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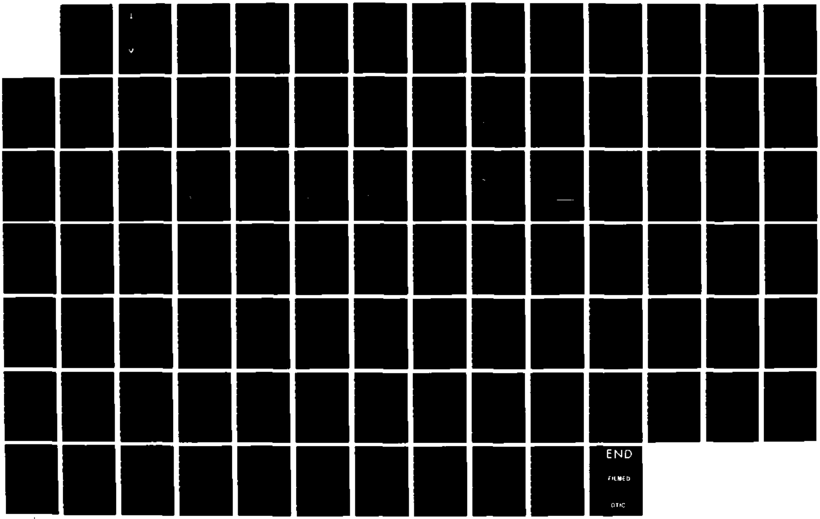

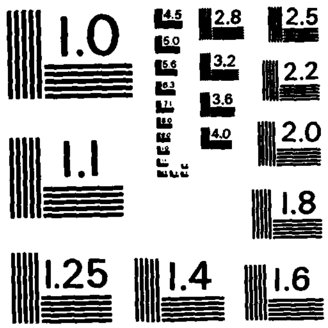
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TECHNICAL REPORT RG-85-10

FURTHER RESULTS FROM APPLICATION OF THE ISOBASIS DESIGN TECHNIQUE OF DISTURBANCE MINIMIZING CONTROL TO A LINEAR, TIME-INVARIANT SECOND-ORDER STATE SET-POINT REGULATOR WITH TIME-VARYING EXTERNAL DISTURBANCE

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DECEMBER 1984

U.S. ARMY MISSILE COMMAND
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Further results on the application of the isobasis design technique of disturbance minimization control to a linear, time-invariant state set-point regulator with time-varying external disturbance inputs are presented. A second-order plant is used as an example. The performance of a disturbance minimizing controller designed using the isobasis technique is presented and is compared to results obtained from a controller designed under the assumption that the external disturbance is constant and to results when no disturbance minimizing control is used. Examples of the utility of time-varying external disturbances

(cont'd)

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE are presented in the Appendix.

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I. INTRODUCTION

Disturbances are defined as the uncontrollable inputs which act on a dynamical system. There are many varieties of disturbance inputs which can be associated with a controlled system and they are, for the most part, completely unpredictable in magnitude and in their arrival times.

In practice, additive disturbances, i.e., disturbances which are represented by terms added to the plant state equation, can arise from motivating effects external to the plant (external disturbances) or from motivating effects arising from the physical characteristics of plant subsystems or internal plant dynamics (internal disturbances). Further, these disturbances can be divided into two categories: (a) noise disturbances, characterized by random and erratic behavior with relatively high-frequency content, and (b) waveform structured disturbances, characterized by a degree of waveform regularity which can be described, piecewise in time, by differential equations forced by sparse sequences of impulses. The nature of these disturbances may be either completely known (through direct prior or real-time observation or test), completely unknown (random-like), or partially known.

Johnson [1-6, 10] introduced the idea of mathematically describing uncertain waveform-structured disturbances by representing them as a weighted linear combination of known basis functions of the form

$$w(t) = \sum_{i=1}^{n} c_i f_i(t), \qquad (1)$$

where $w(t)$ is the plant disturbance vector and is a p-vector and the weighting coefficients c_i are completely unknown constants which can change in magnitude in a random, once-in-a-while fashion. The basis functions $f_i(t)$ are completely known because they are chosen by the designer based on the waveform patterns exhibited (or thought to be exhibited) by the disturbance.

Johnson [1-11] developed a control engineering design technique, referred to as disturbance accommodation, wherein a combination of waveform-mode disturbance modeling and state-variable control methods are utilized to design controllers which will: (1) absorb (counteract), (2) minimize, or (3) constructively utilize the effects of uncertain disturbances on the plant. Three main classes of controllers are considered within the overall cognomen of disturbance accommodating control theory. These are: (1) Disturbance Absorption Controllers (DAC), (2) Disturbance Minimization Controllers (DMC), and (3) Disturbance Utilizing Controllers (DUC). Each class of controller has its own associated design goals and design methodology. The mathematical theories of DAC and DUC were thoroughly developed in References 1 through 12. The theory and techniques associated with DMC were compiled and extended in Reference 13. Additional results pertaining to the application of DMC techniques to a linear, time-invariant, second-order state set-point regulator problem with constant external disturbances were presented in Reference 14.

One of the DMC techniques presented in Reference 13 was the isobasis design technique. This technique makes the a priori assumption that the control vectors will be some function of the disturbance vectors, i.e., they will be composed of some combination of the same basis functions which describe the disturbances. This approach provides a parametric form for the control vectors. By utilizing the preferred minimization method [13], one can design the parameters of the minimization controller to minimize the disturbance effects.

Two examples of the application of the Isobasis technique were presented in Reference 13. One example was for a second-order output servo-command problem with a ramp for the external disturbance and a ramp for the output servo-command. The other example was for a second-order state servo-command problem, again with a ramp for the external disturbance and for the input state servo-command. This report will present results on the application of the Isobasis design technique to linear, time-invariant state set-point regulators with time-varying external disturbances.

II. LINEAR DYNAMICAL SYSTEMS

The class of systems to be considered in this report are "linear, time-invariant, dynamical systems," so called because the vector differential equation for the state x(t) is a linear differential equation, the transformation between the state space and output space is linear, and the elements of the matrices in the plant model are constant with respect to time.

These systems will be represented by equations of the general form

$$\dot{x}(t) = Ax(t) + Bu(t) + Fw(t) \tag{2}$$

$$y(t) = Cx(t) + Eu(t) + Gw(t) \tag{3}$$

where x(t) is the plant state vector and is an n-vector, u(t) is the plant control input vector and is an r-vector, w(t) is the plant disturbance vector and is a p-vector, y(t) is the plant output vector and is an m-vector and A, B, F, C, E, and G are appropriate size, known matrices with time-invariant elements. In addition, the general form of the disturbance state model is [10].

$$w(t) = Hz(t) + Lx(t) \tag{4}$$

$$\dot{z}(t) = Dz(t) + Mx(t) + \sigma(t) \tag{5}$$

where z(t) is the ρ-dimensional disturbance state vector, $\sigma(t)$ is a sparsely populated vector impulse sequence and H, L, D, and M are appropriate size, known matrices.

III. BACKGROUND

In Reference 13, several methods were presented for minimizing, via direct control action, the effects of constant disturbance components which are not completely absorbable on linear, time-invariant state set-point regulators. The metric used for the minimization process is the norm defined by

$$||Ax - b||_Q^2 = (Ax-b)^T Q(Ax-b), \quad Q > 0. \tag{6}$$

The design objective is the minimization of the distance between the attainable and desired set-point, where this distance is defined by the Euclidean norm,

$$d^2 = ||\ \varepsilon\ ||_I^2 = \varepsilon^T \varepsilon , \tag{7}$$

of the error vector between x_{sp} and the plant state x(t), i.e.,

$$\varepsilon(t) = x_{sp} - x(t). \tag{8}$$

An expression for the dynamics associated with this error can be derived by differentiating Equation (8) and substituting in the appropriate terms from Equation (2). The result can be expressed as

$$\dot{\varepsilon}(t) = \dot{x}_{sp} - \dot{x}(t) = A\varepsilon(t) - Bu(t) - Ax_{sp} - Fw(t) , \tag{9}$$

where Ax_{sp} represents the "set-point disturbance" term.

In disturbance accommodating control design, the control vector u(t) is considered to be an ordered collection of the various independent control inputs which are available to accomplish the primary control objective and to "accommodate" the disturbances which are acting on the system. In the design of disturbance minimization controllers, it is common practice to split (allocate) the total control u(t) into two parts as follows,

$$u(t) = u_p(t) + u_d(t) , \tag{10}$$

where $u_p(t)$ is given the task of accomplishing the primary control objective and $u_d(t)$ is given the task of disturbance accommodation. The part $u_d(t)$ can be further allocated into component vectors, as required. For the methods considered in this report, $u_d(t)$ will sometimes be allocated as

$$u_d(t) = u_{ds}(t) + u_{dw}(t) . \tag{11}$$

The component $u_{ds}(t)$ will be designed to accommodate the effects of the set-point disturbance term, while $u_{dw}(t)$ will be designed to accommodate the effects of the external disturbance term. If the plant is completely controllable and is also completely observable, the control $u_p(t)$ can be designed in the form

$$u_p(t) = Kx(t) . \tag{12}$$

Given the allocation of the control vector u(t), Equation 9 can be rewritten as

$$\dot{\varepsilon}(t) = A\varepsilon(t) - Bu_p(t) - Bu_{ds}(t) - Bu_{dw}(t) - Ax_{sp} - Fw(t). \qquad (13)$$

Upon substitution from Equation (4), Equation (13) becomes

$$\begin{aligned}\dot{\varepsilon}(t) = A\varepsilon(t) - Bu_p(t) - Bu_{ds}(t) - Bu_{dw}(t) \\ - Ax_{sp} - FHz(t) - FLx_{sp} - FL\varepsilon(t).\end{aligned} \qquad (14)$$

In the case of Equation (14), one would design $u_p(t)$ in the form

$$u_p(t) = -K\varepsilon(t) , \qquad (15)$$

with K chosen such that the homogeneous system

$$\dot{\varepsilon}(t) = (A + FL + BK)\varepsilon(t) \qquad (16)$$

will yield $\varepsilon(t) \rightarrow 0$ "rapidly." If one lets $\tilde{A} = A+FL+BK$, then Equation (14) can be expressed as

$$\dot{\varepsilon}(t) = \tilde{A}\varepsilon(t) - [(A+FL)x_{sp} + Bu_{ds}(t)] - [FHz(t) + Bu_{dw}(t)] . \qquad (17)$$

IV. PLANT AND EXTERNAL DISTURBANCE MODELS

The plant state and output models to be used in the examples presented in this report are

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} u + \begin{pmatrix} 1 \\ 1 \end{pmatrix} w \tag{18}$$

$$y = (1,\ 0)x\ . \tag{19}$$

The target state set-point vector is given as

$$x_{sp} = (x_{sp,1},\ 0). \tag{20}$$

The plant given by Equations (18) and (19) is completely controllable. For the purposes of the examples it is assumed that all necessary state information is available from an ideal state reconstructor.

Several different types of external disturbance will be applied to the plant and the effectiveness of the disturbance minimization controllers on each will be examined. The disturbances which will be used and the corresponding models, based on Equations (4) and (5) are as follows.

1. $$w(t) = C_o + C_1 t \tag{21}$$

$$w = (1,\ 0)z \tag{22}$$

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \sigma(t) \tag{23}$$

2. $$w(t) = C_1 \sin \alpha t \tag{24}$$

$$w = (1,\ 0)z \tag{25}$$

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -\alpha^2 & 0 \end{bmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \sigma(t) \tag{26}$$

3. $$w(t) = C_o + C_1 e^{\alpha t} \tag{27}$$

$$w = (1,\ 0)z \tag{28}$$

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & \alpha \end{bmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \sigma(t) \tag{29}$$

V. GEOMETRICAL CONSIDERATIONS AND THEOREMS

Since the control distribution matrix B of Equation (18) is a 2x1 matrix of rank 1, it does not span the state space, which is two-dimensional in this example. Hence, $Bu_d(t)$ will have a limited set of attainable points in the state space. Also, the external disturbance distribution matrix is of rank 1 and thus, $Fw(t)$ will have a limited range of action in the state space. As can be seen in Figure 1 the lines of action of Bu_d and Fw are not colinear. Hence, no u_d exists which will completely absorb a non-zero external disturbance or a set-point disturbance resulting from a non-zero target state set-point.

Given that this situation exists and that a design objective is to minimize the effects of the uncancellable disturbances, one thus attempts to design u_d so as to achieve this objective in some fashion. With respect to the vectors Fw_1 and Bu_{d1} shown on Figure 1, one approach to the minimization problem is to first express the vector Fw_1 as the sum of two component vectors, one lying in the column range space of B, R(B), and one lying in the orthogonal complement to the column range space of B, $R(B)^{\perp}$. This makes it easy to see that the component lying in $R(B)^{\perp}$, which is the component that is uncancellable, is minimized if the component lying in R(B) is the orthogonal projection of Fw_1 onto R(B) [13]. How can u_{d1} be chosen such that this is accomplished?

Casting the problem into the form of Equation (6) one wishes to minimize

$$||Bu_{d1} + Fw_1||_Q^2 = (Bu_{d1} + Fw_1)^T Q(Bu_{d1} + Fw_1) . \qquad (30)$$

Consider the following theorems from [15].

<u>Theorem</u>: Let B be an mxn matrix, Fw_1 an m vector an Q a positive definite mxm matrix. Then $||Bu_{d1} + Fw_1||$ is smallest when

$$u_{d1} = XFw_1 , \qquad (31)$$

where X satisfies

$$BXB = B , \quad (QBX)^T = QBX . \,| \qquad (32)$$

<u>Theorem</u>: Let B be an mxn matrix, Fw_1 an m vector and P a positive definite nxn matrix. If $Bu_{d1} = -Fw_1$ has a solution for u_{d1}, the unique solution for which $||u_{d1}||_P$ is smallest is given by

$$u_{d1} = XFw_1 , \qquad (33)$$

where X satisfies

$$BXB = B , \quad (PXB)^T = PXB. \,| \qquad (34)$$

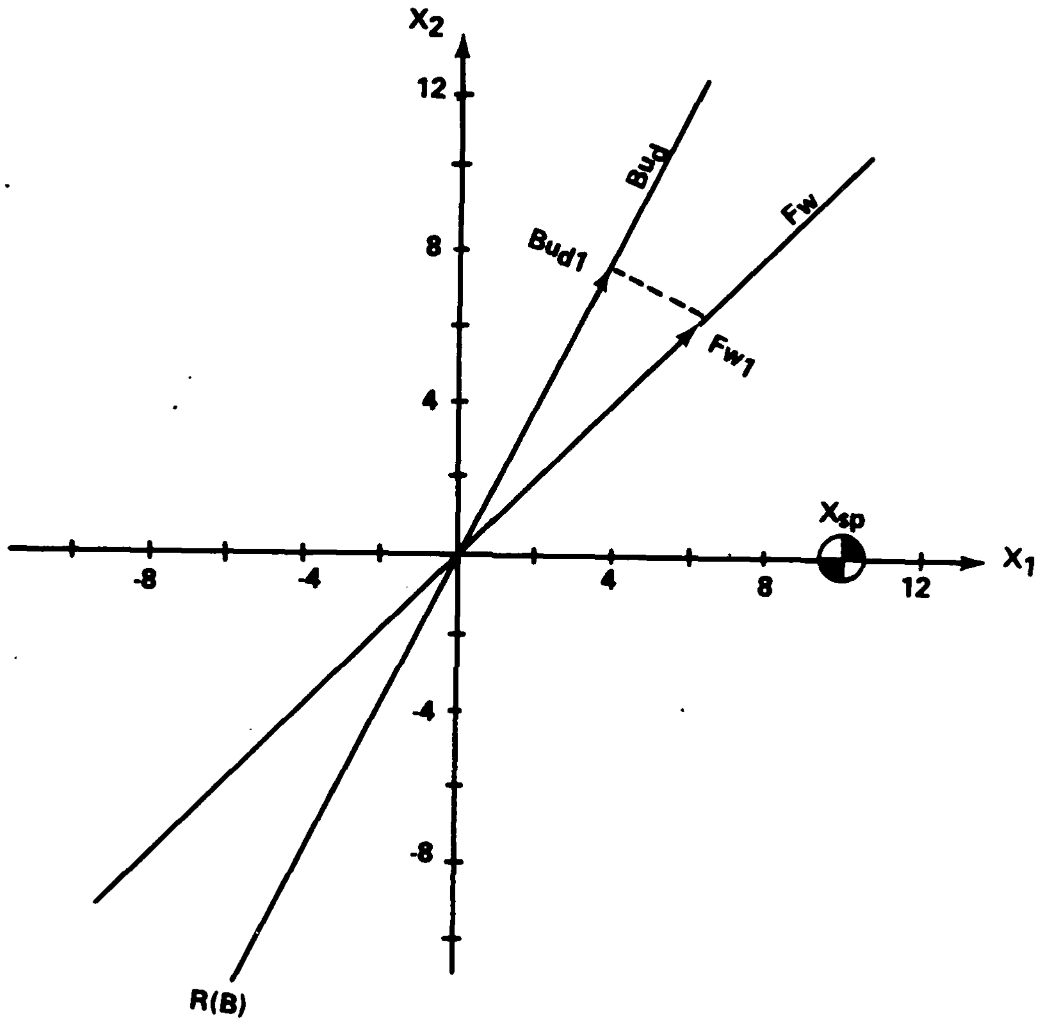

Figure 1. Line of action of external disturbance and control in plant state space.

Corollary: Let B be an mxn matrix, Fw_1 an m vector, Q a positive definite mxm matrix and P a positive definite nxn matrix. Then, there is a unique matrix X satisfying

$$(QBX)^T = QBX \ , \quad (PXB)^T = PXB \ . \tag{35}$$

Moreover, $||Bu_{d1} + Fw_1||_Q$ assumes its minimum value for $u_{d1} = XFw_1$, and in the set of vectors u_{d1} for which the minimum value is assumed, $u_{d1} = XFw_1$ is the one for which $||u_{d1}||_P$ is smallest. |

If the weighting matrices Q and P are chosen to be the identity matrix I, then Equation (30) represents the Euclidean norm and the properties which X satisfies in the above theorems are properties which are satisfied by the Moore-Penrose generalized inverse. Thus, if u_{d1} is chosen as

$$u_{d1} = -B^{\dagger}Fw_1 \ , \tag{36}$$

where $(\cdot)^{\dagger}$ represents the Moore-Penrose generalized inverse of $(\cdot)$, then u_{d1} is the minimum norm controller which minimizes $||Bu_{d1} + Fw_1||$. Also, one has

$$Bu_{d1} + Fw_1 = -BB^{\dagger}Fw_1 + Fw_1 = (I - BB^{\dagger})Fw_1 \tag{37}$$

and $(I-BB^{\dagger})$ is the projector of Fw_1 on $R(B)^{\perp}$ along $R(B)$ and $BB^{\dagger}$ is the projector of Fw_1 on $R(B)$ along $R(B)^{\perp}$. The uncancellable part of Fw_1 is thus the component in $R(B)^{\perp}$.

VI. DISTURBANCE MINIMIZATION WITH $w(t) = c_o + c_1 t$

A. With Allocated Disturbance Minimizing Control Vector

This section will apply the isobasis design technique to a state set-point regulator problem where the external disturbance is given by Equation (21), i.e., a combination of a step and a ramp. The stabilization control u_p, designed according to Equation (15) and Equation (16), is chosen to be [13, 14]

$$u_p(t) = -K\varepsilon(t) = (3., 0.36)\varepsilon(t). \tag{38}$$

Recall Equation (17), which is the general expression for $\dot{\varepsilon}(t)$,

$$\dot{\varepsilon}(t) = \tilde{A}\varepsilon(t) - (Ax_{sp} + Bu_{ds}) - [Fw(t) + Bu_{dw}(t)]. \tag{39}$$

If $w(t) = 0$, a steady-state solution would exist for ε as [14]

$$\varepsilon_{ss} = \tilde{A}^{-1}[Ax_{sp} + Bu_{ds}]. \tag{40}$$

This steady-state error could then be minimized by choosing u_{ds} as

$$u_{ds}^* = (\tilde{A}^{-1}B)^{\dagger}\tilde{A}^{-1}Ax_{sp} \tag{41}$$

which, in this example, becomes [13,14]

$$u_{ds}^* = -2.3447x_{sp,1} \;. \tag{42}$$

In order to develop an expression for $u_{dw}(t)$, with the external disturbance given by Equation (21), the a priori assumption is made that u_{dw} will be of the form

$$u_{dw}(t) = b(c_o + c_1 t) = bw(t). \tag{43}$$

The general solution of Equation (17) for $\varepsilon(t)$ can be written as

$$\varepsilon(t) = e^{\tilde{A}t}\varepsilon(0) - \int_0^t e^{\tilde{A}(t-\tau)}[Ax_{sp} + Bu_{ds}]d\tau - \int_0^t e^{\tilde{A}(t-\tau)}[Fw(\tau) + Bu_{dw}(\tau)]d\tau. \tag{44}$$

If Equation (21) and Equation (43) are substituted into the last term on the righthand side of Equation (44), the error contribution due to the external disturbance term can be expressed as

$$\Delta\varepsilon_w = -\int_0^t e^{\tilde{A}(t-\tau)}(F + Bb)c_o d\tau - \int_0^t e^{\tilde{A}(t-\tau)}(F + Bb)c_1 \tau d\tau$$

$$= -\tilde{A}^{-1}(e^{\tilde{A}t}-I)(F+Bb)c_o - [\tilde{A}^{-2}(e^{\tilde{A}t}-I) - \tilde{A}^{-1}t](F+Bb)c_1 \; . \qquad (45)$$

If one considers the response after the transients have settled out, Equation (45) can be reexpressed as

$$\Delta\varepsilon_w = \tilde{A}^{-1}(F + Bb)c_o + \tilde{A}^{-1}(\tilde{A}^{-1} + It)(F + Bb)c_1 \; . \qquad (46)$$

Upon expanding the right-hand side of Equation (46), one obtains

$$\Delta\varepsilon_w = \begin{bmatrix} 0.0854 & -0.1951 \\ 1.829 & -0.6098 \end{bmatrix} \begin{pmatrix} 1+b \\ 1+2b \end{pmatrix} c_o$$

$$+ \begin{bmatrix} -0.35 + 0.0854t & 0.1023 - 0.1951t \\ -0.9591 + 1.829t & 0.015 - 0.6098t \end{bmatrix} \begin{pmatrix} 1+b \\ 1+2b \end{pmatrix} c_1 \qquad (47)$$

which can be rearranged to the form

$$\Delta\varepsilon_w = \begin{pmatrix} -0.1097w - 0.2477\dot{w} \\ 1.2192w - 0.9441\dot{w} \end{pmatrix} + \begin{pmatrix} -0.3048w - 0.145\dot{w} \\ 0.6094w - 0.9291\dot{w} \end{pmatrix} b \; . \qquad (48)$$

Let Equation (48) be represented as

$$\Delta\varepsilon_w = u_1 b + u_2 \; . \qquad (49)$$

One would like to have

$$u_1 b + u_2 = 0 \qquad (50)$$

but this is not attainable in the case of this example. If the norm minimization criterion is applied, one chooses b such that $||\Delta\varepsilon_w||$ is minimized. Doing this, that b of minimum norm which will minimize the norm of $\Delta\varepsilon_w$ (see Section V) is found to be

$$b^* = -(u_1)^{\dagger} u_2 \qquad (51)$$

where[13]

$$u_1^{\dagger} = (u_1^T u_1)^{-1} u_1^T \; . \qquad (52)$$

When the appropriate substitutions are made from Equation (48) into Equation (52) and Equation (51), the results obtained are as follows,

$$u_1^\dagger = \left[(-0.3048w - 0.1454\dot{w},\ 0.6098w - 0.9291\dot{w}) \begin{pmatrix} -0.3048w - 0.1454\dot{w} \\ 0.6094w - 0.9291w \end{pmatrix}\right]^{-1} u_1^T$$

$$= \left(\frac{-0.3048w - 0.1454\dot{w}}{0.4643w^2 - 1.0438w\dot{w} + 0.8843\dot{w}^2},\ \frac{0.6094w - 0.9291\dot{w}}{0.4643w^2 - 1.0438w\dot{w} + 0.8843\dot{w}^2} \right), \quad (53)$$

$$b^* = \frac{-0.7764w^2 + 1.6167w\dot{w} - 0.9132\dot{w}^2}{0.4643w^2 - 1.0438w\dot{w} + 0.8843w^2}. \quad (54)$$

For the purposes of these examples, it will be assumed that w and $\dot{w}$ are obtained from an ideal disturbance state reconstructor.

A digital simulation was written for this example. Several runs were made with various values for the coefficients describing $w(t)$. The results are shown in Figures 2 through 9. In each case, three curves are given. One curve represents a case where $u_d = 0$. One curve is for the controller designed in this section. A third curve, which is shown for purposes of comparison, is for a case where u_{dw} was designed under the assumption that w was a constant disturbance. With this assumption, when u_{dw} is designed to minimize the contribution of the external disturbance to the steady-state error, the result is

$$u_{dw} = -1.6723z. \quad (55)$$

Figures 2 through 5 show the resulting set-point error magnitude for various target set-points and external disturbances. Figures 6 through 8 show the set-point error magnitude and the error components for a case with zero target set-point and changing external disturbance parameters and Figure 9 shows $w(t)$. As can be seen from the plots, the controller with u_{dw} designed using the isobasis technique with norm minimization does not result in improved performance over the controller designed to minimize ε_{ss} when w is assumed constant.

Another minimization criterion which could be applied would be to design u_{dw} such that $\Delta\varepsilon_{w1}$ or $\Delta\varepsilon_{w2}$ is steered to zero, i.e., design u_{dw} such that the direct effect of the external disturbance on ε_1 or ε_2, respectively, is completely absorbed. If one follows this approach, the parameter b would be designed as follows. First, assume that ε_1 is a critical-state variable and that the direct effect of w on ε_1 is to be absorbed. From Equation (48), one has that

$$\Delta\varepsilon_{w1} = -(0.1097w + 0.2477\dot{w}) - (0.3048w + 0.1454\dot{w})b. \quad (56)$$

In order to obtain $\Delta\varepsilon_{w1} = 0$, one thus requires that

$$b = -\frac{0.1097w + 0.2477\dot{w}}{0.3048w + 0.1454\dot{w}}. \quad (57)$$

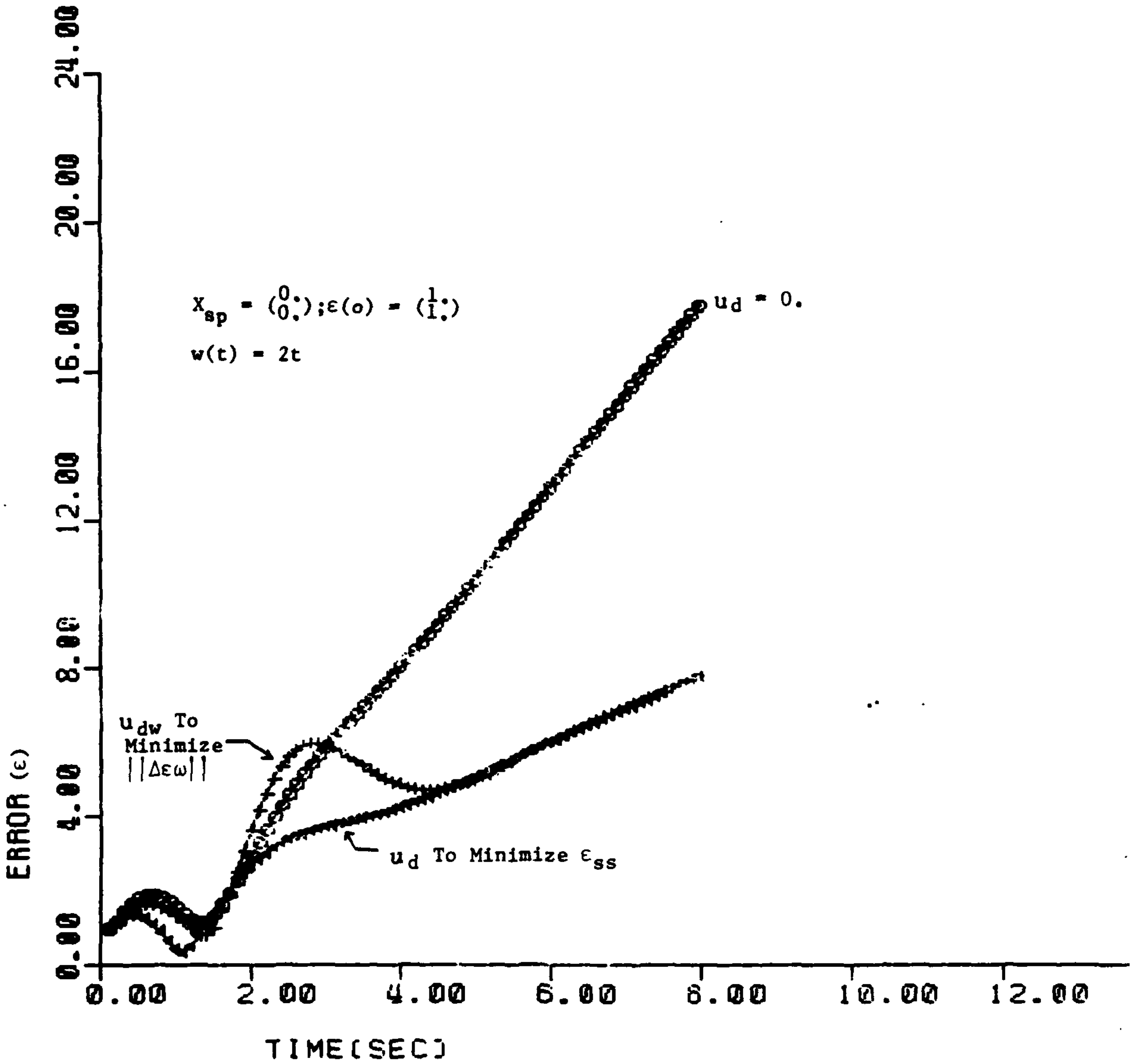

Figure 2. Error versus time, with allocated control vector, $w=2t$, $X_{sp}= (0,0)^T$.

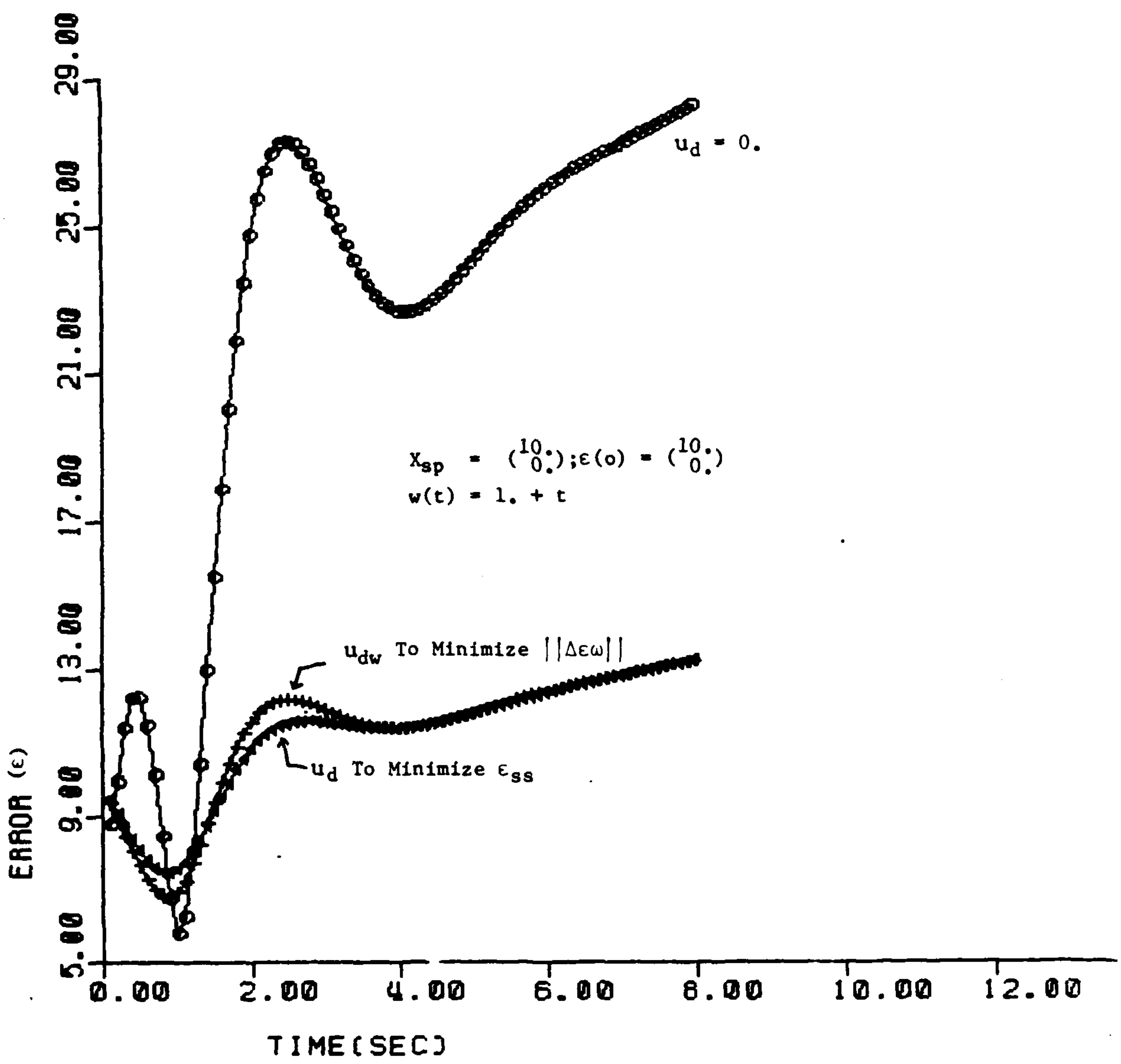

Figure 3. Error versus time, with allocated control vector, $w=1+t$, $X_{sp}= (10,0)^T$.

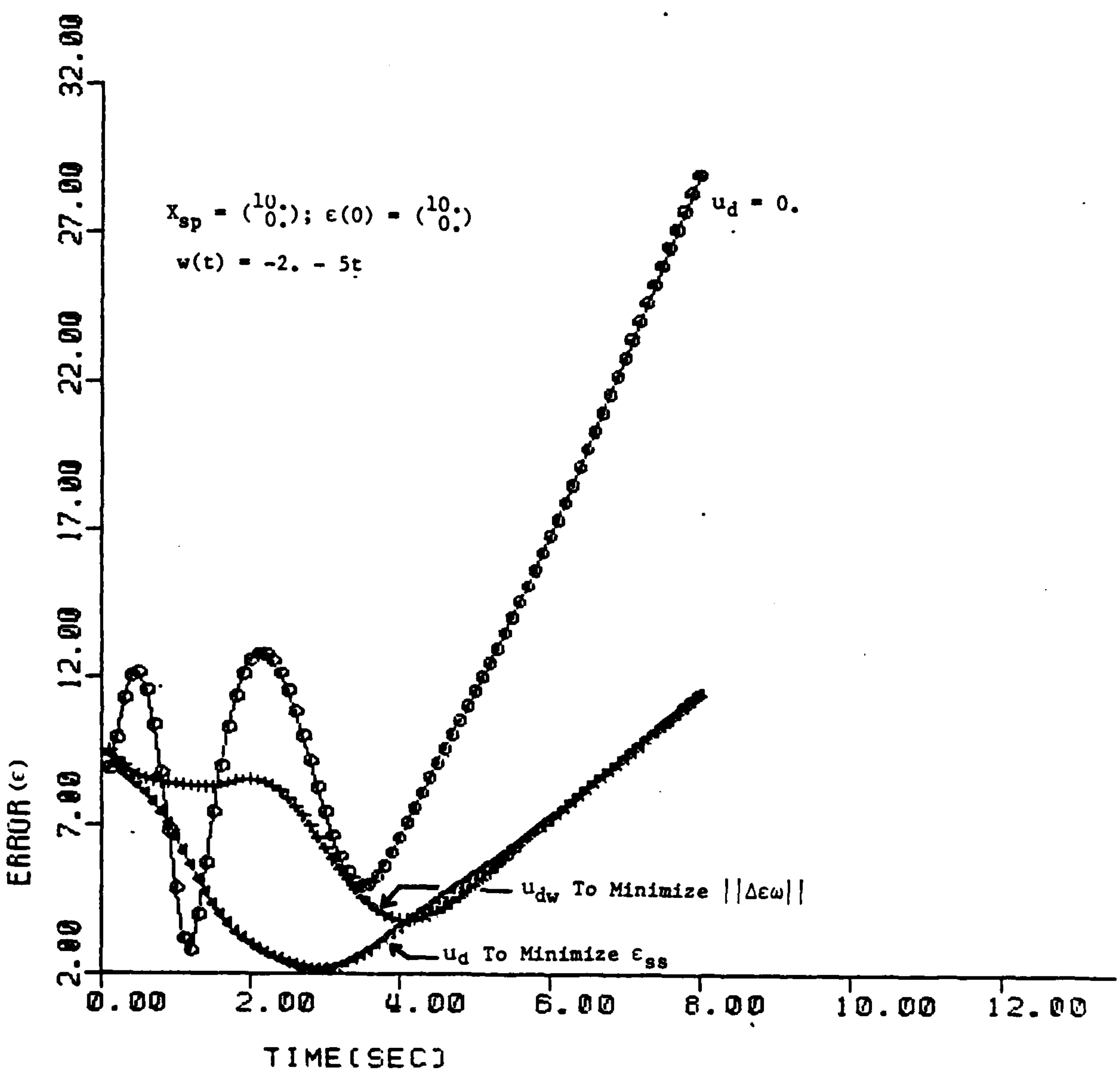

Figure 4. Error versus time, with allocated control vector, $w=-2-5t$, $X_{sp}= (10,0)^T$.

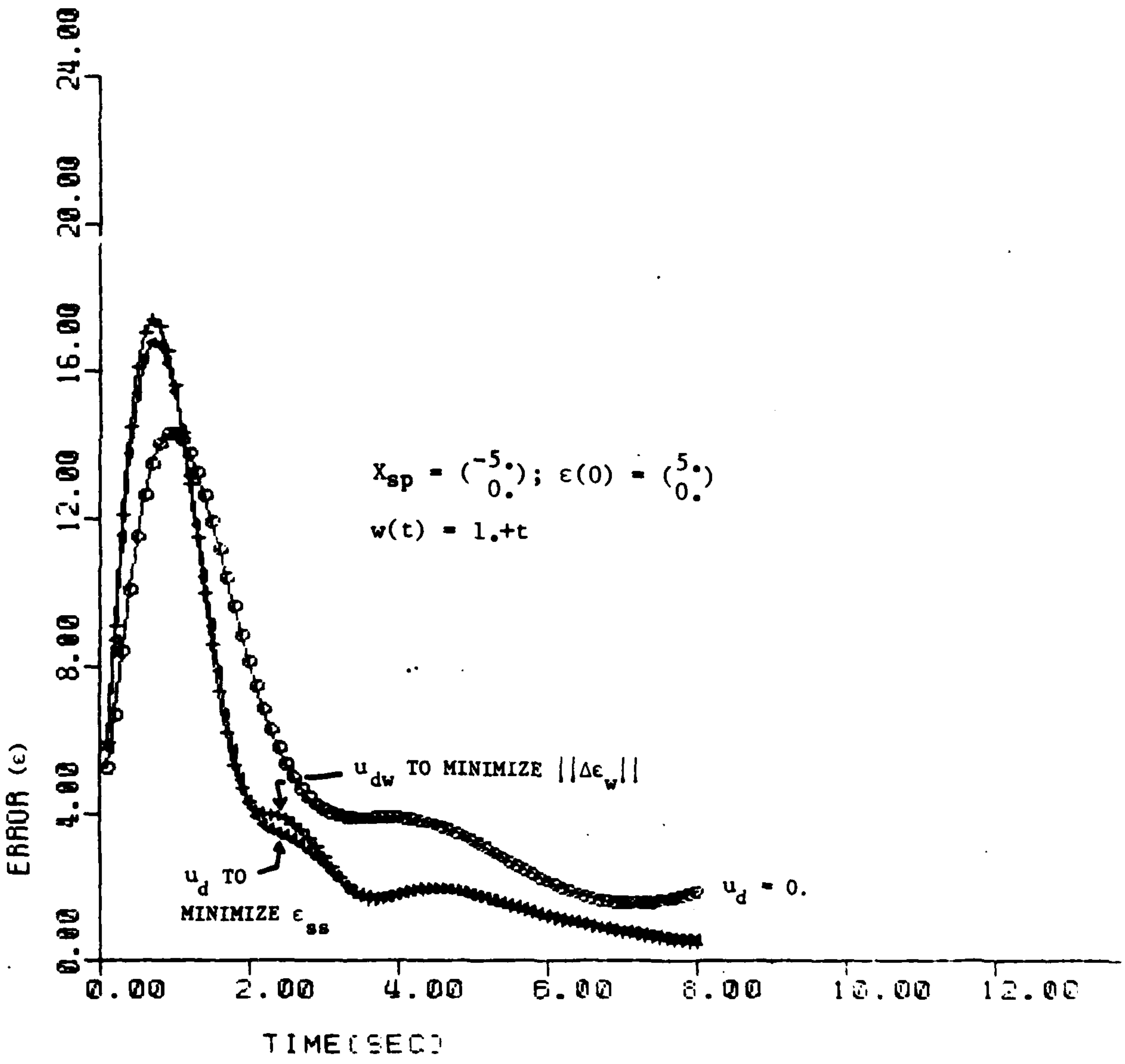

Figure 5. Error versus time, with allocated control vector, $w=1+t$, $X_{sp}=(-5,0)^T$.

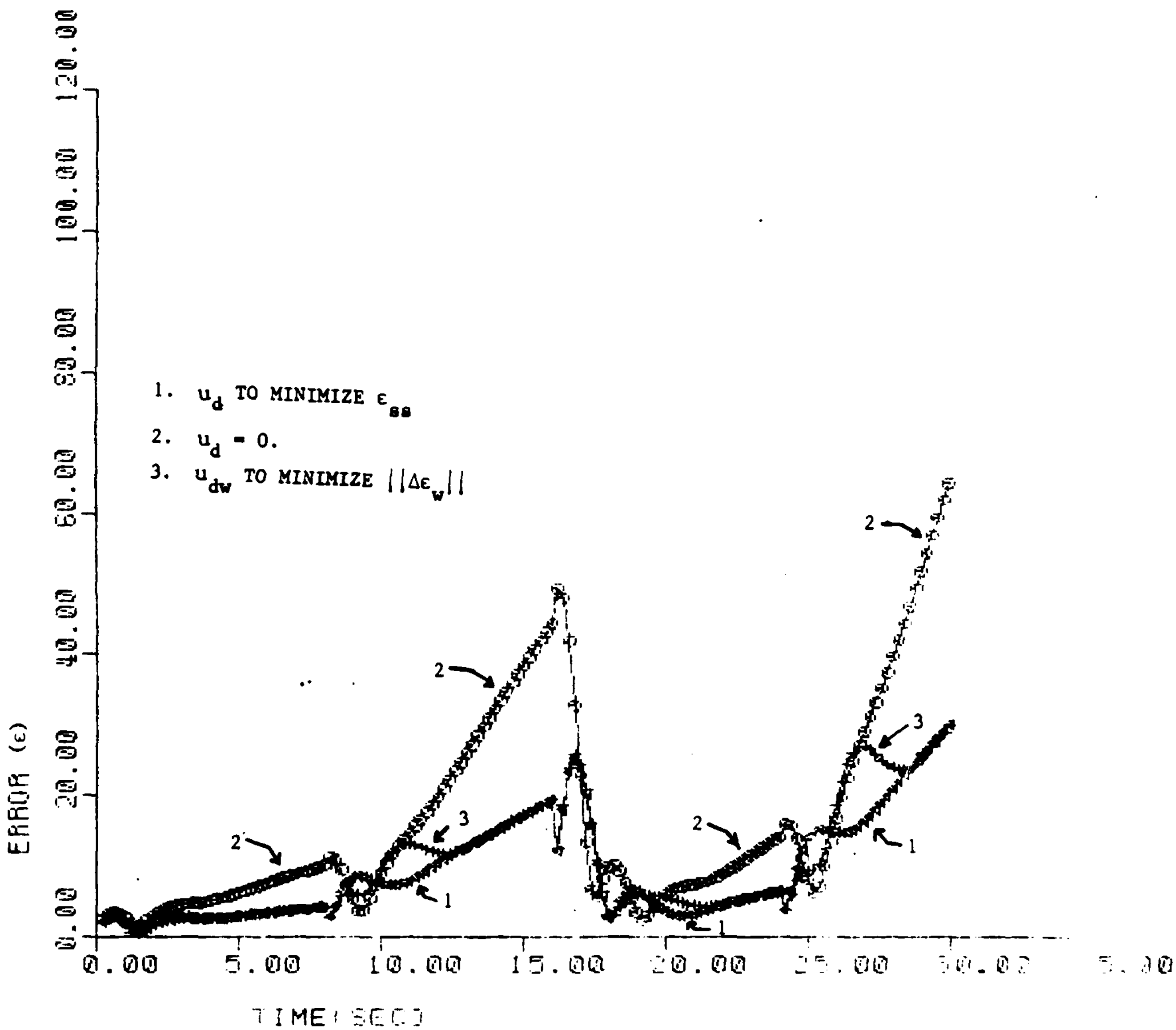

Figure 6. Error magnitude, case with parameter variation in w(t).

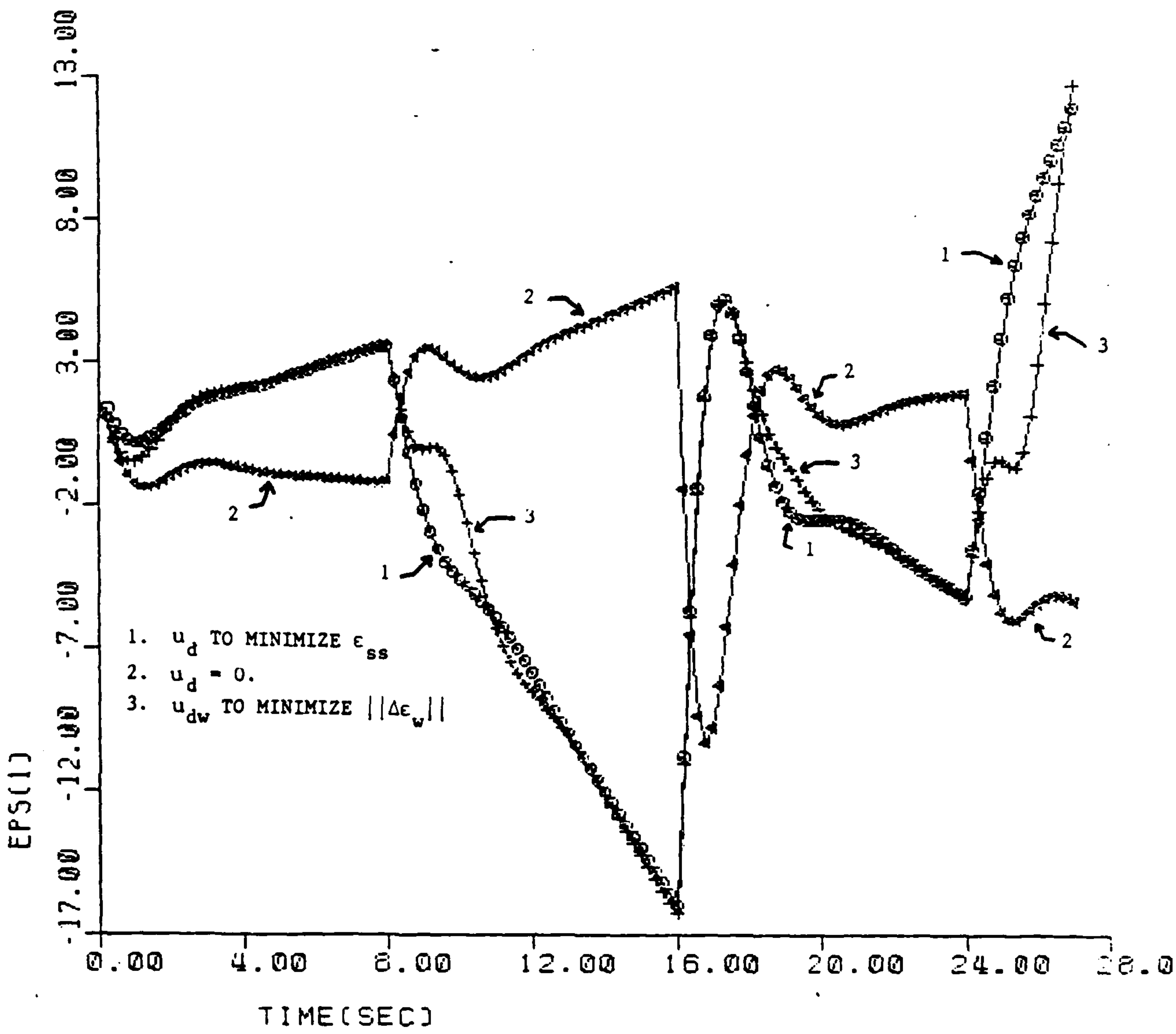

Figure 7. $\varepsilon_1(t)$, case with parameter variation in w(t).

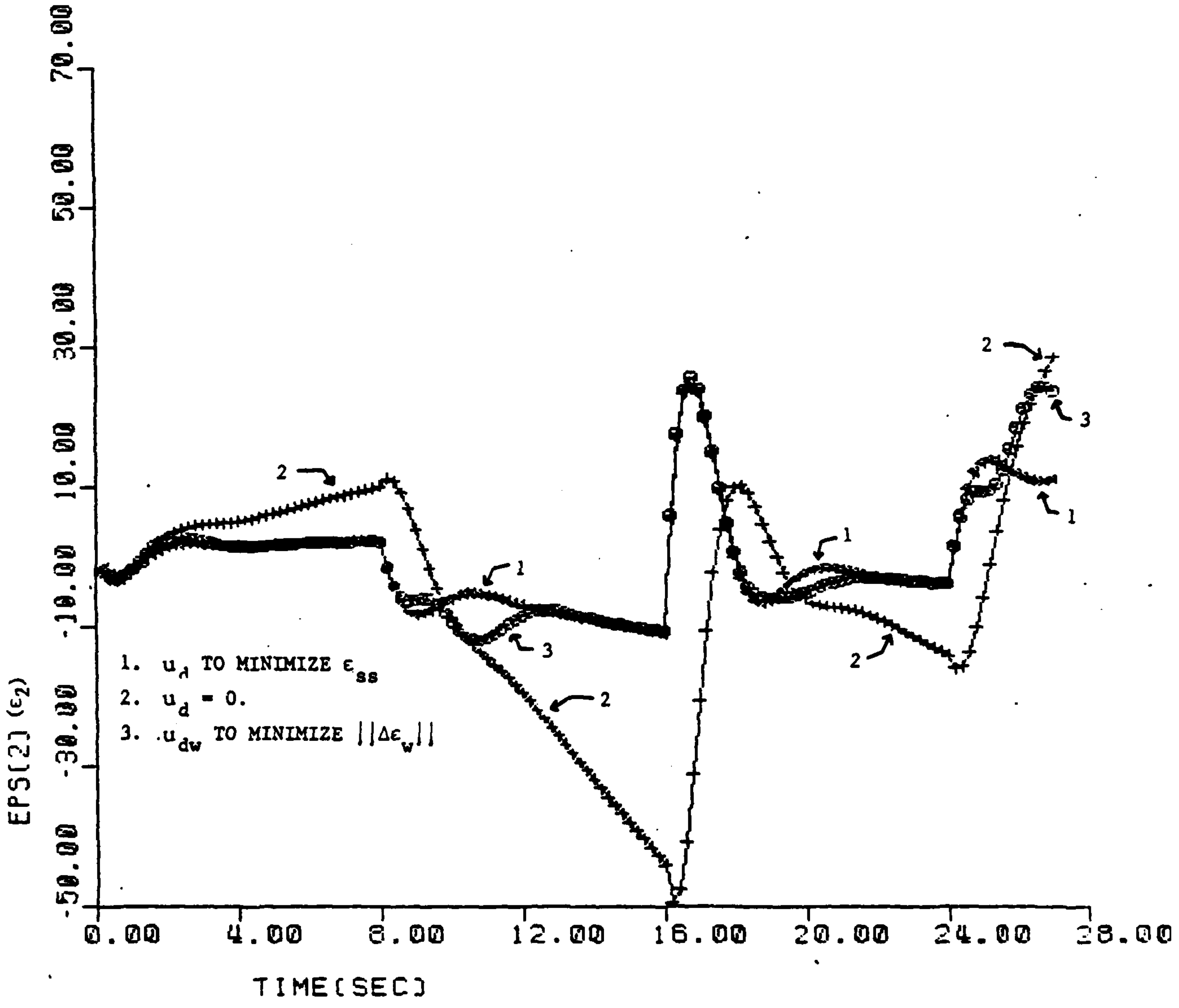

Figure 8. $\varepsilon_2(t)$, case with parameter variation in w(t).

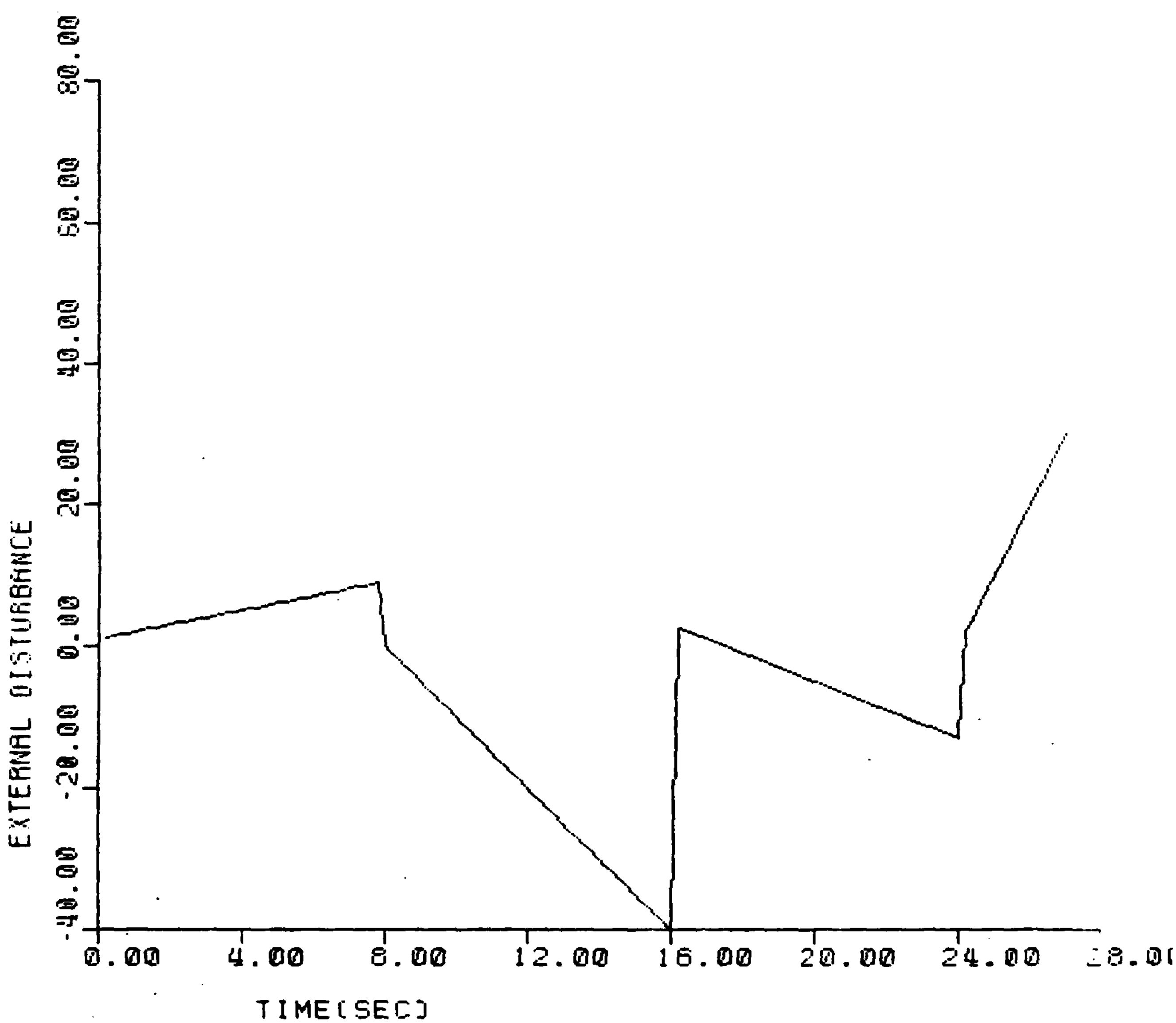

Figure 9. External disturbance time history.

Figures 10 through 15 show results for two cases when Equation (57) is used to define u_{dw}. As can be seen in Figures 11 and 14, in each case $\varepsilon_1(t)$ was steered to zero. However, the total set-point error (Figures 10 and 13) was not improved over that obtained by using u_d designed to minimize ε_{ss}.

Next, assume that ε_2 is the critical-state variable and design u_{dw} to absorb the direct effect of w on ε_2. Again, from Equation (48), one has

$$\Delta\varepsilon_{w2} = (1.2191w - 0.9441\dot{w}) + (0.6094w - 0.9291\dot{w})b. \qquad (58)$$

In order to obtain $\Delta\varepsilon_{w2} = 0$, one requires that

$$b = -\frac{1.2192w - 0.9441\dot{w}}{0.6094w - 0.9291\dot{w}}. \qquad (59)$$

Figure 16 shows results obtained for the case used in Figures 10 through 13 when Equation (59) is used to define u_{dw}. As can be seen, this controller produced unacceptable transients. This points up a caution in the use of this technique.

Since the parameters given by Equation (57) and Equation (59) to define $u_{dw}(t)$ have time varying functions in their denominators, the possibility exists that some combination(s) of the constants c_o, c_1 defining w(t) may result in a zero in the denominator thus causing an infinitely large value to be input by the disturbance minimizing controller. For the two external disturbances used to generate the plots of Figures 10 through 16, i.e.,

$$w(t) = 1 + t \qquad (60)$$

$$w(t) = -2 -5t \qquad (61)$$

a check of the denominator of Equation (57) shows that no positive value of time exists at which u_{dw} would go to infinity. However, the denominator of Equation (59) will go to zero at t = 0.525 seconds for Equation (60) (which gives the resulting peak in the response shown in Figure 16) and at t = 1.125 seconds for Equation (61). A practical implementation would thus require a limit on the allowable magnitude of u_{dw}. Figures 17 through 19 show the results for w as in Equation (60) when the magnitude of u_{dw} is limited to ± 50. From Figure 19 it can be seen that $\varepsilon_2(t)$ is steered to near zero and a comparison of Figures 17 and 10 indicates that choosing u_{dw} to give $\Delta\varepsilon_{w2} = 0$ does reduce the total error over the case where u_{dw} is chosen to give $\Delta\varepsilon_{w1} = 0$. However, in neither case is the total error less than that obtained using the controller designed to minimize ε_{ss}.

B. With Unallocated Disturbance Minimizing Control Vector

In Part A, the disturbance minimizing control u_d was allocated as $u_d = u_{ds} + u_{dw}$ and u_{ds} was designed to minimize the norm of the steady-state set-point error contribution given by Equation (40). As was seen, the performance of the controllers designed in Part A was not as good as that of the controller designed to minimize the steady-state error under the assumption that w(t) was a constant. In this section, the isobasis design technique will be applied with an unallocated disturbance control vector.

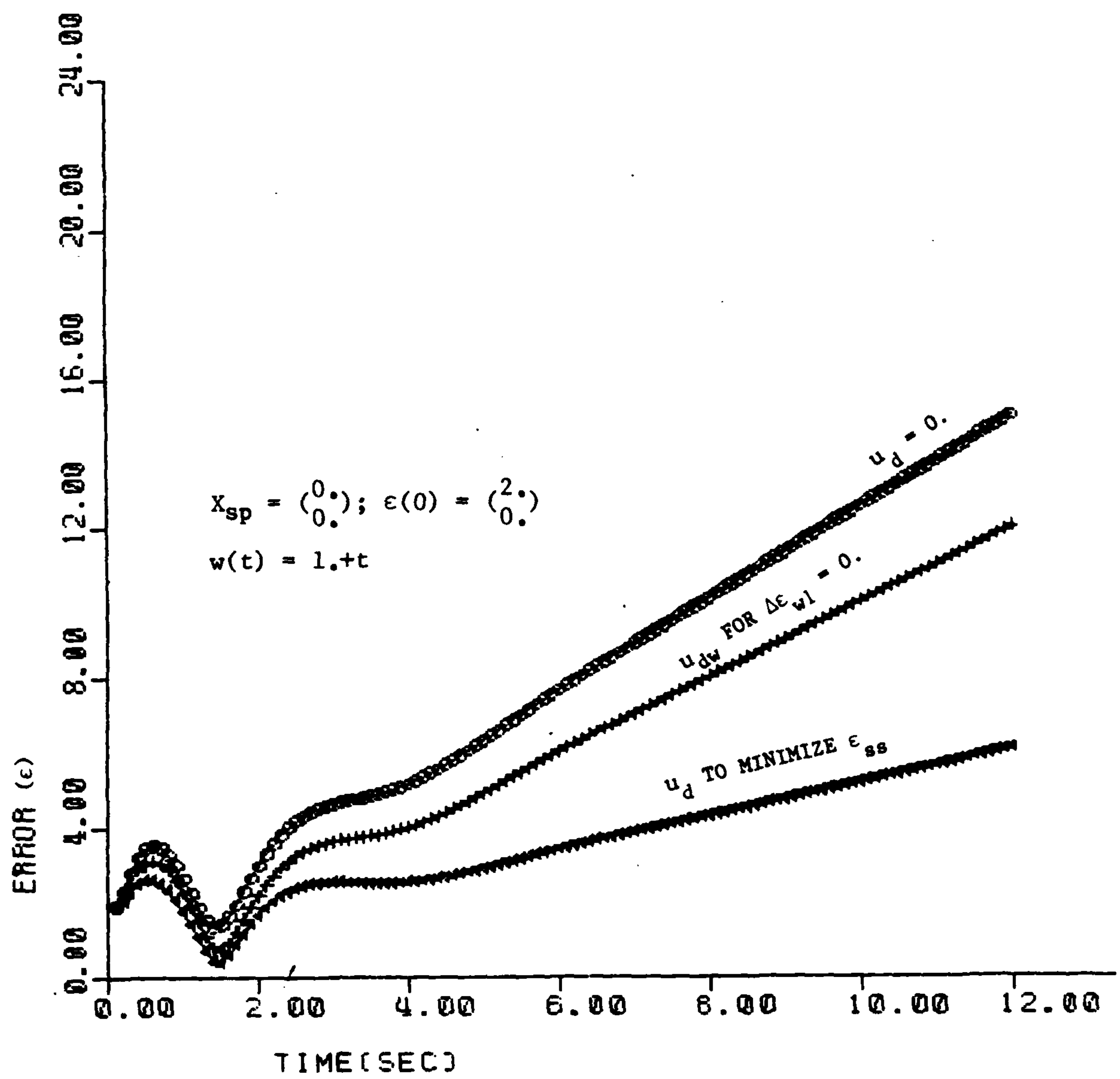

Figure 10. Error versus time, u_{dw} designed to give $\Delta\varepsilon_{wl}$=0.

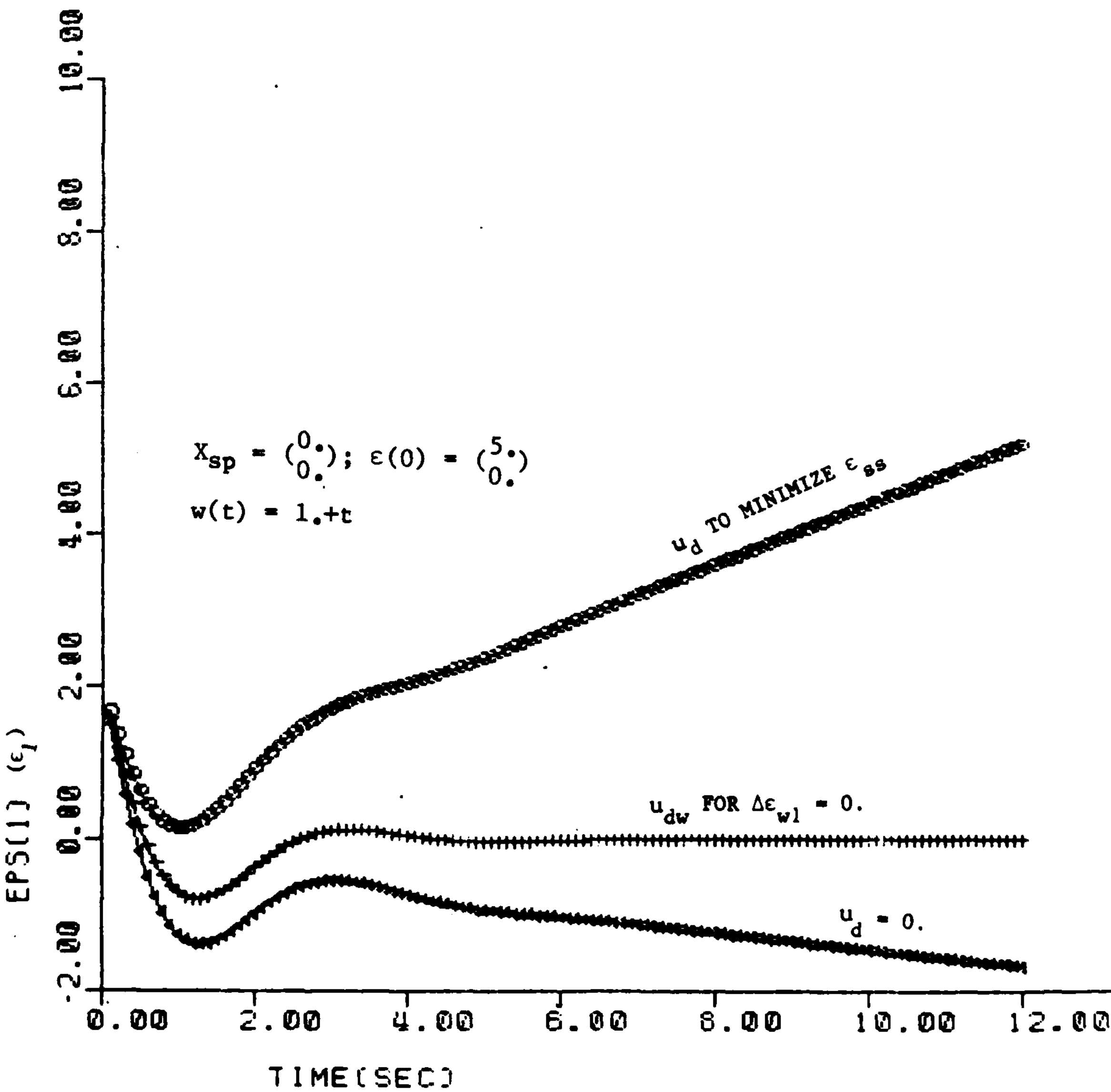

Figure 11. ε_1 (t) with u_{dw} designed to give $\Delta\varepsilon_{w1}$=0.

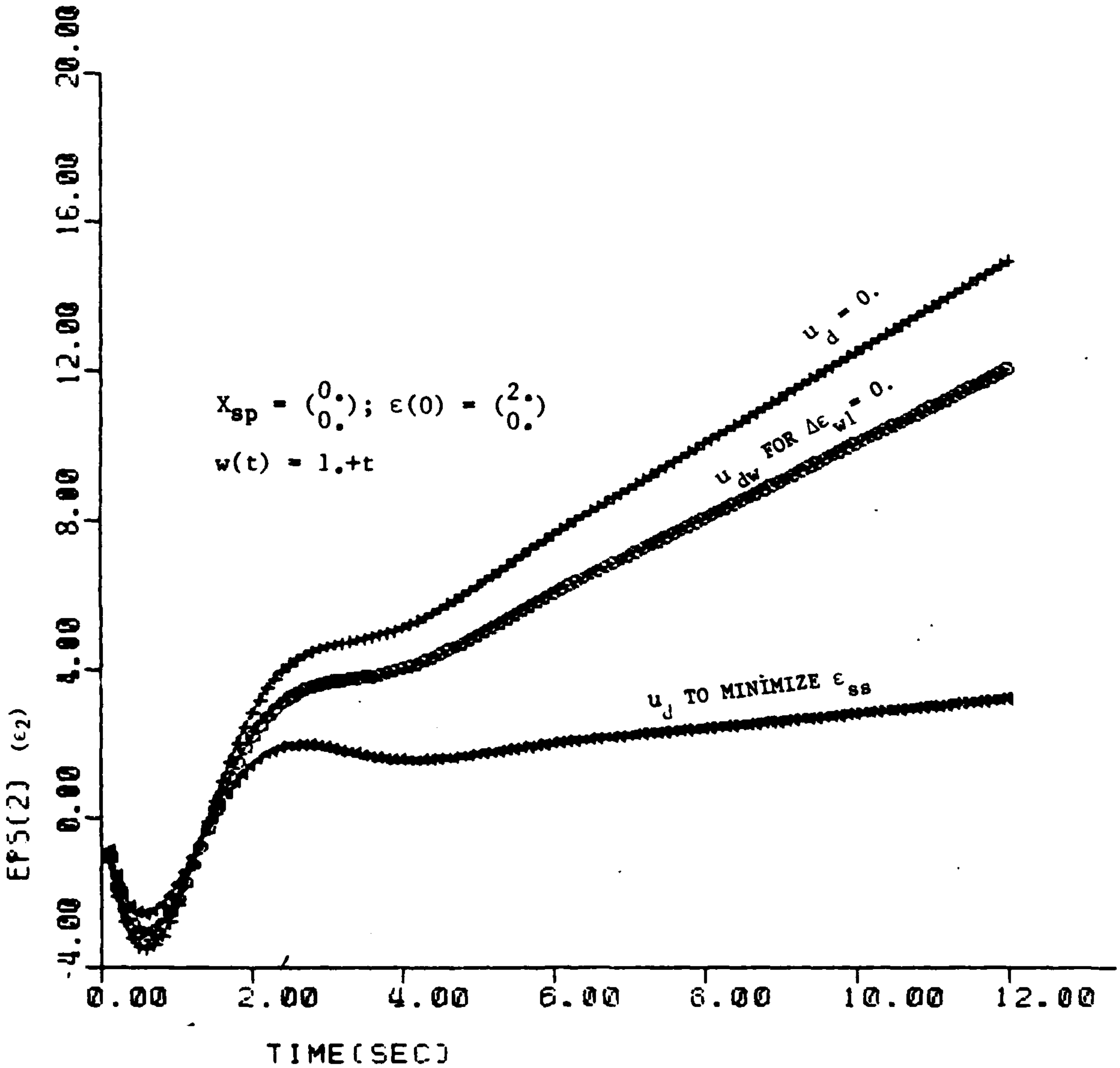

Figure 12. ε_2 (t) with u_{dw} designed to give $\Delta\varepsilon_{w1}=0.$

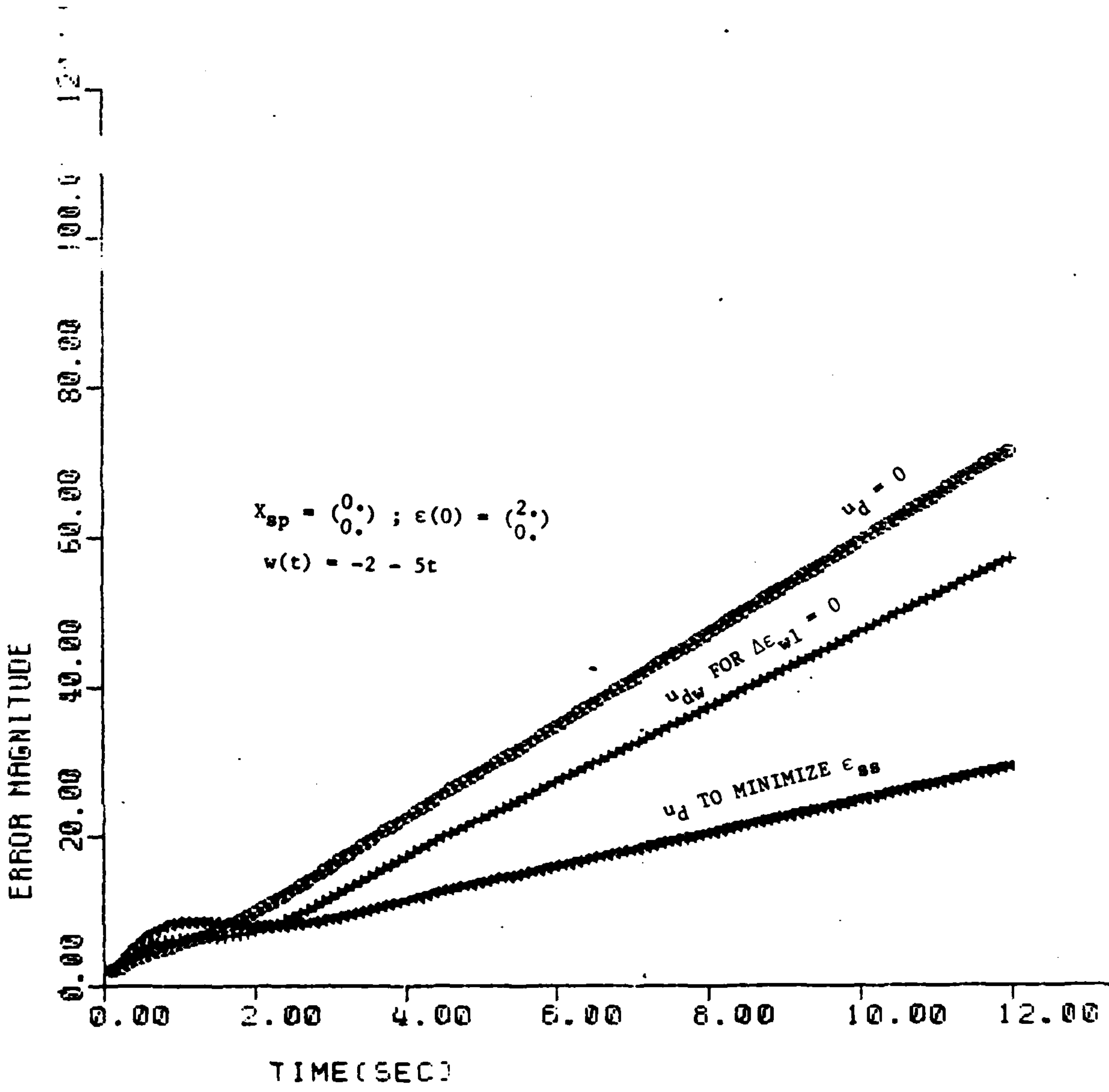

Figure 13. Error magnitude, u_{dw} designed to give $\Delta\varepsilon_{w1} = 0$.

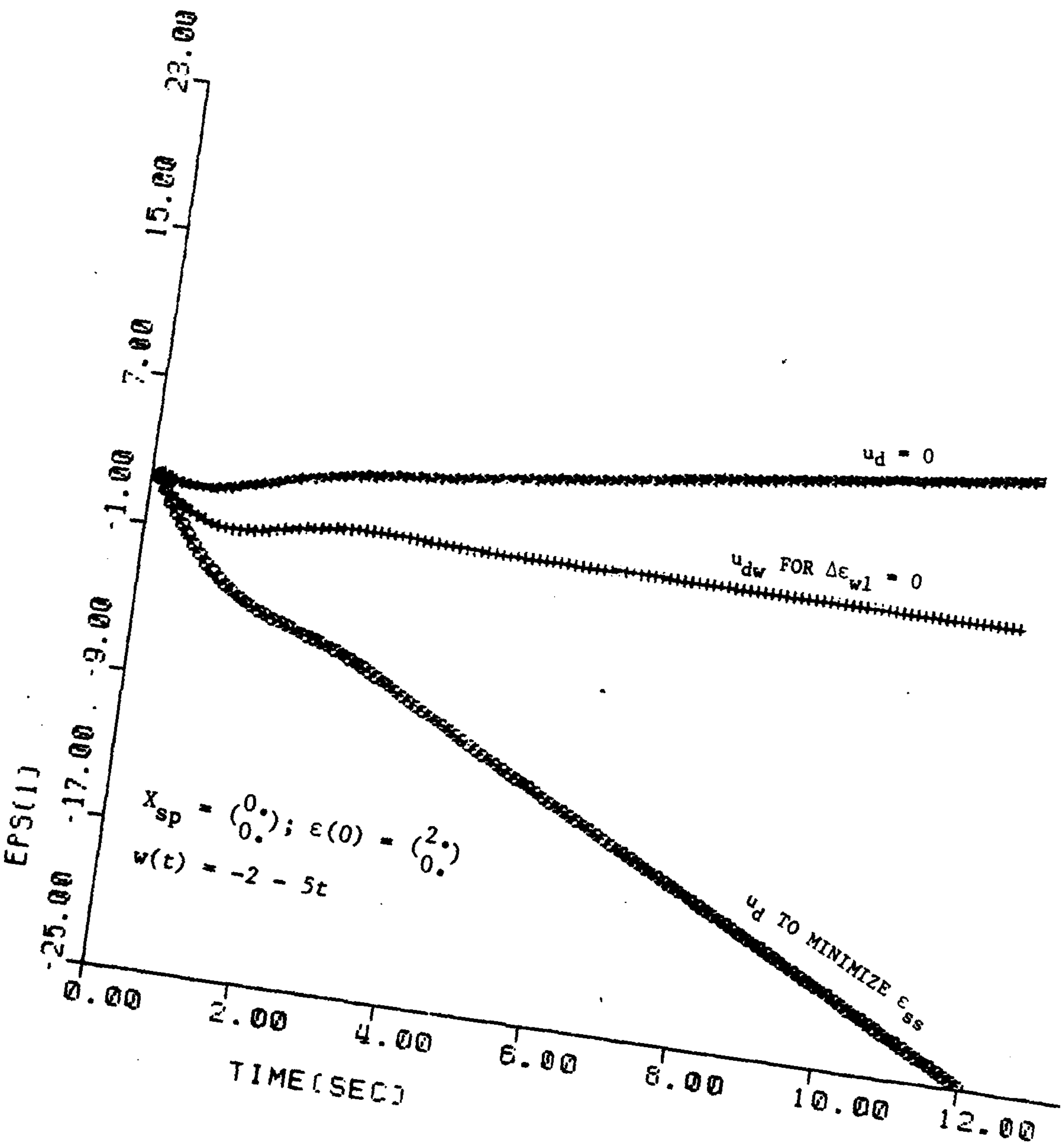

Figure 14. ε_1 (t) with u_{dw} designed to give $\Delta\varepsilon_{w1}$=0.

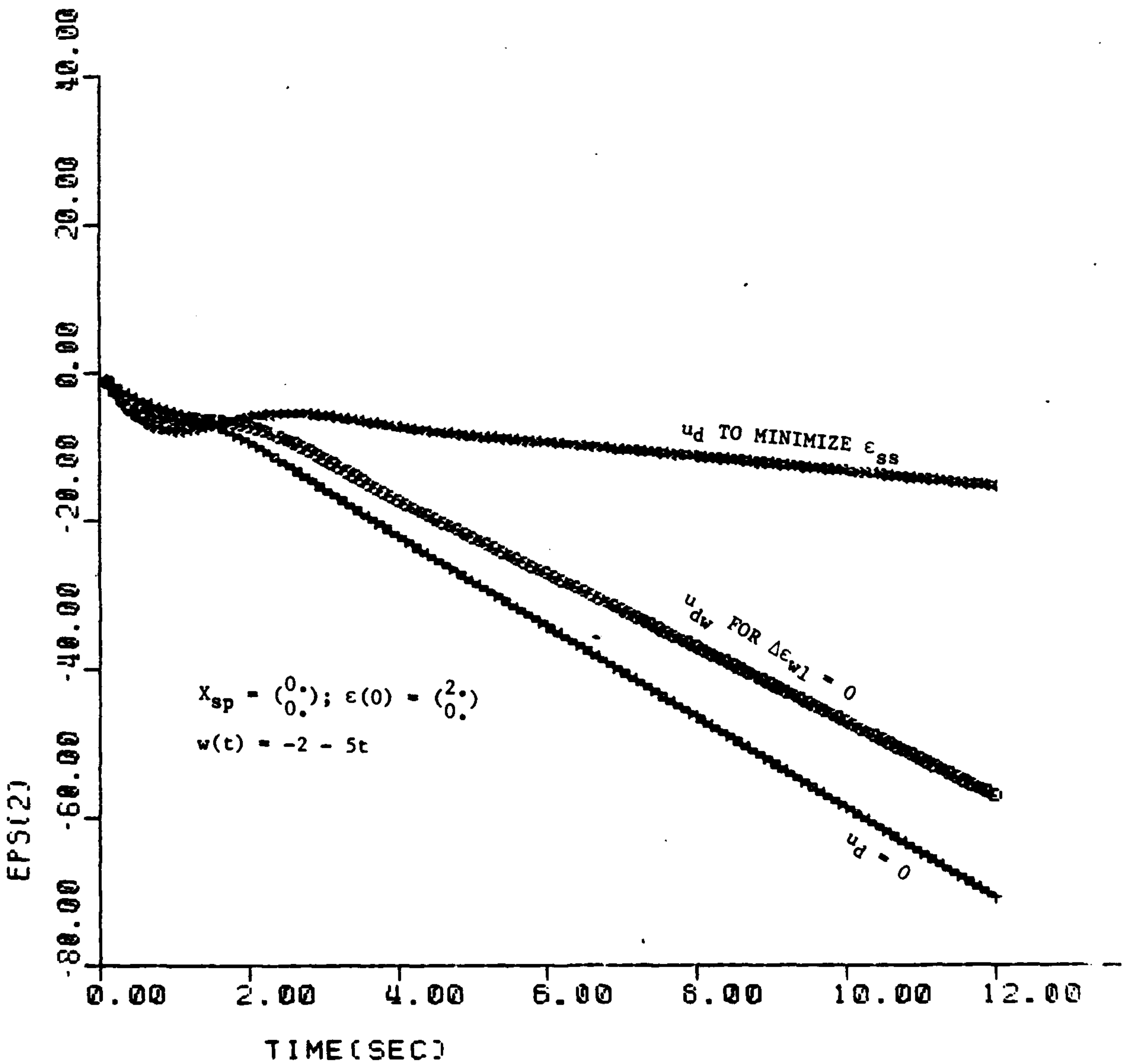

Figure 15. $\varepsilon_2(t)$ with u_{dw} designed to give $\Delta\varepsilon_{w1}=0$.

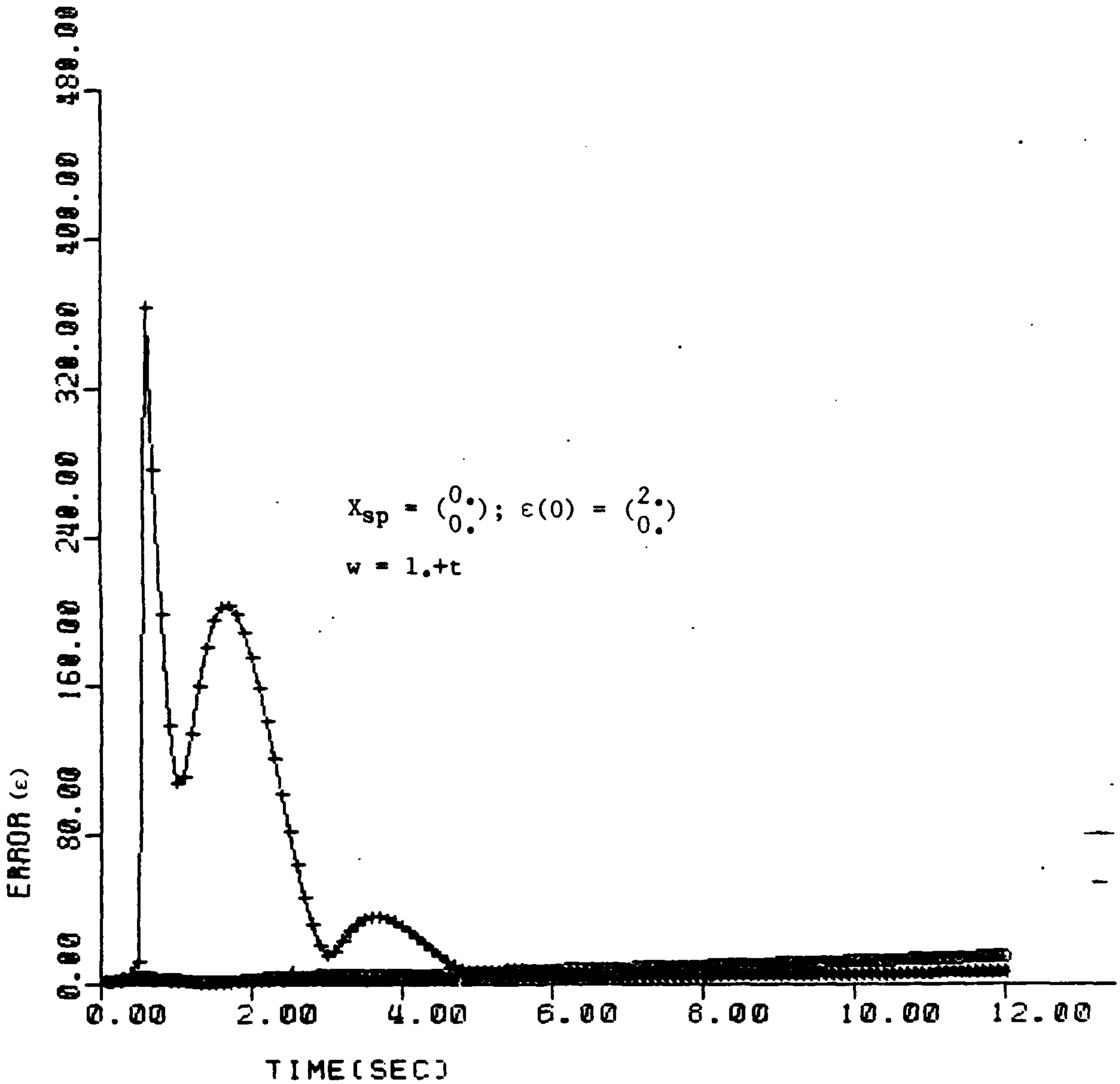

Figure 16. Error versus time, u_{dw} designed to give $\Delta\varepsilon_{w2}=0.$, no limit on u_{dw}.

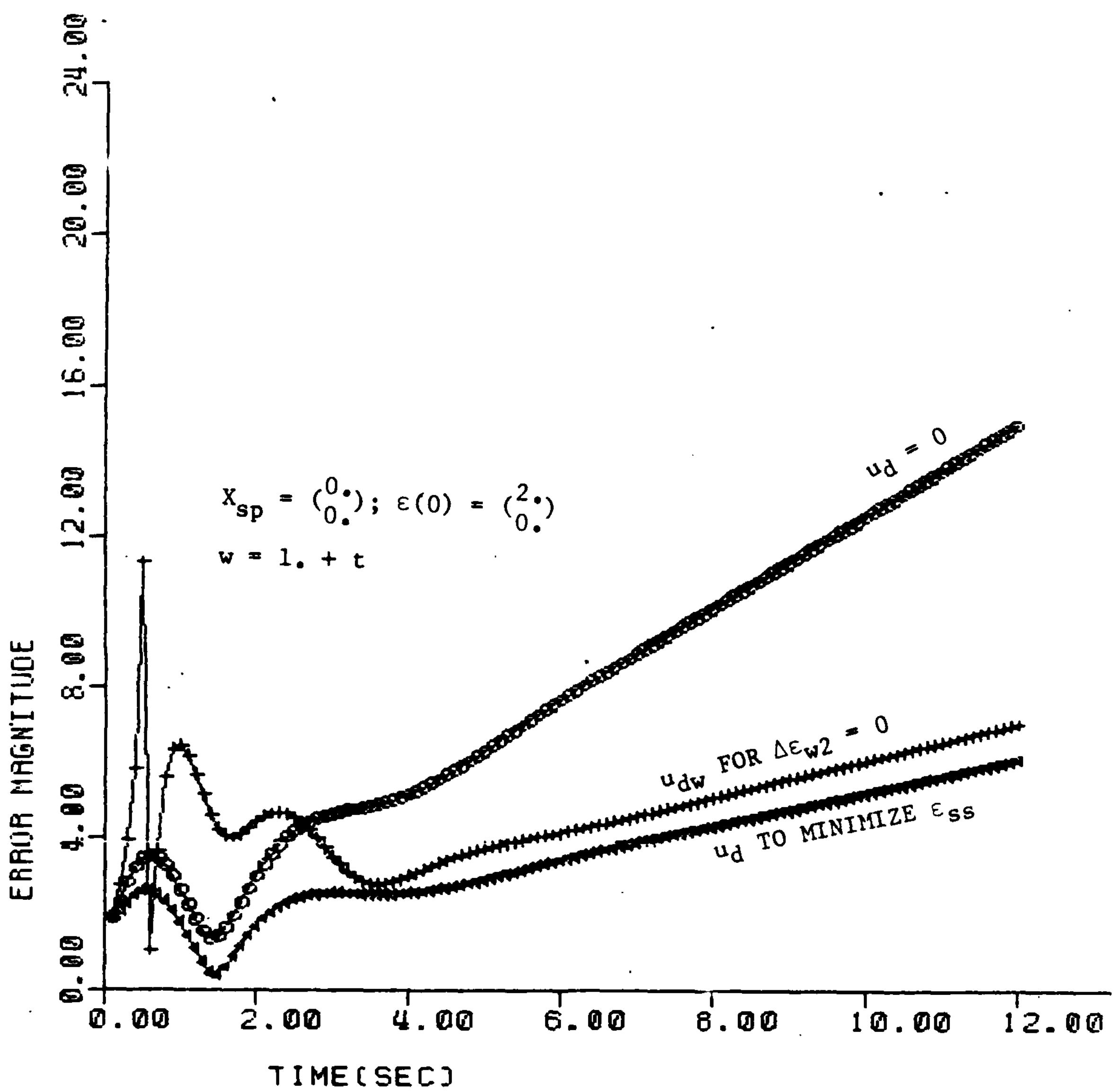

Figure 17. Error versus time, u_{dw} designed to give $\Delta\varepsilon_{w2}$=0., u_{dw} limited to ±50.

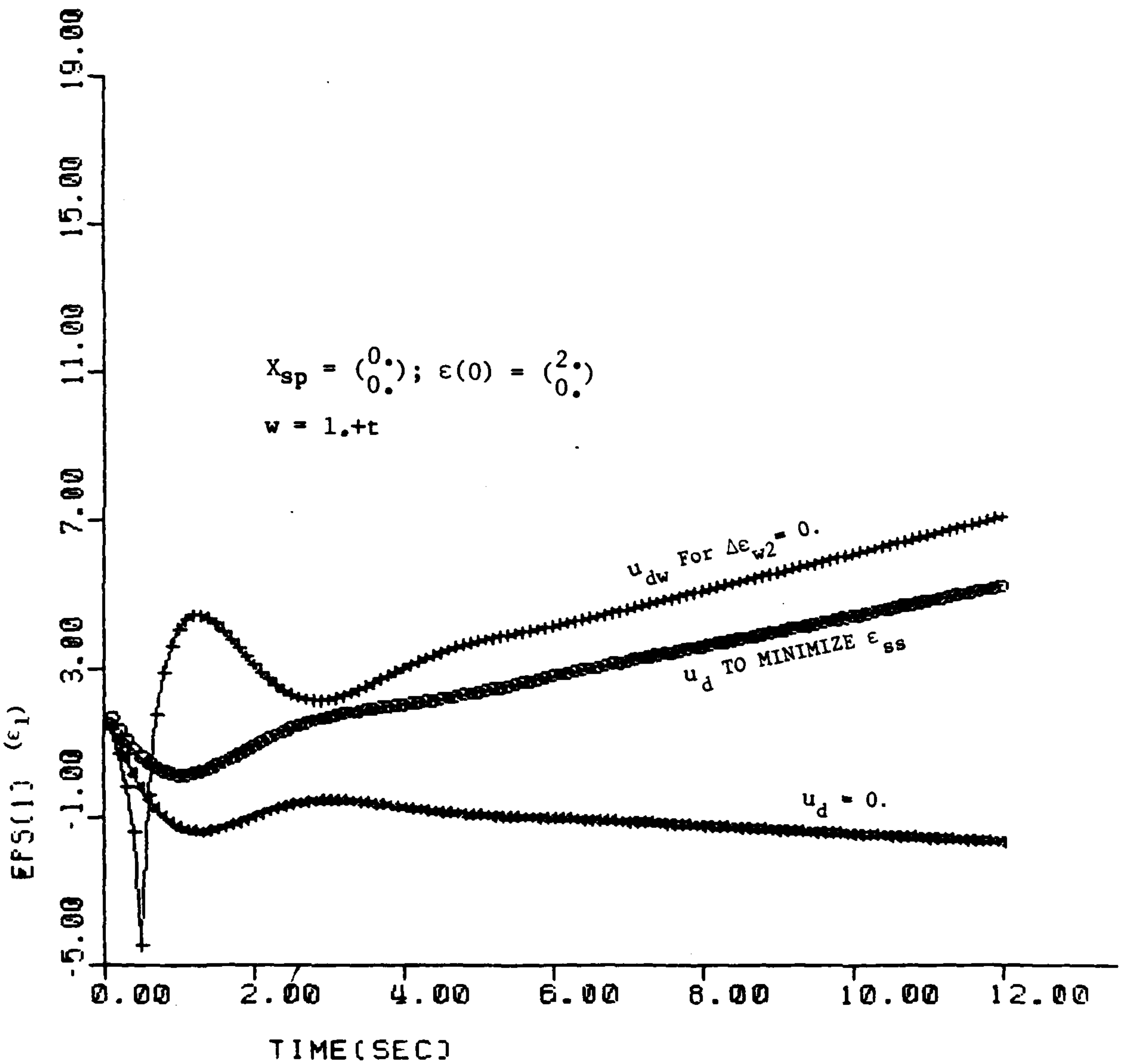

Figure 18. ϵ_1 (t) with u_{dw} designed to give $\Delta\epsilon_{w2}$=0., u_{dw} limited to ± 50.

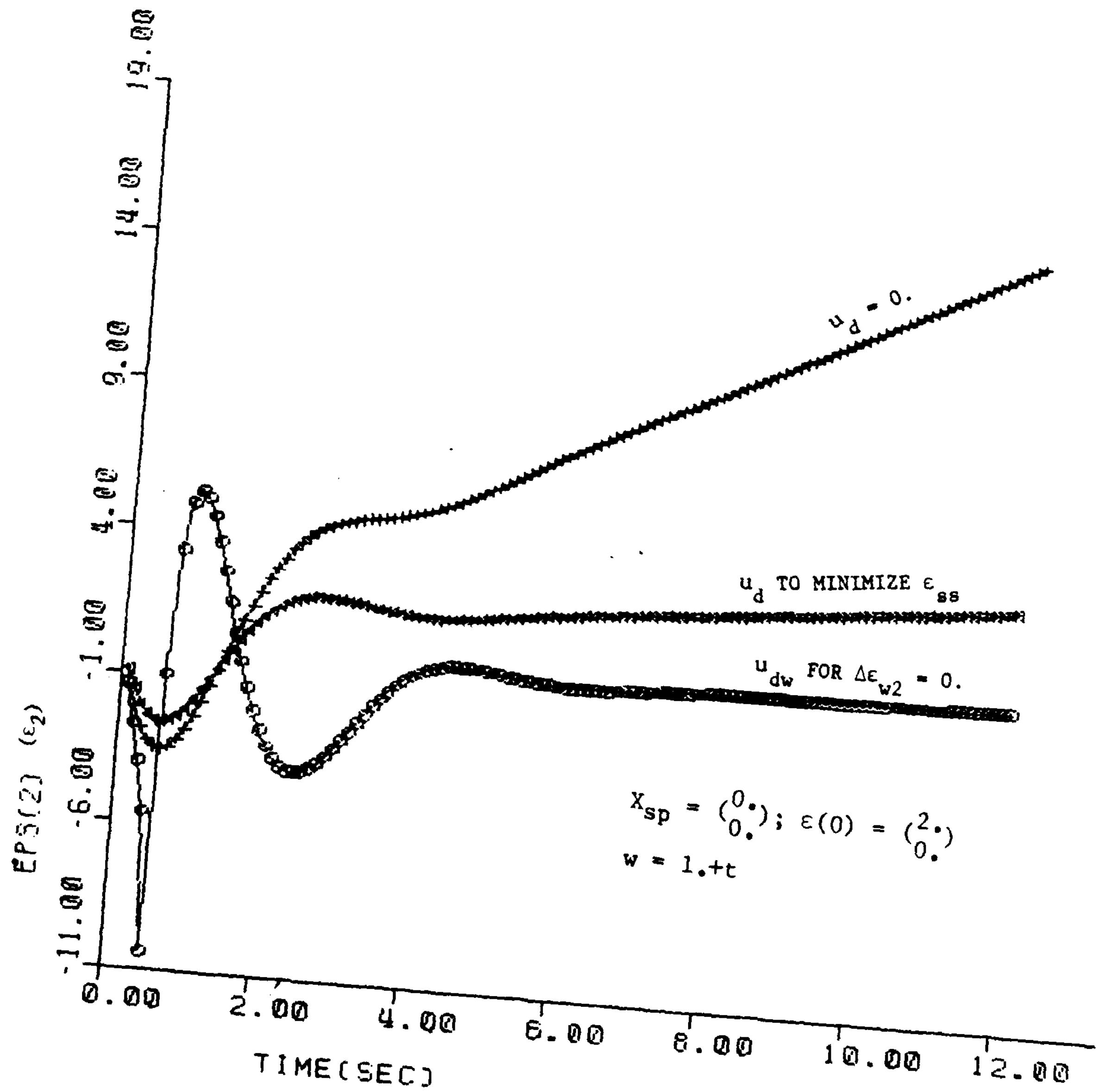

Figure 19. $\varepsilon_2(t)$ with u_{dw} designed to give $\Delta\varepsilon_{w2} = 0.$, u_{dw} limited to +50.

Since the set-point error is a constant, the a priori assumption will be made that u_d is of the form

$$u_d(t) = b(c_o + c_1 t + c_2), \tag{62}$$

where c_2 is included to represent the fact that the set-point disturbance is constant. The general expression for $\dot{\varepsilon}(t)$ is thus given as

$$\dot{\varepsilon}(t) = \tilde{A}\varepsilon(t) - [Ax_{sp} + Fw(t)] - Bu_d(t). \tag{63}$$

The general solution of Equation (63) for $\varepsilon(t)$ can be written as

$$\begin{aligned}
\varepsilon(t) &= e^{\tilde{A}t}\varepsilon(0) - \int_o^t e^{\tilde{A}(t-\tau)}[Ax_{sp} + Fw(\tau) + Bu_d(\tau)]d\tau \\
&= e^{\tilde{A}t}\varepsilon(0) - \int_o^t e^{\tilde{A}(t-\tau)}[Ax_{sp} + Fc_o + Bb(c_o + c_2)]d\tau \\
&\qquad - \int_o^t e^{\tilde{A}(t-\tau)}(Fc_1 + Bbc_1)\tau d\tau \\
&= e^{\tilde{A}t}\varepsilon(0) - \tilde{A}^{-1}(e^{\tilde{A}t} - I)[Ax_{sp} + Fc_o + Bb(c_o + c_2)] \\
&\qquad - [\tilde{A}^{-2}(e^{\tilde{A}t} - I) - \tilde{A}^{-1}t](F + Bb)c_1
\end{aligned} \tag{64}$$

Again, if one considers the response after the transients have settled out, Equation (64) can be reexpressed as

$$\varepsilon(t) = \tilde{A}^{-1}[Ax_{sp} + Fc_o + Bb(c_o + c_2)] + \tilde{A}^{-1}(\tilde{A}^{-1} + It)(F + Bb)c_1 \ . \tag{65}$$

When the appropriate substitutions are made into Equation (65), the result is

$$\begin{aligned}
\varepsilon(t) &= \begin{bmatrix} 0.0854 & -0.1951 \\ 1.829 & -0.6098 \end{bmatrix} \begin{pmatrix} x_{sp,1} + bc_2 + (1+b)c_o \\ 2bc_2 + (1+2b)c_o \end{pmatrix} \\
&\quad + \begin{bmatrix} -0.35 + 0.0854t & 0.1023 - 0.1951t \\ -0.9591 + 1.829t & 0.015 - 0.6098t \end{bmatrix} \begin{pmatrix} 1+b \\ 1+2b \end{pmatrix} c_1 \\
&= \begin{pmatrix} 0.0854x_{sp,1} - 0.1097w - 0.2477\dot{w} \\ 1.829x_{sp,1} + 1.2192w - 0.9441\dot{w} \end{pmatrix} \\
&\quad + \begin{pmatrix} -0.3048c_2 - 0.3048w - 0.1454\dot{w} \\ 0.6094c_2 + 0.6094w - 0.9291\dot{w} \end{pmatrix} b.
\end{aligned} \tag{66}$$

If Equation (66) is written in the form

$$\varepsilon(t) = u_1 b + u_2 \tag{67}$$

then that b of minimum norm which will minimize the norm of $\varepsilon(t)$ is found to be given by

$$b^* = -(u_1)^{\dagger} u_2 = -(u_1^T u_1)^{-1} u_1^T u_2 \; . \tag{68}$$

When Equation (68) is evaluated, the resulting expression for b* is,

$$b^* = \frac{[1.0886(c_2+w)-1.7117\dot{w}]x_{sp,1}+0.7764w^2+(0.7764w-0.4498\dot{w})c_2-1.6166w\dot{w}+0.9132\dot{w}^2]}{0.4643c_2^2+0.9286c_2w-1.0438c_2\dot{w}+0.4643w^2-1.0438w\dot{w}+0.8843\dot{w}^2} \tag{69}$$

It should be noted that if w(t) is assumed to be zero, the corresponding controller using b* from Equation (69) is

$$u_d = -2.3447x_{sp,1} \; , \tag{70}$$

which is the result obtained in Equation (42).

The digital simulation from Part A was used to generate results with b* as given by Equation (69). The two cases given by $u_d = 0$ and u_d designed to minimize ε_{ss} were again included for purposes of comparison. Results are presented in Figures 20 through 24.

As shown in Figure 20, when w(t) is a constant the controller from this section and that designed to minimize ε_{ss} give identical results. Figures 21 through 24 repeat the cases shown in Figures 2 through 5, respectively. A comparison of Figures 21 and 2 shows that with no interaction possible between the set-point and external disturbances (since the set-point is the origin), the disturbance minimizing controllers of Part A and Part B give identical results. When interactions are possible, i.e., neither the set-point nor external disturbances are zero, a diversity of results occurs. Comparing Figures 22 and 3, it can be seen that the controller of Part B gave better overall results than did the controller of Part A. A comparison of Figures 23 and 4 shows that the controller of Part B resulted in much better performance than did the controller of Part A, except around a time of 3.5 seconds, with zero error at two different times. However, in comparing Figures 24 and 5 it can be seen that the controller of Part B gave slightly better results between a time of 2 and 5 seconds, and much worse results between a time of 5 and 9 seconds, as compared to the controller of Part A. It is also apparent that, except possibly for the case shown in Figure 23, the controller designed to minimize ε_{ss} still results in the best overall performance.

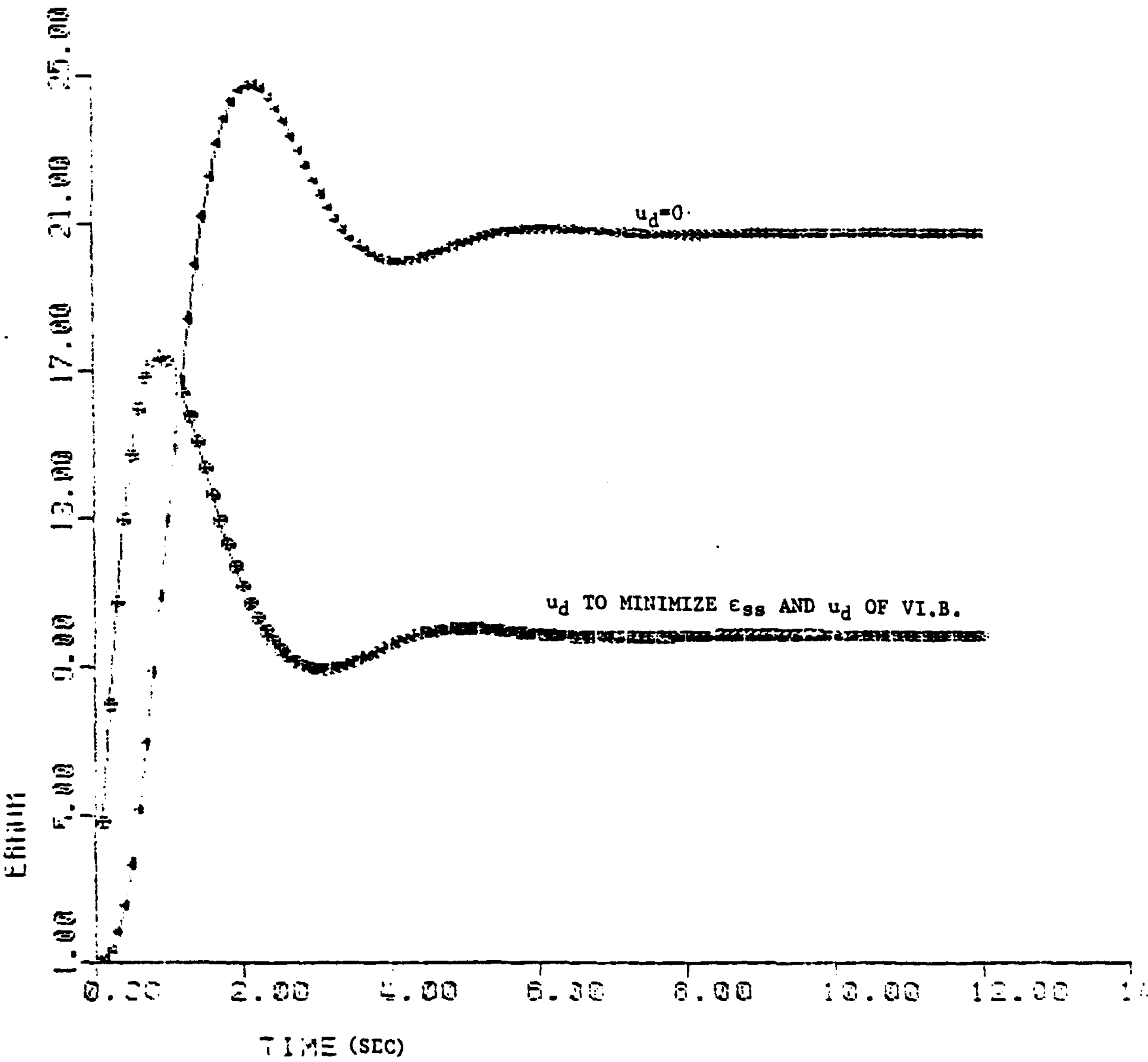

Figure 20. Error versus time, with unallocated control vector, $X_{sp} = (10,0)^T$, w = 2.

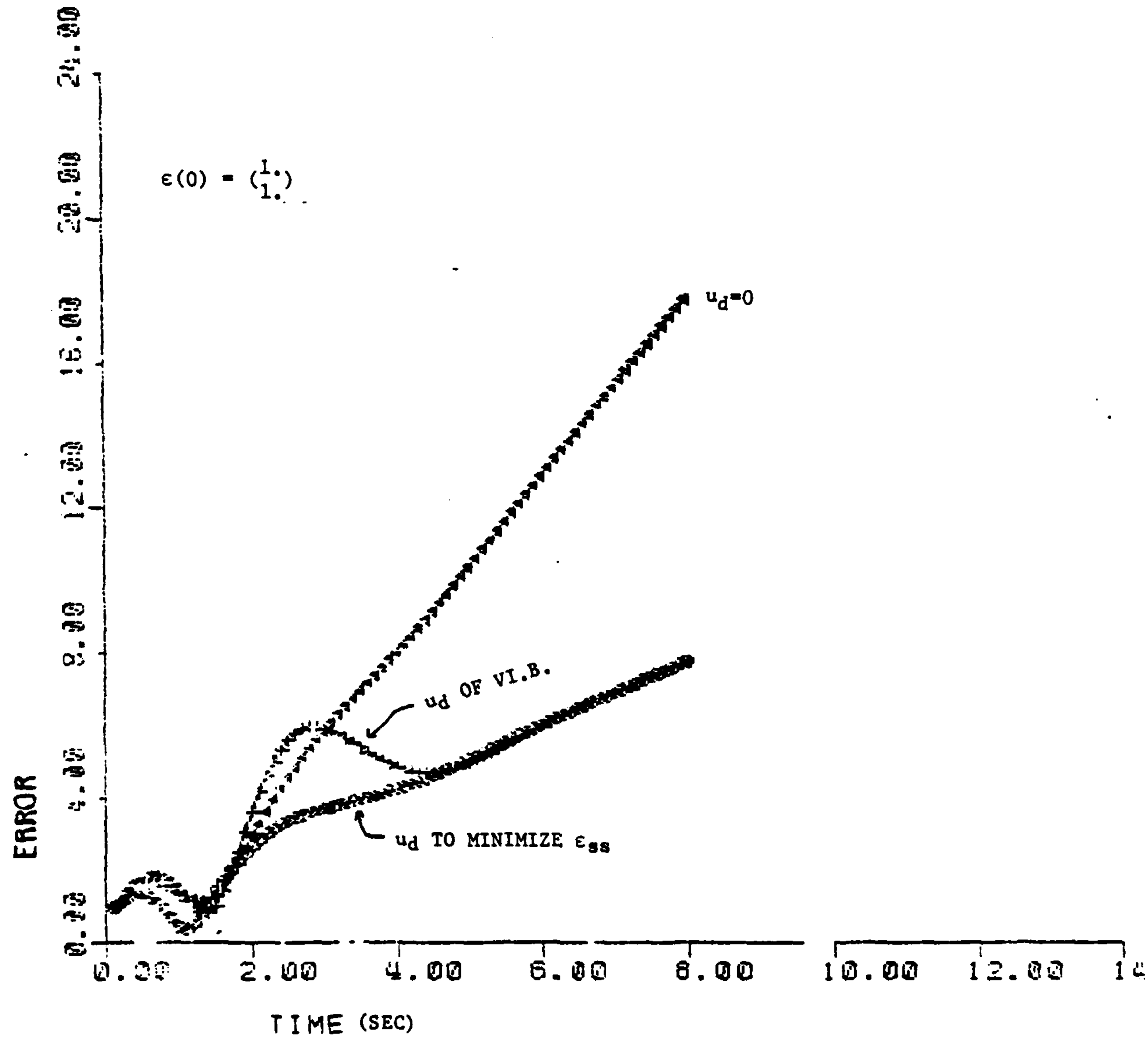

Figure 21. Error versus time, with unallocated control vector, X_{sp}, $(0,0)^T$, $w = 2t$.

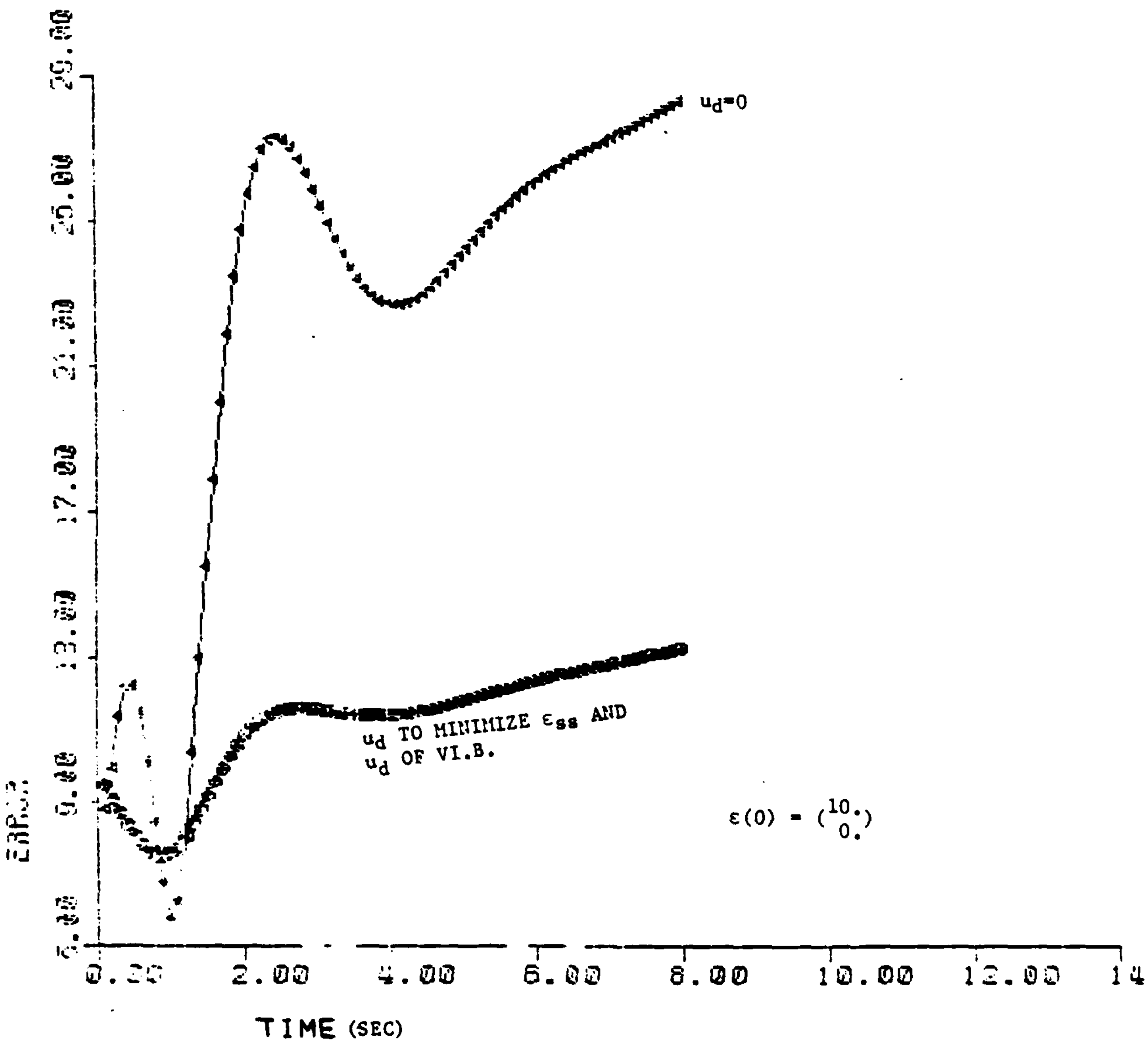

Figure 22. Error versus time, with unallocated control vector, $X_{sp} = (10,0)^T$, $w = 1+t$.

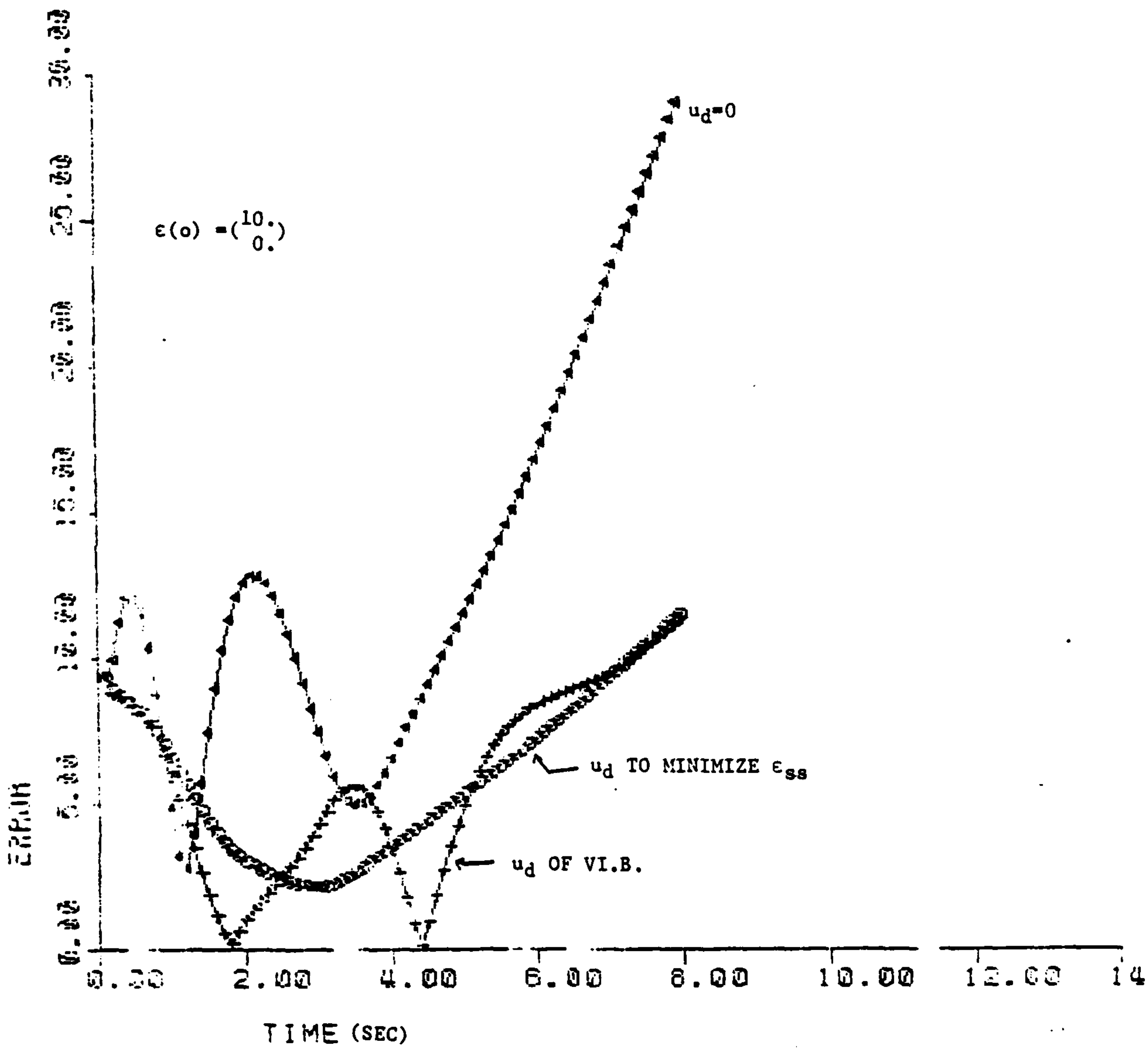

Figure 23. Error versus time, with unallocated control vector, $X_{sp} = (10,0)^T$, $w = -2 - 5t$.

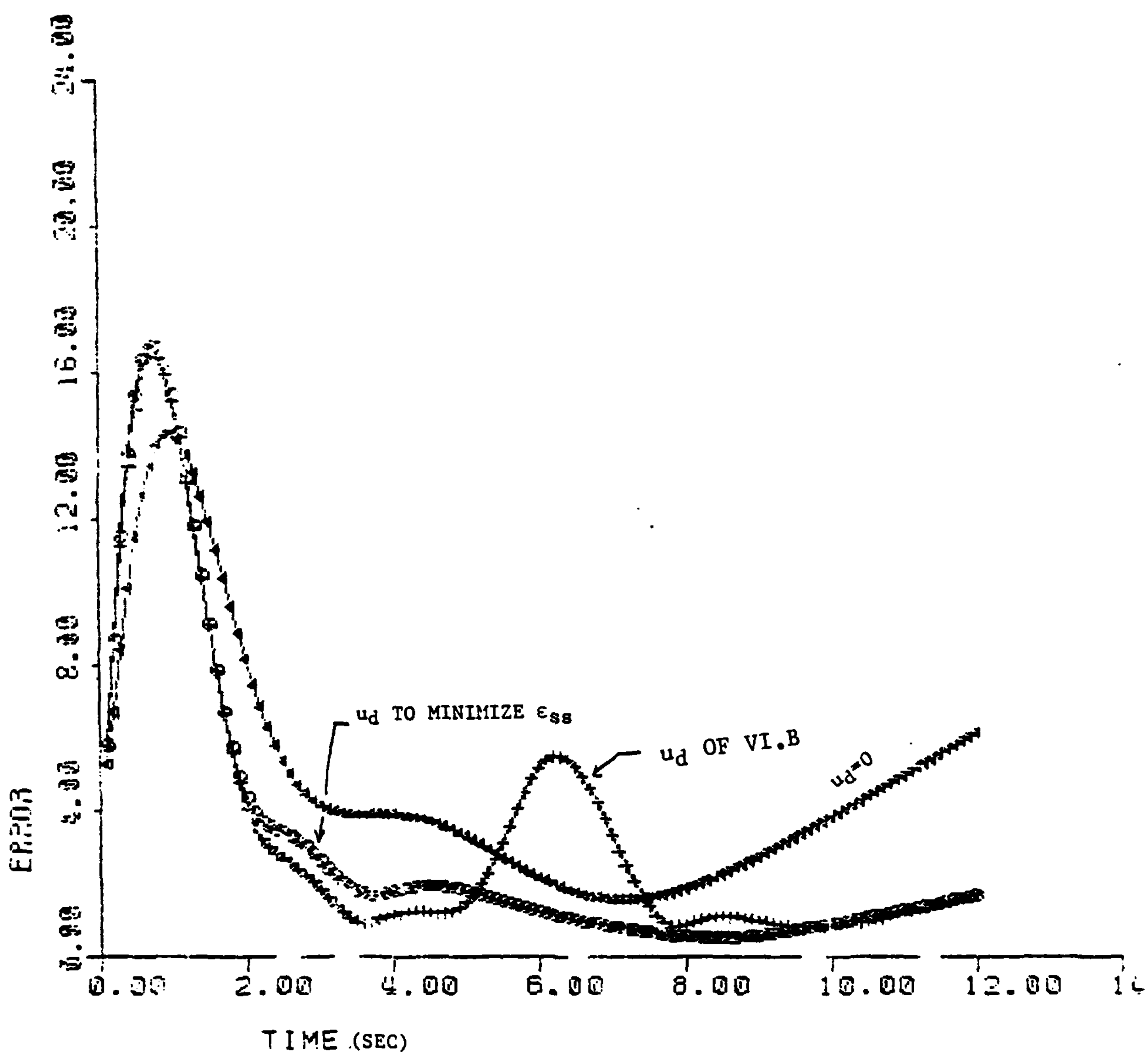

Figure 24. Error versus time, with unallocated control vector, $X_{sp} = (-5,0)^T$, $w = 1+t$.

VII. DISTURBANCE MINIMIZATION WITH $w(t) = c_1 \sin\alpha t$

This section will apply the isobasis design technique to the state set-point regulator example with an external disturbance of the form

$$w(t) = c_1 \sin\alpha t \; . \tag{71}$$

For this case, the a priori assumption is made that u_{dw} is of the form

$$u_{dw}(t) = bc_1 \sin\alpha t = bw(t), \tag{72}$$

and u_{ds} is as shown in Equation (42).

That part of the solution of Equation (17) for $\varepsilon(t)$ which is due to the last term on the right-hand side of Equation (17) will again be found, with u_{dw} as given by Equation (72). In this case then, one has

$$\begin{aligned}\Delta\varepsilon_w &= -\int_0^t e^{\tilde{A}(t-\tau)}[Fw(\tau) + Bbw(\tau)]d\tau \\ &= -\int_0^t e^{\tilde{A}(t-\tau)}(F + Bb)c_1 \sin\alpha\tau d\tau \\ &= -(\tilde{A}^{-2} + \alpha^2 I)^{-1}(-\tilde{A}\sin\alpha t - \alpha I\cos\alpha t + e^{\tilde{A}t}\alpha)(F + Bb)c_1 \; .\end{aligned} \tag{73}$$

The response after the initial system transients have settled out will again be considered in designing u_{dw}, thus, Equation (73) is rewritten as

$$\Delta\varepsilon_w = -\hat{A}^{-1}(-\tilde{A}w - \dot{w}I)(F + Bb), \tag{74}$$

where $\hat{A} = (\tilde{A}^2 + \alpha^2 I)$. From Equation (74) one can find that that b which is itself of minimum norm and which minimizes the norm of $\Delta\varepsilon_w$ is given by

$$b^* = -[\hat{A}^{-1}(-\tilde{A}w - \dot{w}I)B]^{\dagger}\hat{A}^{-1}[-\tilde{A}w - \dot{w}I]F. \tag{75}$$

When the appropriate substitutions are made into Equation (75), the following is obtained,

$$\hat{A}^{-1} = \frac{1}{DET}\begin{bmatrix} -3.762 + \alpha^2 & 1.101 \\ -10.72 & 0.16 + \alpha^2 \end{bmatrix} \tag{76}$$

$$\text{where } DET = \alpha^4 - 3.602\alpha^2 + 10.76 \; , \tag{77}$$

$$\hat{A}^{-1}(-\tilde{A}w - \dot{w}I)F = \frac{1}{DET}\begin{pmatrix} (1.182 + 1.36\alpha^2)w + (2.661 - \alpha^2)\dot{w} \\ (-13.12 + 5.72\alpha^2)w + (10.16 - \alpha^2)\dot{w} \end{pmatrix}, \tag{78}$$

$$\hat{A}^{-1}(-\tilde{A}w - \dot{w}I)B = \frac{1}{DET}\begin{pmatrix} (3.28 + 0.72\alpha^2)w + (1.56 - \alpha^2)\dot{w} \\ (-6.56 + 5.44\alpha^2)w + (10 - 2\alpha^2)\dot{w} \end{pmatrix}, \tag{79}$$

and

$$[\hat{A}^{-1}(-\tilde{A}w-\dot{w}I)B]^{\dagger} = [[\hat{A}^{-1}(-\tilde{A}w-\dot{w}I)B]^T[\hat{A}^{-1}(-\tilde{A}w-\dot{w}I)B]]^{-1}[\hat{A}^{-1}(-\tilde{A}w-\dot{w}I)B] \quad . \tag{80}$$

When Equation (75) is evaluated, one has

$$b^* = -\frac{c_{11}\dot{w}^2 + c_{12}w\dot{w} + c_{13}\dot{w}^2}{c_{21}w^2 + c_{22}w\dot{w} + c_{23}\dot{w}^2} \quad , \tag{81}$$

where

$$\begin{aligned} c_{11} &= 89.95 - 103.59\alpha^2 + 32.1\alpha^4 \\ c_{12} &= -187.28 + 145.55\alpha^2 - 18.96\alpha^4 \\ c_{13} &= 105.75 - 34.54\alpha^2 + 3\alpha^4 \end{aligned} \tag{82}$$

$$\begin{aligned} c_{21} &= 53.79 - 66.65\alpha^2 + 30.11\alpha^4 \\ c_{22} &= -120.97 + 130.73\alpha^2 - 23.2\alpha^4 \\ c_{23} &= 102.43 - 43.12\alpha^2 + 5\alpha^4 \end{aligned} \tag{83}$$

The controller defined by Equation (72) using Equation (81) to Equation (83) was added to the digital simulation of the set-point regulator example of Section VI. Figures 25 through 31 show results obtained with the u_d of this section as compared to results with $u_d = 0$ and u_d designed to minimize ε_{ss} under the assumption that w is a constant. As can be seen from these results, the external disturbance minimizing control vector u_d designed via the isobasis design technique does provide better performance, in this case, than u_d as designed to minimize ε_{ss}, except at low frequencies. An examination of these figures shows that for the low frequency cases (Figures 25, 26, 30 and 31) the controller designed to minimize ε_{ss} provided no, or at least a small, decrease in the peak-to-peak amplitude of the sinusoidal external disturbance effect over the case with $u_d = 0$. However, as the frequency of the external disturbance term increased, this controller resulted in an increase in this peak-to-peak amplitude. The controller designed in this section, on the other hand, generally resulted in a decrease in this amplitude. The exceptions were for the two cases where $\alpha = 1$ rad/sec. As the frequency increased, this decrease became substantial.

In order to obtain a better indication of the performance obtained via use of the isobasis technique for a sinusoidal external disturbance, several sets of runs were made. The first set was for a case with $x_{sp,1} = 10$. Figure 32 shows the percent by which the peak-to-peak magnitude of the sinusoidal component of the error was reduced, as a function of the frequency of w(t), by using u_d as designed in this section. Figures 33 and 34 show the percent reduction, as a function of the amplitude of w(t), for frequencies of 3 and 10 rad/sec, respectively. Figures 35 through 37 present similar data for a case with $x_{sp,1} = -5$. All data in each case was obtained after the initial transients had settled out. As can be seen from examination of Figures 32 and 35, for frequencies above 2 rad/sec (with amplitude held constant) the disturbance minimization controller of this section did reduce the amplitude of the sinusoidal contribution to the total error, as compared to both the $u_d = 0$ case

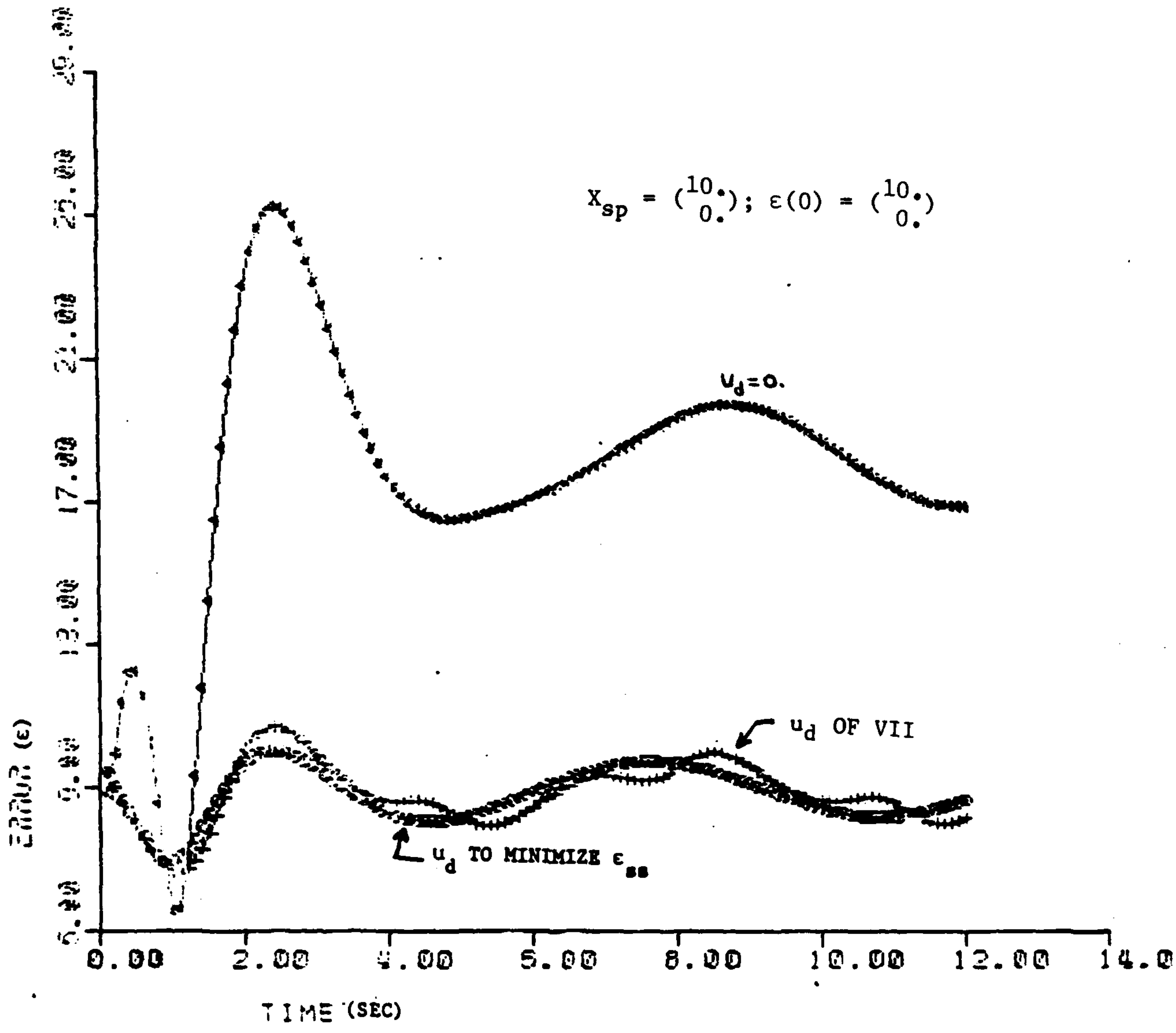

Figure 25. Error versus time, $X_{sp}=(10,0)^T$, w=SIN(t).

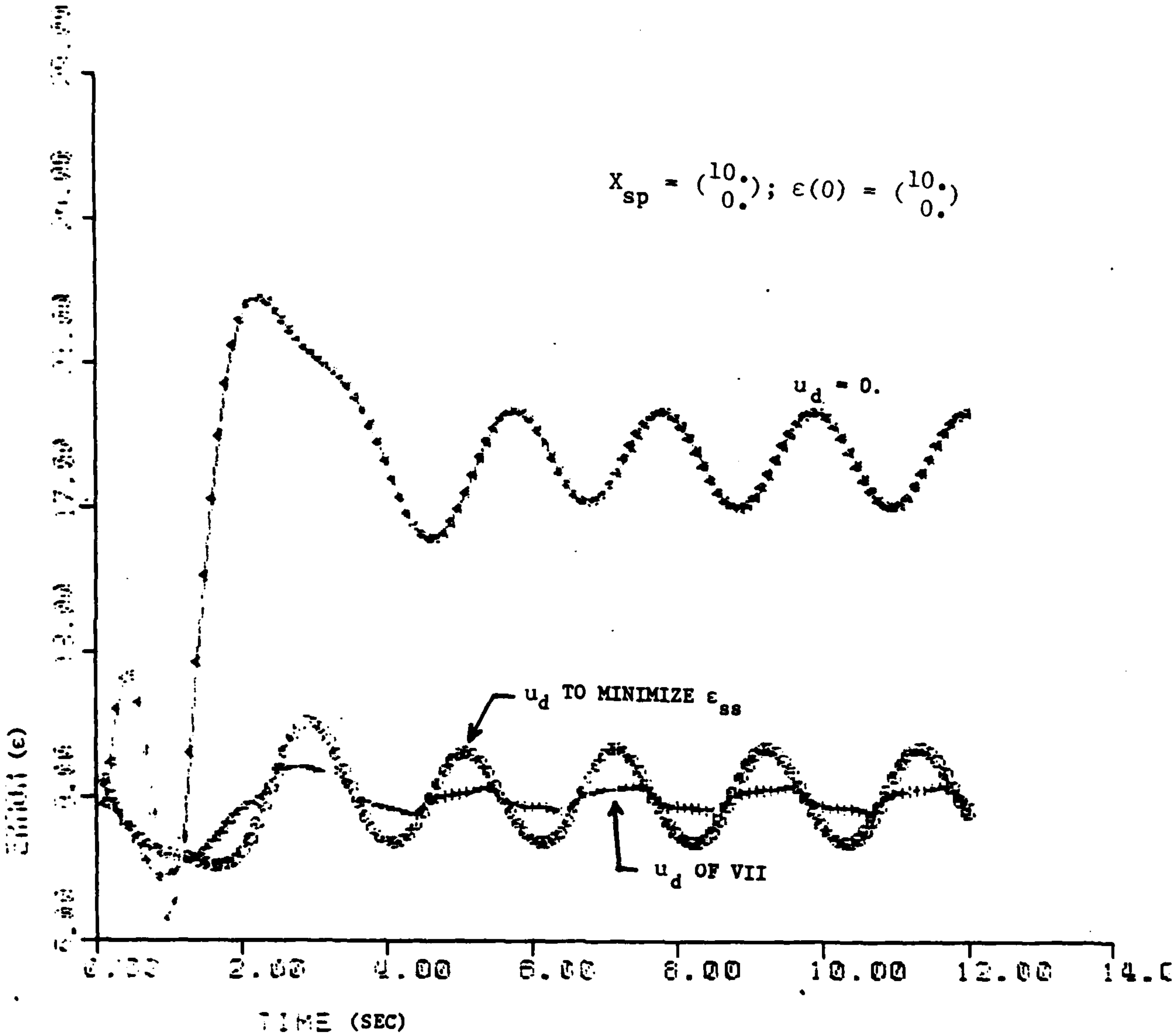

Figure 26. Error versus time, $X_{sp}=(10,0)^T$, w=2SIN(3t).

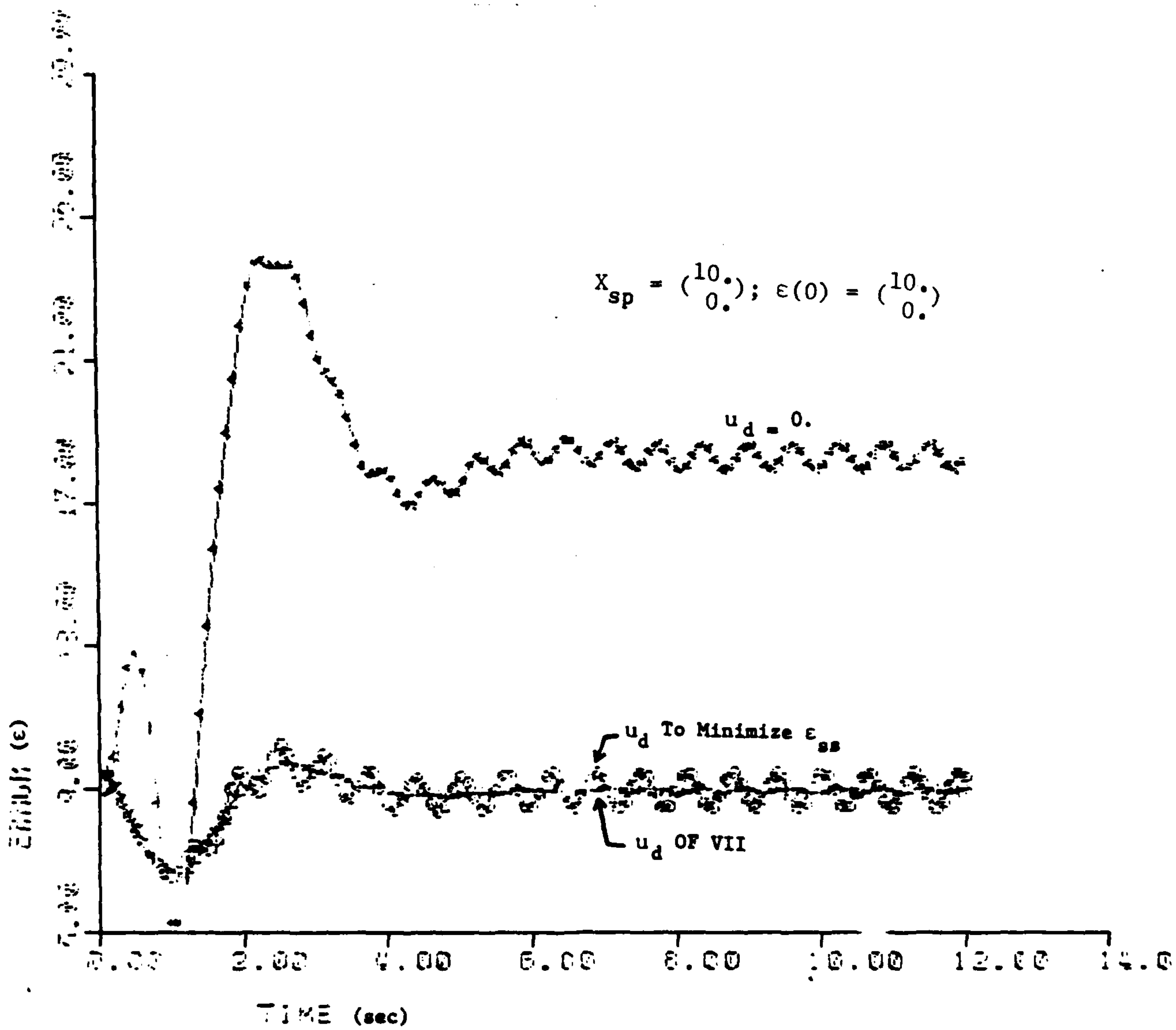

Figure 27. Error versus time, $X_{sp}=(10,0)^T$, $w=3SIN(10t)$.

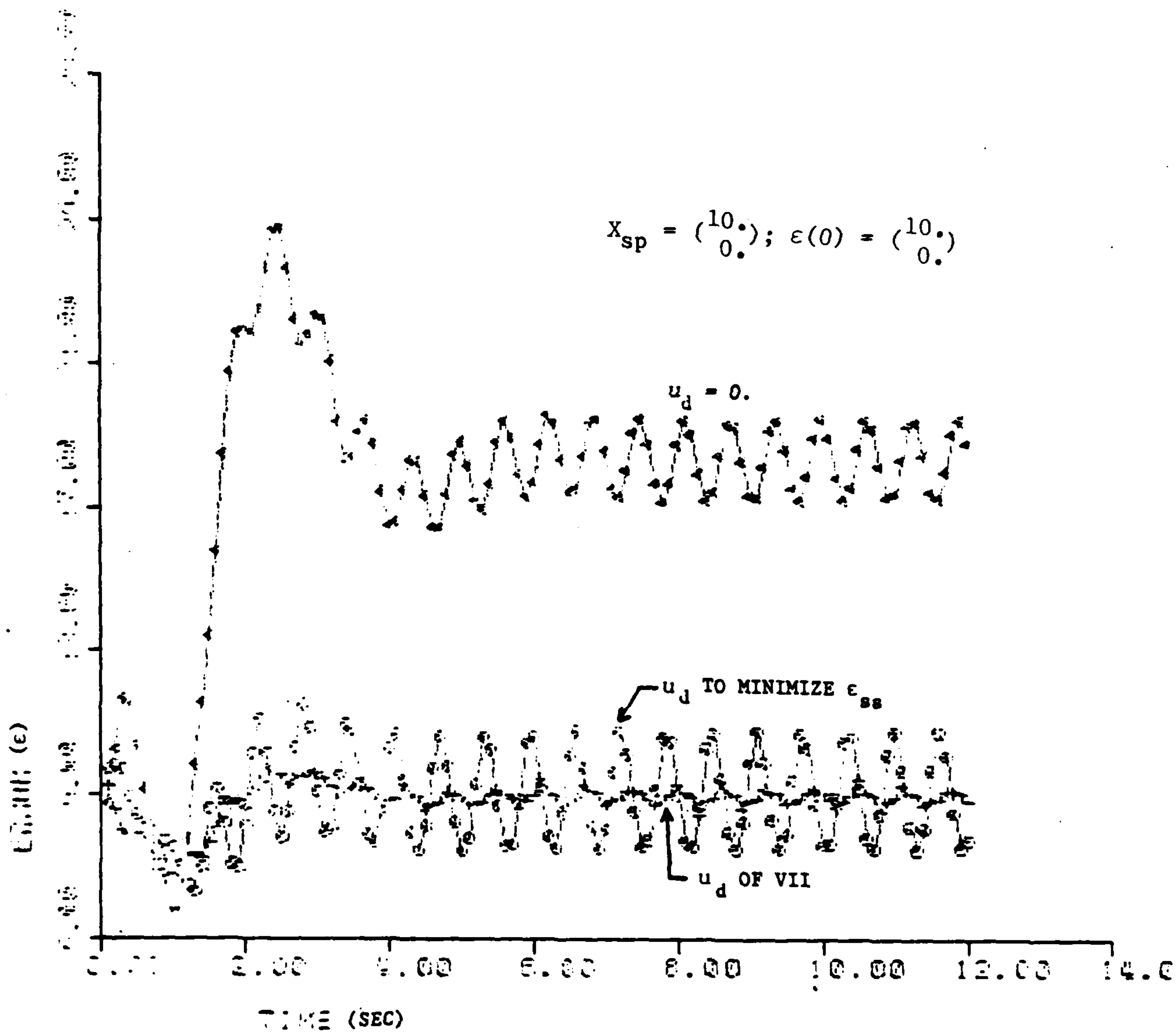

Figure 28. Error versus time, $X_{sp}=(10,0)^T$, w=10SIN(10t).

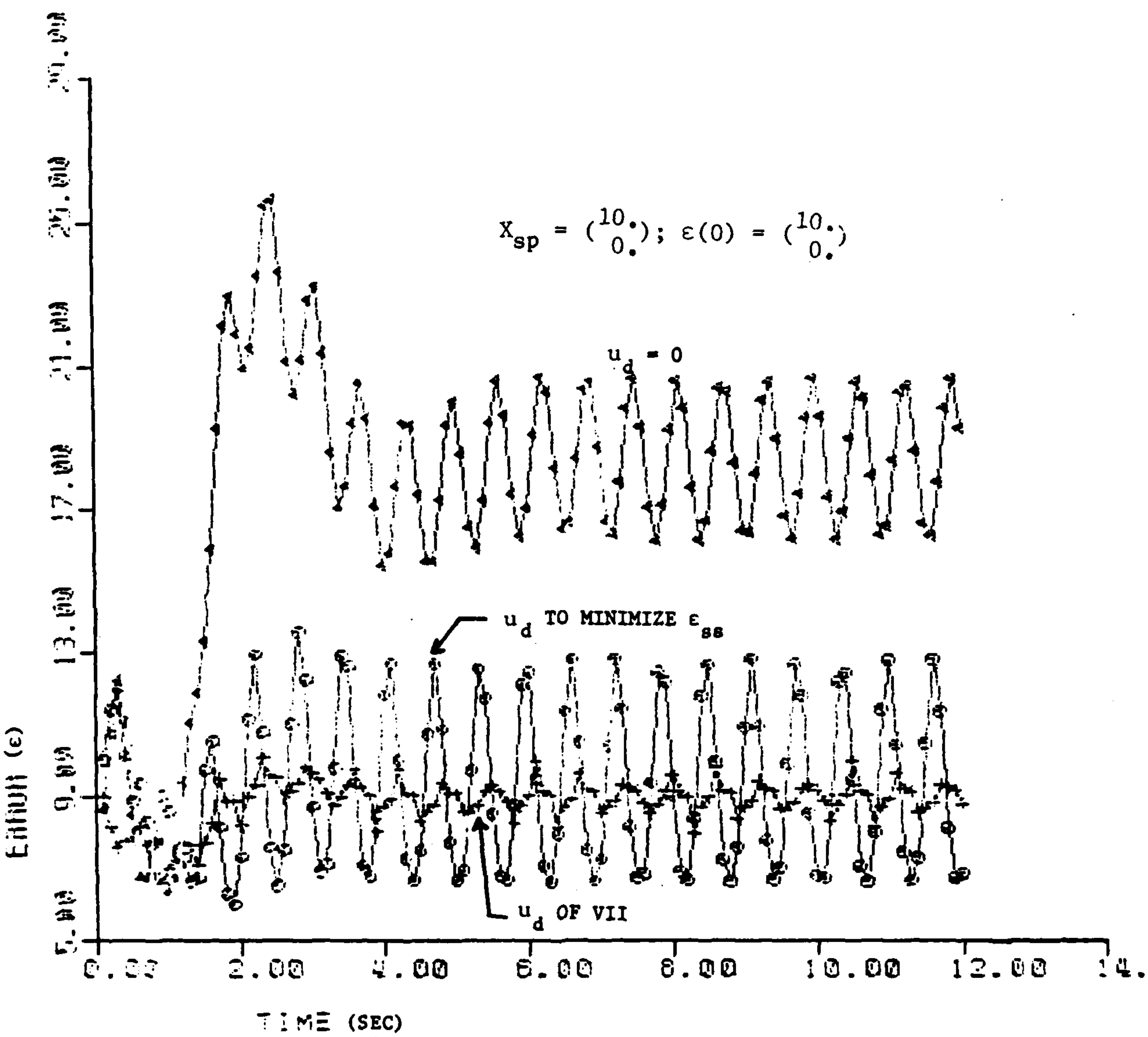

Figure 29. Error versus time, $X_{sp}=(10,0)^T$, w=20SIN(10t).

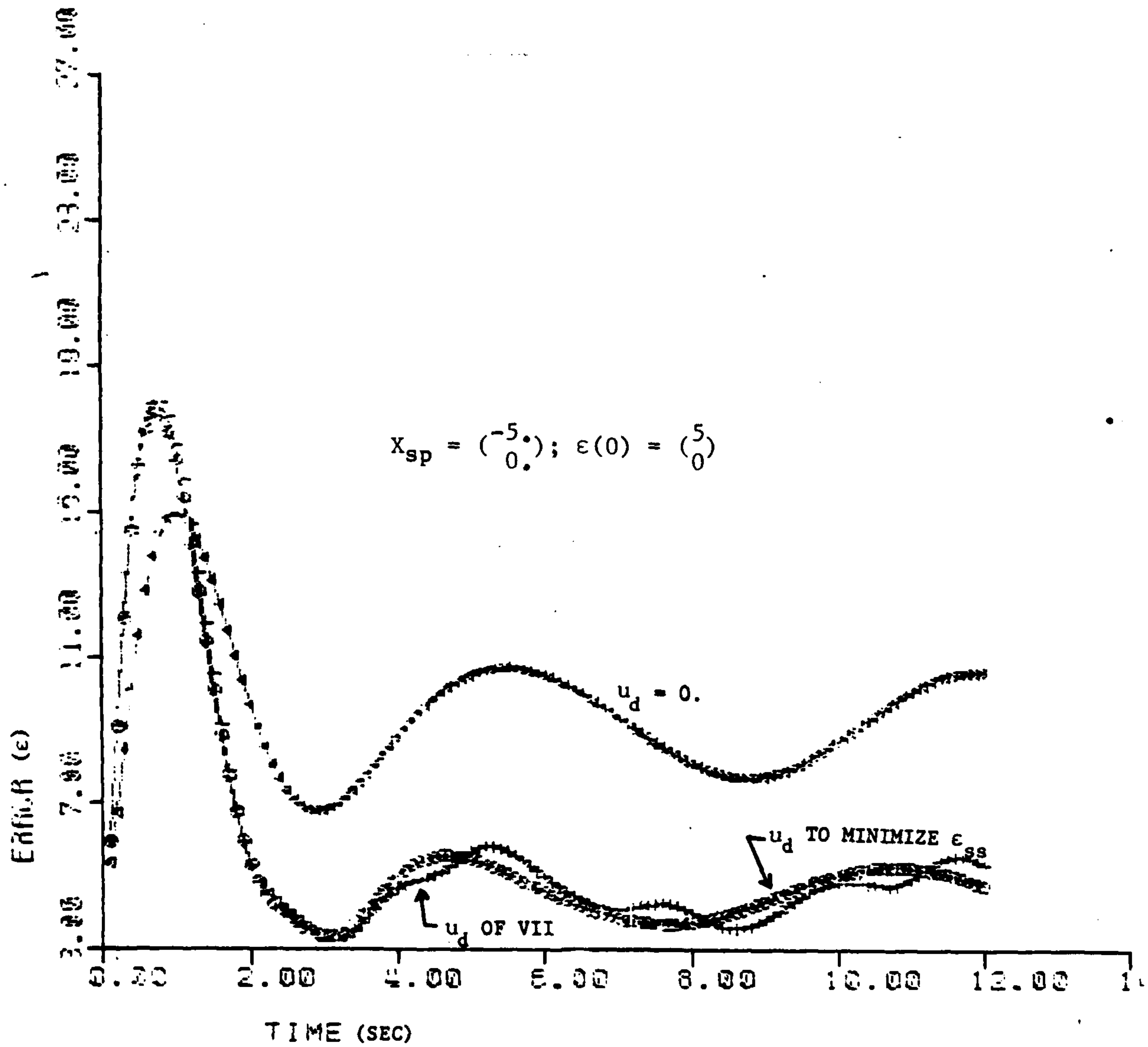

Figure 30. Error versus time, $X_{sp}=(-5,0)^T$, w=SIN(t).

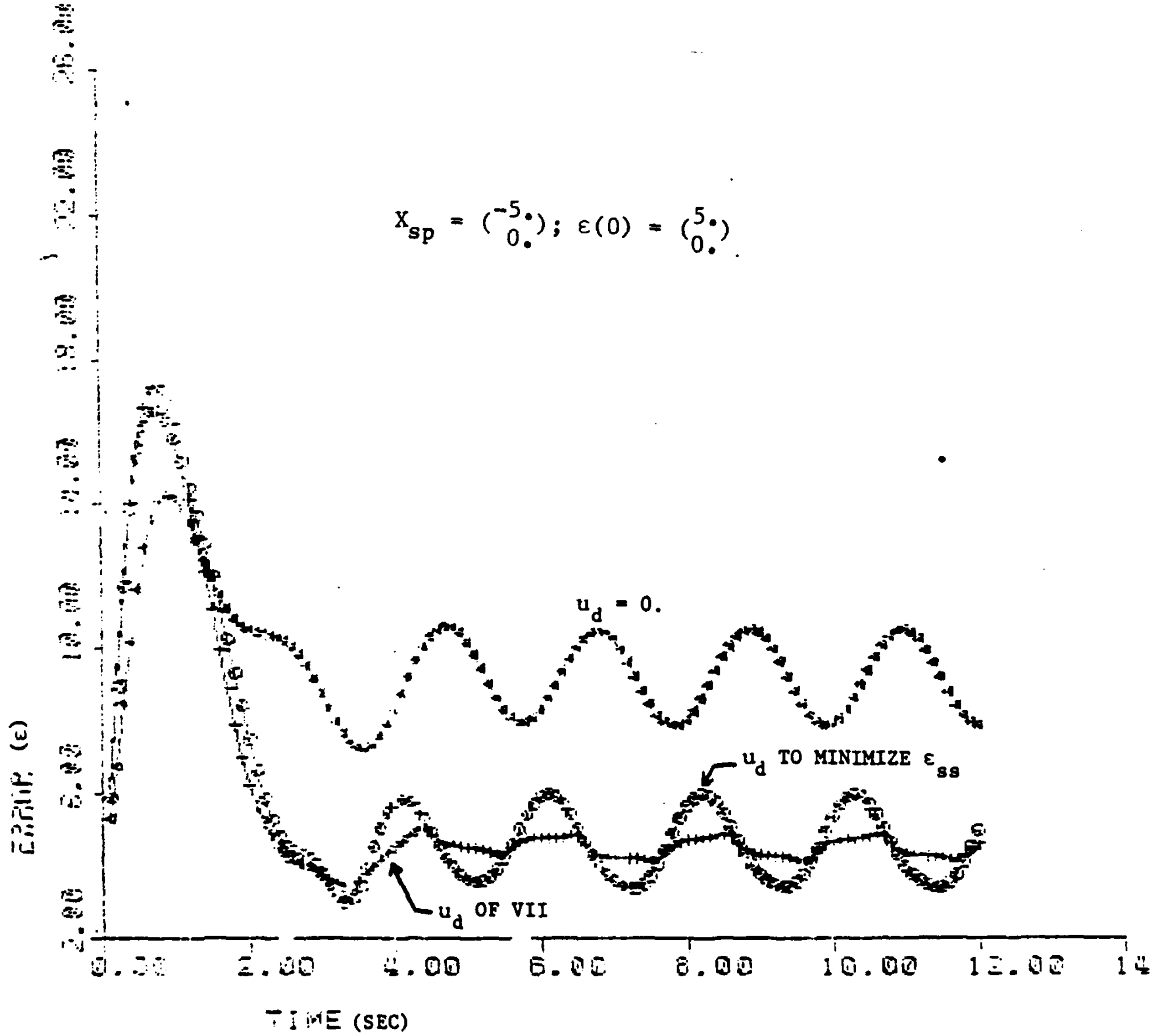

Figure 31. Error versus time, $X_{sp}=(-5,0)^T$, $w=2SIN(3t)$.

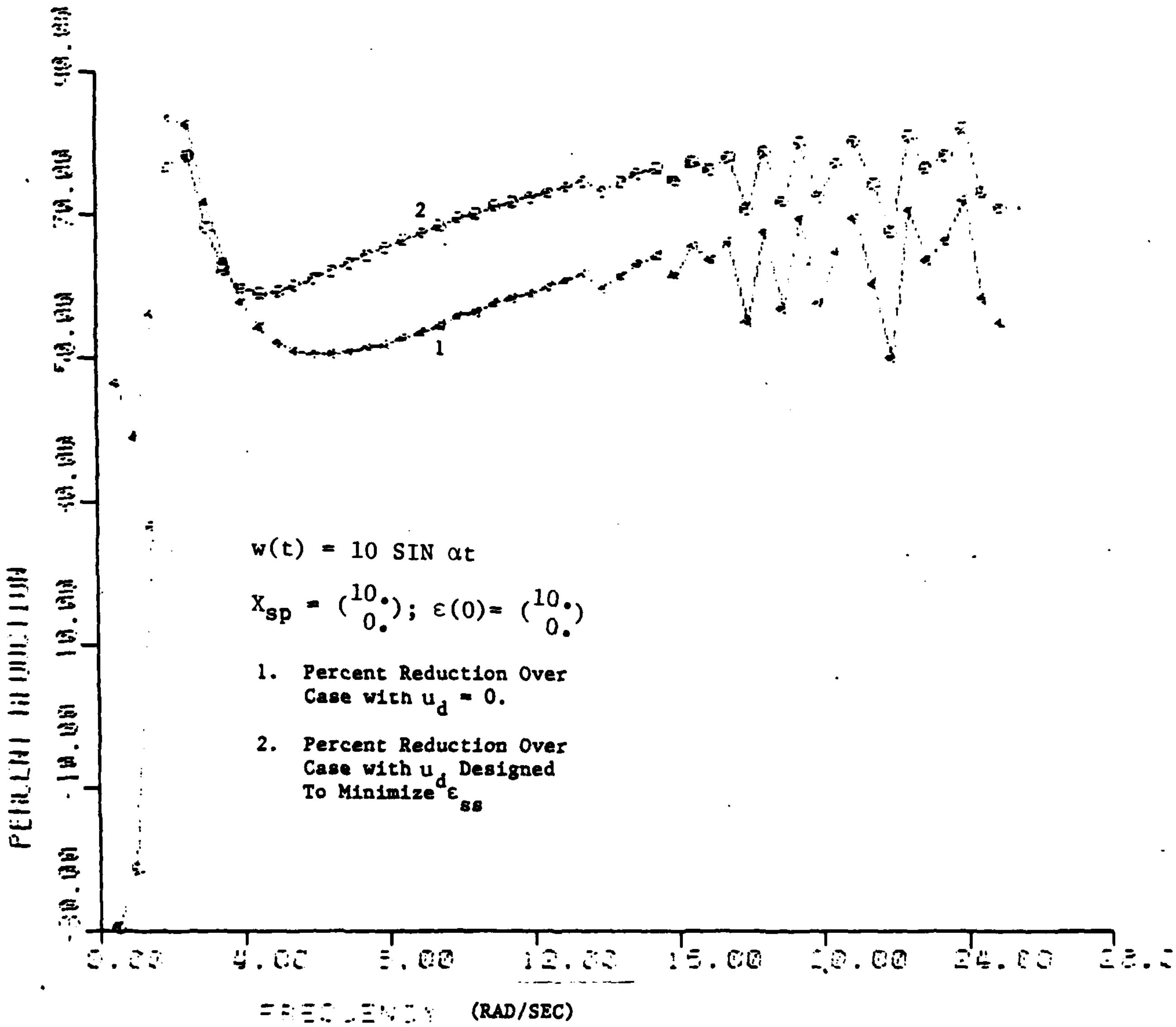

Figure 32. Percent reduction in amplitude of sinusoidal error contribution as function of external disturbance frequency, $X_{sp}=(10,0)^T$.

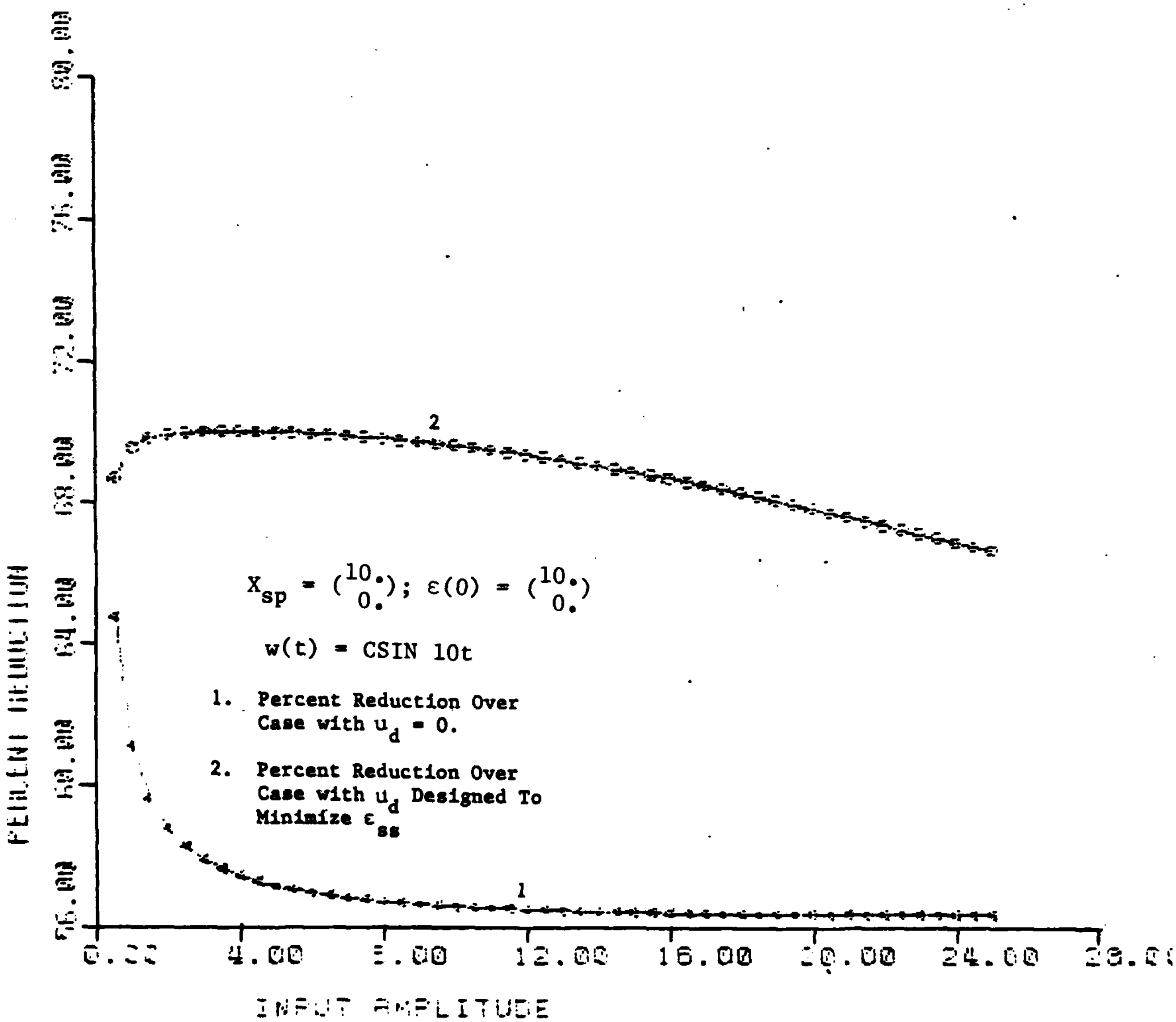

Figure 34. <u>Percent reduction in amplitude of sinusoidal error contribution as function of external disturbance amplitude, α=10 RAD/SEC, $X_{sp}=(10,0)^T$.</u>

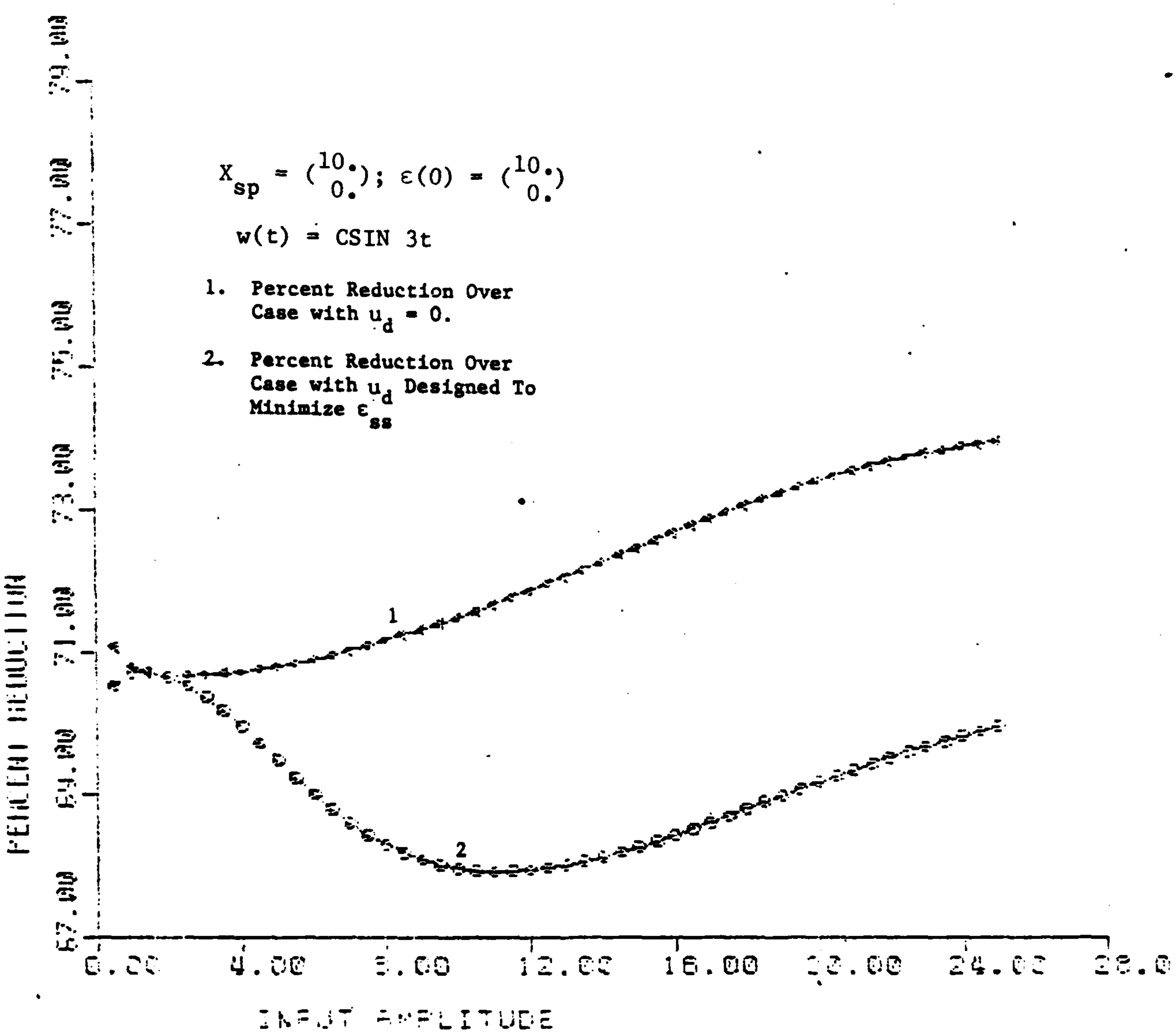

Figure 33. Percent reduction in amplitude of sinusoidal error contribution as function of external disturbance amplitude, α=3 RAD/SEC, $X_{sp}=(10,0)^T$.

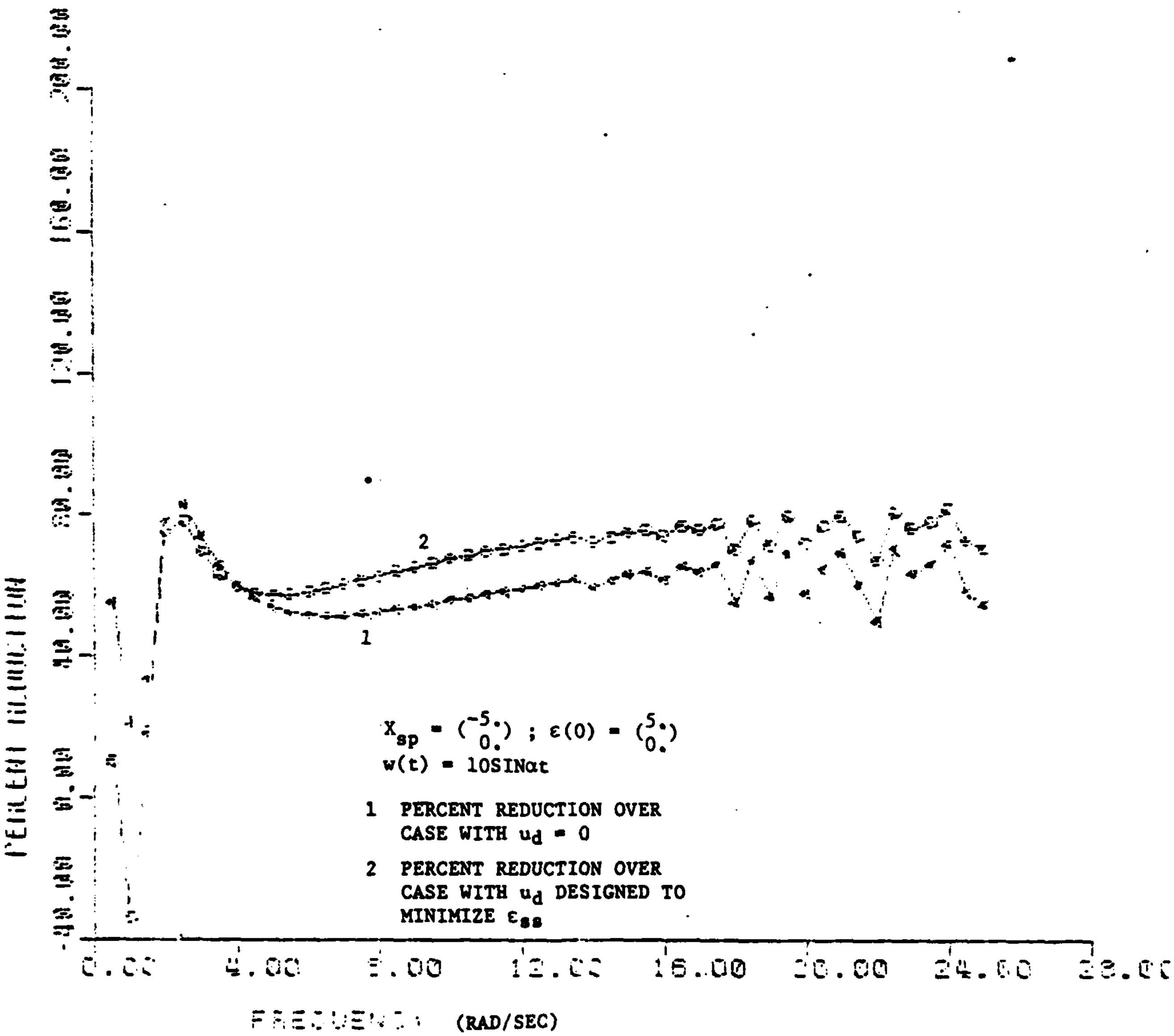

Figure 35. Percent reduction in amplitude of sinusoidal error contribution as function of external disturbance frequency, $X_{sp}=(-5,0)^T$.

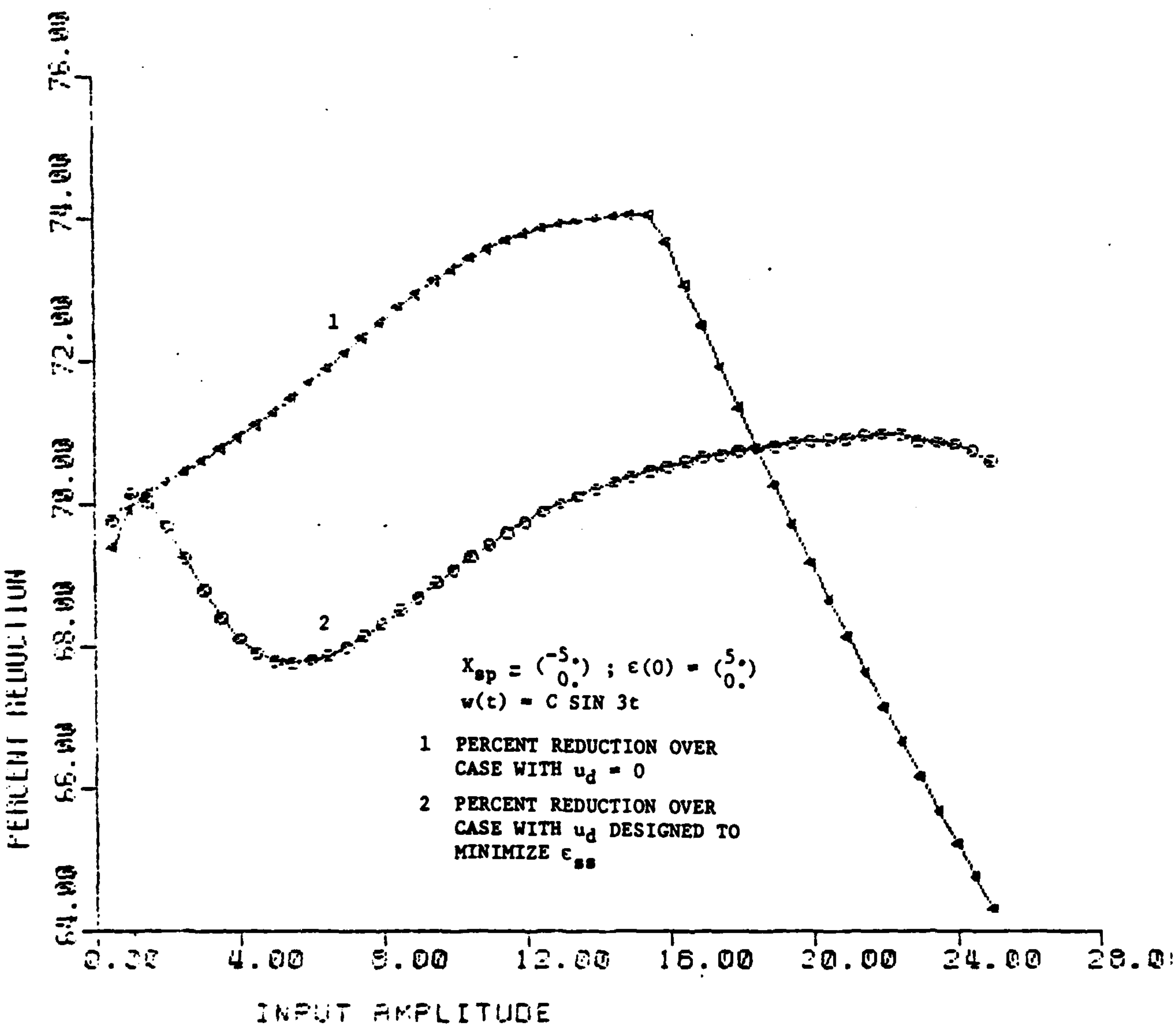

Figure 36. Percent reduction in amplitude of sinusoidal error contribution as function of external disturbance amplitude, α=3 RAD/SEC, $X_{sp}=(-5,0)^T$.

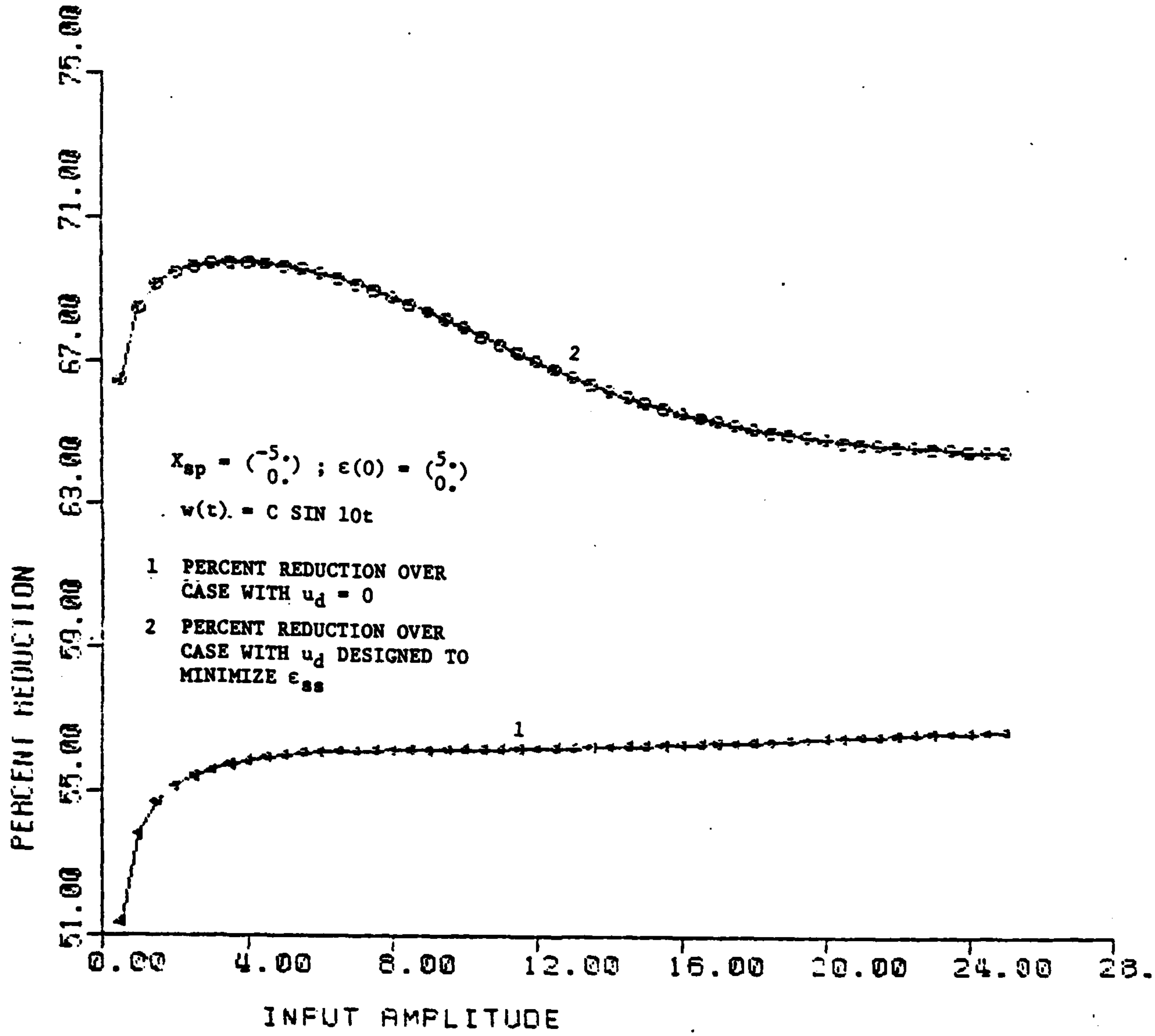

Figure 37. Percent reduction in amplitude of sinusoidal error contribution as function of external disturbance amplitude, α=10 RAD/SEC, $X_{sp}=(-5.,0)^T$.

and the case with u_d designed to minimize ϵ_{ss}. For frequencies below about 1.5 rad/sec, the controller designed to minimize ϵ_{ss} gives better performance. At frequencies above about 25 rad/sec, it was difficult to measure performance accurately due to the small amplitude and increasingly erratic nature of the sinusoidal component of the response produced by the u_d of this section.

VIII. DISTURBANCE MINIMIZATION WITH $w(t) = ce^{\alpha t}$

This section will apply the isobasis design technique to the state set-point regulator example with an external disturbance of the form

$$w(t) = ce^{\alpha t}. \tag{84}$$

For this case, the a priori assumption is made that u_{dw} is of the form

$$u_{dw}(t) = bce^{\alpha t} = bw(t), \tag{85}$$

and u_{ds} is as shown in Equation (42).

That part of the solution of Equation (17) for $\epsilon(t)$ which is due to the last term on the right-hand side of Equation (17) will again be found, with u_{dw} as given by Equation (85). In this case then, one has

$$\Delta\epsilon_w = -\int_0^t e^{\tilde{A}(t-\tau)}(F+Bb)ce^{\alpha\tau}d\tau$$

$$= -(\alpha I-\tilde{A})^{-1}[e^{\alpha It}-e^{\tilde{A}t}](F+Bb)c. \tag{86}$$

If only the response after transients have settled out is considered, Equation (86) can be rewritten as

$$\Delta\epsilon_w = -\hat{A}^{-1}(F+Bb)w \ , \tag{87}$$

where $\hat{A} = \alpha I - \tilde{A}$.

From Equation (87) one finds that that b which is itself of minimum norm and which minimizes the norm of $\Delta\epsilon_w$ is given by

$$b^* = -(\hat{A}^{-1}Bw)^{\dagger}\hat{A}^{-1}Fw. \tag{88}$$

Evaluating the terms in Equation (88) results in

$$\hat{A} = \alpha I - \tilde{A} = \begin{bmatrix} \alpha+2 & -0.64 \\ 6 & \alpha-0.28 \end{bmatrix}, \tag{89}$$

$$\hat{A}^{-1} = \frac{1}{DET} \begin{bmatrix} \alpha-0.28 & 0.64 \\ -6 & \alpha+2 \end{bmatrix} \tag{90}$$

where

$$DET = \alpha^2 + 1.72\alpha + 3.28, \tag{91}$$

$$(\hat{A}^{-1}Bw)^{\dagger} = \left(\frac{\alpha+1}{5\alpha^2-6\alpha+5} \quad , \quad \frac{2(\alpha-1)}{5\alpha^2-6\alpha+5} \right) \quad \left(\frac{DET}{w} \right) \tag{92}$$

and

$$b^* = -\frac{3\alpha^2-8.64\alpha+8.36}{5\alpha^2-6\alpha+5} \tag{93}$$

The controller defined by Equation (85), using Equation (93), and by Equation (42) was added to the set-point regulator digital simulation. Results are presented in Figures 38 to 45 for a case with the origin as the target setpoint. Data is included on each figure for the $u_d = 0$ case and the case with u_d to minimize ε_{ss}. From Figures 38 to 40, for cases with negative exponents in w(t), it can be seen that the isobasis controller design provides the lowest error. From Figure 41, however, it can be seen that as α becomes increasingly negative, the isobasis controller loses its advantage. Figures 42 through 45 present results for cases with positive exponent values in w. The isobasis controller provided the lowest overall error in all cases. For the case in Figure 45, the controller designed to minimize ε_{ss} produced an error twice as large as for the results shown. Figures 46 through 49 present results for two non-zero target set-points. The results are mixed for these four cases, but the isobasis controller can provide smaller errors in some cases.

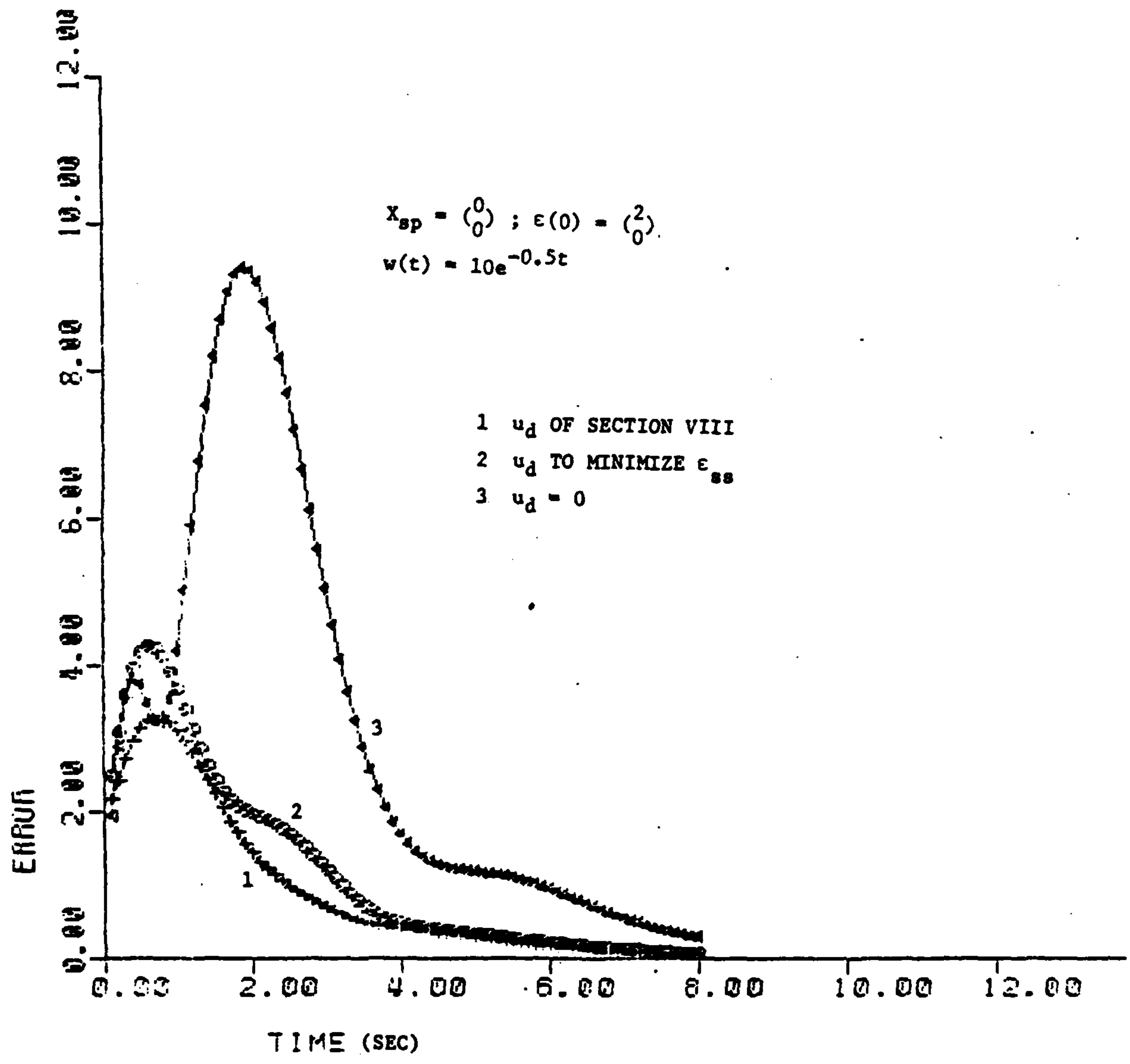

Figure 38. Error versus time, $X_{sp} = (0,0)^T$, $w(t) = 10e^{-0.5t}$.

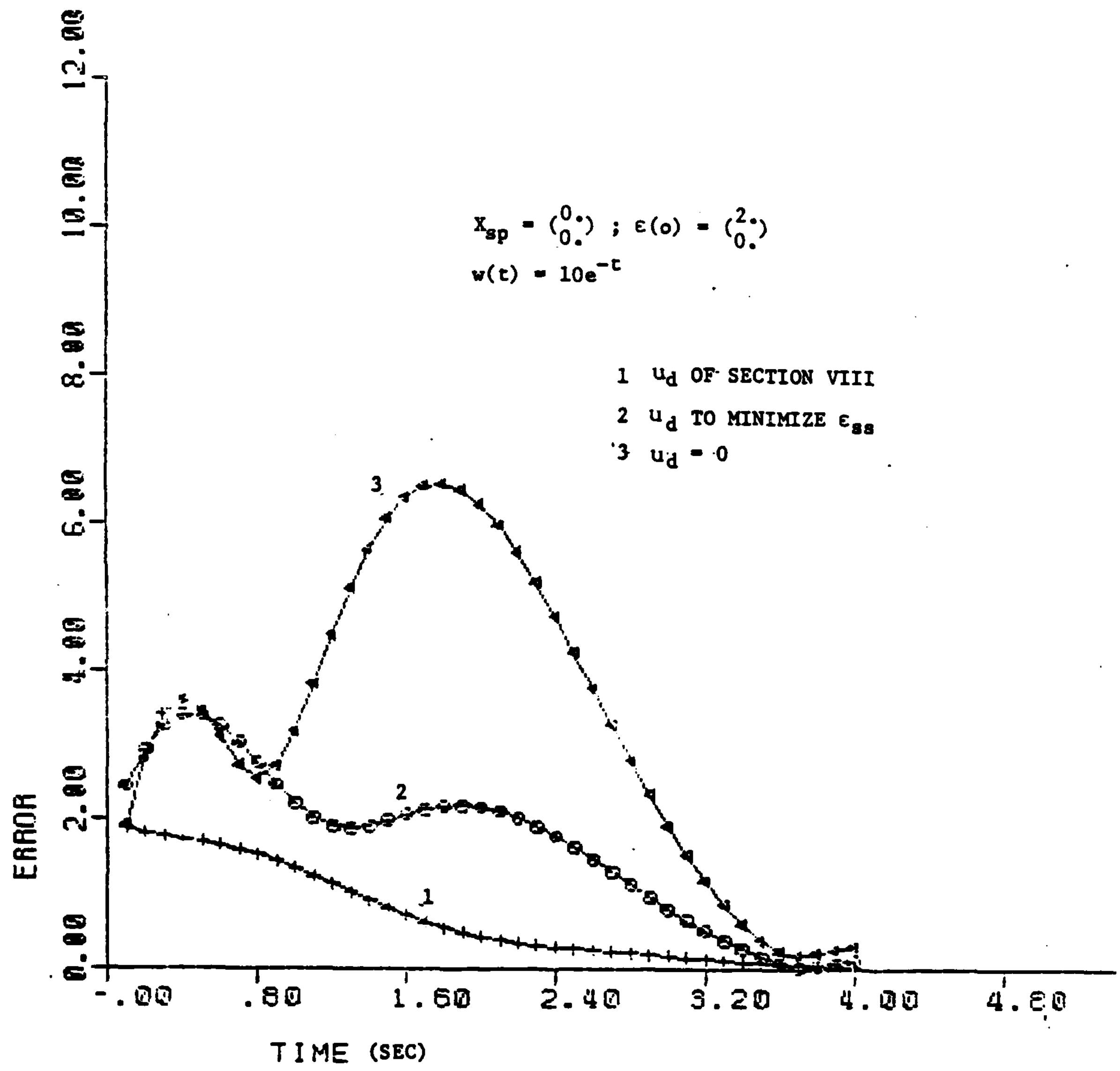

Figure 39. Error versus time, $X_{sp} = (0,0)^T$, $w = 10e^{-t}$.

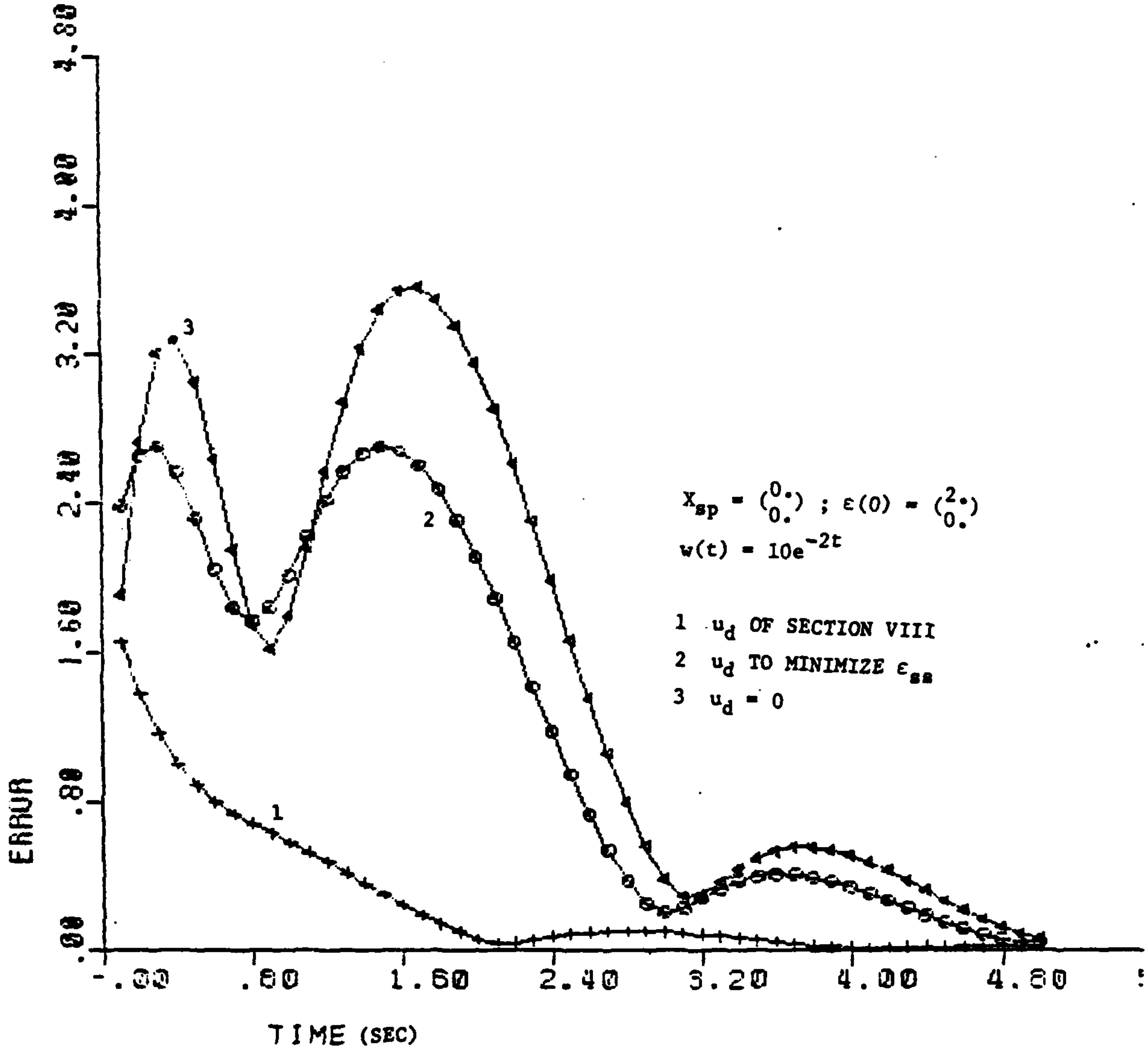

Figure 40. Error versus time, $X_{sp}=(0,0)^T$, $w=10e^{-2t}$.

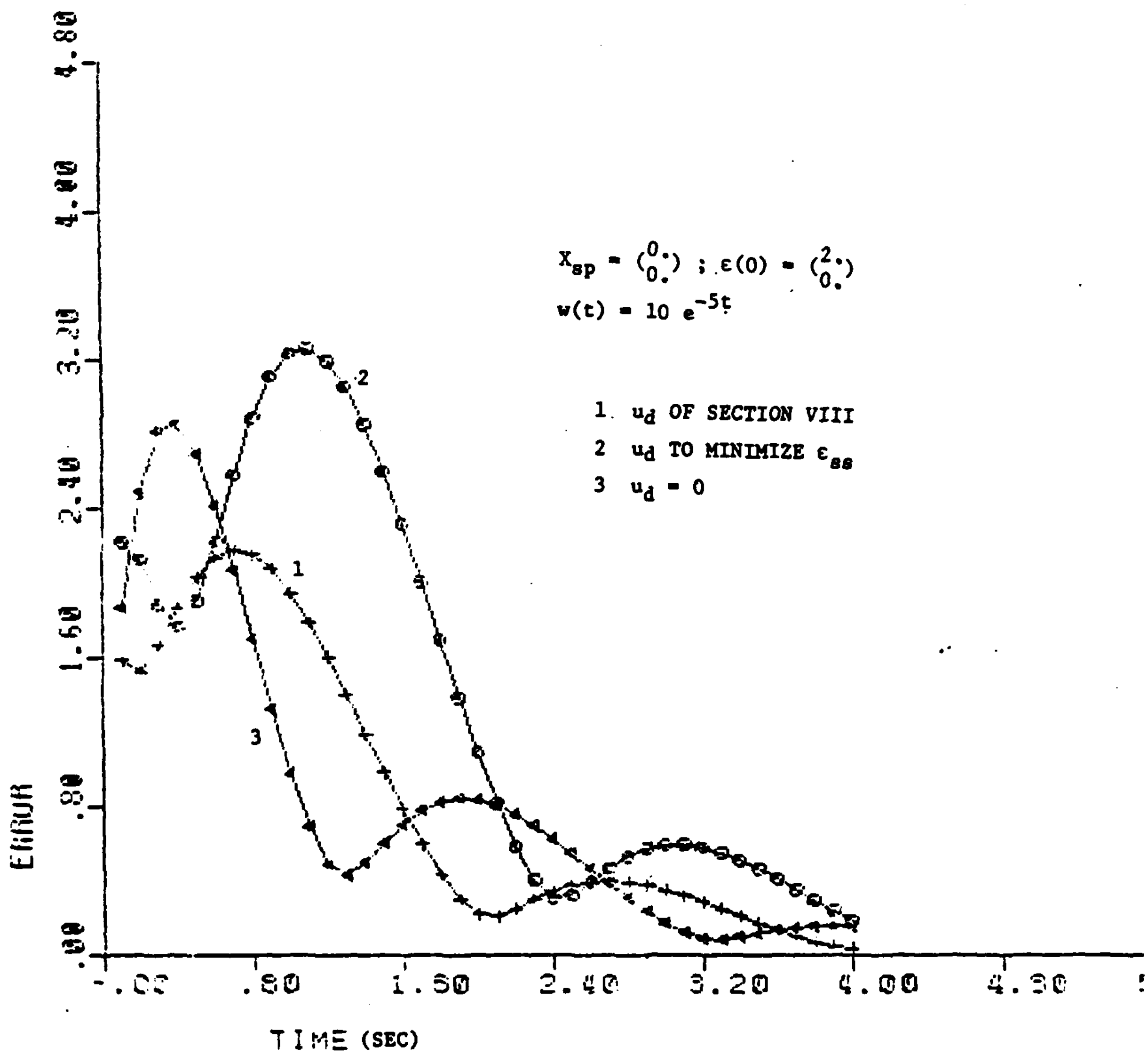

Figure 41. Error versus time, $X_{sp}=(0,0)^T$, $w=10e^{-5t}$.

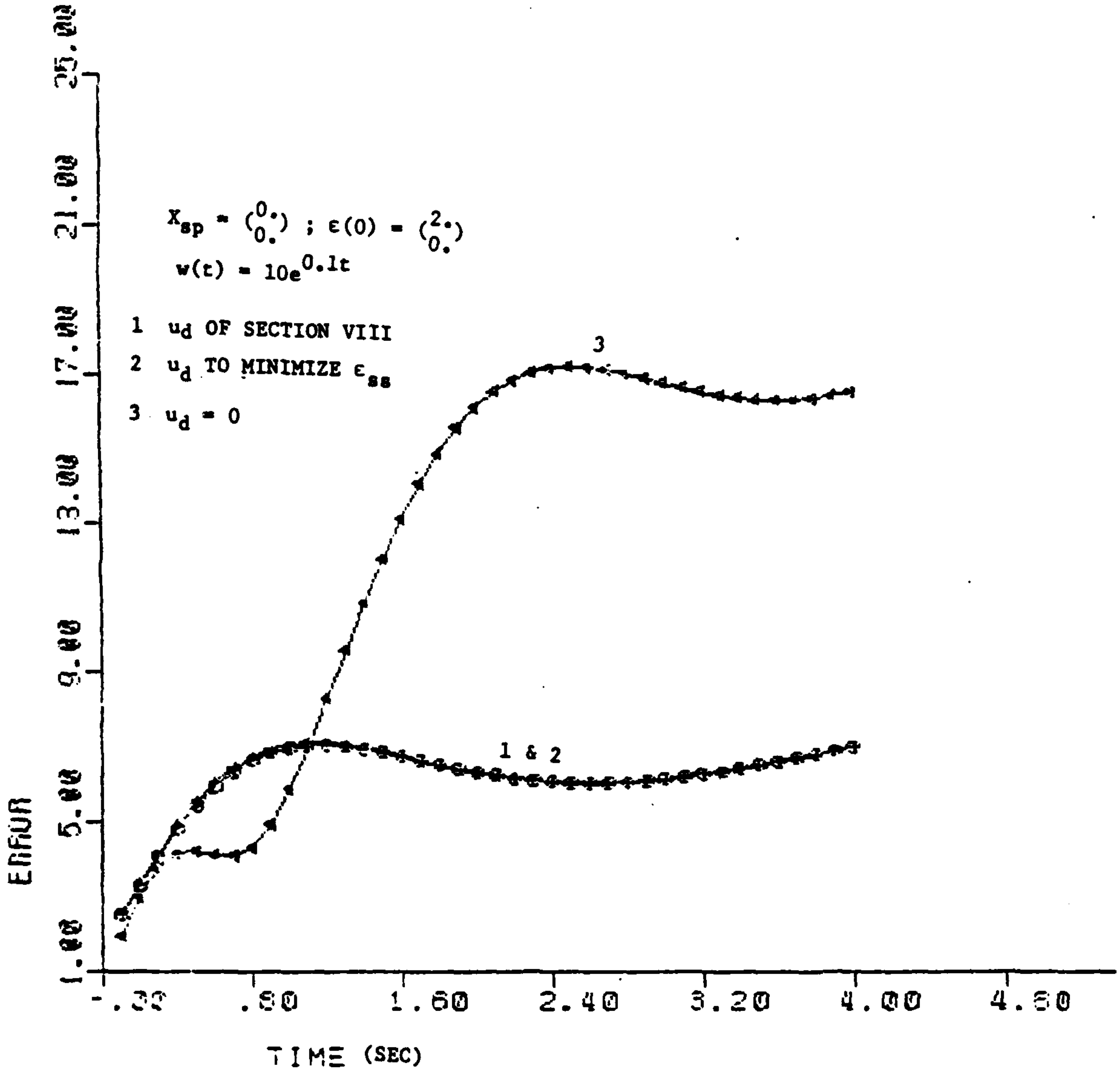

Figure 42. Error versus time, $X_{sp}=(0,0)^T$, $w=10e^{0.1t}$.

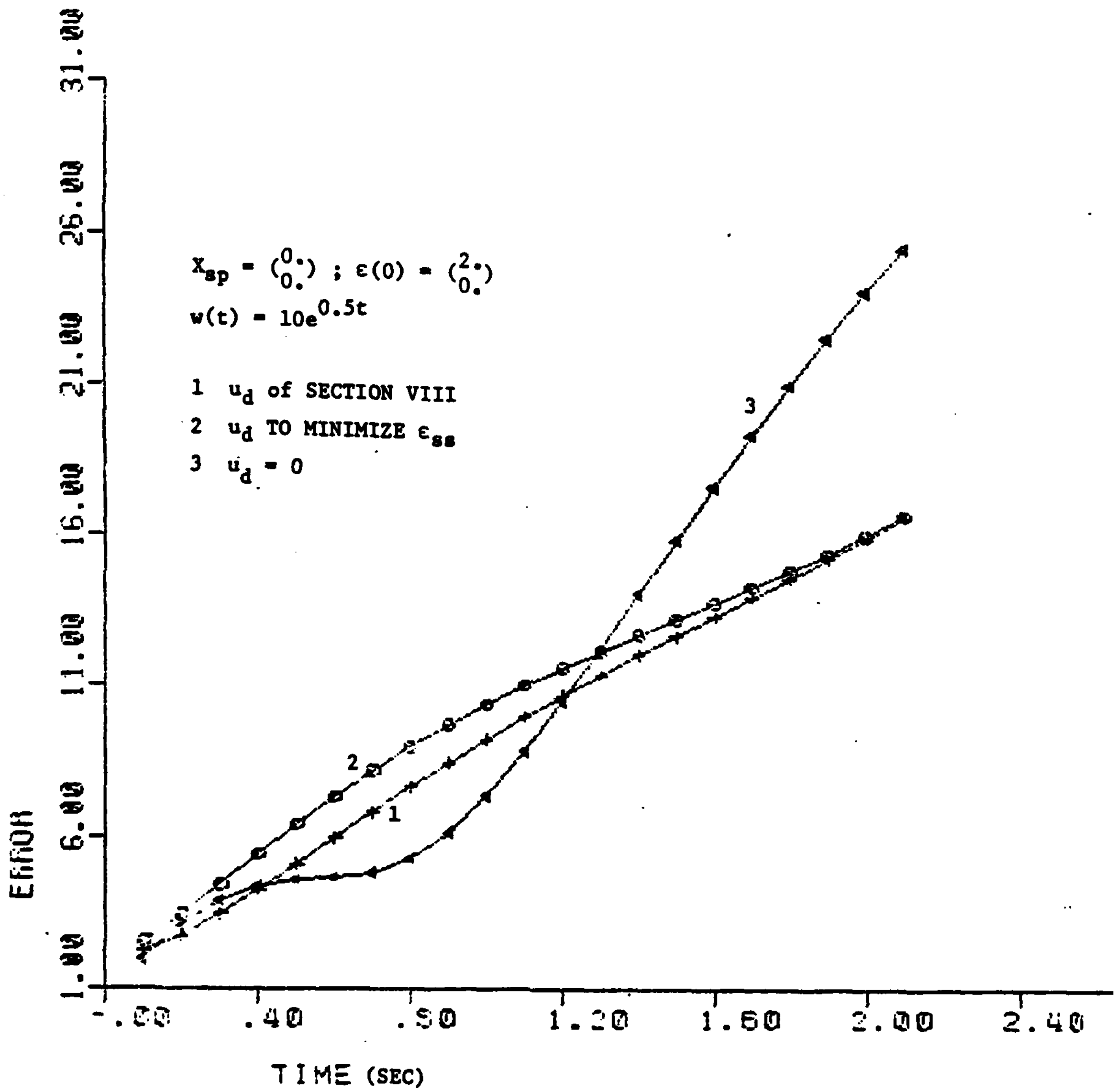

Figure 43. Error versus time, $X_{sp}=(0,0)^T$, $w=10e^{0.5t}$.

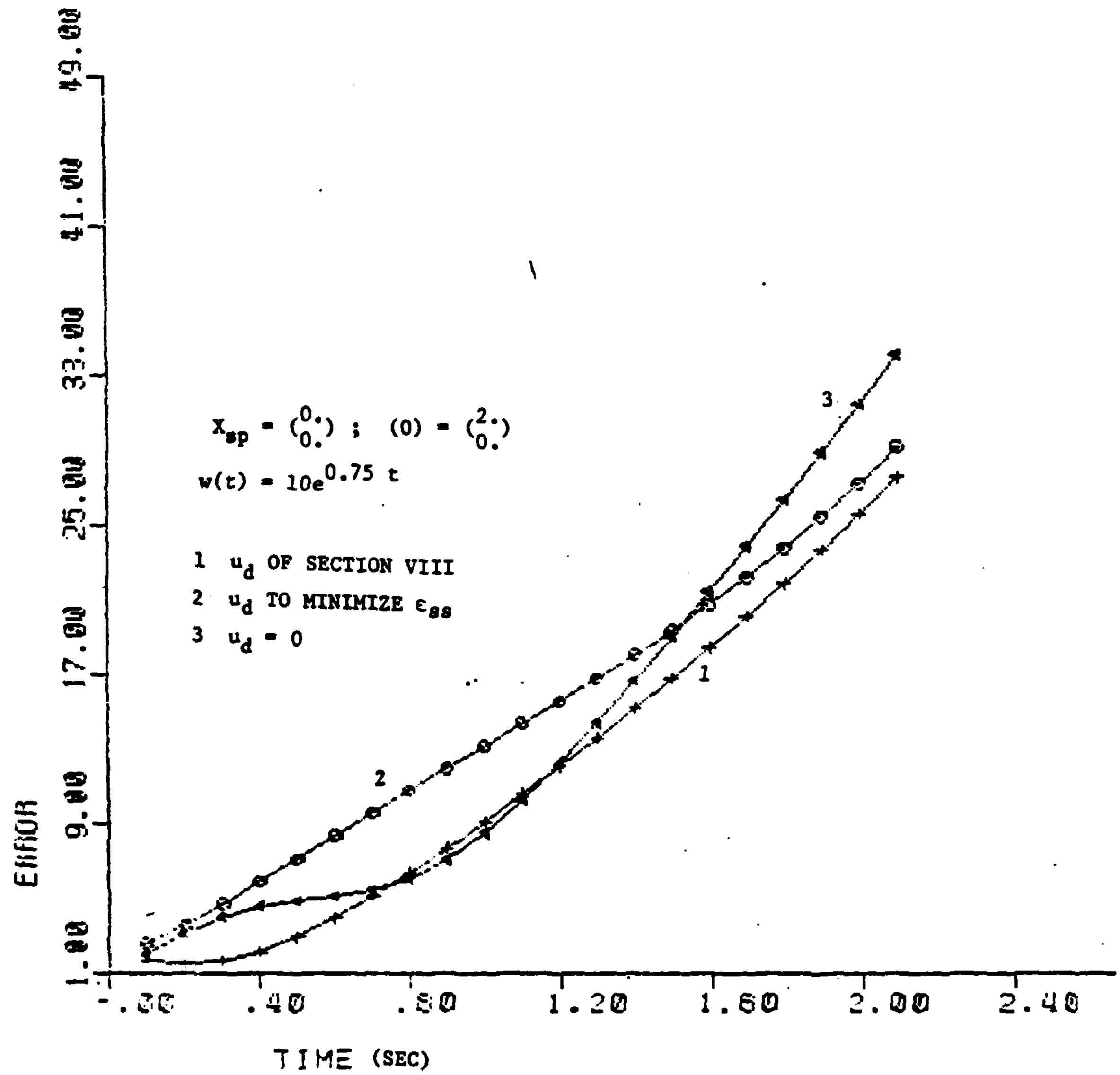

Figure 44. Error versus time, $X_{sp} = (0,0)^T$, w=10e0.75t.

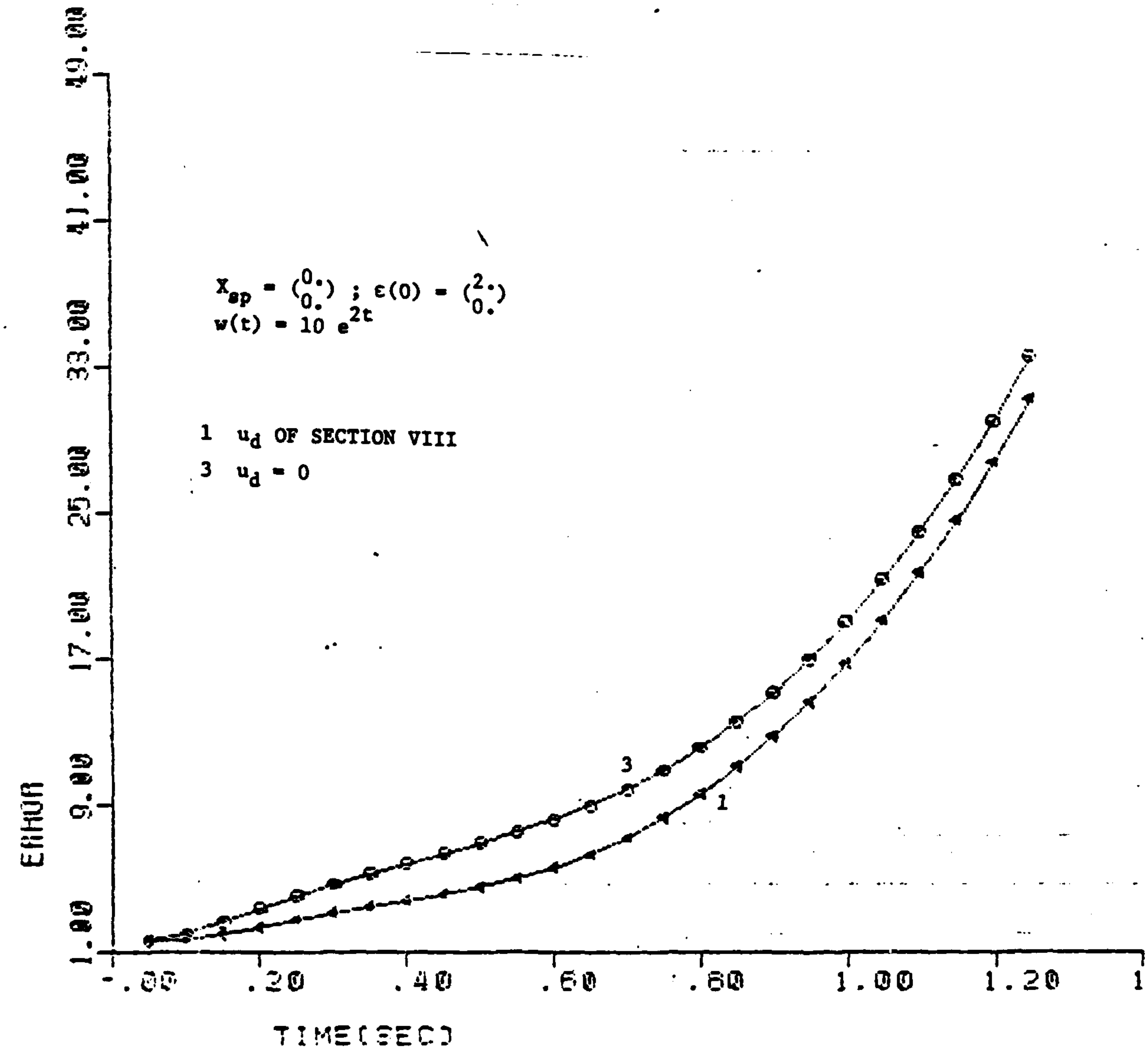

Figure 45. Error versus time, $X_{sp}=(0,0)^T$, $w=10e^{2t}$.

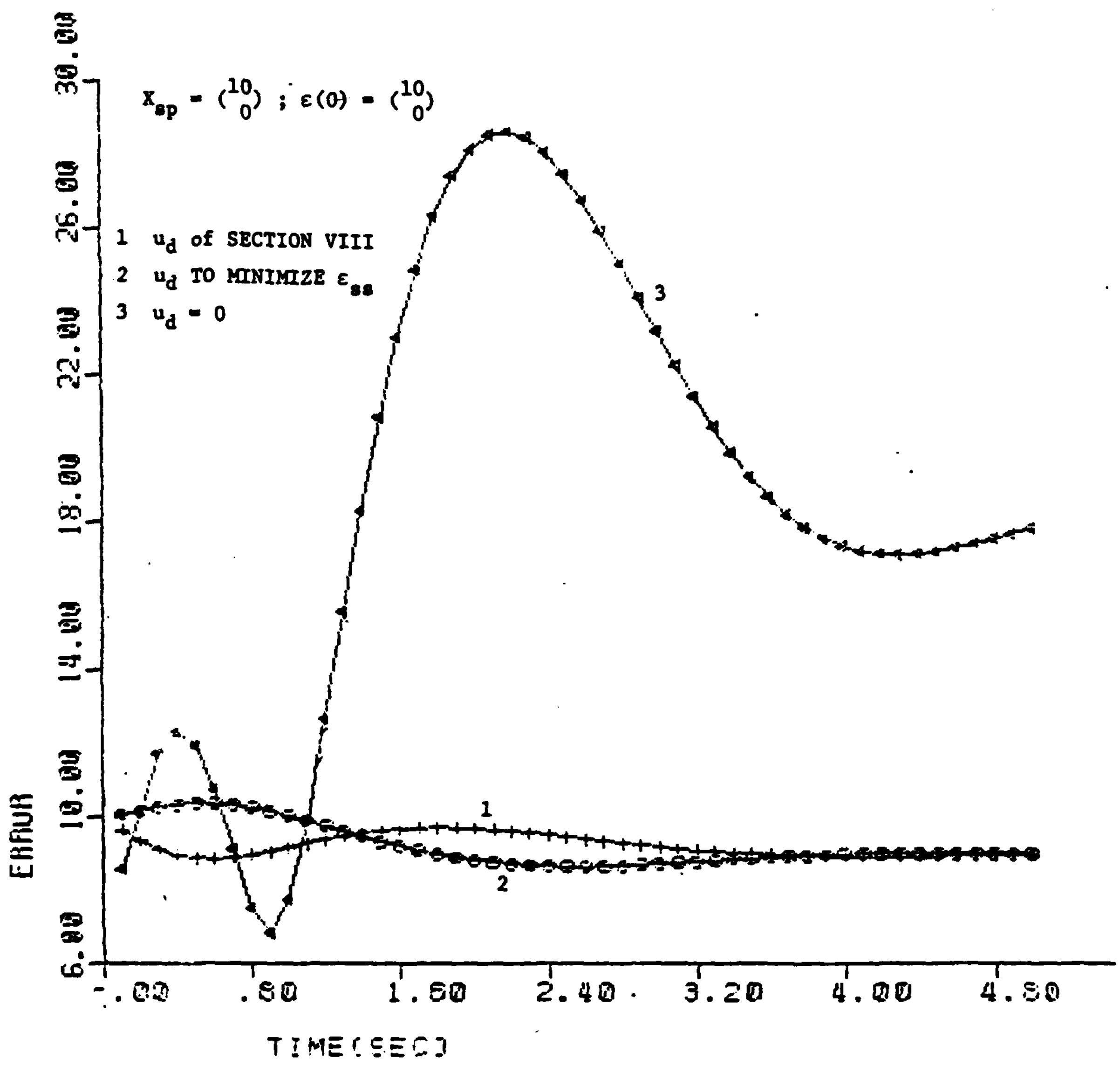

Figure 46. Error versus time, $X_{sp}=(10,0)^T$, $w=10e^{-t}$.

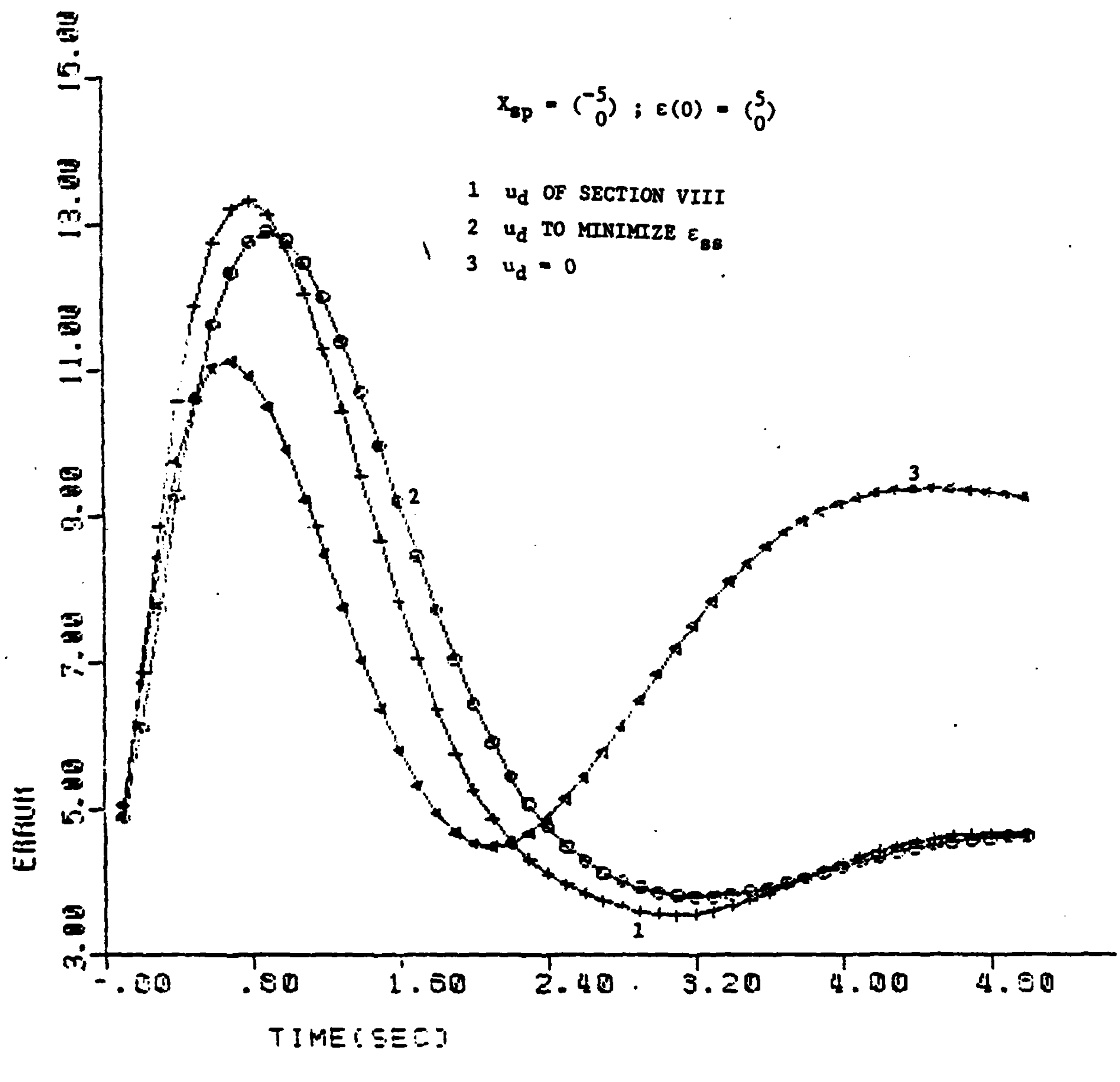

Figure 47. Error versus time, $X_{sp}=(-5,0)^T$, $w=10e^{-t}$.

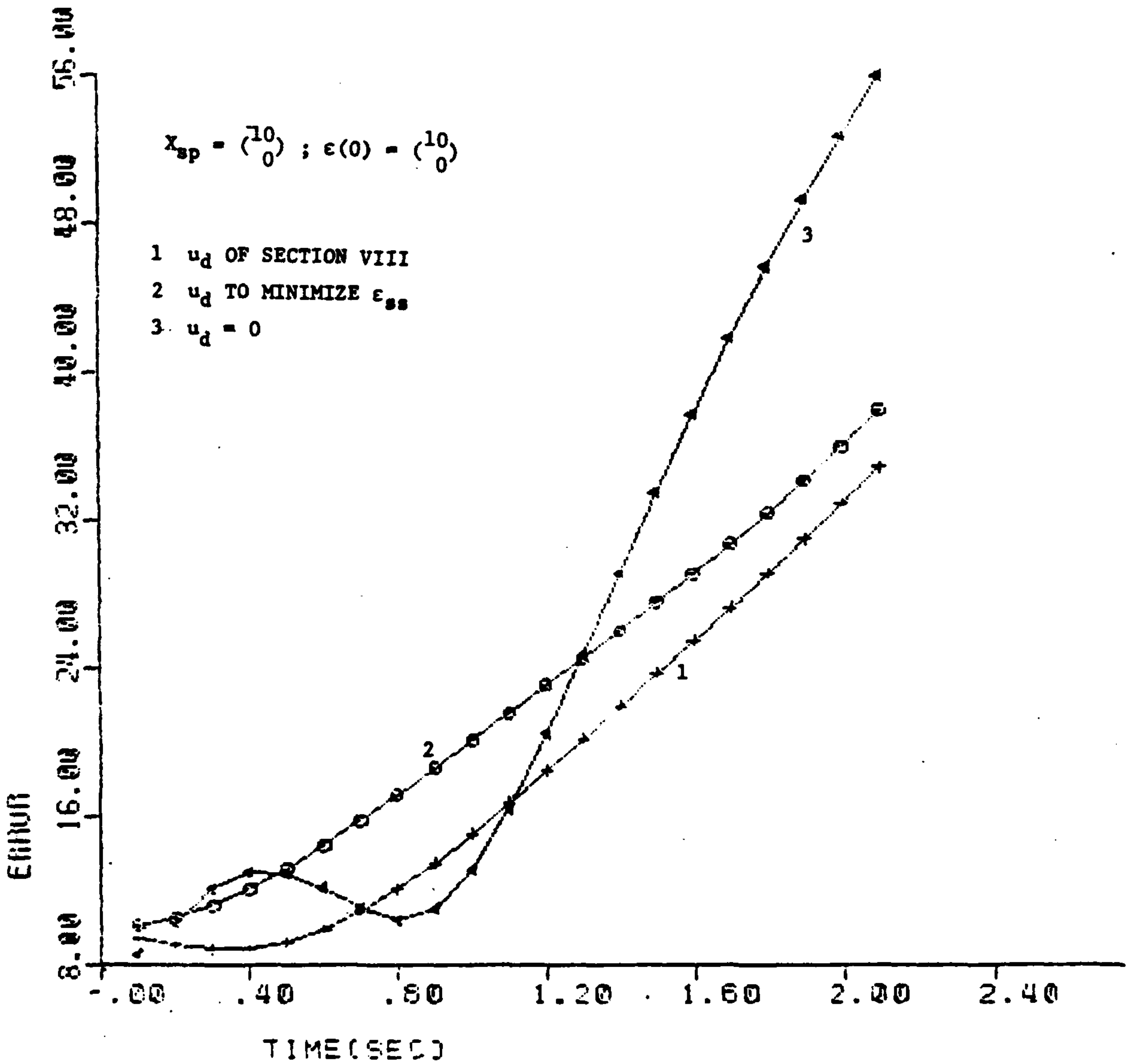

Figure 48. Error versus time, $X_{sp} = (10,0)^T$, $w = 10e^{0.75t}$.

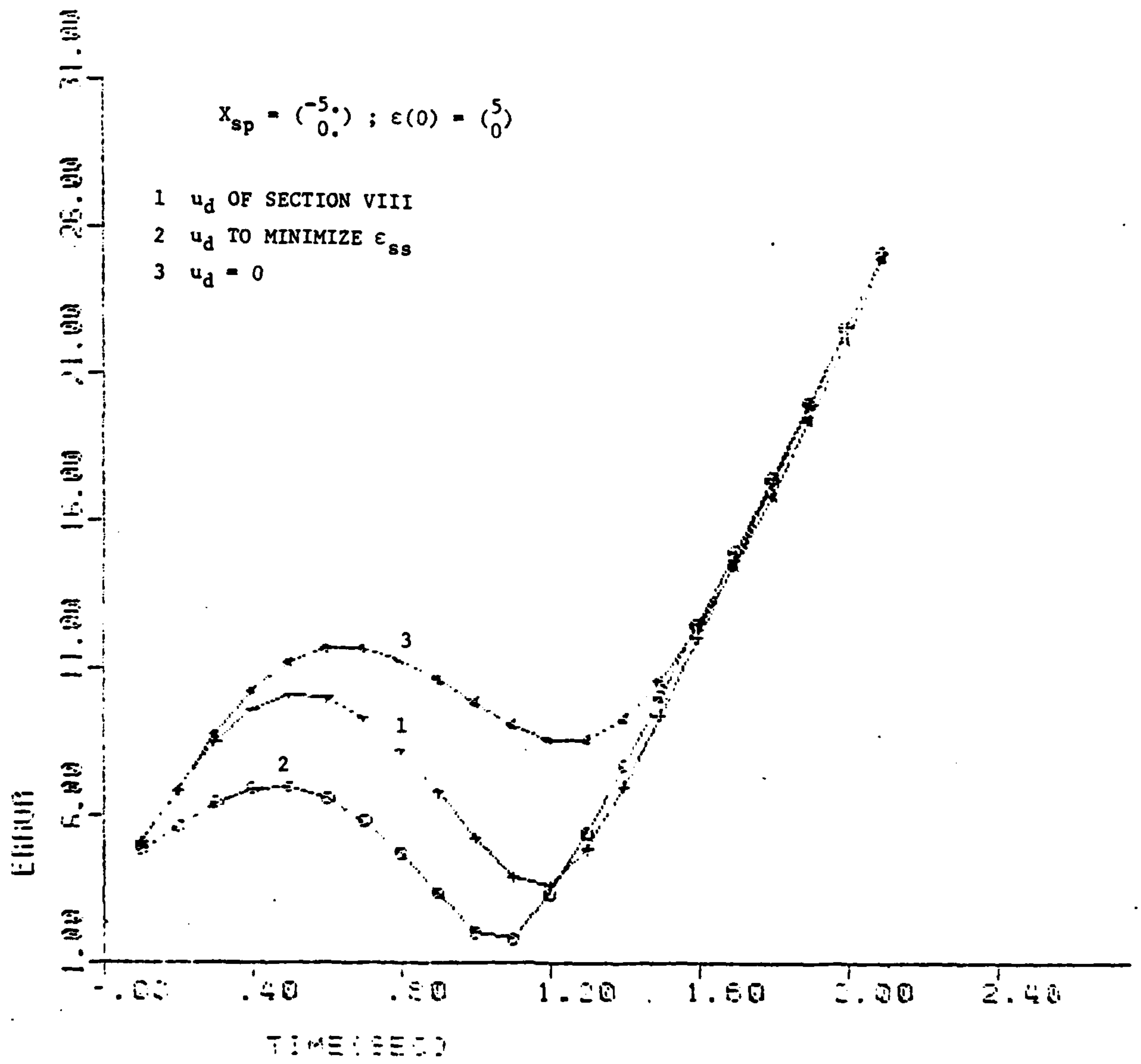

Figure 49. Error versus time, $X_{sp}=(-5,0)^T$, $w=10e^{0.75t}$.

IX. SUMMARY AND CONCLUSIONS

The purpose of this report was to present results of the application of a disturbance minimization control design technique, called the Isobasis design technique, to a state set-point regulator problem with time-varying external disturbance inputs. The isobasis design technique makes the a priori assumption that the disturbance control vector is composed of some combination of the same basis functions which describe the disturbances.

Section VI presented results for disturbance minimization when an external disturbance of the general form $w(t) = c_0+c_1t$ was applied to the plant described in Section IV. In Part A, an allocated disturbance minimizing control vector was developed. The norm minimization and critical-state variable methods were illustrated. It was shown that the isobasis technique provided no advantage over a controller designed under the assumption that w was a constant. In Part B, an unallocated disturbance minimizing control vector was developed. It was shown that with this approach, the isobasis designed controller can provide performance equal to, or better than, that provided by the controller designed with w assumed constant. The performance improvement does not, however, occur in all cases.

Section VII presented results for the isobasis technique when an external disturbance of the form $w(t) = c\sin\alpha t$ was applied to the plant. It was shown that the isobasis designed controller can provide significant reduction in the amplitude of the sinusoidal component of the error. Results were presented for two different target state set-points to show the performance of the controller as a function of the frequency of w(t) and of the input amplitude of w(t).

Section VIII presented results for the isobasis technique when an external disturbance of the form $w(t) = ce^{\alpha t}$ was applied to the plant. It was shown that the isobasis designed controller can result in improved performance for this external disturbance also, but again, not consistently.

As these example results have shown, the isobasis design technique obviously did not produce the best disturbance minimizing controller for all external disturbances examined or even for all target set-points with the same external disturbance. It was shown that it does have the potential of producing a controller which will perform well in reducing the error between the plant state and the target state set-point. Its application should, therefore, be considered in cases involving time-varying external disturbances.

In the Appendix, a definition is given for the "utility" U of time-varying external disturbances. Examples are presented to illustrate the concept of disturbance utility. Two external disturbances were considered: one was $w(t) = c_0 + c_1t$, the other was $w(t) = c\sin(\alpha t)$.

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APPENDIX A

UTILITY OF TIME-VARYING EXTERNAL DISTURBANCES

In the case of state set-point regulation/stabilization problems involving a linear, time-invariant plant and a constant external disturbance, it was shown in Reference 13 that the steady-state error is the residual part of the sum of the set-point and external disturbance vectors which lie in the orthogonal complement of R(B) and is hence unabsorbable by the control vector. It was also shown that the steady-state error may be reduced, independently of the control vectors, by a fortuitous combination of the set-point and external disturbance vectors, except in cases where the set-point is the origin. The "utility" U of the external disturbance was defined in Reference 13 as

$$U = \|\varepsilon_{ss}\|_{w=o} - \|\varepsilon_{ss}\|_{w\neq o}. \tag{A-1}$$

If $U > 0$, then w can actually aid in reducing ε_{ss}. On the other hand, if $U < 0$, then the presence of w increases the value of ε_{ss}. Some examples of disturbance utility were presented in References 13 and 14.

In the case of time-varying external disturbances, as considered in this report, the definition of U given by Equation (A-1) must be modified to

$$U = \|\varepsilon(t)\|_{w=o} - \|\varepsilon(t)\|_{w\neq o}. \tag{A-2}$$

Depending upon the system and the particular w(t), it is possible that U could be always positive, always negative, or positive until a certain time and then negative thereafter, or vice versa. Also, the utility could exhibit a periodic behavior as a function of time, being alternately positive and negative. Several examples will be presented in this section to illustrate disturbance utility for the plant model given by Equation (18) and several types of time-varying external disturbance.

For the first example, the external disturbance will be given by

$$w(t) = 1 + t. \tag{A-3}$$

From Equation (17), with $u_d = 0$, the solution for $\varepsilon(t)$ is found as

$$\varepsilon(t) = e^{\tilde{A}t}\varepsilon(0) - \int_0^t e^{\tilde{A}(t-\tau)}[Ax_{sp} + Fw(\tau)]d\tau\ . \tag{A-4}$$

Let the target set-point be $x_{sp} = (-5,\ 0)^T$. With $w(t) = 0$, one has

$$\|\varepsilon(t)\|_{w=o} = \left\|\int_0^t e^{\tilde{A}(t-\tau)}d\tau(Ax_{sp})\right\| = \|-\tilde{A}^{-1}Ax_{sp}\| \tag{A-5}$$

if the initial transients given by $e^{\tilde{A}t}\varepsilon(0)$ are ignored. Making the appropriate substitutions into Equation (A-5), the result is

$$\|\varepsilon(t)\|_{w=o} = \|(0.427,\ 9.145)^T\| = 9.154. \tag{A-6}$$

With w as given by (A-3), the norm of $\varepsilon(t)$ is found as

$$||\varepsilon(t)||_{w\neq o} = ||\int_o^t e^{\tilde{A}(t-\tau)}[Ax_{sp} + F(1+\tau)]d\tau||$$

$$||-\tilde{A}^{-1}(Ax_{sp} + F) - \tilde{A}^{-1}(\tilde{A}^{-1} + It)F|| \quad \text{(A-7)}$$

$$= \sqrt{1.5t^2 - 21.5t + 79.4} \quad .$$

In order to find the time at which U changes sign one sets

$$||\varepsilon(t)||_{w=o} - ||\varepsilon(t)||_{w\neq o} = 0. \quad \text{(A-8)}$$

Making the substitutions from Equations (A-6) and A-7) into (A-8) results in

$$1.5t^2 - 21.5t - 4.4 = 0, \quad \text{(A-9)}$$

which, when solved, yields a positive time of $t = 14.5$ seconds. In this case then, at $t = 14.5$ seconds the utility of the external disturbance goes from positive to negative, i.e., from aiding in error reduction to increasing the error magnitude. A digital simulation was written for this example and the result is shown on Figure A-1. As can be seen, $\varepsilon(t)$ for the case with $w = 1+t$ exceeds $\varepsilon(t)$ for the case with $w = 0$ after $t = 14.5$ seconds.

If the target set-point is assumed to be $x_{sp} = (10, 0)^T$ then

$$||\varepsilon(t)||_{w=o} = ||(-0.854, -18.29)^T|| = 183.1 \quad \text{(A-10)}$$

$$||\varepsilon(t)||_{w\neq o} = ||(0.1097t - 0.497, -1.219t - 18.566)^T||$$

$$= \sqrt{1.5t^2 + 45.16t + 345.1} \quad . \quad \text{(A-11)}$$

From Equation (A-8) one finds that for this set-point,

$$1.5t^2 + 45.16t + 9.9 = 0, \quad \text{(A-12)}$$

and there are no positive values of t which will satisfy Equation (A-12). For this set-point then, the external disturbance always acts to increase the error (since at $t = 0$, $U = -9.9$). This case was also simulated and the result is shown on Figure A-2. As can be seen, $\varepsilon(t)$ for the case with $w(t) = 1 + t$ is never smaller than $\varepsilon(t)$ for the case with $w = 0$.

As another example, let

$$w(t) = \sin(t) \quad \text{(A-13)}$$

with $x_{sp} = (10, 0)^T$. In this case, one would have

$$||\varepsilon(t)||_{w\neq o} = ||- \int_o^t e^{\tilde{A}(t-\tau)}d\tau(Ax_{sp}) - \int_o^t e^{\tilde{A}(t-\tau)}\sin\tau d\tau(F)||$$

$$= ||\tilde{A}^{-1}Ax_{sp} - \hat{A}^{-1}[-\tilde{A}\sin(t) - I\cos(t)]F||, \quad \text{(A-14)}$$

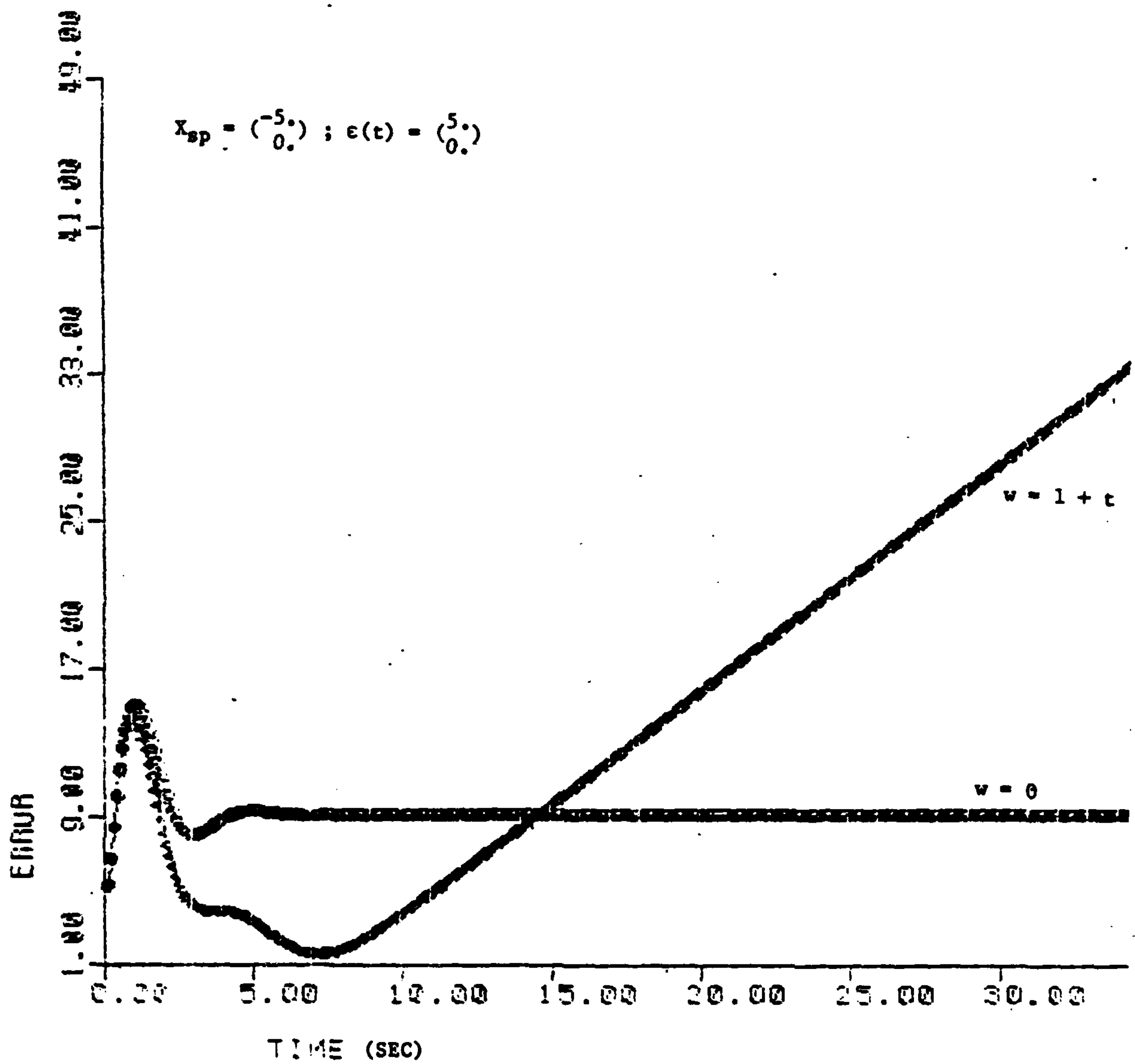

Figure A-1. Error versus time for cases with w=0 and w=1+t., $X_{sp}=(-5.,0.)^T$.

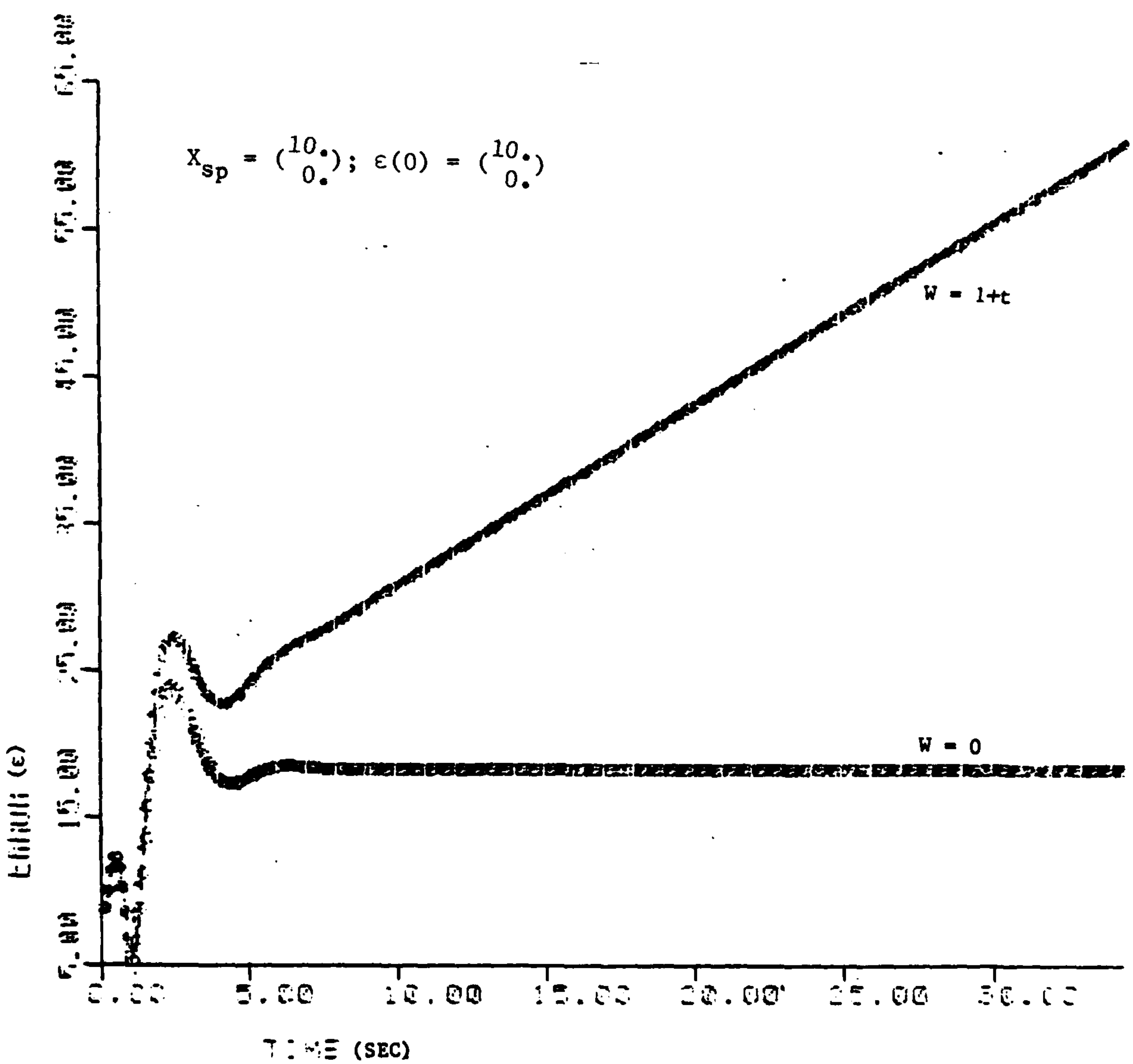

Figure A-2. Error versus time for cases with w=0 and w=1+t., $X_{sp}=(10.,0.)^T$.

where $\hat{A} = \tilde{A}^2 + \alpha^2 I$. When Equation (A-14) is evaluated, the result is

$$\|\varepsilon(t)\|_{w \neq 0} = \left\| \begin{pmatrix} 0.854 - 0.355\sin(t) - 0.232\cos(t) \\ 18.79 + 1.11\sin(t) - 1.336\cos(t) \end{pmatrix} \right\| . \quad \text{(A-15)}$$

The norm of the error when w = 0 is given by Equation (A-10). The utility of the external disturbance of Equation (A-13) and the given set-point is found from Equations (A-15) and (A-10) to be

$$U = 183.1 - \left\| \begin{pmatrix} 0.854 - 0.355\sin(t) - 0.232\cos(t) \\ 18.79 + 1.11\sin(t) - 1.336\cos(t) \end{pmatrix} \right\| . \quad \text{(A-16)}$$

The digital simulation used to generate the data in Figures A-1 and A-2 was modified to include the w(t) as given by Equation (A-13) and to calculate U as given by Equation (A-16). Figure A-3 presents U versus time and Figure A-4 presents the corresponding error versus time data for this second example. If Figures A-3 and A-4 are overlaid, the agreement can be seen between the regions of positive utility and the regions where the error with $w \neq 0$ is less than that with w = 0. Figures A-5 and A-6 present similar data for a case with $w = \sin(t)$ and $x_{sp} = (-5, 0)^T$. Again, it can be seen that the utility of w(t) alternates between positive and negative.

Figure A-7 is a plot of the state space showing the external and set-point disturbance vectors and the line of action, R(B), of the control corresponding to the case of Figure A-5. Note that the vector representing Fw has an arrowhead at each end. This represents the fact that w(t) = sin(t) and w(t) varies between +1 and -1 as a function of time. The component of Ax_{sp} lying in $R(B)^\perp$ is shown and is denoted by $\bar{a}$. Since w(t) varies with time, the component of Fw lying in $R(B)^\perp$ will also vary. This component is also shown in Figure A-6 and is denoted by $\bar{f}$.

In Reference 13, two criteria were given which must be satisfied in order for w(t) to exhibit a positive utility. The first criterion is a magnitude criterion,

$$\|\bar{f}\| < \|2\bar{a}\| , \quad \text{(A-17)}$$

and the second is an angle criterion,

$$90° < \theta < 270° , \quad \text{(A-18)}$$

where θ is the angle between the two vectors $(2\bar{a}+\bar{f})$ and $(\bar{f})$. From Figure A-7, it can be seen that Equation (A-17) is satisfied for all t. The angle criterion, however, is only satisfied when Fw is positive, i.e., when sin(t) is positive. Since $\bar{a}$ is given by [13,14]

$$\bar{a} = (I - BB^\dagger)Ax_{sp} = (-4, 2)^T , \quad \text{(A-19)}$$

the magnitude of $\bar{a}$ is found to be

$$\|\bar{a}\| = 4.47 \quad \text{(A-20)}$$

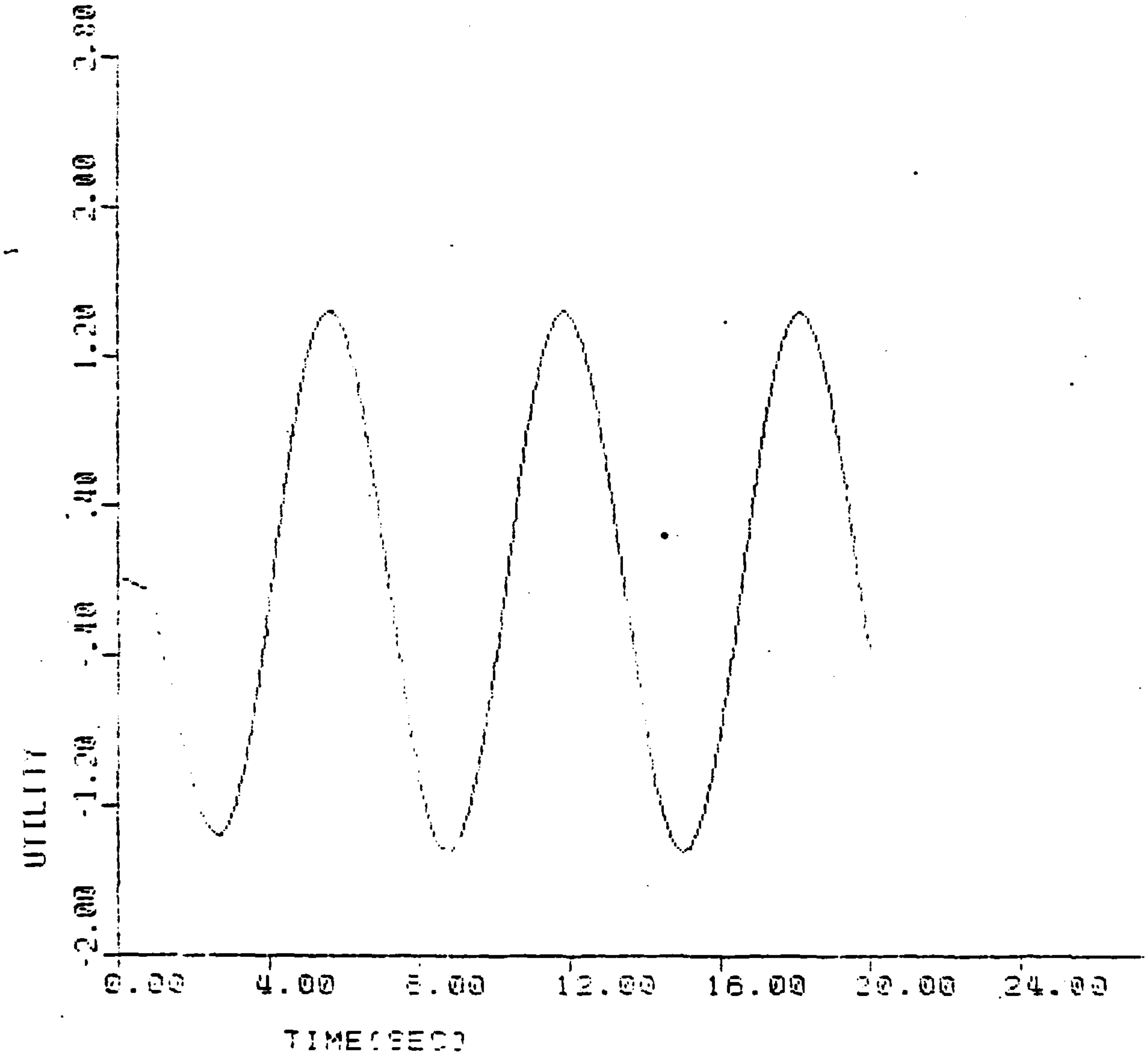

Figure A-3. Utility versus time for $X_{sp}=(10.,0.)^T$, w=SIN(t).

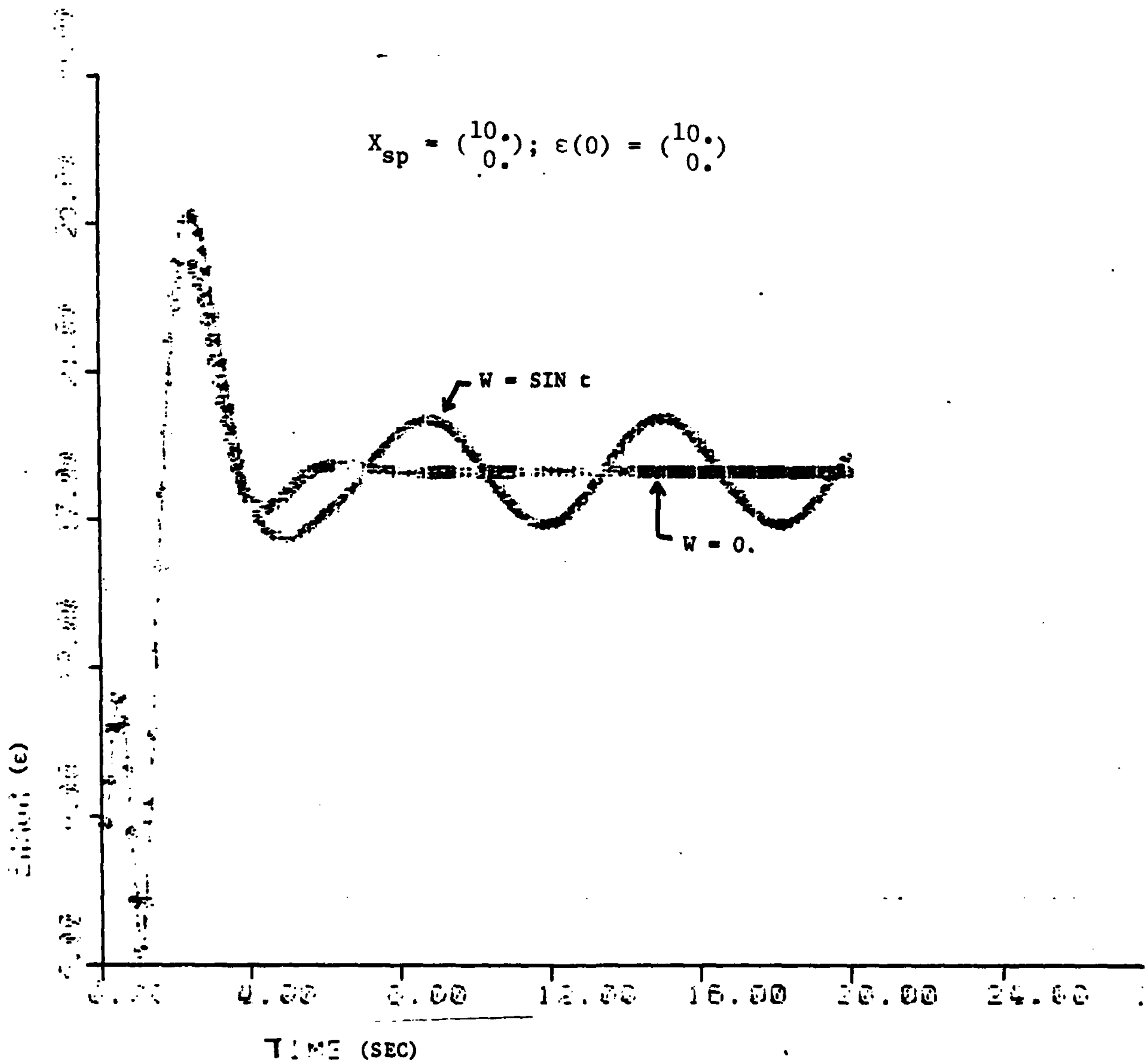

Figure A-4. Error versus time for cases with w=0, w=SIN(t), with $X_{sp}=(10.,0.)^T$.

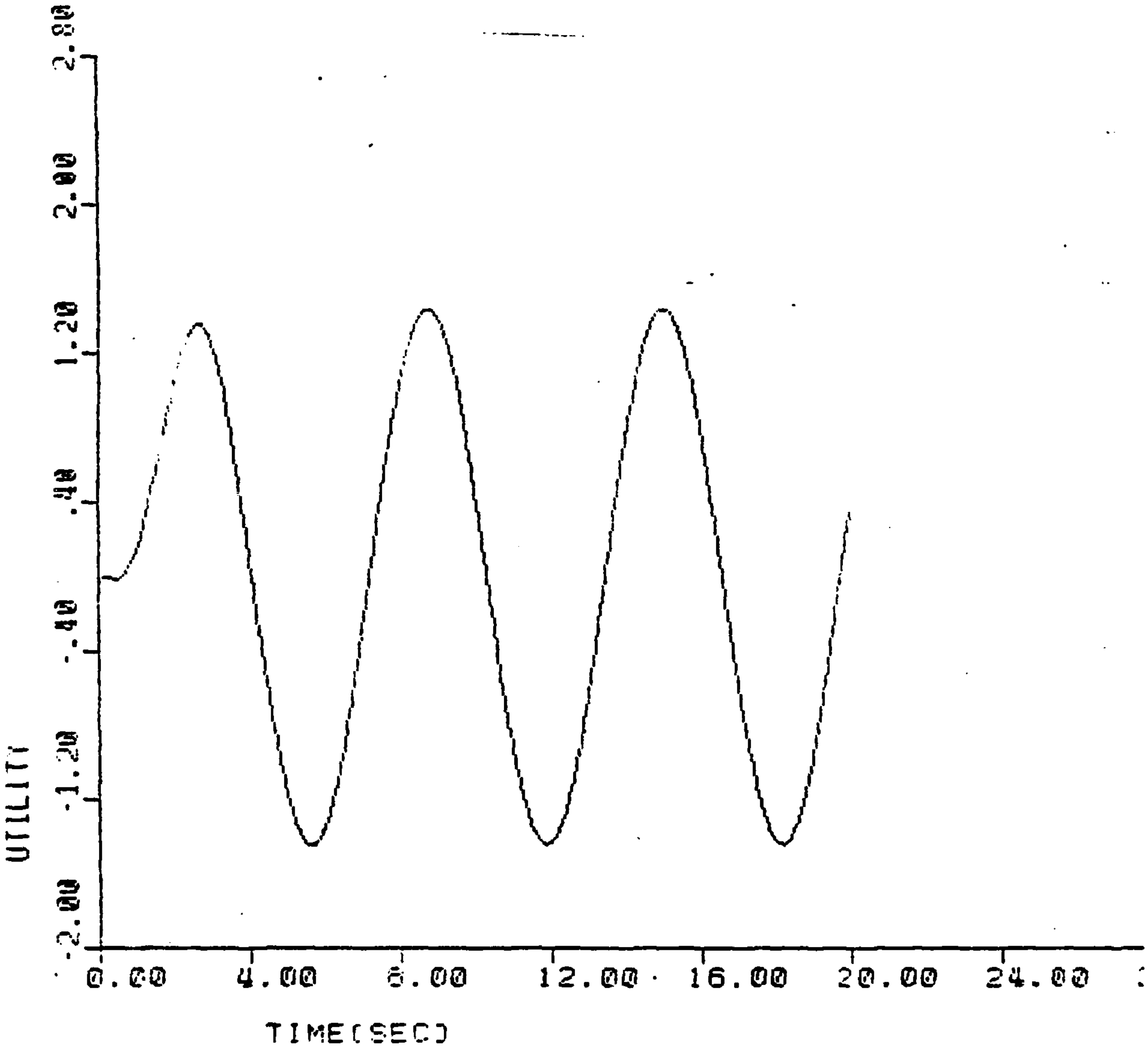

Figure A-5. Utility versus time for $X_{sp}=(-5,0)^T$, w=SIN(t).

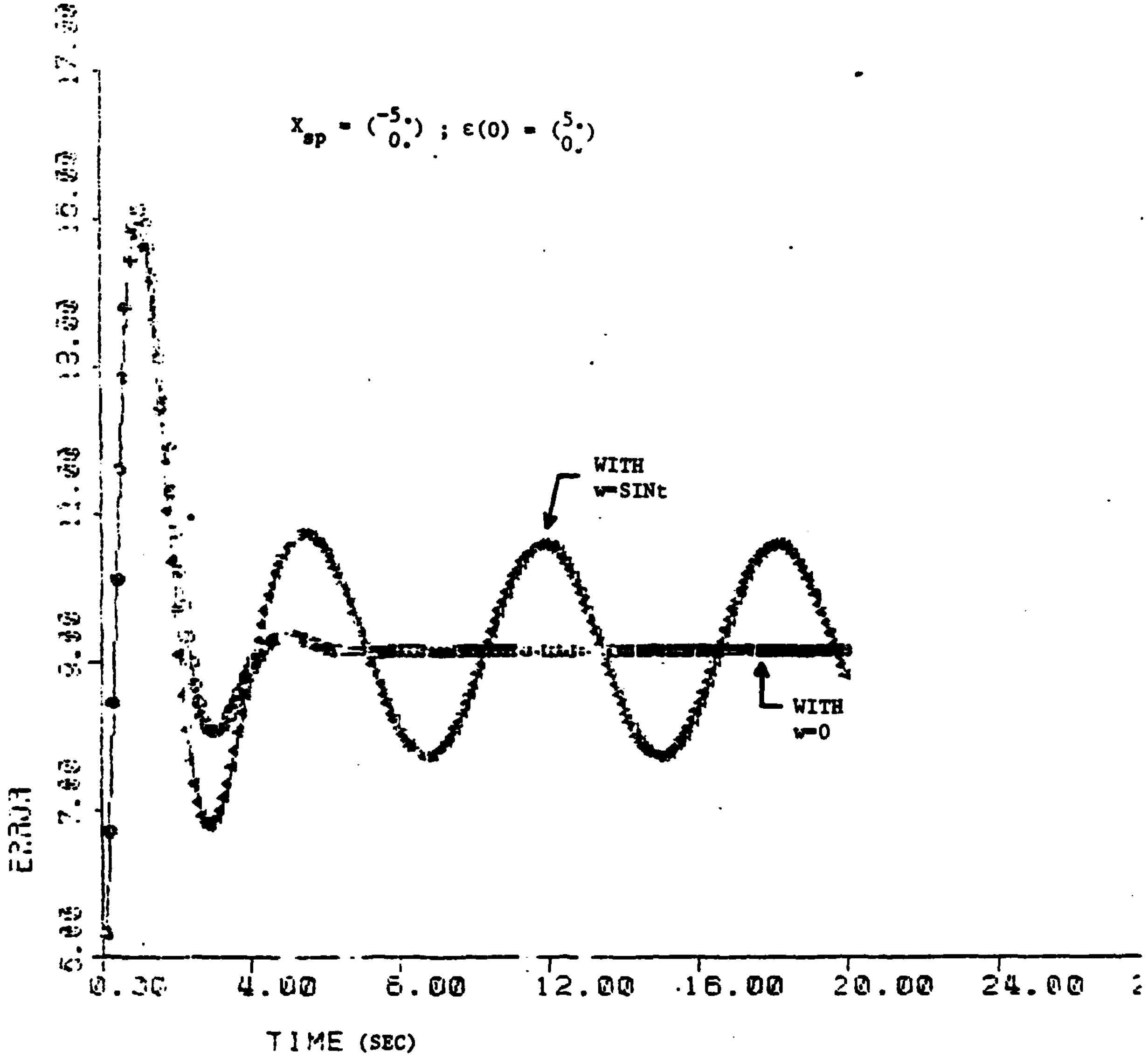

Figure A-6. Error versus time for cases with w=0, w=SIN(t), with X_{sp}=(-5.,0.)T.

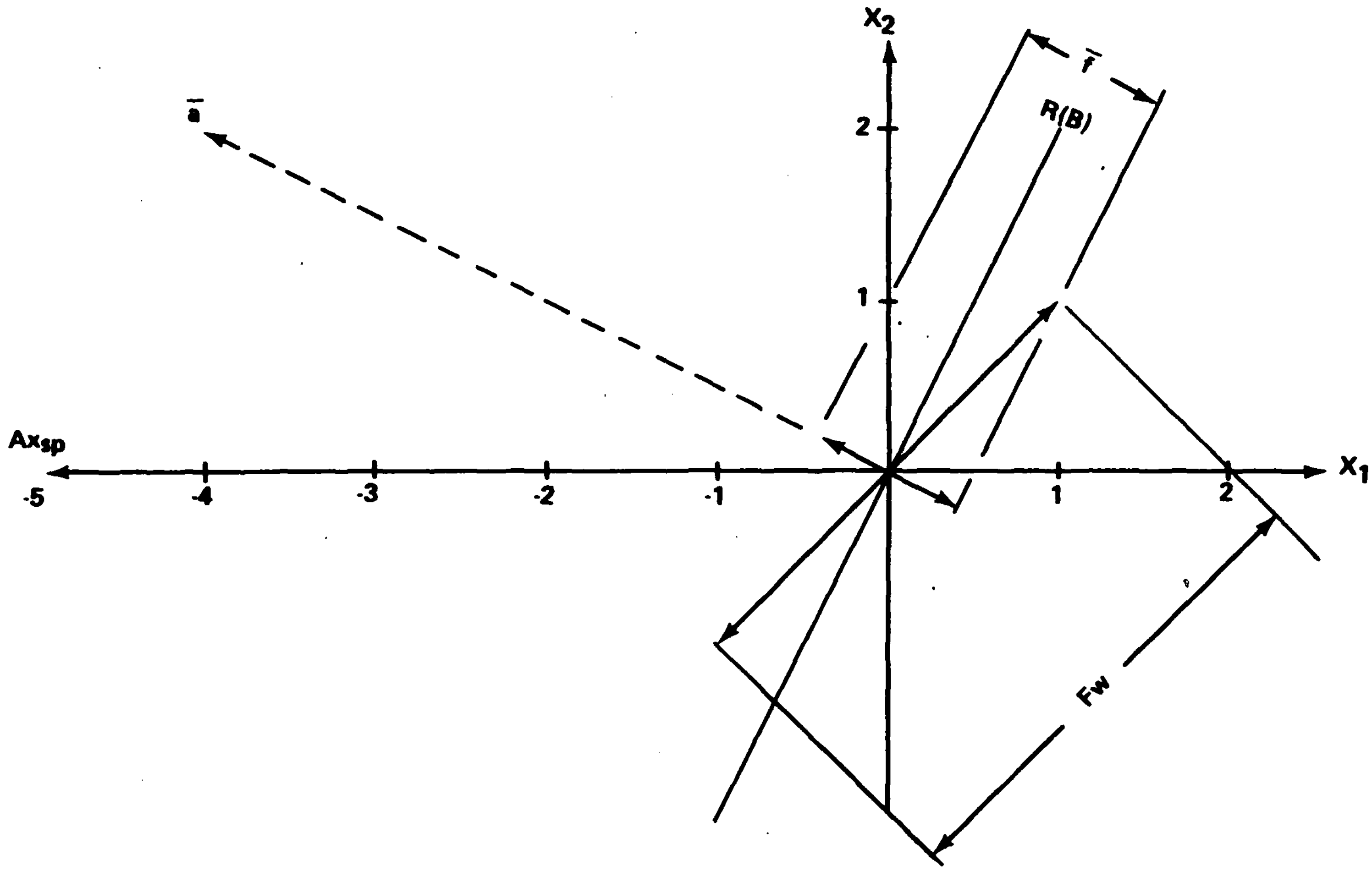

Figure A-7. Geometry of disturbances relative to R(B), $X_{sp} = (-5.,0.)^T$, w = SIN(t).

From the magnitude criterion Equation (A-17), even when θ satisfies the angle criterion, if $\|\bar{f}\|$ is greater than $\|2\bar{a}\|$, the utility of w(t) will be negative.

In order to illustrate how utility changes with w(t), two additional runs were made for the $x_{sp} = (-5, 0)^T$ case. Figure A-8 shows the results for $w(t) = 10\sin(t)$, i.e., $\|\bar{f}\| = 4.47$, and Figure A-9 shows the results for $w(t) = 20\sin(t)$, i.e., $\|\bar{f}\| = 8.94$. As can be seen, as the magnitude of $\bar{f}$ approaches the limit set by Equation (A-17) the utility of w(t) approaches a condition where it will be zero for all t (since the error with $w \neq 0$ is approaching a condition where it will always be greater than for the case with $w = 0$.

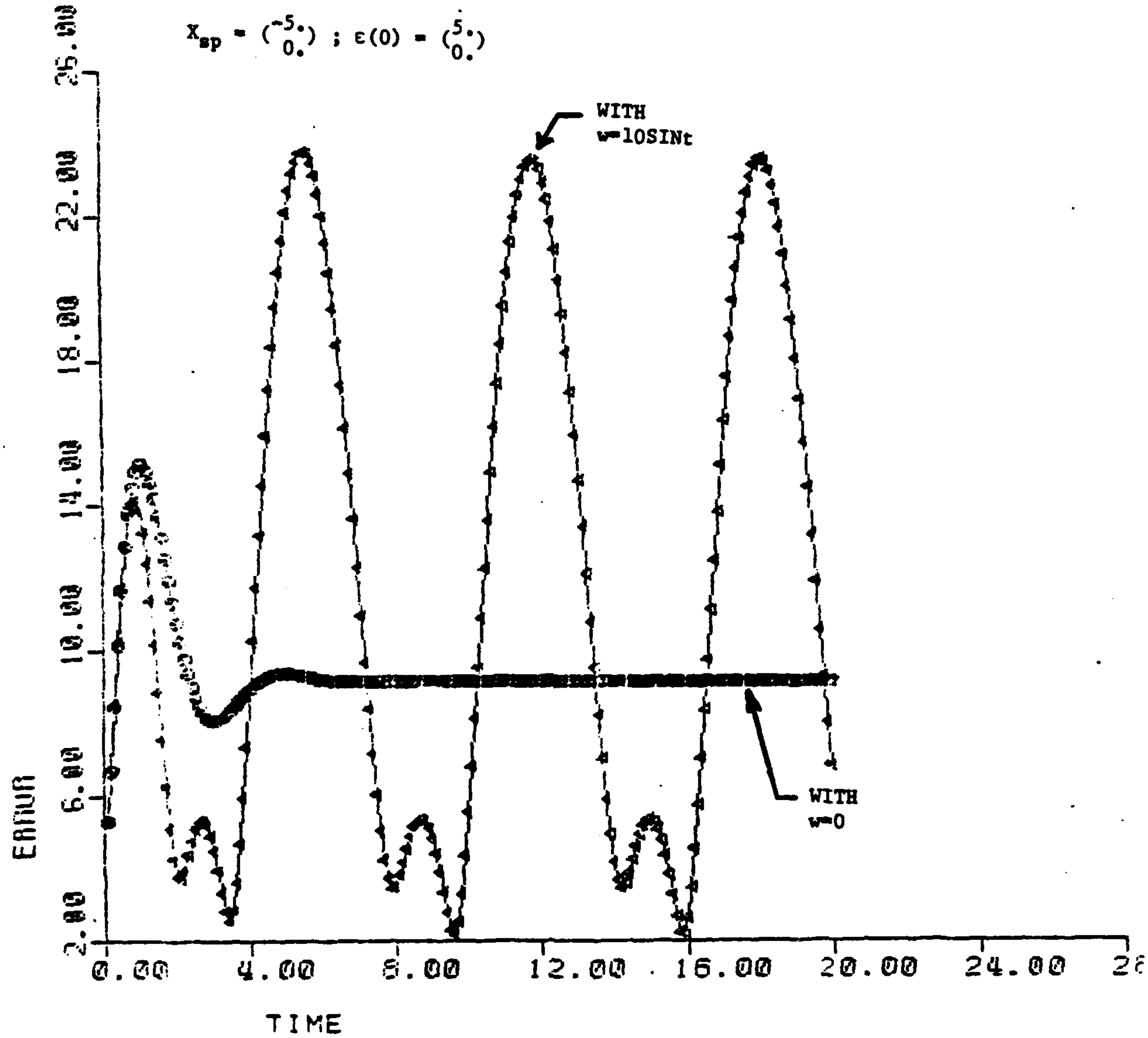

Figure A-8. Error versus time for cases with w=0, w=10 SIN(t), $X_{sp}=(-5.,0.)^T$.

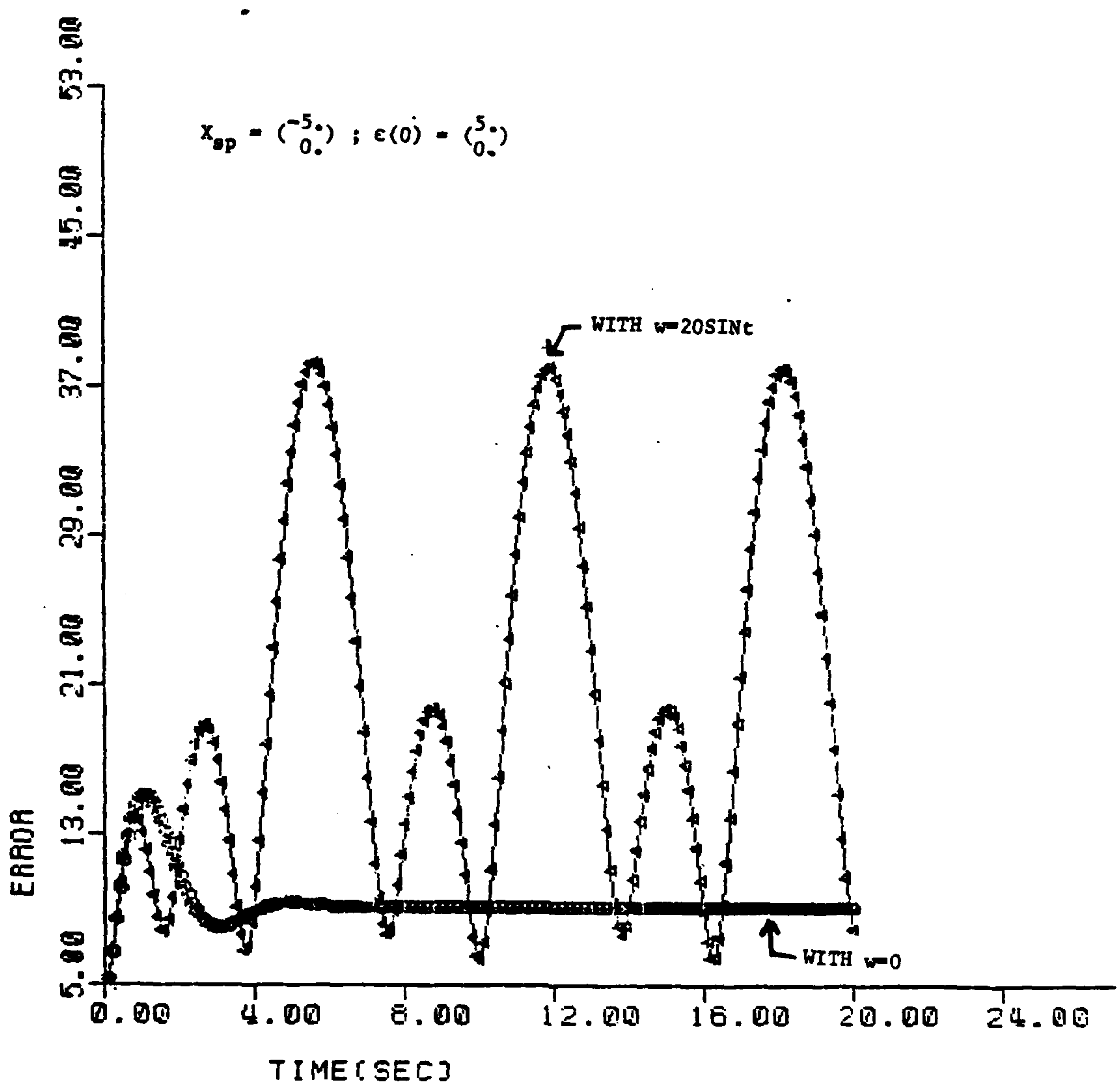

Figure A-9. Error versus time for cases with w = 0, w = 20 SIN(t), $X_{sp} = (-5., 0.)^T$.

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Commander US Army Research Office ATTN: AMXRO-PH, Dr. R. Lontz P. O. Box 12211 Research Triangle Park, NC 27709	1
AMSMI-R, Dr. McCorkle	1
Dr. Rhoades	1
-RG, Dr. Yates	1
-RGN, Mr. McLean	1
Dr. McCowan	10
-RGS, Dr. Kelly	1
-RPT, Record Copy	1
-RPR, Reference	15
-LP, Mr. Voigt	1

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