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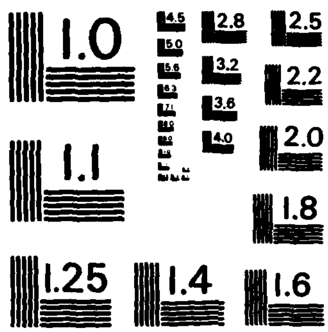

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PRINCIPAL COMPONENT ANALYSIS
UNDER CORRELATED MULTIVARIATE
REGRESSION EQUATIONS MODEL*

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ABSTRACT

In this paper, the authors consider the problem of testing for the equality of the last few eigenvalues of the covariance matrix under correlated multivariate regression equations (CMRE) models. Asymptotic distributions of various test statistics are derived when the underlying distribution is multivariate normal. Some of the distribution theory is extended to the case when the underlying distribution is elliptically symmetric.

1. INTRODUCTION

The classical multivariate regression model plays an important role in the analysis of the data in various disciplines. This model is nothing but a set of correlated univariate regression equations with the same design matrix. But, situations arise often when it is quite unrealistic to assume the same design matrix for each regression equation. For example, it is not realistic to assume, in some situations, that the same independent variables are adequate for prediction of each set of dependent variables. In these situations, we should consider correlated multivariate regression equations (CMRE) with different design matrices. Some work was done in the past (e.g., see Zellner (1962)) on the estimation of location parameters under correlated univariate regression equations model. Recently, Kariya, Fujikoshi and Krishnaiah (1984) considered the problem of testing for the independence of two sets of variables whereas Sarkar and Krishnaiah (1984) considered the problem of testing for sphericity under the CMRE model. In this paper, we consider the problem of testing the hypothesis that the last few eigenvalues of the covariance matrix are equal under the CMRE model.

In Section 2 of this paper, we given some preliminaries which are needed in the sequel. It is complicated to derive the likelihood ratio test statistic under the CMRE model. So, we considered a LRT-like test statistic. When the design matrices in the CMRE model are the same, the above test statistic is equivalent to the usual LRT statistic for testing the hypothesis of the equality of the last few eigenvalues of the covariance matrix. In Section 3, we derived an expression for the null distribution of the LRT-like test statistic under the CMRE model when the sample size tends to infinity. The expression obtained in Section 3 is the same whether the density matrices are equal or not. In Section 4, we derive the asymptotic distribution of the LRT-like test statistic under local alternatives. Asymptotic nonnull distributions of a class of test statistics are derived in

Section 5 under fixed alternatives. The above results are derived under the assumption that the underlying distribution is multivariate normal. In Sections 6 and 7 we extend some of the above results to the case when the underlying distibution is elliptically symmetric. Finally, in Section 8, we discuss applications of the results of this paper in the area of principal component analysis.

2. PRELIMINARIES

Consider two correlated multivariate regression equations given by

$$
\begin{aligned}
Y_1 &= X_1\theta_{11} + E_1 \\
Y_2 &= X_2\theta_{22} + E_2
\end{aligned}
\tag{2.1}
$$

where θ_{11}: $r_1 \times p_1$ and θ_{22}: $r_2 \times p_2$ are unknown, whereas the design matrices X_1: $n \times r_1$ and X_2: $n \times r_2$ are known and are of full rank. We will also assume that the rows of $(E_1\ E_2)$ are distributed independently as multivariate normal with mean vector $\underset{\sim}{0}$ and covariance matrix Σ where

$$
\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}
\tag{2.2}
$$

and Σ_{ij} is of order $p_i \times p_j$. The model given by (2.1) is known as the correlated multivariate regression equations (CMRE) model. Kariya, Fujikoshi and Krishnaiah (1984) considered the problem of testing the hypothesis $\Sigma_{12} = 0$ and derived asymptotic distributions of test statistics associated with testing the above hypotheses. Sarkar and Krishnaiah (1984) derived the asymptotic distributions of test statistics associated with testing the hypothesis that $\Sigma = \sigma^2 I$. In this paper, we derive asymptotic distributions of some test statistics used for testing the hypothesis H_0: $\lambda_{p-q+1} = \cdots = \lambda_p$ where $\lambda_1 \geq \cdots \geq \lambda_p$ are the eigenvalues of Σ. An estimate of Σ is S/n where

$$
S = \begin{bmatrix} Y_1'Q_1Y_1 & Y_1'Q_1Q_2Y_2 \\ Y_2'Q_2Q_1Y_1 & Y_2'Q_2Y_2 \end{bmatrix}
\tag{2.3}
$$

and

$$
Q_i = I_n - X_i(X_i'X_i)^{-1}X_i'.
\tag{2.4}
$$

Unless the design matrices are the same, S/ν is not an unbiased estimate of Σ. An unbiased estimate of Σ is $\hat{\Sigma}$ where $\hat{\Sigma} = (\hat{\Sigma}_{ij})$,

$$\hat{\Sigma}_{ii} = Y_i'Q_iY_i/(n-r_i), \quad \hat{\Sigma}_{12} = Y_1'Q_1Q_2Y_2/t$$

and t is rank of Q_1Q_2. Although $\hat{\Sigma}$ is an unbiased estimate of Σ, it may not be positive definite. So, we will use S/n to estimate Σ in the sequel.

We will now describe a representation of S due to Kariya, Fujikoshi and Krishnaiah (1984) since it is needed in the sequel. Consider the transformation

$$W_i = Z_i'Y_i = \begin{bmatrix} M_i \\ U_i \end{bmatrix} \tag{2.5}$$

where M_i is of order $(r_0-r_i)\times p_i$ and U_i is of order $(n-r_0)\times p_i$. Also, Z_i: $n\times(n-r_i)$ satisfies $Z_i'Z_i = I_{n-r_i}$, $Z_iZ_i' = Q_i$ and is chosen in the following way. Let $Q_0 = I - X(X'X)^{-1}X'$ where $X = [X_1, X_2]$ and $n_0 = n-r_0$. Then, we can express Q_0 as $Q_0 = Z_0Z_0'$ such that $Z_0'Z_0 = I_{n_0}$. Next, let $\bar{Q}_j$ $(j=1,2)$ denote the projection matrices onto $L(X)\cap L(Q_j)$ where $L(A)$ denotes the column space of the matrix A. Then, we can decompose $\bar{Q}_j$ as $\bar{Q}_j = \bar{Z}_j\bar{Z}_j'$ such that $\bar{Z}_j'\bar{Z}_j = I_{r_0-r_j}$. We then choose $Z_1 = (\bar{Z}_1, Z_0)$ and $Z_2 = (\bar{Z}_2, Z_0)$. In this case, $Z_iZ_i' = Q_i$ and $Z_i'Z_i = I_{n_i}$. Using this transformation, Kariya, Fujikoshi and Krishnaiah (1984) decomposed S as follows:

$$S = \begin{bmatrix} W_1'W_1 & W_1'Z_1'Z_2W_2 \\ W_2'Z_2'Z_1W_1 & W_2'W_2 \end{bmatrix} = G+B \tag{2.6}$$

where

$$G = \begin{pmatrix} U_1' \\ U_2' \end{pmatrix}(U_1 \quad U_2) \tag{2.7}$$

$$B = \begin{pmatrix} M_1'M_1 & M_1'KM_2 \\ M_2'K'M_1 & M_2'M_2 \end{pmatrix}, K = \bar{Z}_1'\bar{Z}_2. \tag{2.8}$$

In deriving various asymptotic distributions, we use perturbation technique repeatedly. So, we review this technique briefly.

Consider a diagonal matrix Λ with the ordered latent roots $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p$ and assume that the perturbation of Λ can be expressed as a power series in ε as follows:

$$M = \Lambda + \varepsilon V^{(1)} + \varepsilon^2 V^{(2)} + 0(\varepsilon^3), \tag{2.9}$$

where $V^{(j)}$, $j = 1,2,\ldots$ are symmetric matrices of order $p \times p$ and ε is a small real number and M is a $p \times p$ symmetric matrix whose eigenvalues are $\ell_1 \geq \ell_2 \geq \cdots \geq \ell_p$. Assume λ_α is simple for $\alpha = 1,2,\ldots,p$. Then the perturbation expansion of ℓ_α is given by

$$\ell_\alpha = \lambda_\alpha + \varepsilon v_{\alpha\alpha}^{(1)} + \varepsilon^2 [v_{\alpha\alpha}^{(2)} + \sum_{\beta \neq \alpha}^{p} \lambda_{\alpha\beta} v_{\alpha\beta}^{(1)^2}] + 0(\varepsilon^3) \tag{2.10}$$

where

$$V^{(j)} = (v_{\alpha\beta}^{(j)}) \text{ and } \lambda_{\alpha\beta} = (\lambda_\alpha - \lambda_\beta)^{-1}, \alpha \neq \beta .$$

The above expansion is due to Lawley (1956).

Next consider the case when latent roots of Λ have multiplicities. Suppose $\lambda_{q_1+q_2\ldots+q_{\alpha-1}+1} = \cdots = \lambda_{q_1+\ldots+q_\alpha} = \theta_\alpha$, that is, θ_α has multiplicity q_α for $\alpha = 1,2,\ldots,r$, $q_1+q_2+\ldots+q_r = p$ and $q_o = 0$. Then the mean eigenvalue of M corresponding to θ_α is

$$\bar{\ell}_\alpha = \theta_\alpha + \varepsilon \bar{\ell}_\alpha^{(1)} + \varepsilon^2 \bar{\ell}_\alpha^{(2)} + 0(\varepsilon^3), \tag{2.11}$$

where

$$\bar{\ell}_\alpha^{(1)} = \frac{1}{q_\alpha} \operatorname{tr} V_{\alpha\alpha}^{(1)},$$

$$\bar{\ell}_\alpha^{(2)} = \frac{1}{q_\alpha} \operatorname{tr} [V_{\alpha\alpha}^{(2)} + \sum_{\beta\neq\alpha} \theta_{\alpha\beta}^{-1} V_{\alpha\beta}^{(1)} V_{\beta\alpha}^{(1)}]$$

$$\theta_{\alpha\beta} = (\theta_\alpha - \theta_\beta),$$

with

$$V^{(i)} = \begin{bmatrix} V_{11}^{(i)} & V_{12}^{(i)} & \cdots & V_{1r}^{(i)} \\ \vdots & \vdots & & \vdots \\ V_{r1}^{(i)} & V_{r2}^{(i)} & \cdots & V_{rr}^{(i)} \end{bmatrix}$$

and $V_{\alpha\beta}^{(i)}$ is of order $q_\alpha \times q_\beta$. This result is implicit in Kato (1976). Fujikoshi (1978) also derived it by using a different method.

3. ASYMPTOTIC NULL DISTRIBUTION OF THE LRT-LIKE TEST

The LRT-like test for H_o is given by

$$\Lambda = [\prod_{p-q+1}^{p} \ell_i / (\frac{1}{q} \sum_{p-q+1}^{p} \ell_i)^q]^{n/2},$$

where $\ell_1 \geq \ell_2 \geq \cdots \geq \ell_p$ are the eigenvalues of $S_o = \frac{1}{n_o} S$. We will consider the distribution of

$$\begin{aligned} T &= -2 \log \Lambda \\ &= n[q \log (\mathrm{tr} S_o - \sum_{i=1}^{p-q} \ell_i) - q \log q - \log |S_o| \\ &\quad + \sum_{i=1}^{p-q} \log \ell_i]. \end{aligned} \tag{3.1}$$

For the asymptotic theory we assume, without loss of generality, that the covariance matrix Σ is diagonal. Hence, under H_o,

$$\begin{aligned} \Sigma &= \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_{p-q}, \quad \lambda, \lambda, \ldots, \lambda) \\ &= \Sigma_o, \text{ say.} \end{aligned} \tag{3.2}$$

As mentioned in the preceding section, S can be written as $G + B$ where $G \sim W_p(\Sigma_o, n_o)$ under H_o, and G and B are independently distributed. Define $V = \sqrt{n_o}((G/n_o) - \Sigma_o) = (v_{ij})$, so that

$$S_o = \frac{S}{n_o} = \Sigma_o + \frac{V}{\sqrt{n_o}} + \frac{B}{n_o}. \tag{3.3}$$

We assume that the population roots $\lambda_1, \lambda_2, \ldots, \lambda_{p-q}$ are simple. Then we have from (3.3)

$$\ell_j = \lambda_j + \frac{v_{jj}}{\sqrt{n_o}} + \frac{1}{n_o} \{b_{jj} + \sum_{\substack{k=1 \\ \neq j}}^{p} \lambda_{jk} v_{jk}^2\} + \frac{1}{n_o^{3/2}} Q_j \tag{3.4}$$

for $j = 1, 2, \ldots, (p-q)$ where

$$Q_j = [\sum_{\substack{i=1\\i\neq j}}^{p}\sum_{\substack{k=1\\k\neq j}}^{p}\lambda_{ji}\lambda_{jk}v_{ik}v_{jk}v_{ji} - v_{jj}\sum_{\substack{k=1\\k\neq j}}^{p}\lambda_{jk}v_{jk}^2$$

$$+ 2\sum_{\substack{k=1\\k\neq j}}^{p}\lambda_{jk}b_{jk}v_{jk}],$$

$\lambda_{jk} = (\lambda_j - \lambda_k)^{-1}$ and $\lambda_k = \lambda$ for $k = p-q+1,\ldots,p$. Next, let $\tilde{T} = a \cdot T$, where $a = n_o/n$. Then from (3.1) and (3.4) we get

$$T = T_o + \frac{T_1 + T_2}{\sqrt{n_o}}, \tag{3.5}$$

where

$$T_o = \sum_{j=1}^{p-q}\left(\frac{1}{\lambda_j} - \frac{1}{\lambda}\right)\sum_{\substack{k=1\\k\neq j}}^{p}\lambda_{jk}v_{jk}^2 + \frac{1}{2}\sum_{\substack{i=1\\i\neq j}}^{p-q}\sum_{j=1}^{p-q} v_{ij}^2/\lambda_i^2$$

$$+ \frac{1}{2}\sum_{i=1}^{p-q}\sum_{j=p-q+1}^{p} v_{ij}^2/\lambda_i^2 + \frac{1}{2}\sum_{i=p-q+1}^{p}\sum_{j=1}^{p-q} v_{ij}^2/\lambda^2$$

$$+ \frac{1}{2\lambda^2}\sum_{p-q+1}^{p} v_{ii}^2 + \frac{1}{2\lambda^2}\sum_{p-q+1}^{p}\sum_{\substack{p-q+1\\i\neq j}}^{p} v_{ij}^2 - \frac{1}{2q\lambda^2}\left(\sum_{p-q+1}^{p} v_{ii}\right)^2,$$

$$T_1 = \sum_{j=1}^{p-q}\left(\frac{1}{\lambda_j} - \frac{1}{\lambda}\right)\{\sum_{\substack{i=1\\i\neq j}}^{p}\sum_{\substack{k=1\\k\neq j}}^{p}\lambda_{ji}\lambda_{jk}v_{ik}v_{kj}v_{ji} - v_{jj}\sum_{k\neq j}\lambda_{jk}^2v_{jk}^2\}$$

$$- \sum_{j=1}^{p-q}\frac{v_{jj}}{\lambda_j^2}\sum_{k\neq j}\lambda_{jk}v_{jk}^2 - \frac{1}{3}\operatorname{tr}(\Sigma_o^{-1}v)^3 + \frac{1}{3}\sum_{j=1}^{p-q}v_{jj}^3/\lambda_j^3 + \frac{1}{3q^2\lambda^3}\left(\sum_{p-q+1}^{p}v_{jj}\right)^3$$

$$+ \frac{1}{q\lambda^2}\left(\sum_{p-q+1}^{p}v_{jj}\right)\{\sum_{j=1}^{p-q}\sum_{\substack{k=1\\k\neq j}}^{p}\lambda_{jk}v_{jk}^2\}$$

and

$$T_2 = 2 \sum_{j=1}^{p-q} \left(\frac{1}{\lambda_j} - \frac{1}{\lambda}\right) \sum_{\substack{k=1 \\ k\neq j}}^{p} \lambda_{jk} b_{jk} v_{jk} - \sum_{j=1}^{p-q} \frac{1}{\lambda_j^2} v_{jj} b_{jj}$$

$$- \frac{1}{q\lambda^2} \left(\sum_{p-q+1}^{p} v_{jj} \right) \left(\sum_{p-q+1}^{p} b_{jj} \right) + \sum_{i=1}^{p} \sum_{j=1}^{p} \frac{v_{ij} b_{ji}}{\lambda_i \lambda_j} . \tag{3.6}$$

From (3.5), the characteristic function of $\tilde{T}$ is

$$\phi(t) = \phi_1(t) + \phi_2(t) + O(n_o^{-1}) \tag{3.7}$$

where

$$\phi_1(t) = E[e^{itT_o} \{1 + \frac{it}{\sqrt{n_o}} T_1\}],$$

$$\phi_2(t) = E[e^{itT_o} \frac{it}{\sqrt{n_o}} T_2]. \tag{3.8}$$

Note that the characteristic function of

$$-2(n_o/n) \log [\prod_{p-q+1}^{p} d_i \left(\frac{1}{q} \sum_{p-q+1}^{p} d_i\right)^q]^{n/2}$$

is $\phi_1(t) + O(n_o^{-1})$, where $d_1 \geq d_2 \geq \dots \geq d_p$ are the eigenvalues of G/n_o. We know that [see Fujikoshi, 1977] $\phi_1(t) = (1-2it)^{-f/2}$, where $f = (q(q+1)/2) - 1$. Without loss of generality, suppose $q \geq p_2$ i.e. $p-q < p_1$. Then under H_o,

$$E(b_{ii}) = \begin{cases} (r_o - r_1)\, \lambda_i, & i = 1(1)p-q \\ (r_o - r_1)\, \lambda, & i = p-q+1, \dots, p_1 \\ (r_o - r_2)\, \lambda, & i = p_1+1, \dots, p \end{cases}$$

$$E(b_{ij}) = 0, \; i \neq j. \tag{3.9}$$

Taking conditional expectation with respect to M_i's only we get

$$E_M T_2 = -\sum_{j=1}^{p-q} \frac{v_{jj}}{\lambda j}(r_o - r_1) - \frac{1}{q\lambda}\left(\sum_{p-q+1}^{p} v_{jj}\right)[(r_o - r_1)(q - p_2)$$

$$+ (r_o - r_2)p_2] + \sum_{i=1}^{p-q} \frac{(r_o - r_1)v_{ii}}{\lambda_i} + \sum_{p-q+1}^{p_1} \frac{(r_o - r_1)v_{ii}}{\lambda}$$

$$+ (r_o - r_2)\sum_{p_1+1}^{p} v_{ii}/\lambda$$

$$= C_1 \sum_{p_1+1}^{p} v_{ii} - C_2 \sum_{p-q+1}^{p_1} v_{ii}, \tag{3.10}$$

where

$$C_1 = \frac{(r_1 - r_2)^{q-p_2}}{\lambda q}, \quad C_2 = \frac{(r_1 - r_2)p_2}{\lambda q}.$$

Let

$$\underset{\sim}{v}^{*\prime} = (v_{11}, \ldots, v_{pp}, v_{12}, \ldots, v_{1p}, v_{23}, \ldots, v_{2p}, \ldots, v_{p-1,p}).$$

Then

$$E(\underset{\sim}{v}^*) = 0, \text{ and } \mathrm{Var}(\underset{\sim}{v}^*) = \Delta,$$

where Δ is a diagonal matrix whose elements are given below:

$$\mathrm{Var}(v_{ii}) = \begin{cases} 2\lambda_i^2, & i = 1(1)\, p - q \\ 2\lambda^2, & i = p - q+1, \ldots, p. \end{cases}$$

$$\mathrm{Var}(v_{ij}) = \begin{cases} \lambda_i\lambda_j, & 1 \le i \ne j \le p-q \\ \lambda\lambda_i, & 1 \le i \le p-q,\ p-q+1 \le j \le p \\ \lambda\lambda_j, & 1 \le j \le p-q,\ p-q+1 \le i \le p \\ \lambda^2, & p-q+1 \le i \ne j \le p. \end{cases} \tag{3.11}$$

The limiting distribution of $V = (v_{ij})$ is the distribution of $\bar{V} = (\bar{v}_{ij})$, where $\bar{v}_{ij}$ $(i \leq j)$'s are all independently distributed as normal with mean 0 and variances given by (3.11). This implies that, in the limit, the elements of $\underset{\sim}{v}^*$ are distributed independently as normal with zero means and variances given by (3.11). Now, let $T_o = \underset{\sim}{v}^{*\prime} A \underset{\sim}{v}^*$, where A is a matrix whose elements depend on λ_i's, λ, p and q. Hence,

$$\phi_2(t) = \frac{it}{\sqrt{n_o}} K_1 \int e^{it\underset{\sim}{v}^{*\prime} A \underset{\sim}{v}^* - \frac{1}{2} \underset{\sim}{v}^{*\prime} \Delta^{-1} \underset{\sim}{v}^*} [C_1 \sum_{p_1+1}^{p} v_1^* - C_2 \sum_{p-q+1}^{p_1} v_i^*] d\underset{\sim}{v}^* = 0$$

where $K_1 = (2\pi)^{-p^2/2} |\Delta|^{-1/2}$. So, from (3.7) and (3.8),

$$\begin{aligned} \phi(t) &= \phi_1(t) + 0(n_o^{-1}) \\ &= (1-2it)^{-f/2} + 0(n_o^{-1}), \end{aligned} \tag{3.12}$$

where $f = (q(q+1)/2) - 1$.

Inverting the right side of (3.12) yields the following expression for the asymptotic distribution of Λ:

$$\Pr[-2n_o \log \Lambda / n \leq x] = \Pr(\chi_f^2 \leq x) + 0(n_o^{-1}) \tag{3.13}$$

where $f = [(q(q+1)/2) - 1]$.

4. ASYMPTOTIC NON-NULL DISTRIBUTION OF THE LRT-LIKE TEST UNDER LOCAL ALTERNATIVES

In this section, we derive asymptotic distribution of the LRT-like test statistic under the local alternative H_θ where

$$H_\theta: \quad \lambda_i = \lambda + (\theta_i/\sqrt{n_o}\,) \qquad i = p-q+1,\ldots,p \tag{4.1}$$

where θ_i's are not all equal. Now, define $\tilde{V}$ as $\tilde{V} = \sqrt{n_o}\,((G/n_o)-\Sigma_o)$ so that $\tilde{V} = V+D_\theta^{(0)}$ where $D_\theta^{(0)} = \mathrm{diag}(0,\ldots,0,\theta_{p-q+1},\ldots,\theta_p)$. Hence

$$S_o = \Sigma_o + \frac{\tilde{V}}{\sqrt{n_o}} + \frac{B}{n_o}. \tag{4.2}$$

Note that (4.2) is of the same form as of (3.3), except for the change that V is replaced by $\tilde{V}$. Hence, from Section 3, the characteristic function of $\tilde{T}$ is

$$\phi(t) = \phi(t) + \phi_2(t) + 0(n_o^{-1}),$$

where $\phi_1(t)$, $\phi_2(t)$, T_o, T_1, T_2 are as defined in Section 3, with v_{ij}'s replaced by $\tilde{v}_{ij}$'s. Also note that

$$\tilde{v}_{ii} = v_{ii}, \quad i = 1(1)p-q,$$

$$\tilde{v}_{ij} = v_{ij}, \quad i \neq j, \quad i,j = 1(1)p$$

$$\tilde{v}_{ii} = v_{ii} + \theta_i, \quad i = p-q+1,\ldots,p.$$

Now, define

$$\underset{\sim}{\tilde{V}}^{*\prime} = (\tilde{v}_{11},\ldots,\tilde{v}_{pp},\tilde{v}_{12},\ldots\tilde{v}_{1p},\tilde{v}_{23},\ldots,\tilde{v}_{2p},\ldots,\tilde{v}_{p-1,p}).$$

Then

$$E(\tilde{V}^*) = \mu_\theta , \quad Var(\tilde{V}^*) = \Delta,$$

where

$$\mu_\theta' = (0,\ldots,0, \theta_{p-q+1},\ldots,\theta_p, 0,\ldots,0)$$

and Δ was given in Section 3. As $n_o \to \infty$, $\tilde{V}^*$ is distributed as multivariate normal with mean vector μ_θ and covariance matrix Δ. Further, we have $T_o = \tilde{V}^{*\prime} A\tilde{V}^*$ and

$$E_M(T_2) = C_1 \sum_{i=p_1+1}^{p} \tilde{V}_{ii} - C_2 \sum_{i=p-q+1}^{p_1} \tilde{V}_{ii},$$

where $C_1 = (r_1 - r_2)(q - p_2)/\lambda q$, $C_2 = (r_1 - r_2)p_2/\lambda q$ so that we get

$$\phi_2(t) = \frac{it}{\sqrt{n_o}} |I - 2it\Delta A|^{-1/2} \exp[- \frac{1}{2} \{\mu_\theta' \sum_{r=1}^{\infty} (2it)^r (A\Delta)^{r-1} \Delta\mu_\theta\}]$$

$$x\ c'[\sum_{r=1}^{\infty} (2it)^r (A\Delta)^{r-1} \Delta\mu_\theta], \tag{4.3}$$

where $c' = (0\varepsilon_{p-q}' - C_2\varepsilon_{q-p_2}'\ C_1\varepsilon_{p_2}'\ 0\varepsilon_{p^2-p}')$. Note that the characteristic function of

$$-2a \log [\prod_{p-q+1}^{p} d_i (\frac{1}{q} \sum_{p-q+1}^{p} d_i)^q]^{n/2}$$

is $\phi_1(t) + 0(n_o^{-1})$, where $a = n_o/n$, and $d_1 \geq \ldots \geq d_p$ are the eigenvalues of G/n_o, where under H_θ, $G \sim W_p\ (n_o, \Sigma_o + (D_\theta^{(0)}/\sqrt{n_o})$. Hence (see Fujikoshi, 1981)

$$\phi_1(t) = \psi_f(t, \delta^2/\lambda^2)[1 + \frac{1}{\sqrt{n_o}} \sum_{j=0}^{2} b_j \lambda^{-3} (1 - 2it)^{-j}] + 0(n_o^{-1}) \tag{4.4}$$

where, $\psi_f(t,\Delta)$ is the characteristic function of noncentral χ^2 variable with f df. and noncentrality parameter Δ, and

$$f = \frac{1}{2}(q+2)(q-1)$$

$$\delta^2 = \frac{1}{4}[\mathrm{tr}\Omega^2 - q^{-1}(\mathrm{tr}\Omega)^2],$$

$$\Omega = \mathrm{diag}(\theta_{p-q+1}, \ldots, \theta_p)$$

$$b_o = \frac{1}{6}[2\mathrm{tr}\Omega^3 - 3q^{-1}(\mathrm{tr}\Omega)\mathrm{tr}\Omega^2 + q^{-2}(\mathrm{tr}\Omega)^3],$$

$$b_1 = \frac{1}{2}[-\mathrm{tr}\Omega^3 + 2q^{-1}(\mathrm{tr}\Omega)\mathrm{tr}\Omega^2 - q^{-2}(\mathrm{tr}\Omega)^3],$$

$$b_2 = \frac{1}{6}[\mathrm{tr}\Omega^3 - 3q^{-1}(\mathrm{tr}\Omega)\mathrm{tr}\Omega^2 + 2q^{-2}(\mathrm{tr}\Omega)^3].$$

5. ASYMPTOTIC DISTRIBUTIONS OF A GENERAL CLASS OF TEST STATISTICS

In this section, we derive asymptotic distributions of the function $f(\ell_1,\ldots,\ell_p)$ which is analytic around $(\lambda_1,\ldots,\lambda_p)$ for two cases. In the first case, we assume that the first partial derivatives of $f(\ell_1,\ldots,\ell_p)$ are not all zero. In the second case, we assume that all of the above partial derivatives are zero. The modified LRT statistic, under the null hypothesis, belongs to the second case whereas it belongs to the first case under the alternative hypothesis.

Let $\lambda_i = \theta_i$ for $i = 1,2,\ldots,(r-1)$ and $\lambda_r = \ldots = \lambda_p = \lambda = \theta_r$. Also, let $J_i = \{i\}$ for $i = 1,2,\ldots,(r-1)$ and J_r denote the set $\{r,r+1,\ldots,p\}$. We make the following assumptions:

$$\text{(i)}\quad \left.\frac{\partial f}{\partial \ell_j}\right|_{\underset{\sim}{\ell}=\underset{\sim}{\lambda}} = f_\alpha \quad \text{for } j \in J_\alpha \tag{5.1}$$

$$\text{(ii)}\quad \left.\frac{\partial^2 f}{\partial \ell_j \partial \ell_k}\right|_{\underset{\sim}{\ell}=\underset{\sim}{\lambda}} = f_{\alpha\beta} \quad \text{for } j \in J_\alpha,\ \ k \in J_\beta$$

where $\underset{\sim}{\ell}' = (\ell_1,\ldots,\ell_p)$ and $\underset{\sim}{\lambda}' = (\lambda_1,\ldots,\lambda_{r-1},\lambda,\ldots,\lambda)$. Expanding the function $f(\ell_1,\ldots,\ell_p)$ around $\underset{\sim}{\lambda}' = (\lambda_1,\ldots,\lambda_{r-1},\lambda,\ldots,\lambda)$ and using perturbation expansion of $\ell_1,\ldots,\ell_p$, we arrive at the following:

$$\begin{aligned} L &= \sqrt{n_o}\,[f(\ell_1,\ldots,\ell_p) - f(\lambda_1,\ldots,\lambda_p)] \\ &= \sum_{\alpha=1}^{r} f_\alpha \mathrm{tr} V_{\alpha\alpha} + \frac{1}{\sqrt{n_o}}\Big[\sum_{\alpha=1}^{r} f_\alpha \mathrm{tr} B_{\alpha\alpha} + \sum_{\alpha\neq\beta}^{r}\sum^{r} f_\alpha \theta_{\alpha\beta} \mathrm{tr} V_{\alpha\beta} V_{\beta\alpha} \\ &\qquad + \frac{1}{2}\sum_{\alpha}^{r}\sum_{\beta}^{r} f_{\alpha\beta} \mathrm{tr} V_{\alpha\alpha} \mathrm{tr} V_{\beta\beta}\Big] \\ &\quad + O(n_o^{-1}) \end{aligned} \tag{5.2}$$

where $V = (V_{\alpha\beta})$, $B = (B_{\alpha\beta})$ and V_β and $B_{\alpha\beta}$ are of order $q_\alpha \times q_\beta$ where $q_1 = \ldots = q_{r-1} = 1$. Let $r = p-q+1$. Then,

$$L = T_o + \frac{1}{\sqrt{n_o}}(T_1 + T_2) + O(n_o^{-1})$$

where

$$T_o = \sum_{\alpha=1}^{p-q} f_\alpha v_{\alpha\alpha} + f_{p-q+1} \sum_{p-q+1}^{p} v_{ii},$$

$$T_1 = \sum_{\alpha}^{p-q} \sum_{\beta}^{p-q} f_\alpha \lambda_{\alpha\beta} v_{\alpha\beta}^2 + \sum_{\alpha=1}^{p-q} f_\alpha (\lambda_\alpha - \lambda)^{-1} \left(\sum_{p-q+1}^{p} v_{\alpha i}^2\right)$$

$$+ \sum_{\beta=1}^{p-q} f_r (\lambda - \lambda_\beta)^{-1} \left(\sum_{p-q+1}^{p} v_{\beta i}^2\right)$$

$$+ \frac{1}{2} \sum_{\alpha=1}^{p-q} \sum_{\beta=1}^{p-q} f_{\alpha\beta} v_{\alpha\alpha} v_{\beta\beta} + \frac{1}{2} \sum_{\alpha=1}^{p-q} f_{\alpha r} v_{\alpha\alpha} \left(\sum_{p-q+1}^{p} v_{ii}\right) \tag{5.3}$$

$$+ \frac{1}{2} \sum_{\beta=1}^{p-q} f_{r\beta} v_{\beta\beta} \left(\sum_{p-q+1}^{p} v_{ii}\right) + \frac{1}{2} f_{rr} \left(\sum_{p-q+1}^{p} v_{ii}\right)^2,$$

and

$$T_2 = \sum_{\alpha=1}^{p-q} f_\alpha b_{\alpha\alpha} + f_{p-q+1} \sum_{p-q+1}^{p} b_{ii}.$$

The characteristic function of L, $\phi(t)$, can be written as

$$\phi_1(t) + \phi_2(t) + O(n_o^{-1}),$$

where $\phi_1(t)$, $\phi_2(t)$ have the same expressions as in Section 3, with T_o, T_1, T_2 given by (5.3).

Now, let

$$\underset{\sim}{v}^{*\prime} = (v_{11}, \ldots, v_{pp};\ v_{12}, \ldots, v_{1p},\ v_{23}, \ldots, v_{2p}, \ldots, v_{p-1,p}).$$

Then as $n \to \infty$, $\underset{\sim}{v}^*$ is distributed as multivariate normal with mean vector $\underset{\sim}{0}$ and covariance matrix Δ where Δ is given in Section 3. Also, let

$$\underset{\sim}{a}_o' = (f_1, \ldots, f_{p-q}, \underbrace{f_r, f_r, \ldots, f_r}_{q}, 0, \ldots, 0).$$

Then, $T_o = \underset{\sim}{a}_o' \underset{\sim}{v}^*$. We can write T_1 as $T_1 = \underset{\sim}{v}^{*'} \Lambda_o \underset{\sim}{v}^*$, Λ_o being dependent on f_α's, $f_{\alpha\beta}$'s, λ_α's, p and q. Then we have

$$\phi_1(t) = e^{-t^2/2 \cdot a_o' \Delta a_o}[1 + \frac{it}{\sqrt{n_o}} \operatorname{tr}(\Lambda_o \Delta) + \frac{(it)^3}{\sqrt{n_o}} \{\underset{\sim}{a}_0' \Delta \Lambda \Delta \underset{\sim}{a}_0$$

$$+ \frac{4}{3}(\sum_1^{p-q} f_i^3 \lambda_i^3 + q\lambda^3 f_{p-q+1}^3)\}] + 0(n_o^{-1}). \tag{5.4}$$

To evaluate $\phi_2(t)$, we take conditional expectation of T_2 with respect to M_i's and get

$$E_M(T_2) = (r_o - r_1) \sum_{\alpha=1}^{p-q} \lambda_\alpha f_\alpha + f_r \lambda[(r_1 - r_2)p_2 + (r_o - r_1)q]$$

$$= K_2 \text{ (say)}$$

assuming that $q \geq p_2$ without loss of generality. Hence,

$$\phi_2(t) = \frac{it}{\sqrt{n_o}} K_2 \, E[e^{it a_o' v^*}]$$

$$= \frac{it}{\sqrt{n_o}} K_2 \, e^{-t^2/2 \cdot a_o' \Delta a_o}. \tag{5.5}$$

Finally,

$$\phi(t) = e^{-t^2\tau^2/2}[1 + \frac{it}{\sqrt{n_o}} g_1 + \frac{(it)^3}{\sqrt{n_o}} g_3] + 0(n_o^{-1}) \tag{5.6}$$

where

$$\tau^2 = a_o' \Delta a_o = 2[\sum_{I}^{p-q} f_i^2 \lambda_i^2 + q\lambda^2 f_{p-q+1}^2],$$

$$g_1 = K_2 + tr(\Lambda_o \Delta),$$

$$g_3 = a_o' \Delta \Lambda_o \Delta a_o + \frac{4}{3}(\sum_{I}^{p-q} f_i^3 \lambda_i^3 + q\lambda^3 f_{p-q+1}^3) \tag{5.7}$$

Inverting the right hand side of (5.6), we get the following

Theorem 5.1. Let

$$L^* = \sqrt{n_o}\,\{f(\ell_1,\ldots,\ell_p) - f(\lambda_1,\ldots,\lambda_p)\}/\tau \tag{5.8}$$

where $\tau > 0$ and $\lambda_1 > \lambda_2 > \ldots > \lambda_{p-q} > \lambda_{p-q+1} = \ldots = \lambda_p = \lambda$. Then

$$P[L^* \leq x] = \Phi(x) - \frac{1}{\sqrt{n_o}}\,[(g_1/\tau)\Phi^{(1)}(x) + (g_3/\tau^3)\Phi^{(3)}(x)] + 0(n_o^{-1}) \tag{5.9}$$

where $\Phi^{(j)}(x)$ is the j^{th} derivative of the standard normal distribution function $\Phi(x)$ and τ, g_1 and g_3 are given by (5.7).

When $q = 1$ in the expression (5.8), we obtain the asymptotic distribution of L^* when $\lambda_1 > \ldots > \lambda_p$.

When $f_1,\ldots,f_r$ in (5.2) are simultaneously equal to zero, we cannot obtain the asymptotic distribution of $f(\ell_1,\ldots,\ell_p)$ from (5.8). For example, let

$$f(\ell_1,\ldots,\ell_p) = T$$

where T was defined by (3.1). Then

$$\left.\frac{\partial f}{\partial \ell_j}\right|_{\underset{\sim}{\ell} = \underset{\sim}{\lambda}} = 0 \tag{5.10}$$

for $j = 1,2,\ldots,p$ when $\lambda_{p-q+1} = \ldots = \lambda_p$. So, the asymptotic distribution of the LRT-like test statistic cannot be obtained from Theorem 5.1.

Now let

$$L^{**} = n_o\{f(\ell_1,\ldots,\ell_p) - f(\ell_1,\ldots,\ell_p)\}$$

where $\lambda_1 > \lambda_2 > \ldots > \lambda_{p-q} > \lambda_{p-q+1} = \ldots = \lambda_p = \lambda$ and we assume that $f(\ell_1,\ldots,\ell_p)$ satisfies the conditions (5.1). Also, we assume that $f_1 = \ldots = f_{p-q+1} = 0$ and $f'_{\alpha\beta}$ are not all equal to zero simultaneously. Then

$$L^{**} = \frac{1}{2} \sum_{\alpha=1}^{r} \sum_{\beta=1}^{r} f_{\alpha\beta} \operatorname{tr} V_{\alpha\alpha} \operatorname{tr} V_{\beta\beta}$$

$$= \underset{\sim}{V}' F \underset{\sim}{V} \tag{5.11}$$

where $\underset{\sim}{V}' = (v_{11},\ldots,v_{(p-1)(r-1)}, \operatorname{tr} V_{rr})$, $r = p-q+1$, and $\operatorname{tr} V_{rr} = v_{rr} + \ldots + v_{pp}$, and $F = \frac{1}{2}(f_{\alpha\beta})$. But, we know that $v_{11},\ldots,v_{pp}$ are distributed asymptotically as independent normal variables with zero means and variances given by $\operatorname{var}(v_{ii}) = 2\lambda_i^2$ for $i = 1,2,\ldots,p-q$ and $\operatorname{var}(v_{ii}) = 2\lambda^2$ for $i = p-q+1,\ldots,p$. So, L^{**} is asymptotically distributed as a linear combination of independent chi-square variables with one degree of freedom.

6. ASYMPTOTIC DISTRIBUTION OF AN ANALYTIC FUNCTION WHEN THE UNDERLYING DISTRIBUTION IS ELLIPTICALLY SYMMETRIC

Now we will give asymptotic distributions of the statistics L^* and L^{**} defined in Section 5, when the underlying distribution is elliptically symmetric. Here we note that $\underset{\sim}{x}$: $n\times 1$ is said to be elliptically symmetric if its characteristic function is of the form $\exp(i\underset{\sim}{t}'\underset{\sim}{\mu})\phi(\underset{\sim}{t}'\Omega\underset{\sim}{t})$, where $\underset{\sim}{\mu}$: $n\times 1$, Ω: $n\times n$, and $\Omega \geq 0$ and we write $\underset{\sim}{x} \sim EC_n(\underset{\sim}{\mu},\Omega;\phi)$. Multivariate normal, multivariate t and some other distributions belong to the family of elliptically symmetric distributions. For a discussion on the elliptically symmetric distributions, the reader is referred to Kelker (1970).

In this section, we assume as before, that the rows of E are independently distributed with the same dispersion Σ and also

$$\underset{\sim}{e}_j \sim EC_n(\underset{\sim}{0},\lambda_j^* I_n;\phi) \tag{6.1}$$

where $\lambda_j^* \propto \lambda_j$, $j = 1,2,\ldots,p$.

For simplicity of notation, let $r_1 = r_2 = r$ and $n_o = n-r$. Also let $Q_1 = (q_{ij}^{(1)})$, $Q_2 = (q_{ij}^{(2)})$. We assume that each of $\sum_{\alpha=1}^{n} q_{\alpha\alpha}^{(1)} q_{\alpha\alpha}^{(2)}/n_o$, $\sum_{\alpha=1}^{n} q_{\alpha\alpha}^{(j)2}/n_o$, $j = 1,2,3$ and $\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} q_{\alpha\beta}^{(j)2}/n_o$, $j = 1,2,3$ are of $O(1)$ and we write for large n

$$\sum_{\alpha} {q_{\alpha\alpha}^{(j)}}^2/n_o = K_1^{(j)} \qquad j = 1,2,3$$

$$\sum_{\alpha\neq\beta}\sum {q_{\alpha\beta}^{(j)}}^2/n_o = K_2^{(j)} \qquad j = 1,2,3$$

and

$$\sum_{\alpha} q_{\alpha\alpha}^{(1)} q_{\alpha\alpha}^{(2)}/n_o = K_3. \tag{6.2}$$

Then it can be shown that the limiting distribution of $Z = \sqrt{n_o}\,(\frac{S}{n_o} - D_\lambda)$ is the same as that of $\bar{Z} = (\bar{z}_{ij})$, where

$$\bar{Z}_{ii} \sim N(0, 2\lambda_i^2 \phi),$$

$$\bar{Z}_{ij} \sim N(0, \lambda_i \lambda_j \psi), \quad i \neq j \tag{6.3}$$

where

$$\phi = \begin{cases} 1 + \frac{3}{2}\,(\phi''(o)/\phi'(o)^2 - 1)K_1^{(1)}, & i = 1,2,\ldots,p_1 \\ 1 + \frac{3}{2}\,(\phi''(o)/\phi'(o)^2 - 1)K_1^{(2)}, & i = p_1+1,\ldots,p \end{cases}$$

and

$$\psi = \begin{cases} [K_2^{(1)} + \frac{\phi''(o)}{\phi'(o)^2} K_1^{(1)}] & i,j = 1,\ldots,p_1 \\ [K_2^{(2)} + \frac{\phi''(o)}{\phi'(o)^2} K_1^{(2)}], & i,j = p_1+1,\ldots,p \\ [K_2^{(3)} + \frac{\phi''(o)}{\phi'(o)^2} K_1^{(3)}], & i = 1,\ldots,p_1;\ j = p_1+1,\ldots,p \\ & j = 1,\ldots,p_1;\ j = p_1+1,\ldots,p. \end{cases}$$

Also

$$\mathrm{Cov}(\bar{Z}_{ii}, \bar{Z}_{jj}) = \lambda_i \lambda_j \left(\frac{\phi''(o)}{\phi'(o)^2} - 1\right)C, \quad i \neq j \tag{6.4}$$

where

$$C = \begin{cases} K_1^{(1)}, & i,j = 1(1)p_1 \\ K_1^{(2)}, & i,j = p_1+1,\ldots,p \\ K_3, & i = 1(1)p_1;\ j = p_1+1,\ldots,p. \end{cases}$$

All other elements of $\bar{Z}$ are uncorrelated.

We define $f(\ell_1,\ldots,\ell_p)$ the same way as in Section 5 and make the same assumptions on it. Σ also has the same structure as in Section 5, i.e. $\lambda_{p-q+1}=\ldots=\lambda_p=\lambda$.

Let

$$\underset{\sim}{Z}' = (Z_{11},\ldots,Z_{pp},Z_{12},\ldots,Z_{1p},Z_{23},\ldots,Z_{2p},\ldots,Z_{(p-1)p}).$$

Then for large n,

$$\underset{\sim}{Z} \sim (0,\Omega), \tag{6.5}$$

$$\text{where } \Omega = (w_{ij})$$

w_{ij}'s are given by (6.2), (6.3) and (6.4), with the restriction $\lambda_{p-q+1}=\ldots=\lambda_p=\lambda$. We get the following result for the asymptotic distribution of $f(\ell_1,\ldots,\ell_p)$.

<u>Theorem 6.1</u>. Let

$$L^* = \sqrt{n_o}\,\{f(\ell_1,\ldots,\Omega_p) - f(\lambda_1,\ldots,\lambda_p)\}/\tau$$

where $\tau > 0$ and $\lambda_1>\ldots>\lambda_{p-q} = \lambda_{p-q+1}=\ldots=\lambda_p=\lambda$. Then

$$P[L^* \leq x] = \Phi(x) + 0(n_o^{-1/2}) \tag{6.6}$$

where $\Phi(x)$ is defined before, and $\tau^2 = \underset{\sim}{a}_o'\Omega\underset{\sim}{a}_o$, where $(f_1,\ldots,f_{p-q},f_{p-q+1},\ldots,f_{p-q+1},0,\ldots,0)$. When $q = 1$, we get from (6.6) the asymptotic distribution of L^* when $\lambda_1>\lambda_2>\ldots>\lambda_p$.

If $f_\alpha = 0\ \forall\,\alpha$ and $f_{\alpha\beta}$'s are not all zero simultaneously, we can write $L^{**} = n_o\{f(\ell_1,\ldots,\ell_p) - f(\lambda_1,\ldots,\lambda_p)\}$ as $\underset{\sim}{Z}'\Gamma\underset{\sim}{Z} + 0(n_o^{-1/2})$, where Γ is a function of $f_{\alpha\beta}$'s, p and q. Then the characteristic function of L^{**} is

$$|I - 2it\Gamma\Omega|^{-1/2} + O(n_o^{-1/2}),$$

which implies that for large n, L^{**} is distributed as a linear combination of χ^2 variables with 1 d.f. We can get the asymptotic null and nonnull distributions of the LRT-like test as special case of the above results.

7. ASYMPTOTIC DISTRIBUTIONS OF THE LRT-LIKE TEST WITH ELLIPTICALLY SYMMETRIC ERRORS

In this section, we derive the asymptotic distributions of the LRT-like test under the following assumption on the errors:

$$\underset{\sim}{e} \sim EC_{np}(0, I_n \otimes \Sigma^*; \phi) \tag{7.1}$$

where $\underset{\sim}{e}$ = Vec E' and Σ^* is proportional to Σ. Then we can write (e.g., see Anderson and Fang (1982))

$$E \overset{d}{=} RUA,$$

where $A'A = \Sigma^*$, A; p×p, U: n×p, Vec U = $\underset{\sim}{u}^{(np)}$, distribution function of R is related to ϕ and R is independent of U. Here "$X \overset{d}{=} Y$" means that the distribution of X is the same as that of Y. Let us write $A = (A_1\ A_2)$ where A_i is of order $n\times p_i$, $i = 1,2$, and $p = p_1+p_2$; then we have $E_1 \overset{d}{=} RUA_1$, $E_2 \overset{d}{=} RUA_2$. Since

$$S = \begin{pmatrix} E_1'Q_1E_1 & E_1'Q_1Q_2E_2 \\ E_2'Q_2Q_1E_1 & E_2'Q_2E_2 \end{pmatrix}$$

we get

$$S \overset{d}{=} R^2 \begin{pmatrix} A_1'U'Q_1UA_1 & A_1'U'Q_1Q_2UA_2 \\ A_2'U'Q_2Q_1UA_1 & A_2'U'Q_2UA_2 \end{pmatrix}.$$

Using this it can be shown that Λ is independent of R^2. Hence the distribution of Λ will be the same as in the normal case. Thus the asymptotic null and non-null distributions of Λ under assumption (7.1) are the same as in Sections 3 and 4 respectively.

8. APPLICATIONS

The motivation behind the study in this paper is to derive some asymptotic results useful in the area of principal component analysis under the CMRE model. The object of the principal component analysis is to select a small number of important linear combinations of the variables which will best describe the variation among experimental units. The variance of i-th most important principal component is given by λ_i. If λ_i is very small, then the corresponding principal component is not important. So, it is of importance to test whether the magnitude of λ_i or the relative magnitude of λ_i with respect to $\lambda_1+\ldots+\lambda_p$ is significant. In many of the practical situations, the last principal component is not significant. In these situations we may test the hypothesis $\lambda_{p-q+1}=\ldots=\lambda_p$ and if this hypothesis is accepted, we conclude that the last q principal components are not important. One possible test for H_0 under the CMRE model is the modified LRT test statistic described in Section 3. Other possible test statistics are ratios of the roots like, for example, $\ell_{p-q+1}(\ell_{p-q+1}+\ldots+\ell_p)$, ℓ_{p-q+1}/ℓ_p, etc.

The results of this paper are useful in implementation of the above procedures.

The results in this paper are also useful in studying certain structures of the covariance matrix. For example, the problem of testing the hypothesis that Σ has the intraclass correlation structure can be handled by testing the hypothesis that $\lambda_2=\ldots=\lambda_p$. If Σ has particular structure, we may take advantage of the structure to improve the efficiency of the estimates and power of the tests.

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In this paper, the authors consider the problem of testing for the equality of the last few eigenvalues of the covariance matrix under correlated multivariate regression equations (CMRE) model. Asymptotic distributions of the various test statistics are derived when the underlying distribution is multivariate normal. Some of the distribution theory is extended to the case when the underlying distribution is elliptically symmetric.

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