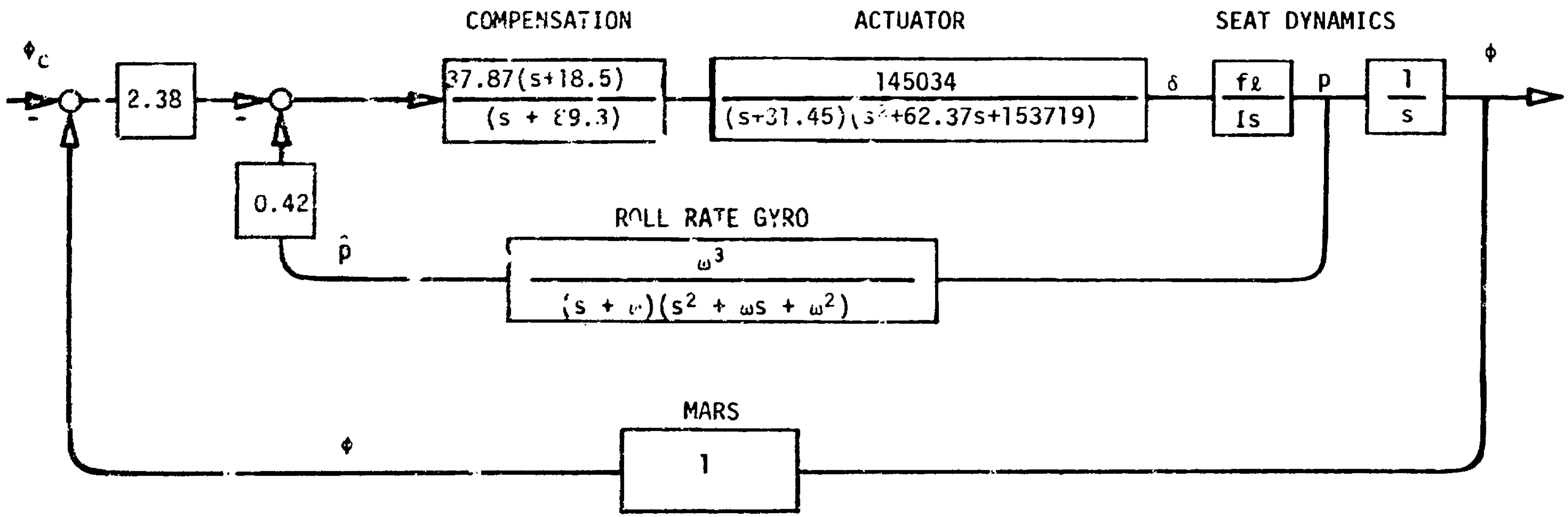

FIGURE 1. BLOCK DIAGRAM OF ROLL CONTROL SYSTEM LINEAR MODEL.

represent a model with only the velocity loop closed or one with both the attitude and velocity loops closed. Equations 1-4 represent a linear model of the roll control system for small perturbations from the vertical. It should be noted that this model neglects aerodynamic moments acting about the roll axis, as the most significant of these, due to linear velocity, is unknown.

SEAT ROTATIONAL DYNAMICS

A simple planar model of the seat body rotational dynamics consists of a torque T applied to an inertia I. This model can be represented as two first order differential equations.

$$\dot{\phi} = p \tag{5}$$

$$\dot{p} = \frac{T}{I} \tag{6}$$

This model neglects aerodynamic torques acting about the roll axis.

THREE DOF SIMULATION

A simple planar three DOF simulation model is formed by combining equations of the seat body dynamics with a model of the control system.

The dynamic equations are obtained using the coordinates and nomenclature shown in Figure 2.

$$\ddot{y}_i = \frac{f}{m} \sin(\phi - \delta) \tag{7}$$

$$\ddot{z}_i = \frac{-f}{m} \cos(\phi - \delta) + g \tag{8}$$

$$\ddot{\phi} = \frac{f\ell}{I} \sin \delta \tag{9}$$

A simple control system model is obtained by approximating the rate gyro feedback control loop as a first order transfer function relating angular velocity to its commanded value. Such a model is shown in Figure 3. A problem with this model is that it will produce unrealistically large angular accelerations as δ is not calculated and therefore cannot be limited. As a result, this model is modified by expanding the transfer function block

BACK VIEW

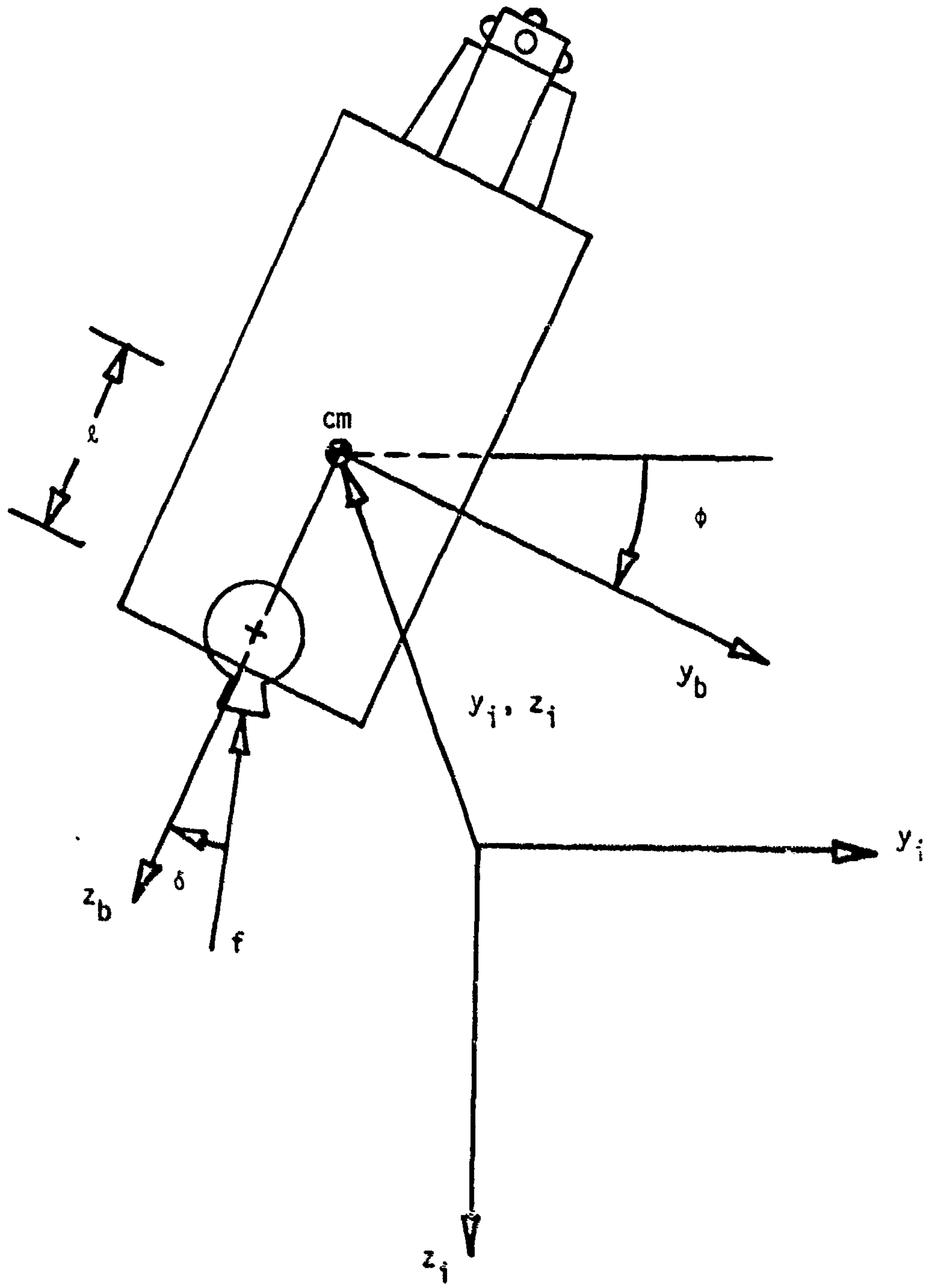

FIGURE 2. COORDINATE SYSTEMS AND NOMENCLATURE
FOR 3-DOF SIMULATION MODEL.

diagram and using Equation 9 so that δ is now calculated explicitly and limited as shown in Figure 4. The simulation is formed by combining the equations corresponding with the block diagram in Figure 4 with Equations 7 and 8. It should be noted that this simulation neglects aerodynamic forces and moments.

The velocity control loop time constant corresponding to the existing hardware is obtained by approximating the step response of the linear control system model with an exponential curve. The step response of the model in Figure 1 is shown in Figure 5, along with the response corresponding to $\tau = 0.03$ sec. This time constant corresponds to that of the existing hardware.

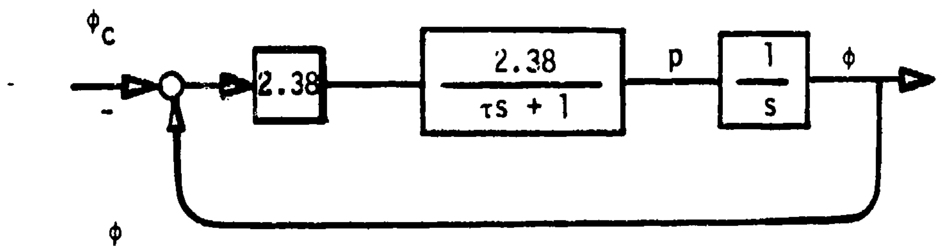

FIGURE 3. BLOCK DIAGRAM OF SIMPLIFIED CONTROL SYSTEM MODEL.

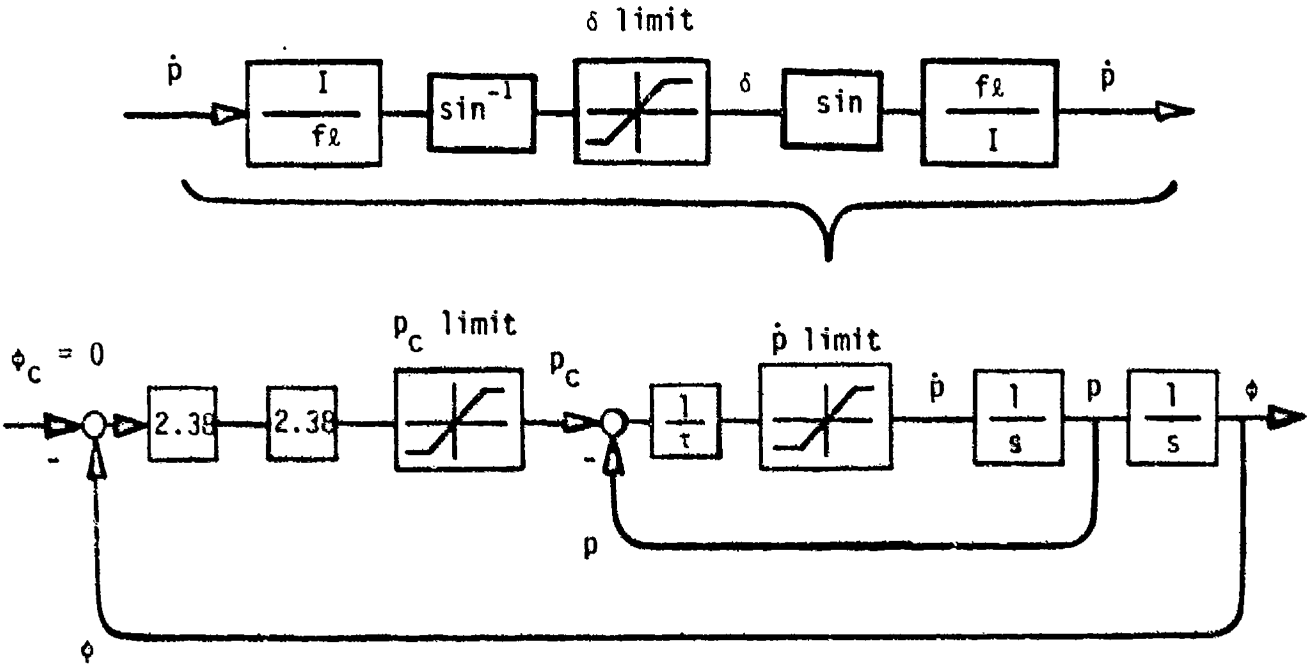

FIGURE 4. EXPANDED BLOCK DIAGRAM OF SIMPLIFIED CONTROL SYSTEM MODEL WITH LIMITS.

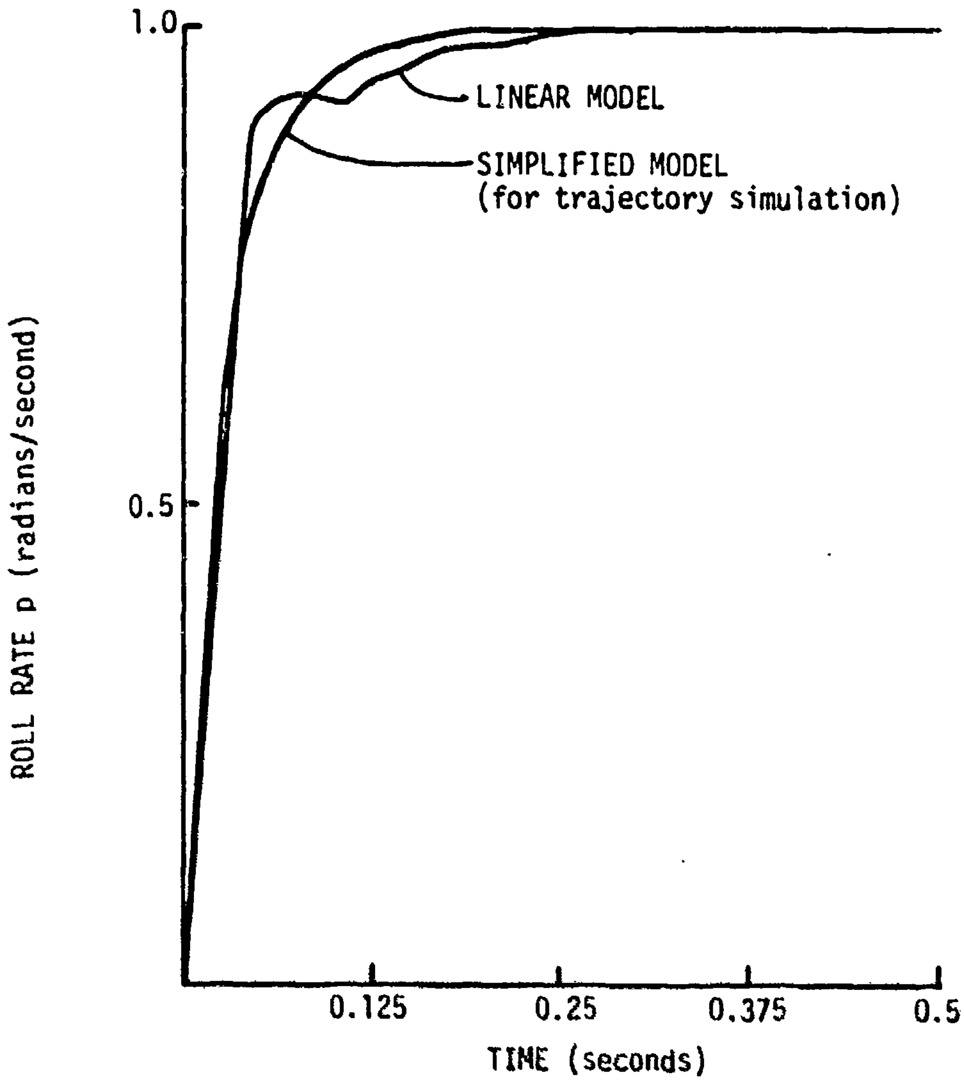

FIGURE 5. STEP RESPONSES OF LINEAR AND SIMPLIFIED CONTROL SYSTEM MODELS.

SPECIFICATIONS FOR ANGULAR VELOCITY MEASUREMENTS

The current VSS control system, Figure 1, contains rate gyroscopes which provide angular velocity measurements. These measurements are used in a rate (angular velocity) feedback control loop. An alternative to using rate gyros or other rate sensors is to estimate seat angular velocity, in the onboard microprocessor, from the MARS attitude measurements. The design and even the feasibility of such an estimator will be dependent upon the required fidelity of the velocity estimate. Since the estimator will be implemented in the microprocessor, the rate feedback loop is digital and, therefore, velocity estimate specifications should be established using a discrete time analysis. Generally, discrete analyses are more complicated, and provide less physical insight, than continuous time analyses. As a result, preliminary specifications for the angular velocity signal are determined assuming that it is an output of a continuous rate sensor.

Specifications are evaluated for the velocity signal bandwidth, bias, noise, and time delay.

BANDWIDTH

The velocity signal bandwidth affects the bandwidth of the closed rate feedback loop which, in turn, affects the VSS trajectory. Since performance or trajectory requirements have been defined, a logical approach is to first determine the minimum rate loop bandwidth which will satisfy these requirements, and then determine the minimum rate signal bandwidth which will yield this minimum loop bandwidth.

The effect of rate loop bandwidth on the trajectory is evaluated using the three DOF simulation model described previously. The simulation is initialized with the seat inverted, ϕ = 180 degrees, and with no linear or angular velocity. Trajectories are then generated with various values of rate loop bandwidth and the results are summarized in Figure 6. These results indicate that very little improvement in altitude loss or vertical velocity at burn-out is obtained as the bandwidth is increased beyond 2 Hertz. Care should be taken in relating this frequency to the required bandwidth, as these results were obtained using a very simple planar model which neglects aerodynamic forces and moments and cross coupling terms

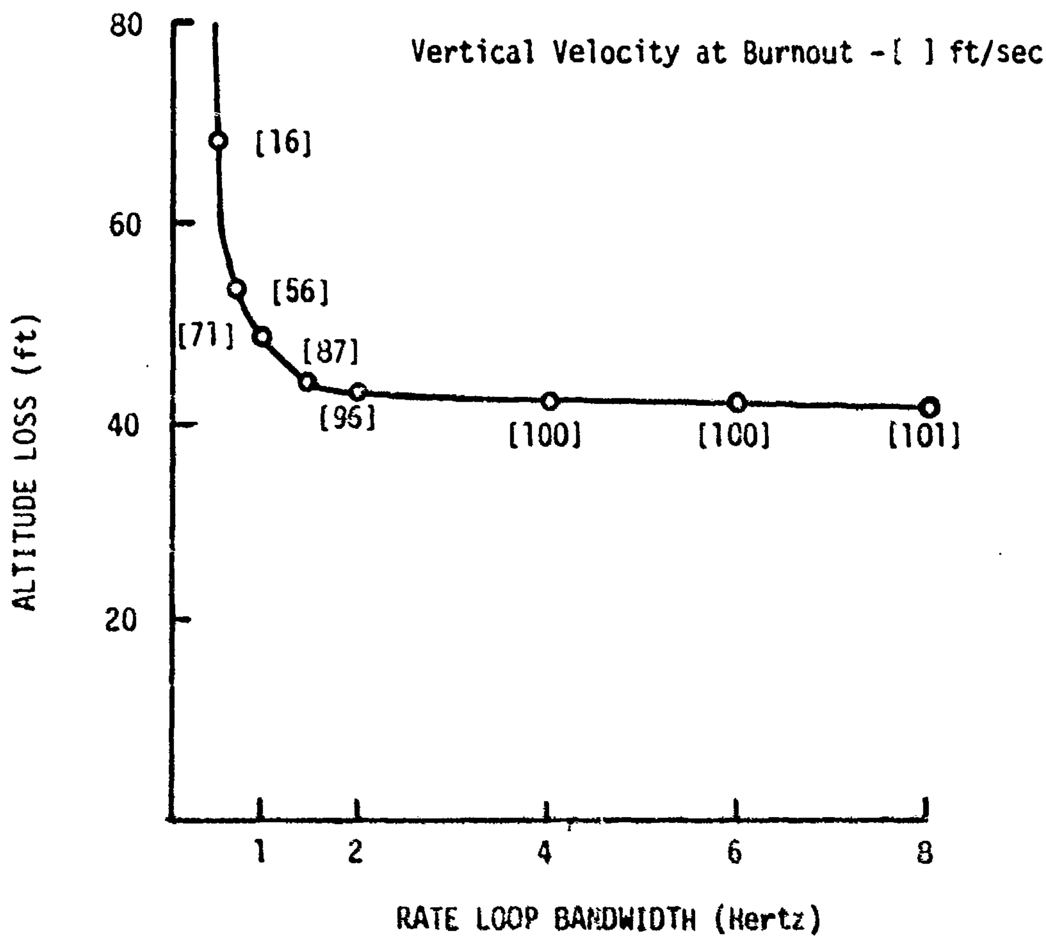

FIGURE 6. SIMULATION RESULTS FROM 3-DOF MODEL WITH VARIOUS RATE LOOP BANDWIDTHS. (For stationary, inverted ejection with a 500 degree/second roll rate limit)

between axes. As a result, the bandwidth should be higher than 2 Hertz. A rate loop bandwidth 50 to 100 percent above 2 Hertz should provide an adequate safety margin and good performance. Thus the 6 Hertz bandwidth of the existing rate loop provides a very comfortable margin.

The rate signal bandwidth should be significantly higher than the required rate loop bandwidth. Generally, good stability and response characteristics are obtained from such systems if the rate signal bandwidth is three times that of the closed rate loop. Thus, for a 4 Hertz rate loop, the signal bandwidth should be at least 12 Hertz.

The above requirements were established using a completely continuous system model. Some insight into the effects of sampling on the bandwidth requirements is obtained using a model with a continuous rate sensor whose output is sampled. This model, which is based on the control system model described previously (Figure 1), is used to determine the sample rate required for a specified gain margin, given the rate sensor (signal) bandwidth. The trade-off between bandwidth and sample rate, for a 6 db margin, is shown in Figure 7. These results indicate that for reasonable sample rates, less than 100 samples/second, the required rate signal bandwidth is significantly higher than that established for the completely continuous model.

BIAS

A constant bias in the rate signal will cause a steady state error in the seat attitude. The steady state attitude will, therefore, deviate from vertical by the sum of this error, a similar error caused by aerodynamic moments, and the attitude measurement error. Since the aerodynamic moments may be large, resulting in large attitude errors, the error due to the bias should be a small fraction of the allowable 10 degrees of attitude error. The steady state attitude error due to a rate signal bias B is a function of the rate and attitude gains.

$$\phi = \frac{0.420}{2.38} B \qquad (10)$$

If this error is to be kept smaller than 2 degrees, then the maximum allowable bias is 11.3 degrees/second.

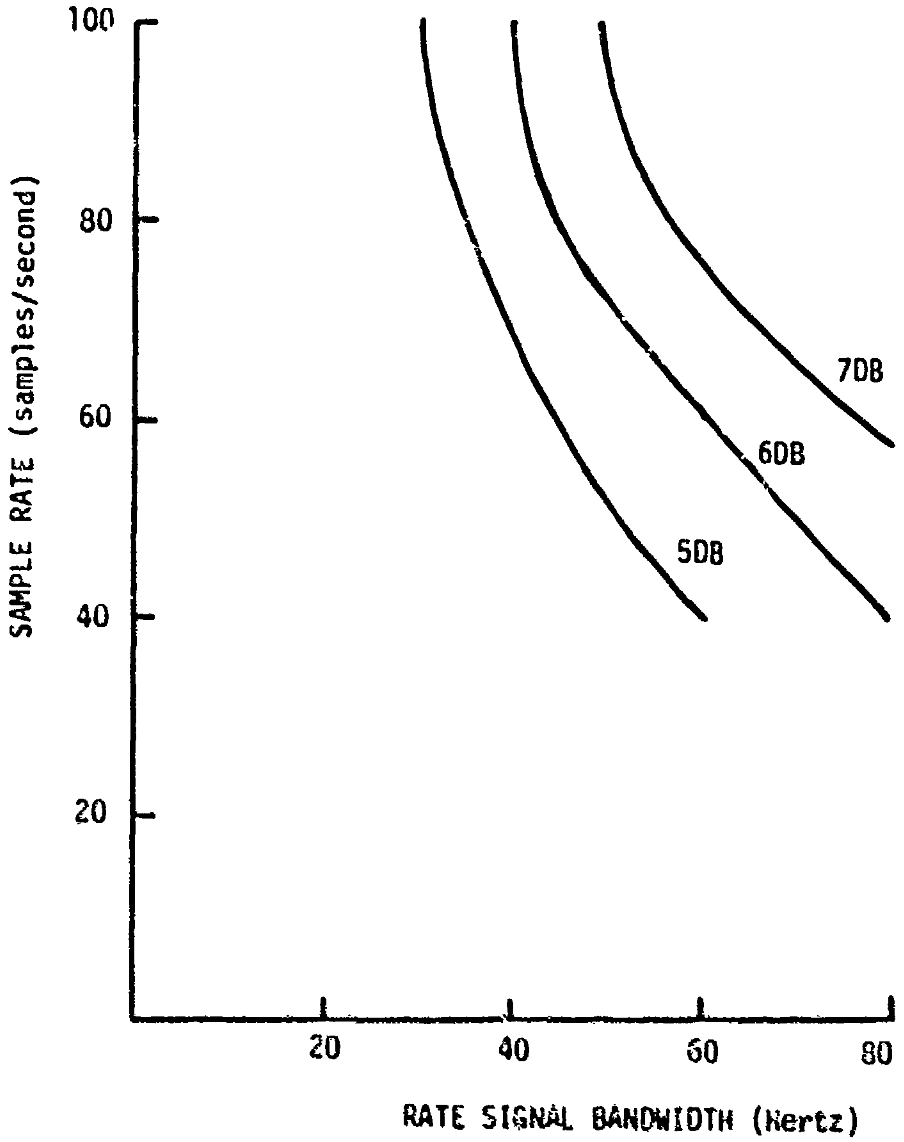

FIGURE 7. GAIN MARGIN OF CONTROL SYSTEM MODEL AS A FUNCTION OF SAMPLE RATE AND RATE SIGNAL BANDWIDTH.

NOISE CHARACTERISTICS

Noise in the rate measurement may cause the closed-loop system to exhibit sustained oscillations. Since these oscillations will involve the seat attitude and the thrust vector control (TVC) system (and may lead to actuator damage), it is important that the magnitude or root mean square (RMS) value of the oscillations be kept within allowable limits. The noise RMS should be specified so that the RMS response of the system is acceptable. That is, the RMS attitude error should be a small fraction of the allowable 10 degrees and the RMS actuator (TVC) response should be "very" small, probably less than 0.3 degrees.

The RMS response of the continuous control system model described by Equations 1-4, driven by zero mean, Gaussian noise in the feedback u can be calculated using (Reference 4)

$$[F - GC]X + X[F - GC]^T + GRG^T = 0 \tag{11}$$

and

$$U = CXC^T \tag{12}$$

where

X = state covariance matrix

U = control covariance matrix

R = measurement noise power spectral density (PSD)

Equations 11 and 12 can be solved for the state and control variances (and covariances) which are the squares of the RMS responses. Solution of these equations requires knowledge of the measurement noise PSD, which is defined as

$$R = 2\tau x \tag{13}$$

where

τ is the correlation time

x is the noise variance

Thus, calculation of this system's RMS response requires knowledge of the measurement noise variance or RMS value and the correlation time. Since this correlation time is not currently available, the RMS system response is not calculated, and no specification is placed on the measurement noise RMS value.

TIME DELAY

A time delay between the taking of a measurement and use of that measurement for application of control introduces a phase lag which reduces the system gain and phase or stability margins. In order to minimize the margin reductions, the phase lag should be small at the system gain and phase cross-over frequencies. The phase lag is the product of the delay time and the frequency, and the cross-over frequencies are functions of the system dynamics.

Another method of minimizing the effects of time delay is to require it to be very small relative to the characteristic times (time constants) of the closed loop system. Thus, if the closed loop bandwidth is 4 Hertz which corresponds approximately to a 0.04 second time constant, then time delays less than 0.004 seconds should have minimal effect on the system response characteristics.

SPECIFICATIONS FOR ATTITUDE MEASUREMENTS

The VSS control system, Figure 1, contains a MARS which provides measurements of attitude relative to vertical. These measurements are used in a position (attitude) feedback control loop which is closed around the previously described rate loop. Since the MARS outputs are sampled and processed in a microprocessor, and a velocity estimator may be implemented in the microprocessor, both feedback loops may be digital and, therefore, attitude measurement specifications should be established using a discrete analysis. Generally, discrete analyses are more complicated, and provide less insight, than continuous analyses. As a result, preliminary specifications for the attitude signal are determined assuming it is an output of a continuous sensor. Insight obtained from this analysis will facilitate the discrete time analysis, which is discussed later.

Specifications are evaluated for the attitude signal bandwidth, bias, noise, and time delay.

BANDWIDTH

The attitude signal bandwidth affects the characteristics of the rate loop command, and ultimately the VSS trajectory. This bandwidth should, therefore, be high enough so that the maximum rate is commanded when the system is enabled after an inverted ejection. Since the current system requires 88 degrees of attitude error to produce the maximum angular velocity command of 500 degrees/second and the entire system is enabled about 0.25 seconds after ejection, a minimum bandwidth is approximately

$$\frac{-1}{0.25(2\pi)} \ln \left(1 - \frac{88}{180}\right) = 0.43 \text{ Hertz} \qquad (14)$$

There are other criteria which should also be considered in specifying a minimum bandwidth. One of these is the adequate rejection of system disturbances. The seat will be subjected to aerodynamic moment disturbances at frequencies corresponding with the angular velocity. The maximum angular velocity is 500 degrees/second which corresponds with a frequency of

$$\frac{500}{360} = 1.4 \text{ Hertz} \qquad (15)$$

The attitude signal and loop bandwidths should, therefore, both be higher than this frequency.

Another criterion is that the attitude signal bandwidth should be significantly higher than the required closed loop bandwidth. Generally, good stability and response characteristics are obtained for this type of system if the attitude signal bandwidth is three times that of the closed attitude loop. Thus the signal bandwidth should be at least 4.2 Hertz.

BIAS

A constant bias in the attitude signal will cause a corresponding steady state error in the seat attitude. The steady state attitude will, therefore, deviate from vertical by the sum of this error and similar errors caused by aerodynamic moments and rate signal biases. Since the aerodynamic moments may be large, resulting in large attitude errors, the error due to this bias should be a small fraction of the allowable 10 degree error. This error should therefore be limited to 2 degrees for attitudes near vertical.

NOISE CHARACTERISTICS

The effects of attitude measurement noise are similar to those previously described for rate measurement noise and are, therefore, not repeated here.

TIME DELAY

The effects of a time delay between an attitude measurement and use of the measurement for application of control are similar to those previously described for rate measurements. If the closed attitude loop bandwidth is 1.4 Hertz, which corresponds approximately to a 0.10 second time constant, then time delays less than 0.01 seconds should have minimal effect on the system response characteristics.

ESTIMATION OF ANGULAR VELOCITY

The current VSS control system uses measurements of angular velocity obtained from rate gyros. An alternative to using rate gyros or other rate sensors is to estimate the pitch and roll angular velocities from attitude measurements provided by the MARS. If the estimates are "sufficiently accurate," they may be used for feedback in place of the gyro measurements.

The estimation is to be performed in the onboard microprocessor. As a result, the estimate algorithms must be discrete and, therefore, their performance will not only be dependent upon the attitude measurement characteristics and the estimate bandwidth required, but also upon the rate at which the algorithms are processed. Since this complicates the design and analysis of discrete time estimators and continuous time estimators are generally better understood, several continuous methods of estimating velocity from position measurements are evaluated initially. Insight obtained from these evaluations is used to select the most promising method. A discrete version of this estimator is then designed and analyzed.

CONTINUOUS ESTIMATORS

Three continuous methods of estimating velocity from position measurements are presented and evaluated.

Kinematic Estimator

A classical method of estimating velocity is based on the kinematics, which indicate that the position measurement should be differentiated and filtered. This operation is expressed in transfer function form as

$$\hat{p}(s) = \frac{\omega^2 s}{s^2 + 2\xi\omega s + \omega^2}\,\phi(s) \tag{16}$$

where $\hat{p}$ is the estimated velocity and ϕ is the position measurement. At low frequencies, $s << \omega$, this transfer function can be approximated as

$$\hat{p}(s) = s\,\phi(s) \tag{17}$$

and the estimate is equal to the derivative of the measurement. Note that any measurement noise at these frequencies will also be differentiated and, therefore, amplified. At high frequencies, s << ω, Equation 16 can be approximated as

$$\hat{p}(s) \approx \frac{\omega^2 \phi(s)}{s} \tag{18}$$

and the estimate is a function of the measurement integral. Note that measurement noise at these frequencies will also be integrated and, therefore, attenuated.

Equations 17 and 18 indicate that the estimate bandwidth is a function of ω, and ω should be selected as a trade-off between sufficient estimate bandwidth and adequate noise attenuation.

Dynamic Estimator

The classical estimator is kinematic in the sense that it essentially differentiates the measurements which are in the frequency range of interest. It seems logical that an estimator based on the system kinematics and dynamics should provide improved estimates. This is the basis of a dynamic estimator (observer) which consists of a system model driven by the error between the measurement and its estimated value.

A simple planar model of the ejection seat rotational dynamics is given in Equations 5 and 6. A dynamic estimator for this model can be expressed in matrix form as

$$\begin{bmatrix} \dot{\hat{\phi}} \\ \dot{\hat{p}} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\phi} \\ \hat{p} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/I \end{bmatrix} T + \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} [\phi - \hat{\phi}] \tag{19}$$

where ^ indicates an estimate, T is the applied torque, and ϕ is the position measurement. The estimator design now consists of selecting the gains K_1 and K_2 to obtain desired response characteristics. Insight into the effects of K_1 and K_2 is obtained by letting

$$\begin{aligned} K_1 &= 2\xi\omega \\ K_2 &= \omega^2 \end{aligned} \tag{20}$$

and solving Equation 19 for the transfer functions between the inputs and the estimates.

$$\hat{\phi} = \frac{(2\xi\omega s + \omega^2)}{D}\,\phi + \frac{T}{ID} \qquad (21)$$

$$\hat{p} = \frac{\omega^2 s}{D}\,\phi + \frac{(s+2\xi\omega)}{ID}\,T \qquad (22)$$

where

$$D = s^2 + 2\xi\omega\, s + \omega^2$$

Note that for simplicity the Laplace transform notation has been dropped.

At low frequencies, $s \ll \omega$, these transfer functions can be approximated as

$$\hat{\phi} \simeq \phi + \frac{T}{I\omega^2} \qquad (23)$$

$$\hat{p} \simeq s\phi + \frac{2\xi}{I\omega}\,T \qquad (24)$$

The last terms in these two equations are very small, since

$$I\,\ddot{\phi} = T \qquad (25)$$

or

$$\frac{T}{I} = s^2\,\phi \qquad (26)$$

and s is small, so that the position estimate is equal to the measurement and the velocity estimate is equal to the derivative of the measurement. Again note that any measurement noise at these frequencies will also be differentiated and, therefore, amplified.

At high frequencies, $s \gg \omega$, Equations 21 and 22 can be approximated as

$$\hat{\phi} = \frac{2\xi\omega}{s}\,\phi + \frac{T}{I\,s^2} \qquad (27)$$

$$\hat{p} = \frac{\omega^2}{s}\,\phi + \frac{T}{I\,s} \qquad (28)$$

so that the estimates are functions of the measurement integral and, therefore, measurement noise at these frequencies will be attenuated. The estimates also contain terms involving the applied torque. Insight concerning the contribution of these terms is obtained by substituting Equation 26 into Equations 27 and 28. This indicates that these terms drive the position and velocity estimates to follow the measurement and its derivative, respectively, at high frequencies.

Equations 23, 24, 27, and 28 indicate that at frequencies lower than ω, the measurement (ϕ) terms dominate the estimator response and at frequencies higher than ω, the applied torque (T) terms dominate the response. Therefore ω should be selected so that measurement noise is adequately attenuated. Note that noise in torque input is always attenuated for reasonable values of ω, and is significantly attenuated at high frequencies as the torque is integrated.

It is interesting to compare the characteristics of the velocity estimates obtained from the kinematic and dynamic estimators. Comparison of Equations 17 and 24 indicates that the low frequency estimates are essentially equal as the term involving the applied torque in the latter equation is very small. Comparison of Equations 18 and 28 indicates that the high frequency estimates differ significantly. The estimate provided by the kinematic estimator, Equation 18, approaches zero as frequency increases, but the estimate provided by the dynamic estimator maintains fidelity due to the applied torque term. This is the major advantage of a dynamic estimator. Another advantage is that it provides filtered estimates of the measurement which can be used for feedback in place of the actual, noisy, measurement.

Reduced Order Dynamic Estimator

In one sense the dynamic estimator seems redundant in that it provides estimates of a measurement which is already available. This redundancy can be eliminated by using a reduced order dynamic estimator. As the name implies, this estimator is also based on the system kinematics and dynamics, but it only provides estimates of variables which are not measured. This estimator consists of a reduced order system model driven by the error between the measurement time derivative and its estimate.

A reduced order estimator for the dynamic model used previously is

$$\dot{\hat{p}} = \frac{T}{I} + K(\dot{\phi} - \hat{p}) \tag{29}$$

To avoid the need for differentiating the measurement, define

$$p' = \hat{p} - K\phi \tag{30}$$

so that the estimator can be formulated as

$$\dot{p}' = \frac{T}{I} - K\hat{p} \tag{31}$$

and

$$\hat{p} = p' + K\phi \tag{32}$$

The estimator design now consists of selecting the gain K to obtain the desired response characteristics. Insight into the effects of K is obtained by letting

$$K = \omega \tag{33}$$

and solving Equations 31 and 32 for the transfer functions between the inputs and the estimate.

$$\hat{p} = \frac{\omega s\phi}{s+\omega} + \frac{T}{I(s+\omega)} \tag{34}$$

At low frequencies, $s \ll \omega$, this transfer function can be approximated as

$$\hat{p} \approx s\phi + \frac{T}{I\omega} \tag{35}$$

The last term in this equation is small, as indicated by Equation 26 and the fact that s is small. Therefore, the velocity estimate is equal to the derivative of the measurement.

At high frequencies, $s \gg \omega$, Equation 34 can be approximated as

$$\hat{p} \approx \omega\phi + \frac{T}{Is} \tag{36}$$

so that the estimate is a function of the measurement and, therefore, measurement noise at these frequencies is fed directly into the estimate. The estimate also contains a term involving the applied torque. This term is identical to one in the dynamic estimator Equation 28, and its contribution to the estimate is the same as discussed previously.

Equations 35 and 36 indicate that at frequencies lower than ω, the measurement (ϕ) term dominates the estimator response and at frequencies higher than ω, the measurements and applied torque (T) terms both contribute to the response. Therefore, the frequency ω should be selected much lower than the measurement noise frequencies so that the noise will not be differentiated (amplified).

It is interesting to compare the characteristics of the velocity estimates obtained from the reduced order and dynamic estimators. Comparison of Equations 24 and 35 indicate that the low frequency estimates are essentially equal. Comparison of Equations 28 and 36 indicates that the high frequency estimates differ significantly, as the reduced order estimator feeds the measurement directly into the estimate. Thus the major disadvantage of the reduced order estimator is that measurement noise is not filtered, but is passed directly into the estimate.

DISCRETE ESTIMATORS

The comparisons of the three continuous estimators indicate that the dynamic estimator has several advantages.

1. It attenuates high frequency measurement noise.
2. It provides good estimates at high frequencies.
3. It not only provides an estimate of velocity, but also a filtered estimate of position which can be used by the controller in place of the actual (noisy) measurement.

Thus the dynamic estimator is the most promising method of estimating velocity from position measurements. A discrete version of this estimator is now presented and analyzed.

Estimator Equations

If the dynamic estimator equation is expressed as

$$\dot{\hat{x}}_e = F_e \hat{x}_e + G_e T + K [\phi - \hat{\phi}] \tag{37}$$

where the vectors and matrices x_e, G_e, K, and F_e are defined as their corresponding quantities in Equation 19, then a discrete prediction version of this estimator is

$$\hat{x}_e(n+1) = \Phi_e \hat{x}_e(n) + \Gamma_e' T(n) + K[\phi - \hat{\phi}](n) \tag{38}$$

where

$$\Phi_e = e^{F_e t} \tag{39}$$

$$\Gamma_e' = \int_0^t e^{F_e \eta} d\eta \; G_e \tag{40}$$

and t is the sample period and n indicates the nth sample or iteration.

The estimator design now consists of selecting values for the elements in the gain matrix K. As mentioned previously, one criterion for selecting the gains is to minimize the effects of measurement noise. Another criterion is the speed with which the estimates converge to their actual values.

Estimator Error Dynamics

It can be shown (Reference 5) that the equation for the estimate error dynamics is

$$\tilde{x}_e(n+1) = [\Phi_e - KH] \tilde{x}'(n) \tag{41}$$

where

$$H = [1 \quad 0] \tag{42}$$

and

$$\tilde{x}_e = \hat{x}_e - x_e \tag{43}$$

is the estimate error. The estimate error dynamics are governed by the eigenvalues of the bracketed matrix in Equation 41. Therefore, gain selection results from a trade-off between noise attenuation, which suggests a low bandwidth estimator, and rapid convergence, which suggests a high bandwidth estimator.

Gain Specification

The gains are calculated by comparing the coefficients of the estimator characteristic equation with those of a desired characteristic equation. The estimator characteristic equation is

$$z^2 + (K_1 - 2)\, z + (1 - K_1 + K_2\, t) = 0 \tag{44}$$

where z is the discrete operator. Now the gains are selected to place the z-plane poles in locations which yield the desired response characteristics. Since the estimator is a second order system, its response can be characterized by a natural frequency and damping ratio. The characteristic equation of a discrete system whose response is similar to that of a continuous second order system, with natural frequency ω and damping ratio ξ, is

$$z^2 - (2\, R \cos\theta)\, z + R^2 = 0 \tag{45}$$

where

$$\theta = \omega t\, (1 - \xi^2)^{1/2} \tag{46}$$

$$R = e^{-[\theta\xi/(1-\xi^2)^{1/2}]} \tag{47}$$

Comparing coefficients of Equations 44 and 45 yields

$$K_1 = 2 - 2\, R \cos\theta \tag{48}$$

$$K_2 = (R^2 - 1 + K_1)/t \tag{49}$$

Equations 46-49 are used to design a discrete estimator whose (estimate) convergence and (measurement) filtering characteristics correspond to the response of a second order system defined by ξ and ω. These characteristics are for an estimator whose outputs or estimates are not used for feedback. The effects of feeding back the estimates are discussed in the next section.

COMBINED ESTIMATOR AND CONTROL SYSTEM

The angular velocity estimates are to be used for feedback in place of the rate gyro measurements. Since the estimator also provides a filtered attitude estimate, it is logical to use this estimate for feedback in place of the MARS measurement. Since the estimator is implemented in the microprocessor, the control system is digital and, therefore, a discrete analysis of the closed loop system is performed. Initially, the closed loop system with estimator is described, and the differences between ideal and non-ideal estimators discussed. Next, the estimator design, which consists of sample rate and gain selection, and an associated discrete analysis are presented. Finally, specifications for attitude measurement bias, bandwidth, time delay and noise are discussed.

It is emphasized that the analysis and evaluations are performed assuming the gains, actuator, and actuator compensation are those of the original system model.

CLOSED LOOP SYSTEM

The closed loop system model, shown in Figure 8, is identical to the original system model, Figure 1, except that the feedback variables are now provided by the estimator. Previously, the estimator was analyzed assuming that it is based on an exact model of the system and that its estimates are not used for feedback. The effects of feeding back estimates are analyzed using a closed loop discrete model of the control system and an ideal estimator. The effects of modeling errors are determined using a closed loop model with a non-ideal estimator. Discrete models of the control system, ideal estimator, and non-ideal estimator are now derived.

Control System Model

A continuous open loop model of the original control system, including gyro dynamics, is defined by Equations 1 and 3. Since the gyro is not used in this portion of the study, its dynamics are eliminated from the model. The resulting model, which contains the dynamics of the seat inertia, actuator, and actuator compensator, is expressed as

$$\dot{x}_c = F_c x_c + G_c u \qquad (50)$$

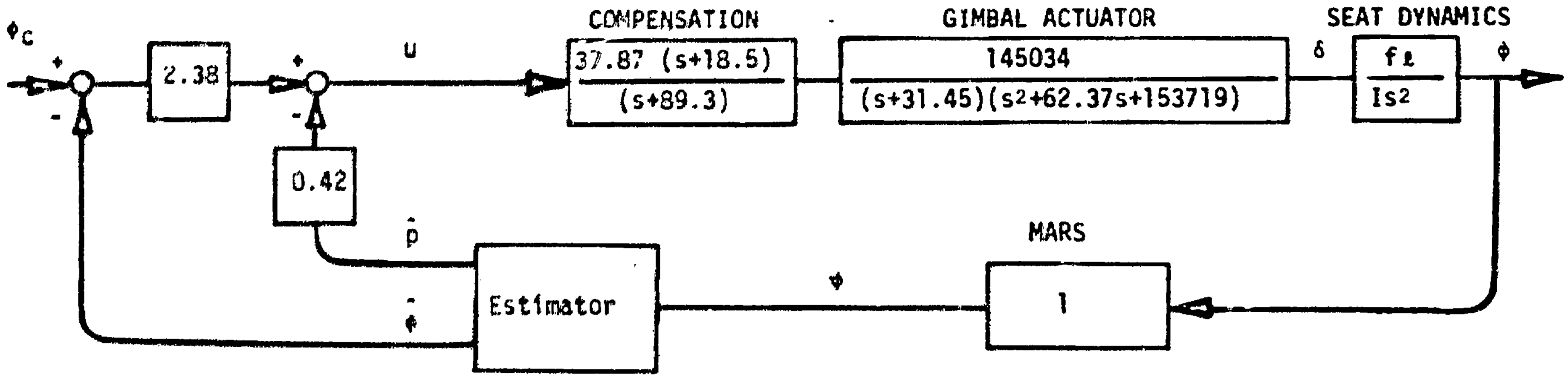

FIGURE 8. BLOCK DIAGRAM OF CONTROL SYSTEM MODEL WITH ESTIMATOR.

where

$$x_c = [\phi \ p \ \delta \ \dot{\delta} \ \ddot{\delta} \ \delta_f]^T \tag{51}$$

This model is discretized using Equations 39 and 40, with F_c and G_c, to obtain

$$\begin{bmatrix} x_e \\ x \end{bmatrix}_{(n+1)} = \begin{bmatrix} \Phi_e & \Phi_e' \\ \Phi' & \Phi \end{bmatrix} \begin{bmatrix} x_e \\ x \end{bmatrix}_{(n)} + \begin{bmatrix} \Gamma_e \\ \Gamma \end{bmatrix} u(n) \tag{52}$$

where

$$x_e = [\phi \ p]^T \tag{53}$$

and

$$x = [\delta \ \dot{\delta} \ \ddot{\delta} \ \delta_f]^T \tag{54}$$

Equations 52-54 represent a discrete model of the open loop control system. Note that the state vector is separated into those variables which will be estimated, x_e', and the remaining state variables x.

Ideal Estimator Model

An ideal estimator is one which is based on an exact model of the system. Although all models of physical systems contain errors, in many practical situations these errors are small so that an "essentially" ideal estimator can be designed.

The ideal estimator for the control system model defined in Equation 52 is

$$\hat{x}_e(n+1) = \Phi_e \hat{x}_e(n) + z(n) + \Gamma_e u(n) + K(z_e - H_e \hat{x}_e)(n) \tag{55}$$

where

$$\begin{bmatrix} z_e \\ z \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & \Phi_e' \end{bmatrix} \begin{bmatrix} x_e \\ x \end{bmatrix} \tag{56}$$

$$H = [1 \ 0] \tag{57}$$

Note that this estimator requires not only a measurement of attitude z_e, but also a measurement of x, in order to calculate z, if Φ_e' is non-zero.

The estimator error equation is obtained by subtracting the first row of Equation 52 from Equation 55, and substituting Equation 56.

$$\tilde{x}_e(n+1) = [\Phi_e - KH]\, \tilde{x}_e(n) \tag{58}$$

where

$$\tilde{x}_e = \hat{x}_e - x_e \tag{59}$$

The closed loop system model is formed by combining the control system model (Equation 52) with the estimator model (Equations 55 and 56) and defining the feedback variable as

$$u = -C\, \hat{x}_e \tag{60}$$

However, insight into the effects of feeding back estimates is provided if the closed loop model is expressed in terms of the control system model state and the estimate error.

$$\begin{bmatrix} x_e \\ x \\ \tilde{x}_e \end{bmatrix}(n+1) = \begin{bmatrix} \Phi_e - \Gamma_e C & \Phi_e' & -\Gamma_e C \\ \Phi' - \Gamma C & \Phi & -\Gamma C \\ 0 & 0 & \Phi_e - KH \end{bmatrix} \begin{bmatrix} x_e \\ x \\ \tilde{x}_e \end{bmatrix}(n) \tag{61}$$

The eigenvalues of the closed loop system are those of the matrix in Equation 61. Inspection of this matrix reveals that, due to the zero elements in the last row, its eigenvalues are those of Equation 58 and those of Equation 52 with

$$u = -C\, x_e \tag{62}$$

Thus the eigenvalues are those of the estimator and those of the control system assuming that the actual states ϕ and p are fed back. Therefore, feeding back estimates from an ideal estimator does not affect the eigenvalues of the control system. This separation of eigenvalues property allows the controller (C) and the estimator (K) to be designed separately, but used together.

Non-Ideal Estimator Model

Implementation of an ideal estimator, Equations 55-57, requires knowledge of z or equivalently, measurements of the elements of x which contribute to z. Unfortunately, all elements of Φ'_e are non-zero and, therefore, all elements of x must be measured if the estimator is to be ideal. Considering Equation 54, the only element of x which can be easily measured is the actuator deflection δ. Therefore, a logical approach is to measure δ and use an estimator of the form of Equation 55, with

$$z = \begin{bmatrix} \Phi'_e(1,1) \\ \Phi'_e(2,1) \end{bmatrix} \delta \tag{63}$$

where the numbers in parentheses indicate matrix elements. The problem with this approach is that the matrix elements in Equation 63 are dependent upon the rocket motor thrust (force) f, which varies as a function of time. The thrust variation is significant and should be accounted for. However, Equation 63 provides little insight into how to accomplish this.

An alternate approach results from realizing that the z term in Equation 63 accounts for the effects of the actuator deflection at the beginning of the sample period on the torque applied to the seat over the entire sample period. Since the instantaneous torque is ℓf, an approximation to the average torque over the sample period is

$$z \approx [\ell f] \frac{f(n+)}{f} \delta \tag{64}$$

where f is the constant thrust used in calculation of Φ'_e and f(n+) is the expected average or mean thrust over the sample period. The bracketed terms in Equations 63 and 64 correspond so that z may be approximated as

$$z \approx \begin{bmatrix} \Phi'_e(1,1) \\ \Phi_e(2,1) \end{bmatrix} \frac{f(n+)}{f} \delta \tag{65}$$

An approximate average or mean thrust f(n+) can be obtained by storing the expected thrust profile in the microprocessor.

The non-ideal estimator consists of Equation 55, the first row of Equation 56, Equations 57 and 65. The error equation for this estimator is

$$\tilde{x}_e(n+1) = [\Phi_e - KH]\,\tilde{x}_e(n) + z(n) - \Phi_e'\,x(n) \tag{66}$$

A comparison of Equations 66 and 58 indicates that the non-ideal error equation is not uncoupled from the system states as is the ideal error equation. As a result, the closed loop system equations are coupled, that is they do not have the zero elements which are in Equation 61, and therefore the closed loop eigenvalues differ from those of the control system and estimator. The amount of this difference depends upon the non-ideal effects. If these effects are small the closed loop eigenvalues may be very close to those of the control system and estimator. However, if these effects are significant, the eigenvalues will be significantly different which may lead to undesirable or even unstable response characteristics. Thus the control system and estimator should not be designed separately. The design of a system with a non-ideal estimator is, therefore, somewhat more complicated than that of a system with an ideal estimator.

There are two additional modeling errors which should be mentioned. The system model, Equation 52, is based on the assumption that the only torque acting on the seat is that due to actuator (thrust vector) deflection. Actually there will also be aerodynamic torques which are functions of the linear and angular velocities of the seat. These torques are not modeled since the torque due to linear velocity is significantly larger than that due to angular velocity, and the linear velocity is not measured or estimated.

ESTIMATOR DESIGN

Since the estimator is non-ideal, its dynamics are coupled with those of the control system and, therefore, the estimator and control system should not be designed separately. However, the scope of this study does not include redesign of the control system, so an estimator will be designed for use with the existing control system. The estimator design consists of selecting the sample rate and gains.

Sample Rate

Generally, several criteria are considered in selecting a sample rate for a digital controller and estimation system. Criteria considered in determining a minimum sample rate are; (1) response to control inputs and initial conditions, (2) response to external disturbances, (3) sensitivity to parameter variations, and (4) roughness of response to control inputs. As the sample rate is lowered, the achievable system bandwidth is reduced due to limitations implied by both the Sampling Theorem and those of approximations associated with discretizing a continuous design. Note that the latter limitation is eliminated by designing in the discrete domain. Criteria considered in determining a maximum sample rate are; (1) computation errors due to finite word length, (2) computation time required for control algorithms, (3) computation time required for functions other than control, and (4) cost and availability of processing hardware.

For the VSS system the major competing criteria are obtaining required bandwidth and response characteristics, implying a high sample rate, and having time for processing the MARS measurements, implying a low sample rate. Thus the sample rate should be the minimum at which the required system bandwidth and response characteristics can be obtained.

Previous experience indicates that sampling at least 10 times the required closed loop bandwidth usually provides "good" response characteristics. The required closed loop bandwidth of the rate loop was earlier defined as 4 Hertz, so that the sample rate should be at least 40 samples/second. However, the bandwidth of the existing rate loop is 6 Hertz, so the associated sample rate should be at least 60 samples/second. Another consideration in selecting the sample rate is that the undesirable effects of using a non-ideal estimator become less significant as the sample rate is increased. These considerations along with the fact that the MARS measurements are updated 80 times/second, lead to the selection of an 80 samples/second sample rate.

Gains

The gains affect the estimate convergence dynamics and the measurement filtering characteristics or, equivalently, the estimator bandwidth. Previously, equations (46-49) were derived for the gains which provide estimator convergence and filtering characteristics similar to the response of a second order system defined by ξ and ω. These equations were derived for an estimator whose outputs are not used for feedback. Subsequent analysis revealed that an ideal estimator can be designed separately from the control system, so these equations are also valid for an ideal estimator. If a non-ideal estimator is used and the non-ideal effects are significant, the closed loop eigenvalues will differ significantly from those corresponding with ξ and ω. One approach in this situation is to initially select ξ and ω corresponding with the desired response characteristics of the estimator, solve Equations 46-49 for the gains, and determine the eigenvalues of the resulting closed loop system. If these eigenvalues are not satisfactory, new values of ξ and ω are selected and the procedure repeated until satisfactory eigenvalues are obtained.

The three estimator characteristics considered in the initial selection of ξ and ω are convergence, bandwidth, and noise attenuation. The initial damping ratio ξ is 0.707, as this value is generally associated with good response characteristics. If the estimates are to converge in 0.05 seconds, which is one-tenth the time that the existing system requires to reach vertical after an inverted ejection, then the minimum natural frequency is

$$\omega = \frac{4}{0.707(0.05)} = 113 \text{ radians/second}$$

or, equivalently, 18 Hertz. A standard practice in estimator design is to place the estimator bandwidth about a factor of 4 higher than the desired closed loop bandwidth, Reference 5. If the 6 Hertz bandwidth of the existing rate loop is to be retained, then the estimator bandwidth (natural frequency) should be 24 Hertz or, equivalently, 150 radians/second. Thus the minimum natural frequency is defined by bandwidth requirements. Since measurement noise is attenuated at frequencies above the estimator bandwidth, the effects of noise are minimized if the estimator bandwidth is minimized. Therefore, the initial natural frequency is 150 radians/second. Simulation of this closed loop model indicates that the response is satisfactory and is similar to that of the continuous model of the existing system.

A detailed description of this estimator design is given in Appendix A.

SPECIFICATIONS FOR ATTITUDE MEASUREMENTS

Specifications for the attitude measurements were previously evaluated assuming the control system and measurements are continuous. These specifications are now evaluated assuming that control system, estimator, and measurements are discrete.

Bandwidth

Previously, a minimum attitude signal bandwidth of 4.2 Hertz was established by considering the closed loop bandwidth requirements of a continuous attitude loop model. Since the closed loop bandwidth requirements are not dependent upon whether the system is continuous or discrete, the minimum signal bandwidth remains 4.2 Hertz. However, a maximum signal bandwidth should now also be specified in order to minimize the effects of aliasing in the discrete system. Aliasing occurs when the measurement contains frequencies above one-half the sample rate, Reference 5. Typically, aliasing effects are minimized by passing the continuous measurement through an analog low pass filter prior to sampling. This prefilter is designed to provide sufficient noise attenuation above half the sample rate so the effects of aliasing on the closed loop system are not significant. Therefore the prefilter bandwidth, or equivalently the maximum bandwidth of the measurement, is dependent upon the order of the prefilter. If a first order filter is used, its bandwidth should be significantly lower than half the sample rate. If higher order filters are used, their bandwidth can be increased as long as they provide adequate attenuation at half the sample rate.

The actual design of a prefilter will be dependent upon the noise characteristics of the continuous measurements and additional filtering and processing performed prior to sampling. Since these noise characteristics are not known and the scope of this study does not include analysis of the measurement filtering and processing in the existing system, a prefilter is not designed. However, once the measurement noise, filtering, and processing are evaluated and analyzed, a prefilter can be designed using the guidelines outlined above.

Bias

The effects of a constant bias in the discrete model are the same as in the continuous model and, therefore, the bias should be kept below 2 degrees for attitudes near vertical, as established previously.

Noise

Attitude measurement noise may cause the closed loop system to exhibit sustained oscillations. Since these oscillations will involve the seat attitude and the TVC system (and may lead to actuator damage), it is important that the magnitude or RMS value of the oscillations be kept within allowable limits. Therefore, the measurement noise RMS should be specified so the system RMS response is acceptable, that is, the RMS attitude error should be a small fraction of the allowable 10 degrees and the RMS actuator (TVC) deflection should be very small, probably less than 0.3 degrees.

The RMS response of the closed loop system can be evaluated using a discrete model of the combined estimator and control system. This model, which is formed by combining the estimator equations (55, the first row of 56, and 63) with the control system equations (52-54 and 62), can be expressed in the form

$$x_a(n+1) = \Phi_{cl}\, x_a(n) + \Gamma_{cl}\, v(n) \tag{67}$$

where Φ_{cl} is the closed loop system matrix, and v is the measurement noise, and

$$x_a = [x_e \quad x \quad \hat{x}_e]^T \tag{68}$$

$$\Gamma_{cl} = [0 \qquad K]^T \tag{69}$$

If the noise v is a Gaussian purely random sequence with constant covariance X, then the resulting mean square state (covariance) can be calculated as follows (Reference 6).

$$X_a(n+1) = \Phi_{cl} X_a(n) \Phi_{cl}^T + \Gamma_{cl} X \Gamma_{cl}^T \quad (70)$$

Equation 70 can be solved for the steady state values of the state variances (and covariances), which are the squares of the RMS responses.

However, the MARS attitude measurements are prefiltered and processed so the noise v is not a Gaussian purely random sequence, but is a Gauss-Markov random sequence. As a result, the prefiltering and processing should be accounted for in the determination of the system RMS response, and Equation 70 should not be used directly. Since the scope of this study does not include analysis and evaluation of the measurement filtering and processing, the system RMS response is not evaluated.

Time Delay

Previously it was established for a continuous attitude loop that time delays less than 0.01 seconds should have minimal effect on the system response characteristics. Since the discrete system contains a "predictive" dynamic estimator, Appendix A, which uses <u>current</u> measurements to calculate (predict) estimates and the feedback control u at the <u>next</u> sample time, there is a time delay of almost one sample period or 0.0125 seconds. This time delay is not a complete sample period as some time is required for calculation of the estimates. Therefore the allowable time delay is the sample period minus the time required for calculating the estimates. Delays longer than this will degrade the system stability and response characteristics.

SUMMARY

The existing Vertical Seeking Seat (VSS) control system uses angular velocity and attitude measurements provided by rate gyroscopes and the Micrad Attitude Reference System (MARS), respectively. A study has been conducted to determine the feasibility of estimating angular velocity from the MARS attitude measurements, and using these estimates in place of the measurements currently provided by rate gyros. This estimation is to be performed in the on-board microprocessor.

The design and performance of an angular velocity estimator will depend upon the sample or iteration rate of the microprocessors, the allowable errors and bandwidth requirements of the velocity estimate, and the errors and bandwidth characteristics of the attitude measurement, all of which are dependent upon the required bandwidth, stability, and response characteristics of the complete VSS system. Therefore, the first phase of this study included a literature search and technical discussion with appropriate personnel at the Naval Weapons Center to determine the design goals and preliminary specifications for the control system and sensor bandwidths. It was learned that the existing system was designed to optimize performance using the available sensing and actuating hardware, and not to meet quantitative bandwidth requirements. Since the control system and sensor bandwidth requirements are not known, but are necessary for determining the feasibility of using an angular velocity estimator, they are evaluated.

Several important assumptions are made in order to facilitate the analyses and evaluations performed in this study. One assumption is that analyses and evaluations of the roll axis control system are applicable to the pitch axis control system, as these systems are independent and very similar. A second assumption is that the MARS provides measurements of roll (and pitch) attitude relative to the vertical. Another assumption is that the control gains, actuator, and actuator compensation are those of the existing system model, since the scope of this study does not include redesigning the control system. The analyses and evaluation results obtained using all these assumptions are now summarized. Methods of determining the effects of these assumptions are presented later, along with other recommendations.

Since the MARS outputs are sampled and the estimator is to be implemented in the microprocessor, the control system is digital and specifications should be evaluated using a discrete time analysis. However, preliminary evaluations are performed using a continuous time analysis rather than a discrete time analysis, as the former is generally less complicated and provides more physical insight than the latter. Here it is assumed that the angular velocity and attitude measurements are outputs of continuous sensors. These analyses indicate that system performance requirements can be met with an angular velocity feedback loop bandwidth of 4 Hertz and an attitude feedback loop bandwidth of 1.4 Hertz. The feedback signal bandwidth should be about three times that of the corresponding loop, so that the angular velocity and attitude measurement bandwidths should be at least 12 Hertz and 4.2 Hertz, respectively. The effects of measurement bias and time delay are also evaluated. The effects of measurement bias are similar to those of aerodynamic moments in that they both contribute to the steady state attitude error of the seat. Since the aerodynamic moments may be large, resulting in large attitude errors, the errors due to bias should be a small fraction of the allowable 10 degrees of attitude error. If the errors resulting from angular velocity and attitude biases are each to be less than 2 degrees, then the maximum biases are 11.3 degrees/second and 2 degrees, respectively. The effect of a time delay between the taking of a measurement and the use of that measurement for application of control is to reduce the system stability margins. This reduction is minimal if the time delay is very small relative to the characteristic time (dominant time constant) of the closed loop system. If the angular velocity and attitude loop bandwidths are 4 Hertz and 1.4 Hertz, respectively, then associated measurement time delays less than 0.004 seconds and 0.01 second, respectively, should have minimal effect on the system stability and response characteristics.

Preliminary insight into the effects of sampling on the bandwidth requirements is obtained using a model with continuous angular velocity and attitude sensors whose outputs are sampled. A discrete time analysis of this model indicates that for sample rates below 100 samples/second, the angular velocity signal bandwidth must be at least 40 Hertz, which is approximately 3 times that required for the completely continuous model. However, the actual bandwidth requirements are evaluated later using a discrete time model of the control system and angular velocity estimator.

Three methods of estimating angular velocity are presented and analyzed. The first method is a kinematic estimator which is based only on kinematics. The second method is a dynamic estimator which is based on the system kinematics and dynamics. This estimator provides filtered estimates of angular velocity and attitude. The third method is a reduced order dynamic estimator which provides only angular velocity estimates. Analysis of these methods indicates that the dynamic estimator has several advantages in that it attenuates high frequency measurement noise, provides good estimates at high frequencies, and also provides a filtered estimate of attitude which can be used for feedback in place of the actual (noisy) measurement. As a result, this dynamic estimator is used in the discrete time analysis.

The dynamic estimator is based on a second order planar model of the seat rotational dynamics. If this model is "essentially" exact the estimator is ideal and the separation property allows the estimator and control system to be designed separately, but used together. However, the model used in this study is inexact as it neglects aerodynamic moments and cross coupling terms. As a result, the estimator is non-ideal and, therefore, the estimator and control system should not be designed separately. Since the scope of this study does not include redesign of the control system, an estimator is designed for use with the existing control system.

Estimator design and the evaluation of attitude measurement specifications are performed using a discrete time model of the combined estimator and control system. Design of the estimator consists of selecting the sample rate and gains. Sample rate selection results from a trade-off between obtaining required bandwidth and response characteristics, implying a high sample rate, and having time for processing the MARS measurements, implying a low sample rate. The sample rate should be at least ten times the required closed loop bandwidth. Since the required bandwidth of the rate loop is 4 Hertz, the sample rate should be at least 40 samples/second. However, the bandwidth of the existing rate loop is 6 Hertz, so the associated sample rate should be at least 60 samples/second. Since the MARS measurements are updated 80 times/second, a sample rate of 80 samples/second is selected. Estimator gain selection results from a trade-off between obtaining fast estimate convergence, implying high gains and bandwidths, and having adequate attentuation of measurement noise, implying low gains and bandwidths. The estimator bandwidth should be about four times the required closed loop bandwidth. Since the required rate loop bandwidth is 4 Hertz, the estimator should have a 16 Hertz bandwidth. However, the existing rate loop bandwidth is 6 Hertz, so an associated estimator should have a 24 Hertz bandwidth. The gains are select: ' to provide this bandwidth. A digital computer simulation of the resulting closed loop model indicates that the response is satisfactory and is similar to that of the continuous model of the existing system. A detailed description of the estimator is given in Appendix A.

The attitude measurement specifications are re-evaluated using the discrete time model of the combined estimator and control system. The minimum bandwidth is the same as established using a continuous model, that is, 4.2 Hertz. However, a maximum bandwidth must now be specified in order to minimize the effects of aliasing in the discrete system. Typically, aliasing effects are minimized by passing the continuous measurement through a low pass analog prefilter prior to sampling. The design of a prefilter, which determines the maximum bandwidth, is dependent upon the noise characteristics of the continuous measurements and additional filtering and processing performed prior to sampling. Since these noise characteristics are not known and the scope of this study does not include analysis of the measurement filtering and processing, a specific prefilter s not designed, but a design approach is discussed. The steady state effects of a constant bias are the same as for the continuous model and, therefore, the bias should still be kept below 2 degrees for attitudes near vertical. Since measurement noise may cause sustained oscillations of the actuator (TVC) and seat attitude, it is important that the noise RMS be specified so the RMS actuator deflection and attitude are within allowable limits. A complete evaluation of the system response to noise should include effects of measurement filtering and processing. Since evaluation of these effects is beyond the scope of this study, the system response to measurement noise is not determined. The allowable time delay is one sample period (0.0125 seconds) minus the time required for calculating the estimates. Delays longer than this will degrade the stability and response characteristics of the system.

RECOMMENDATIONS

Several recommendations for further evaluation and analysis are made based on the results and assumptions of this study. Recommendations are presented for further evaluation of the estimator, estimator modifications and alternate estimators, and an integrated discrete design of the entire system.

The estimator should be further evaluated using the NWC six degree of freedom (DOF) VSS digital computer simulation. Results from this simulation will determine the effects of aerodynamic torques, cross coupling between the pitch and roll axes, coupling due to yaw angular velocity, and MARS errors. It should be possible to determine each of these effects individually, by proper selection of the simulation initial conditions. If one or more of these effects are significant, it may be necessary to modify the estimator.

There are several estimator modifications which should be evaluated. One modification involves alternate methods of approximating the applied control torque over the sample period. The method described in this study involves using a measurement of actuator deflection along with a stored profile of the expected thrust as a function of time. Alternate methods include using previous and current deflection measurements to predict deflections over the next sample period, using the commanded deflection instead of the measured deflection, and measuring thrust using one or more accelerometers. Another modification is to estimate, and thereby compensate for, aerodynamic torques. Other modifications include the addition of cross coupling terms between the roll and pitch axis estimators, and possibly the addition of yaw axis coupling terms.

There are also alternatives to using or modifying the original estimator. One is to use a full dynamic estimator which includes models of the actuator compensation and actuator. This may eliminate the need for measuring the actuator deflection. A second alternative is to use a "current dynamic estimator" in place of the "predictive dynamic estimator," Reference 5. This may reduce the sensitivity to errors in measuring the applied control torque, but will reduce the time available for processing the MARS measurements. A third alternative is to estimate angular velocity and linear and angular acceleration using outputs of linear accelerometers, Reference 7. It may be possible to use the acceleration estimates to advantage in the VSS system. Another alternative is to design an estimator which uses MARS and accelerometer measurements.

A discrete time estimator has been designed for use with an existing continuous time control system, which consists of feedback gains and actuator compensation, and MARS, which includes measurement filters and processing. Since the estimator design results from an exact discrete time analysis of a combined estimator and control system model, it is dependent upon the independently designed control system. The performance of this system should, therefore, not be as good as that of a system in which the estimator, control system, and measurement filter-processing are all designed considering that they will be used together. Thus, it is recommended that such a design be performed. This design should be based on an exact discrete time analysis of the entire system, and should include evaluation of the alternate estimator designs discussed above.

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APPENDIX A

DISCRETE ESTIMATOR EQUATIONS

Given the continuous time model

$$\begin{bmatrix} \dot{\phi} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \phi \\ p \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{f\ell}{I} \end{bmatrix} \delta \tag{A1}$$

with

$f = 3000$ lbf
$\ell = 1.0$ ft
$I = 12.6$ slug-ft

the corresponding discrete time estimator is

$$\begin{bmatrix} \hat{\phi} \\ \hat{p} \end{bmatrix}_{(n+1)} = \begin{bmatrix} -0.874 & 0.0119 \\ -76.3 & 0.822 \end{bmatrix} \begin{bmatrix} \hat{\phi} \\ \hat{p} \end{bmatrix}_{(n)} + \begin{bmatrix} 0.0174 \\ 2.56 \end{bmatrix} \delta(n) + \begin{bmatrix} 1.87 \\ 75.3 \end{bmatrix} \phi(n) \tag{A2}$$

where ϕ is the measurement. The sample rate used in obtaining Equation A2 is 80 samples/second.