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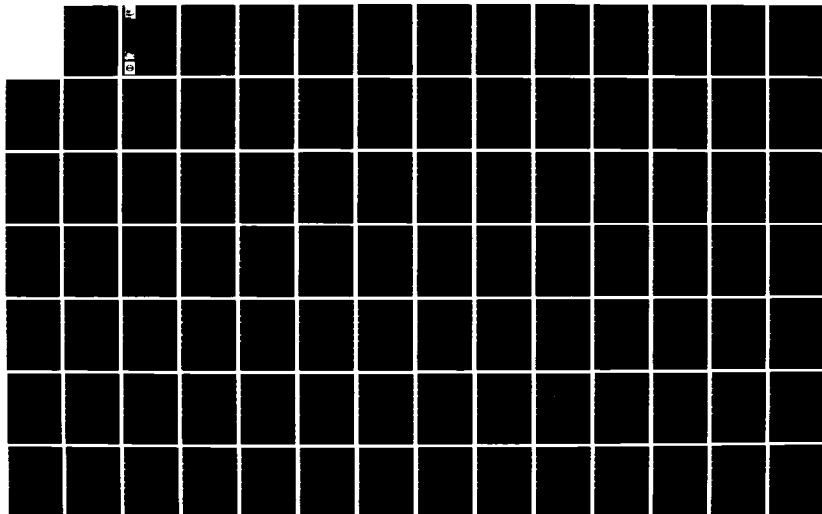
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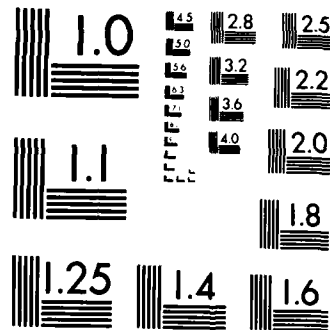
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ENVIRONMENTAL IMPACT  
RESEARCH PROGRAM

MISCELLANEOUS PAPER EL-85-2

SELECTION OF TURBULENCE AND MIXING  
PARAMETERIZATIONS FOR ESTUARY WATER  
QUALITY MODELS

by

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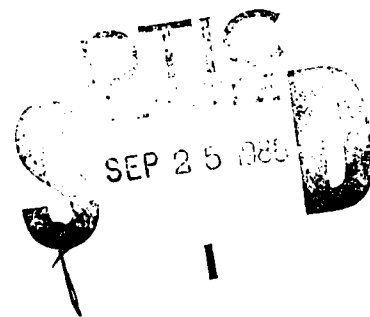


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transport models consistent with this variability are generalized. The equations for one-dimensional, two-dimensional (vertical and horizontal), and three-dimensional models are presented with particular attention paid to the terms requiring closure. Turbulence models are reviewed in the context of a hierarchical approach with commonly used functional forms and coefficient values tabularized. Finally, selection guidelines for the various closure forms are summarized.

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## PREFACE

This report was prepared by Dr. Keith W. Bedford for the US Army Engineer Waterways Experiment Station (WES), Vicksburg, Miss. The study was sponsored by the Office, Chief of Engineers (OCE), under the Environmental Impact Research Program (EIRP). Dr. John Bushman and Mr. Earl Eiker were OCE Technical Monitors. Mr. Dave Mathis was Water Resources Support Center Technical Monitor. Dr. Roger T. Saucier was EIRP Program Manager.

This report provides a critical review of the methods and assumptions by which spatially and temporally simplified estuarine water quality models are structured and mixing processes parameterized. The intended audience is familiar with numerical hydrodynamic or water quality modeling; however, sections of the report provide a general introduction to numerical estuarine modeling.

Mr. Ross W. Hall, Water Quality Modeling Group (WQMG), monitored the study under the supervision of Mr. Mark S. Dortch, Chief, WQMG; Mr. Donald L. Robey, Chief, Environmental Research and Simulation Division; and Dr. John Harrison, Chief, Environmental Laboratory.

During the preparation of this report, COL Tilford C. Creel, CE, and COL Robert C. Lee, CE, were Commanders and Directors of WES and Mr. F. R. Brown was Technical Director. At the time of publication, COL Allen F. Grum, USA, was Director and Dr. Robert W. Whalin was Technical Director.

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CONVERSION FACTORS, U.S. CUSTOMARY TO METRIC (SI)  
UNITS OF MEASUREMENT

U.S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

<u>Multiply</u>	<u>By</u>	<u>To Obtain</u>
inches	2.54	centimeters
feet	0.3048	meters
yards	0.9144	meters
fathoms	1.8288	meters
miles (U.S. statute)	1.609344	kilometers
miles (U.S. nautical)	1.852	kilometers
square feet	0.09290304	square meters
square miles (U.S. statute)	2.589988	square kilometers
cubic feet	0.02831685	cubic meters
cubic yards	0.7645549	cubic meters
miles (U.S. statute) per hour	0.44704	meters per second
knots (international)	0.5144444	meters per second
foot-pounds (force)	1.355818	newton-meters
degrees (angular)	0.01745329	radians
gram/liter	1	kilogram/cubic meter
pound/cubic foot	16.02	kilogram/cubic meter

# SELECTION OF TURBULENCE AND MIXING PARAMETERIZATIONS FOR ESTUARY WATER QUALITY MODELS

## PART I: INTRODUCTION

### Background

1. The assessment and management of water quality changes in estuaries are aided in large part by the use of numerical models. Numerical models are based upon the reduction of a set of dynamic nonlinear differential equations to a system of solvable algebraic equations and the differential equations must include all the operable physical, biological, and chemical mechanisms extent in the simulation to be performed. By use of a time and space grid the reduction to an algebraic solution involves the use of discretizations which requires assumptions about how these processes behave at scales below the time and space scales of the nodal points. The use of such grid systems involves a loss of information from the system but the effects of such nonresolved processes are felt by the resolved processes and must be specified by empirical functions of the resolved scale variables.

2. Estuaries display the most complicated interaction of hydrodynamic, chemical, and biological processes possible in surface water flows. The length and time scale variability of the processes ranges continuously from fractions of centimeters and seconds involved in turbulence processes, sediment, and organism sizes to the years and kilometers typical of changes in the coastal and ocean factors imposed upon the estuary. In order to achieve economically manageable models, spatially and temporally simplified models (i.e. one-dimensional, two-dimensional, or steady-state) have been used. Such models are derived by averaging the governing differential equations, which results in a

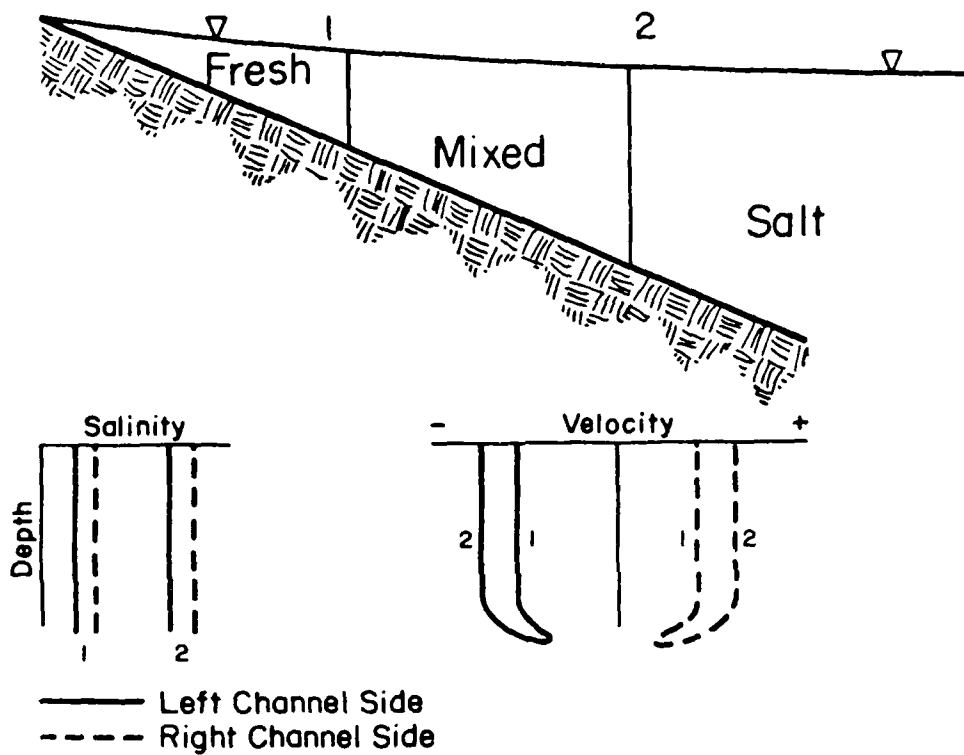
series of correlation terms involving subaveraging scale variables. The specification of the correlations in terms of resolved scaled variables is called the "closure problem" and the functional forms for closure range in complexity from the simplest empirical forms to extremely elegant descriptions in terms of nonlinear dynamic partial differential equations. It is common in hydraulic practice to refer to these functions as mixing, dispersion, and/or diffusion parameterizations and for estuaries it is no exaggeration to say that there are hundreds of different forms and coefficients for these parameterizations.

### Objectives

3. It is the objective of this report to review the classes and forms of the various estuary mixing parameterizations. To do so requires a review of the estuarine physical processes which contribute to mixing and dispersion; a description of the resulting transport equations and terms requiring closure; a review of the various parameterization forms; and, finally, an assessment of the current state of practice as regards estuarine water quality modeling with the various closure forms.

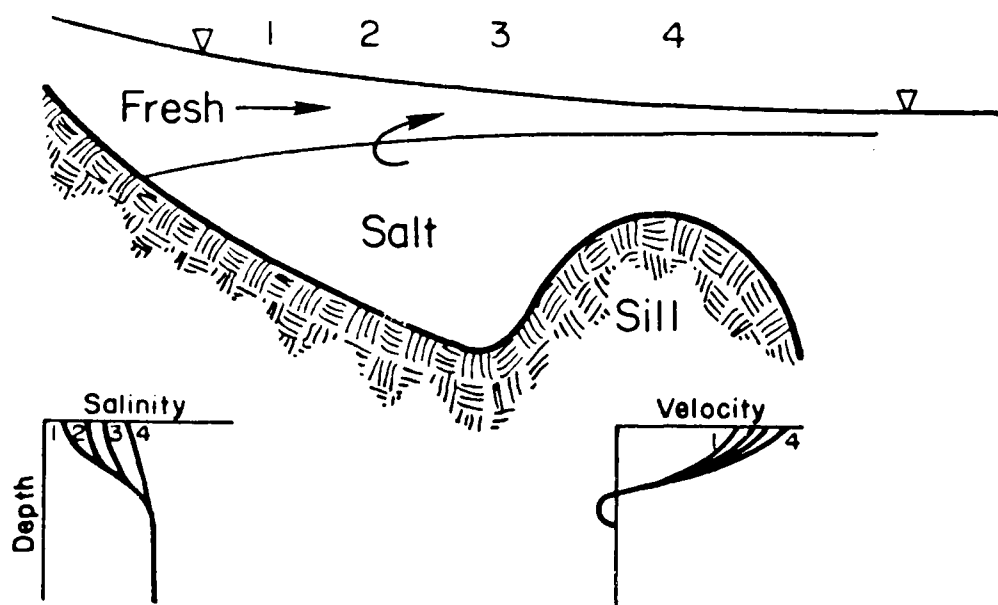
### Rationale

4. The viewpoint or rationale of this report is quite simple. It is the firm belief of the author that fundamental to the success of any surface water quality model calculation is a properly formulated model which includes all the necessary hydrodynamic mechanisms. Further, the numerical method used to formulate the model must be correct. As to the first assertion, one need only look at the evidence now being accumulated which clearly shows that the flow field physics play a dominant role in determining the contaminant and ecological character of estuaries. For

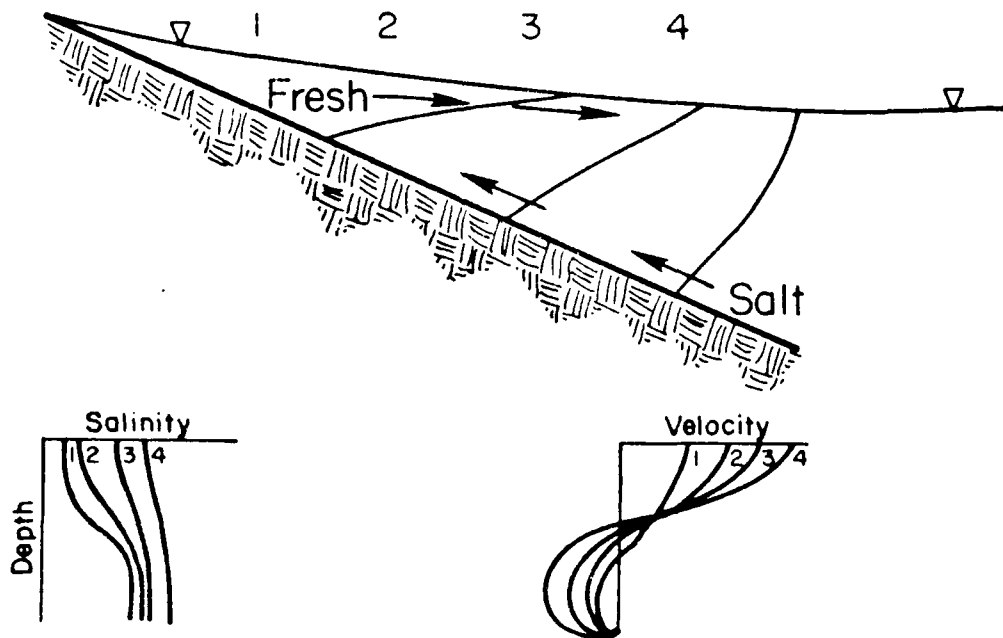


### e. Homogeneous With Lateral Variation

Figure 2. (Concluded)



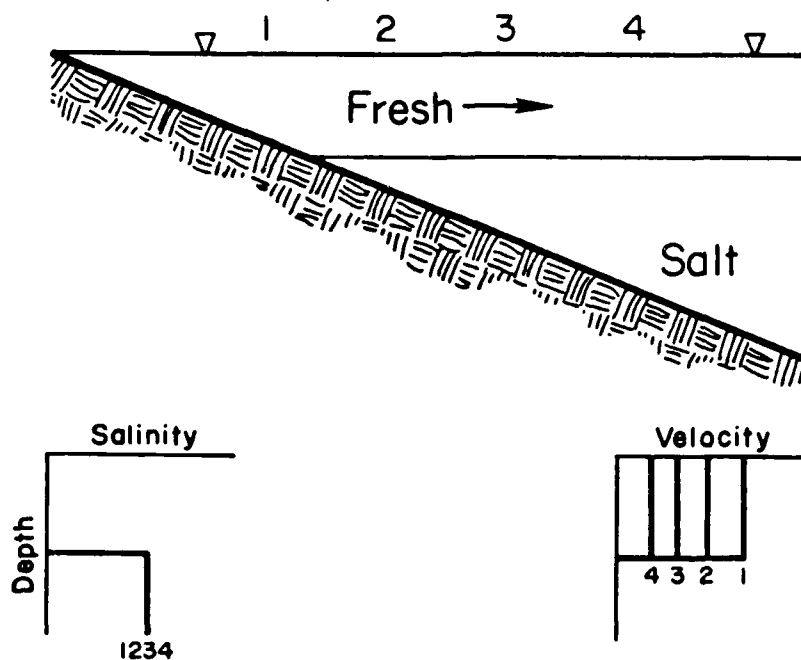
c. Fjord Type



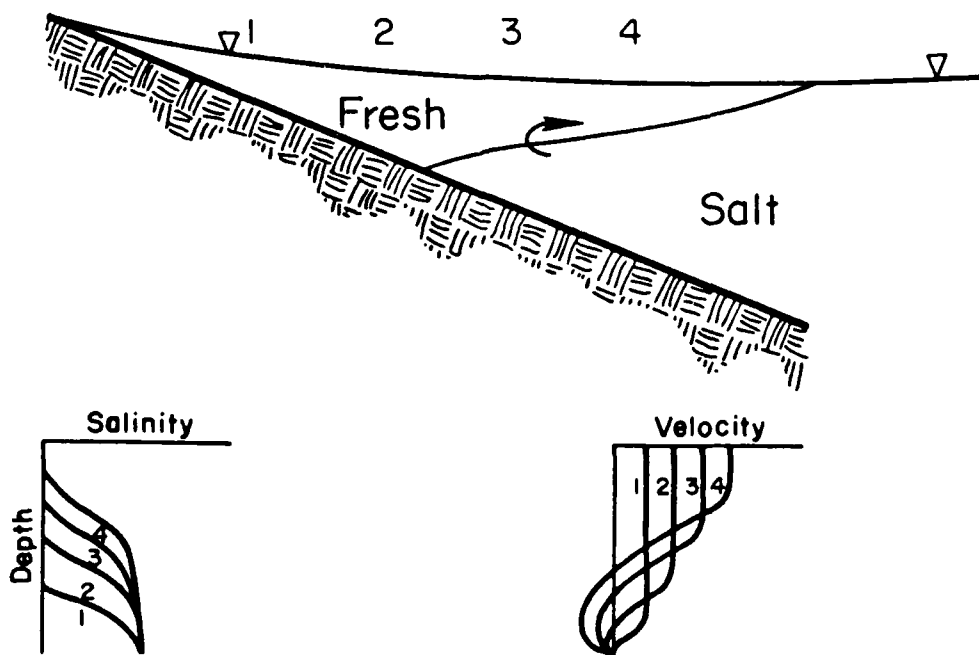
d. Partially Mixed

Figure 2. (Continued)





a. Salt Wedge Without Friction



b. Salt Wedge With Friction

Figure 2. Typical estuary configurations by salinity and velocity structure (after Dyer 1973)  
(continued)

### Classification by density structure

37. Often, in coastal ocean areas, freshwater/salt-water mixing within an estuary can result in certain identifiable salinity structures within an estuary. As salinity is recognized as an indication of density structure, these schemes are also recognized as classifying estuarine behavior according to density structure. Dyer (1973), after Pritchard (1955) and Cameron and Pritchard (1963), gives four types of estuaries commonly encountered. These are the highly stratified salt wedge type, the fjord type, the partially mixed type, and the homogeneous type. The homogeneous type is further subdivided into vertically, longitudinally, and sectionally homogeneous categories.

38. The highly stratified salt wedge type occurs when the ratio of the river flow to tidal flow is relatively large and the width-to-depth ratio is relatively small. If the fluid is considered frictionless, then the fresh water, being lighter, flows outward over the seawater, the surface velocity decreasing towards the mouth as the channel diverges. If the sea is assumed tideless, then the interface between two waters (sea and fresh) will be nearly horizontal, and extend upstream to the point where the channel bottom elevation is at mean sea level. Velocity and salinity profiles would resemble those of Figure 2a.

39. If friction is introduced in the form of viscosity, shear stresses will result at the interface due to the motion of the freshwater passing over the seawater, and the interface will be pushed downstream to a point where the slope of the interface would be sufficient to balance the shear. If the shear is sufficient, then internal waves develop at the interface, the breaking of these waves resulting in a net entrainment of water into the upper layer. This requires a net inflow in the lower layer to counterbalance the increased outflow in the upper layer. A

### Others

34. Estuarine formations due to tectonic displacement, landslides, volcanos, and other similar catastrophic occurrences are also observed. These are by far the minority in total numbers, although they may exhibit some of the most striking features of estuarine systems. One such example is the tectonically produced estuary of the San Francisco Bay System (California, U.S.A.).

### Classification Schemes

35. The estuarine literature affords a vast abundance of classification schemes which various authors have promoted. These schemes base classifications on both observed and derivable quantities of estuarine circulation, stratification, and mixing types. Included are the effects of density structure, tidal inflow volumes, fresh water inflow, body force, and channel curvature effects. Although some classification schemes attempt to approach the problem simply, by considering only one facet of estuarine physics, other schemes, such as Hansen and Rattray's (1966) stratification-circulation diagrams, attempt to consider many more. The end result of most of these schemes is the definition of certain dimensionless numbers, which, it is argued, can segregate various estuarine types.

36. As these schemes are nearly as numerous as the authors who have devised them, only a few examples of the many available are discussed herein. Where more than one author has proposed a similar scheme, similarities and differences, and the criteria which must be satisfied in order for each to apply, will be examined. Full descriptions of these schemes may be found by consulting the cited references.

inlet at the mouth, inlet velocities are higher than flow velocities up-estuary of the mouth. The mouth bar and inlet are subject to gross seasonal changes, both position and size, as sediment availability and wind wave patterns on the sea are seasonally variable. Bars often disappear at times of high river outflow (due to washout), reestablishing themselves when the river flow decreases. Conversely, at times of low river outflow, the bar may develop entirely across the mouth, the beach effectively damming the river, and thus prohibiting the river and the sea from interacting as an estuary system.

32. Bar built estuaries are commonly found in tropical areas, or where the tidal range of the sea is small, and along coasts which experience active sediment deposition. Examples include the Roanoke River (U.S.A.) and the Vellar Estuary (India).

#### Inundated floodplains

33. Although similar to coastal plain and bar built estuaries, the inundated floodplain would differ in that its occurrence would be restricted to the case where the river would be of a much shallower depth than the adjoining sea. Such would be the case of many small streams emptying into the Great Lakes. In such a case, periodic rise and fall of the sea level would cause a periodic inundation/drainage pattern in the floodplain of the creek. This would not represent a true estuarine behavior. However, if the mean sea level were to rise for a longer time than the periodic sea surface fluctuations, a "temporary" estuary may develop, the floodplain remaining inundated for a long enough period of time that something similar to an estuarine system may result. The Old Woman Creek Estuary (Huron County, Ohio, U.S.A.) emptying into Lake Erie, is of this type.

aries. Therefore, the topography of these estuaries is very similar to that of a river valley.

26. Coastal plain estuaries generally widen and deepen towards the mouth. The formation of splits and deltas could alter this pattern to varying degrees. Cross sections are usually triangular and width-to-depth ratios generally are large. These patterns can vary, depending upon the particular landform of the area in which they are found.

27. Bottom deposits usually consist of a variety of muds, silts, clays, and sands. Commonly observed is a gradual transition from mud to sand bottom as one progresses down (from estuary head to mouth).

28. Coastal plain estuaries are commonly found in temperate climates. Here, tidal forces are generally large so that the river discharge, and hence the sediment arriving from the river, are generally much less than the tidal prism volume and the littoral transport, respectively.

29. Examples of coastal plain estuaries are the Chesapeake Bay System (U.S.A.) and the Mersey (England).

#### Bar built estuaries

30. Bar built estuaries are similar to coastal plain systems in that they, too, have experienced inundation from the rising seas in postglacial times. Unlike the coastal plain estuaries, in bar built estuaries sediment delivered from land drainage is large, so that sedimentation has kept pace with inundation. Characteristic bars often develop across the mouths of these systems. This bar is usually a break-point bar, i.e., the bar developed where the waves break upon the beach. This process of bar building requires that the tides be small and that there is a large sediment supply to the region.

31. Bar built estuaries are usually only a few meters deep and contain extensive systems of back lagoons and shallow waterways. Since there is normally a constricted

as the sea forcing may completely dominate circulation and the freshwater source may be solely derived from direct land runoff.

#### Fjords

21. Fjords are found where ice cover from prehistoric times have deepened and widened preexisting river valleys. Rock bars (or sills), generally left intact, are found at the mouth of the fjord and at the intersections of adjoining fjords. The sills are usually quite shallow compared to the average fjord depth.

22. Fjords are characterized by rock covered floors and walls. Sediment layers are generally thin, and are usually restricted to areas near the freshwater source(s), where sediment from land runoff settles out.

23. Freshwater inflow to fjords is generally very small compared to the total volume of the water in the fjord. Freshwater inflow is, however, often much greater than the tidal contributions, due to the restricted passage of tidal flow imposed by the sill at the mouth.

24. Width-to-depth ratios in fjords often are much smaller than in other types of estuaries (10:1 compared to 100:1). Their occurrence is restricted to the northern latitudes where the Pleistocene ice was present, and to mountainous regions near the seas. Examples of fjords are Songe Fjord (Norway), Puget Sound (Washington State, U.S.A.), and Milford Sound (New Zealand).

#### Drowned river mouths or coastal plain estuaries

25. These types of estuaries were formed in post-glacial times when the melting glaciers caused a rise in the mean sea level (MSL) and subsequently inundated rivers emptying into the prehistoric seas. Since the sediment quantity delivered by the rivers is generally small, sedimentation has not kept pace with inundation in these estu-

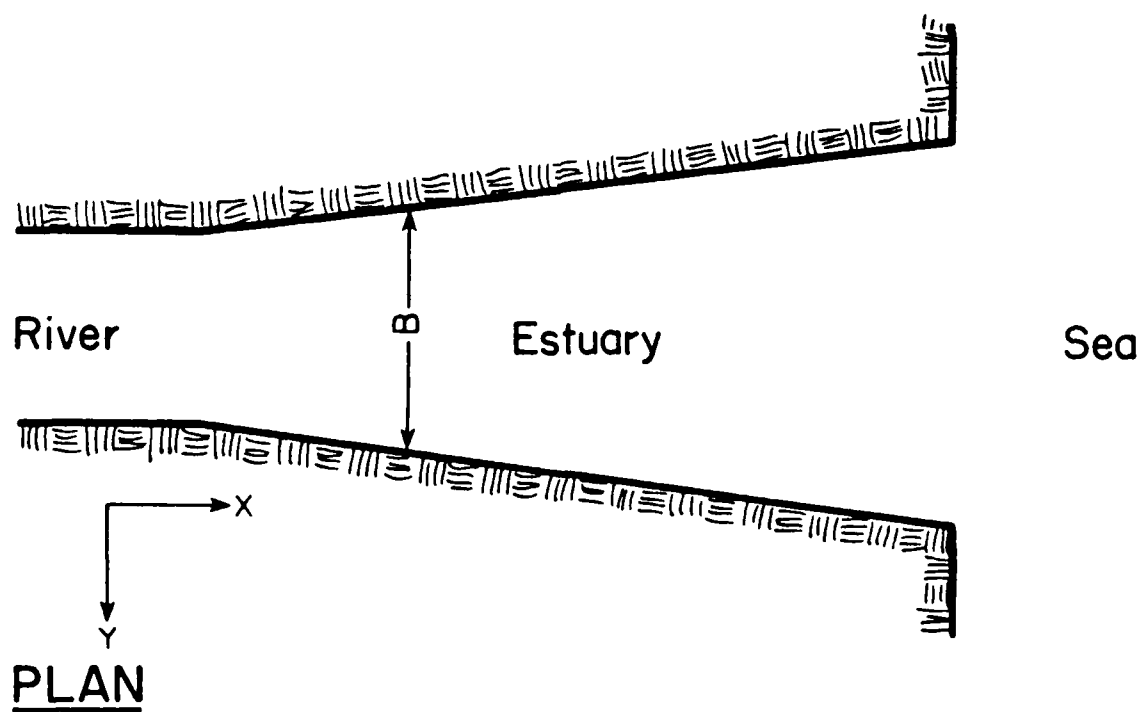
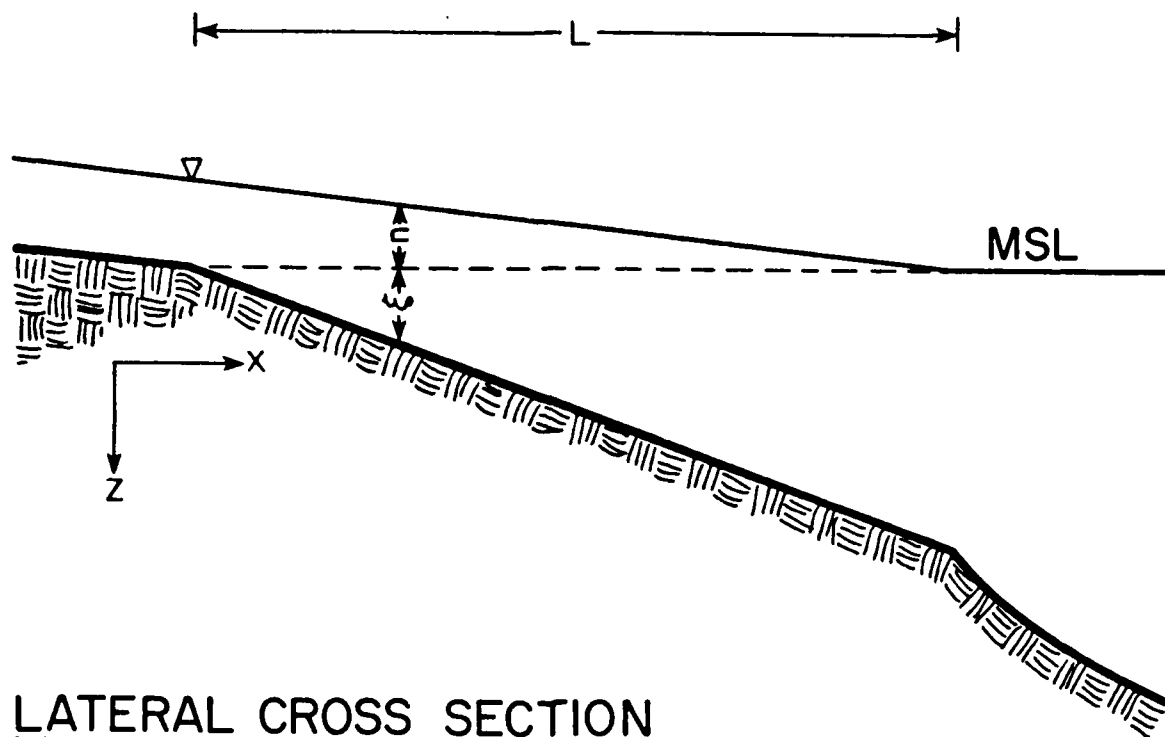


Figure 1. A typical estuary

logic" may be applicable to rivers which empty into freshwater seas.

16. Herein, an estuary is defined as any body of water, which at some time experiences freshwater inflow, and is in communication with a sea, such that physical processes with the sea (waves, currents, water level elevations, etc.) directly affect the physical processes within the estuary. This definition explicitly excludes the freshwater source and the sea from the estuary, so that conditions imposed by these sources will enter into the considerations only as boundary conditions to the estuary.

17. For clarity in further discussions, the head of the estuary is defined as the point where the freshwater source completely dominates the physics of the flow. The mouth is where sea influences (unaffected by channel convergence, beach inlets, etc.) completely dominate. Figure 1 shows a typical estuary in plan and cross section.

#### Formation Mechanisms

##### Geologic formulations

18. The information in this section is directly abstracted from Pritchard (1955) and Dyer (1973).

19. Estuaries are commonly segregated according to geologic configuration and formation. The several types observed are referred to as fjords, drowned river mouths (or coastal plain estuaries), and bar built estuaries. This report also shows how the general definition of an estuary presented herein can be extended to include inundated floodplains.

20. The general definition can be extended even further to include some bays and lagoons, or parts thereof, as exhibiting estuarine characteristics. The boundary conditions of these systems, however, may be much harder to define (e.g., the point of seawater or freshwater influence)



important physical parameters the tidal motion of the adjoining sea, salinity (density) gradients which may be present, and freshwater inflow from the river.

12. McDowell and O'Conner (1977) state that the estuary is "that part of the river system in which the river widens under the influence of tidal action...." Along these same lines, Brandt and Herdendorf (1972) define an estuary as "that part of the lower river course that is affected by mixing of water from the stream with that from the receiving sea."

13. Dyer (1973) cites the interaction of freshwater and saltwater as a criterion for estuarine description, and quotes Cameron and Pritchard (1963), who state that "An estuary is a semi-closed coastal body of water which has a free connection with the open sea and within which sea water is measurably diluted from land drainage." Dyer goes further to describe a positive estuary as one in which freshwater inflow exceeds evaporation and a negative estuary as one in which evaporation exceeds freshwater inflow.

14. Common to all of these definitions is the condition that an estuary is a region where a river meets and interacts with a sea or large body of water. Under these conditions, therefore, some part of the river will be influenced by the boundary conditions imposed at the open water and at the point where upstream river hydraulics dominate. The influence is seen directly in water level elevations, flow reversal, and mixing/circulation processes within the estuary. Proper insight also determines those biological and chemical processes which could be used to delineate a particular estuary.

15. For the purposes of this study, estuaries are defined such that the definition need not be limited to the case where a salinity gradient is present. Such a general definition will help to define situations where "estuarine

## PART II: ESTUARY ORIGINS AND CLASSIFICATION

8. Estuarine studies comprise a large portion of those conducted in the coastal zones. Estuaries are recognized as regions where a river meets a sea, their waters being free to interact. Mixing, circulation, biological diversity, and chemical interactions have all been focal points in past studies. Reviews of basic estuary physics are found in Dyer (1973), McDowell and O'Conner (1977), and Officer (1976).

9. This chapter is concerned with the general aspects of estuarine behavior. Definitions, formation mechanisms, and classification schemes are all compiled from the above literature. Complete discussions of physical processes and interactions are saved for Chapter III.

### Definitions

10. Estuarine definitions have been proposed by a number of authors. Depending upon the particular author's background and interests, the definition may be based on such distinctions as geology, hydraulic behavior, biology, and chemistry. Since many of the studies have been conducted along ocean coasts, nearly all of these definitions include some consideration of the saltwater/freshwater mixing scenarios within the estuary. Consideration of these saltwater effects may be implicit, such as the biologist's concern of community diversity, to explicit, such as the hydraulicist's interest in buoyancy effects due to density gradients. Since this report is concerned with the physical hydraulic processes contributing to water quality, the following definitions are limited to the latter case.

11. Hansen and Rattray (1966) define estuaries as "regions of transition from river to ocean." They cite as

proper incorporation of the hydrodynamic mechanisms and the precise representation of the continuum must be done before any mixing representation can be successfully selected and used. Accomplishing these two preliminary considerations reduces error and permits the mixing model to be used as intended.

7. The following sections concentrate on delineating the status of various estuarine transport processes and their variability. The material in Parts II and III has been in part extracted from a report prepared by Bedford et al. (1983). Before the examination of the mixing models, both the governing equations and their relation to the physics must be examined.

example, Lewis and Platt (1982) document such efforts for large-scale flows, especially interactions with continental shelf circulation, while Bedford, Sykes, and Libicki (1982) demonstrate the influence of small-scale turbulence properties in determining the rate constants for water quality model source/sink terms. It is the belief of this author that proper model portrayal of hydrodynamics will reduce error in calculated water quality variables and also reduce the discrepancies between coefficient values necessary to make the answers "come out right" or compare favorably with theoretically expected values in the literature. Therefore, this report will concentrate on delineating physical transport processes and their model parameterization as regards turbulence and mixing.

#### Scope

5. If allowed, a thorough review of all mixing and dispersion relations would consume several years time; therefore, the scope of the review must be limited. First, although it is a most critical problem, this review will not consider any numerical methods. The misuse or incorrect application of numerical methods can result in serious errors. Some of the turbulence models to be reviewed here require very accurate numerical representation and when used permit very accurate answers obtained with theoretically expected coefficients. Secondly, it will not be possible to review every single mixing and turbulence parameterizations and corresponding coefficients. Therefore, the author has opted to present all the classes of functional forms and representative examples of their use.

6. In summary, mixing and dispersion models are to be reviewed in this document. By their empirical nature they have always been a source of constant controversy as regards both the proper functional forms and coefficient value. The

net circulation, with inflow in the lower layer and outflow in the upper layer, would result. Typical profiles are shown in Figure 2b.

40. The fjord type estuary is similar to the salt wedge type, except that the depth of the estuary is much greater. Because of the presence of the sill at the mouth, tidal flow is generally much less than freshwater flow, and the mixing involved would again result from entrainment of higher density lower water into the surface water layer. The big difference here is that in fjords, the freshwater depth is nearly constant throughout, being limited by the sill depth, so that the interface between the two waters is almost always horizontal. Partial homogeneity may arise, depending upon the volume of freshwater inflows. When this inflow is high, the maximum density gradient has been found to be at a lower depth than when the inflow is low (Dyer 1973).

41. Circulation over the fjord sill is yet another feature which distinguishes it from the salt wedge type. Since inflow comes from the coastal zone, the inflowing water may have a density less than that observed in the open ocean (but always greater than that of the fresh water). Also, if the sill depth is very shallow, inflow over the sill may be effectively curtailed, thus rejuvenation of the lower water may not take place, and it is possible for the lower layers to become anoxic. Figure 2c shows a typical fjord.

42. The partially mixed estuary occurs when a tide is allowed to interact with the freshwater inflow to the estuary. The introduction of tidal motion at the mouth causes the entire estuary to oscillate, and through the forces of viscous dissipation, the saltwater mixes upward, as in the salt wedge type. In addition, freshwater also mixes downward. This creates a situation where the net

density of the seaward flow will be increased as one progresses towards the mouth of the estuary, and undiluted fresh water will be seen only at the head of the estuary. Thus, horizontal salinity gradients will be observed at all depths within the well-mixed estuary.

43. Freshwater inflow in these estuaries is generally much less than the tidal volume. Because of this, and depending upon the specific relationship between the two in a particular estuary, certain circulation patterns will result, highly dependent upon the phasing of the tidal wave within the estuary. A detailed discussion of these phenomena is held until Part III. Figure 2d shows a typical partially mixed estuary.

44. The homogeneous estuary generally occurs when the cross section is small, thus enabling velocity shear to completely mix the water column (or section). These estuaries are characterized by a tidal flow which is much larger than the river flow.

45. In a vertically homogeneous estuary that is very wide, Coriolis or channel curvature effects may cause flow separation, such that seaward flowing water would be observed on one side of the estuary, with landward flowing water on the other. Thus, circulation would be in the horizontal rather than (or in addition to) the vertical direction.

46. If, however, the estuary is narrow, lateral shear may be sufficient to completely mix the waters across the width of the estuary. This would result in a laterally homogeneous or sectionally homogeneous estuary. Figure 2e shows a typical homogeneous estuary.

47. It must be noted here that no consideration has been given to estuarine systems where more than one of the above types of density structures may result. Obviously, if the topographic or bathymetric situations were of the proper

kind, several different density structures may arise within the same estuary. Thus, it may be that near the freshwater source, the estuary may exhibit characteristics of the salt wedge type, and, as one progresses down-estuary, gradually change so that as one nears the mouth, the estuary becomes more and more homogeneous. Thus, the above classifications are seen only as idealizations to what we may expect to see in the future.

#### Classification by parameter estimation

48. Several authors have attempted to classify estuaries according to quantities which could either be easily derived or observed. Schemes have been promoted which include the effects of freshwater inflow, tidal prism volume, density effects, and energy considerations. Although these are too numerous to detail in their entirety, some of the more popular techniques are examined herein.

49. One of the earliest attempts to parameterize estuaries was to define the flushing time and tidal prism of a particular estuary. The flushing time of an estuary is defined as the period of time it takes the freshwater volume to wash out of the estuary. The flushing time  $t$  is given by:

$$t = V_f / R \quad (1)$$

where  $V_f$  = volume of freshwater in the estuary and,  $R$  = river runoff rate.

50. The river discharge rate  $Q_D$  is given by:

$$Q_D = R / f \quad (2)$$

where  $f = V_f / V_{TOT}$ , and  $V_{TOT}$  = total flow volume of water in the estuary. Therefore,  $1/f$  represents a measure of the combined dispersion effects within the estuary.

51. If we define a tidal prism, denoted  $\Omega$ , as the volume of water entering the estuary over one tidal cycle, then we can write:

$$\Omega = \bar{Q}_T T \quad (3)$$

where  $\bar{Q}_T$  = an averaged tidal flow, and  $T$  = tidal period. If we assume complete mixing over the tidal cycle, then we can write, for the total flow volume,  $V_{TOT}$ ,

$$V_{TOT} = \Omega + V_f \quad (4)$$

where  $V_f = RT$ . This assumes that the total volume of water in the estuary remains unchanged when averaged over the tidal cycle, or that the total volume of water is the same at the beginning and end of the tidal cycle.

52. If some tracer exists (such as salt in the ocean environment), then an appropriate mass balance gives:

$$V_{TOT} \bar{S} = (\Omega + V_f) \bar{S} = \Omega \sigma \quad (5)$$

where  $\bar{S}$  is the average salinity in the estuary and  $\sigma$  is the average salinity in the ocean. Solving Equations 2 and 5 for  $\bar{f}$  yields:

$$\bar{f} = 1 - \frac{\bar{S}}{\sigma} = \frac{V_f}{\Omega + V_f} \quad (6)$$

Thus, the tidal prism and the flushing time are related by the parameter  $\bar{f}$ . The term  $\bar{f}$  represents the amount of energy available for mixing from both tidal and freshwater inflows.

53. Although these concepts are very simple and direct, it must be emphasized that complete mixing is assumed in the derivation. Since this is not the case in the majority of estuaries studied, caution must be exercised in application of the above concepts. Ketchum (1957) tried to circumvent this problem by assuming that an estuary can be divided into segments. In each section the assumption of complete mixing is made. Unfortunately, this assumption



is also of dubious validity, and the same cautions in applying Ketchum's work must be exercised as in the earlier development.

54. Simmons (in Ippen 1966), in a paper concerning field experience in estuaries, used the concepts of freshwater flow volumes and tidal prisms to place gross restrictions on estuarine mixing types. His observations led him to conclude that for the ratio  $V_f / \Omega \approx 1$ , the estuary is likely to be highly stratified; for  $0.2 < V_f / \Omega < 0.5$ , the estuary is likely to be partially mixed; and for  $V_f / \Omega < 0.1$ , the estuary is likely to be well mixed (or homogeneous). He also notes that, between these values, estuaries seem to exhibit characteristics of transition from one mixing type to another. These "transitional" estuaries seem to be more affected by changes in freshwater inflow than by changes in channel dimensions or configurations (Simmons 1966).

55. Other authors have promoted the use of a densimetric Froude number to indicate the effects of density stratification on shear-generated turbulence and mixing within an estuary. Among those who have approached the problem in this way are Pritchard (1955), Fischer (1976), and Officer (1976).

56. The densimetric Froude Number is defined as follows (denoted  $F_{Rd}$ ):

$$F_{Rd} = U_f / \sqrt{gh\Delta\rho/\rho} \quad (7)$$

where  $U_f$  = surface water layer velocity,  $h$  = water depth of surface layer,  $\Delta\rho$  = density difference between river and sea water; and  $\rho$  = density of lower layer. A value of  $F_{Rd} = 1$  indicates that a critical velocity of  $U_f$  has been reached; i.e., at  $F_{Rd} = 1$ ,  $U_f = U_{crit} = \sqrt{gh\Delta\rho/\rho}$ , which is recognized as the celerity of a gravity wave propagation along the density interface. At  $U_f$  crit, internal waves develop at

the interface and break, thus mixing the waters of the surface and bottom layers. For  $U_f < U_{f \text{ crit}}$ , waves do not develop, and hence mixing of the two waters would be less profound. For  $U_f > U_{f \text{ crit}}$ , turbulence would grow in proportion to  $U_f$ , and hence mixing would be more profound.

57. In determining  $F_{RD}$ , it is interesting to note that  $g$ ,  $\rho$ , and  $\Delta\rho$  are directly calculable from observable quantities. However,  $U_f$  and  $h$  are much harder to determine.  $U_f$  is defined as the surface layer water velocity. If this layer is assumed to be completely made up of river flow, then an average  $U_f$  could be determined from the river discharge rate,  $Q_d$ , divided by an average cross section area within the estuary. In general, however, the average cross sections in an estuary vary from point to point, and from time to time within the tidal cycle. Therefore,  $U_f$  could at best be a gross approximation.

58. Also, if the estuary was of a partially mixed character, then the surface water velocity would not be a simple function of the river discharge, but also of the entrainment volume from the lower layer. Thus, calculation of  $F_{RD}$  under these circumstances indicates that  $F_{RD}$  would only be an appropriate representation of mixing (or stratification) in the case of a truly stratified estuary, i.e., for the case  $F_{RD} < 1$ .

59. Also noted here is that the surface layer depth,  $h$ , is, in most estuaries, a fairly undeterminable quantity. We would like to say that, when using the Froude number approach,  $h$  has an average value over the entire estuary. However, arguments similar to those presented for  $U_f$  can be made. The term  $h$  is seen as not having a true average over the estuary, either in time or space.

60. Although severe, these arguments point out the difficulty in trying to assign an "average" number to represent the mixing (turbulent) properties in an estuarine

environment. A more detailed critique of averaging in the estuarine environment is presented in Parts IV and V.

61. Harleman and Abraham (1966) define an estuary number,  $E$ , and state that  $E$  represents the relationship between the energy available from the tidal activity and the energy available from the freshwater inflow. In the notation used herein, the estuary number is:

$$E = \Omega U_0^2 / g d U_f T \quad (8)$$

where  $U_0$  = peak flood velocity of the tide;  $d$  = estuary depth; and  $T$  = tidal period. In a later study, Harleman and Ippen (1967) state that the transition from strongly stratified to well-mixed estuaries lies in the range of  $0.03 < E < 0.30$ . This indicates that as the available tidal energy increases, given by the numerator (Equation 8) above, turbulence-generated mixing becomes more and more important, until the situation approaches that of a well-mixed estuary.

62. The Richardson number has also been proposed as a means of classifying estuarine behavior. In general, the Richardson number  $R_i$  is given by the following:

$$R_i = (g \partial \rho / \partial z) / \bar{\rho} (\partial \bar{u} / \partial z)^2 \quad (9)$$

where  $\partial \rho / \partial z$  = density change with depth;  $\bar{\rho}$  = average density in the estuary, and  $\partial \bar{u} / \partial z$  = average horizontal change with depth. Notice that in this general form, the changes of  $\rho$  and  $\bar{u}$  with respect to  $z$  have been written as partial derivatives since no assumption as to the functional relationship with other coordinate axes has been forwarded.

63. Using the concept of the Richardson number, Fischer (1976) proposed the Estuarine Richardson number,  $R$ , given by:

$$R = \Delta \rho g Q_D / \rho b U^3 \quad (10)$$

where  $U$  = rms tidal velocity; and  $b$  = estuary width. Thus, Fischer's  $R$  represents a consideration of both the effects of density stratification ( $\Delta\rho/\rho$ ) and the relationship between freshwater and tidal energies. Fischer contends that the transition from highly stratified to well-mixed estuaries will occur in a range of approximately  $0.08 < R < 0.80$ .

64. It is instructive at this time to examine the relationships between these various dimensionless numbers. Assuming that stratified flow is observed, and that the surface layer velocity is such that instability of the interface is imminent, the river discharge is:

$$Q_D = U_f b h \quad (11)$$

and  $F_{RD} = 1$  implies that  $U_f = \sqrt{gh\Delta\rho/\rho}$ . Substituting Equation 11 into Fischer's  $R$  (Equation 10):

$$R = \Delta\rho g (U_f b h) / \rho b U^3 \quad (12a)$$

$$\text{or } R = \Delta\rho g \sqrt{gh\Delta\rho/\rho} b h / \rho b U^3 \quad (12b)$$

or upon simplification,

$$R = \Delta\rho^{3/2} g^{3/2} h^{3/2} / \rho^{3/2} U^3 \quad (12c)$$

By taking  $R^{2/3}$ , it is recognized:

$$R^{2/3} = \Delta\rho g h / \rho U^2 \quad (13a)$$

or

$$R^{2/3} = (\Delta\rho/h) g / \rho (U/h)^2, \quad (13b)$$

which is the same form as the Richardson number,  $R_i$ , defined in Equation 9. By taking the square of the Froude number

(Equation 7):

$$F_{Rd}^2 = U_f^2 / (gh\Delta\rho/\rho) \quad (14a)$$

which, upon rearrangement, becomes

$$F_{Rd}^2 = (U_f^2/h^2) / g(\Delta\rho/h). \quad (14b)$$

Therefore, the functional relationship between  $R$ ,  $R_1$ , and  $F_{RD}$  is:

$$R^{2/3} = C_1 R_1 = C_2 (F_{Rd})^{-2} \quad (15)$$

where  $C_1$  and  $C_2$  are proportionality constants relating  $R$ ,  $R_1$ , and  $F_{RD}$ .

65. A similar argument leads to a representation of  $E$ , the estuarine number. If it is assumed that the tidal prism can be approximated by integrating the rms tidal velocity over the estuarine width, a characteristic depth, and the tidal cycle, then  $\Omega$  can be written:

$$\Omega = U b z T \quad (16)$$

where  $z$  is a characteristic depth associated with the tide. Furthermore, assuming that the peak flood velocity is some linear function of the rms tidal velocity, then

$$U_0^2 = C_3 U^2 \quad (17)$$

Substituting these expressions back into Equation 8 gives:

$$E = UbzTC_3U^2/gdU_fT \quad (18a)$$

or

$$E = bzC_3U^3/gdU_f. \quad (18b)$$

Therefore,  $E$  is of the same form as  $R$ ,  $R_i$ , and  $F_{Rd}$ , only that density effects have been ignored.

66. Other relationships which have been proposed, similar to the above, are Abbot's (1966) and Ippen's (1966). Abbot proposes that the number  $K$ , given by:

$$K = (gh)^{3/2} \partial \rho / \partial x / \sigma U_0^2 \rho \quad (19)$$

where  $\sigma = 2\pi/T$ , the tidal frequency, could be used to determine the longitudinal effects of density gradients in the flow field. His work suggests that large values of  $K$  indicate circulation patterns determined largely by the effects of density gradients in the flow field. Small  $K$  indicates that the nonlinear convective effects would dominate the physics of the circulation.

67. Ippen suggests the use of a stratification number, given by  $G/J$ , where:  $G$  = rate of turbulent energy dissipation and  $J$  = gain of potential energy of the flow field. It is recognized that both  $G$  and  $J$  are difficult, if not impossible, parameters to measure. However, if it is assumed that turbulent energy dissipation is proportional to some power and a characteristic velocity in the flow field, and that gain of potential energy is proportional to the density times some water level elevation change, then the stratification number is seen as one more representation of the combined effects of flow and density fields.

68. Hansen and Rattray (1966) attempt to combine the effects of stratification and circulation by means of a gross parameterization in a stratification/circulation diagram. This scheme divides estuaries into three classes or types, listed below:

Type I: the net estuarine flow is seaward at all depths;

Type II: there is a flow reversal at depth, such that above a particular depth the net flow is seaward, and below that depth the net flow is landward;

Type III: density effects dominate the circulation pattern, such that the majority of the circulation is gravity induced.

These three types are subsequently divided into subtypes, according to the relative degree of stratification associated with each.

69. This work led them to conclude that single parameter representation of estuarine physics is at best a gross approximation. The study shows that most estuaries cannot be represented by a point (single-parameter), but that most estuaries are represented by a straight line on their diagram. This implies that an estuary cannot be classified as a single type, but that type actually changes from point to point within a particular estuary. Indeed, change in the physical parameters (such as river inflow) has the same general effect as change in position along the estuary (Hansen and Rattray 1966).

70. It must be noted at this point that since most of the work done on estuaries is concerned with the coastal-plain types (as these are the most prevalent in the highly technological Northern Hemisphere), most of the verifications of the parameters proposed have been limited to these particular estuaries. It is unknown to this author (at this time) whether any work has shown these parameterizations to be adequate for other types of estuaries, in other topographic regions. A compilation of these schemes can be found in Table 1.

71. As alluded to above, it must be recognized that none of these parameterizations consider topographical or bathymetric effects. It has been shown by several authors (Kjerfve 1978, Stommel and Farmer 1953, Kjerfve and Proehl 1979, Murty et al. 1980, Elliot et al. 1978, Ianniello 1977) that indeed topograph and bathymetric influences are

Table 1. Summary of estuary classification schemes

Parameter Name	Definition	Reference to Text	Original Reference
Flushing Time	$f = 1 - \frac{\bar{S}}{\sigma} = V_f / (\Omega + V_f)$	Eq. 6	
Inflow vs. Tidal Prism	$V_f / \Omega = 1$ stratified $0.2 < V_f / \Omega < 0.5$ partially mixed $F_f / \Omega < 0.1$ Homogeneous		Simmons (1966)
Densimetric Froude Number	$F_{RD} = U_f / \sqrt{gh\Delta\rho/\rho}$ $F_{RD} \geq 1$ stratified $F_{RD} < 1$ mixed	Eq. 7	Pritchard (1955) Fischer (1976) Officer (1976)
Estuary Number	$E = U_0^2 / (gdU_f T)$ $E > 0.3$ mixed $E < 0.3$ stratified	Eq. 8	Harleman and Abraham (1966)
Richardson Number	$R_i = (g\Delta\rho/\rho z) / (\overline{\rho}(\partial\bar{u}/\partial z)^2)$	Eq. 9	

Continued



Table 1. Concluded

Parameter Name	Definition	References to Text	Original Reference
Estuarine Richardson Number	$R = (\Delta \rho g D_g) / \rho b U^3$	Eq. 10	Fischer (1976)
Stratification Number	$G / \beta$		Ippen (1966)
Stratification/ Circulation Diagram		Fig. 3	Hansen and Rattray (1966)

important in determining estuarine circulation and mixing patterns. Their exclusion from the above formulations prohibits a general application of these concepts. Also, since only density and flow effects have been considered, other systems which may be affected by omniscimatic conditions, such as large-scale pressure systems or surface wind stress, cannot be adequately described by such parameterizations.

72. Therefore, although geological descriptions provide for adequate delineation of estuaries, they provide no indication of the physical flow structure, mixing, or circulation patterns. Also, although the parameterizations mentioned do provide some indication of estuarine physics, their constraints must be critically reviewed before application to a particular estuary.

73. There is no clear-cut classification scheme which sufficiently describes all of the various aspects of estuarine behavior or a priori indicates clearly the importances of the various processes.

108. Within the Great Lakes intense changes in wind shear aided by unstable air/lake temperature differences (Schwab 1978) propagate across the lake surface during the passage of weather systems. For Lake Erie, which is oriented in a southwest to northeast position, the direction of dominant storm tracks is down the longitudinal axis of the lake and very intense surges with amplitudes as high as 3 meters (Hamblin 1979) can occur at the downstream boundary of the freshwater estuary adjacent to the lake. Rao (1967) explained the occurrence of such surges as a resonance phenomena wherein the wind shear front travels with the same speed as the average gravity wave speed of the lake. It should be pointed out that this effect is attributed to the Great Lakes and its possible effects in other estuaries have not been investigated. After passage of the shear and release of the surge, the seiche die-off as Kelvin waves causes tidal-like flushing of the adjacent river with long period 14 hour water level oscillations.

109. Dingman and Bedford (1984) have also documented the only observed case of pressure suction effects in the Great Lakes. These effects occurred during the cyclone of 1978 when a 28.0-in. Hg low passed over Lake Erie.\*

#### Combined meteorological and boundary effects

110. Further evidence for the role of meteorological events on estuarine flows is found in the works of Elliot (1978) and Elliot et al. (1978). These works examine the circulation within the Chesapeake Bay and the adjacent Potomac River Estuary. Current speeds at 7- and 15-ft depths, wind data, and river inflow (Susquehanna River) were collected for a position near the head of Chesapeake Bay (Elliot et al. 1978). In addition, surface pressure and water level elevations were collected at various points within the Potomac River (Elliot 1978). Elliot et al. (1978) indicate that for the Upper Chesapeake Bay, 75% of

\* A table of factors for converting US customary units of measurement to metric (SI) is presented on page vi.

with special emphasis made on other studies which indicate similar results.

#### Meteorological forcing functions

105. Meteorologic effects have been examined in a variety of estuaries. The studies indicate that the effects of weather systems range from barotropic, such as sea level variability due to surface pressure gradients, which affects water level elevations and hence hydraulic flow through the estuary mouth (N. Smith 1978), to baroclinic, such as the influence of wind stress on near-surface currents (Elliot 1978).

106. N. Smith (1978) shows that the effect of surface pressure gradients ( $\Delta P$ ) is to produce observable water level deviations from that predicted by the tidal activity. His study covers a 185-day period in which surface pressure and water levels are observed at Corpus Christi, Texas, and Tampa, Florida, in the Gulf of Mexico. Because tidal ranges in the Gulf are small (approx. 0.75 m), the effect of surface  $\Delta P$  is most evident. This study indicates that for periods of 2-6 days, coherence between surface  $\Delta P$  and water levels is high. Although coherence at 8, 12, and 24 hr is also high, Smith attributes this to simultaneous oceanic and atmospheric tides, and thus not a meteorological effect. He gives an approximate relationship of -1 cm/mb between surface P and water level, thus indicating the importance of this input on sea level boundary conditions.

107. In a companion study, N. Smith (1977) examines the response of Corpus Christi Bay current velocities to surface P in the Gulf. He finds that the dominant ebb and flood velocities at the mouth of the bay exhibit periodic fluctuations on the order of 305 days. Although exact correlations are not presented, the implication of the effects of surface P on the estuary flow field is apparent.

observations agree closely with Kolmogoroff's (1941) theory of inertial subrange turbulence, with the spectra agreeing closely with Kolmogoroff's predicted slope of  $k^{-5/3}$  in the inertial subrange.

102. The companion study of Grant and Moilliet (1962) arrives at similar conclusions concerning the cross-channel turbulent component. They note, however, that the flow field is highly anisotropic, such that correlations between longitudinal and cross-sectional mixing are not determinable. They also note that much of the data do not agree with Kolmogoroff's predicted slope, but that during many of the observations, cross-channel Reynold's numbers were sufficiently low to pre-empt Kolmogoroff's theory.

103. These studies indicate the necessity for a better understanding of the turbulent structure of estuarine flows. Additional observations and theory are presented in the following sections. A concise presentation in text of many of the above and other early studies can be found in Officer (1976). The reader is directed to this book for details.

#### Review of Recent Studies

104. The last eight years (1975-1983) have been very productive times for the analysis and understanding of estuarine transport. Many studies have been conducted which indicate the importance of various large- and small-scale activities. These range from the influence of river inflow and meteorological activity on the overall long-term (residual) circulation to the importance of channel curvature and estuary size and configuration on combined dispersion/diffusion effects. Although these studies are too numerous to review in their entirety, several key features from the more insightful studies are presented,

term effects due to wind and surface pressure gradients. He notes that lack of significant stratification, although not a formal requirement, is implied, since significant stratification tends to inhibit complete mixing. When the criteria are met, however, this simple methodology may be appropriate for engineering approximations.

98. The second approach is to assume average dispersion/diffusion coefficients, and to use these in a numerical or analytical study. These studies are often supplemented with field observation, laboratory flume analysis, and/or dye tracer studies. A more detailed discussion of these approaches is presented later.

99. Although circulation and mixing effects are explicitly considered in many early studies, the turbulent structure of the flow, although recognized, is often relegated as a small-scale effect, and not treated explicitly. Turbulent activity is implicitly filtered from the other activities by virtue of the temporal and spatial resolution of these early studies.

100. The effects of turbulence are seen in the reduction of local velocity and density gradients, sediment resuspension, and combined dispersion/diffusion effects. The mechanisms responsible for turbulence are not well understood. Turbulence theory has arisen from the observation of effects rather than the delineation of causes.

101. Two early studies of turbulence in tidal streams are those of Grant et al. (1962) and Grant and Moilliet (1962). These examine the one-dimensional spectra of turbulence in longitudinal and lateral directions. Grant et al. (1962) observed that, in general, at high Reynold's number ( $Re=10^8$ ), the time scale associated with the turbulence generally decreases with increasing wave number,  $k$ , where  $k = 2\pi/\lambda$ , and  $\lambda$  = wavelength. They note that their

of energy). This effect is most apparent near solid boundaries, where velocity gradients can be quite large (Ketchum 1957, Abbott 1960). Channel transitions, curvature, etc., can also produce similar effects (Stewart 1957; Stommel and Farmer 1953).

95. Two other mechanisms which promote mixing are referred to as "trapping" and "pumping" (Okubo 1973). Trapping refers to the effect of small side embayments, which periodically fill and empty with the rising and falling tides. Pumping refers to the interaction of the tidal wave with the bathymetry. These interactions are nonlinear, thus promoting advective transport within the estuary. More will be said about these effects later on.

96. The mixing of waters within an estuary results from the interactions of river input, tides, winds, and bathymetry. Because of the complexity of these interactions, a consistent evaluative approach is not available at this time. Instead, analysis of mixing usually progresses along one of the two lines of thought. One procedure assumes complete mixing within sections with net dispersion resulting from mean flows (Arons and Stommel 1951; Ketchum 1957). Ketchum assumes complete mixing at a cross section, and that mean motion can be determined from the river discharge. The movement (dispersion) of any pollutant is then calculated as an average over the tidal cycle. Stommel and Farmer (1953) suggest that in this procedure, the estuary may be divided into segments, each as long as the tidal excursion, in which complete mixing is assumed. Conservative pollutants are easily handled by this procedure. Nonconservative pollutants require the inclusion of some type of decay/growth coefficient.

97. Fischer (1976) points to several difficulties in applying these concepts to real systems. Most notably is nonconsideration of tidal cycle fluctuations, and longer

discussed earlier. Longitudinal pressure gradients result from the density distribution and the surface slope.

91. Surface stress from wind also adds to the inter-estuarine mixing. Wind speed, direction, duration, and estuary size and configuration are all factors to consider when examining the wind-induced mixing.

92. The effects of wind are twofold. Firstly, wind produces waves, the characteristics (period, height, etc.) of which are a function of wind speed and direction. Waves exhibit some type of orbital motion, which helps to promote vertical mixing. The extent and effectiveness of the mixing depends on wave height and water depth. Functional relationships for wind induced waves can be found in Volume I of the Shore Protection Manual (U.S. Army Coastal Engineering Research Center 1977). Csanady (1975) notes that in deep lakes, vertical mixing from wave fields is highly dependent on ambient stratification. Fischer (1976) notes that this may also be true in estuaries, although the complex nature of estuarine interactions precludes any analytical solution.

93. Secondly, currents develop in the surface layers under the influence of a sustained wind. Along open coasts, these currents may be quite strong. The interaction between the current and the tide produces dramatic alterations in observed tidal amplitudes (Dronkers and Schonfeld 1955). This superposition is quite complex, not lending itself to a simple analysis. Therefore, none will be presented herein, recognizing that a careful analysis on a case-by-case basis should be attempted to determine if these effects are important in a particular area of interest.

94. Bathymetry also affects local mixing through non-linear interactions with the flow field. Local velocity gradients promote exchange between streamtubes, such that the system approaches an equilibrium situation (conservation



activity. A critical freshwater flow velocity is defined when the velocity of the surface layer is equal to the celerity of a gravity wave travelling along the interface. This is given by:

$$V_{f \text{ crit}} = \sqrt{gh\Delta\rho/\rho} \quad (21)$$

where  $V_{f \text{ crit}}$  is the celerity;  $g$  is the gravitational constant;  $h$  is the depth of the surface water layer;  $\Delta\rho$  is the difference in density between the surface and lower layers; and  $\rho$  is the density of the lower layer. Defining the freshwater flow velocity  $V_f$  as:

$$V_f = Q_R/A_{\text{sur}} \quad (22)$$

where  $Q_R$  is the river discharge rate and  $A_{\text{sur}}$  is the surface layer cross-sectional area, allows comparison of  $V_f$  and  $V_{f \text{ crit}}$ . When  $V_f > V_{f \text{ crit}}$ , internal waves break along the interface. Water from the lower layer is entrained, or trapped, in the seaward flowing surface layer. The surface layer thus gains mass in the down-estuary direction, a net upward flux occurs to supply this mass; and a net inflow develops in the lower layer to compensate for mass lost to the surface layer. Thus, the traditional circulation pattern envisioned by Pritchard is established.

90. Tidal activity also supplies energy to promote internal mixing. This arises both from increased shear, associated with the sliding wedge under the surface layer, and with increased vertical velocities, associated with the rising and falling tides. The combined effect of tides and freshwater inflow can be so intense as to completely disrupt the stratification. In such a case, nearly homogeneous density can be observed in the vertical, with density gradients in the longitudinal and lateral direction only (Figure 3d). Such is the case of the well-mixed estuary

the most profound changes correspond with the longitudinally directed winds. Seaward directed winds tend to reduce the observed salinity in the surface waters. Landward directed winds have the opposite effect. Barlow attributes these observations to surface current generation and increased vertical mixing associated with the wind. He notes that the wind effects are most apparent when the salinity is examined on a seasonal basis.

87. Topography and bathymetric changes are also recognized as significantly affecting estuarine circulation. Pritchard (1957) attributes current deflection in the James River as resulting from a balance between Coriolis force and lateral shearing stress. Stewart (1957), in a note on Pritchard's work, indicates that the observed deflection can be attributed to the channel curvature. The current deflection results from centrifugal acceleration attributed to the meandering channel. Stewart's calculations indicate that the observed deflection can be almost entirely accounted for as resulting from the channel meander.

88. The effect of mouth constrictions is examined in Stommel and Farmer (1953). The theoretical analysis of conditions at the mouth indicates that there is a point where mixing is so complete that no further reduction in the density structure is possible. They call this condition "overmixing," making special note that this can only occur in estuaries exhibiting some type of hydraulic control at the mouth. "Hydraulic control" refers to some type of inlet constriction. Long wave propagation through the inlet in this case needs to be considered.

89. Mixing and dispersion in estuaries has also been approached by a number of authors. One of the most discussed theories of estuarine mixing deals with density-induced mixing across the interface. Shear at the interface develops in proportion to freshwater discharge and tidal

83. Increasing the tidal amplitude results in even more enhanced exchange between the waters of the two layers. A point is reached where this exchange is so intense that the stratified nature of the estuary is completely disrupted (see Part II). In such a case, the estuary is said to be well mixed. Density is fairly constant with depth. However, with regard to Figure 3d, longitudinal gradients are present. Density increases in the down-estuary direction. Velocities are unidirectional, landward with the rising tide and seaward with the falling tide.

84. The above analysis (after Pritchard 1955) is very simplified. Only the effects of river inflow, density gradients, and tidal activity are considered. Pritchard notes that a particular estuary may exhibit one or more of these characteristic circulation scenarios. The transition from stratified to well-mixed conditions can be seen with decreased river inflow, increased tidal velocities, increasing width, and decreasing depth. Hansen and Rattray (1966) expand on Pritchard's classifications by considering the combined effects of stratification and circulation. Their criteria were examined in Part II.

85. To this point, circulation has been approached as resulting from the interactions of tide, river inflow, and density gradients. Obviously, other influences within the estuary may contribute to the overall circulation pattern. These include the effects of weather (wind and pressure), bathymetry, and topographic (channel convergence, inlet damping, etc.) transitions. Few early works examine these influences in detail. Some that do are discussed below.

86. Barlow (1956) examines the effect of wind speed and direction on the salinity distribution in Great Pond, Woods Hole, Massachusetts. Although tidal action and freshwater inflow are observed to affect salinity distributions,

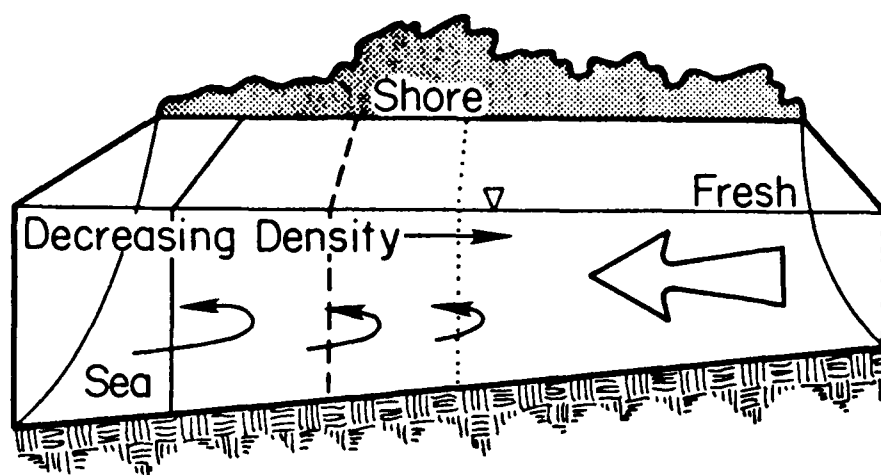
estuary. Opposing pressure gradients now develop at depth. In the surface layer, pressure distributions at depth result from the mean surface slope; below the density interface, the slope of the pressure gradient changes due to the internal slope of the density interface (Figure 3b). As can be seen from Figure 3b, a net circulation results from these pressure gradients. Net outflow (seaward) is observed in the surface layer with net inflow (landward) in the lower layer.

81. The introduction of tides in the sea results in a more realistic situation, as most seas experience tidal, or similar, activity at some time or another. Tides add significant momentum input due to the hydraulically induced flow, and also affect the mean pressure distributions due to the changing elevations at the estuary mouth. The overall affect depends on the tidal range and the temporal characteristics of the tide.

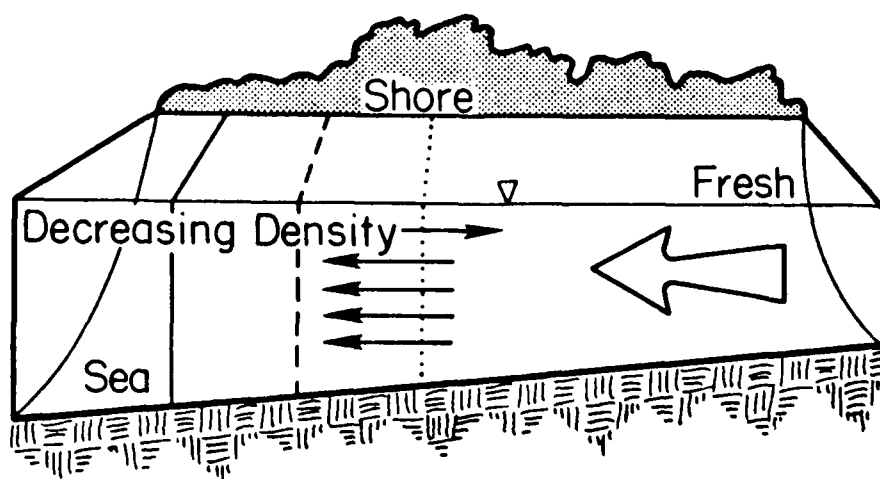
82. If the tide is small to moderate in amplitude, the wedge tends to move up and down the estuary with rising and falling tide, respectively. Increased momentum associated with the tide results in increased interfacial shear resulting in an enhanced exchange of waters between the two layers. The exchange increases are due to the more energetic interfacial wave breaking associated with the tidal activity. As the lower seawater layer moves landward, the effective velocity (vector sum) between the two layers increases. The Newtonian stress relationship:

$$\tau = \mu \frac{dv}{dy} \quad (20)$$

shows the shear to be directly proportional to the velocity gradient. Pritchard (1955) observes that increased exchange enhances discharge through the surface layer by as much as 40 times the river inflow (Figure 3c).

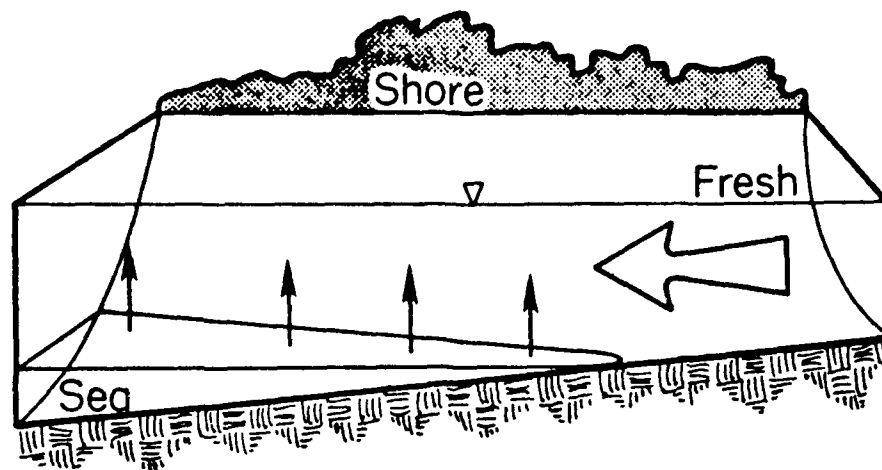


c. Type C estuary

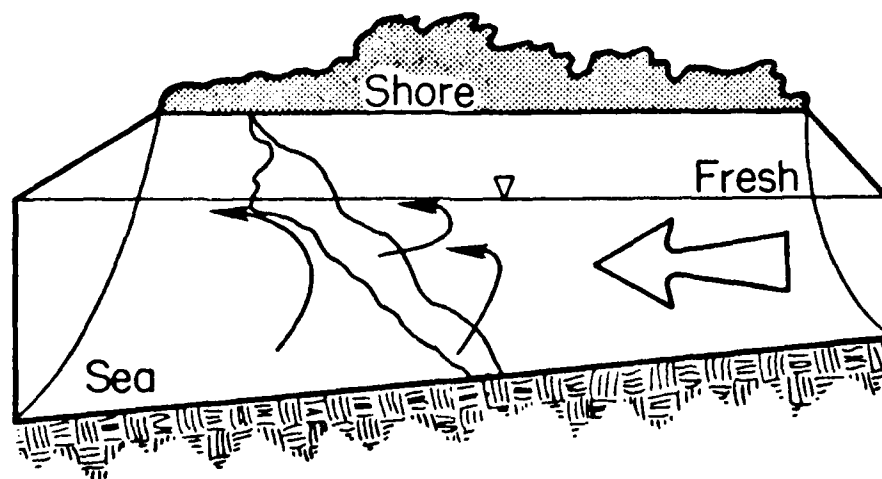


d. Type D estuary

Figure 3. (Concluded)



a. Type A estuary



b. Type B estuary

Figure 3. Pritchard's estuarine circulation patterns  
(Continued)

such intercomparisons and are widely known. Methods of digital signal analysis and spectral analysis are also used. Since these methods will be used in the following discussions, Appendix A has been included to provide the basic elements of digital signal processing and spectral analysis.

77. Early estuary research is concerned mainly with the overall, or average, flow or circulation patterns. Circulation patterns are distinctive, developing in a particular estuary as a function of energy input, bathymetry, and nonlinear interactions within the estuary over a variety of time and space scales.

78. One of the earliest studies is by Pritchard (1955), who identifies four characteristic estuarine circulation patterns. The following discussion comes from this work and the reader is referred to Figures 3a-3d.

79. Assume a typical estuary in which river inflow is small, the sea is tideless, the seawater is of a higher density than the river, and meteorological effects are ignored. Although ideal, this provides a clear starting point for discussion. Energy input in this situation results from the river inflow. A wedge of more dense sea water forms under the seaward flowing freshwater layer. The up-estuary intrusion of the seawater reaches the point where the bottom depth equals the mean sea level. Flow velocity in the surface layer is expressed as a continuity relationship, equal to the river discharge divided by the surface layer cross-sectional area. In a tideless sea, the freshwater/seawater interface is nearly horizontal. Figure 3a portrays this situation.

80. As the river inflow increases, interfacial friction increases between the seawater and freshwater layers. This tends to decrease the stability at the interface, and to move the up-estuary intrusion point down-

### PART III: ESTUARY TRANSPORT PROCESSES AND VARIABILITY

74. Studies of estuarine hydraulics are severely complicated by several characteristic features of the flow field. Three key features are identified in the literature as circulation, mixing, and turbulence. The time and space scales associated with these range from large to small, respectively. Segregation of these features, however, is not easy; for example, what is identified as mixing in one work may be called turbulence in another. Only the examination of the particular temporal and spatial resolution associated with these activities lends coherence to the many studies thus far conducted.

75. This chapter attempts to clarify some of these concepts. First, a general overview of some early studies is presented. These examine some of the more apparent mechanisms responsible for observable estuarine transport. Theories are presented to explain the various mechanisms acting within the estuary, and general features of estuarine motion are identified. Secondly, a review of recent studies examines results indicating the importance of several forcing and response mechanisms not explicitly considered in many of the early works. The influence of these mechanisms on transport is presented in both theory and observation. Finally, after a discussion of turbulence, a summary of the temporal and spatial attributes of key features in estuarine flow fields is presented.

#### Overview of Early Studies

76. Methods for presenting and intercomparing the physical processes of estuary transport must at minimum specify the length and time scale, the various magnitudes of each process, and the relative importance. Methods of variance and nondimensional analysis are important tools in



the total current energy occupies time scales longer than 3 days. They attribute this to local winds, hence a baroclinic activity. At time scales longer than 5 days, the importance of the Susquehanna River discharge is indicated.

111. The companion paper (Elliot 1978) comes from a 1-year study in the Potomac Estuary. It indicates that the low-pass (nontidal) currents have fluctuating components at periods of 2-5 days. Local winds account for nearly 55% of the total record variance. This is interpreted as a baroclinic response. However, of almost equal importance is the barotropic response due to interactions at the mouth of the Potomac with the Chesapeake Bay. The authors note the fluctuating nature of the residual current and the seemingly large standard deviations associated with many of the measured variables. These indicate high variability in the estuarine flow patterns, even at very long intervals when compared with the tidal activity. The inability to accurately determine the estuarine sequence is apparent.

112. From this study Elliot identifies six circulation types in the Potomac River. This study supports the earlier observations of Pritchard (1955) and Hansen and Rattray (1966) concerning the changing nature of estuarine circulation patterns.

113. Kjerfve et al. (1978) found that the response of North Inlet, S.C., displayed characteristics similar to those noted in the Potomac River. Kjerfve cites Weisberg (1976), who shows that 48% of the current variance in a 51-day record can be attributed to activity which is subtidal in frequency. Weisberg contends that this variance can be attributed to the local winds, which are periodic at approximately 4- to 5-day intervals. This and the studies mentioned earlier indicate that meteorological events can

explain much of the flow variability noted in estuaries that are subtidal in frequency.

#### Bottom topography

114. In addition to meteorological influences, recent studies have indicated the importance of bathymetry in net circulation and mixing processes. The major response examined herein is the influence of tidal wave interaction with bottom topography as it affects the residual current structure. This is the pumping mechanism alluded to above.

115. Kjerfve (1978) examines bottom topography in North Inlet, S.C., concluding that bathymetry may well indicate overall estuarine circulation patterns. Although his study lacks quantitative arguments, Kjerfve points to several qualitative aspects of the estuary, such as the development of separate flood and ebb channels, as substantial proof of his contention.

#### Residual currents, mixing, and diffusion

116. The distinction between residual currents, mixing, and dispersion gets particularly fuzzy in the context of a model. Simply, residual currents are what is left over after the tidal effects have been filtered out. Probably the most quantitative treatment of the role of tidally induced residual currents can be found in the works of Ianniello (1977) and Murty et al. (1980). In both studies, the effects of these currents are examined through the use of numerical models. Output with and without the inclusion of these advective terms is compared.

117. Ianniello (1977) notes that the inclusion of these nonlinear effects becomes important when the residual velocities approach those of other dominant mechanisms, "most notably those due to density gradients and river discharge." He shows that for a typical estuary, these currents can be of the order of  $10 \text{ cms}^{-1}$ . This could constitute a significant portion of the mean flow. He

further states that it is important to consider both breadth and depth variations if these currents are to be adequately predicted.

118. Murty et al. (1980) lend further support to the importance of these currents in the mixing and dispersion of contaminants. Inclusion of these terms in their study causes surface contaminants to penetrate and spread more through the estuary over the tidal cycle. It is interesting to note that the inclusion of the Coriolis force has little, if any, effect on the calculations. One study is, however, no basis to suggest that these effects may be neglected in dispersion studies. Murty's situation is, after all, an idealized one, and by no means representative of true estuarine behavior.

119. Often studies conducted in estuaries are prompted by the desire to understand pollutant transport, sediment resuspension, and similar mass species phenomena. Although these studies are too numerous to review in detail, some general considerations are presented herein.

120. Diffusion is a process controlled by gradients on the molecular level. In quiescent flow, diffusion may be the dominant mechanism responsible for the reduction of concentration gradients. In turbulent flow, however, diffusion processes are significantly overshadowed by convective transport activity. As most natural, and most certainly estuarine, flows are turbulent, molecular diffusion can be assumed negligible, such that the convective processes dominate dispersion effects. In traditional field studies, little regard for model structure or use is taken.

121. Evaluation of mass dispersion has traditionally been handled by the definition of an overall dispersion coefficient. This is usually expressed as a Fickian-type equation (McDowell and O'Connor 1977) such that:

$$U_x C = E_x (dC/dx) \quad (23)$$

where  $U_x$  is the mean velocity in the x-direction;  $C$  is the mean concentration;  $dC/dx$  is the concentration gradient in the x-direction; and  $E_x$  is the combined diffusion/dispersion coefficient. Similar equations lead to the definition of  $E_y$  and  $E_z$ .

122. Unfortunately, the estuarine environment is not so simple that realistic mean values for  $U_x$  and  $C$  can be determined. This arises from the fluctuating nature of the flows. The decomposition of the flow field into mean and fluctuating components results in the formation of several cross-product terms. After McDowell and O'Connor (1977) these cross products can be expressed as the form:

$$-\overline{u'c'} = E_{xx} d\bar{c}/dx + E_{xy} d\bar{c}/dy + E_{xz} d\bar{c}/dz \quad (24)$$

where variables are as previously defined, the overbar ( $\bar{\quad}$ ) denotes average quantities, and the prime ( $'$ ) denotes fluctuations from the mean. Equation 24 is a 3-D representation of Equation 23. Similar equations can be obtained for components of the y- and z-directions. McDowell and O'Connor (1977, Chapter 3) present a discussion on these coefficients.

123. Most appropriately, estuarine dispersion studies should consider the full 3-D flow field. As this is a tedious, if not impossible task, assumptions reducing the dimensionality of the equations are often proposed. These approximations are discussed in detail in Part IV of this report.

124. Studies to determine dispersion characteristics of estuaries have been conducted in both in situ and in the

laboratory flumes. Fischer (1976) cites the U.S. Army Engineer Waterways Experiment Station (WES) and Delft Laboratory flume experiments as dominant in their class. Both experiments show that longitudinal dispersion in tidal channels is proportional to some form of the gradient Richardson number ( $\Delta\rho g Q_f / \rho b U^3$ ). Longitudinal dispersion is recognized as having dependence on mean flow ( $u$ ), density gradients, freshwater discharge ( $Q_f$ ), and some mean length scale ( $b$ ). The major drawback to these studies is that they were conducted in straight rectangular channels. The direct applicability of these experiments to natural channels is therefore questionable.

125. In situ measurements show a wide range of dispersion coefficients for all three coordinate directions. Table 2 gives typical observed values for longitudinal dispersion coefficients. Fischer (1976) also presents excellent discussions on the observed variability in longitudinal, transverse, and vertical directions. The interested reader should consult this reference for details.

126. Recently, Wilson and Okubo (1978) have re-evaluated data collected from a dye release study conducted in the York River, Chesapeake Bay, in 1960. They neglect lateral variations in their analysis, and conclude that longitudinal dispersion is significantly affected by vertical shear effects. The dye was initially released on slack tide before the flood. It is interesting to note the similarities in their observations with the predictions of Murty et al. (1980).

127. Neglecting lateral variations may lead to erroneous results, as suggested in Fischer (1976) and supported by Smith (1977). Smith approaches this problem mathematically and shows that for a narrow estuary, lateral shear can affect longitudinal dispersion much more than vertical shear. He notes that the results of any study are

Table 2. Observed longitudinal dispersion coefficients in various estuaries (after Fischer et al. 1979)

Estuary	Value or Range of Dispersion <sup>2</sup> Coefficients (m <sup>2</sup> /sec)	Original Source	Comments
Hudson River	160	Thatcher and Harleman (1972)	Values may be high due to the nature of the formulation
Rotterdam	280		
Potomac Delaware	55 5-15(10 <sup>2</sup> )		
San Francisco Bay	200	Cox and Macola (1967)	Value used in numerical model
Delaware	100	Paulson (1969)	
Potomac	20-100	Hetling and O'Connell (1966)	Value computed from dye release experiment
Mersey (England)	160-360	Bowden (1963)	

transferable to other estuaries only if topography is similar. Also, the presence of dominant salinity effects requires the Richardson number to be identical.

128. Other studies (Murray and Siripong 1978; Dyer 1973) have shown similar results. The interested reader is directed to these works for details.

129. Energy dissipation in estuaries is mathematically analogous to dispersion. Energy dissipation in most natural flows results from the turbulent interactions within the flow field, and from boundary shearing stresses. Analytic techniques require determination of the turbulent characteristics of the flow field. Since these characteristics are highly variable in time and space, exact determinations are not possible. For example, Heathershaw (1976) observes that 60% of the Reynold's stress occupies 10% of the time.

130. Brown and Trask (1980) have examined tidal energy dissipation in a shallow, well-mixed estuary. They use one-dimensional momentum and mass balance formulations to arrive at the following representation of the bottom stress:

$$[\tau_b] = -\rho h \frac{\partial [u]}{\partial t} + \frac{\partial (u^2/2)}{\partial x} + g \frac{\partial \eta}{\partial x} \quad (25)$$

where  $\tau_b$  is the bottom stress,  $\rho$  is the mean density,  $h$  is the mean depth,  $g$  is the gravitational constant,  $u$  is the mean cross-sectional velocity,  $\eta$  is the surface elevation, and the brackets  $[ ]$  indicate an integration over the longitudinal distance,  $x$ . Furthermore, energy dissipation is calculated by:

$$[\phi] = [\tau_b][u] \quad (26)$$

where  $[\phi]$  is the area-averaged dissipation rate per unit area.

## Comments on Turbulence

### Eddies

131. The mixing field studies referred to in the previous subsection address one small aspect of the modeling problem discussed here. The distinction between turbulence, mixing and dispersion will become clearer after discussing the model structure. It is necessary to discuss what current thinking is on just what is turbulence. This section is not definitive in that only highlights of modern concepts are discussed. The fine articles by Woods (1977) and Nihoul (1978) serve as an excellent review. This section also takes material from Svensson (1981), Krauss (1980), Tennekes and Lumley (1972), Hinze (1975), and Bedford (1981). The reader is directed to these sources for more in-depth treatment. Estuarine turbulence is found in three layers: (a) the surface layer dominated by wind created surface waves which is a highly agitated layer consisting of velocity shear created turbulence; (b) a similar shear dominated bottom layer containing high vorticity and turbulence; and (c) an internal layer in which turbulence appears intermittently as patches. These intermittent patches result from the nonlinear interaction of various trains of internal waves which, as a result of increased shear and reduced Richardson Number, cause Kelvin Helmholtz instabilities. Breaking internal waves also cause such internal layer turbulence patches to occur. Therefore, whereas turbulence was thought to be chaotic or random it was at least supposed to be uniformly distributed within the flow field. Newer intermittency and wave-wave interactions abrogate this picture. Woods (1977) and Nihoul (1978) review these features in great detail.

132. According to Woods the distribution of turbulence is thought to consist of eddies with scales that



continuously and nonlinearly range from the largest scales of the problem (i.e. kilometers and days) to sizes governed by viscosity (i.e. centimeters and seconds). Generally small eddies have short lifetimes and large eddies have large lifetimes. Using Wood's (1977) distinctions, the following classifications are made. The largest eddies occur in the region where the Rossby No.,  $R_0$ , is less than unity. The variable  $R_0$  is defined as in Table 3, and these eddies are strongly influenced by the earth's rotation. Next smaller in size are eddies occurring in the range where  $R_0$  and  $R_i$ , the Richardson number, are greater than unity. These eddies are in the buoyant range. Eddies with  $R_i$  values much less than unity are divided into three classes: Kelvin Helmholtz billows; eddies with  $R_i < 1$  and  $R_e > 1$  which correspond to the inertial subrange turbulence associated with the Kolmogorov inertial subrange spectrum; and viscous subrange eddies.

133. Nihoul, using a wave-wave interaction model, develops the following model of turbulence at large scales. A dispersion relation relating the wave frequency,  $\omega$ , to the wave number,  $k$  ( $k = 2\pi/\lambda$ ,  $\lambda$  = wavelength), is established and, after application to the equations of motion and accounting for harmonics and modes, a dispersion band is established. Nonlinear interactions are displayed by an interaction band in the  $\omega - k$  plane by accounting for velocity effects in the dispersion. Whenever an interaction band and a dispersion band cross, one expects turbulent eddies at the time and length boundaries of the intersection. Turbulent eddies then are free to propagate energy along the dispersion band by inertial interactions between eddies of various sizes. Instead of Wood's eddies Nihoul distinguishes between three types of macroscale waves.

Table 3. Definitions of dimensionless parameters

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$R_o$ , Rossby No.	$Uf^{-1}L^{-1}$
$R_i$ , Richardson No.	$H^2N^2U^{-2}$
$R_e$ , Reynolds No.	$LU/\nu$

where,

$U$  = horizontal velocity scale  
 $f$  = Coriolis frequency  
 $L$  = horizontal length scale  
 $H$  = depth scale  
 $\nu$  = kinematic viscosity  
 $N$  = Brunt Väisälä frequency  

$$= \left( -\frac{g}{\rho} \frac{\partial \rho_o}{\partial z} \right)^{1/2}$$
 $\rho$  = local density  
 $\rho_o$  = average density  
 $g$  = acceleration of gravity

---

134. In both viewpoints the plethora of waves existing in the flow field are free to pass through various turbulent eddies without interaction. It also is not possible in many instances to separate such wave-eddy interactions in measured data. Indeed it is now thought that all data previously measured as turbulence might contain a large fraction of the signal derived from such wave activity. Future developments bear close observation as our concepts of turbulence and therefore our parameterizations must properly reflect the physics. It is apparent that far more coherent motions such as waves now make up what used to be considered as turbulence. Parameterizations should (but do not) reflect this modified viewpoint.

The energy cascade and variance spectrum

135. The horizontal variance spectra will show distinct peaks at the various scales of the turbulent windows (Nihoul 1978). Also these peaks will correspond to distinct energy inputs from the various eddies as per Woods (1977). At scales small enough to be in the inertial subrange the turbulence is homogeneous and therefore the spectra decreases continuously until the viscous dissipation of all energy. As noted by Nihoul any one spectra will show many peaks and valleys. However, the ensemble average of many spectra taken at different times produces a smoothed plot tending towards a  $k^{-P}$  dependency with increasing values of  $k$ . At scales greater than a kilometer,  $P = 3.0$  and energy may be transferred either up to larger scales (the "red" cascade) or down to smaller scales. At subkilometer scales  $P \approx 5/3$ , corresponding to the famous Kolmogorov spectrum. Here energy is transmitted to only smaller scales. In theory, field work, and models it has been customary to distinguish "two" and "three" dimensional turbulence by the slope of the spectral shape.

### Summary of Processes and Time and Space Variability

136. It is readily apparent from the preceding review that the interaction of effective transport processes is complex and spreads over several orders of magnitude in time and space. It is important then to try and summarize all this activity.

137. The early works of Ketchum, Stommel, and Pritchard all consider the tidal cycle as the temporal scale, and the tidal excursion as the spatial scale, when determining mean flow characteristics. This is evident in the dispersion studies which propose complete cross-sectional mixing, and averaging techniques which propose that mean flows may be calculated over a tidal cycle (Pritchard 1955). In light of recent studies, it seems that these techniques may lead to false estimates of mean flow characteristics.

138. Partch and Smith (1978) observe that data collected in the Duwamish River Estuary, Puget Sound, indicate nonstationarity in frequencies both longer and shorter than the tidal cycle. They state that this "lack of stationarity at any time scale precludes the determination of a well-defined mean from which to measure turbulent fluctuations." Thus, a review of techniques to determine various time scales seems in order.

139. Cannon (1971) examines the energy density of the flow field in the Patuxent River and these fit nicely into Kolmogorov's theory which predicts the  $-5/3$  slope for flows where energy dissipation is the only significant external parameter. Peaks in the spectra at 4 to 8 min are attributed to cross-channel seiche, which in the Patuxent has a period of approximately 7.5 min. The  $-1$  slope in some of the spectra is attributed to aliasing by the surface wave field.

140. Weisberg (1976), in a review of data collected on several estuaries, observes wide variability in current speed and direction. He concludes that mean flows cannot be determined by sampling over a few tidal cycles. Instead, he suggests that an adequate sampling interval can be determined by:

$$J = \psi^2 / 2B_e T' \epsilon^2 \hat{U}^2 \quad (27)$$

where  $J$  is the number of tidal cycles which must be observed;  $B_e$  is an effective spectral bandwidth;  $T'$  is the tidal period;  $\epsilon$  is the normalized acceptable error;  $\hat{U}$  is a mean value estimate of the current speed; and  $\psi^2$  is the total signal variance. For a typical estuary with an estimated mean flow of approximately  $10 \text{ cm}^{-1}$ , he shows that a minimum of 18 days of data is required to determine mean flow to a tolerance of  $0.2 \text{ cms}^{-1}$ .

141. Other suggestions for adequate temporal resolution are given by Bowden (1978) and Chatwin and Sullivan (1978). Bowden examines mixing processes and indicates that the effect of shear in the direction of flow in an oscillatory current in dispersing contaminants may be quite large. He states that the ratio  $T/T_c$ , where  $T$  is the period of oscillation (tidal cycle),  $T_c = h^2/K_z$ ,  $h$  is the mean depth, and  $K_z$  is the effective vertical eddy diffusivity, can be used to determine the effect of shear. For  $T/T_c \ll 1$ , the shear effect is negligible; for  $T/T_c \gg 1$ , the effect of the oscillatory current may be just as effective as a steady current in directing longitudinal dispersion. Here, the effect of vertical mixing on longitudinal dispersion is evident.

142. Chatwin and Sullivan (1978) identify four characteristic time scales of dispersion which they

attribute to "statistical unsteadiness in the relative velocity field." These are shown in Table 4. They state that by comparison,  $T = 1/3 (t-t_0)P$ , where  $t-t_0$  indicates an observation period, the relative importance of the effect given in Table 4 may be judged. For typical values in the Tay Estuary, England, the dispersion is chiefly dominated by the effect of local turbulence (see Chatwin and Sullivan 1978 for detail).

143. Partch and Smith (1978) observe that for the Duwamish River Estuary, 50% of the vertical turbulent salt flux occupies 20% of the time. This they attribute to periods of intense mixing associated with the development of an internal hydraulic jump during periods of the ebb flow.

144. Kjerfve and Proehl (1979) examine the velocity variability in the North Inlet, S.C. Their data are interesting in that significant variability is seen over successive tidal cycles in the net flows. Also evident is the development of characteristic flood and ebb channels. They note that even in a well-mixed estuary such as North Inlet, the velocity structure is quite complex and "should not be modeled with an area-averaged velocity mean."

145. All of these studies indicate the wide range of temporal and spatial resolution necessary to adequately describe estuarine flows. At the present, there is no concise, integrating theory which can be used to determine the necessary resolution of temporal and spatial characteristics in the estuarine flow field.

146. Table 5 summarizes much of the information thus far available. Included are both theoretical and observational studies. Time and space scales associated with the various activities are indicated, with reference to the studies in which they are presented. Figure 4 presents a graphical compilation of these processes. In both the table and figure surface waves have been excluded as they

Continuity:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial(Hu)}{\partial x} + \frac{\partial(Hv)}{\partial y} = 0 \quad (38)$$

X-Momentum:

$$\begin{aligned} \frac{\partial(Hu)}{\partial t} + \frac{\partial(Huu)}{\partial x} + \frac{\partial(Huv)}{\partial y} &= -gH \frac{\partial H}{\partial x} \\ &+ \frac{1}{\rho} \frac{\partial(H\tau_{xx})}{\partial x} + \frac{1}{\rho} \frac{\partial(H\tau_{xy})}{\partial y} \quad (1) \\ &+ \frac{\tau_{sx} - \tau_{bx}}{\rho} + \frac{1}{\rho} \frac{\partial}{\partial x} \int_{-h}^{\zeta} \rho(u'u') dz + \frac{1}{\rho} \frac{\partial}{\partial y} \int_{-h}^{\zeta} \rho(u'v') dz \quad (2) \end{aligned} \quad (39)$$

Y-Momentum:

$$\begin{aligned} \frac{\partial(Hv)}{\partial t} + \frac{\partial(Huv)}{\partial x} + \frac{\partial(Hvv)}{\partial y} &= -gH \frac{\partial H}{\partial y} + \frac{\tau_{sy} - \tau_{by}}{\rho} \\ &+ \frac{1}{\rho} \frac{\partial(H\tau_{yx})}{\partial x} + \frac{1}{\rho} \frac{\partial(H\tau_{yy})}{\partial y} + \frac{1}{\rho} \frac{\partial}{\partial x} \int_{-h}^{\zeta} \rho u'v' dz \quad (1) \\ &+ \frac{1}{\rho} \frac{\partial}{\partial y} \int_{-h}^{\zeta} \rho v'v' dz \quad (2) \end{aligned} \quad (40)$$

underscored with the brackets. The S is, for the rest of the section, the source/sink term.

157. The formulation of an equation for the Reynolds averaged pressure,  $P$ , or free surface,  $\delta$ , field is performed in one of two ways, either as a free surface or a rigid lid. Regardless of which method, the pressure field is formed from requiring the vertically integrated momentum equation to conserve mass at each time step. The rigid lid approach is valid under conditions set forth by Bennett (1974) and Haq and Lick (1975) and can be formulated either as an elliptic differential equation (Liggett and Hadjitheodorou 1968) or performed numerically (Paul 1983). The rigid lid does permit potentially longer time steps to be used, but such models are not in general use for estuaries.

158. Free surface versions of estuary models exploit the hydrostatic pressure equation to make a direct relation between free surface elevation and the pressure field. Implicit free surface models are attractive formulations due to less severe time step restrictions (Paul 1983).

#### Two-dimensional depth-averaged circulation and transport

159. By assuming hydrostatic pressure variations and weakly varying dependent variables in the vertical coordinate direction, a dynamic two-dimensional model may be created. After Reynolds averaging the depth average is defined and applied for any variable  $\beta^*$  as:

$$\beta^* = \bar{\beta} + \beta' \quad (36)$$

where the overbar now stands for the depth average quantity, i.e.

$$\bar{\beta} = \frac{1}{(h+\zeta)} \int_{-h}^{+\zeta} \beta^* dz \quad (37)$$

Dropping the overbar, the equations governing this class of models are:



Y-Momentum:

$$\frac{\partial v}{\partial t} + \frac{\partial(uv)}{\partial x} + \frac{\partial(vv)}{\partial y} + \frac{\partial(wv)}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial y} - fu$$

$$\left[ -\frac{\partial(\overline{u'v'})}{\partial x} - \frac{\partial(\overline{v'v'})}{\partial y} - \frac{\partial(\overline{w'v'})}{\partial z} \right] \quad (1) \quad (32)$$

Z-Momentum:

$$\frac{\partial P}{\partial z} = -pg \quad (33)$$

Transport:

$$\frac{\partial \alpha}{\partial t} + \frac{\partial(u\alpha)}{\partial x} + \frac{\partial(v\alpha)}{\partial y} + \frac{\partial(w\alpha)}{\partial z}$$

$$= \left[ -\frac{\partial(\overline{u'\alpha'})}{\partial x} - \frac{\partial(\overline{v'\alpha'})}{\partial y} - \frac{\partial(\overline{w'\alpha'})}{\partial z} \right] + S \quad (34)$$

(2)

156. In the above equations Reynolds averaging has been used to prepare the above mean flow equations. The dependent variables are the average quantities defined for any variable  $\beta^*(x,t)$  as

$$\beta^* = \bar{\beta} + \beta' \quad (35)$$

but the bars have been dropped from the above set of equations. The correlated quantities are the Reynolds stresses and fluxes resulting from Reynolds temporal averaging and are the terms requiring closure. They are

these models, the horizontal coordinates are  $x$  and  $y$  and have velocity components  $u$  and  $v$ . The vertical coordinate is  $z$  which is positive upward. The origin is embedded in the undisturbed free surface. The term  $\delta(x,y,t)$  is the elevation of the free surface while  $h(x,y)$  is the location of the time invariant bottom. Wind shear can be specified at the surface and the water body is considered shallow such that the vertical momentum equation reduces to hydrostatic pressure. No spatial averaging is used to prepare the momentum and transport equations and the molecular viscosity terms are assumed unimportant for such flows and are neglected.

155. The equations governing such a model are as follows:

Continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (30)$$

X-Momentum:

$$\begin{aligned} \frac{\partial u}{\partial t} + \frac{\partial(uu)}{\partial x} + \frac{\partial(uv)}{\partial y} + \frac{\partial(uw)}{\partial z} = - \frac{1}{\rho} \frac{\partial P}{\partial x} + fv \\ \left[ - \frac{\partial(\overline{u'u'})}{\partial x} - \frac{\partial(\overline{u'v'})}{\partial y} - \frac{\partial(\overline{u'w'})}{\partial z} \right] \quad (1) \end{aligned} \quad (31)$$

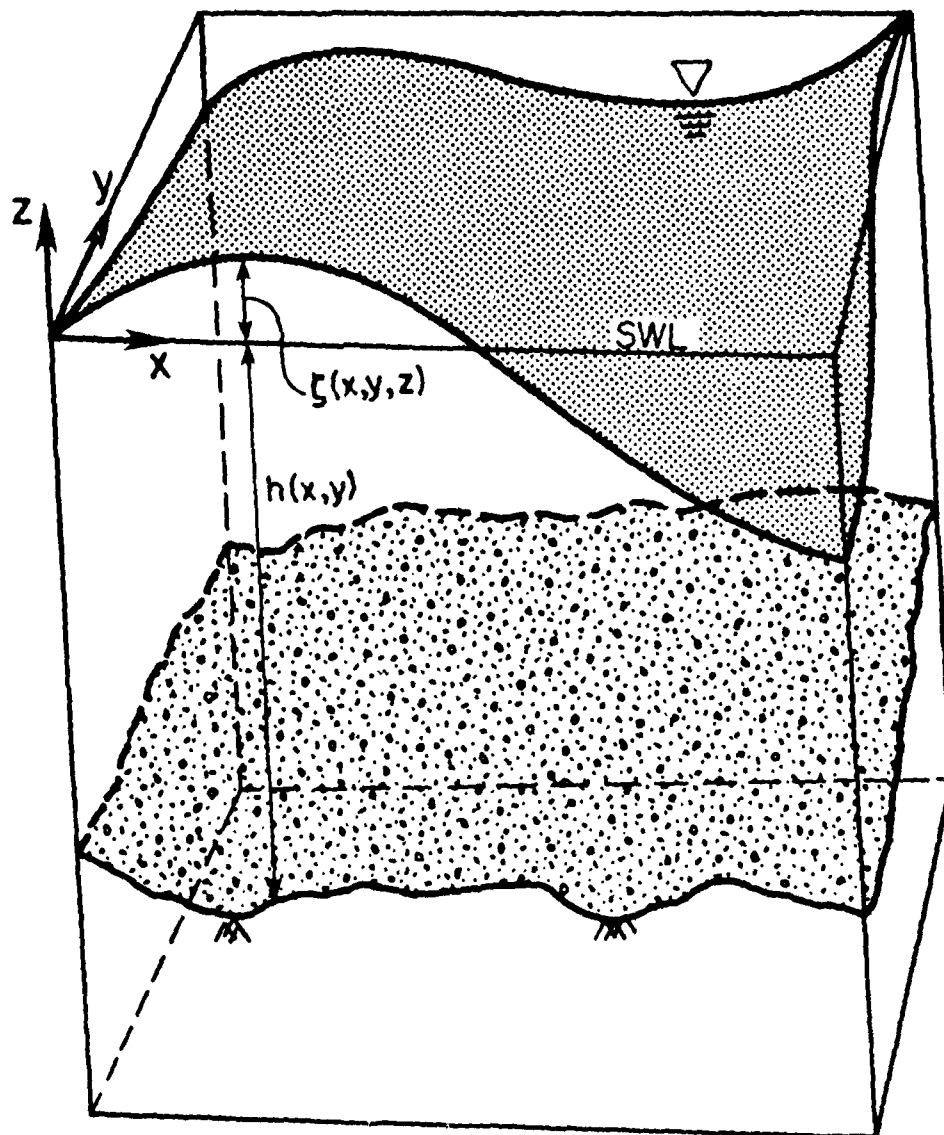


Figure 6. Coordinate notation

have been developed (Figure 5, columns 3 and 4) and used, but only in rectangular domains.

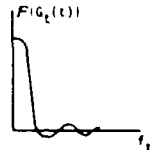
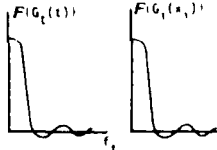
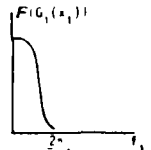
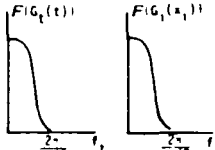
152. Since environmental hydraulics models are dominated by irregular domains and since the higher order averaging has not yet been adapted for use in such problems, the basic and turbulence models developed here will be derived from the Reynolds averaging point of view. The correlated terms requiring closure generated from such averaging are therefore functions of the uniform weight function and subject to the limitations inherent in this approach. The use of higher order averaging may become a factor but not in the foreseeable future as far as the practitioner is concerned.

#### Model Equations

153. This section presents the dynamic governing equations for the various classes of estuary models to which the reviewed turbulence models apply. It is assumed that Reynolds averaging in time has removed the smaller scale temporal fluctuations from the equations. Four classes of model equations are presented: wind driven three-dimensional circulation and transport; two-dimensional vertically averaging circulation and transport; two-dimensional laterally averaged flow and transport; and one-dimensional area-averaged flow and transport. All equations are prepared for arbitrary planform and bottom topography.

##### Three-dimensional circulation and transport

154. First solved for arbitrary bottom and planform by Liggett and Hadjithodorou (1969), the quasi-three-dimensional shallow wind driven circulation model equations have been extended to include nonlinear terms and allow application in a variety of estuary and lake recirculation problems as well as open coast transport calculations. A coordinate system is defined as in Figure 6. As for all

	Reynolds Temporal Averaging	Spatial-Temporal Averaging With Constant Filter	Leonard's Averaging	Suggested by the authors
	1882 to Present	1950 to Present	1974 to Present	1982
Definitions of the large scale component, equation (1).	$\bar{u}(\underline{x}, t) = \frac{1}{\Delta_t} \int_{-\Delta_t/2}^{\Delta_t/2} u(\underline{x}, t') dt'$	$\bar{u}(\underline{x}, t) = \frac{1}{\Delta_t} \int_{-\Delta_t/2}^{\Delta_t/2} \frac{1}{\Delta_i} \int_{-\Delta_i/2}^{\Delta_i/2} u(\underline{x}', t') dx' dt'$	$\bar{u}(\underline{x}, t) = \frac{1}{\Delta_t} \int_{-\Delta_t/2}^{\Delta_t/2} u(\underline{x}, t') dt'$	$\bar{u}(\underline{x}, t) = \frac{1}{\Delta_t} \int_{-\Delta_t/2}^{\Delta_t/2} \frac{1}{\Delta_i} \int_{-\Delta_i/2}^{\Delta_i/2} u(\underline{x}', t') dx' dt'$
Definition of the filter function, equation (2).	$G(\underline{x}, t) = G_t(t)$ $G_t(t) = \frac{1}{\Delta_t}$ for $-\frac{\Delta_t}{2} \leq t \leq \frac{\Delta_t}{2}$ $= 0$ otherwise	$G(\underline{x}, t) = G_t(t) \prod_{i=1}^n G_i(x_i)$ where: $G_t(t) = \frac{1}{\Delta_t}$ for $-\frac{\Delta_t}{2} \leq t \leq \frac{\Delta_t}{2}$ $= 0$ otherwise $G_i(x_i) = \frac{1}{\Delta_i}$ for $-\frac{\Delta_i}{2} \leq x_i \leq \frac{\Delta_i}{2}$ $= 0$ otherwise	$G(\underline{x}, t) = G(\underline{x}) = \prod_{i=1}^n G_i(x_i)$ where $G_i(x_i) = \sqrt{\frac{\gamma}{\pi}} \frac{1}{\Delta_i} e^{-\gamma x_i^2 / \Delta_i^2}$ for all $x_i$	$G(\underline{x}, t) = G_t(t) \prod_{i=1}^n G_i(x_i)$ where $G_t(t) = \sqrt{\frac{\gamma}{\pi}} \frac{1}{\Delta_t} e^{-\gamma t^2 / \Delta_t^2}$ for all $t$ $G_i(x_i) = \sqrt{\frac{\gamma}{\pi}} \frac{1}{\Delta_i} e^{-\gamma x_i^2 / \Delta_i^2}$ for all $x_i$
Fourier Transform of the filter function	$F[G(\underline{x}, t)] = F[G_t(t)]$ $= \frac{\sin(f_t \Delta_t / 2)}{(f_t \Delta_t / 2)}$	$F[G(\underline{x}, t)] = F[G_t(t)] \prod_{i=1}^n F[G_i(x_i)]$ $= \frac{\sin(f_t \Delta_t / 2)}{(f_t \Delta_t / 2)} \prod_{i=1}^n \frac{\sin(f_i \Delta_i / 2)}{(f_i \Delta_i / 2)}$	$F[G(\underline{x}, t)] = F[G(\underline{x})] = \prod_{i=1}^n F[G_i(x_i)]$ $= \prod_{i=1}^n \exp[-f_i^2 \Delta_i^2 / 4\gamma]$	$F[G(\underline{x}, t)] = F[G_t(t)] \prod_{i=1}^n F[G_i(x_i)]$ $= \exp[-f_t^2 \Delta_t^2 / 4\gamma] \prod_{i=1}^n \exp[-f_i^2 \Delta_i^2 / 4\gamma]$
				

Legend:  $\Delta_t$  = temporal averaging scale  
 $\Delta_i$  = spatial averaging scale in the  $x_i$  direction  
 $\gamma$  = dimensionless constant with optimum value of 6  
 $f_t$  = frequency =  $2\pi/T$  where  $T$  = wave period  
 $f_i$  = wave number in the  $x_i$  direction =  $2\pi/L_i$  where  $L_i$  = wave length in the  $x_i$  direction

Figure 5. History of the averaging procedures

varying variable  $\alpha(\underline{x},t)$  is defined by the convolution integral as:

$$\bar{\alpha}(\underline{x},t) = \int_{-\infty}^{+\infty} G(\underline{x}-\underline{x}',t-t') (\underline{x}',t') d\underline{x}' dt'. \quad (28)$$

The underbar stands for the vector coordinate quantity i.e.  $\underline{x} = x,y,z$ .  $G(x,t)$  is a weight function defined such that:

$$\int_{-\infty}^{+\infty} G(x,t) dx dt = 1. \quad (29)$$

150. The forms for the filter function are relatively few and simple. The time honored functional form is the top hat or uniform weight function. The first use of the uniform weight function was by Reynolds who filtered in time over a time period  $t$ . Figure 5, column a, contains this filter. The averaging length  $\Delta t$  for Reynolds averaging is conceived as being finite and larger than the time scale of the smallest turbulence fluctuation and smaller than the time scale of the largest fluctuation which are controlled by the problem geometry and joining function (Hinze 1975). The uniform weight function was extended to spatial filtering about 20 years ago via Smagorinsky et al. (1965) and Deardorff (1970). These filter characteristics are summarized in column b in Figure 5.

151. Several fundamental deficiencies in the uniform weight function approach were alleviated via the use of higher order averaging. As initiated by Leonard (1974) and reviewed by Ferziger (1981), Gaussian filters and the higher order decomposition of the inertia or advection terms permitted more accurate calculation of flows in regular domains. Extension of these concepts to surface water flows has been by Bedford and Babajimopoulos (1980), Babajimopoulos and Bedford (1980), and Findikakis and Street (1982a, 1982b). Both time and space higher order filters

#### PART IV: TRANSPORT MODEL EQUATIONS

147. The unquestioned starting point for deriving a set of model equations is the Navier-Stokes equations and accompanying transport equation. Reference is made to Batchelor (1967), Monin and Yaglom (1975), or Hinze (1975) for background information. Two procedures need to be applied to these equations to arrive at a form reflecting the model physics of the problems being considered here. Both procedures involve simplification of the complexity in the equations and both procedures involve averaging.

148. The first procedure is to reduce the equations in their spatial structure. It is possible to assume hydrostatic pressure variation for the vertical momentum balance and thereby achieve models that are either two-dimensional in plan, vertical plane two-dimensional, or quasi-three-dimensional. Methods of depth and lateral or width averaging are used to spatially simplify such equations. The second procedure is necessary to reduce the complexity and scale of the turbulent motion expressed in the equations to a level consistent with that being resolved in the model. The traditional form of such averaging is called Reynolds averaging. Before summarizing the equations for each class of model a brief review of averaging is presented.

##### Averaging

149. The removal or suppression of turbulent fluctuations from the governing equations as a means of analyzing turbulent flows has been accepted strategy for over 100 years since the work of O. Reynolds (1894). The effect of such averaging is to derive a set of governing equations for the behavior of the average or mean flow field variables. The mean flow field for any time and space

are not resolvable by the models being considered here. Processes 1, 2, and 6 constitute composed boundary loadings on the estuary.



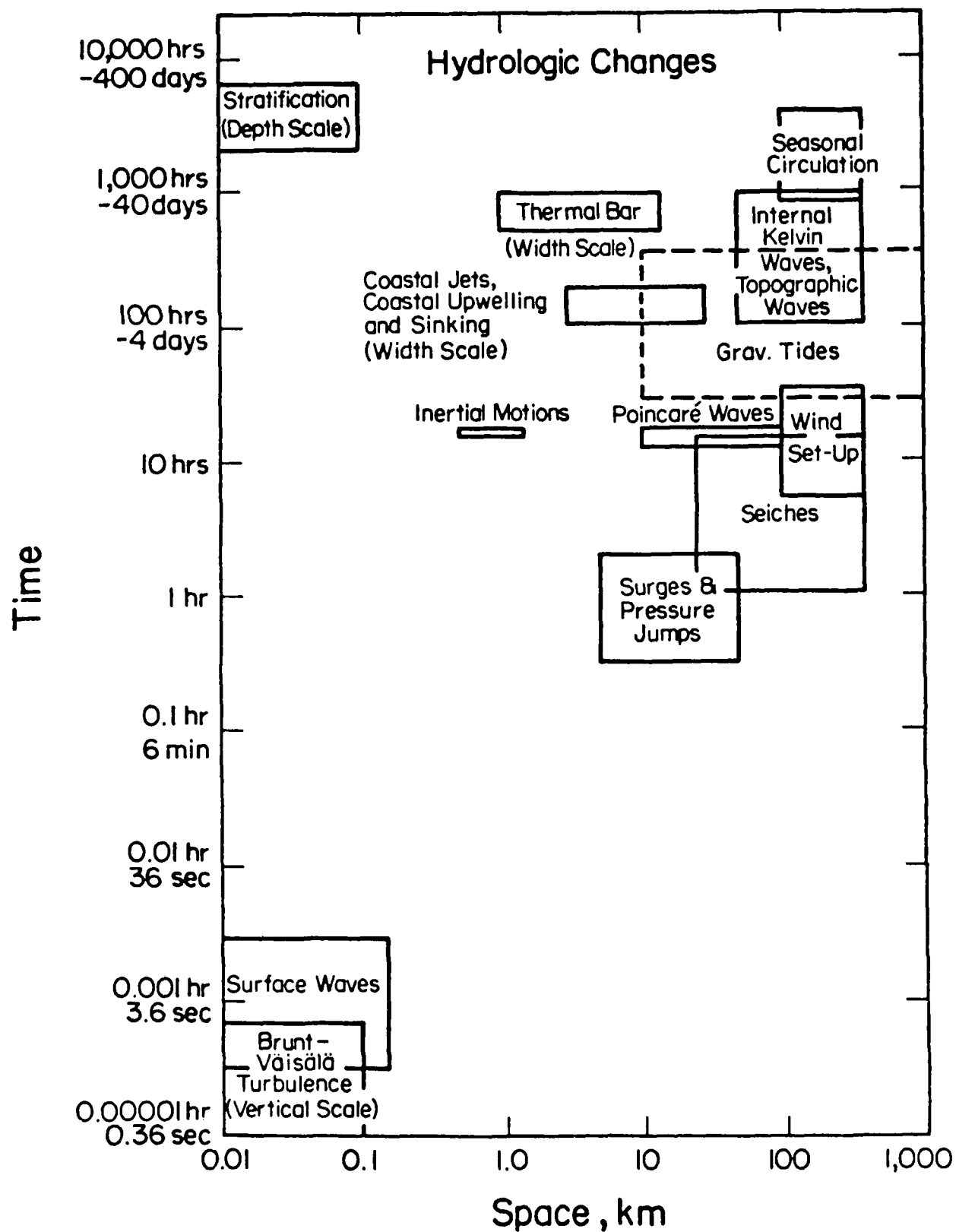


Figure 4. Scales of physical estuary phenomena

Table 5. Concluded

No.	Activity/Process	Time/Length Scale	Reference
9.	Velocities a. Tidal b. Residual c. Turbulent	T = 2 - 10 days T = 18 days $u' = u(10^4)$	Elliot (1978) Weisberg (1976) Pritchard (1971)
10.	Salinity intrusion	$10^2 < L/H < 10^4$ (ratio of salt intrusion distance to estuary depth)	Rattray and Mitsuda (1974)
11.	Internal waves	T = 2-11 hr L = 10 - 1000 m H = 1 m	Mortimer (1968) Garrett and Munk (1979)
12.	Kelvin, topographic waves	T = 14 - 30 hr L = 10 - 1000 m	Mortimer (1968) Pedlosky (1979)
13.	a) Coastal jets b) Upwelling	T = 1 - 2 days L = 100 - 1000 km	Csanady (1982)
14.	Thermal stratification	D = 10 - 100 m T = 100 - 200 days	Turner (1973)
15.	Vertical shear	D = 10 - 100 m T = 10 sec	Fischer et al. (1979)
16.	Inertial motions	$D = 6.7 W \sqrt{\sin \phi}$ T = 14 hr	Pedlosky (1979)

Table 5. Summary of length and time scales associated with various activities and influences within the estuaries

No.	Activity/Process	Time/Length Scale	Reference
1.	Wind stress	T = 3 days D = $6.7 W / \sqrt{\sin \phi}$	Elliot et al. (1978) Eckman depth of influence
2.	Far-field forcing (coastal current, storm surge, etc.)	T = 10-20 days	Elliot et al. (1978)
3.	Tidal activity	Diurnal/semidiurnal ranges variable (>1 to >40 ft)	
4.	Local turbulence (Brunt-Väisälä)	T = 1 min - 1 hr T = 1 min L = 1 m (vertical scale) L = f(Ri)	Cannon(1971) Nihoul (1978) Kent and Pritchard (1959)
5.	Residual circulation	T = 20 days (overall) T = 1-2.5 days(type)	Elliot and Wang (1978) Elliot et al. (1978)
6.	River inflow	T = 5 days - 10 days Coherent with sea level at periods of 2.5 - 20 days	Elliot and Wang (1978) Elliot and Wang (1978)
7.	Surface slope (set up)	4.2 days (barotropic)	Kjerfve et al. (1978)
8.	Sea level	a) Coherent with wind at 9.2 days (surges) creation time ~6 hr  b) Coherent with surface N. Smith (1978) pressure at 6 days (pressure jumps) creation time ~6 hr	Kjerfve et al. (1978)

(Continued)

Table 4. The relative time scales of turbulent flows in estuaries as suggested by Chatwin and Sullivan (1978)

Time Scale	Explanation
$T_1 = \left( \frac{1}{\underline{v}} \frac{\partial \underline{v}}{\partial t} \right)^{-1}$	An unsteadiness in the mean flow field measured relative to a fixed point (Eulerian).
$T_2 = \frac{\underline{v}(x,t)}{\underline{v}(x,t)}$	Convective effects due to the movement of a water mass into a region with different characteristics
$T_3 = \frac{10d(x,t)}{u_*}$	Ratio of the depth (d) to the local friction velocity ( $u_*$ ). This indicates the effects of large eddies on convective transport processes.
$T_4 = \left  \frac{\partial \underline{v}}{\partial z} \right ^{-1}$	Indicates the convection of mass through a region of appreciable shear.

## Transport

$$\frac{\partial(H\alpha)}{\partial t} + \frac{\partial(Hu\alpha)}{\partial x} + \frac{\partial(Hv\alpha)}{\partial y} = \underbrace{\frac{1}{\rho} \frac{\partial(HN_x)}{\partial x} + \frac{1}{\rho} \frac{\partial(HN_y)}{\partial y}}_{(3)} + \frac{q_s}{\rho} + \underbrace{\frac{1}{\rho} \frac{\partial}{\partial x} \int_{-h}^{\zeta} \rho(u'\alpha') dz + \frac{1}{\rho} \frac{\partial}{\partial y} \int_{-h}^{\zeta} \rho(v'\alpha') dz}_{(4)} + HS \quad (41)$$

160. In Equation 41 the term  $q_s/\rho$  provides for heat or mass flux at the water surface. The terms  $\tau_{sx}$ ,  $\tau_{bx}$ ,  $\tau_{sy}$ , and  $\tau_{by}$  are boundary stresses, either at the surface,  $s$ , or bottom,  $b$ , in the  $x$ - and  $y$ -coordinate. These may be specified a priori as in the case of the surface or wind stresses. It is customary to parameterize the bottom stresses by a friction factor parameterization in terms of the mean flow.

161. The bracketed terms labeled as (1) in the  $x$ - and  $y$ -momentum equation are the depth-averaged Reynolds stress ( $\tau_{ij}$ ) quantities resulting from the Reynolds averaging. Of similar origin, bracketed term (3) is the depth averaged Reynolds heat or mass flux ( $N_i$ ) resulting from Reynolds averaging. Bracketed terms (2) and (4) represent correlations resulting from depth averaging the inertia and advection terms and are commonly called the dispersion terms. As has been noted by Rodi (1980), the integrated stresses in large flows are often negligible and therefore not parameterized; however, the dispersion and integrated flux terms are all-important and need to be retained and parameterized in terms of mean flow variables. Elaboration of the equation derivation is found in Abbott (1979).

## Two-dimensional laterally averaged flow and transport

162. Developed primarily to investigate the

interaction between long wave affected mixing of water masses and vertical density stratification these models have been used primarily to model mixing zones or salt wedges in seiche or tide affected estuaries. Though few in number these models seek to replicate very complicated stratified flow phenomena, a most difficult task. The models of Parrels and Karelse (1981) and Boericke and Hogan (1977) are the two earliest efforts. The Parrels and Karelse paper reviews all existing vertical plane models. These models consider lateral variations to be small in comparison to vertical and horizontal variations. The bottom may vary with respect to horizontal location. With the exception of wind driven cavity models (see for example Findikakis and Street 1982a), most vertical plane models used in practice assume hydrostatic pressure. Further, flow may enter at either the upstream or downstream end of the flow field thereby replicating the effects of freshwater inflow to the estuary and the possible effects of tidal fluctuations. The width  $B(x,z)$  is assumed to vary with length and depth although assuming regular channel cross sections such as trapezoids or rectangles will reduce the complications involved in making detailed specifications of depth variations for  $B$ . Again Reynolds averaging plus lateral averaging are performed; the space averaging is defined as:

$$\beta^* = \bar{\beta} + \beta' \quad (42)$$

with

$$\bar{\beta} = \frac{1}{B(x,z)} \int_{y_1(x)}^{y_2(x)} \beta dy. \quad (43)$$

Here  $y_1$  and  $y_2$  represent lateral distances from the origin to each side of the channel at any point  $x$ .

163. With these concepts the governing equations are  
Continuity:

$$\frac{\partial \zeta}{\partial x} + \frac{1}{B_{\zeta}} \frac{\partial}{\partial x} \int_{-h}^{\zeta} B u dz = 0 \quad (44)$$

X-Momentum:

$$\begin{aligned} \frac{\partial (Bu)}{\partial t} + \frac{\partial (Bu u)}{\partial x} + \frac{\partial (Bu w)}{\partial z} &= -gB \frac{\partial \zeta}{\partial x} - gB \int_{-h}^{\zeta} (\zeta - z) \frac{\partial \alpha}{\partial z} dz \quad (1) \\ (2) \quad &+ \frac{1}{\rho} \frac{\partial (B \tau_{xx})}{\partial x} + \frac{1}{\rho} \frac{\partial (B \tau_{xz})}{\partial z} + \frac{\tau_w}{\rho} \\ (3) \quad &+ \frac{1}{\rho} \frac{\partial}{\partial x} \int_{y_1}^{y_2} \rho u' u' dy + \frac{1}{\rho} \frac{\partial}{\partial z} \int_{y_1}^{y_2} \rho u' w' dy \quad (45) \end{aligned}$$

Z-Momentum:

$$\frac{\partial P}{\partial z} = -\rho g \quad (46)$$

Equation of State:

$$\rho = \rho_0 + \rho \alpha \quad (47)$$

Transport:

$$\begin{aligned} \frac{\partial (B\alpha)}{\partial t} + \frac{\partial (Bu\alpha)}{\partial x} + \frac{\partial (Bw\alpha)}{\partial z} &= \\ \frac{\partial q_s}{\partial t} + \frac{1}{\rho} \frac{\partial (BN_x)}{\partial x} + \frac{1}{\rho} \frac{\partial (BN_z)}{\partial z} & \quad (4) \end{aligned}$$

$$+ \frac{1}{\rho} \frac{\partial}{\partial x} \int_{y_1}^{y_2} \rho(u'\alpha') dy + \frac{1}{\rho} \frac{\partial}{\partial z} \int_{y_1}^{y_2} \rho(w'\alpha') dy + BS \quad (48)$$

-----! ⑤

As in the previous set of equations, bracketed terms ② and ④ represent the stresses and fluxes resulting from Reynolds averaging, while bracketed terms ③ and ⑤ correspond to the dispersion terms resulting from the spatial averaging. Bracketed term ① explicitly accounts for stratification effects either due to temperature or a dissolved substance such as salt or sediment. The equation of state dictates the form of the coefficient C in term ①.

#### One-dimensional area-averaged flow and transport

164. By far the most routinely used set of model equations, the one-dimensional equations averaged over a cross section area  $A(x,t)$ , suppresses any lateral or vertical variation of the dependent variable. Various derivation strategies have been employed (see Harleman 1971 or Liggett 1975) but all are based upon a post-Reynolds averaging area average defined as  $\beta^* = \bar{\beta} + \beta'$  where  $\bar{\beta}$  is defined as:

$$\bar{\beta} = \frac{1}{A(x,t)} \int_A \beta^* dA \quad (49)$$

#### Continuity:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \quad (50)$$



### Momentum:

$$\frac{\partial Q}{\partial t} + \frac{\partial (Qu)}{\partial x} + gA \frac{\partial (h+\xi)}{\partial x} = gA(S_0 - S_f) \quad (51)$$

$$\textcircled{1} \left[ \frac{1}{\rho} \frac{\partial (A\tau_{xx})}{\partial x} \right] + \frac{1}{\rho} \frac{\partial}{\partial x} \int_A \rho u' u' dA \textcircled{2}$$

### Transport:

$$\frac{\partial (A\alpha)}{\partial t} + \frac{\partial (Q\alpha)}{\partial x} = - \frac{1}{\rho} \frac{\partial (AN_x)}{\partial x} - \frac{1}{\rho} \frac{\partial}{\partial x} \int_A (u'\alpha') dA + AS \quad (52)$$

$$\textcircled{3} \quad \textcircled{4}$$

165. Bracketed term  $\textcircled{1}$  is the area averaged stress  $\tau_{xx}$  resulting from Reynolds averaging while terms  $\textcircled{2}$  and  $\textcircled{4}$  result from the area averaging procedure. The variable  $N_x$  is the area average flux term resulting from Reynolds averaging. The variable  $S_0$  is the gravity slope term while  $S_f$  is the bottom friction, a friction slope term usually parameterized by a Chezy or Mannings relation with the average velocity.

### Tidal-averaged models

166. It is sometimes desirable to determine the distribution of hydrodynamic and water quality information over periods longer than a complete tidal cycle. The residual field models are prepared by averging the equations over a preselected time period,  $T$ , which is usually two tidal cycles or longer, i.e. for any variable  $\beta^*$  the definition is:

$$\beta^* = \beta_T + \beta'_T \quad (53)$$

where

$$\langle \beta^* \rangle = \beta_T = \frac{1}{T} \int_{\tau}^{\tau+T} \beta^* d\tau \quad (54)$$

and  $\beta_T'$  is the deviation from the tidal average. For the one-dimensional area-averaged equations (50 through 52) this averaging yields the following set of equations for the species transport equation (Okubo 1964):

$$\begin{aligned} & \frac{\partial(A_T \alpha_T)}{\partial t} + \frac{\partial(U_e A_T \alpha_T)}{\partial x} \\ &= - \frac{\partial(A_T U_T' \alpha_T')}{\partial x} - \frac{\partial}{\partial x} \left( \langle A N_x \rangle + \langle \frac{1}{\rho} \int_A \rho(u' \alpha') dA \rangle \right) + \langle A S \rangle \quad (55) \end{aligned}$$

where

$$U_E = U_T + \langle U_T' A_T' / A_T \rangle; \quad (56)$$

and the tidal average velocity  $U$  at a cross section is

$$U_T = Q_f / A_T - \langle U_T' A_T' A_T \rangle - \frac{1}{A_T} \frac{\partial V_0}{\partial t} \quad (57)$$

where  $Q_f$  is the freshwater discharge through the section and  $V_0$  is the tidal-average volume of water stored upstream. If successive tides are equal, the  $\partial V_0 / \partial t = 0$ .

167. A hydrodynamic model can be constructed to calculate  $U_E$  but these equations are not routinely in use. For multidimensional versions of this model see the works of Tee (1981).

#### Steady-state analogs

168. All the above models can be converted from dynamic to steady-state models by simply eliminating the

time derivative. However, since the very reason estuaries need such models is their unsteadiness, it will be assumed that dynamic models are preferred and are to be used subsequently.

#### Equation of state

169. In tidal flows with not only heat but also salt, a relationship between the density and contributing variables must be established. A simple linear relation is customary in modeling, i.e.

$$\rho = \rho_0(1 + \gamma C - \beta T) \quad (58)$$

where  $\rho_0$  is a reference density,  $T$  is the temperature, and  $C$  is the concentration. Interpolation for the variables  $\gamma$  and  $\beta$  is necessary as a function of the range of values considered. Should a very broad range of  $T$  or  $C$  be considered, then improved parabolic or higher order polynomial relations must be used in place of Equation 58.

#### Model Equation Classifications and First Assessment of Limitations

170. The spatial simplifications introduced in these equations result in not only the new bracketed terms which must be specified but also eliminate to a greater or lesser extent the transport phenomena displayed in Figure 4. Using this distinction, therefore, each model can be classified based upon spatial structure. Each type of model, by prohibiting various mechanisms from being predicted or accounted for in the calculation, can be tailored to the problem at hand; i.e., the most spatially complicated model, having fewer unresolved mechanisms, can be applied to the more complicated problems containing the most variability. The model equations defined above can be further subdivided by a number of distinctions. Since this

report concentrates on the physics contributing to and the forms of subgrid scale modeling the models will be further subdivided based upon the physical differences. For obvious reasons in all cases only models employing free surface effects are considered.

171. Two principal subdivisions are common in the literature. First a distinction between models is made depending upon the time scale over which dynamic activity is to be resolved. Of course, a number of time periods can be used but only the distinction between subtidal and supertidal seems to endure. Models seeking to predict only dynamic variations over periods longer than tidal periods are used for long-term effects studies. Residual circulation or transport models, because they suppress small-scale activity, are only valid for assessing variability at time periods of weeks or longer. Oil spills, storm events, and sewage or heat disposal events are all activities whose dynamic character is very much affected over small time periods; therefore, models with subtidal time resolutions are necessary for these simulations.

172. Residual models require data collected over long time periods to establish credibility and the interpretation and parameterization of all the terms requiring closure are very much different than those required for the more realistic subtidal scale models. As pointed out in the previous section, residual models are based upon the presumption that the long period tidal effects can be linearly "subtracted" from the equations. The results in the previous section also indicate that this is a false notion and a significant source of error which accrues nonlinearly in the assessment of model validity.

173. The second significant subdivision is based upon the presence of thermal or salt stratification. Such stratification requires the simultaneous or coupled solution

of the momentum equation and the salt and/or heat equation. Stratification automatically implies the necessity of vertical processes resolution and the presence of internal Kelvin, Poisson, and topographic waves.

#### Model Summary

174. Without regard to closure or numerical solution structure, Table 6 details an acronym for each model type, the equations of which it is composed, the processes resolved in the equation (see Table 5 and Figure 4), and those processes excluded from the model equations. It is noted that all models resolve the fluctuating free surface and permit variable planform and bottom topography. As a final comment it should be brought to the reader's attention that in the model and process summary the division of processes between tidal and residual time scale models is not at all clear. Note that, if variability is to be calculated over time steps of 1 day or more, possible internal wave or Kelvin wave activity might be partially resolved. It should, therefore, be remembered that the selection of a time scale of resolved activity should be made such that all processes are clearly resolved or not resolved. In other words, the time scale should be selected to fall in the space/time gaps between processes.

#### Model Resolvability Limits From Spatial Simplifications

175. In addition to model limitations on resolvability imposed by the inclusion or exclusion of certain physical processes as discussed in the previous section, very definite resolvability limits occur due to dimensionally simplified structures in the governing equations. Based upon arguments presented in Fischer et al. (1979) the following limits are known.

Table 6. Model classification and effective transport processes

Acronym	Description	Eq. No.	Processes Included/ Resolved (Table 5)	Processes Excluded (Table 5)
1DT	One-dimensional, resolves tidal activity	50-52	1, 2, 6, 3, 5, 7, 9a, 9b, 10a	4, 8, 9, 11, 12, 13, 14, 15, 16, 10b
1DR	One-dimensional residual flow and transport	55-57	1, 2, 6, 5, 7, 9b, 10a	3, 4, 8, 9a, 9c, 11, 12, 13, 14, 15, 16, 10b
2DHT	Two-dimensional, horizontal plane, tidal resolution	38-41	1, 2, 3, 5, 6, 7, 8, 9a, 9b, 10a, 12, 13a, 16	4, 9c, 11, 13, 14, 15, 10b
2DHR	Two-dimensional, horizontal plane, residual resolution	38-41	1, 2, 5, 6, 7, 8, 9b, 10a, 12 (partially), 13a, 16	3, 4, 9a, 9c, 11, 13b, 14, 15, 10b, 12
2DVTH	Two-dimensional, vertical plane, tidal resolution, homogeneous	44-48	1, 2, 3, 5 (poorly), 6, 7, 8, 9a, 9b, 15	4, 9c, 10a, 10b, 11, 12, 13, 14, 16
2DVTS	Two-dimensional, vertical plane, tidal resolution, stratified	44-48	1, 2, 3, 4, 5, 6, 7, 8, 9, 10a, 10b, 11, 14, 15	12, 13, 16

(Continued)

Table 6. (Concluded)

Acronym	Description	Eq. No.	Processes Included Resolved (Table 5)	Processes Excluded (Table 5)
2DVRH	Two-dimensional, vertical plane, residual, homogeneous	44-48	1, 2, 5 (poorly), 6, 7, 8, 9b, 14, 15	3, 4, 9a, 9c, 10a, 10b, 11, 12, 13, 16
2DVRs	Two-dimensional, vertical plane, residual, stratified	44-48	1, 2, 5, 6, 7, 8, 9b, 10a, 10b, 11 (partially), 14, 15	3, 4, 12, 13, 16
3DTH	Three-dimensional, tidal, homogeneous	30-34	1, 2, 3, 4, 5, 6, 7, 8, 9a, 9b, 9c, 12, 13, 14, 15, 16	10, 11
3DTS	Three-dimensional, tidal stratified		1, 2, 3, 4, 5, 6, 7, 8, 9a, 9b, 9c, 10a, 10b, 11, 12, 13, 14, 15, 16	
3DRH	Three-dimensional, residual, homogeneous		1, 2, 5, 6, 7, 8, 9b, 10a, 10b, 11 (partially), 13, 12 (partially), 15, 16P	3, 4, 9a, 9c, 11, 12, 14
3DRS	Three-dimensional, residual, stratified		1, 2, 5, 6, 7, 8, 9a, 9c, 10, 11 (partially), 12 (partially), 13, 14, 15, 16	3, 4, 11, 12

176. The basic assumption implied in the one-dimensional analysis is that the quantity is relatively well mixed throughout the vertical extent of the cross-section area in question. Therefore, the time period and spatial extent over which vertical well mixedness occurs becomes the minimum time and space scale of resolvability in longitudinal variations predicted by the model. Lateral mixing takes longer than vertical mixing; therefore, the time to lateral mixing and the corresponding length scale become conservative scale estimators. Table 7 contains these estimates as abstracted from Fischer. The same line of reasoning holds in the analysis of the 2DH class of models. In this class of models the time and space extent to vertical mixing become the controlling limits of model resolvability. Similarly in the 2DV model class the time and space extent to lateral well mixedness is the minimum resolution permitted in the models.

177. In all these models, if for each model class the time scale to well mixedness is greater than the tidal period, then the tidal period resolution limit of residual class of models is superseded by the mixing time. It should be emphasized that these are guidelines and are by no means definitive. These estimates are based upon mixing models for very regular channels. Some decreases in time or space resolvability limits are permissible for transport in highly irregular flow domains. See Fischer et al. (1979) for a more thorough discussion.

#### Model Resolvability Limits Due to Problem Specification

178. The modeler is very rarely told to use such and such a set of model equations. Rather the analyst is given a problem and left to his or her own devices to select the proper set of equations. Typical problems might include:



Table 7. Minimum length/time resolvability due to spacial simplification

Model Class Table	Controlling Dif- fusion Process	Minimum Length, $L^*$	Minimum Time, $T^*$	Definitions
1D	Transverse mixing	$L^* \approx 0.1 \frac{\bar{u} W^2}{\epsilon_t}$	$T^* \approx \frac{W^2}{\epsilon_t} \quad 0.1$	$\epsilon_v$ = vertical eddy diffusivity $\epsilon_T$ = transverse eddy diffusivity $d$ = depth $u^*$ = friction velocity = $\sqrt{gds}$ $S$ = bed slope $W$ = full width $\bar{u}$ = average longitudinal velocity
	Vertical mixing $\epsilon_v \approx 0.07 \, du^*$	$L^* \approx .001 \frac{u W^2}{\epsilon_v}$	$T^* \approx \frac{D^2}{\epsilon_v} \quad (0.001)$	
2DV	Transverse mixing as in 1D	See 1D	see 1D	

determining the near field transport of a wastewater treatment plant discharge due to plant by passing of storm water; calculating the effect on the estuary of a storm event; or analyzing the long-term water quality response of the estuary resulting from the building of a pair of 600 megawatt nuclear reactors on the estuary. The distinctions between these model classes are often derived from the time and length scale of the variability of a boundary condition whose effect on the estuary is to be tested. The minimum variability desired in the calculated variables also is a criteria for distinguishing amongst these problem classes. For each problem a decision must be made as to what minimum time and space resolution is necessary to completely satisfy the problem solution requirements. The model equations selected to perform the problem solution must be capable of resolving this minimum set of length and time scales while including all the relevant transport processes. If the vertical mixing time scale is less than the lateral mixing time, then certainly 2DV and 1DH models will be unsatisfactory. Further, if the required problem resolution is less than the vertical mixing times, then only the 3D class of model will be tentatively acceptable.

179. Finally, it should also be noted that the minimum variability imposed by the problem statement dictates that the minimally acceptable variability must exist in both model output calculations and the boundary and initial conditions. One cannot calculate variability to any more refined degree than the variability of the information imposed at the boundary. A corollary is that if the calculation requires variability, more coarse than that provided at the boundaries, then one must low pass filter the boundary condition data until consistency in variability is obtained between model equations, problem specification, and available field data for boundary information.

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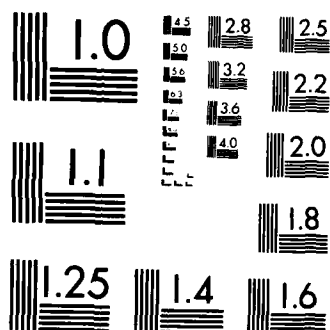
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## PART V: SUBGRID SCALE PARAMETERIZATIONS

180. The following sections review the various functional forms for parameterizing activity that is not resolved by the grid or averaging procedure. For each class of turbulence or mixing models, the function will be presented along with what, in the author's opinion, are the acceptable range of empirical coefficients. One or two references from each functional form will be presented.

181. By no means can an exhaustive review of each published paper and a summary of every coefficient be obtained. With very little effort this author has collected over 300 articles and reports from the last 15 years of publications. It is certain that a collection of worldwide papers would easily double that number. Therefore, this document is subjective in the sense that in the author's experience the various forms presented here represent the most accurate or time-tested forms.

### Taxonomy of Subgrid Scale Models

182. There are now so many forms of mixing, dispersion, and turbulence models that some effective means of creating coherent classifications is necessary. Kline (1978) proposed a five-level scheme which was modified by the addition of one more level for the AFOSR-HTTM Stanford Conference (Kline and Cantwell 1981). Agreeing with Ferziger (1982) it can be said that the first three of the original five are practical and the last two are research oriented. Table 8 contains a brief summary of the approaches. The increase in levels occurs along with an increase in the generality of the turbulence model and in the cost as well. Level three is the region of most interest in this report and is subdivided into two further levels. Levels 3a and 3b are distinguished by the

Table 8. Turbulence model classification

Level No.	Name	Concept	Comments
1	Correlations	Relationships between various data collected in experiments	Not predictive; no generality to new experience without new data collection
2	Integral equation	Control volume concept	Needs rederivation for every new situation
3	Reynolds equations	Based on solution of Reynolds equations with functions for turbulence quantities	Most extensively used level of turbulence simulation in practice
	a. Phenomenological	Empirical formula with coefficients for mixing	
	b. Analytic	Solution of transport equations for turbulence quantities	
4	Large Eddy Simulation	Higher order treatment of averaging. Full simulation of large-scale activity	Poised for practical contributions
5	Full Turbulence Simulations	Solution of full unaveraged Navier Stokes Transport Equations	No computer large enough to compute geophysical flows

difference in the amount of physics used to create the turbulence model. The last two levels are research levels although aspects of Large Eddy Simulations have been used in geophysical flows (Bedford 1980, 1981, 1983; Findikakis and Street 1982a, 1982b). They will not be dealt with in this report. Also, full turbulence simulations are not yet feasible for geophysical flows.

183. Since so much attention is directed at level 3 in this report and in practical computations, it is necessary to make more refined classifications about the turbulence model structure. We employ here Rodi's (1980) distinctions which are based upon the number of equations used to express turbulence quantities. Table 9 contains the classification scheme. The first four subclassifications are all based on the concept of the eddy viscosity or diffusivity. The first classification is the phenomenological classification involving empirical functions while the remaining methods involve solving transport equations for the turbulence quantities. The fifth classification involves writing and solving a transport equation for each turbulent stress or flux component. This last turbulence modeling approach has not yet been used in geophysical or surface water models and therefore will be dealt with no further. Launder and Spalding (1972), Reynolds (1976), and Rodi (1980) review the stress flux equations in more detail.

#### Eddy Viscosity/Diffusivity and Dispersion Coefficients

184. The basic method for closing the model equations presented in Part IV is the eddy viscosity or diffusivity approach. For the general case of three-dimensional flow, the general momentum flux definition in indicial notation is:

Table 9. Turbulence model subclassifications for level three

Name	Reference/Origin	Concept
1. Zero Equation		
a) Constant Eddy Viscosity/ Diffusivity	Boussinesq (1877)	Eddy viscosity, $E$ ; constant throughout flow field
b) SGS grid size dependent	Smagorinsky et al. (1965)	$E \propto \Delta$ To grid volume and energy dissipated in volume
c) Empirical functions	Prandtl (1945) Munk and Anderson (1945)	$E \propto (l_m) (V)$ ; $l_m$ = mixing length, $V$ = velocity scale $E \propto (l_m, R_i)$ ; $R_i$ = Richardson's Number
2. One Equation	Kolmogorov (1968)	$E = \epsilon_1 \sqrt{kL}$ $k$ = velocity scale; found from diff. equation $L$ - empirically specified; length scale
3. Two Equation	Chou (1945)	$E = \epsilon_1 \sqrt{kT}$ ; both $k$ and $L$ found from diff. transport equations
4. Turbulent Stress/ Flux Models	Chou (1945)	No viscosity/diffusivity; diff. equations solved for each flux or stress component



$$-\overline{u'_i u'_j} = E_t \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} k \delta_{ij} . \quad (59)$$

Here  $E_t$  is the turbulent eddy viscosity and  $\delta_{ij}$  is the Kronecker Delta function equal to one when  $i = j$  and 0 when  $i \neq j$ . The primed and unprimed quantities are as in the definition of the mean and fluctuation quantities. The variable  $K$  is the kinetic energy of the fluctuation motion, i.e.

$$k = \frac{1}{2} (\overline{u'^2} + \overline{v'^2} + \overline{w'^2}) \quad (60)$$

The eddy viscosity is a function of the state of turbulence and is not a property of the fluid.

185. The eddy diffusivity is an analogous concept and in the general three-dimensional case:

$$-u'_i \alpha' = K^* \frac{\partial \alpha}{\partial x_i} \quad (61)$$

It is customary to relate the turbulent eddy viscosity and diffusivity by the turbulent Prandtl (for temperature) or Schmidt (mass species) Number,  $\sigma_t$ :

$$\sigma_t = \frac{E^*}{K^*} ; \quad K^* = \frac{E^*}{\sigma_t} \quad (62)$$

186. These definitions originally were applied to the Reynolds stress and transport fluxes occurring from the time averaging of the model equations, i.e. terms ① and ② in Equations 31, 32, and 34; terms ① and ③ in Equations 39, 40, and 41; terms ② and ④ in Equations 45 and 48; and terms ① and ③ in Equations 51 and 52. Recognizing that turbulence at large scales is anisotropic, the eddy viscosity/

diffusivity is often different in magnitude and functional form in different coordinate directions.

187. The spatial averaging performed in the development of the one- and two-dimensional models also results in terms requiring closure. The Boussinesq concept above has been extended to close these terms by defining dispersion coefficients which relate the subaverage scale correlations to spatial gradients of the mean flow or transport variables. As an example from the one-dimensional model, term ④ in Equation 52 is closed as:

$$- \int (u' \alpha') dA = - D^* \frac{\partial \alpha}{\partial x} \quad (63)$$

where  $\alpha$  is the area-time average value of  $\alpha$ , and  $D$  is the dispersion coefficient. Should the distinction between all these terms be retained in the model, a very large number of empirical viscosity/diffusivity and dispersion coefficients need to be specified. In practice, not all these terms are important: for instance, in the mass transport equation the dispersion terms are dominant and the combination of such terms into one empirical relation proportional to a spatial gradient is often made. In the review of subgrid scale models in the next section, models retaining the distinction between Reynolds and dispersion terms will be reviewed. However, it is not common practice to model the separate terms. Table 10 summarizes the functional definitions for the model closures. The next section reviews models for the viscosity/dispersion terms.

#### Functional Forms for the Eddy Viscosity/Diffusion and Dispersion

188. This section details the forms used in specifying the coefficients defined in Table 10. Along with

Table 10. Functional definitions for model closure

Model Type	Eq. No.	Term No.	Reynolds Stress/Flux Closure	Eq. No.	Term No.	Dispersion Closure	Combined Form
1DR	51	1	$\tau_{xx} = \rho E^* \partial u / \partial x$	51	2	$\int \overline{u' u'} dA = -D^* \partial u / \partial x$	$E \partial u / \partial x = (E^* + D^*) \partial u / \partial x$
	52	3	$N_x = \rho K^* \partial \alpha / \partial x$	52	4	$\int \overline{u' \alpha'} dA = -D^* \partial \alpha / \partial x$	$K \partial \alpha / \partial x = (K^* + D^*) \partial \alpha / \partial x$
2DHT	39, 40	1	$\tau_{xx} = \rho E_{xx}^* \partial u / \partial x$	39, 40	2	$1/\rho \int \rho (\overline{u' u'}) dz = D_{xx}^* \partial u / \partial x$	$E_{xx} \partial u / \partial x = (E_{xx}^* + D_{xx}^*) \partial u / \partial x$
	39, 40	1	$\tau_{xy} = \rho E_{xy}^* \partial u / \partial y$	39, 40	2	$1/\rho \int \rho (\overline{u' v'}) dz = D_{xy}^* \partial u / \partial y$	$E_{xy} \partial u / \partial y = (E_{xy}^* + D_{xy}^*) \partial u / \partial y$
	39, 40	1	$\tau_{yy} = \rho E_{yy}^* \partial v / \partial y$	39, 40	2	$1/\rho \int \rho (\overline{v' v'}) dz = D_{yy}^* \partial v / \partial y$	$E_{yy} \partial v / \partial y = (E_{yy}^* + D_{yy}^*) \partial v / \partial y$
	39, 40	1	$\tau_{yx} = \rho E_{yx}^* \partial v / \partial x$	39, 40	2	$1/\rho \int \rho (\overline{v' u'}) dz = D_{yx}^* \partial v / \partial x$	$E_{yx} \partial v / \partial x = (E_{yx}^* + D_{yx}^*) \partial v / \partial x$
2DVRH	41	3	$N_x = \rho K_x^* \partial \alpha / \partial x$	41	4	$1/\rho \int \rho (\overline{u' \alpha'}) dz = D_x^* \partial \alpha / \partial x$	$K_x \partial \alpha / \partial x = (K_x^* + D_x^*) \partial \alpha / \partial x$
	41	3	$N_y = \rho K_y^* \partial \alpha / \partial y$	41	4	$1/\rho \int \rho (\overline{v' \alpha'}) dz = D_y^* \partial \alpha / \partial y$	$K_y \partial \alpha / \partial y = (K_y^* + D_y^*) \partial \alpha / \partial y$
2DVTH	45	2	$\tau_{xx} = \rho E_{xx}^* \partial u / \partial x$	45	3	$1/\rho \int \rho (u' u') dy = D_{xx}^* \partial u / \partial x$	$E_{xx} = (E_{xx}^* + D_{xx}^*) \partial u / \partial x$
2DVTS	45	2	$\tau_{xz} = \rho E_{xz}^* \partial u / \partial z$	45	3	$1/\rho \int \rho (u' w') dy = D_{xz}^* \partial u / \partial z$	$E_{xz} = (E_{xz}^* + D_{xz}^*) \partial u / \partial z$
2DVRH	48	4	$N_x = \rho K_x^* \partial \alpha / \partial x$	48	5	$1/\rho \int \rho (u' \alpha') dy = D_x^* \partial \alpha / \partial x$	$K_x \partial \alpha / \partial x = (K_x^* + D_x^*) \partial \alpha / \partial x$
2DVRH	48	4	$N_z = \rho K_z^* \partial \alpha / \partial z$	48	5	$1/\rho \int \rho (v' \alpha') dy = D_z^* \partial \alpha / \partial z$	$K_z \partial \alpha / \partial z = (K_z^* + D_z^*) \partial \alpha / \partial z$
3DTH	31, 32	1	$\overline{u' u'} = E_{xx} \partial u / \partial x$				
3DTS	31, 32	1	$\overline{u' u'} = E_{xy} \partial u / \partial y$				
3DRH	31, 32	1	$\overline{u' w'} = E_{xz} \partial u / \partial z$				
3DRS	31, 32	1	$\overline{v' u'} = E_{yx} \partial v / \partial x$				
	31, 32	1	$\overline{v' v'} = E_{yy} \partial v / \partial y$				
	31, 32	1	$\overline{v' w'} = E_{yz} \partial v / \partial z$				
	34	2	$\overline{u' \alpha'} = K_x \partial \alpha / \partial x$				
	34	2	$\overline{v' \alpha'} = K_y \partial \alpha / \partial y$				
	34	2	$\overline{w' \alpha'} = K_z \partial \alpha / \partial z$				

NOT APPLICABLE

NOT APPLICABLE

the functional forms will be an estimate of the values of the coefficients in the functional form. In all these models, except the constant viscosity model, the basic relationship is that  $E$  or  $K$  is proportional to the product of a velocity scale,  $V$ , and a length scale,  $L$ . Much of this material has been assimilated from a variety of reference texts, particularly Launder and Spalding (1972), Rodi (1980), Fischer et al. (1979), various reports, and over 300 papers. Absolutely no effort has been made to determine in any statistical or measured fashion the "best" coefficient for each model. Rather the values are those reported by the principal author or user of the model. This might be discouraging, particularly in the case of the zero equation models which tend to be less general in functional form and contain empirical coefficients which are often site specific. It is this author's belief that if all other sources of model errors are controlled, the model users' experience becomes an integral part of the model calculation in that based on the use of the reported coefficients selective user judgments concerning changes in coefficient values become the key to the adequate use of the zero equation models. Therefore, the reported values are to be considered by the user as good starting values and the user should be prepared to have sufficient data to validate the coefficient selection and perform sensitivity studies. A summary table for each major class of model function will accompany a brief verbal description of the function. Basic assumptions necessary for accurate application will also be summarized as will references to recent works reporting the use of the form being presented. Stress/flux transport models are not reviewed. The last part of this report offers an assessment of and selection of guidelines for these models as well as a summary of the requirements for the use of higher order turbulence models.

### Zero Equation Models

189. A summary of this class of models is found in Table 11.

#### Constant viscosity/diffusivity/dispersion (No. 1)

190. In its most general form, the assumption of this model is that there is no functional dependence used to calculate the coefficients. Two methods can then be used to specify the values. First, values used from flow fields that are similar in size or values from previous onsite model studies may be adapted. The second method is to simply perform a field study at the scales to be resolved in the model and measure the values. Such studies are very expensive. In both methods comparisons of calculations with field data are required for tuning; therefore, moderate to heavy data collection is required for validation. There is little predictive value in these coefficients; however, successful hindcasts have been performed when sufficient field data are available.

#### Grid size dependent dissipation schemes (No. 2)

191. Originally developed and used by meteorology modelers (Smagorinsky et al. 1965), the basis of these models is that the energy dissipation within the grid volume must be correct such that the cascade process can be replicated properly in the resolved flow field. The major assumption is that the production and dissipation of turbulence are taking place within the grid volume. No transport of turbulence quantities is assumed. Although three-dimensional (Smagorinsky et al. 1965) and two-dimensional forms (Lilly 1969) are available, these models for some reason have not been used very heavily in multidimensional surface water flows. Works by Bedford (1981, 1980) and Liu and Leendertse (1981) are representative. In these works the models have been shown to be particularly effective in simulating the large

Table 11. Zero equation turbulence models

Model no	Model Application or Use (Table 6)	Reference	Function form	Definitions	Coefficient Values	Comments
1a	All	Not applicable	$\epsilon$ or $k = \epsilon$	$\epsilon = \text{constants}$	User adapted	Isotropic turbulence/mixing
1b	All	Not applicable	$E_x$ or $E_y = E$ or $K_x = K_y = E$ $E_z$ or $D_z = E_z$	$E_1$ and $E_2 = \text{constants}$	User adapted	An isotropic mixing for shallow water
2	2DHT, 2DHR, 2DVTM, 2DVTs, 2DVRM, 2DVRs, 3DVM, 3DVS, 3DVRM, 3DVRs, (1981)	Seagorinsky et al. (1965) Bedford (1981) Liu and Leendertse (1981)	$E_x = E_y = E = C \Delta x \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)$	$C = \text{coefficient}$ $\Delta x = \text{length scale}$ $\Delta E_3 = (\Delta x^2 + \Delta y^2 + \Delta z^2)^{1/2}/2$ $\Delta E_2 = (\Delta x^2 + \Delta y^2)^{1/2}$ $w = \text{vorticity}$	$C = 0.2 - 0.4$	Horizontal eddy viscosities and diffusivities. Will marginally work in 3D homogeneous flows
3a	2DHT, 2DHR, 2DVTM, 3DTH, 2DVS, 3DVS, 2DVRM, 3DVRM, 2DVRs, 3DVRs, (1967)	Okubo (1974) Okubo (1968) Yudelison (1967) Batchelor (1967)	$E_x = A_1 \Delta x^{4/3} = E_y$ $K_x = B_1 \Delta x^{4/3} = K_y$	$\Delta E_3 = (\Delta x^2 + \Delta y^2 + \Delta z^2)^{1/2}/2$ $\Delta E_2 = (\Delta x^2 + \Delta y^2)^{1/2}$ $A_1 = \text{coefficients for dissipation}$ $B_1 = \text{coefficients for dissipation}$	$A_1 = B_1 = 0.1(10^{-2})^2 \text{ m}^2/\text{sec}^3$	Use for horizontal flows with relatively isotropic turbulence in the plane
3b	same as 3a	Okubo (1976)	$E_x = K_x = 0.068 \Delta x^{1.15}$ $E_y = K_y =$	$\Delta x$ as in 3a		Same as 3a, i.e. horizontal viscosity/diffusivity
4	2DHT, 2DHR, 2DVTM, 2DVS	Okoye (1970)	$K_x = K_y = C u^* H$ $E$ and $K$ are related by Prandtl $\epsilon, \alpha_c$	$C = \text{coefficient}$ $u^* = \text{friction velocity}$ $H = \text{total depth}$ $C_f = \text{friction coefficient}$	$C = 0.067$	Dispersion in horizontal 2D flows dominated by bed shear generated turbulence
5	All except 1DT, 1DR	Hinze (1975) McComell and O'Connor (1977)	$E_z = k u^* z(1-z/H)$ $K_z$ related to $E_z$ by Prandtl No. $\alpha_c$	$H = \text{total depth}$ $K = \text{von Karman's coefficient}$ $u^* = \text{friction velocity (see No. 4)}$ $R_H = \text{hydraulic radius}$ $S = \text{slope hydraulic grade line}$	$K = 0.4$	Used in neutral conditions i.e. $E_z = E_{z0}$ $K_z = K_{z0}$
6a	All except 2DHT, 2DHR, 2DVTM, 1DT, 2DVS, 1DR	Munk and Anderson (1948) Rossby and Montgomery (1935)	$E_z = E_{z0} (1 + \frac{1}{2} R_1)^{-\alpha_1}$ $K_z = K_{z0} (1 + \frac{1}{2} R_1)^{-\alpha_2}$	$R_1 = \text{Richardson No.}$ $\frac{g \Delta \rho}{\rho \Delta z}$ $\frac{(g \Delta z)^2}{\rho \Delta z}$ $\rho = \text{density}$ $q = \text{Vector sum horizontal velocity}$ $= (u^2 + v^2)^{1/2}$	$B_1 = 10; \alpha_1 = 0.5$ $B_2 = 3.33; \alpha_2 = 1.5$	$E_{z0}, K_{z0} = \text{neutral stability values. Thus, form not usable for unstable stratification}$

(Continued)

Table 11. (Concluded)

Model No.	Model Application or Use (Table 6)	Reference	Function Form	Definitions	Coefficient Values	Comments
6b	same as 6a	Mannayev (1958)	$E_z = E_{z0} e^{-E_z R_1}$ $K_z = E_{z0} e^{-E_z R_1}$	As above	As above	Will handle instability during surface cooling
6c	same as 6a	Ellison (1957)	$E_z = \frac{b(1-R_f/R_{fc})}{(1-R_f)^2}$	$b = K_z/E_z$ in neutral conditions $R_f = \text{flux Richardson No.} = (b_z/E_z) R_1$ $R_{fc} = \text{critical } R_f \text{ where } Ku = 0$	$R_{fc} = 0.15$ $R_{fc} = 0.05$	
6d	same as 6a	Kullenberg et al. (1973)	$K_z = \frac{W^2}{R_1(dq/dz)}$	$W = \text{wind speed}$	$\xi = \text{adjustable coefficient}$	For surface mixing initiated by winds greater than 4-5 m/sec
7a	1D 1DR	Harleman (1966)	$K_x = 63 n U_H^5 / b_z^2 (\text{m}^2/\text{sec})$ $E_x = 0$	$n = \text{Manning's roughness coefficient}$ $U = \text{Velocity, cross-section average}$ $R_h = \text{hydraulic radius}$		Used in homogeneous, non-stratified portions of estuary
7b	1D 1DR	Fischer (1979)	$K_x = 0.011 U^2 w^2 / du^*$ $E_x = 0$	$W = \text{full stream width}$ $d = \text{depth}$ $u^* = \text{friction velocity, } \sqrt{g h S_0}$ $U^2 = \text{average velocity.}$		Also used in homogeneous portion of estuary
7c	1D 1DR	Holley et al. (1970) Thatcher and Harleman (1972)	$K_x = K^*(3S/\bar{X}) + K_{xm}$	$K^* = 0.002 U_{L2}(E_s) - 0.25$ $S' = S/S_0$ ; $S_0 = \text{ocean salinity}$ $\bar{X} = x/L_2$ ; $L_2 = \text{estuary length}$ $K_{xm} = 3K_x$ ; $K_x$ as in 7a $U_0 = \text{max cross-sectional mean tidal velocity at estuary entrance}$ $E_s = \text{estuary No. (Eq. 8 in Part II).}$		Can't be used at downstream end of estuary where estuary is wide or if estuary is very irregular. Excessive lateral transport results in higher $K_x$ which is not reflected in these formulae

horizontal scales of motion which, for estuaries, are several orders of magnitude larger than the depth scale. Coefficient values are determined once for each basin and therefore neither require nor permit any tuning. It should be noted that this is one of the few models which explicitly accounts for the grid size influence on the eddy viscosity value. This reflection of reality is not often observed in other turbulence models.

General empirical functions (Nos. 3, 4, 5, and 6)

192. Again with reference to Table 11 a number of forms are available for specifying the coefficient values. Again, all these forms presuppose that production and dissipation are equal and taking place at the point in question. The method of structuring such a model involves assuming one or perhaps two dominant physical mechanisms that cause the turbulence which is to be modeled. Then, under these simplified conditions, it is possible to create a mathematical formula representing the turbulence. Such formulae cannot represent more than two or so physical mechanisms, are site specific, and do not work well in highly complicated flows or domains.

193. In so far as the specification of horizontal activity is concerned, it is noticed that forms 3a and 3b are power law expressions also incorporating the grid size. It should be noted that for homogeneous flows, very few functional forms for traditional horizontal mixing and dispersion coefficients exist apart from Nos. 1, 2, and 3. For the special case of the 2DH model where a thin shear exists whose turbulence is controlled by bottom friction, turbulence model No. 4 is available.

194. With regard to the vertical mixing, dispersion, and turbulence models, it should be noted that the physics contributing to them is compressed into a relatively thin vertical extent and stratification must be represented



sought. Second, a transient model is often used to calculate the short time transport pattern modifications introduced immediately after an impulsive change to a new steady rate in the imposed boundary loadings or ambient flow field condition.

213. Some might say environmental hydrodynamics and transport models are far field models of the entire estuary (or river, lake, or coastal region). Here the natural geometry of the estuary controls the circulation as opposed to inertia. Boundary loadings are usually nonsteady as is the model calculation itself. All the various turbulence processes are effective and range through several decades of size and frequency. The horizontal extent of the problem is at minimum several orders of magnitude greater than the vertical extent, yet the effect on the horizontal transport of the vertical stratification is crucial to the proper calculation of the estuary transport.

#### Engineering Hydraulics - Assessment

214. There can be really no significant disagreement that the introduction of the two equation turbulence modeling approach has resulted in very accurate calculations of near field transport. Rodi's monograph (1980) is as complete a documentation of the success of near-field K-E calculations as will be found. The papers presented at the Jackson, Mississippi (1982), and MIT (1983) ASCE conferences also further confirm this success. Buoyancy-affected flows with and without recirculation are for the first time readily calculated and with little controversy in the selection of coefficients. Channel cross flows and secondary circulations are also for the first time now readily and accurately calculable, but extensions to stress equations are necessary. Both two- and three-dimensional, steady and unsteady versions are available and have been

## PART VII: TURBULENCE MODEL ASSESSMENT AND SELECTION GUIDELINES

### Turbulence Model Assessment

212. As has been mentioned before, the transport model equations described in Part IV may be tailored to fit a variety of problems. In this author's opinion, it is useful to distinguish between transport models that are concerned with engineering hydraulics and those dealing with environmental hydrodynamics and transport. The distinction between these application areas are manyfold; however, the following general distinctions are useful. Applications for engineering hydraulics are calculations performed to analyze the transport in, near, or around an engineered public works or the nearfield impact of same. Examples include nearfield sewage or cooling water discharges, flow in straightened channels or bends, irrigation works, construction of river regulation systems such as dams or locks, design and installation of tidal control works, and harbor design and channel dredging and maintenance. In such problems the transport is often initially dominated by advection or inertia very near the structure with turbulence and mixing playing increasingly more important roles with increasing distance from the structure. Flow interaction with boundaries is very important and often results in recirculation zones. Very important is the horizontal scale of the problem, which is in many cases only one or two orders of magnitude greater than the depth; therefore, model resolution becomes more fully three-dimensional. Stratification becomes a controlling influence in the transport near the hydraulic works. Two time periods are of concern in Engineering Hydraulics models. First is steady-state where the boundary flow loading, say for example the discharge, might be assumed steady and a resulting plume pattern

are quite simply derived as are the resulting mixing models. Partial resolution of a process is a situation where the above relationship is quite complex, often unknown, and quite difficult to summarize into empirical turbulence models.

time and space will greatly determine the functional form for the parameterization of the correlations in terms of grid scale. Model resolution limits are also set by the selection of these model features. The grid also imposes further limits to model resolution. As a rule of thumb when erecting a nodal grid for discretization, the Nyquist sampling theorem (Otnes and Enochson 1972) says that the absolute minimum number of nodal points necessary to reconstruct a signal with period or time base  $T$  or wavenumber or spatial scale  $L$  is three. In practice, the number is often greater than four, particularly for lower order interpolations. If this be so, then the minimum time step  $\Delta t$  or grid size  $\Delta x$  necessary to resolve a variable process on a model grid is  $\Delta t = T/2$  or  $\Delta x = L/2$ . It is now possible to preliminarily relate the grid size to the minimum size of process to be resolved. Obviously in so doing one also determines what processes are not resolved. Since the subgrid scale model must summarize the nonresolved processes, the complexity of the nonresolved processes determines to a great extent the form of the subgrid parameterization.

#### Grid Selection and Process Resolution

211. It is not at all wise to select a grid spacing that only partially resolves a coherent process. This is particularly the case for periodic or intermittent phenomena such as Kelvin or internal waves, or Kelvin Helmholtz instabilities. In other words, a grid must be selected which falls in the spectral gap or space-time gap in the process diagram. All the functional subgrid scale parameterization forms have been developed for cases where assumptions about the production and dissipation of energy at the grid scale favor a spectral gap. At the spectral gap, the relationships between turbulence production and dissipation

## PART VI: COMMENTS ON THE NUMERICAL SOLUTION

208. After selecting the model equation structure and the turbulence or mixing model the numerical integration procedure must be selected and applied. This selection is not within the scope of this report however three aspects of the numerical integration procedure do affect the resolution of the model and the successful selection and use of a turbulence model. These three items are commented on because they are a source of significant error in the calculation and often these errors are not repaired but covered up by changing empirical coefficients in the mixing models. Such tuning gets the correct answer but does not result in a correct model.

### Numerical Approximation and False Diffusion

209. It is very well known (Mesinger and Arakawa 1976) that incorrect approximations to derivatives in the model equations can lead to truncation errors which are in some cases the same size as the terms being approximated. This is particularly the case for the advection or inertia term. A review of the proper methods, constraints, and procedures for handling such false diffusion problems is found in Gosman and Lai (1982). Very careful attention to the numerical structure of the derivative approximations is the single most important way to reduce the variance between model calculations and data.

### Model Resolution Limits Due to Grid Size

210. The procedures of Reynolds averaging and depth, lateral or area averaging result in terms composed of correlations of subgrid or averaging scale variables. As discussed before, the selection of the type of spatial average in the model and the selection of the grid size in

1/2, the superequilibrium version corresponds to Mellor's level 2-1/4. Both such efforts are unique in that within one "program" a suite of evermore complex turbulence models can be selected and tailored to the problem at hand.

Table 13. Two equation turbulence models

Model No.	Model Application or Use	Reference	Definitions	Equation/Function Form	Eq. No.	Coefficient Values
10	All Models in Table 6	Rodi (1980, 1982)		$E = \epsilon_1 k^2/c$ $\text{Transport Eq. for } k:$ $\frac{\partial k}{\partial t} + u_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \epsilon \frac{\partial k}{\partial x_j} \right) + \epsilon \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i}$ $+ \frac{\epsilon}{g} \frac{\partial \omega}{\partial x_j} - \epsilon P$ $P = \text{stress production term (Eq. 1)}$ $G = \text{buoyancy production term (Eq. 2)}$ $R_f = -1/2 G \omega^2 / (P+G)$ $= \text{flux Richardson No.}$ $\text{Transport Eq. for } \epsilon:$ $\frac{\partial \epsilon}{\partial t} + u_j \frac{\partial \epsilon}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \epsilon \frac{\partial \epsilon}{\partial x_j} \right) + \epsilon_{1c} \frac{\epsilon}{k} (P + G)$ $- (1 + \epsilon_{3c} R_f) - \epsilon_{2c} \frac{\epsilon^2}{k}$	(1)	$\epsilon_1 = 0.09$  $\epsilon_{1c} = 1.44$ $\epsilon_{2c} = 1.92$ $\epsilon_{3c} = 0$ for nonstratified flows $\epsilon_{3c} = 1.3$ for stratified flows using Rodi's (1980) remedy; $C_{3c} = 0.8$ (No-sain 1977)
11	All 2DH class models	Rastogi and Rodi (1978)		$E = \epsilon_1 \frac{k}{c}$ $\text{Transport Eq. for } k:$ $\frac{\partial k}{\partial t} + u \frac{\partial k}{\partial x} + v \frac{\partial k}{\partial y} = \frac{\partial}{\partial x} \left( \epsilon \frac{\partial k}{\partial x} \right) + \frac{\partial}{\partial y} \left( \epsilon \frac{\partial k}{\partial y} \right) + P_H + P_{kv} - c$ $\text{Transport Eq. for } \epsilon:$ $\frac{\partial \epsilon}{\partial t} + u \frac{\partial \epsilon}{\partial x} + v \frac{\partial \epsilon}{\partial y} = \frac{\partial}{\partial x} \left( \epsilon \frac{\partial \epsilon}{\partial x} \right) + \frac{\partial}{\partial y} \left( \epsilon \frac{\partial \epsilon}{\partial y} \right) + \epsilon_{1c} \frac{\epsilon}{k} P_H - \epsilon_{cv} - \epsilon_{2c} \frac{\epsilon^2}{k}$ $P_H = \epsilon \frac{2}{k} \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} + \frac{\partial v}{\partial y} \frac{\partial v}{\partial y} + \frac{\partial u}{\partial y} \frac{\partial v}{\partial x}$ $P_{kv} = \epsilon_k \frac{u^3}{H}$ $P_{cv} = \epsilon_c \frac{u^4}{H^2}$ $u^* = C_f (u^2 + v^2)$ $\epsilon_k = 1/\sqrt{C_f}; \epsilon_c = 3.6 \frac{\epsilon_{2c}}{C_f} \sqrt{\epsilon_1}$	(1)	$\epsilon_{1c} = 1.44$ $\epsilon_{2c} = 1.92$

where the bar indicates Reynolds averaging. An exact equation for  $E$  can then be derived from the Navier Stokes. The model form of the  $E$  equation is as in Model No. 10, Equations (2) and (3), in Table 13. This is called the  $k-\epsilon$  model.

205. A 2DH version is also available and is presented in Model No. 11, Equations (2) and (3) in Table 13. Model No. 10 has also been modified for stratified effects as per Rodi (1980). Raithby and Schneider (1980) present a  $k-E$  model that contains a more general approach to the algebraic equation stratification method of McGuirk and Rodi (1979). This model and a later version (Elliot and Raithby 1981) have been applied to the calculation of surface discharge cooling water jets both into ambient water and in a coastal discharge situation with currents.

#### Higher Order Turbulence Models

206. attempts at using turbulence models with more detailed transport equation expressions for each term requiring closure are just now being applied to surface water or estuary flows.

207. Of note here are the hierarchical schemes being used by G. Mellor at Princeton (see Mellor and Yamada 1982, 1974) and the invariant modeling hierarchy of turbulence models used by Aeronautical Research Associates of Princeton (ARAP) group also in Princeton, New Jersey. Relevant papers include Sheng (1982), Lewellen et al. (1980), and Lewellen (1977). Mellor has recently applied the level 2-1/2 model to the Hudson/Raritan River Estuary. Although Mellor's full model, i.e., level 4, is a stress equation model, it has not been used for estuary or surface water model simulations. The ARAP model also has a built-in suite of turbulence models to use and they correspond to Mellor's, i.e. ARAP's quasi-equilibrium version corresponds to Mellor's level 2-



water discharge (Rodi 1982); coastal sediment transport and vegetative canopy boundary layers (Sheng 1982); ocean Ekman boundary layers (Weatherly 1982); New York Bight/Harbor estuary models (Mellor et al. 1983); heated effluent discharges (Raithby 1983), channel expansions (Chapman 1983); and canopy boundary layers (Burke and Stolzenbach 1983).

203. Conceptually, a second equation for the length scale, or as Rodi points out any value  $Z$  of the form  $Z = k^m L^n$ , is written. The general transport equation for  $Z$  is (Rodi 1980):

$$\underbrace{\frac{\partial Z}{\partial t}}_{\text{rate of change}} + \underbrace{u_i \frac{\partial Z}{\partial x_i}}_{\text{convection}} = \underbrace{\frac{\partial}{\partial x_i} \left( \frac{\sqrt{k} L}{\sigma_Z} \frac{\partial Z}{\partial x_i} \right)}_{\text{diffusion}} + \underbrace{C_{Z1} \frac{Z}{k} P}_{\text{production}} - \underbrace{C_{Z2} Z \frac{\sqrt{k}}{\partial L}}_{\text{destruction}} + S \quad (68)$$

where  $r_Z$  is an empirical coefficient as are  $C_{Z1}$  and  $C_{Z2}$ . The variable  $P$  is the production as defined in Equation 65, while  $S$  is the source term whose form depends on the definition of  $Z$ .

204. Under high Reynolds Number conditions with local isotropy the dissipation can be expressed in terms of vorticity as:

$$\epsilon = \gamma \frac{\overline{\partial u_i^2}}{\partial x_j^2} \quad (69)$$

Leendertse model has been extensively applied and tested and the turbulence model is unique in that it combines the best features of the subgrid scale models (i.e. No. 2, Table 11), which are excellent for the larger scale horizontal flows, with the turbulence model approach for vertical exchange or mixing. Several evolutions of this turbulence model have been applied and tested. The most significant advance occurred with the elimination of a Richardson Number dependent approach (No. 6a - 6d) for handling stratification.

201. Very few whole estuary one equation models exist other than these.

Two Equation Turbulence Models (Nos. 10 and 11; Table 13)

202. Without doubt the single most influential development in parameterizing mixing and turbulence is the use of two equation turbulence models. The prime mover in introducing these methods into the hydraulics community has been W. Rodi, whose review on turbulence modeling has already been cited. In this work a significant review of actual test cases is presented and tabularized. It should be noted that the cases reviewed are for the most part confined to engineering hydraulics, i.e., flows either free stream or recirculating, that are confined to simple geometries. These include: wakes, jets, channel flows with buoyancy, and cross flow effects variously affecting these flows. Examples of more recent works have been presented in a series of conference sessions at the American Society of Civil Engineers (ASCE) Hydraulics Division Conferences in Jackson, Mississippi (Smith 1982), and Massachusetts Institute of Technology (MIT) (Shen 1983). A review of the two equation models presented at these review sessions shows they have been applied to: one- and two-dimensional analyses of sedimentation basins (Schamber 1982; Schamber and Larock 1980); side channel discharge into a channel and cooling

Table 12. One equation turbulence models

Model No.	Model Application or Use, (Table 6)	Reference	Definitions	Equation/Function Form	Eq. No.	Coefficient Values
8	All 2DV Model Classes	Smith and Dyer (1978)	$\beta$ = coefficient of thermal expansion or salt coefficient depending on stratification $\alpha$ = concentration, S, or temperature, depending on stratification erf = error function; $R_1, R_y, R_{fc}$ as in Model 6c, Table 11.	$E = \epsilon_1 \sqrt{K} L$ <p>Transport Eq. for <math>k</math>:</p> $\frac{\partial k}{\partial t} + u_1 \frac{\partial k}{\partial x_1} = \frac{\partial}{\partial x_1} \left( \epsilon \frac{\partial k}{\partial x_1} \right) + \epsilon \frac{\partial u_1}{\partial x_j} \frac{\partial u_j}{\partial x_1} + \beta g \frac{\partial \alpha}{\partial x_1} - \epsilon_D \frac{k^{3/2}}{L}$ <p>Length Scale Eq.</p> $L = \frac{.105H \operatorname{erf}(\delta z/H)}{1 + 0.24 \beta_1}$ $\frac{\sigma_{t_0}}{\sigma_t} = \frac{1 - R_y/R_{fc}}{1 - 2R_y}$	(1)   	

$$\underbrace{-\beta g_i \overline{u'_i \alpha'}}_{G=\text{buoyant production/destruction}} - \underbrace{\gamma \frac{\overline{\partial'_i}}{\partial x_j} \frac{\overline{\partial'_i}}{\partial x_j}}_{E=\text{viscous dissipation}} \quad (65)$$

Many of these terms require expression in terms of the mean flow field dependent variables; therefore, it is customary (see Rodi) to say that:

$$u'_i \left( \frac{u'_j u'_j}{2} + \frac{p'}{\rho} \right) = \frac{E}{\sigma_k} \frac{\partial k}{\partial x_i} \quad (66)$$

where  $\sigma_k$  is an empirical coefficient. Also,

$$\epsilon = C_D \frac{k^{3/2}}{L} \quad (67)$$

199. The specification of the length scale must be done empirically. Rodi reviews various methods for these scales particularly as regards "relatively" simple flows such as jets and thin shear layers.

200. Table 12 contains two examples of existing applications to large-scale estuary or estuary type flows (as opposed to the smaller scale flows such as jets, etc.). The length scales in both models are mixing length type formulations; note that L in No. 9 is Model No. 5 in Table 11. Note also that model No. 8 treats buoyancy effects within the length scale while model No. 9 treats such effects directly within a differential equation (Equation No. 2 Table 12, Model No. 9). The Smith-Dyer model has been used to calculate vertical profiles of velocity, etc., in United Kingdom estuaries, while the Liu-Leendertse model is a 3D model used for a variety of estuaries. The Liu-

# One Equation Turbulence Models (Nos. 8 and 9; Table 12)

197. One equation models and the two equation models to be reviewed in the following section are distinguished from the empirical/mixing length models because they permit, to a greater or lesser extent, turbulence quantities to be transported. This is an attempt to undo the assumption in the previous models that production and destruction of turbulence occur at the same point in space. Permitting advection of turbulence quantities is more realistic in that it now permits more unsteady flows to be accurately calculated.

198. Using the basic eddy viscosity assumption, the eddy viscosity,  $E$ , is set equal to (Prandtl 1945; Kolmogorov 1942):

$$E = \phi \sqrt{k} L \quad (64)$$

where  $\phi$  is a coefficient,  $L$  is a length scale, and  $k$  (Equation 60) is the kinetic energy density of the velocity fluctuations in the three directions. The one equation model uses a transport equation for determining  $k$  and uses an empirical function for determining  $L$ . One additional differential equation is therefore required for the model. Following Rodi's (1980) excellent review, the general equation for  $k$  is derived directly from the general 3D Navier Stokes equation as:

$$\underbrace{\frac{\partial k}{\partial t}}_{\text{rate of change}} + \underbrace{\frac{u_i \partial k}{\partial x_i}}_{\text{trans- port}} = \underbrace{\frac{\partial}{\partial x_i} \left[ u_i' \frac{u_j' u_j'}{2} + \frac{p'}{\rho} \right]}_{\text{diffusive transport}} - \underbrace{\frac{u_i' u_i'}{2} \frac{\partial u_i}{\partial x_j}}_{P=\text{shear production}}$$

properly. For homogeneous thin shear layer conditions, Prandtl's mixing length theory (Model No. 5) has been used in estuary studies. The modifications for stratification effects have assumed a relation between the suppression of turbulence by stratification and the production of turbulence by vertical shear. Models 6a - 6d all represent various methods for this relation. The most popular form is 6a. It suppresses turbulence properly when the stratification is high; however, it will not properly portray the conditions existing when cold water overlays warm water. Model 6b is an attempt to do so although it has not been used that often. Models 6c and 6d are refinements based upon more complicated summaries of the assumed physics. Model 6d is used particularly in situations where the surface layer mixing is directly due to wind. The value of  $E_{z0}$  and  $K_{z0}$  in these equations is specified from Models 1, 3, or 5. The combination of 5 with 6a has been used quite successfully by Heinrich, Lick, and Paul (1983) in lake simulations.

#### One-dimensional mixing and dispersion (No. 7)

195. Models for one-dimensional mixing and dispersion are found in Nos. 7a - 7c. These parameterizations have been quite numerous and subject to considerable debate. It should be noted that these models are so heavily averaged that the SGS models are called upon to represent a myriad of eliminated physics. The empirical approach will be quite taxed as they cannot parameterize more than a few dominant mechanisms.

196. Significant publications in this area exist, particularly the work by Fischer et al. (1979). Fischer's work contains compilations of both functional forms and values reported from field studies. The models reported here have been extensively used and represent excellent techniques for estimating an initial sequence of values.

validated. Clearly, one-dimensional models are of no use in the calculation of engineering hydraulics problems. The author feels very strongly that the optimal use of K-E models is for this class of problem and recommends their use as in Table 13, Nos. 10 and 11. It should be noted that for problems where very strong boundary curvature is present and/or secondary currents, it will be necessary to consider going to a more complicated stress equation model. This calculation is just beginning to be readily performed.

#### Environmental Hydraulics and Transport

215. The recommendation of refined turbulence modeling for engineering hydraulics problems does not necessarily extend to full basin unsteady flow problems. The horizontal scale of the problem is usually quite large in comparison to the depth and, consequently, horizontal turbulent fluctuations as mentioned before tend to be small. In practice, the horizontal turbulence is usually the same size as or smaller than the numerical dispersion introduced by the numerical method. However, because of the shallow depth, it is crucial to include refined modeling of the turbulence/ shear processes. Therefore, for all 3D and 2DV models with stratification, it is recommended that the one equation approach, i.e. Nos. 8 and 9 in Table 12, be used when performing whole basin calculations. The vertical mixing is then being modeled by a refined turbulence model while a subgrid scale (Table 11, No. 2) is used for the horizontal subgrid scale energy dissipation.

216. For the spatially simplified cases of model classes 2DH and 1D, it will not be possible to adequately address the problem of stratification. Looking at the 1D case first, a whole variety of complications call into question the use of such models for estuaries. First, irregular boundaries, meandering thalwegs, Coriolis forces,

and storms propagating across estuaries all contribute to the highly nonuniform distribution of transport at a cross section. Although in principle the averaging procedure should account for some of these phenomena, in practice they become much too difficult to parameterize in a subgrid scale model. These problems are exacerbated at the downstream end of estuaries which are very wide. Secondly, the necessity of dealing with the stratification issue is also quite difficult. Despite model 7c in Table 11, it has not been demonstrated that there is any precision or predictability in this approach, as it is only geared for the case of salinity effects. Additional difficulties are encountered with thermal stratification. By this we are not speaking of heated effluents but rather the discharge into the head of the estuary of water at a different temperature than the estuary water. Such is the case in almost all Great Lakes freshwater estuaries (Bedford et al. 1983) where the water travels at different temperatures into the lake receiving water without mixing completely. The homogeneous stretches of estuaries can be effectively modeled using the remaining models in Table 11, with No. 7 as starting values. However, even the problem of simple bends proves elusive for quantitative dispersion coefficients (see Fischer et al. 1979). In summary, there is a place for such models, particularly in relatively narrow sections where the channel is well defined, stratification effects are relatively unimportant, and vertical and horizontal mixing is relatively quick. This last point is important in that all too often the mixing length and time required for the cross section mixing inherent in the model is large in comparison to the time and space grid to be used in the 1D class of model. Such effects used to be modeled by completely mixed or hybrid elements and often this step is improperly



executed thus causing significant error in the 1D portion of the calculation.

217. For the 2DH class of models, several factors are at work. Foremost is that stratification effects are suppressed. Therefore, these models should not be used where this is a problem. However, for homogeneous cases where basin-wide long-wave effects are important, these are quite effective models. Representing a shallow, fast mixing situation, the turbulence is very much controlled by bottom/boundary turbulence. Therefore, using relatively simple mixing length arguments, one can parameterize the horizontal mixing using the form of model No. 4, Table 11. To remedy excessive coefficient modifications for the subgrid effects neglected in this model, the use of one of the grid size dependent forms, i.e. No. 2, Table 11, is also recommended. This type of parameterization is all that is necessary for such flows where the horizontal scales are quite excessive. Especially in predicting surface displacements, the 2DH models have quite a good performance record. This is achieved with such simple nonrefined turbulence and mixing models as long as the models are not applied in a fashion that abrogates the basic assumption inherent in the subcomponents aggregated in the model.

218. The number of 2DV and 3D models for estuary water quality studies is a testimonial to the difficulties of dealing with the stratification problem. Refined turbulence models in 3D calculations do exist (see references in Part V), but the horizontal scales are effectively parameterized by the forms recommended at the beginning of this subsection.

#### Summary Selection Guidelines

219. The outline of this report is organized to first consider the physics of estuaries followed subsequently by

transport model selection and selection of the closure model. It is the author's belief that the selection of the closure model is one of the last considerations after a number of crucial decisions have been made regarding the more fundamental model structure, the relationship of the selected model to the physics, and the numerical solution algorithm. Prior to selection of the closure model, it is also necessary to ascertain the sufficient availability and density of field data necessary to both justify the resolution of the selected model and validate the results. Major inconsistencies occur between mixing model coefficients necessary to calculate distributions of observed water quality variables and theoretically or experimentally observed values. These discrepancies are a source of controversy in that they compromise model predictability and perhaps overestimate the degree to which management controls must be placed on proposed receiving water uses. It is the author's belief that these discrepancies are, in most instances, not the fault of the mixing model but rather occur as a result of incomplete attention to the above-mentioned considerations. Thorough attention to all the required model structure decisions plus use of turbulence models in situations for which their original derivations applied are the keys to obtaining as much success as possible from one particular mixing model. The following paragraphs contain a series of functions in the model development which requires careful decision and whose result directly bears upon the selection of the turbulence or mixing model and ultimately its performance (vis a vis coefficient discrepancies).

#### Determination of modeling objective

220. In consultation with all the affected personnel, a decision must be made on the intended purpose of the simulation. From this consultation must come a decision on

what phenomena are to be predicted and what is the maximum time and space extent of the calculation. Also from this consultation must come a decision as to what is to be the smallest scale of resolved activity in the critical model calculated variable, i.e., what minimum time and space period is to be calculated by the model. Remembering Wood's (1977) constraint that only two decades separating the largest and smallest scales is practical in a model, a decision must be made as to these limits. A modeling objective must always be established before proceeding. A model of an estuary cannot be universal. The tendency towards one universal model for an estuary is one very major reason for the variability in mixing model coefficients because each mixing model has a different set of physical processes and assumptions involved in the model derivation. Therefore, each transport model should be adapted after a sound modeling objective with desired resolution limits and processes delimited.

Assessment of available data and collection  
of necessary data

221. Given the objective defined above, it is necessary to determine what field data exist and define what data are to be collected for hindcasting and coefficient tuning followed by the validity check in a forecasting mode. This is a crucial step for several reasons. First, the minimum resolution in the model is only as valid as the minimum resolution used to impose boundary conditions. Second, the validity of model calculations over the maximum desired resolution must be determined with data at minimum collected with the same maximum dimensions. Third, the validity of the model calculations must be determined with data adequately sampled to resolve the major processes to be resolved (as per step one).

### Analysis of data and process identification

222. Based upon the collection of data in step two, it is important to process these data by correlation and/or spectral analyses to determine what processes are at work in the tentative model resolution band width hypothesized in step one and to determine to some extent a length/time process library. This is an important step in that the numerical grid which defines the work to be done by the turbulence and mixing model must fall between processes or in the spectral gap. Also the length/time diagram helps in determining whether there is relatively complete mixing in either the vertical or transverse scale and thereby helps in determining what transport model equations are permissible. Finally, this step is important because empirical turbulence models are derived for use in the spectral gap where turbulence and production are assumed in equilibrium. Therefore, knowledge of the space-time or spectral gaps is quite important.

### Selection of transport model equations

223. By comparing the minimum desired resolution in step one to see if it is consistent with the minimum transverse or vertical times and the resolution in the available data and after ensuring the degree to which the desired resolved processes in step one are consistent with the data analysis in step three and after determining which model type, i.e. 3D, 2DH, 2DV, etc., is consistent with all the above, a selection of the model structure can now be made. This step also determines how much the mixing models are asked to do and it is important to reconcile and be consistent with the decisions made in the previous three steps. This is a particularly important step as regards the 1D and 2DH class of models which cannot handle the stratification effects present in most estuaries. In the 1D, 2DH, and 2DV model classes various minimum mixing times

serve as minimum barriers to valid resolution and these minimum resolutions must be equal to or less than the resolution time in steps one and three for these classes of models to be adequately applied.

#### Selection of numerical method and discretization

224. This step, for reasons discussed earlier is crucial as the numerical method incurs turbulence-like errors which must be controlled. This author believes that in addition to the inconsistencies outlined in steps one through four, this step produces errors equally as large.

225. The selection of the grid must be dense enough to permit the resolution of the minimum desired resolution in step one. The Nyquist relation examined in Part VI is a guideline for this selection.

#### Selection of turbulence model

226. Using the summary tables in Part V and the first section of Part VII, the turbulence model can now be selected.

227. Consideration of these decisions will minimize the inconsistencies between the physics, math, and data interactions in the creation of the model. To be sure, even when all these precautions have been observed, errors in mixing coefficients will occur. But, if these other errors have been controlled, then and only then can the mixing model be held accountable and valid coefficient changes be justified. The importance of educated modelers cannot be overemphasized. This author is of the opinion that no numerical model and no turbulence model is so universal and/or so user friendly as to permit an inexperienced user to achieve valid answers. Every one of the six considerations outlined above must be actively initiated by the modeler who must apply his or her experience to the decision.

228. Even the simplest turbulence model can be used effectively if and only if it is used for the purpose or conditions the derivation is intended. Further there should be an experienced modeler in control of the exercise who recognizes that no model is universal and therefore each problem should be approached as a new modeling exercise.

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points ( $2m < n$ ), then the averaged value at the  $i^{\text{th}}$  node is:

$$\bar{\alpha}_i = \sum_{j=-m}^m w_j \alpha_{i-j} \quad (\text{A3})$$

with the restriction that

$$\sum_{i=1}^n w_i = 1. \quad (\text{A4})$$

4. The selection of the weighting function is dictated by a variety of circumstances. Principally, the chief criterion is that variability be removed sufficiently to meet the goals of the analysis, i.e., enough information must remain to display trends. In this most general form, averaging (or smoothing) is called filtering. Procedures for filtration have been developed primarily by signal analysts, and only recently have these methods begun to appear in more widespread use. Texts by Oppenheim and Schaffer (1975) and Otnes and Enochson (1972, 1978) are excellent signal processing works, but the book by Hamming (1978) provides the most accessible information about filtration to the layman. On a much less rigorous but nevertheless extraordinarily sophisticated level, the book by Tukey (1977) is recommended, particularly because the methods are adaptable to data and signals which are obtained at unevenly spaced intervals.

5. It is not the author's intent to thoroughly review filtration theory; however, as it will be used subsequently, it is necessary to explain the method for evaluating a filter and to demonstrate how, by the proper construction of this averaging, the analyst can significantly control the variability of information in the problem and model.

6. The primary filter performance index is called the transfer function. It is determined by seeing how a simple

## APPENDIX A: DIGITAL SIGNAL PROCESSING AND SPECTRAL ANALYSIS

1. This brief Appendix discusses (a) basic digital signal analysis, i.e. how to determine the time and length scales of processes, (b) extraction of process time and length scales, and (c) spectral analysis.

### Basic Digital Signal Processing Procedures

2. Crucial to the creation of satisfactory models is a knowledge of time and length scales over which processes occur. The principal statistics for quantitatively determining typical transport process scales are deviations as measured by correlation and spectral methods. Before discussing their determination, a small introduction to averaging must take place.

3. Averages are numbers created to reduce the variability in noisy information. In their most general form, averages are created by running integrations called convolution integrals, i.e. if  $\alpha(x,y,z,t)$  is the time and space distribution of a pollutant concentration,  $\alpha$ , then the most general averaging that can be applied is (Monin and Yaglom 1975)

$$\bar{\alpha}(x) = \iiint_{-\infty}^{+\infty} w(x-x', y-y', z-z', t-t') \alpha(x', y', z', t') dx' dy' dz' dt' \quad (A1)$$

where  $w(x,y,z,t)$  is the weighting function selected such that,

$$\iiint_{-\infty}^{+\infty} w(x,y,z,t) dx dy dz dt = 1. \quad (A2)$$

If instead of analog or continuous data, we have (as is the more practical case) data taken at discrete points, then digital averaging is performed in a similar manner. Therefore, given that  $\alpha$  is known at  $i$  points for  $n$  total nodes and the weighting function  $w_j$  is defined over  $2m$

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known signal is modified by the filter and by the ratio of filtered to unfiltered signal. Assume that a signal  $\alpha(t)$  can be formulated by a Fourier series, i.e.:

$$\alpha(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos \frac{\pi}{n} kt + \sum_{k=1}^{\infty} b_k \sin \frac{\pi}{n} kt \quad (A5)$$

where

$$a_k = \frac{1}{n} \int_{-n}^n \alpha(t) \cos \frac{\pi}{n} t dt; \text{ and} \quad (A6)$$

$$b_k = \frac{1}{n} \int_{-n}^n \alpha(t) \sin \frac{\pi}{n} t dt \quad (A7)$$

$n$  = interval of  $t$ ;  $t < |n|$

In complex notation:

$$\alpha(t) = \sum_{k=-\infty}^{\infty} c_k e^{(\pi i/n)kt}; \quad (A8)$$

where

$$c_k = \frac{1}{2n} \int_{-n}^n f(t') e^{-(\pi i/n)kt'} dt' \quad (A9)$$

$$c_k = \begin{cases} \frac{a_k - ib_k}{2} & k > 0 \\ a_0 & k = 0 \\ \frac{a_k + ib_k}{2} & k < 0 \end{cases} \quad (A10)$$

Then, since the series is linear, we may analyze our filter with one term of the series and, because of the linearity, make accurate estimations as to the filters effect on the entire series. Therefore, let us analyze (as in Hamming 1977) one frequency (spatial or temporal) of the series for  $\alpha$  ; i.e., let's look at the  $k$ th frequency relation for:

$$\alpha_k = e^{2\pi i \sigma_k t} = e^{i \omega_k t} \quad (A11)$$

where  $i = \sqrt{-1}$ ;  $t$  = time (or one space coordinate);  $\alpha_k$  =  $k$ th frequency in Hertz; and  $\omega_k = 2\pi\sigma =$  frequency radians/second. Further let us analyze a typical filter (or smoothing) operation, three-point smoothing. For three equally spaced points, a smoothed estimator of  $\alpha$  at point  $k$  is  $\beta_k$ , defined as:

$$\beta_k = \text{smoothed value} = \frac{\alpha_{k-1} + \alpha_k + \alpha_{k+1}}{3} \quad (A12)$$

If

$$\alpha_k = e^{2\pi i \sigma k} \quad (A13)$$

then

$$\beta_k = e^{i \omega k} \left[ \frac{e^{-i \omega} + 1 + e^{i \omega}}{3} \right] \quad (A14a)$$

$$\beta_k = e^{i \omega k} \left[ \frac{1 + 2 \cos \omega}{3} \right] \quad (A14b)$$

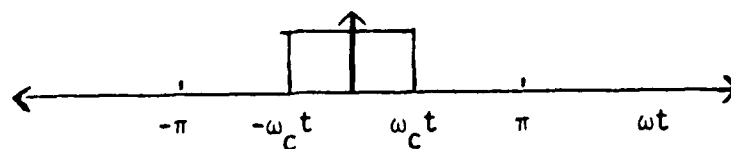
$$7. \text{ If } H(\omega) = \beta_k / \alpha_k \quad (A15)$$

and is defined as the transfer function, then from above:

$$H(\omega) = \frac{1 + 2 \cos \omega}{3} = \frac{\sin (3\omega/2)}{3 \sin (\omega/2)} . \quad (A16)$$

Figure A1 shows the plot of the transfer function vs. frequency for five through eleven point averages. Notice that the averaged quantity  $\beta_k$  is composed of low frequency information as the filtration will not permit passage of the high frequency noisy data. In general, filter (average) transfer functions decrease with increasing frequency (small wavelength). The more restrictive the averaging, the further towards the origin the  $H(\omega)$  curve will be shifted, and the lower will be the frequency range from which the averaged parameter is composed. Notice also in Figure A1 that the rapid decline in  $H(\omega)$  occurs at increasingly lower frequencies as the number of points in the average increases.

8. It is possible to design filters to exclude any information at a frequency or wavelength range greater or noisier than some cutoff value  $\omega_c$ . This is called a sharp-cutoff, low-pass filter as low frequency or longer wavelength information is retained by the filtering operation. Assume (Hamming 1978) a transfer function as follows:



where

$$H(\omega) = \begin{cases} 1 & |\omega t| \leq \omega_c t \\ 0 & |\omega t| > \omega_c t \end{cases} .$$

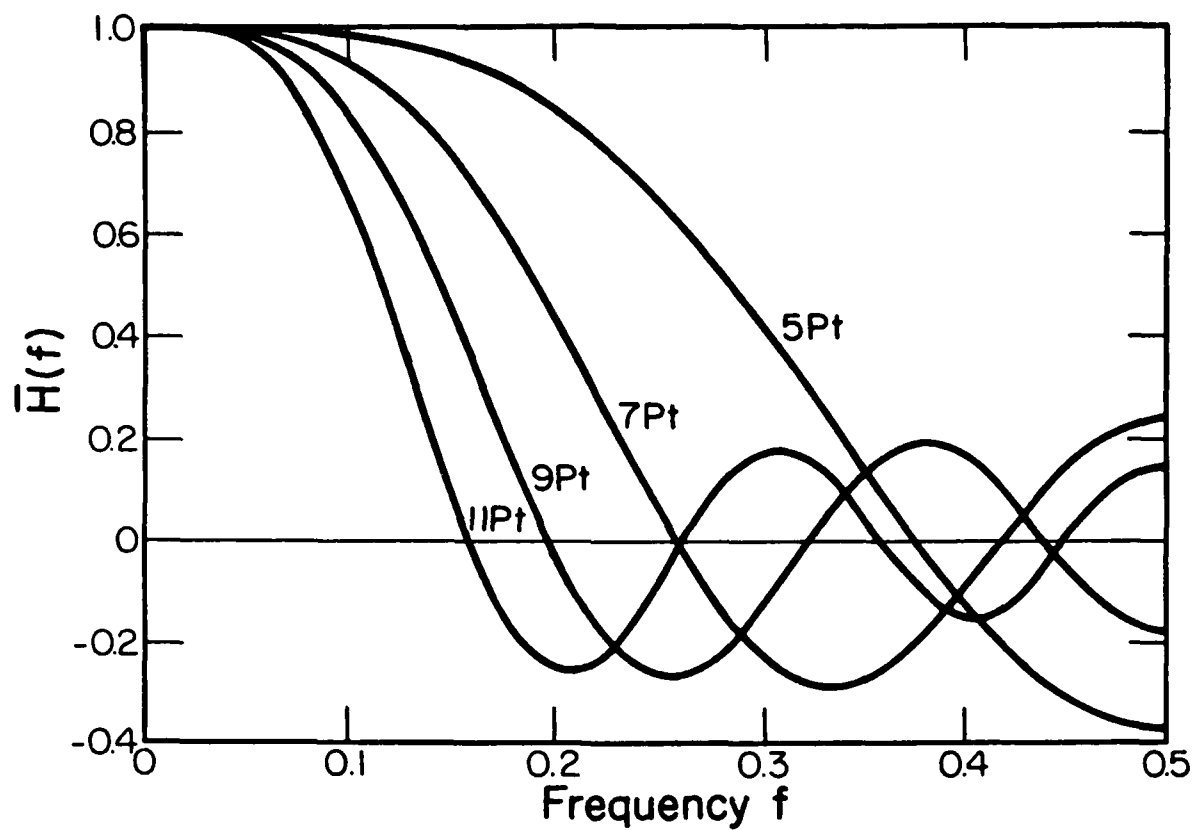


Figure A1. Transfer function for smoothing by least-square quadratics

The objective is to now find  $\omega_m$  such that:

$$\beta_k = \sum_{m=-j}^j \omega_m \alpha_{n-m}. \quad (A17)$$

Taking a Fourier transform (Hamming 1978), it is shown that:

$$\omega_m = \omega_{-m} = \frac{t}{\pi} \int_0^{\pi/t} H(\omega) \cos(m\omega t) d\omega. \quad (A18)$$

Therefore,

$$\beta_k = \frac{\omega_c t}{\pi} \sum_{m=-j}^{+j} \frac{\sin(m\omega_c t)}{m\omega_c t} \alpha_{n-m} \quad (A19)$$

and by Fourier expansion,

$$\bar{H}(\omega) = \frac{\omega_c t}{\pi} \left( 1 + 2 \sum_{m=-j}^{+j} \frac{\sin(m\omega_c t)}{m\omega_c t} \cos(m\omega t) \right). \quad (A20)$$

As an example, consider three-point cutoff smoothing. The above formulae give:

$$\beta_k = \frac{\alpha_k}{2} + \frac{1}{\pi} (\alpha_{k-1} + \alpha_{k+1}) \quad (A21)$$

for a cutoff of  $\pi/2 = \omega_c t$  and

$$H(\omega) = \frac{1}{2} + \frac{2}{\pi} \cos\left(\frac{\omega\pi}{2}\right). \quad (A22)$$

As a second example, choose seven point smoothing with  $\omega_c t = 2\pi/3$  (i.e. include more noise):

$$\beta_k = \frac{2}{3} \alpha_k + \frac{\sqrt{3}}{2\pi} (f_{k+1} + f_{k-1}) - \frac{\sqrt{3}}{4} (f_{k+2} + f_{k-2}). \quad (A23)$$

Notice that when more noise is included, less weight is attached to terms away from the center term.

9. This brief excursion into averaging and filtration is important; it demonstrates a systematic procedure for processing information and equations in such a fashion that only the level of information necessary for the goals of the analysis is retained, and is retained properly. Most importantly it also demonstrates that significant control over the flow of information both within and into the model can be achieved once the analyst has decided what frequency or time and length scale is to be retained in the model. Therefore, if each process in the model has a dominant time and space scale, we can include or exclude the effect of the process by simply designing filters with cutoff wavelengths or frequencies approximately equal to the typical time and length scale of the process and then averaging the variability out of the analysis.

#### Extraction of Process Length and Time Scales

10. It is necessary to demonstrate how to determine the time and length scales of variable processes. These could include even the simplest geometric measurements for the most complex, turbulent signals. The key is that variability exists in the process or configuration being measured.

11. The primary quantitative tool for these calculations is the correlation function, which is a measure



of the spatial or temporal interdependence of a state variable with itself or the interdependence of two variables.

12. The most general form of the correlation function is an average of the product of two data signals, spatially or temporally lagged by an ever increasing amount. For any two data sets  $\alpha(x,y,z,t) = \alpha(\underline{x},t)$  and  $\beta(\underline{x},t)$ , the following definitions apply:

$$\mu_{\alpha}(\xi_1) = \lim_{\Delta\xi \rightarrow \infty} \frac{1}{\Delta\xi} \int_{\xi_1}^{\xi_1 + \Delta\xi} \alpha(\xi) d\xi \quad (A24)$$

and

$$R_{\alpha\beta}(\xi_1, \Delta\xi) = \lim_{\Delta\xi \rightarrow \infty} \int_{\xi_1}^{\xi_1 + \Delta\xi} \alpha(\xi) \beta(\xi + \Delta\xi) d\xi \quad (A25)$$

where  $\mu_{\alpha}$  = the mean of  $\alpha$  over the interval  $\xi$  and  $R_{\alpha\beta}$  = the covariance of signals  $\alpha$  and  $\beta$  lagged by  $\Delta\xi$ .

Many special cases apply: (a) Point correlations between two signals  $\alpha$  and  $\beta$  generated simultaneously at spatial point  $\underline{x}_p$ . This for example could be lake level and flow, velocity and temperature, DO and temperature, or coliform and outfall discharge; (b) Autocorrelations between the values one variable has, separated by a constant time interval  $\tau$ . This procedure has long been used in the analysis of lake levels, harbor oscillations, temperature fluctuations, sewerage outflows, etc.; (c) Time correlations or cross correlations between two different signals originating at the same point separated by a lag  $\tau$ ; and (d) Spatial correlations between two signals generated simultaneously

from  $t_p$  but separated by a distance  $r$ . The same property,  $\alpha$ , measured at the two points is direct correlation, while spatially separated correlation between any two different signals is cross correlation.

13. For the general case of time correlations, Equation A25 is written as

$$R_{\alpha\beta}(t_1, \tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{t_1}^{t_1+T} \alpha(\underline{x}, t) \beta(\underline{x}, t+\tau) dt. \quad (A26)$$

The digital analog of this expression is for  $n$  points:

$$\hat{\mu}_\alpha = \frac{1}{n} \sum_{i=1}^n \alpha_i \quad (A27)$$

and

$$R_{\alpha\beta}^{\wedge}(k) = \frac{1}{n-k} \sum_{i=1}^{n-k} (\alpha_i - \hat{\mu}_\alpha)(\beta_{i+k} - \hat{\mu}_\beta) \quad (A28)$$

$$k = 0, 1, 2 \dots$$

The caret denotes digital estimators of the correlation functions.

14. If instead of two different series we write:

$$R_\alpha(t_1, \tau) = \lim_{T \rightarrow \infty} \int_{t_1}^{t_1+T} (\underline{x}, t) \alpha(\underline{x}, t+\tau) dt \quad (A29)$$

then  $R_\alpha(t, \tau)$  is called the autocovariance function. If it is normalized with  $R_\alpha(t, 0)$ , it is the autocorrelation coefficient:

$$\rho_{\alpha}(\tau) \triangleq \frac{R_{\alpha}(\tau)}{R_{\alpha}(0)} \quad (A30)$$

with the corresponding value range from  $-1 < \rho < 1$ . For the case of stationary statistics,  $\rho_{\alpha}(\tau)$  approaches 1 as  $\tau$  approaches 0. Also, note that  $R_{\alpha}(0)$  is the variance of the variable process.

15. In general, the deterministic data will have a persistent autocorrelation function over all lags. In contrast, for random data,  $R_{\alpha}(\tau)$  approaches zero for large lags.

16. Two different time scales are defined from  $\alpha(\tau)$ . The first is an average or integral scale,  $\tau_e$ , which is found as

$$\tau_e = \int_0^{\infty} \rho_{\alpha}(\tau) d\tau . \quad (A31)$$

This represents an average time over which  $\alpha$  or  $\alpha(\bar{x}, t)$  and  $\beta(\bar{x}, t)$  remain correlated. The second time scale is determined by fitting a parabola to  $R_{\alpha}(\tau)$  at  $\tau = 0$ . This provides a measure of the characteristic time of the instantaneous fluctuation in the signal  $\alpha$ . This time, called the microscale time,  $\tau_E$ , is found by expanding  $\rho_{\alpha}(\tau)$  in a parabolic series about  $\rho_{\alpha}(0)$  and determining the time at which this curve crosses the  $\tau$  axis. This is depicted in Figure A2 and is found mathematically as:

$$\tau_E = \left[ \frac{2 \overline{\alpha^2}}{(2\alpha/2t)^2} \right]^{1/2} . \quad (A32)$$

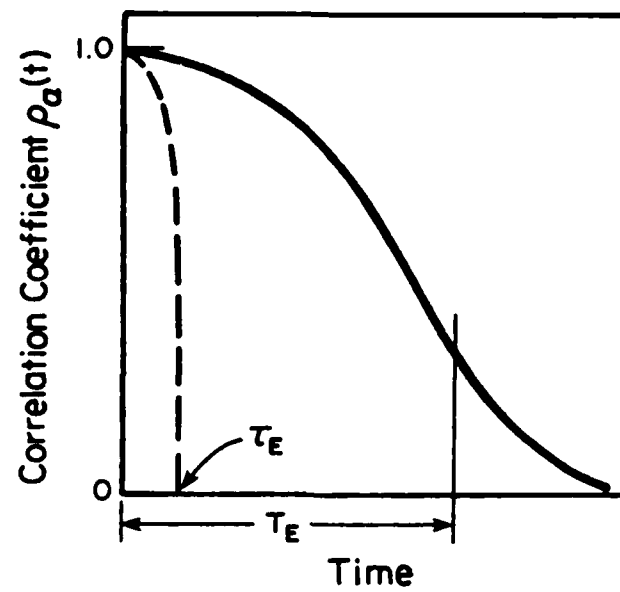


Figure A2. Time scale definition

The variable  $\tau_E$  is simply the ratio of the root mean square values of  $\alpha$  divided by the root mean square value of the derivative. Note that Equation A32 implies that if a process with signal  $\alpha(\underline{x}, t)$  and  $\tau_E$  exists, then the variability of the signal can be removed only by averaging over time period  $\tau$  such that  $\tau \gg \tau_E$ .

17. Spatial correlations and scales are found in a similar fashion. From Equation A28 the spatial correlation is written as:

$$R_{\alpha\beta}(\underline{x}_1, r) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{t_1}^{t_1+T} (\alpha(\underline{x}_1, t) \beta(\underline{x}_1 + r, t)) dt \quad (A33)$$

where  $\underline{x}$  is the vector location of the sampling point; i.e.,  $x_i + y_j + z_k$ ; and  $r$  is the separation between  $\alpha$  and  $\beta$ .

Again, digital expressions for  $N$  points are formed as:

$$\hat{R}_{\alpha\beta}(\underline{x}_1, r) = \frac{1}{N-k} \sum_{i=1}^{N-k} (\alpha_i - \bar{\alpha})(\beta_{i+k} - \bar{\beta}) \quad (A34)$$

for  $k = 0, 1, \dots, N$ .

If  $R_{\alpha\beta}$  or  $R_{\alpha\tau}$  is normalized with  $R_{\alpha\beta}(\underline{x}_1, 0)$ , the range of  $R$  is from -1 to 1.

18. Length scales for both the integral and microscale are also formed from knowledge of  $\rho_{\alpha\beta}(\underline{x}_1, r)$ , i.e.:

$$L_E = \int_0^\pi \rho_{\alpha\beta}(\underline{x}_1, r) dr \quad (A35)$$

and

$$\lambda = \left[ \frac{\overline{2\alpha^2}}{(2\alpha/2r)^2} \right] \quad \text{at } r=0. \quad (\text{A36})$$

19. Relationships between spatial and temporal autocorrelations are formed from Taylor's Hypothesis, which is described in Hinze (1975).

#### Spectral Analysis

20. A useful method in the analysis of equations and signals has been the introduction of wave space statistics to help identify time and length scales. The assumption inherent in this procedure is that an arbitrarily variable signal is a summation of periodic sines and cosines (or eddies). Each term of the series has a different amplitude,  $A_0$ , frequency,  $\omega$ , or wavelength,  $\lambda$  (or wave number  $k = 2\pi/\lambda$ ). High variability (noise) is associated with short-wavelength, high-frequency components, while the longer wavelength components provide the coherent, deterministic structure. The advantage of this viewpoint is that in Cartesian space statistics are generated by averaging the entire range of observed data, but in the Fourier viewpoint a mathematical structure for the signal is assumed and those segments of the structure that contribute to the observed behavior of the signal can be determined. In other words it is now possible to see how components of the motion result from the interaction of the eddies.

21. A plot of the correlation structure between eddies in wave space is called a spectrum, and spectral plots are either angular frequency,  $\omega = 2\pi/T$ , or wave number,  $k = 2\pi/\lambda$ , dependent.

22. The spectral density of a signal  $\alpha(\underline{x}, t)$  is found by the cosine Fourier transform of the covariance function

and in analog form is given as:

$$G_{\alpha\beta}(\omega) = 4 \int_0^{\infty} R_{\alpha\beta}(t_1, \tau) \cos(\omega\tau) d\tau. \quad (A37)$$

For autocovariance:

$$G_{\alpha}(\omega) = 4 \int_0^B R_{\alpha}(t_1, \tau) \cos(\omega\tau) d\tau. \quad (A38)$$

23. The discrete forms are found from Equation A38 as

$$\hat{G}_{\alpha}(j) = \sum_{k=0}^m R_{\alpha}(k) \cos(kh\omega_j) \quad (A39)$$

where  $\omega_j$  is the frequency at the  $j$ th point. Note that  $\omega_j$  is only defined at 0, 1, ...,  $m$  points.

24. Spectral plots may be found by transform procedures for any correlation function. Spectral techniques have been presented in many texts on digital signal analysis and random phenomena. The work of Otnes and Enochson (1972) is typical.

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