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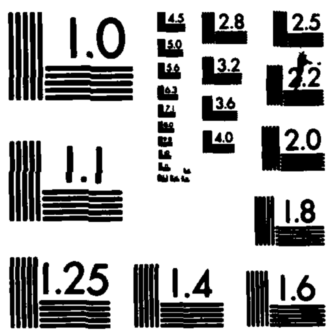

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STRUCTURES REPORT 416

INTEGRAL EQUATION FORMULATION FOR THREE-DIMENSIONAL UNSTEADY TRANSONIC FLOW

by

J. A. GEAR

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SUMMARY

The unsteady transonic small perturbation differential equation is converted into an integro-differential equation by application of the classical Green's function method. It is shown that no contribution from shock waves explicitly appears in this integral equation, due to the shock capturing properties of the Green's function method. After assuming that the motion consists of small infinitesimal perturbations around a thin nearly-planar body, a simplified integral equation for the streamwise velocity component is obtained, which is suitable for fast numerical computations.

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CONTENTS

NOTATION

Symbol	*Definition*
C_P	pressure coefficient
$\bar{C}_P$	scaled pressure coefficient for $M < 1$
$\tilde{C}_P$	scaled pressure coefficient for $M > 1$
E	domain function
G	Green's function
H	Heaviside step function
K_1, K_2, K_3	coefficients for subsonic case
$\tilde{K}_1, \tilde{K}_2^{\pm}, \tilde{K}_3^{\pm}$	coefficients for supersonic case
l	typical chord length
M	free stream Mach number
$\tilde{\mathbf{N}}$	unit normal in Prandtl-Glauert variables
$\tilde{\mathbf{n}}$	unit normal in Eulerian variables
r_α	$\{(x-x_1)^2-\alpha^2[(y-y_1)^2+(z-z_1)^2]\}^{\frac{1}{2}}$
r_β	$\{(x-x_1)^2+\beta^2[(y-y_1)^2+(z-z_1)^2]\}^{\frac{1}{2}}$
R	$\{(X-X_1)^2-(Y-Y_1)^2-(Z-Z_1)^2\}^{\frac{1}{2}}$
R_B	projection of wing surface onto $z = 0$ plane
R_W	projection of wake surface onto $z = 0$ plane
S	equation defining wing, wake and shock surfaces
U	streamwise perturbation velocity ($=\partial\Phi/\partial x$)
U_P	$= \partial\Phi_P/\partial x$
U_∞	free stream velocity
x, y, z, t	Eulerian variables
X, Y, Z, T	Prandtl-Glauert variables
X_T	position of trailing edge
α	$\sqrt{M^2-1}$
β	$\sqrt{1-M^2}$
γ	ratio of specific heats
γ^*	$2-(2-\gamma)M^2$
δ	thickness ratio, Dirac delta function
θ	accustic time delay between two points
Θ	accustic time delay in Prandtl-Glauert variables

Λ	$\lvert X-X_1\rvert-\{(Y-Y_1)^2+(Z-Z_1)^2\}^{\frac{1}{2}}$
ρ_∞	free stream density
Σ	surface surrounding wing, wake and shock waves
ϕ	perturbation potential
Φ	scaled perturbation potential
Φ_P	solution for purely symmetrical flow
χ	$M^2(1+\gamma^*)\phi_x\,\phi_{xx}$
ψ	velocity potential

1. INTRODUCTION

The integral equation method in steady transonic flow problems was first used by Oswatitsch [13]. Since then, the method has been extended and modified by Spreiter and Alksne [15], Norstrud [11], Nixon [9] and Piers and Sloof [14]. Nixon [8], [10] extended the integral equation method to harmonically oscillating two-dimensional airfoils, by perturbing from the steady solution. A weakness of this formulation is its inability either to create or eliminate shock waves as a result of the unsteady motion. In this paper we use a method initially developed by Morino [6], [7] for purely subsonic and supersonic oscillatory flows. The method uses the classical Green's theorem approach to derive an integro-differential equation for the perturbation velocity potential for unsteady flow about a general body. Recently the technique was adapted to unsteady transonic flow by Tseng and Morino [16].

In the following, the isentropic inviscid flow of a perfect gas, initially irrotational, is considered. Under this hypothesis, the existence of a velocity potential ψ, may be assumed such that the fluid velocity $\mathbf{v} = (u, v, w) = \nabla\psi$. The most basic approximation of inviscid aerodynamics is that of a small disturbance. Thus if U_∞ is the free stream velocity (which is assumed to be directed solely in the x-direction), then a perturbation potential ϕ can be defined such that

$$\psi(x, y, z; t) = U_\infty[x+\phi(x, y, z;\ t)], \tag{1}$$

where x, y, z represent a rectangular cartesian co-ordinate system and t is the time variable. Note that subsequent equations are expressed in non-dimensional co-ordinates based on a length scale l, a typical value of the airfoil chord length, a velocity scale U_∞, and a density scale ρ_∞, a typical value of the density at infinity. The time variable is then scaled with l/U_∞ and the pressure with $\rho_\infty U_\infty^2$.

In terms of the dimensionless Eulerian co-ordinates x, y, z and t, the governing equation for the perturbation velocity potential can be written as

$$\nabla^2\phi - M^2\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}\right)^2\phi = \chi\ , \tag{2}$$

where M is the free stream Mach number and χ includes all the nonlinear terms. For simplicity the discussion will be restricted to the nonlinear term

$$\chi = M^2(1+\gamma^*)\,\phi_x\,\phi_{xx}, \tag{3a}$$

with

$$\gamma^* = 2-(2-\gamma)M^2, \tag{3b}$$

where γ is the ratio of specific heats. The procedure used to derive the approximate equation [(2) with χ given by (3*a*)], when the flow is transonic, is based on the small parameter δ representing the ratio of airfoil thickness to chord length. The main assumptions are that as $\delta \to 0$, the quantities $(1-M^2)/\epsilon(\delta)$, $\phi/\epsilon(\delta)$, $y/\mu(\delta)$, $z/\mu(\delta)$ and $t/\mu^2(\delta)$ all remain fixed, whereas $\epsilon(\delta)$, $\mu(\delta) \to 0$ (cf. with [1] and [2]). If terms smaller than $\epsilon\mu^4$ and ϵ^2 are neglected then χ is given by equation (3*a*). Note that the coefficient of $\phi_x\,\phi_{xx}$ in (3*a*) is usually written as $M^2(1+\gamma)$. The modification in (3*a*) comes forward if the small perturbation equation is derived directly from the conservation of mass equation (see [17]).

The effect of this modification is that the relations for a normal shock show better agreement with those of full potential theory. It also leads to a better agreement of the critical pressure coefficient with its exact isentropic value (i.e. when the steady local fluid speed equals the local sonic speed) (see [3] and [4]).

In the subsequent analysis a general theory of potential aerodynamic flow around thin lifting bodies having arbitrary shape and motion is presented. In Section 2, for subsonic free stream Mach numbers, the small perturbation differential equation (2), is converted into an integro-differential equation for the velocity potential, by application of the classical Green's theorem approach. It is also shown that no contribution from shocks explicitly appears in the subsequent integral equation for ϕ, due to the shock capturing nature of the method. In Section 3 and 4 the integral equation formulation is simplified by assuming that the motion consists of small infinitesimal perturbations around a steady thin nearly-planar body. Eventually we obtain, a simplified integral equation for the streamwise perturbation velocity component, which is suitable for fast numerical calculations. In the Appendix a similar integral equation is derived for the case when the free stream Mach number is supersonic.

2. SOLUTION BY NONLINEAR GREEN'S FUNCTION METHOD

The method of solution for (2) is based upon the well-known Green's function technique. The Green's function for the equation of potential is the solution of the problem

$$\nabla^2 G - M^2\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}\right)^2 G = \delta(x-x_1, y-y_1, z-z_1, t-t_1) \tag{4a}$$

with

$$G = 0 \qquad \text{at infinity,} \tag{4b}$$

where δ is the Dirac delta function. The solution of (4*a* and 4*b*) for the subsonic case ($M < 1$) is given by (see Morino [7])

$$G(\mathbf{x}-\mathbf{x}_1, t-t_1) = -\frac{1}{4\pi r_\beta(\mathbf{x}, \mathbf{x}_1)}\delta(t_1-t+\theta), \tag{5a}$$

where

$$r_\beta(\mathbf{x}, \mathbf{x}_1) = \{(x-x_1)^2+\beta^2[(y-y_1)^2+(z-z_1)^2]\}^{\frac{1}{2}}, \tag{5b}$$

$$\beta^2 = 1-M^2 \tag{5c}$$

and

$$\theta = \frac{M}{\beta^2}[r_\beta(\mathbf{x}, \mathbf{x}_1)-M(x-x_1)]. \tag{5d}$$

The corresponding solution for the supersonic case ($M > 1$) is given in the Appendix.

Using the procedure developed in [7] in order to obtain a representation of the perturbation potential in terms of its value and the values of its derivatives on the wing body, the wake and the shock waves (if any) we find it convenient to define a domain function E such that

$$E(x, y, z; t) = \begin{cases} 1 & \text{outside } \Sigma \\ \frac{1}{2} & \text{on } \Sigma \\ 0 & \text{inside } \Sigma, \end{cases} \tag{6}$$

where Σ is a surface which completely surrounds the wing, its wake and any shocks, and which is defined by the equation

$$S(x, y, z; t) = 0. \tag{7}$$

Following the general Green's theorem method we now multiply the equation of aerodynamic potential (equation (2)) by the Green's function G and subtract equation (4a) multiplied by ϕ. After suitable manipulations we find that

$$\nabla_1 \cdot (G\nabla_1\phi - \phi\nabla_1 G) - M^2 \frac{d}{dt_1}\left(G\frac{d\phi}{dt_1} - \phi\frac{dG}{dt_1}\right)$$

$$= G\chi - \phi\delta(x-x_1, y-y_1, z-z_1; t-t_1), \tag{8a}$$

with

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{\partial}{\partial x}. \tag{8b}$$

In (8a) the arguments of ϕ and its derivatives are x_1, y_1, z_1 and t_1, while the arguments of G and its derivatives are $x-x_1$, $y-y_1$, $z-z_1$ and $t-t_1$. If equation (8a) is multiplied by the domain function E, and integrated over the whole four-dimensional space-time, it is found that the potential satisfies (see Morino [6], [7]),

$$4\pi E(\mathbf{x}; t)\phi(\mathbf{x}; t) = -\iiint_{-\infty}^{\infty} \frac{[E\chi]^\theta}{r_\beta} dV(\mathbf{x}_1)$$

$$-\iint_{\Sigma^\theta}\left[\nabla_1 S \cdot \nabla_1\phi - M^2 \frac{ds}{dt_1}\frac{d\phi}{dt_1}\right]^\theta \frac{1}{r_\beta|\nabla_1 S^\theta|} d\Sigma^\theta(\mathbf{x}_1)$$

$$+\iint_{\Sigma^\theta}\left[\nabla_1 S \cdot \nabla_1\left(\frac{1}{r_\beta}\right) - M^2 \frac{dS}{dt_1}\frac{\partial}{\partial x_1}\left(\frac{1}{r_\beta}\right)\right]^\theta \frac{\phi^\theta}{|\nabla_1 S^\theta|} d\Sigma^\theta(\mathbf{x}_1)$$

$$-\frac{\partial}{\partial t}\iint_{\Sigma^\theta}\left[\nabla_1 S \cdot \nabla_1\theta - M^2 \frac{ds}{dt_1}\left(1 + \frac{\partial\theta}{\partial x_1}\right)\right]^\theta \frac{\phi^\theta}{r_\beta|\nabla_1 S^\theta|} d\Sigma^\theta(\mathbf{x}_1). \tag{9}$$

In equation (9) the superscripted θ indicates evaluation at time $t_1 = t-\theta$, where θ is defined by (5d), thus Σ^θ indicates the surface defined by $S^\theta = S(x_1, y_1, z_1; t-\theta) = 0$. Therefore Σ^θ is a surface of the three-dimensional space (x_1, y_1, z_1), which depends parametrically upon x, y, z and t. Note that the surface Σ is composed of three branches. The first, Σ_B, is the surface of the body. The second is the surface Σ_W, of the wake. Note that this surface Σ_W is considered twice, since Σ is a closed surface. In other words, the two sides of the wake are considered to be independent surfaces having the same equation but opposite outwardly directed normals. The third one is the surface, Σ_S, of any shock waves, for which similar considerations are valid. In the subsequent analysis it is convenient to isolate the contribution of each of these surfaces.

If we now substitute for χ in (9) with its approximate value (3a), we see that the volume integral on the right of (9) can be integrated by parts in the x-direction. This integration by parts delivers integrals over the bounding surfaces of the volume. If we separate the contribution of each of these surfaces it is found that (9) reduces to

$$4\pi E(\mathbf{x};t)\phi(\mathbf{x};t) = \frac{M^2(1+\gamma^*)}{2}\iiint_{-\infty}^{\infty}\left\{\left[E\phi_{x1}{}^2\right]^\theta \frac{\partial}{\partial x_1}\left(\frac{1}{r_\beta}\right) - \frac{1}{r_\beta}\frac{\partial \theta}{\partial x_1}\frac{\partial}{\partial t}\left[E\phi_{x1}{}^2\right]^\theta\right\} dV(\mathbf{x}_1)$$

$$-\iint_{(\Sigma_B+\Sigma_W)^\theta}\left[\nabla_1 S\,.\,\nabla_1\phi - M^2\frac{ds}{dt_1}\frac{d\phi}{dt_1} - \frac{M^2(1+\gamma^*)}{2}\phi_{x1}{}^2\frac{\partial s}{\partial x_1}\right]^\theta \frac{1}{r_\beta|\nabla_1 S^\theta|}d\Sigma^\theta(\mathbf{x}_1)$$

$$+\iint_{(\Sigma_B+\Sigma_W)^\theta}\left[\nabla_1 S\,.\,\nabla_1\left(\frac{1}{r_\beta}\right) - M^2\frac{ds}{dt_1}\frac{\partial}{\partial x_1}\left(\frac{1}{r_\beta}\right)\right]^\theta \frac{\phi^\theta}{|\nabla_1 S^\theta|}d\Sigma^\theta(\mathbf{x}_1)$$

$$-\frac{\partial}{\partial t}\iint_{(\Sigma_B+\Sigma_W)^\theta}\left[\nabla_1 S\,.\,\nabla_1\theta - M^2\frac{ds}{dt_1}\left(1+\frac{\partial\theta}{\partial x_1}\right)\right]^\theta \frac{\phi^\theta}{r_\beta|\nabla_1 S^\theta|}d\Sigma^\theta(\mathbf{x}_1). \qquad (10)$$

The contribution from shocks does not explicitly appear in (10), this is due to the shock boundary conditions.

$$\Delta\phi = 0, \qquad (11a)$$

$$-M^2\Delta\left(\phi_x+\phi_t\right)\frac{\partial s}{\partial t}+\Delta\left[\left(1-M^2\right)\phi_x - \frac{M^2(1+\gamma^*)}{2}\phi_x{}^2 - M^2\phi_t\right]\frac{\partial s}{\partial x}$$

$$+\Delta\left(\phi_y\right)\frac{\partial s}{\partial y}+\Delta\left(\phi_z\right)\frac{\partial s}{\partial z} = 0, \qquad (11b)$$

where Δf represents the jump in f across the shock. The condition (11*a*) follows from the fact that there can be no circulation around infinitesimal paths threading the shock front, while (11*b*) is found by integrating the conservation or divergence form of (2) (with χ given by (3*a*)) across a jump discontinuity. The lack of an integral over the surface Σ_S, does not necessarily signify that shocks do not contribute to the potential ϕ. As shown by Tseng and Morino [16] the contribution from shock waves is now effectively imbedded in or captured by the volume integral in (10).

3. QUASI-STEADY APPROXIMATION

In practical applications it is convenient to introduce a restrictive assumption, which allows a considerable simplification of equation (10). In particular the unsteady motion of the aircraft can be assumed to consist of small infinitesimal perturbances around the steady state configuration, so that, for the purpose of evaluating the integrals in (10), Σ and S are assumed to be fixed.

However, when the boundary conditions on the body are applied, the time dependence of the surface position must be taken into consideration, in order to produce any unsteady motion. It is also convenient to introduce the Prandtl-Galuert variables

$$X = x, \quad Y = \beta y, \quad Z = \beta z$$

and

$$T = \frac{\beta^2}{M} t \tag{12a}$$

with

$$\Phi = \frac{M^2(1+\gamma^*)}{\beta^2}\phi. \tag{12b}$$

Substituting (12*a* and 12*b*) into (10) and applying the quasi-steady approximation, it is found that the perturbation potential satisfies

$$E(\mathbf{X})\Phi(\mathbf{X}, T)$$

$$= \frac{1}{8\pi}\iiint_{-\infty}^{\infty} E(\mathbf{X}_1)\left\{U^2 \frac{\partial}{\partial X_1}\left(\frac{1}{|\mathbf{X}-\mathbf{X}_1|}\right) - \frac{1}{|\mathbf{X}-\mathbf{X}_1|}\frac{\partial\Theta}{\partial X_1}\frac{\partial}{\partial T_1}U^2\right\}\Bigg|_{T_1 = T-\Theta} dV(\mathbf{X}_1)$$

$$-\frac{1}{4\pi}\iint_{\Sigma_B+\Sigma_W} \frac{\partial\Phi}{\partial N}(\mathbf{X}_1, T-\Theta)\frac{1}{|\mathbf{X}-\mathbf{X}_1|} d\Sigma(\mathbf{X}_1)$$

$$+\frac{1}{4\pi}\iint_{\Sigma_B+\Sigma_W} \left\{2M\Phi_T + \frac{U^2}{2}\right\}\Bigg|_{T_1 = T-\Theta} \frac{(\tilde{\mathbf{N}}-\mathbf{i})}{|\mathbf{X}-\mathbf{X}_1|} d\Sigma(\mathbf{X}_1)$$

$$+\frac{1}{4\pi}\iint_{\Sigma_B+\Sigma_W} \left\{\Phi\frac{\partial}{\partial N}\left(\frac{1}{|\mathbf{X}-\mathbf{X}_1|}\right) - \frac{\Phi_T}{|\mathbf{X}-\mathbf{X}_1|}\frac{\partial\Theta}{\partial N}\right\}\Bigg|_{T_1 = T-\Theta} d\Sigma(\mathbf{X}_1) \tag{13a}$$

where

$$\Theta = |\mathbf{X}-\mathbf{X}_1| - M(X-X_1), \tag{13b}$$

$$|\mathbf{X}-\mathbf{X}_1| = \{(X-X_1)^2+(Y-Y_1)^2+(Z-Z_1)^2\}^{\frac{1}{2}}, \tag{13c}$$

$$U = \Phi_x \tag{13d}$$

and

$$\frac{\partial}{\partial N} f = \tilde{\nabla} f . \tilde{\mathbf{N}}.$$

Here $\tilde{\Delta}$ and $\tilde{\mathbf{N}}$ are the gradient operator and unit normal to the surface in Prandtl–Galuert space. It should be noted that all the terms on the right hand side of (13*a*) have a physical interpretation analogous to potential theory. The first integral on the right (of (13*a*), which originates from the nonlinearity on the right of (2), has the form of a distribution of streamwise directed doublets of strength $(U^2(\mathbf{X}, T)/8\pi$ per unit volume. The second and third integrals represent distributions of sources over the wing and the wake with strengths given by $-1/4\pi\partial\Phi/\partial N$ and $(2M\Phi_T + U^2/2)$ $(\bar{\mathbf{N}} \cdot \mathbf{i})/4\pi$ per unit area. The last integral on the right of (13*a*) corresponds to a distribution of doublets, directed normally to the wing and the wake, with strength $\Phi(\mathbf{X}, T)/4\pi$ per unit area. Hence (13*a*) asserts that the disturbance outside the closed surfaces Σ_B and Σ_W, is the same disturbance that would be produced by a fictitious distribution of sources and doublets. Also, it should be noted that Θ (see (13*b*)) is equal to the time necessary for a disturbance to propogate from $\mathbf{X}_1$ to $\mathbf{X}$ at the speed of sound. In other words, Θ is equal to the usual acoustic time delay.

At this stage in the analysis we should note that the volume and surface integrals in (13*a*) have singular contributions when $\mathbf{X}_1$ equals $\mathbf{X}$. In the preceding and subsequent analysis it is assumed that these singular points are excluded from the integration region by suitable principal value definitions. For the volume integral we define a principal value by surrounding the singularity with a sphere of small radius and take the limit as the radius tends to zero. When $\mathbf{X}$ is on either the body or the wake then the surface integrals are singular when $\mathbf{X}_1$ equals $\mathbf{X}$. In this case we define the principal value by surrounding the singularity with a circle of small radius and take the limit as the radius tends to zero.

The integral equation for the perturbation potential can be further simplified by applying the relevant boundary conditions on the body and the wake. Firstly we consider the kinematic condition on a surface in an invisied fluid. This condition amounts to the statement that a fluid particle initially in a surface, remains in that surface throughout the motion. If the surface is defined by $s(\mathbf{x};\ t) = 0$ then the kinematic condition specifies that

$$\frac{D}{Dt}(S(\mathbf{x};\ t)) = S_t + S_x + \phi_x S_x + \phi_y S_y + \phi_z S_z = 0$$

on

$$S(\mathbf{x};\ t) = 0, \tag{14}$$

where D/Dt is the Eulerian derivative. Now consider $\partial\Phi/\partial N$, the normal derivative of Φ in Prandtl–Glauert variables. Using (12*a* and 12*b*), $\partial\Phi/\partial N$ can be written as

$$\frac{\partial\Phi}{\partial N}(\mathbf{X}, T) = -\frac{M^2}{\beta^2}(\bar{\mathbf{N}} \cdot \mathbf{i})U(\mathbf{X}, T) + \frac{M^2(1+\gamma^*)}{\beta^4}\frac{\partial\phi}{\partial n}(\mathbf{x}, t)\frac{|\nabla S|}{|\bar{\nabla} S|} \tag{15}$$

where $\partial\phi/\partial n$ is the normal derivative of ϕ in Eulerian coordinates, and is known from (14) when the surface S is specified.

In the wake the usual Kutta condition applies. The wake is a surface of discontinuity in ϕ, where the two sides of the wake are independent surfaces having the same equation. If the boundary condition (14) is applied to both sides of the wake we have that

$$\Delta\left(\frac{\partial\phi}{\partial n}\right) = 0$$

on

$$S_W(\mathbf{x}, t) = 0, \tag{16}$$

where $\Delta(\ \ )$ now represents the value on the upper side minus the value on the lower side and S_W specifies the wake surface. Equation (16) simply states that there is no jump in normal velocity across the wake. Also from the Kutta condition we know that the flow leaves the trailing edge smoothly. Consequently there can be no discontinuity in pressure at the trailing edge and throughout the wake. When the flow is isentropic the appropriate approximate expression for the pressure coefficient is

$$C_P = -2(\phi_x + \phi_t). \tag{17}$$

Thus across the wake we have that

$$\Delta(\phi_x)+\Delta(\phi_t) = 0$$

on

$$S_W(\mathbf{x}, t) = 0 \tag{18}$$

or in Prandtl–Glauert variables

$$-\frac{M^2}{\beta^2}\Delta U = M\Delta\Phi_T$$

on

$$S_W(\mathbf{X}, T) = 0. \tag{19}$$

If we now apply (16) and (19) to (15) we find that across the wake,

$$\Delta\left(\frac{\partial\Phi}{\partial N}\right) = (\tilde{\mathbf{N}} \,.\, \mathbf{i})M\Delta\Phi_T$$

on

$$S_W(\mathbf{X}, T) = 0. \tag{20}$$

Equation (13*a*) can now be simplified by substituting (15) and (20) into the second integral on the right hand side (i.e. (15) is used over Σ_B and (20) over Σ_W). However, it is convenient at this stage, due to the discontinuity in ϕ, to assume that the wake can be represented by a vortex sheet parallel to the XY-plane. Thus the normal direction to the wake is parellal to the Z-axis and terms involving $\tilde{\mathbf{N}} \,.\, \mathbf{i}$ (in (13*a*)), one zero over the wake. Also, since the fluid is all moving at approximately the same stream speed, a vortex element at a general point $\mathbf{X}$ of the wake was shed in the past at a moment determined by the time interval $\beta^2(X-X_T(Y))/M$ required for it to reach $\mathbf{X}$.
Hence at a general wake point $\mathbf{X}$ and time T.

$$\Delta f(X,\ Y, Z, T) = \Delta f(X_T(Y),\ Y, Z,\ T-\beta^2(X-X_T(Y))/M), \tag{21}$$

where $X_T(Y)$ specifies the position of the trailing edge and f in (21) represents Φ and its derivatives. Thus using (21) integrals over the wake can be evaluated once Φ is known at the trailing edge.

Now using (15) and (20), in the second integral on the right of (13*a*), together with (21) on the horizontal wake, we find, after a few minor manipulations, that the integro-differential equation for Φ becomes

$$E(\mathbf{X})\Phi(\mathbf{X}, T) = \Phi_p(\mathbf{X}, T)$$

$$+\frac{1}{8\pi}\int\int\int_{-\infty}^{\infty} E(\mathbf{X}_1)\left\{U^2\frac{\partial}{\partial X_1}\left(\frac{1}{|\mathbf{X}-\mathbf{X}_1|}\right)-\frac{1}{|\mathbf{X}-\mathbf{X}_1|}\frac{\partial\Theta}{\partial X_1}\frac{\partial U^2}{\partial T_1}\right\}\Bigg|_{T_1 = T-\Theta} dV(\mathbf{X}_1)$$

$$+\frac{1}{8\pi}\int\int_{\Sigma_B}\left\{-M^2\tilde{C}_p/\beta^2+U^2\right\}\Bigg|_{T_1 = T-\Theta}\frac{(\tilde{\mathbf{N}} \,.\, \mathbf{i})}{|\mathbf{X}-\mathbf{X}_1|}d\Sigma(\mathbf{X}_1)$$

$$+\frac{1}{4\pi}\iint_{\Sigma_B}\left\{\Phi\frac{\partial}{\partial N}\left(\frac{1}{|\mathbf{X}-\mathbf{X}_1|}\right)-\frac{\Phi_T}{|\mathbf{X}-\mathbf{X}_1|}\frac{\partial}{\partial N}(|\mathbf{X}-\mathbf{X}_1|)\right\}\Bigg|_{T_1=T-\Theta} d\Sigma(\mathbf{X}_1)$$

$$+\frac{Z}{4\pi}\iint_{R_W}\left\{\frac{\Delta\Phi(X_T(Y_1), Y_1, 0, T-\Gamma)}{|\mathbf{X}-\mathbf{X}_1|^3}+\frac{\Delta\Phi_T(X_T(Y_1), Y_1, 0, T-\Gamma)}{|\mathbf{X}-\mathbf{X}_1|^2}\right\}dX, dY_1, \quad (22a)$$

where

$$\Phi_p(\mathbf{X}, T) = -\frac{M^2(1+\gamma^*)}{4\pi\beta^4}\iint_{\Sigma_B}\frac{\partial\phi}{\partial n}(\mathbf{x}_1, t-\theta)\frac{|\nabla_1 S|}{|\mathbf{X}-\mathbf{X}_1|\,|\tilde{\nabla}_1 S|}d\Sigma(\mathbf{X}_1) \quad (22b)$$

$$\bar{C}_p = -2(U+\beta^2\Phi_T/M) = M^2(1+\gamma^*)C_p/\beta^2 \quad (22c)$$

and

$$\Gamma = \Theta+\beta^2(X_1-X_T(Y_1))/M. \quad (22d)$$

In (22*a*) Φ_p represents the solution for the purely symmetrical problem of linearized flow past a thin symmetrical wing at zero incidence. Once the shape of the body is specified Φ_p is completely determined by (22*b*) and (14). In the last integral of (22*a*) R_W represents the projection of the wake surface Σ_W onto the $X-Y$ plane, which is assumed to be the mean horizontal surface of the wake. Also, we note here that Γ, see (22*d*), is the time necessary for a disturbance to be advected downstream from the trailing edge to the wake point $\mathbf{X}_1$ at free stream speed, plus the acoustic time delay between $\mathbf{X}_1$ and the field point $\mathbf{X}$.

In order to solve for the potential it is necessary, in general, to obtain a numerical approximation for (22*a*). We note that (22*a*) determines the potential, at time T, on the body, the wake and in the fluid from known values of Φ, Φ_T and U at times less than T. The evaluation of Φ_T and U from Φ involves the use of finite differences. When evaluating U, due care must be used to ensure that the correct differencing scheme is used in regions where the flow is locally supersonic. It should be noted however, that, the necessity to find U by finite differencing can be eliminated by differentiating (22*a*) with respect to X (taking due care of the singularity in the volume integral). Thus equation (22*a*) and the integro-differential equation for U can then be used with time differencing, to step forward in time evaluating Φ and U at all points of the flow. However at this stage it is worthwhile making a further approximation which in effect assumes that the wing is a thin nearly planar body.

4. THIN BODY APPROXIMATION

The determination of the original equation (2) (with (3*a*)) was based upon the smallness of the parameter δ (which represents the ratio of airfoil thickness to chord length). Consequently, it is consistent to assume that the wing is a thin nearly-planar body with normal direction parallel to the Z-axis. Applying this approximation to the surface integrals in (22*a*), but not Φ_p, produces a significant simplification. The third term on the right of (22*a*) becomes zero and the last two integrals may be integrated by parts in the X-direction, to produce an integral over a vortex sheet. Note that this is the same as analytically continuing the integrands through the surface of the body onto a mean horizontal plane and then ignoring terms of second order in δ.

Applying the thin body approximation to the surface integrals in (22*a*) and subsequently integrating by parts the last two integrals, we find that the integro-differential equation for Φ can be written as

$$E(\mathbf{X})\Phi(\mathbf{X}, T) = \Phi_p(\mathbf{X}, T)$$

$$+\frac{1}{8\pi}\int\int\int_{-\infty}^{\infty} E(\mathbf{X}_1)\left\{U^2\frac{\partial}{\partial X_1}\left(\frac{1}{|\mathbf{X}-\mathbf{X}_1|}\right)-\frac{1}{|\mathbf{X}-\mathbf{X}_1|}\frac{\partial\Theta}{\partial X_1}\frac{\partial}{\partial T_1}U^2\right\}\Bigg|_{T_1=T-\Theta} dV(\mathbf{X}_1)$$

$$+\frac{Z}{4\pi}\int\int_{R_B}\frac{dX_1dY_1}{[(Y-Y_1)^2+Z^2]}\left[1+\frac{(X-X_1)}{|\mathbf{X}-\mathbf{X}_1|}\right]\left\{\Delta U(X_1,Y_1,0,T-\Theta)+(1-M)\Delta\Phi_T(X_1,Y_1,0,T-\Theta)\right\}$$

$$+\frac{Z}{4\pi}\frac{1}{(1+M)}\int\int_{R_W}\frac{dX_1dY_1}{[(Y-Y_1)+Z^2]}\left[1+\frac{(X-X_1)}{|\mathbf{X}-\mathbf{X}_1|}\right]\Delta U(X_T(Y_1),Y_1,0,T-\Gamma), \tag{23}$$

where R_B represents the projection of Σ_B onto the $X-Y$ plane and (19) has been used to eliminate $\Delta\Phi_T$ on the wake. Note that the last two integrals in (23) represent lifting effects, and correspond to a vortex distribution over the $X-Y$ plane. It should also be noted that if the flow is steady, then derivatives with respect to T are zero, ΔU is zero at the trailing edge and throughout the wake, and (23) then reduces to the formula derived by Klunker [5].

The integral equation (23) for the reduced velocity potential contains derivatives of the reduced velocity potential under the integral signs and as such, is an integro-differential equation for Φ. Although it is quite conceivable to solve this equation by straight forward numerical means it is convenient to derive an integral relationship for the streamwise velocity component U. The relevant integral equation is found by differentiating (23) with respect to X, taking appropriate care of the dipole singularity in the volume integral at the pivotal point (X, Y, Z). The form of the nonlinear singular integral equation for U depends upon the definition of the principal value of the singular volume integral. Following Ogana [12] we define the principal value of this integral by surrounding the singularity with a sphere of small radius and take the limit as the radius tends to zero. It follows from (23) on differentiation with respect to X, that

$$E(\mathbf{X})U(\mathbf{X},T) = E(\mathbf{X})U^2(\mathbf{X},T)/6 + U_p(\mathbf{X},T)$$

$$+\frac{1}{8\pi}\int\int\int_{-\infty}^{\infty} E(\mathbf{X}_1)\left\{U^2(\mathbf{X}_1,T-\Theta)K_1(\mathbf{X},\mathbf{X}_1)+\frac{\partial}{\partial T}U^2(\mathbf{X}_1,T-\Theta)K_2(\mathbf{X},\mathbf{X}_1)\right.$$

$$\left.+\frac{\partial^2}{\partial T^2}U^2(\mathbf{X}_1,T-\Theta)K_3(\mathbf{X},\mathbf{X}_1)\right\}dV(\mathbf{X}_1)$$

$$+\frac{Z}{4\pi}\int\int_{R_B+R_W}\left\{\frac{\Delta U(X_1,Y_1,0,T-\Theta)}{|\mathbf{X}-\mathbf{X}_1|^3}+\frac{\Delta U_T(X_1,Y_1,0,T-\Theta)}{|\mathbf{X}-\mathbf{X}_1|^2}\right\}dX_1dY_1, \tag{24a}$$

where

$$K_1(\mathbf{X},\mathbf{X}_1) = \{(Y-Y_1)^2+(Z-Z_1)^2-2(X-X_1)^2\}/|\mathbf{X}-\mathbf{X}_1|^5, \tag{24b}$$

$$K_2(\mathbf{X},\mathbf{X}_1) = 2M(X-X_1)/|\mathbf{X}-\mathbf{X}_1|^3+|\mathbf{X}-\mathbf{X}_1|K_1(\mathbf{X},\mathbf{X}_1), \tag{24c}$$

$$K_3(\mathbf{X},\mathbf{X}_1) = -[M|\mathbf{X}-\mathbf{X}_1|-(X-X_1)]^2/|\mathbf{X}-\mathbf{X}_1|^3 \tag{24d}$$

and

$$U_p(\mathbf{X},T) = \frac{\partial}{\partial X}\Phi_p(\mathbf{X},T). \tag{24e}$$

In (24*a*) $U_p(\mathbf{X},T)$ is uniquely determined from (24*e*), (22*b*) and (14) once the shape and motion of the body are specified. It is easily seen that equation (24*a*) involves only U, U_T and U_{TT}. If we specify that U is zero everywhere for T less than zero and start the body moving at T equals zero, then using an appropriate numerical approximation, (24*a*) can be used to step forward in time evaluating U at all points of the flow. Equation (23) can be used on the same way to evaluate Φ and Φ_T, while the pressure coefficient can be found from (22*c*).

5. CONCLUSIONS

We have obtained an integro-differential equation for the streamwise velocity component U, which asserts that the disturbance outside the wing and wake, is the same disturbance that would be produced by a fictitious distribution of sources on the wing surface, a vortex sheet on the $Z = 0$ plane and a volume distribution of streamwise directed doublets. It should be noted that if the flow is steady, then derivatives with respect to time are zero and (24*a*) then reduces to the formula given by Ogana [12].

Both (23) and (24*a*) are suitable for fast numerical computations. It has been stated that these equations may be solved by a time-stepping routine which evaluates U or Φ at a given point of the flow from values obtained at earlier times. It should be noted that (23) and (24*a*) may also be solved by assuming that the motion consists of a steady part plus a small harmonically oscillating unsteady part. Thus time-independent integro-differential equations would be obtained which can be solved by an iterative technique.

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APPENDIX

Integral Equation Formulation for Supersonic Mach numbers

When the free stream Mach number is supersonic ($M > 1$) then the Green's function for (4*a* and 4*b*) is (see Morino [7])

$$G(\mathbf{x}-\mathbf{x}_1, t-t_1) = \frac{-1}{4\pi r_\alpha(\mathbf{x},\mathbf{x}_1)}\{\delta(t_1-t+\theta^+)+\delta(t_1-t+\theta^-)\}, \tag{25a}$$

where

$$r_\alpha(\mathbf{x},\mathbf{x}_1) = \{(x-x_1)^2-\alpha^2[(y-y_1)^2+(z-z_1)^2]\}^{\frac{1}{2}}, \tag{25b}$$

$$\alpha^2 = M^2-1 = -\beta^2 \tag{25c}$$

and

$$\theta^\pm = \frac{M}{\alpha^2}[M(x-x_1)\pm r_\alpha(\mathbf{x},\mathbf{x}_1)]. \tag{25d}$$

In (25*a*, 25*b* and 25*d*) it has been assumed that

$$|x-x_1| > \alpha[(y-y_1)^2+(z-z_1)^2]^{\frac{1}{2}}. \tag{26}$$

If (26) is not satisfied then $G(\mathbf{x}-\mathbf{x}_1, t-t_1) = 0$.

An integral equation for the reduced velocity potential when the free stream Mach number is supersonic, can be derived by following a similar procedure to that described in Section 2. That is, multiply (8) by the domain function E (see (6)), substitute for G from (25*a*) and integrate over both space and time. The resulting integral equation is similar to (9) (or (10)) except that twice as many terms appear on the right hand side due to the two time delay terms in (25*a*). Note, as in the subsonic case, the contribution from shock waves will not explicitly appear in the integral equation formulation due to the shock capturing nature of the method.

Applying the quasi steady approximation as derived in Section 3, together with the relevant boundary conditions (cf. with Section 3) and introducing the apposite Prandtl-Galuert variables (for $M > 1$) (cf. with (12*a* and 12*b*)),

$$X = x,\ \ Y = \alpha y,\ \ Z = \alpha z \ \text{ and } \ T = \frac{\alpha^2 t}{M} \tag{27a}$$

with

$$\Phi = M^2(1+\gamma^*)\phi/\alpha^2, \tag{27b}$$

we find that the perturbation potential satisfies (cf. with (22*a*, 22*b* and 22*c*) and (13*b*, 13*c* and 13*e*)),

$$E(\mathbf{X})\Phi(\mathbf{X},T) = \Phi_p^+(\mathbf{X},T)+\Phi_p^-(\mathbf{X},T)$$

$$+\frac{1}{8\pi}\int\int\int_{-\infty}^{\infty} H(\Lambda)\left\{\left[(U^2)^{\ominus+}+(U^2)^{\ominus-}\right]\frac{\partial}{\partial X_1}\left(\frac{1}{R}\right)-\frac{1}{R}\frac{\partial\Theta^+}{\partial X_1}\frac{\partial}{\partial T}(U^2)^{\ominus+}\right.$$

$$\left.-\frac{1}{R}\frac{\partial\Theta^-}{\partial X_1}\frac{\partial}{\partial T}(U^2)^{\ominus-}\right\}dV(\mathbf{X}_1)$$

$$+\frac{1}{8\pi}\int\int_{\Sigma_B}\frac{H(\Lambda)(\tilde{\mathbf{N}}\,.\,\mathbf{i})}{R}\left\{-\frac{M^2}{\alpha^2}[\tilde{C}_p^{\ominus+}+\tilde{C}_p^{\ominus-}]+(U^2)^{\ominus+}+(U^2)^{\ominus-}\right\}d\Sigma(X_1)$$

$$+\frac{1}{4\pi}\int\int_{\Sigma_B+\Sigma_W} H(\Lambda)\left\{\left(\Phi^{\ominus+}+\Phi^{\ominus-}\right)\frac{\tilde{\partial}}{\partial N}\left(\frac{1}{R}\right)-\frac{(\Phi_T^{\ominus+}-\Phi_T^{\ominus-})}{R}\frac{\tilde{\partial}}{\partial N}(R)\right\}d\Sigma(\mathbf{X}_1), \qquad (28a)$$

where

$$\Phi_p^{\pm}(\mathbf{X},T) = -\frac{M^2(1+\gamma^*)}{4\pi\alpha^2}\int\int_{\Sigma_B}\frac{H(\Lambda)}{R|\tilde{\nabla}_1 S|}\frac{\partial\phi}{\partial n}(\mathbf{x}_1,t-\theta^{\pm})\,|\nabla_1 S|\,\mathrm{d}\,\Sigma(\mathbf{X}_1), \qquad (28b)$$

$$\tilde{C}_p = -2(U+\alpha^2\Phi_T/M) = M^2(1+\gamma^*)C_p/\alpha^2, \qquad (28c)$$

$$\Theta^{\pm} = M(X-X_1)\pm R, \qquad (28d)$$

$$R = \{(X-X_1)^2-(Y-Y_1)^2-(Z-Z_1)^2\}^{\frac{1}{2}}, \qquad (28e)$$

$$\frac{\tilde{\partial}}{\partial N}f = \left(-\frac{\partial f}{\partial X},\frac{\partial f}{\partial Y},\frac{\partial f}{\partial Z}\right).\,\tilde{\mathbf{N}}, \qquad (28f)$$

$$\Lambda = |X-X_1|-\{(Y-Y_1)^2+(Z-Z_1)^2\}^{\frac{1}{2}} \qquad (28g)$$

and

$$f^{\ominus\pm} = f(\mathbf{X}_1,T-\Theta^{\pm}). \qquad (28h)$$

Here as in Section 3, $\tilde{\nabla}$ and $\tilde{\mathbf{N}}$ are the gradient operator and unit normal to the surface in Prandtl–Glauert space and H is the Heaviside step function.

Following Section 4 it is consistent to assume that the wing is a thin nearly planar body with normal direction parallel to the Z-axis. Subsequently we find that equation (28*a*) reduces to (cf. with (23))

$$E(\mathbf{X})\Phi(\mathbf{X},T) = \Phi_p^+(\mathbf{X},T) + \Phi_p^-(\mathbf{X},T)$$

$$+\frac{1}{8\pi}\int\int\int_{-\infty}^{\infty} E(\mathbf{X}_1)H(\Lambda)\left\{[(U^2)^{e+}+(U^2)^{e-}]\frac{\partial}{\partial X_1}\left(\frac{1}{R}\right)-\frac{1}{R}\frac{\partial\Theta^+}{\partial X_1}\frac{\partial}{\partial T}(U^2)^{e+}\right.$$

$$\left.-\frac{1}{R}\frac{\partial\Theta^-}{\partial X_1}\frac{\partial}{\partial T}(U^2)^{e-}\right\}dV(\mathbf{X}_1)$$

$$+\frac{Z}{4\pi}\int\int_{R_B+R_W}\frac{dX_1dY_1H(\Lambda)}{[(Y-Y_1)^2+Z^2]}\left(1+\frac{(X-X_1)}{R}\right)\left\{\Delta U(X_1,Y_1,0,T-\Theta^+)\right.$$

$$\left.+\Delta U(X_1,Y_1,0,T-\Theta^-)+(M+1)\Delta\Phi_T(X_1,Y_1,0,T-\Theta^+)+(M-1)\Delta\Phi_T(X_1,Y_1,0,T-\Theta^-)\right\}, \quad (29)$$

where R_B and R_W represent the projection of Σ_B and Σ_W onto the $X-Y$ plane, and, $\Delta(\ )$ represents the value on the upper side of body or wake minus the value on the lower side.

The appropriate integral relationship for the streamwise velocity component U, can now be determined by differentiating (29) with respect to X, taking due care of the dipole singularity in the volume integral at the pivotal point (X,Y,Z). It follows from (29) then, that (cf. with (24*a*–24*e*))

$$E(\mathbf{X})U(\mathbf{X},T) = E(\mathbf{X})U^2(\mathbf{X},T)/3 + U_p^+(\mathbf{X},T) + U_p^-(\mathbf{X},T)$$

$$+\frac{1}{8\pi}\int\int\int_{-\infty}^{\infty} E(\mathbf{X}_1)H(\Lambda)\left\{[(U^2)^{e+}+(U^2)^{e-}]\tilde{K}_1(\mathbf{X},\mathbf{X}_1)+\frac{\partial}{\partial T}(U^2)^{e+}\tilde{K}_2^+(\mathbf{X},\mathbf{X}_1)\right.$$

$$\left.+\frac{\partial}{\partial T}(U^2)^{e-}\tilde{K}_2^-(\mathbf{X},\mathbf{X}_1)+\frac{\partial^2}{\partial T^2}(U^2)^{e+}\tilde{K}_3^+(\mathbf{X},\mathbf{X}_1)+\frac{\partial^2}{\partial T^2}(U^2)^{e-}\tilde{K}_3^-(\mathbf{X},\mathbf{X}_1)\right\}dV(\mathbf{X}_1)$$

$$-\frac{Z}{4\pi}\int\int_{R_B+R_W} dX_1dY_1H(\Lambda)\left\{\frac{[\Delta U(X_1,Y_1,0,T-\Theta^+)+\Delta U(X_1,Y_1,0,T-\Theta^-)]}{R^3}\right.$$

$$\left.+\frac{[\Delta U_T(X_1,Y_1,0,T-\Theta^+)-\Delta U_T(X_1,Y_1,0,T-\Theta^-)]}{R^2}\right\}, \quad (30a)$$

where

$$\tilde{K}_1(\mathbf{X},\mathbf{X}_1) = -2\{2(X-X_1)^2+(Y-Y_1)^2+(Z-Z_1)^2\}/R^5, \tag{30b}$$

$$\tilde{K}_2{}^{\pm}(\mathbf{X},\mathbf{X}_1) = -2M(X-X_1)/R^3 \pm R\tilde{K}_1(\mathbf{X},\mathbf{X}_1), \tag{30c}$$

$$\tilde{K}_3{}^{\pm}(\mathbf{X},\mathbf{X}_1) = -[MR\pm(X-X_1)]^2/R^3 \tag{30d}$$

and

$$U_p{}^{\pm}(\mathbf{X},T) = \frac{\partial}{\partial X}\Phi_p{}^{\pm}(\mathbf{X},T) \tag{30e}$$

As in equation (24*a*), (30*a*) involves only U, U_T and U_{TT}. Then using an appropriate numerical approximation, (30*a*) can be used, stepping forward in time, to evaluate U at all points of the flow. Equation (29) can be used in the same way to evaluate Φ and Φ_T, while the pressure coefficient can be found from (28*c*).

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The unsteady transonic small perturbation differential equation is converted into an integro-differential equation by application of the classical Green's function method. It is shown that no contribution from shock waves explicitly appears in this integral equation, due to the shock capturing properties of the Green's function method. After assuming that the motion consists of small infinitesimal perturbations around a thin nearly-planar body, a simplified integral equation for the streamwise velocity component is obtained, which is suitable for fast numerical computations. Keywords;

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