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FIRST PASSAGE TIMES IN STOCHASTIC DIFFERENTIAL
EQUATIONS OF MATHEMATICAL. (U) NORTHWESTERN UNIV
EVANSTON IL DEPT OF ENGINEERING SCIENCE AND.

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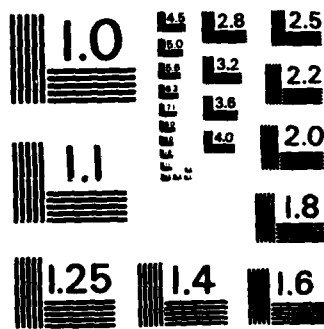
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SECURITY CLASSIFICATION OF THIS PAGE

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REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION Unclassified			1b. RESTRICTIVE MARKINGS		
2a. SECURITY CLASSIFICATION AUTHORITY			3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited		
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE			4. PERFORMING ORGANIZATION REPORT NUMBER(S)		
5a. NAME OF PERFORMING ORGANIZATION Northwestern Univ.			5b. OFFICE SYMBOL (If applicable)		
6a. ADDRESS (City, State and ZIP Code) Evanston, IL 60201			7a. NAME OF MONITORING ORGANIZATION AFOSR/NM		
7b. ADDRESS (City, State and ZIP Code) Bldg. 410 Bolling AFB, D.C. 20332-6448			8. MONITORING ORGANIZATION REPORT NUMBER(S) AFOSR-TR- 85-0767		
9a. NAME OF FUNDING/SPONSORING ORGANIZATION AFOSR			9b. OFFICE SYMBOL (If applicable) NM		
10a. ADDRESS (City, State and ZIP Code) Bldg. 410 Bolling AFB, DC 20332			9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR-83-0086		
11. TITLE (Include Security Classification) First Passage Times Stochastic Differential Equations of Mathematical Physics and Engineering			10. SOURCE OF FUNDING NOS.		
PERSONAL AUTHOR(S) B. J. Matkowsky			PROGRAM ELEMENT NO. 61102F		
12. TYPE OF REPORT Final			PROJECT NO. 2304		
13b. TIME COVERED FROM 4/1/83 TO 3/30/85			TASK NO. A4		
14. DATE OF REPORT (Yr., Mo., Day) April 1985			WORK UNIT NO.		
15. PAGE COUNT 5					
16. SUPPLEMENTARY NOTATION					
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)		
FIELD	GROUP	SUB. GR.	random phenomena, asymptotic methods, atomic migration chemical reaction rates		
19. ABSTRACT (Continue on reverse if necessary and identify by block number) The goal of our study was to develop new asymptotic and singular perturbation methods for the analysis of random phenomena, and their application in various areas of science and engineering. We have studied the problems of (1) atomic migration in crystals, (2) ionic and super-ionic conductivity, (3) chemical reaction rates, surface desorption rates, and more general activated rate processes, (4) noise effects on the hysteretic Josephson junction, D.C.-SQUID, and other tunnel junction devices, (5) stability and reliability of randomly loaded elastic structures, (6) relative stability of various multi-stable systems, among others. Considerable success has been achieved, not only in developing new mathematical methods, but in solving a number of problems whose solution has been long outstanding. These include the Kolmogorov Exit Problem and the Kramers nonlinear diffusion problem, among others.					
DTIC FILE COPY					
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT UNCLASSIFIED/UNLIMITED <input checked="" type="checkbox"/> SAME AS RPT. <input type="checkbox"/> DTIC USERS <input type="checkbox"/>			21. ABSTRACT SECURITY CLASSIFICATION Unclassified		
22a. NAME OF RESPONSIBLE INDIVIDUAL Dr. R. N. Buchal			22b. TELEPHONE NUMBER (Include Area Code) (202) 767-4939		
			22c. OFFICE SYMBOL NM		

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AD-A160 323

FINAL REPORT

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

AFOSR 83-0086



Accession For	
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DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
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Distribution/ _____	
Availability Codes	
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"FIRST PASSAGE TIMES IN STOCHASTIC DIFFERENTIAL EQUATIONS
OF MATHEMATICAL PHYSICS AND ENGINEERING"

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Approved for public release;
distribution unlimited.

April 1985

85-10 11 168

Final Report

Air Force Office of Scientific Research Grant AFOSR 83-0086, entitled
"First Passage Times in Stochastic Differential Equations of Mathematical
Physics and Engineering"

Principal Investigator: Prof. Bernard J. Matkowsky
Dept. of Engineering Sciences and Applied
Mathematics
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Under this grant, we supported the successful collaboration between Professors B. J. Matkowsky and Z. Schuss on Singular Perturbation Methods in Stochastic Differential Equations and Applications to various scientific disciplines. We made a number of breakthroughs on some fundamental problems, both in science and in mathematics.

The goal of our study was to develop new asymptotic and singular perturbation methods for the analysis of random phenomena, and their application in various areas of science and engineering. We have studied the problems of (1) atomic migration in crystals, (2) ionic and super-ionic conductivity, (3) chemical reaction rates, surface desorption rates, and more general activated rate processes, (4) noise effects on the hysteretic Josephson junction, D.C.-SQUID, and other tunnel junction devices, (5) stability and reliability of randomly loaded elastic structures, (6) relative stability of various multi-stable systems, among others. Considerable success has been achieved, not only in developing new mathematical methods, but in solving a number of problems whose solution has been long outstanding. These include the Kolmogorov Exit Problem and the Kramers nonlinear diffusion problem, among others. The problems above, are modelled by stochastic differential equations, describing continuous dynamical systems.

We also developed new methods for the study of stochastic difference and integro-differential-difference equations describing discrete systems, modelled by master equations. We studied new applications of these methods to (A) large communication and computer networks, and their scheduling, and (B) problems of large deviations in statistical testing, estimation and sequential sampling. In addition, master equations serve as a more fundamental description of the physical and chemical processes described above. In certain instances, the continuous description can be shown to be a limiting approximation of the discrete system, the so-called diffusion approximation. In other instances, however, this approximation procedure, though widely used, can be shown to yield results which are incorrect by orders of magnitude. We have given conditions for the validity, as well as the limitations, of this approximation, and when the approximation is not valid, we have provided new procedures for dealing directly with the underlying stochastic difference equations. A list of publications and abstracts now follows.

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH (AFOSR)
FUNDAMENTAL RESEARCH PROGRAM
STOCHASTIC PROCESSES
MATTHEW J.
Chief, Technological Institute Division

1. "Uniform expansion of the transition rate in Kramers problem," J. Statis. Physics 35, (1984) pp. 443-456.

Kramers' model of diffusion over potential barriers, e.g. chemical reactions, based on the noise activated escape of a particle from a potential well, is considered. Kramers derived escape rates valid for intermediate and large damping, and in a separate analysis, for small damping. In the small damping limit, Kramers' intermediate result reduces to the transition-state rate which does not agree with the small damping result. A new escape rate is derived that is uniformly valid for all values of the damping coefficient. The new rate reduces to Kramers' results in the appropriate limits and, in particular, connects Kramers' intermediate and small damping results.

2. "Solution of Kramers Moyal equations for problems in chemical physics," J. Chemical Physics 81 (1984) pp. 1285-1293.

We derive asymptotic solutions of Kramers-Moyal equations (KMEs) that arise from master equations (MEs) for stochastic processes. We consider both one step processes, in which the system jumps from x to $x + \epsilon$ or $x - \epsilon$ with given probabilities, and general transitions, in which the system moves from x to $x + \epsilon \xi$, where ξ is a random variable with a given probability distribution. Our method exploits the smallness of a parameter ϵ , typically the ratio of the jump size to the system size. We employ the full KME to derive asymptotic expansions for the stationary density of fluctuations about, as well as for the mean lifetime of stable equilibria. Thus we treat fluctuations of arbitrary size, including large fluctuations. In addition we present a criterion for the validity of diffusion approximations to master equations. We show that diffusion theory can not always be used to study large deviations. When diffusion theory is valid our results reduce to those of diffusion theory. Examples from macroscopic chemical kinetics and the calculation of chemical reaction rates ("Kramers" models) are discussed.

3. "Asymptotic analysis of the optimal filtering problem for one-dimensional diffusions measured in a low noise channel, Part II", SIAM J. Appl. Math. 44, (1984) pp. 1176

We consider the problem of filtering one-dimensional diffusions with nonlinear drift coefficients, transmitted through a nonlinear low noise channel. We construct an asymptotic solution to Zakai's equation for the unnormalized conditional probability density of the signal, given the noisy measurements. This expansion is used to find the asymptotic expansion of the minimum error variance filter and its mean square estimation error (MSEE). We construct approximate filters whose MSEE agrees with that of the optimal one to a given degree of accuracy. The dimension of the approximate filter increases with the required degree of accuracy. Similarly, we expand the maximum a posteriori probability estimator and the minimum energy estimator and compare their performance. We also discuss some extended Kalman filters and present some examples.

4. "Asymptotic solution of the Kramers-Moyal equation and first passage times for Markov jump processes," Physical Rev. A 29 (1984), pp. 3359

We calculate the activation rates of metastable states of general one-dimensional Markov jump processes by calculating mean first-passage times. We employ methods of singular perturbation theory to derive expressions for these rates, utilizing the full Kramers-Moyal expansions for the forward and backward operators in the master equation. We discuss various boundary conditions for the first-passage-time problem, and present some examples. We also discuss the validity of various diffusion approximations to the master equation, and their limitations.

5. "First passage times for processes governed by Master Equations," Springer Proceedings in Physics #1, (1984) pp. 19-36.

We calculate the activation rates of metastable states of processes governed by Master Equations, by calculating mean first passage times. We employ methods of singular perturbation theory to derive expressions for these rates, utilizing the full Kramers-Moyal expansions for the forward and backward operators in the Master Equation. In addition we discuss the validity of various diffusion approximations to the Master Equation, showing that such approximations are not valid in general.

6. "Noise-induced transitions in multi-stable systems," Springer Proceedings in Physics #1 (1984) pp. 79-94.

The damped physical pendulum driven by constant torque serves as a model for many physical systems (e.g., the motion of an ion in a crystal subject to a uniform electrostatic field, the point Josephson junction driven by constant current, charge density waves, etc.). For certain ranges of parameters it has both stable equilibrium states and a stable non-equilibrium state. In the presence of a random driving force of thermal or shot noise type there are transitions between the stable states of the pendulum. We calculate the steady state distribution of fluctuations about the stable states and the transition rates between them. For the point Josephson junction at very low temperatures we postulate the existence of "self-generated" shot noise and obtain transition rates which agree with the experimental results of Voss and Webb. This paper summarizes the work of Ben-Jacob, Bergman, Imry, Knessl, Matkowsky and Schuss.

7. "Frequency fluctuations in noisy oscillators," SIAM J. Appl. Math., to appear.

We study frequency and period fluctuations in a nonlinear oscillator driven by Gaussian white noise. We define the random period as the random time between two consecutive zero crossings by the random phase plane trajectory, and the random frequency as the number of such zero crossings per unit of time. These quantities are shown to be related by renewal theory. We find asymptotic expressions for the means and variances of the random period and random frequency, for small damping and small noise. The formulas are particularly useful for oscillators with high frequency.

8. "An asymptotic theory of large deviations for Markov jump processes," SIAM J. Appl. Math., to appear.

We present new asymptotic methods for the analysis of Markov jump processes. The methods, based on the WKB and other singular perturbation techniques, are applied directly to the Kolmogorov equations and not to approximate equations that come e.g. from diffusion approximations. For time homogeneous processes, we construct approximations to the stationary density function and the mean first passage time from a given domain. Examples involving a random walk and a problem in queueing theory are presented to illustrate our methods. For a class of time inhomogeneous processes, we construct long time approximations to the transition probability density function and the probability of large deviations from a stable state. The law of large numbers is obtained as a special case.

9. "Asymptotic analysis of a state-dependent M/G/1 queueing system," submitted for publication.

We present new asymptotic methods for the analysis of queueing systems. These methods are applied to a state-dependent M/G/1 queue. We formulate problems for and compute approximations to (i) the stationary density of the unfinished work; (ii) the mean length of time until the end of a busy period; (iii) the mean length of a busy period; (iv) the mean time until the unfinished work reaches or exceeds a specified capacity; and (v) the distribution of the maximum. The methods are applied to the full Kolmogorov equations, scaled so that the arrival rate is rapid and the mean service is small. Thus, we do not truncate equations as in diffusion approximations. We specialize the results to state-dependent M/M/1 queues. For state-independent M/M/1 queues, we demonstrate their accuracy by comparison with known exact solutions and results obtained using diffusion approximations.

10. "System crash in a finite capacity M/G/1 queue," submitted for publication.

We analyze a finite capacity, M/G/1 queueing system in which an arrival that causes the system capacity of unfinished work to be exceeded results in the loss of all jobs circulating in the system, i.e., the system "crashes". Problems are formulated for the stationary density of unfinished work, the system utilization, the mean length of the busy period, the mean total unfinished work in the system during a busy period, and the probability that the system crashes before completing a busy period. We construct exact solutions for these quantities for an M/M/1 queue with constant arrival and service rates. Approximate expressions for these quantities are constructed for M/G/1 queues in which the arrival rates and service densities depend on the amount of unfinished work in the system at the instant a new customer enters the system. These approximate expressions, which are obtained by using singular perturbation techniques, are shown to agree with the exact results for state-independent M/M/1 queues.

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