AD-A160 616 ASSESSING SYSTEM RELIABILITY USING CENSORING METHODOLOGY(U) STANFORD UNIV CA DEPT OF STATISTICS H DOSS ET AL. 12 SEP 85 TR-364 N00014-76-C-0475 1/1

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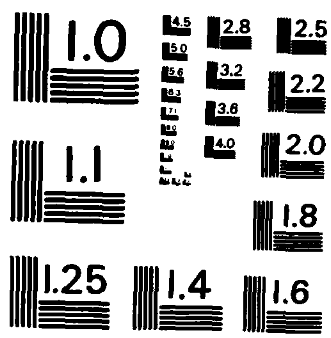

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ASSESSING SYSTEM RELIABILITY USING CENSORING METHODOLOGY

BY

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TECHNICAL REPORT NO. 364
SEPTEMBER 12, 1985

Prepared Under Contract
N00014-76-C-0475 (NR-042-267)
For the Office of Naval Research

Herbert Solomon, Project Director

DEPARTMENT OF STATISTICS
STANFORD UNIVERSITY
STANFORD, CALIFORNIA

1. INTRODUCTION

The following problem is of practical and theoretical importance in reliability.

Under study is a coherent structure consisting of n independent components. The structure is observed continuously until it fails. For components failing before or at system failure time, we observe *complete* lifelengths; for components still functioning at system failure time we observe *censored* lifelengths. For the system we observe system lifelength. From a sample of m such structures we wish to estimate the system lifelength distribution.

As far as we know, this problem has not been solved or even treated in the literature.

In this paper we propose an estimator of the system lifelength distribution. As a by-product, we also obtain estimates of the mean and quantiles of the system lifelength and of all the corresponding quantities for component lifelengths. In Section 2 we give several asymptotic results concerning the estimators. In Section 3 we present and discuss two bootstrap schemes that are used in small samples to assess the variability of our estimates and to construct confidence intervals.

NOTATION

We assume that the system and each component are in either a functioning state or a failed state. Thus, we can define the lifelengths of the components and of the system. Let

X_{ij} = the lifelength of component j in system i;

T_i = the lifelength of system i;

$Z_{ij} = \min(X_{ij}, T_i)$;

$\delta_{ij} = I(X_{ij} \le T_i)$;

F_j be the distribution function of X_{ij}, and the component reliability function be $S_j(t) = 1 - F_j(t)$;

F_s be the distribution function of T_i, and the system reliability function be $S_s(t) = 1 - F_s(t)$;

H_j be the distribution function of Z_{ij}, and $\bar{H}_j(t)$ denote $1 - H_j(t)$.

In the definitions above the letter i indexes systems and the letter j indexes components. Throughout the paper i ranges from 1 to m, and j from 1 to n. The random variables X_{ij} are not observed. We observe only the Z_{ij}'s and δ_{ij}'s.

It is helpful to keep in mind a concrete example. Figure 1 below shows diagrammatically a simple structure of 3 components, arranged neither in series nor in parallel. We carry this example throughout the paper. In the example (the subscript i indexing systems has been suppressed) $T = X_1 \wedge (X_2 \vee X_3)$, where $x \wedge y = \min(x, y)$ and $x \vee y = \max(x, y)$.

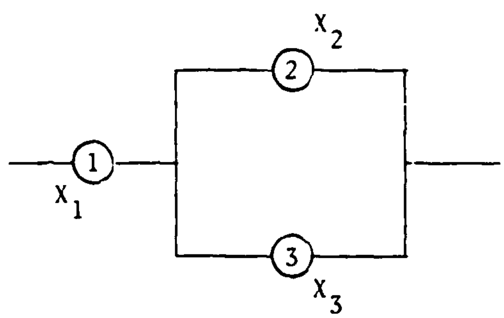


Figure 1 X_i = lifelength of component i, i = 1, 2, 3.

We note that F_s can be estimated naively by the empirical distribution function of the T_i's:

(1) $$\hat{F}_s^{emp}(t) = \frac{1}{m} \sum_{i=1}^{m} I(T_i \le t) \quad \text{for } t \ge 0.$$

However, it is clear that this estimator does not use all the information available in the sample.

THE ESTIMATOR

For any coherent structure of n independent components there corresponds a function $h: [0, 1]^n \to [0, 1]$, such that

$$S_s(t) = h(S_1(t), \ldots, S_n(t)) \quad \text{for } t \geq 0. \tag{2}$$

See Chapter 2 of Barlow and Proschan (1975) for details. In the example given by Figure 1, we have

$$S_s(t) = S_1(t)[1 - (1 - S_2(t))(1 - S_3(t))],$$

so that

$$h(u_1, u_2, u_3) = u_1[1 - (1 - u_2)(1 - u_3)] \quad \text{for } u_1, u_2, u_3 \in [0, 1].$$

To construct our estimator, we first estimate the $S_j(t)$'s by the Kaplan-Meier estimates

$$\hat{S}_j(t) = \prod_{i:\, Z_{(i)j} \leq t} \left(\frac{m - i}{m - i + 1}\right)^{\delta_{(i)j}} \quad \text{for } t \geq 0. \tag{3}$$

($Z_{(1)j} < \ldots < Z_{(m)j}$ denote the ordered values of $Z_{1j}, \ldots, Z_{mj}$, and $\delta_{(1)j}, \ldots, \delta_{(m)j}$ are the δ's corresponding to $Z_{(1)j}, \ldots, Z_{(m)j}$, respectively) and then substitute $\hat{S}_j(t)$ for $S_j(t)$ in (2). The estimator is defined by

$$\hat{S}_s(t) = \begin{cases} h(\hat{S}_1(t), \ldots, \hat{S}_n(t)) & \text{if } t < T_{(m)} \\ 0 & \text{if } t \geq T_{(m)}. \end{cases} \tag{4}$$

Here, $T_{(m)} = \max_i T_i$.

We have shown that the estimate $\hat{F}_s (= 1 - \hat{S}_s)$ is the nonparametric mle of F_s. We do not present a formal proof here, but instead offer the following heuristic argument. It is well known (Kaplan and Meier, 1958; Johansen, 1978) that the Kaplan-Meier estimate is the nonparametric mle of a distribution function when the data is randomly right censored. An extension of this result is that $(\hat{S}_1, \ldots, \hat{S}_n)$ is the nonparametric mle of $(S_1, \ldots, S_n)$; the invariance principle for mle's implies that $h(\hat{S}_1 \ldots, \hat{S}_n)$ is the nonparametric mle of $h(S_1, \ldots, S_n)$.

Equation (4) provides the basis for the estimation of functionals of F_s such as quantiles and the mean. For example, we can estimate $F_s^{-1}(p)$ by $\hat{F}_s^{-1}(p)$, for $p \in (0, 1)$. Similarly we can estimate $\mu_s = \int_0^\infty t dF_s(t)$ by $\hat{\mu}_s = \int_0^\infty t d\hat{F}_s(t)$.

2. ASYMPTOTIC RESULTS

In this section we describe the limiting behavior of the estimates of the system and component reliability functions. As an application, we show how our results can be used to improve the system reliability. The proofs of the theorems stated below will appear elsewhere.

We begin with an important result concerning the simultaneous estimation of the component reliability functions. For $T > 0$, $D[0, T]$ denotes the space of real valued functions on $[0, T]$ that are right continuous and have left limits, with the Skorohod metric topology (see Chapter 3 of Billingsley, 1968), and $D^n[0, T]$ denotes the product metric space.

THEOREM A. Suppose $F_1, \ldots, F_n$ are continuous, and let T be such that $F_j(T) < 1$ for $j = 1, \ldots, n$. Then as $m \to \infty$

$$m^{\frac{1}{2}}(\hat{S}_1 - S_1, \hat{S}_2 - S_2, \ldots, \hat{S}_n - S_n) \to (W_1, W_2, \ldots, W_n)$$

in $D^n[0, T]$, where $W_1, \ldots, W_n$ are *independent* mean 0 Gaussian processes, with covariance structure given by

$$\mathrm{Cov}(W_j(t_1),\ W_j(t_2)) = \tag{5}$$
$$S_j(t_1)\,S_j(t_2)\int_0^{t_1}\frac{dF_j(u)}{\bar{H}_j(u)S_j(u)} \quad \text{for } 0 \le t_1 \le t_2 \le T.$$

The weak convergence of the Kaplan-Meier estimator to a Gaussian process has been well-established in the literature (Breslow and Crowley, 1974; Gill, 1983) under the assumption that the lifelengths and the censoring variables are independent. In our situation the component lifelengths are censored by the system lifelength, and the independence condition is clearly violated. We can, however, redefine the censoring variables to bypass this difficulty. This is easiest to explain in terms of the example given by Figure 1. Consider Component 1. Clearly, X_1 is censored by $Y_1 = X_2 \vee X_3$, which *is* independent of X_1. Similarly, X_2 is censored by $Y_2 = X_1$, and X_3 by $Y_3 = X_1$. The construction is general: for an arbitrary system, X_j is censored by Y_j = lifelength of system if X_j is replaced by ∞. *One can check* that

(i) X_j and Y_j are independent,

(ii) $\min(X_j,\ T) = \min(X_j,\ Y_j)$.

Thus, the known weak convergence results for the Kaplan-Meier estimate apply to the individual $\hat{S}_j$'s.

For fixed m, the $\hat{S}_j$'s are in general dependent. This is easily seen in the example given by Figure 1, in which Components 2 and 3 are both censored by Component 1. For complicated systems the dependence may be complex. Thus, the novel results given by Theorem A are first, the joint asymptotic convergence of the $\hat{S}_j$'s and second, their asymptotic independence.

The asymptotic independence of the $\hat{S}_j$'s is interesting. Before proving Theorem A we conjectured this result by considering the two special cases of parallel and series systems. These are often viewed as extreme cases in reliability theory, and an analysis of these cases may shed light on the dependence structure between the $\hat{S}_j$'s. For parallel systems there is no censoring at all; the $\hat{F}_j$'s are the usual empirical cdf's and are trivially independent for every m. For a series system (say of just two components) we have

$$(6) \qquad m \operatorname{Cov}(\hat{F}_1(t), \hat{F}_2(t)) = m\, E\hat{F}_1(t)\hat{F}_2(t) - m\, E\hat{F}_1(t)E\hat{F}_2(t).$$

Since $\hat{F}_1(t)\hat{F}_2(t) = \hat{F}_s^{emp}(t)$ (see (1)), the first term on the right side of (6) is $m\, F_s(t)$. Consider now the second term on the right side of (6). From Efron (1967) we obtain the bounds

$$(7) \qquad 0 \le F_j(t) - E\hat{F}_j(t) \le S_j(t)e^{-m\bar{H}_j(t)} \quad \text{for } j = 1, 2.$$

(Actually, Efron, 1967, has the inequalities reversed. This is because he uses the version of the Kaplan-Meier estimate that is defined to be 0 past the last observation, whether or not it is censored.) Combining (6), (7) and the fact that $F_1(t)F_2(t) = F_s(t)$, we obtain for series systems that

$$(8) \qquad m \operatorname{Cov}(\hat{F}_1(t), \hat{F}_2(t)) \to 0 \quad \text{exponentially fast as } m \to \infty.$$

This proves that $\hat{S}_1(t)$ and $\hat{S}_2(t)$ are asymptotically uncorrelated and hence asymptotically independent, assuming that joint asymptotic normality has been established. Since intuitively the series structures give rise to maximum possible dependence among the $\hat{S}_j$'s, we were led to conjecture the asymptotic independence of the $\hat{S}_j$'s for general structures. Our proof of Theorem A does indeed give the result (8) for arbitrary systems.

To prove Theorem A we show that for each m, the vector of processes

$$\left\{ m^{\frac{1}{2}} \left(\frac{\hat{S}_1(t) - S_1(t)}{S_1(t)}, \ldots, \frac{\hat{S}_n(t) - S_n(t)}{S_n(t)} \right); \ t \in [0, T] \right\}$$

is approximately (as $m \to \infty$) a martingale with respect to the σ-field generated by all uncensored component deaths observable bv time t. We then apply an appropriate martingale central limit theorem, via the Cramèr-Wold device, to deduce the result.

The next theorem gives the asymptotic normality of our estimator of system reliability.

THEOREM B. Suppose $F_1, F_2, \ldots, F_n$ are continuous, and suppose T is such that $F_j(T) < 1$ for $j = 1, 2, \ldots, n$. Then as $m \to \infty$

$$m^{\frac{1}{2}}(\hat{S}_s - S_s) \to W \quad \text{weakly in } D[0, T],$$

where W is a mean 0 Gaussian process with covariance structure given by

$$\mathrm{Cov}(W(t_1), W(t_2)) = \sum_{j=1}^{n} \left(\frac{\partial h}{\partial u_j}(u_1, \ldots, u_n) \Bigg|_{\substack{(u_1, \ldots, u_n) = \\ (S_1(t_1), \ldots, S_n(t_1))}} \right)$$

$$\left(\frac{\partial h}{\partial u_j}(u_1, \ldots, u_n) \Bigg|_{\substack{(u_1, \ldots, u_n) = \\ (S_1(t_2), \ldots, S_n(t_2))}} \right)$$

$$S_j(t_1)S_j(t_2) \int_0^{t_1} \frac{dF_j(u)}{\bar{H}_j(u)S_j(u)} \quad \text{for } 0 \le t_1 \le t_2 \le T.$$

For fixed $t \in [0, T]$, the asymptotic normality of $m^{\frac{1}{2}}(\hat{S}_s(t) - S_s(t))$ follows from Theorem A and an application of the delta method. Thus, the proof of Theorem B consists of a straightforward generalization of this for the process $\{m^{\frac{1}{2}}(\hat{S}_s(t) - S_s(t));\ t \in [0, T]\}$.

Greenwood's formula can be used to estimate the variance of $\hat{S}_j(t)$.

Since this estimate is well known to be consistent, it is clear that one can consistently estimate the asymptotic variance of $\hat{S}_s(t)$ given by Theorem B. This enables the construction of asymptotic confidence intervals for $S_s(t)$.

We close this section with an applicati~n of Theorem A to the joint estimation of the reliability importance of components. The reliability importance $I_j(t)$ of component j at time t is defined by

$$I_j(t) = \frac{\partial}{\partial u_j} h(u_1, \ldots, u_n)\Big|_{\substack{(u_1, \ldots, u_n) = \\ (S_1(t), \ldots, S_n(t))}} .$$

Let $\varepsilon_1, \ldots, \varepsilon_n$ be small numbers. Note that

$$h(S_1(t) + \varepsilon_1, \ldots, S_n(t) + \varepsilon_n) - h(S_1(t), \ldots, S_n(t)) \doteq \sum_{j=1}^{n} \varepsilon_j I_j(t).$$

Thus, the reliability importance of components may be used to evaluate the effect of an improvement in component reliability on system reliability, and can therefore be very useful in system analysis in determining those components on which additional research can be most profitably expended. For details, see pp. 26-28 of Barlow and Proschan (1975), and the review by Natvig (1984).

Notice that

$$I_j(t) = h_j(S_1(t), \ldots, S_n(t)),$$

where h_j: $[0, 1]^n \to [0, 1]$ is some smooth function. Thus, to estimate $I_j(t)$, a natural choice is

$$\hat{I}_j(t) = h_j(\hat{S}_1(t), \ldots, \hat{S}_n(t)).$$

THEOREM C. Suppose $F_1, \ldots, F_n$ are continuous and $T > 0$ is such that $\max_{1 \le j \le n} F_j(T) < 1$. Then as $m \to \infty$

$$m^{\frac{1}{2}}(\hat{I}_1 - I_1, \ldots, \hat{I}_n - I_n) \to (Y_1, \ldots, Y_n)$$

weakly on $D^n[0, T]$, where $(Y_1, \ldots, Y_n)$ is a vector of mean 0 Gaussian processes whose covariance structure is given by

$$\mathrm{Cov}(Y_{j_1}(t_1), Y_{j_2}(t_2)) =$$

$$\sum_{\substack{k=1 \\ k \ne j_1, j_2}}^{n} \left(\frac{\partial^2 h}{\partial u_{j_1} \partial u_k} \Bigg|_{\substack{(u_1, \ldots, u_n) = \\ (S_1(t_1), \ldots, S_n(t_1))}} \right) \left(\frac{\partial^2 h}{\partial u_{j_2} \partial u_k} \Bigg|_{\substack{(u_1, \ldots, u_n) = \\ (S_1(t_2), \ldots, S_n(t_2))}} \right)$$

$$S_k(t_1) S_k(t_2) \int_0^{t_1} \frac{dF_k(u)}{\bar{H}_k(u) S_k(u)} ,$$

for $0 \le t_1 \le t_2 \le T$ and $j_1, j_2 = 1, \ldots, n$.

For fixed t, the asymptotic normality of $m^{\frac{1}{2}}(\hat{I}_1(t) - I_1(t), \ldots, \hat{I}_n(t) - I_n(t))$ follows from Theorem A and an application of the delta method. The theorem follows from an easy extension of this argument to the process $\{m^{\frac{1}{2}}(\hat{I}_1(t) - I_1(t), \ldots, \hat{I}_n(t) - I_n(t));\ t \in [0, T]\}$.

3. BOOTSTRAPPING SCHEMES

Although the estimates $\hat{F}_s(t)$, $\hat{F}_s^{-1}(p)$, and $\hat{\mu}_s$ are easy to describe and their asymptotic distributions relatively simple, their finite sample distributions (particularly for $\hat{F}_s^{-1}(p)$) are completely intractable. In practice, we will need to supplement any estimate with an estimate of its standard error, and in fact an estimate of its whole distribution. In this section we discuss bootstrapping schemes for estimating the distribution

<u>METHOD 2</u>. In this estimate of the model P, component j has lifelength distribution $\hat{F}_j$, for $j = 1, \ldots, n$; so we construct artificial systems by resampling component lifelengths from the Kaplan-Meier estimates $\hat{F}_j$.

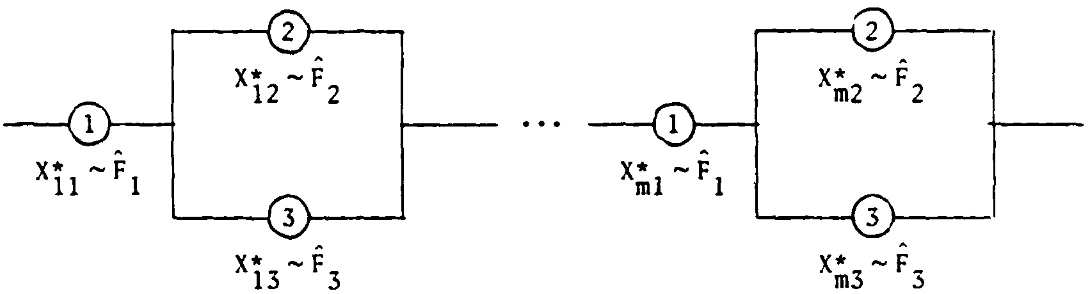
A formal description of the algorithm is

(1) Generate $X_j^* \sim \hat{F}_j$ for $j = 1, 2, \ldots, n$ independently. This gives one artificial system which we denote by Sys^{*1}.

(2) Repeat Step 1 independently m times giving one sample of artificially constructed systems, denoted $\mathrm{Sys}^{*1}, \ldots, \mathrm{Sys}^{*m}$.

(3) Compute η^* based on the m systems in Step 2.

(4) Repeat Steps 1, 2 and 3 independently B times, obtaining $\eta^{*1}, \eta^{*2}, \ldots, \eta^{*B}$. Figure 2 schematically describes this method.



Sys^{*1} has lifelength T_1^* ... Sys^{*m} has lifelength T_m^*

<u>Figure 2</u>. In Method 2, the estimate η^* is based on the data $Z_{ij}^* = \min(X_{ij}^*, T_i^*)$, $\delta_{ij}^* = I(X_{ij}^* \le T_i^*)$.

Note that our version of $\hat{S}_j$ given by (3) is strictly positive for $t \ge \max_i Z_{ij}$ if $\max_i Z_{ij}$ corresponds to a censored observation. In this case, we view $\hat{F}_j$ as giving mass to the point ∞; thus X_j^* is equal to ∞ with positive probability.

In applying either method, the η^{*b}'s are used in the usual way to make inference about G. Let G_m^* equal the empirical cdf of the η^{*b}'s. The standard deviation of G is estimated by

(9) $$\hat{\sigma} = \{\frac{1}{B-1} \sum_{b=1}^{B} (\eta^{*b} - \eta^{*\cdot})^2\}^{\frac{1}{2}}$$

where $\eta^{*\cdot} = \frac{1}{B} \sum_{b=1}^{B} \eta^{*b}$. The standard deviation is not a particularly meaningful quantity if G is asymmetric or far from normal, which may occur when the sample size is small. In that case the intervals $\eta \pm z_\alpha \hat{\sigma}$, where z_α is the $100 \cdot \alpha$ percentile point of a standard normal variate and $\hat{\sigma}$ is given by (9) are essentially useless. As alternatives, we can use the percentile intervals of level α ($\alpha \in (0, 1)$) given by

(10) $$[G_m^{*-1}(\alpha), G_m^{*-1}(1-\alpha)],$$

and the more elaborate bias corrected and BC_a intervals (see Efron 1984a, b), all based on G_m^*.

In comparing the two methods of bootstrapping, it is helpful to make an analogv with the regression model in which we observe random pairs (Y_i, X_i), $i = 1, \ldots, m$, where

(11) $$Y_i = X_i'\beta + \varepsilon_i,$$

X_i is a p dimensional vector of covariates, β is a p dimensional vector of unknown coefficients, to be estimated, and ε_i are iid from an unknown distribution F on R^1, centered at 0 in some sense. Let $\hat{\beta}$ be an estimate of β, whose variability we wish to assess.

One way of bootstrapping is to resample the pairs (Y_i, X_i), and construct an artificial value β^* based on the resampled pairs. This corresponds to our Method 1.

Another way to bootstrap is to resample the residuals

$$\hat{\varepsilon}_i = Y_i - X_i'\hat{\beta}.$$

Let F_m be the empirical cdf of $\hat{\varepsilon}_1, \ldots, \hat{\varepsilon}_m$, and let $\varepsilon_1^*, \ldots, \varepsilon_m^*$ be iid $\sim F_m$. We construct $Y_i^* = X_i'\hat{\beta} + \varepsilon_i^*$, $i = 1, \ldots, m$, from which we can obtain an artificial value $\hat{\beta}^*$. This corresponds to our Method 2.

Method 2 makes more use of the structure in our model, in particular the assumption of independence of the component lifelengths. We view this both as a strength and a weakness.

We expect Method 2 to be "preferable", but we have not carried out any studies to assess the two methods. We hope to pursue this problem and report the results in a future paper.

Efron (1981) has discussed use of the bootstrap on censored data. He considered the standard setup for randomly right censored data, which corresponds to two components arranged in series in our model. He showed that for this case, the two methods of bootstrapping are identical. The two methods are not always the same in our situation, as can easily be seen by considering a parallel structure of two components.

It would be of interest to study the asymptotic behavior of the bootstrap in our problem. This would determine whether confidence intervals based on the bootstrap are asymptotically valid. To carry out such a study, it would be necessary to select some relatively simple estimates; $\hat{F}_s(t)$ where t is fixed is a prime choice. Let $\hat{F}_s^*(t)$ denote a bootstrap replication of $\hat{F}_s(t)$, obtained by one of the two methods. Suppose we can show that for almost every sample sequence $\{(Z_{ij}, \delta_{ij});\ j = 1, \ldots, n,\ i = 1, 2, 3, \ldots\}$,

$$m^{\frac{1}{2}}(\hat{F}_s^*(t) - \hat{F}_s(t)) \text{ and } m^{\frac{1}{2}}(\hat{F}_s(t) - F(t)) \tag{12}$$

have the same asymptotic distribution.

It would then follow that the simple percentile intervals (10) are asymptotically correct to the first order, in the terminology of Efron (1984b).

Hjort (1985) has shown that in the standard setup for randomly right censored data, the bootstrap approximation is asymptotically correct to the first order. More specifically, suppose that X_i are iid $\sim F$, Y_i are iid $\sim G$, independently of the X's, and that the data is $(X_i \wedge Y_i, I(X_i \le Y_i))$, $i = 1, \ldots, m$. Let $\hat{F}$ be the Kaplan-Meier estimator, and let $\hat{F}^*$ be the empirical cdf of a random sample of size m from $\hat{F}$. Hjort (1985) showed that under certain regularity conditions, for fixed t, with probability one,

(13) $m^{\frac{1}{2}}(\hat{F}^*(t) - \hat{F}(t))$ and $m^{\frac{1}{2}}(\hat{F}(t) - F(t))$ have the same asymptotic distribution.

This result offers hope that (12) is true under some reasonable set of assumptions.

Ackowledgements.

We thank Ian McKeague and Rob Tibshirani for helpful comments.

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4. TITLE (and Subtitle): Assessing System Reliability Using Censoring Methodology	5. TYPE OF REPORT & PERIOD COVERED: TECHNICAL REPORT
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): Hani Doss, Steven Freitag, and Frank Proschan	8. CONTRACT OR GRANT NUMBER(s): N00014-76-C-0475
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Department of Statistics, Stanford University, Stanford, CA 94305	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: NR-042-267
11. CONTROLLING OFFICE NAME AND ADDRESS: Office of Naval Research, Statistics & Probability Program Code 411SP	12. REPORT DATE: September 12, 1985
	13. NUMBER OF PAGES: 18
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)	15. SECURITY CLASS. (of this report): UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

APPROVED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

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Also issued as Florida State University Statistic Report M711 supported by Air Force Office of Scientific Research.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Censored Data, Reliability, Bootsrap Procedures.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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TECHNICAL REPORT NO. 364

20. ABSTRACT

Suppose that we have a sample of iid systems each consisting of independent components. Let F denote the distribution of system lifelength. Each system is continuously observed until it fails. For every component in each system, either a failure time or a censoring time is recorded. A failure time is recorded if the component fails before or at the time of system failure; otherwise a censoring time is recorded. We introduce a method for finding estimates of F(t) based on the mutual censorship of the component lifelengths inherent in this model. We present limit theorems that enable the construction of confidence intervals in large samples. For small samples, we describe and discuss bootstrap schemes that can be used to implement the method.

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