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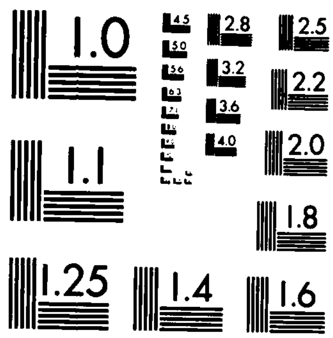

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THESIS

COMPARISON OF THE METRIC AND
HEURISTIC N.P.S. INVENTORY MODEL

by

Ko, Kyung Yoon

June 1985

Thesis Advisor: F. Russell Richards

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Because of the desire to improve operational readiness and to simultaneously reduce support costs, there is a great deal of interest in the military services in implementing multi-echelon models for determination of adequate but economical stocking levels for spare parts. Two models presently used--METRIC and MOD-METRIC --are inefficient and requir excessive time for computation. In an attempt to solve these deficiencies a heuristic model was

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Comparisons with METRIC revealed that the heuristic model was much more efficient computationally, but the solution was frequently far inferior to that obtained by METRIC. The comparison indicate strongly that base(shipboard) stock levels as determined by existing allowance models are larger than are needed in an integrated system.

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Comparison of the METRIC and Heuristic N.P.S. Inventory Model

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ABSTRACT

Because of the desire to improve operational readiness and to simultaneously reduce support costs, there is a great deal of interest in the military services in implementing multi-echelon models for determination of adequate but economical stocking levels for spare parts. Two models presently used--METRIC and MOD-METRIC--are inefficient and require excessive time for computation. In an attempt to solve these deficiencies a heuristic model was developed at the Naval Postgraduate School. The main purpose of this thesis is to compare the characteristics and performance of the simple heuristic model with the MTERIC solution.

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I. INTRODUCTION

A. BACKGROUND

"No matter how large our forces or how modern our military equipment, if our forces are not ready to fight, or if they cannot be sustained once engaged, we have no real combat capability" [Ref. 1]. These words were used in the Secretary of Defense's annual report to Congress(FY83) to promote the idea that a balance must be maintained force modernization and support of existing forces [Ref. 2]. The support of existing forces is an issue of high interest to military managers. Continually they try to find the most efficient way to allocate the budget that is assigned for spare parts. Many inventory models have been built to help determine the proper stockage levels for recoverable items which maximize performance subject to a given spares investment. One of these models is METRIC(Multi-Echelon Technique for Recoverable Items Control).

METRIC is a mathematical model translated into a computer program, capable of determining base and depot stock levels for a group of recoverable items. Its governing purpose is to optimize system performance for specified levels of system investment. METRIC is designed for application at the weapon system level, where a particular line item may be demanded at several bases and the bases are supported by one central depot [Ref. 3: p. 123].

Another model for spare parts allocation is MOD-METRIC which is a model for a multi-item, two-echelon two-indenture inventory system. MOD-METRIC is an extension of METRIC, which permits the explicit consideration of a hierarchical parts structure. The hierarchical parts structure means that the recoverable items themselves contain recoverable items.

The objectives of the MOD-METRIC model are to describe the logistic relationship between the components and the final assembly, and to compute base and depot spare stock levels for all items with explicit consideration of this logistics relationship [Ref. 4: p. 472]. The difference between the METRIC and MOD-METRIC models is the manner in which the average resupply time is expressed. Chapter II of this paper provides a detailed review of these two models.

Both models are excellent tools for finding efficient spare parts stockage solutions for small-sized problems. However, because they use a recursive solution technique, each run takes much computation time, making the models infeasible to use for large-sized problems. To overcome these computational difficulties, and to provide a workable solution for practical problems an heuristic stockage model has been developed at the Naval Postgraduate School [Ref. 5].

B. PURPOSE

The main purpose of this paper is to compare the solutions given by the simple heuristic N.P.S. model to the optimal solutions given by the METRIC. The comparison of the models will look at both efficiency in terms of computational time and an the quality of the solutions obtained.

C. PREVIEW

Chapter II of this paper provides a detailed review of the METRIC and MOD-METRIC models. Included in that chapter are the assumptions of the models and the solution techniques used for each.

Chapter III addresses the problem of making the METRIC and the N.P.S. models more efficient by approximating the expressions for base backorder days and by estimating base stock levels using simple multiple regression equation instead of the recursive computations required for the exact solutions. The regression equations use as inputs only the ready rate, and the demand rate.

Chapter IV describes the assumptions and solution procedure of the heuristic N.P.S. model for solving the same stockage allocation problem.

Chapter V presents the results of example computer runs which illustrate how close the approximate solutions cf the heuristic N.P.S. model are to the optimal METRIC solution. The comparisons include a modification of the N.P.S. solution using marginal analysis to incorporate unit costs.

Chapter VI summarizes the thesis and presents.

If we can find the expected base resupply time of the end item at base j, then we can use equation 2.1 for the base backorders of the end item. An expression for T is as follows;

$$T_{0j} = r_{0j}R_{0j} + (A_{0j} + \text{expected depot delay time}).$$

When the depot has S_{00}, the expected depot delay time per demand is very similar to equation 2.3 . Let

$$\delta(S_{00})D_{00} = \text{Expected depot delay per demand.}$$

Then,

$$\delta(S_{00})D_{00} = \frac{1}{\lambda_{00}} \sum_{x=S_{00}+1}^{\infty} (x - S_{00})p(x|\lambda_{00}D_{00}): \qquad \text{(eqn 2.14)}$$

where $\lambda_{00} = \sum_{j=1}^{n} (1-r_{0j})\lambda_{0j}$ is the expected depot demand.

Average base repair time for end item at base j, R_{0j}, is equal to the average remove and replace time, given the necessary module is available, plus the expected delay due to the unavailability of the module which is required to repair the engine. Therefore $R_{0j} = B_{0j} + \Delta_{0j}$ where

B_{0j} = the average repair time at base i if modules are available;

Δ_{0j} = the average delay in base engine repair due to the unavailability of a needed module.

Let the expected delay in engine base repair time due to a backorder on module i at base j be represented by Δ_{ij} . Then

$$\Delta_{ij} = \frac{1}{\lambda_{ij}} \sum_{x=S_{ij}+1}^{\infty} (x_{ij} - S_{ij})\ p(x_{ij}|\lambda_{ij}T_{ij}) \qquad \text{(eqn 2.15)}$$

R_{ij} = Average base repair time for module i at base j;

A_{ij} = Average order and ship time for module i at base j to the depot;

D_{i0} = Average depot repair time of module i;

C_i = Cost of module i (C_0 means cost of engine);

S_{ij} = Stock level for module i at location j, j = 0,1,..,n;

T_{ij} = Average resupply time for module i at base j.

3. Assumptions

The assumptions stated in METRIC are also applicable to MOD-METRIC except 1 and 8. Instead of compound Poisson demand in METRIC, MOD-METRIC assumes that the demand process is the simple Poisson process. METRIC assumes that each item has the same military essentiality. In MOD-METRIC the essentialities of end items and modules are explicitly expressed through the equation that represents each item's contribution to the end item's resupply time. Furthermore, MOD-METRIC assumes that if an engine requires repair and that repair is made at the base level, the probability that more than one module requires repair is zero.

4. Objective Function

As in METRIC, the objective is to minimize the total expected engine's (or end item) base backorder days summed over all bases. As before, the minimization is subject to a budget constraint. In deriving the objective function only one end item and its modules are considered.

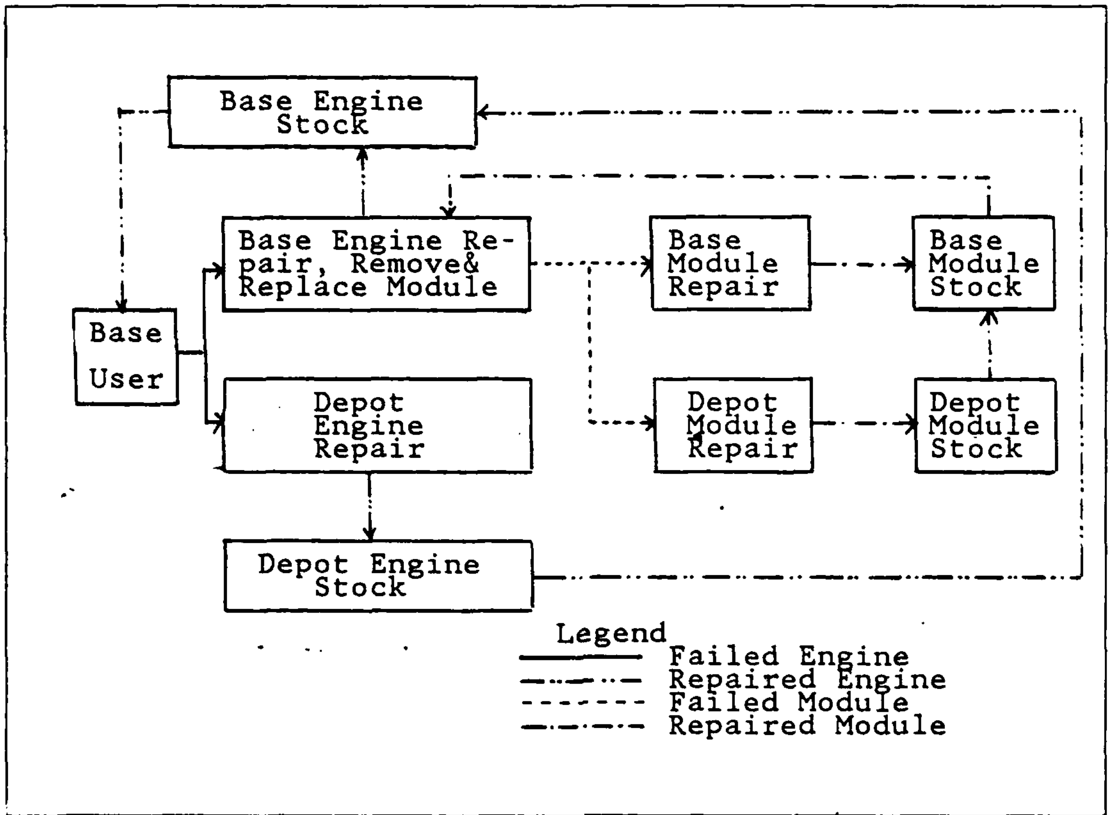

Figure 2.2 MOD-METRIC Repair Process.

2. Data Requirements and Notation

The data items required by MOD-METRIC are listed below.

m = Number of modules associate with the end item. The subscript i will index modules, i = 0 represents the end item;

n = Number of bases. The subscript j will index bases, j = 0 represents the depot;

λ_{ij} =Average number of daily removals of module i at base j;

r_{ij} = Probability a failure of module i at base j requires base repair;

which are discussed in Chapter V suggests that S_{i0} need never be larger than the greatest integer less than $\lambda_{i0} D_i$ to accomplish mean supply response time goals in the neighborhood of 5 days.

The minimum should be taken to be 0, and the search conducted over the values $0,1,2,\ldots,(\lambda_{i0} D_i)$ to find the METRIC solution.

B. MOD-METRIC

1. Maintenance System Structure

Consider a maintenance system consisting of two echelons--depot and bases. Assume that an end item is divided into several repair modules. For example, if an end item is an aircraft engine it may have, modules for intake, combustion and exhaust. If an engine fails at a base, it is replaced by a serviceable engine from base stock. The failed engine then goes immediatly to either base repair or is shipped to the depot and a resupply request is sent from the baseto the depot.

When the engine is repaired at the base, it is assumed that one of the modules is faulty. A serviceable module from the base stock, if available, will replace the failed module, and the repaired engine is placed in base engine stock. The failed module that is removed from the engine at a base is repaired at the base or is sent to the depot and a resupply request is submitted to the depot. This entire repair process is shown in figure 2.2 .

Now, there are several different stock levels which affect system performance. Since engine and modules have different functions in the repair process, each stock level affects the system performance differently.

6. Choice of a Multiplier and Depot Stock Level Range

Substitutuon of equation 2.5 into equation 2.11 gives

$$\sum_{x=S_{ij}}^{\infty} \{x - (S_{ij} + 1)\ p(x|\lambda T(S_{i0}))\}$$

$$- \sum_{x=S_{ij}}^{\infty} \{(x - S_{ij})\ p\{x|\lambda T(S_{i0})\} \geq \theta_k C_i$$

Which reduce to

$$\sum_{x=S_{ij}}^{\infty} p(x|\lambda T(S_{i0}) \geq \theta_k C_i$$

Add 1.0 to each side gives

$$1.0 - \sum_{S_{ij}}^{\infty} p\{x|\lambda T(S_{i0})\} \geq 1.0 + \theta_k C_i$$

Finally, we rewrite the left side:

$$\sum_{x=0}^{S_{ij}} p\{x|\lambda T(S_{i0})\} \geq 1.0 + \theta_k C_i . \qquad \text{(eqn 2.13)}$$

If we define the "ready rate" to be probability that the quantity of an item demanded during a resupply time is less than or equal to the stock level S_{ij} , we see from equation 2.13 that the METRIC solution forces the ready rate for each item to be at least as large as 1.0 + $\theta_K C_i$. Therefore, if we choose a minimum ready rate, a lower bound on θ_K can be estimated from inequality 2.13 . This minimum value of can be used as the starting Lagrange multiplier in step 1 of the solution technique.

We also need to establish a range of values for depot stock level S_{i0} . Because the depot demand rate is and repair time is D_i , the average number of units of items in depot resupply is $\lambda_{i0} D_i$. Empirical evidence in the runs

$B_{ij}(S_{ij},S_{i0})$ is discretely convex for a given S_{i0} (see [Ref. 9: p. 260]):

$$B(S_{ij},S_{i0}) - \theta_k C_i S_{ij} \le B_{ij}(S_{ij}+1,S_{i0}) - \theta_k C_i (S_{ij}+1).$$

or

$$B_{ij}(S_{ij}+1,S_{i0}) - B_{ij}(S_{ij},S_{i0}) \ge \theta_k C_i. \qquad \text{(eqn 2.11)}$$

To find the optimum stock level for each base, given depot stock S_{i0} and θ_k, increase S_{ij} from 0 to the smallest integer which satisfies inequality 2.11 .

Step 4. Increase depot stock S_{i0} by one unit, and return to step 3. Continue increasing depot stock until it reaches the upper bound established in step 2.

Step 5. Choose optimal S_{i0} for each item. The optimum S_{i0} and its corresponding base stock levels are those values which minimize problem 2.12 .

$$\text{Min} \sum_{j=1}^{n} (B_{ij}(S_{ij},S_{i0}) - \theta_k C_i S_{ij} - \theta_k C_i S_{i0}). \qquad \text{(eqn 2.12)}$$

Step 6. Change the item and go to step 2.

Step 7. Compute the required total cost for buying all S_{ij} for a given θ_k. If total cost is less than $|\varepsilon|$ (where ε is an acceptable prespecified difference total cost and budget), then stop. The current Lagrange multiplier gives optimal solution for those resources actually required by the solution S_{ij}. Otherwise choose a new θ_k and go tostep 3. A bisection search procedure should be used to determine a new value for θ_k.

If an x can be found which maximizes equation 2.8, then this x is also the solution for the constrained problem, equation 2.7 .

Everett's Theorem is useful for solving the METRIC problem. According to Everett's Theorem we can express our original problem, equation 2.6 as follows;

$$\text{Min} \sum_{i=1}^{m} \sum_{j=1}^{n} B_{ij}(S_{ij}, S_{i0}) - \theta \sum_{i=1}^{m} \sum_{j=0}^{n} C_i S_{ij} \qquad \text{(eqn 2.9)}$$

where $\theta \leq 0.0$.

Different choices of θ lead to different resource levels, and it is necessary to adjust them by trial and error to achieve a given constraint. Therefore we need to solve problem 2.9 for several values of the multiplier θ. That value which provides the resource level that is closest to the budget constraint will be chosen.

Since our problem is separable in the items, problem 2.9 can be solved for each item separately. That is, we can solve the m subproblems,

$$\text{Min} \sum_{j=1}^{n} (B_{ij}(S_{ij}, S_{i0}) - C_i S_{ij} - C_i S_{i0}). \qquad \text{(eqn 2.10)}$$

The solution technique is outlined below:

Step 1. In the description θ_k refers to the trial value for the Lagrange multiplier at step k.
Select a starting value θ_0 for the multiplier; where $\theta_0 < 0$. A reasonable choice for θ_0 will be presented later.

Step 2. Establish an upper bound on S_{i0}. This will be presented later also.

Step 3. Given S_{i0} and θ_k, determine the base stock level for each base(S_{i0} may start at zero). Since

As mentioned above, the objective of METRIC is to minimize the sum of backorders for all items i and for all bases j within a budget constraint. The METRIC problems is then to find S is greater than or equal to zero for all i and j which

$$\text{Minimize} \sum_{i=1}^{m} \sum_{j=1}^{n} B_{ij}(S_{ij}, S_{i0}) : \qquad \text{(eqn 2.6)}$$

$$\text{Subject to} \sum_{i=1}^{m} \sum_{j=0}^{n} C_i S_{ij} \leq B$$

where B is the available budget.

5. <u>Solution Technique</u>

The METRIC problem can be solved by using either marginal allocation or the generalized Lagrangian Multiplier method [Ref. 3: p. 133]. We describe the generalized Lagrange multiplier method since that is what is suggested by the authors of METRIC. First, we state the theorem by Everett which is the basis for the generalized Lagrange multiplier method.

Everett theorem [Ref. 8]:

Let S be a set(completely arbitrary) of possible strategies or actions and let H(x) be the pay off (or utility) which accrues from employing the strategy $x \in S$. Let C(x) be the resource required by strategy $x \in S$, and let C be the maximum amount of the resource available. We want to

$$\text{Maximize } H(x): \text{ for all } x \in S \qquad \text{(eqn 2.7)}$$

$$\text{Subject to } C(x) \leq C.$$

This problem can be expressed as an unconstrained problem for a given Lagrangian multiplier $\theta > 0$ as follows:

$$\text{Maximize } H(x) - \theta C(x). \qquad \text{(eqn 2.8)}$$

$$T_{ij} = r_{ij} \cdot R_{ij} + (1 - r_{ij}) \cdot (A_{ij} + \text{expected depot delay time}).$$

Except for the expected depot delay time all the variables are assumed known. The depot delay time is zero if the depot has infinite stock. If the depot has no stock, the time is D(depot repair time). If the depot has finite stock S then the expected depot backorders is

$$\sum_{x=S_{i0}+1}^{\infty} (x - S_{i0}) p(x \mid \lambda_{i0} D_i) \qquad \text{(eqn 2.2)}$$

where $\lambda_{i0} = \sum_{j=1}^{n} (1 - r_{ij}) \lambda_{ij}$.

Equation 2.2 can be interpreted as depot backorder days per day [Ref. 7]. Thus , when we divide this number by depot demand per day(λ_{i0}) we get the expected depot back-order days per demand:

$$\frac{1}{\lambda_{i0}} \sum_{x=S_{i0}+1}^{\infty} (x-S_{i0}) p(x \mid \lambda_{i0} D_i). \qquad \text{(eqn 2.3)}$$

Sherbrooke argues that this average delay will be some fraction of the depot repair time D and uses the notation $\delta(S_{oo})D$ for this expression to emphasize this fact. And so the resupply time can be expressed as follows;

$$T_{ij}(S_{i0}) = r_{ij} R_{ij} + (1-r_{ij})(A_{ij} + \delta_i(S_{i0}) D_i). \qquad \text{(eqn 2.4)}$$

Now we can write the equation for the expected number of backorders for item i at base j when the depot has S_{i0} and the base has S_{ij} stock levels;

$$B_{ij}(S_{ij}, S_{i0}) = \sum_{x=S_{ij}} (x-S_{ij}) \, p(x \mid \lambda_{ij} T_{ij}(S_{i0})). \qquad \text{(eqn 2.5)}$$

value. The actual resupply time distribution is not required.

Feeney and Sherbrooke [Ref. 6] extended Palm's Theorem for the case where demands are compound Poisson distributed under the assumption that all demands placed by a given customer have the same resupply time. They show, in this case, that the resulting distribution of the number of units in resupply is compound Poisson with parameter λT.

In addition, Feeney and Sherbrooke looked at the special case in which the demands per customer are logarithmically distributed with variance-to-mean ratio q. They showed that for this special case the resulting distribution for the number of units in resupply is negative binomial with parameters q and $k = \lambda T / \ln q$; i.e.

$$p(x \text{ units in resupply}) = \frac{(k+x-1)!\ (q-1)^x}{(k-1)!\ x!\ q^{k+x}}\ ; \quad (x = 0,1,2..,\ q > 1,\ k > 0).$$

Using the result given by Palm's Theorem, we can then compute easily the expression for the steady state expected number of backorders. Let S be the number of units of stock allocated to a base and let T be the mean resupply time for the base. The number of backorders at a given time will then be zero if the demand during the resupply time is less than or equal to S and the number of backorders will be (x-S) if the demand is larger than S. The expected number of backorders at any given time, denoted B(S,T), is then given by;

$$B(S,T) = \sum_{x=s+1}^{\infty} (x-S)p(x|\lambda T) \qquad \text{(eqn 2.1)}$$

Since the value of λ_{ij} for a base is assumed known, we need to compute only the mean resupply time T_{ij} for item i at base j.

7. The level at which repair is performed depends only on the complexity of the repair.
8. Items and bases may have different military essentialities; however, items are normally considered to be equally essential. [Ref. 4: p. 474]

4. Objective Function

The objective function used by METRIC is to minimize the sum of expected backorder days on all recoverable items at all bases pertinent to a specific weapon system. A base backorder for an item exists any time there is an unsatisfied demand for that item at the base level. Depot backorders are of interest only insofar as they affect base backorders.

To derive the expression for expected backorder days the following mathematical background is needed.

A key result for both METRIC and MOD-METRIC concerns the probability distribution of the number of units in resupply. For the case where demands are Poisson distributed, Palm's Theorem gives this distribution under the assumptions stated for the METRIC and MOD-METRIC model.

Palm's theorem(stated in the context of the stockage problem);

> Let s be the spare stock for an item where demands are Poisson distributed with customer arrival rate λ. Let $\psi(t)$ be the probability distribution of resupply time with mean T. Then, in the backorder case, with and (S-1,S) stockage policy the steady-state probability of x units in resupply is Poisson distributed with parameter λT; i.e.

$$h(x) = p(x \text{ units in resupply}) = \frac{(\lambda T)^x \exp(-\lambda T)}{x!}$$

$x = 0, 1, 2, \ldots.$

Thus, the distribution of the number of units in resupply depends on the resupply time only through its mean probability that item i cannot be repaired at base j.

A_{ij} = Expected order and ship time for item i from base j to the depot(days).

R_{ij} = Expected base repair time of item i at base j(days).

D_i = Expected depot repair time of item i(days).

λ_{ij} = Expected number of demands for item i at location j(demand/day) j = 0,1,2,...n.

S_{ij} = Stock level for item i at location j; j = 0,1,2,....n.

T_{ij} = Expected resupply time for item i at base j.

B_{ij} = Backorders for item i at location j ; j = 0,1,2,...n.

Note; subscript i is used to index the items and subscript j refers to the different bases. The subscript j=0 refers to the depot.

3. Assumptions

1. A stationary compound Poisson probability distribution describes the demand process for each item(for our comparisons later we consider only the case in which demands are standard Poisson distributed).
2. There is no lateral resupply between bases.
3. There are no condemnations(all failed parts are repaired).
4. A failure of one type of item is statistically independent of those that occur for any other type of item.
5. Repair times are statistically independent.
6. There is no waiting or batching of items before repair is started on an item(infinite channel queuing assumption).

a repair is accomplished. This process is shown in figure 2.1 .

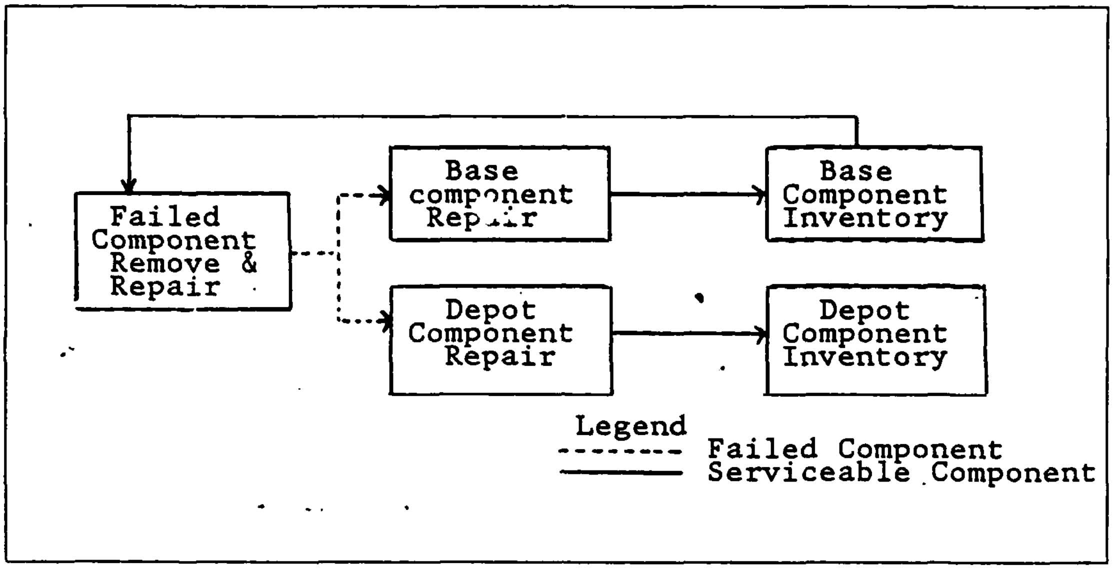

Figure 2.1 METRIC Repair Process.

The depot and base stock level of an item will affect the performance of the system. As stock levels increase the average resupply time of the failed item will decrease. The objective of the METRIC model is to determine the base and depot stock levels of every item for a given budget constraint such that the total backorder delay at the bases is minimized.

2. Data Requirements and Notations

The METRIC model requires several input data items for implementation. The required data are listed below;

m = The number of recoverable items.

n = Number of bases.

C_i = The cost of item i.

r_{ij} = Probability that a failure of item i at base j can be repaired by base; it follows that $1 - r_{ij}$ is

II. DISCUSSION OF METRIC AND MOD-METRIC

This chapter describes the METRIC and MOD-METRIC inventory models. For each model the following subjects are discussed: maintenance system structure, assumptions, data requirements and notation, the objective function and solution techniques. METRIC is more fully described by Sherbrooke [Ref. 3], and MOD-METRIC by Muckstadt [Ref. 4].

A. METRIC

1. Maintenance System Structure

Consider the multi-echelon maintenance structure as used in the METRIC model. In a multi-echelon structure stocking/maintenance facilities are organized in a hierarchical structure according to supply/maintenance flows which are represented as an arborescent network.

When a unit fails at base level there is a probability r that it can be repaired at the base, and a probability 1-r that it must be returned to the depot. Because of the typical high costs and low demand for items, the inventory stockage policy is (S-1,S), which means that items are not batched for repair or resupply request. If there is inventory available at the base, a serviceable item replaces the failed item. If no inventory is available, the equipment will be inoperable until an item is repaired. In either case, the failed item immediately begins base repair; or is sent to the depot. When the failed item is sent to the depot a resupply request to the depot is issued. If the depot has a serviceable part, it will send the item to the base immediately. If the depot does not have the item available, it will send a serviceable unit to the base after where

$$T_{ij} = r_{ij}R_{ij} + (1 - r_{ij})(A_{ij} + \frac{1}{\lambda_{i0}} \sum_{x=S_{i0}+1}^{\infty} (x - S_{i0})\, p(x|\lambda_{i0}D_{i0})) \qquad \text{(eqn 2.16)}$$

and

$$\lambda_{i0} = \sum_{j=1}^{n} (1 - r_{ij})\, \lambda_{ij} .$$

The expected delay in engine repair at base j due to modules is;

$$\lambda_{i0} = \frac{1}{r_{0j}\, \lambda_{0j}} \sum_{i=1}^{m} \lambda_{ij}\, \Delta_{ij} . \qquad \text{(eqn 2.17)}$$

Thus we have shown that the average resupply time for an engine, T_{0j} , can be expressed as:

$$T_{0j} = r_{0j}(B_{0j} + \Delta_{0j}) + (1 - r_{0j})(A_{0j} + \delta(S_{00})D_{00}). \qquad \text{(eqn 2.18)}$$

Now, the problem of MOD-METRIC is to find $S_{ij} \geq 0$ shich can be expressed mathematically.

$$\text{Minimize} \sum_{j=1}^{n} \sum_{x=S_{0j}}^{\infty} (x - S_{0j}) P(x|\lambda_{0j}T_{0j}) \qquad \text{(eqn 2.19)}$$

$$\text{Subject to} \sum_{i=0}^{m} \sum_{j=0}^{n} C_i S_{ij} \leq B .$$

5. Solution Technique

The solution technique suggested by Muckstadt [Ref. 4] is outlined in this section.

Problem 2.19 is not separable because T_{0j} is a complex function of the S_{ij}. The approach taken by Muckstadt was to partition the problem into two subproblems--the module subproblem and the end item subproblem. The solution algorithm is as follows;

Step 1. Set up minimum investment levels for modules and end items. Let those be g and f, respectively. Let z be the total expected backorders for the end item when the base stock levels for the end item are S_{0j}. Set a budget increment for modules, b. Let g' be a trial value for total system modules investment and let $z' = \infty$.

Step 2. Solve the module subproblem given the minimum budget $g = g'$. That is, find $S_{ij} \geq 0$ which

$$\text{Minimize } \sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{x=S_{ij}}^{\infty} (x-S_{ij})\ P(x|\lambda_{ij}T_{ij}(S_{i0}))$$

$$\text{Subject to } \sum_{i=1}^{m} \sum_{j=0}^{n} C_i S_{ij} \leq g'.$$

Step 3. Solve the end item subproblem. Find $S_{0j} \geq 0$ as in the METRIC solution which

$$\text{Minimize } z = \sum_{j=1}^{n} \sum_{x=S_{0j}}^{\infty} (x - S_{0j})(x|\lambda_{0j}T_{0j}(S_{00}))$$

$$\text{Subject to } \sum_{j=0}^{n} C_0 S_{0j} \leq B - g'$$

where T_{0j} is calculated using the module stock levels determined in step 2 and given S_{00}. Compute also the value of z, the value of z associated with this optimal solution.

Step 4. If $z > z'$, go directly to step 5. Otherwise let $z' = z$ and retain the corresponding stock levels as the incumbent stock levels. Then go to step 5.

Step 5. Increase g' by b. If $B - g' < f$, then terminate. Otherwise return to step 2. If the algorithm stops on this step, the optimal stock levels and the associated minimum expected backorders will be those saved from step 4. In step 2 an optimization problem in solved in which a portion of the budget g', is allocated among the modules to determine depot and base module stock levels. The objective of this subproblem is to determine that division of g' which minimize the sum of the expected delay function. The optimal value of S_{00} is found by trial and error by searching through the integers.

Note that the form of the suboptimization problem in step 2 is exactly the same as the METRIC problem. Thus, step 2 can be solved using the techniques that were used in METRIC.

The engine delay time due to unavailable modules at a base(Δ_{oj}) is fixed by solving step 2 for a trial module budget. This means that the expected engine repair time at a base is fixed in problem 2.18 . Thus step 3 is also exactly same as the METRIC problem for only one item. Consequently, the METRIC solution technique can be used in step 3 again.

III. REGRESSION TECHNIQUE

As observed in the METRIC and MOD-METRIC solution techniques, a lot of calculations are required to compute B (S_{ij}, S_{i0}) and to find S_{ij} (i =1,2,...m, j =0,1,2...n). Because of the computation time it is difficult to determine the spares allocation for thousands of items. It would be much faster if the computationally heavy recursive calculations could be replaced by a non-recursive procedure. This chapter presents results of an effort to approximate the exact computations using multiple regression equations.

For estimation of the regression equations the following procedure was used(the i,j subscripts are suppressed since the results apply to each base and item separately):

1. Select $\mu = \lambda T(S)$.
2. Vary the base stock level over the integers; S=0 1,2,..
3. For each value of S, find the true ready rate, $RR(s) = (x \leq S|\mu)$, and backorder function $\sum_{x=S}^{\infty} (x-S)p(x|\mu)$.
4. Select those values of S for which the ready rate is in a predetermined interval like, say 80% to 95%.
5. Repeat steps 1-4 a range of values of μ.

The set of values selected at step 4 constituted the regression data base for the predetermined ready rate subinterval . Now, with this data base, MINITAB was used for two separate regression analyses.

1). Base stock, S, was regressed on μ and ready rate RR.
2). Base backorders, B, was regressed on μ, μ^2, S, S^2, and RR.

Tables I and II show the regression output from MINITAB. They show how each variable contributes to the prediction of S and B.

TABLE I

Regression Equation for Base Stock
($5.0<\mu<7.5$, $.85<RR<.9$)

```
THE REGRESSION EQUATION IS
Y = -  11.3 +  1.23 X1 +  14.1 X2
  * Y is a expected base stock
                                        ST. DEV.
                      COEFFICIENT       OF COEF.
INTERCEPT              -11.31897         0.09860
X1(μ )                   1.22801         0.00222
X2(RR)                  14.0667          0.1138

WITH (  72 - 3) =  69 DEGREES OF FREEDOM
R-SQUARED =100.0 PERCENT
```

Separate equations were estimated for arbitary selected subintervals for μ and RR. We selected 5 subintervals for μ and 5 subintervals for RR giving a total of 25 possible sets of equations.

Table III shows how accurately the regression equations estimate the actual S and B (S, μ, RR) for a given μ , and RR.[1]

Table III reads as follows; if a base has 7 spare parts and its μ is 5.15, then its ready rate is 85% and backorder days are 0.2927. A regression equation is also used to estimate the base stock and backorder days assuming an 85% ready rate and the μ from the first column of Table III .

---

[1]Table III uses the equation that appeared at table I and table III .

TABLE II

Regression Equation for Backorders
($5.0<\mu<7.5$, $.85<RR<.9$)

```
THE REGRESSION EQUATION IS
Y=    2.74 - 0.156 X1 +0.0076 X2
   + 0.147 X3 -0.0051 X4 -   3.09 X5
```

* Y is a expected backorders(B).

	COEFFICIENT	ST. DEV. OF COEF.
INTERCEPT	2.73925	0.02929
X1(μ)	-0.155809	0.005349
X2(μ^2)	0.0075982	0.0001835
X3(S)	0.147205	0.004226
X4(S^2)	-0.0050734	0.0001005
X5(RR)	-3.08886	0.03673

WITH (72- 6) = 66 DEGREES OF FREEDOM

R-SQUARED =100.0 PERCENT

TABLE III

Estimates of S , B by Regression Equations

Actual Data				Regression Estimation	
μ	S	B	RR	S	B
5.150	7.	0.2927	0.8505	7.0265	0.2892
5.600	8.	0.2223	0.8856	8.0761	0.2175
5.775	8.	0.2598	0.8695	8.0643	0.2551
5.950	8.	0.3013	0.8523	8.0368	0.2966
6.375	9.	0.2260	0.8878	9.0599	0.2208
6.550	9.	0.2617	0.8730	9.0665	0.2564
6.725	9.	0.3011	0.8572	9.0590	0.2956
7.100	10.	0.2187	0.8942	10.0416	0.2123
7.275	10.	0.2518	0.8808	10.0678	0.2456
7.450	10.	0.2882	0.8664	10.0810	0.2821

Table III shows that the regression equation gives very good estimates of the values for both the base stock and the backorder days. Thus, if we are able to use such equations in solving multi echelon inventory problems, we can reduce the computation time very much. The N.P.S. model does use these equations and which will be presented at the next chapter.

IV. HEURISTIC MODEL

A. BACKGROUND

As we have seen in the chapter II and III, the METRIC and MOD-METRIC solutions require extensive recursive computation. For example, the first recursive process involves a search for the optimal base stock level given the depot stock level and a value for the Lagrangian multiplier. This process is repeated every time the depot stock is changed--a second recursion. Furthermore, the above two recursive processes are repeated for each item and each value of the Lagrangian multiplier. Because of these recursions, much computational time is required, and it could be difficult to find the optimal solution for a system having thousands of items(the typical aircraft has about 2,000 work unit coded repairable item [Ref. 10]).

Recall that in the METRIC model it was shown that the optimal base stock level was the smallest integer value of S_{ij} which satisfies

$$P(x_{ij} \le S_{ij}) \ge 1 + \theta C_i .$$

If we ignore the integrality of demand we can interpret this inequality as providing the same ready rate for each base. Now, this has the altinative interpretation that given a value of the Lagrange multiplier and a fixed depot stock level, the METRIC solution corresponds to the determination of the largest ready rate which is budget feasible.

Now, aside from minimizing base backorder days, it is frequently the case that a base may wish to achieve a specified ready rate assuming no support from a higher echelon(depot). Such is the case, for example, with shipboard(base) allowance list models which stock to provide a 90% ready rate for a 90 day period of time.

The N.P.S. stockage model incorporates this notion of a specified base ready rate and attempts to determine the depot stock level S_0 required to achieve a mean supply response time(MSRT) goal. For our study, the goal was selected arbitrarily to be 125 hours(5.2 days).

B. SOLUTION TECHNIQUE

The base ready rate is a function of the amount of depot stock(the depot stock determines the T_{ij}) and the amount of base stock. Also the optimal base stock is a function of the ready rate and depot stock. Furthermore, as shown in equation 2.4, the base backorder is a function of S_{ij} and $T_{ij}\lambda_{ij}(S_{i0})$. Therefore, the first step of the heuristic model is to determine the base stock as a function of fixed depot stock and ready rate. The second step is to determine the value of the base backorder days as a function of the fixed depot stock and the base stock which was found in the first step.

The necessary values can be found recursively using the same expressions used in METRIC or they can be approximated very accurately and rapidly using regression functions. The N.P.S. model saves much computation time when solving for the optimal base stock levels and base backorders by using the regression equations described in the previous Chapter.

The depot stock leves for a given item by beginning at 0 and incrementing depot stock by one unit until the average base MSRT first reaches the specified MSRT goal. Thus S_{00} is the smallest integer such that

$$\sum_{j=1}^{n} B_{ij}(S_{0j},S_{00}) \;/\; \sum\lambda_{ij} \le MSRT_g .$$

Where $B_{ij}(S_{oj},S_{00})$ is th total expected base backorder days for item i at base j when base j ha S_{ij} units and the depot has S_{i0} units of stock. $MSRT_g$ is the specified MSRT goal.

This procedure is repeated for each item one-by-one.

The big difference between the METRIC and N.P.S. models is that METRIC minimizing the expected number of base backorder days for a fixed budget, whereas the N.P.S. model attempts to find the minimum depot stock levels required to achieve a desired supply response time goal for each item. A comparison of the METRIC and N.P.S. models is presented in chapter 5.

C. ALGORITHM

Below is an outline of the solution technique used by the N.P.S. model. Since no preference is given to any item by the N.P.S. model and since there is no budget constraint, no consideration need be given to the unit cost. Therefore the same solution procedure is repeated for each item.

Step 1. Find the smallest non-negative integers S_{ij}, $j=1,2,\ldots,n$, such that

$$P(x(T) \le S_{ij}) \ge 0.9$$

where $T = 90$ days. The regression equations described in chapter III can be used for this step. Set the depot stock S equal to 0.

Step 2. Determine $T_j(S_0)$ and $B_j(S_j, S_0)$ for $j = 1,2,\ldots,n$. (The regression equations can be used to approximate the backorder functions required by this step).

Step 3. Compute the demand-weighted average MSRT across the n bases.

$$MSRT = \frac{\sum_{j=1}^{n} \lambda_j MSRT_j}{\sum_{j=1}^{n} \lambda_j} = \frac{\sum_{j=1}^{n} B_j(S_0, S_j)}{\sum_{j=1}^{n} \lambda_j}$$

If MSRT is less than 125 hours then stop. The current S_0 is the desired solution. If MSRT greater than 125 hours, increase S_0 by one unit and go to step 2.

Step 4. Repeat for each item.

V. COMPARISON OF THE MODELS

In order to evaluate the heuristic, non-optimal, N.P.S. model we selected several sample data sets and computed the N.P.S. solution for each. We then repeated the computations using the METRIC solution procedure. Since the METRIC solution is optimal, we can evaluate the quality of the N.P.S. solution by comparing its performance, to that of the METRIC model. The results of several comparisons are contained in this chapter.

Example problems consist of three items which are to be stocked at one depot and three bases. For the purpose of illustration, four data sets are constructed. First the N.P.S. model solves the problem and computes the associate budget, then this budget is used as a constraint in the METRIC model.

A. DATA SET 1: LONG ORDER-SHIP TIME AND LOW PROBABILITY OF BASE REPAIR

Table IV gives the complete input data for the first data set. The first example considers long order-ship times and zero probability of base repair. Thus, all repairs must take place at the depot. Item 1 has a low demand rate, item 2 has a medium demand rate and item 3 has a high demand rate. The demand rates are consistent through the four sample data sets.

Recall from chapter II the following notations;

r_{ij} = probability that item i can be repaired at base j,

R_{ij} = the expected repair time in days for item i at base j,

A_{ij} = the average order and ship time in days from base j to the depot for item i,

TABLE IV

INPUT DATA 1

Item	Base	λ_{ij}/day	r_{ij}	R_{ij}	A_{ij}	D_i	Cost
1	1	0.044	0.0	0.0	90.0	20.0	$200.0
	2	0.056	0.0	0.0	90.0		
	3	0.067	0.0	0.0	90.0		
2	1	0.111	0.0	0.0	90.0	25.0	750.0
	2	0.133	0.0	0.0	90.0		
	3	0.167	0.00	0.0	90.0		
3	1	0.222	0.0	0.0	90.0	30.0	1500.0
	2	0.244	0.0	0.0	90.0		
	3	0.278	0.0	0.0	90.0		

D_i = the average depot repair time in days for item i,

λ_{ij} = the failure rate(failures per day) of item i at base j.

Tables V and VI give the N.P.S. and METRIC results. Observe that there are significant differences in the stockage allocations between the two models. However, the difference in performance(MSRT) is not so large. Notice also the large difference in computational times; 0.27 seconds for N.P.S. solution and over 20 times as long(5.87 seconds) for the METRIC solution.

B. DATA SET 2: LONG ORDER-SHIP TIME AND HIGH PROBABILITY OF BASE REPAIR

Table VII gives the data for the second test data set. Characteristics of this data set are long ordering and shipping times and a high probability of base repair. Thus, the bases will rarely need to rely on the depot for support. Intuitively, very little stock will be needed at the depot level.

TABLE V

Results of N.P.S. Model for Data 1

	Depot Stock	Base 1 S	Base 1 RR	Base 2 S	Base 2 RR	Base 3 S	Base 3 RR	MSRT
Item 1	1	7	0.906	8	0.863	9	0.832	4.8467
Item 2	4	14	0.799	16	0.755	20	0.763	4.9172
Item 3	9	26	0.707	28	0.673	32	0.684	5.1011

Total Cost : $188000.0
System Average MSRT : 5.01178 days
Time for Computer Run : 0.27 seconds

TABLE VI

Results of METRIC Model for Data 1

	Depot Stock	Base 1 ST.	Base 1 RR.	Base 2 ST.	Base 2 RR.	Base 3 ST.	Base 3 RR.	MSRT
Item 1	2	8	0.966	10	0.973	11	0.961	1.0455
Item 2	7	14	0.859	16	0.828	20	0.842	3.0891
Item 3	16	23	0.644	25	0.625	29	0.663	5.8286

Total Cost : $188450.0
System Average MSRT : 4.37275 days
Time for Computer Run : 5.87 seconds

As expected, both solutions place little stock at the depot. The N.P.S. model puts zero stock there for each item. Observe also that the system average MSRT values are much lower than the response time goal of 5.2 days(125 hours). This is because of the self-supporting capability of the bases. As before, the N.P.S. solution required a small fraction of the time required by the METRIC solution.

```
DO WHILE (ABS(BUDGET -SUMCOST)>749.0);
 LAM (LAMBDA) =QK;
 PUT SKIP EDIT(QK)(X(1),F(11,8));
 SUMCOST1 =0.0;
 SUMCOST =0.0;
 TOTAL_DEMAND =0.0;
 TOTAL_MSRT = 0.0;
 SUM_DEMAND =0.0;
 DO HAN = 1 TO ID;
   TOTAL_DEMAND = 0.0;
   DDEM =0;
   DO J = 1 TO BASE;
    PP =BDEM(HAN,J,3) * (1-BPROB(HAN,J,5));
    DDEM = DDEM +PP;
    TOTAL_DEMAND = TOTAL_DEMAND + BDEM(HAN,J,3);
   END;
    U =DDEM *DREP(HAN,1,6);
    SPACE1 = 1000.0;
     DO WHILE (SIO <18);
       PX = EXP(-U);
       SUM2 = U - SIO;
       IF SIO > 0
         THEN DO;
           DO X = 0 TO SIO-1;
             SUM2 =SUM2 +(SIO - X) * PX;
             PX = U * PX / (X+1);
           END;
         END;
         DDELY =SUM2 /DDEM;
         SUM3 =0.0;
         SUM4=0.0;
         SUM5 = 0.0;
```

APPENDIX A

COMPUTER PROGRAM FOR METRIC MODEL BY PL/I

```
*PL/C ATR SOURCE L=9000 P=150 T=(1,30)XREF
 BBO:PROCEDURE OPTIONS (MAIN);
  DCL(COST(15,10,1),BREP(10,10,2),BDEM(10,10,3),bbod(20),
      bord(10,10,4),bPROB(10,10,5),DREP(10,10,6))FLOAT;
  DCL(DDEM,UB(20),RS,LS,QK,P(70),SPACE(20),BRST(20),DDELY,
      SUMCOST,BODD,RR(30,6,45,3),MSRT)FLOAT;
  DCL(PP,PPP,M,WIJ(30, 6,45, 3),U,KO,LAM(70),PX,POISSON,
           BUDGET,QK1,SUM5,BOD1,SUM6,READY_RATE(15))FLOAT;
  DCL(Y3,SUM1,SUM2,MU(20),BOD2,QK2,TOTAL_DEMAND)FLOAT
      initial(0.0);
  DCL(I,J,K,L,X,Y,Z,V,F,HAN,LAMBDA,NUMBER)FIXED;
  DCL(SIO,SIJ,ID,BASE)FIXED INIT(0);
  DCL(SUMCOST1,SUMCOST2(40),SPACE1,SUM3,SUM4,BOD,W1,W(40))
       flost;
  DCL (MMSRT,TOTAL_AVG_MSRT,TOTAL_MSRT,SUM_DEMAND)FLOAT;

  LAMBDA = 1;
  QK = (0.6-1.0)/ 200;
  ID =3;
  BASE =3;
  GET LIST (BUDGET);

  DO I =1 TO ID;
    DO J= 1 TO BASE;
     K = 1;
       GET LIST(COST(I,J,K),BREP(I,J,K+1),BDEM(I,J,K+2),
           BORD(I,J,K+3), BPROB(I,J,K+4),DREP(I,J,K+5));
    END;
  END;
   SUMCOST = 0.0;
```

consequences on presently used allowance list models used by the military services.

An evaluation of the computation times required for determining the N.P.S. and METRIC solutions showed the times for the METRIC solution to be as much as 20 times as great as those required for the N.P.S. solution. Thus, the N.P.S. solution does hold promise for implementation in the real world with large weapon systems.

In general, the performance of the N.P.S. model was somewhat disappointing in comparison to the METRIC solution. The results of this thesis indicate that the N.P.S. model requires additional work in the areas mentioned above.

the mean demand, the stockage level, and the ready rate. Regression equation were also developed for estimation of the base stock levels, as a function of the mean demand and the ready rate. The accuracy of both sets of equations was very high. This suggests that significant reductions in computer time in the METRIC model(or others)could be achieved by replacing the exact recursive computations with the approximate regression equations.

In order to accomplish the third objective, several sample data sets were created for use in testing the METRIC and N.P.S. models. Four of the data sets are described in Chapter V. Analysis of the results of the sample data sets revealed some shortcomings of the proposed N.P.S. model. One of those shortcomings, complete disregard for item costs was corrected by modifying the N.P.S. model to include costs in a marginal analysis procedure for determining depot stock allocations. This modification achieved only minor improvement but since it was restricted to the depot stock decisions it requires additional study. Analysis also revealed that the base stock levels are consistently too high in the N.P.S. model. Significant reductions in the base ready rate from 90% to 70% and less were made with the support system still able to achieve specified mean supply response time goals at significantly reduced costs. We also observed that the ready rate protection period, nominally stated as 90 days seems longer than necessary with the types of values used for order and ship times and repair times in our examples. Clearly, both the ready rate and the length of the protection period should be a function of the other input parameters of the model, like order and ship times, base repair probability, and base and depot repair times. Selection of a uniform "90% protection for 90 days" policy for determining shipboard stock will sometimes over protect or under protect significantly. This might have dramatic

VI. CONCLUSIONS

There were basically three major objectives of this thesis. The first was to review the literature and to understand the existing models for controlling the stockage decisions in multi-item, multi-echelon inventory system for repairable items. The second objective was to develop the computer programs needed to implement the most promising of the existing algorithms. The last objective was to evaluate the heuristic N.P.S. model by comparing it to the baseline solution given by the existing algorithm.

The review of the literature revealed that basically all of the existing models are derived from the METRIC family developed at RAND in 1968. Therefore, the METRIC model was selected as our baseline. That algorithm was programmed and is operational on the N.P.S. IBM 370 model 3033 computer. Appendix A of this thesis contains a listing of the PL/I source code for that program. The N.P.S. model proposed by Apple [Ref. 5] was also programmed in PL/I on the N.P.S. computer system and is included in this thesis as Appendix B.

Execution of the METRIC and N.P.S. models revealed extensive recursion in the computations of stockage levels and the backorder expression. Since it is well known that one of the major problems preventing widespread acceptance of the METRIC family of models is the extensive computational time required to solve for the optimal solution for reasonably-sized weapon systems, we sought to determine if accurate approximations could be used in place of much of the recursion.

Multiple regression equations were developed for estimation of the expected base backorder days as a function of

TABLE XVI

Results of the Marginal Analysis

	Depot Stock	Base 1 S	Base 1 RR	Base 2 S	Base 2 RR	Base 3 S	Base 3 RR	MSRT
Item 1	4	7	0.944	8	0.919	9	0.901	2.6312
Item 2	6	14	0.841	16	0.807	20	0.819	3.6043
Item 3	7	26	0.663	28	0.626	32	0.634	6.1941

Total Cost : $187100.0
System Average MSRT : 4.93885 days
Time for Computer Run : 0.58 seconds
*N.P.S. model : MSRT = 5.011 Total Cost = $188000.0
*METRIC MODEL : MSRT = 4.373 Total Cost = $188450.0

As may seen from this Table, the marginal analysis modification provided only a small improvement over the N.P.S. model(MSRT decreased from 5.011 days to 4.939 days and costs were reduced by $900) and performance still falls significantly short of the METRIC solution. Observe also, as expected, that the marginal analysis solution did increase the stockage levels of the less expensive items and decreased the stockage level of the most expensive item. This is evident from the item MSRT values. While it is clear that any solution for which budgets and unit costs are considered should improve overall effectiveness in a budget constrained environment, it may not be desirable to allow the more expensive items to suffer in terms of stockage support. Incorporation of a workable essentiality coding policy could be used to override the impact observed above.

While the increased performance obtained by using the marginal analysis modification was not significant, keep in mind that the marginal allocation was applied only to the depot stock levels. It is reasonable to assume that additional improvement could be obtained with similar modification to the base stock allocation procedure.

the depot per unit time for item i; D_i is the average depot repair time for item i; and A_{ij} is the order and ship time from the depot to base j.

Define $\Delta B_i(d_i) = B_i(d_i) - B_i(d_i+1)$

to be the reduction in total base backorder delay for item i if the depot stock level is increased from d units to d_i+1 units. Because the impact of a change in depot stock on the total base backorders is felt only indirectly through a reduction in $T_{ij}(d_i)$, no useful analytical simplification of $\Delta B_i(d_i)$ could be determined. Nevertheless, it can be computed easily directly.

Finally, let $\lambda = \sum_{i=1}^{m} \sum_{j=1}^{n} \lambda_{ij}$ be the total expected demand over all items and bases. We are now ready to describe the marginal analytic procedure.

Step 1. Determine the base stock levels S_{ij} as before from ready rate considerations. Set the initial depot stockage vector to be (0,0,...,0); i.e. $d_i = 0$ for i = 1,2,...m.

Step 2. Compute $T_{ij}(d_i+1)$ for i=1,2,..,m and j=1,2,..,n

Step 3. Compute $B_i(d_i)/C_i$ for i=1,2,..,m and let k be that index for which this ratio is maximum.

Step 4. Let $d_k = d_k + 1$.

Step 5. Compute MSRT = $(\sum_{i=1}^{m} B_i(d_i))/\lambda$.

Step 6. If MSRT ≤ Goal, stop. Otherwise go to step 2.

3. Example of the Marginal Analysis Procedure

The marginal analysis modification was applied to the data set 1 examined in the previous section. The resulting allocation are presented in Table XVI .

for multiple items an iterative process was used to select sequentially that item at each step which provides the greatest reduction in total base backorder days per dollar invested. The necessary dollars are allocated to purchase one unit of the selected item and the process is repeated until the overall MSRT objective was achieved.

2. Mathematical Description

Let $(d_1, d_2, \ldots, d_n)$ be the depot stock levels for items 1,2,...n, respectively. (Note: This is a change in the notation used previously to emphasize that the only decision variables incorporated in the marginal analytic solution are the depot stock levels for the n items). Let $B_i(d_i)$ be the total base backorder days for item i when d units are stocked at the depot. Recall that the depot stock level influences the base backorder expression only through the depot resupply time T_{ij}. To emphasize this, we express the depot resupply time for item i and base j as $T_{ij}(d_i)$. The total expected base backorder days for item i can then be written as:

$$B_i(d_i) = \sum_{j=1}^{n} \sum_{x=S_{ij}}^{\infty} (x-S_{ij}) P_{ij}(x \mid \lambda_{ij} T_{ij}(d_i)). \qquad \text{(eqn 5.1)}$$

Now, we have seen previously that the average depot delay per demand in satisfying base j when the depot has d_i units of stock is given by:

$$T_{ij}(d_i) = A_{ij} + \{\sum_{x=d_i}^{\infty} (x-d_i) \; P(x \mid \lambda_i D_i)\} \; / \; \lambda_i ;$$

where $\lambda_i = \sum_{j=1}^{n} (1-r_{ij}) \lambda_{ij}$ is the total expected demand at

Observe in the METRIC solution that the base stock allocations for item 1 are identical to those in the N.P.S. solution. Since the METRIC Lagrange multiplier solution gives each base the same ready rate for a given item it is clear that the ready rate selected by METRIC for item 1 is 70%, the same as that used by the N.P.S. model for each item. However, METRIC selected ready rates for items 2 and 3 which were smaller than 70%(the base stockage levels for items 2 and 3 are smaller than those given by the N.P.S. solution). These smaller ready rates are a reflection of the higher costs for items 2 and 3. Thus, the METRIC solution illustrates that the ready rates selected for the base stock levels in the N.P.S. model should not necessarily be the same for each item, but should be a function of the unit costs. The cheaper items should get higher ready rates than the more expensive items.

E. MODIFIED N.P.S. MODEL

1. Marginal Analytic Solution

The sample analyses discussed in the previous section suggested that the N.P.S. heuristic model could perhaps be improved if the item mean supply times were allowed to vary depending on unit costs. Thus, it might be better to provide greater protection for the less expensive items and less protection for the more expensive items. The METRIC solution does discriminate in this fashion, attempting to provide the greatest performance per dollar invested. Therefore, in an effort to improve the efficiency of the N.P.S. model and to incorporate consideration of the unit costs, the N.P.S. model was modified by using marginal analysis to determine the depot stockage levels.

The modified model determined the base stock levels just as before. Then, to determine the depot stock levels

TABLE XIV

Results of N.P.S. Model for Data 4

	Depot Stock	Base 1 S	Base 1 RR	Base 2 S	Base 2 RR	Base 3 S	Base 3 RR	MSRT
Item 1	2	5	0.892	6	0.875	7	0.882	3.7507
Item 2	2	12	0.793	14	0.777	17	0.749	4.7321
Item 3	9	22	0.670	24	0.668	28	0.702	4.9468

Total Cost : $162250.0
System Average MSRT : 4.72896 days
Time for Computer Run : 0.34 seconds

TABLE XV

Results of METRIC Model for Data 4

	Depot Stock	Base 1 S	Base 1 RR	Base 2 S	Base 2 RR	Base 3 S	Base 3 RR	MSRT
Item 1	6	5	0.968	6	0.967	7	0.971	0.84274
Item 2	17	8	0.837	10	0.879	12	0.863	2.21882
Item 3	17	19	0.630	21	0.640	24	0.647	5.51025

Total Cost : $161550.0
System Average MSRT : 3.89729 days
Time for Computer Run : 1.75 seconds

stock levels are too high. Even with the reduction in base ready rate to 70%, the solutions above show that the METRIC solution puts less stock at the bases and more at the depot than does the N.P.S. solution. The solution obtained by METRIC is nearly 20% better than the N.P.S. solution. Thus, additional modification of the N.P.S. model to reduce base stock even more should be considered.

bases to be justified by an MSRT goal of 125 hours.[2] As a result of the above observations, the N.P.S. model was modified for the last data test case by reducing the base ready rate from 90% to 70%.

D. DATA SET 4: ZERO BASE REPAIR AND LONG DEPOT REPAIR TIME

Table XIII gives the data for the fourth test data set. Characteristics of this data set are short ordering and shipping times, zero probability of base repair and long depot repair time.

TABLE XIII

INPUT DATA 4

Item	Base	λ_{ij}/day	r_{ij}	R_{ij}	A_{ij}	D_i	Cost
1	1	0.044	0.0	24.0	45.0	40.0	$200.0
	2	0.056	0.0	25.0	45.0		
	3	0.067	0.0	23.0	45.0		
2	1	0.111	0.0	29.0	45.0	50.0	750.0
	2	0.133	0.0	27.0	45.0		
	3	0.167	0.0	28.0	45.0		
3	1	0.222	0.0	33.0	45.0	60.0	1500.0
	2	0.244	0.0	35.0	45.0		
	3	0.278	0.0	34.0	45.0		

A comparison of the results above to those obtained for data set 3 shows that the MSRT goal of 125 hours(5.2 days) can be achieved even with longer depot repair times and lower base ready rates(70% versus 90%) at a reduced investment cost. This supports the argument above that the base

[2]This comment is contingent on the validity of the assumptions made by METRIC model. In particular, the (S-1,S) ordering policy assumption and the Poisson demand assumption.

TABLE XI

Results of N.P.S. Model for Data 3

	Depot Stock	Base 1 S	Base 1 RR	Base 2 S	Base 2 RR	Base 3 S	Base 3 RR	MSRT
Item 1	0	7	0.991	8	0.987	9	0.986	0.3213
Item 2	0	14	0.986	16	0.985	20	0.991	0.1721
Item 3	0	26	0.988	28	0.987	32	0.991	0.1209

Total Cost : $171300.0
System Average MSRT : 0.16212 days
Time for Computer Run : 0.20 seconds

TABLE XII

Results of METRIC Model for Data 3

	Depot Stock	Base 1 S	Base 1 RR	Base 2 S	Base 2 RR	Base 3 S	Base 3 RR	MSRT
Item 1	3	10	1.000	9	0.999	10	0.999	0.01529
Item 2	9	16	0.998	14	0.996	17	0.998	0.03050
Item 3	20	24	0.996	21	0.994	23	0.993	0.04811

Total Cost : $171750.0
System Average MSRT : 0.03849 days
Time for Computer Run : 6.64 seconds

The results from analyses of the last two data sets show that the system mean supply response times can be made significantly smaller than the 125 hour goal, even with zero stock at the depot. This is because the stockage levels at the bases are very high. This example points out a shortcoming of the N.P.S. model(and perhaps of the current Navy allowance list models); too much stock is positioned at the

TABLE IX

Results of METRIC Model for Data 2

	Depot Stock	Base 1 S	Base 1 RR	Base 2 S	Base 2 RR	Base 3 S	Base 3 RR	MSRT
Item 1	0	9	1.000	10	1.000	11	1.000	0.0001
Item 2	1	15	1.000	17	1.000	18	1.000	0.0002
Item 3	4	24	1.000	28	1.000	29	1.000	0.0003

Total Cost : &171750.0
System Average MSRT : 0.00025 days
Time for Computer Run : 13.57 seconds

TABLE X

INPUT DATA 3

Item	Base	λ_{ij}/day	r_{ij}	R_{ij}	A_{ij}	D_i	Cost
1	1	0.044	0.0	0.0	45.0	20.0	$200.0
	2	0.056	0.0	0.0	45.0		
	3	0.067	0.0	0.0	45.0		
2	1	0.111	0.0	0.0	45.0	25.0	750.0
	2	0.133	0.0	0.0	45.0		
	3	0.167	0.0	0.0	45.0		
3	1	0.222	0.0	0.0	45.0	30.0	1500.0
	2	0.244	0.0	0.0	45.0		
	3	0.278	0.0	0.0	45.0		

As with the previous data set we see that the N.P.S. model gives zero stock at the depot. METRIC, on the other hand, carries less stock at the bases and positive stock levels at the depot. Both models easily satify the MSRT goal of 125 hours.

TABLE VII

INPUT DATA 2

Item	Base	λ_{ij}/day	r_{ij}	R_{ij}	A_{ij}	D_i	Cost
1	1	0.044	0.85	24.0	90.0	20.0	$200.0
	2	0.056	0.90	25.0	90.0		
	3	0.067	0.90	23.0	90.0		
2	1	0.111	0.85	29.0	90.0	25.0	750.0
	2	0.133	0.80	27.0	90.0		
	3	0.167	0.90	28.0	90.0		
3	1	0.222	0.80	33.0	90.0	30.0	1500.0
	2	0.244	0.75	35.0	90.0		
	3	0.278	0.80	34.0	90.0		

TABLE VIII

Results of N.P.S. Model for Data 2

	Depot Stock	Base 1 S	Base 1 RR	Base 2 S	Base 2 RR	Base 3 S	Base 3 RR	MSRT
Item 1	0	7	0.999	8	0.999	9	0.999	0.0037
Item 2	0	14	0.999	16	0.999	20	1.000	0.0009
Item 3	0	26	1.000	28	0.999	32	1.000	0.0006

Total Cost : $171300.0
System Average MSRT : 0.00112 days
Time for Computer Run : 0.20 seconds

C. DATA SET 3: SHORT ORDER-SHIP TIME AND ZERO PROBABILITY OF BASE REPAIR

Table X gives the data for the third sample data set. This set is characterized by short ordering and shipping times and low probabilities of base repair. Table XI and XII are the N.P.S. and METRIC model results.

```
RS = QK * COST(HAN,1,1);
BOD=0.0;
DO I = 1 TO BASE;
   SPACE(I) =0.0;
END;

DO   G = 1 TO BASE;
  SIJ =0;
  DO Y =1 TO BASE;
   M =BREP(HAN,Y,2) *BPROB(HAN,Y,5) + (1.0-
      bprob(han,y,5))*(BORD(HAN,Y,4)+DDELY);
   BRST(Y) = M;
  END;
  PUT SKIP(1) LIST('THE BASE MU');
  DO Z = 1 TO BASE;
    UB(Z) = BDEM(HAN,Z,3) *BRST(Z);
  END;

  SIJ = 0;
  K =1;
  Y3 =1.0;
  POISSON = EXP(-UB(G));
  PROB_SUM = POISSON;
  RR(LAMBDA,HAN,SIO+1,G) = PROB_SUM;
  DO WHILE (PROB_SUM < 1.0+RS);
    Y3 = Y3 *UB(G) / K;
    PROB = POISSON *Y3;
    PROB_SUM = PROB_SUM + PROB;
    K = K + 1;
    SIJ = SIJ +1;
  END;
    RR(LAMBDA,HAN,SIO+1,G) =  PROB_SUM;

  WIJ(LAMBDA,HAN,SIO+1,G) = SIJ;
  PX = EXP (-UB(G));
  SPACE(G) = UB(G) -SIJ;
```

```
  IF SIJ > 0
    THEN DO;
      DO X=0 TO SIJ -1;
         SPACE(G) =SPACE(G) + (SIJ - X)*PX;
         PX = UB(G) * PX/(X+1);
      END;
    END;

  SUM5 =SUM5 + SPACE(G);
  BOD = BOD+SPACE(G) -QK*COST(HAN,1,1)*SIJ;
END;
DO Z=1 TO BASE;
     PUT EDIT(UB(Z))(X(2),F(9,5));
END;
PUT EDIT(SUM5)( X(1),F(11,5));
MMSRT = SUM5 / TOTAL_DEMAND;
G =1;
BODD=BOD - QK*COST(HAN,1,1)*SIO;
PUT EDIT(MMSRT,BODD)(X(2),F(9,5),X(2),F(9,5));
IF BODD<SPACE1
 THEN DO;
  DO I = 1 TO BASE;
     READY_RATE (I) =
          RR(LAMBDA,HAN,SIO+1,I);
  END;
  BOD1 = SUM5;
  SPACE1 =BODD;
  DO I = 1 TO BASE;
   BBOD(I) =SPACE(I);
  END;
  W1 =SIO;
  NUMBER =0.0;
  DO I = 1 TO BASE ;
    W(I)=WIJ(LAMBDA,HAN,SIO+1,I);
    NUMBER=W(I)+NUMBER;
```

```
        END;
       END;
      SIO =SIO + 1;
      END;

   SUMCOST1=COST(HAN,1,1)*(NUMBER+W1);
   BOD2 = BOD2 + BOD1;
   PUT SKIP EDIT(HAN,W1)(X(10),F(2),X(2),F(2));
   DO I = 1 TO BASE;
     PUT EDIT(W(I),READY_RATE(I))
              (X(2),F(2),X(2),F(5,3));
   END;
   MSRT = BOD1/TOTAL_DEMAND;
   PUT EDIT(MSRT,SUMCOST1)
     (X(2),F(9,5),X(3),F(9));
   SUMCOST =SUMCOST + SUMCOST1;
   SIO =0.0;
   SUM_DEMAND = SUM_DEMAND + TOTAL_DEMAND;
   TOTAL_MSRT = TOTAL_MSRT + MSRT * TOTAL_DEMAND;
END;

   PUT EDIT (SUMCOST)(X(7),F(9));
   TOTAL_AVG_MSRT = TOTAL_MSRT / SUM_DEMAND;
   SUMCOST2(LAMBDA) =SUMCOST;

 BOD2=0.0;
 LAMBDA = LAMBDA +1;

 IF SUMCOST <BUDGET
    THEN DO;
      QK1 = QK;
      QK  =0.5*(QK1 +QK2);
    END;
    ELSE DO;
      QK2 = QK;
      QK =0.5*(QK1 + QK2);
    END;
```

```
    END;
  PUT SKIP EDIT('TOTAL AVERAGE MSRT IS',
                total_avg_msrt)(a(23),x(2),F(10,5));

END BBO; *DATA
```

APPENDIX B

COMPUTER PROGRAM FOR N.P.S. MODEL BY PL/I

```
*PL/C  ATR SOURCE L=4000 P=70 T=(3,30) XREF
OPTIMAL : PROCEDURE OPTIONS(MAIN);
 DCL (ITEM, ITEMS, BASE, BASES, I, X, K, BASE_STOCK(6,5),
     depot_stock,OPTI_DEPOT_STOCK(6),SUM,TOTAL_STOCK)FIXED;
 DCL (COST(9,9,6),BASE_REP_TIME(9,9,6),BASE_DEMAND(9,9,6),
      ORDER_TIME(9,15,6),dEPOT_REP_TIME(9,15,6),
      READY_RATE(5),TIJ,NEW_PROB,base_prob(9,15,6))FLOAT;
 DCL (DEPOT_DEMAND,SUM_DEMAND,MU,Y3,PX,POISSON,PROB_SUM,
      msrt,SUM9,SUM10,BASE_MU,SUM_BACKORDER,BASE_BACKORDER,
      TOTAL_COST,LAMBDA_T,EXPECTED_BACKORDER)FLOAT;
 DCL (TOTAL_DEMAND, TOTAL_MSRT,TOTAL_AVG_MSRT,item_cost,
      base_res_time(7))float;
 ITEMS = 3;
 BASES = 3;
 TOTAL_COST =0.0;
 DO I =1 TO ITEMS;
    DO J = 1 TO BASES;
       K = 1;
       GET LIST(COST(I,J,K),BASE_REP_TIME(I,J,K+1),
               BASE_DEMAND(I,J,K+2),ORDER_TIME(I,J,K+3),
               BASE_PROB(I,J,K+4),DEPOT_REP_TIME(I,J,K+5));
    END;
 END;
 TOTAL_DEMAND = 0.0;
 TOTAL_MSRT = 0.0;
 DO ITEM = 1 TO ITEMS;
    SUM = 0;
    DEPOT_DEMAND = 0.0;
    SUM_DEMAND = 0.0;
```

```
 DO BASE =1 TO BASES;
    TIJ =(1.0-BASE_PROB(ITEM,BASE,5))*90 +
      BASE_PROB(ITEM,BASE,5)*BASE_REP_TIME(ITEM,BASE,2);
    MU =TIJ*BASE_DEMAND(ITEM,BASE,3);
    PUT SKIP LIST('TIJ IS ',TIJ);
    PUT SKIP LIST('BASE MU WHEN TIJ DAYS',MU);
    NEW_PROB = .74;

    PUT SKIP LIST('NEW PROBILITY IS',NEW_PROB);
    K = 1;
    Y3 = 1.0;
    POISSON = EXP(-MU);
    PROB_SUM = POISSON;
    DO WHILE (PROB_SUM <= NEW_PROB);
       Y3 = Y3 * MU/K;
       PROB = POISSON * Y3;
       PROB_SUM = PROB_SUM + PROB;
       K = K + 1;
    END;
    BASE_STOCK(ITEM,BASE) = K -1;


    SUM = SUM + BASE_STOCK(ITEM,BASE);
    DEPOT_DEMAND =DEPOT_DEMAND + BASE_DEMAND
           (item,base,3)*(1.0 -BASE_PROB(ITEM,BASE,5));
    SUM_DEMAND = SUM_DEMAND + BASE_DEMAND(ITEM,BASE,3);
END;

DEPOT_STOCK =0;
MSRT = 5.5;
DO WHILE (MSRT >= 5.2);
   LAMBDA_T = DEPOT_DEMAND * DEPOT_REP_TIME
              (item,bases,6);
   PX = EXP(-LAMBDA_T);
   SUM10 = LAMBDA_T - DEPOT_STOCK;
   IF DEPOT_STOCK > 0
```

```
   THEN DO;
     DO X = 0 TO DEPOT_STOCK-1;
        SUM10 = SUM10 + (DEPOT_STOCK - X) * PX;
        PX = LAMBDA_T * PX /(X+1);
     END;
   END;
   EXPECTED_BACKORDER = SUM10;
   DEPOT_DELAY =EXPECTED_BACKORDER / DEPOT_DEMAND;
   SUM_BACKORDER = 0.0;


   DO BASE = 1 TO BASES;
    READY_RATE(BASE)=0.0;
    BASE_RES_TIME(BASE) =BASE_REP_TIME(ITEM,BASE,2)*
           BASE_PROB(ITEM,BASE,5) + (1 -
           BASE_PROB(ITEM,BASE,5))*(ORDER_TIME(ITEM,
           BASE,4)+DEPOT_DELAY);
    BASE_MU = BASE_RES_TIME(BASE) * BASE_DEMAND
              (item,base,3);
    PUT SKIP LIST('BASE MU IS',BASE_MU);
    PX = EXP(-BASE_MU);
    SUM9 =BASE_MU - BASE_STOCK(ITEM,BASE);

    IF BASE_STOCK(ITEM,BASE)>0
      THEN DO;
        DO X =0 TO BASE_STOCK(ITEM,BASE);
           SUM9 = SUM9 +(BASE_STOCK(ITEM,BASE) - X) *
                  px;
           READY_RATE(BASE) = READY_RATE(BASE)+PX;
           PX = BASE_MU * PX /(X+1);
        END;
      END;
      BASE_BACKORDER = SUM9;
      SUM_BACKORDER = SUM_BACKORDER + BASE_BACKORDER;
 END;
 MSRT =SUM_BACKORDER /SUM_DEMAND;
```

```
        PUT SKIP LIST('MSRT IS',MSRT);
        DEPOT_STOCK = DEPOT_STOCK +1;
     END;
     TOTAL_DEMAND = TOTAL_DEMAND + SUM_DEMAND;
     TOTAL_MSRT = TOTAL_MSRT + MSRT*SUM_DEMAND;

     OPTI_DEPOT_STOCK(ITEM) = DEPOT_STOCK -1;
     TOTAL_STOCK = SUM +OPTI_DEPOT_STOCK(ITEM);
     ITEM_COST = TOTAL_STOCK * COST(ITEM,BASES,1);

     PUT SKIP EDIT(OPTI_DEPOT_STOCK(ITEM))(X(2),F(6));
     DO BASE = 1 TO BASES;
       PUT EDIT(BASE_STOCK(ITEM,BASE),READY_RATE(BASE))
            (X(2),F(6),X(2),F(6,4));
     END;
     PUT EDIT (MSRT)(X(2),F(9,4));
     PUT SKIP(3);
     TOTAL_COST = TOTAL_COST + ITEM_COST;
   END;
   PUT SKIP EDIT('TOTAL COST IS',TOTAL_COST)(X(3),A(14),
              f(9,2));
   TOTAL_AVG_MSRT = TOTAL_MSRT / TOTAL_DEMAND;
   PUT SKIP EDIT('TOTAL AVERAGE MSRT IS',TOTAL_AVG_MSRT)
              (X(3),A(20),X(2),F(9,5));
END OPTIMAL; *DATA
```

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