

Relativistic Effects in the Global Positioning System

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1.0 SUMMARY AND CONCLUSIONS

The GPS system (or NAVSTAR/GPS system) of the DOD, developed and deployed by the USAF, is a system of satellites which transmit navigation signals to users on or near the surface of the Earth. Users with suitable receivers can navigate to ~ 10 m accuracy in near real time. The complete system will comprise 21 satellites for near-continuous coverage of the whole Earth with navigation signals; currently (April 1985) there are 6 functioning GPS satellites in orbit.

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The principle of operation is range determination by precise timing of signals, similar to LORAN but more precise. A GPS receiver determines its range from several (at least 4) GPS satellites by measuring the time of flight of the UHF (L-band) signals from satellite to receiver. Then a microprocessor in the receiver calculates its position by triangulation from the known satellite positions. Each GPS satellite carries one or more highly precise atomic clocks so that the time of transmission of the signals is precisely known.

If (as is not usually the case) a GPS receiver contains its own clock, it can directly measure the times of flight from 3 satellites and thereby determine its 3 spatial coordinates of position. (Each satellite broadcasts ephemeris data sufficient to determine its own position.)

If the receiver does not contain an atomic clock, it must also determine the precise time, and therefore the signal time of flight, by observation of 4 satellites and solution of 4 equations in 4 unknowns; the four unknowns are the 3 spatial coordinates of the receiver and the 1 time reference. The receiver reads out its position every 6 seconds. A receiver may weigh <30 lbs and may cost <33,000. In the future, a receiver on a couple of chips should be available, at greatly reduced cost and weight.

Determining position to an accuracy of 10 m requires that (1) the satellite positions be known to 10 m; and that (2) the clock times be accurate to \sim 30 ns (since radio signals travel very nearly 9 m in 30 ns).

No space qualified clock can maintain an accuracy of 30 ns for more than a few days; moreover, the satellite orbit changes. Therefore it is necessary to update the information broadcast by each satellite about once a day. This is done by a ground station.

The requirement on clock accuracy is so great that the effects of relativity theory on clock rate must be taken into account in system design; otherwise the system would not work as specified. Einstein predicted in 1905 that moving clocks would run slower, as a consequence of the special theory of relativity ("time dilation"); later he predicted that a clock in a gravitational potential well would run slower also, as a consequence of the general theory of relativity ("gravitational redshift"). These effects have been amply confirmed in many experiments, and are considered to be well understood by most physicists. In particular, these effects make it impossible to synchronize globally a system of clocks without relativistic corrections. Most specialists in atomic clocks and time transfer consider that the necessary relativistic corrections are well understood, and they routinely make them every day when intercomparing clocks in different locations on the Earth or in space.

A group of scientists (A. Skalafuris, NRL; J. Cohen and H. Moses, U. Pennsylvania; A. Rosenblum, Max-Planck Institute in West Germany) have for a number of years been claiming that relativistic corrections are not taken into account in GPS system design. Rather, they claim that there are fundamental errors in system design having to do with relativity theory, and that the system will consequently fail to meet its specifications.

A. Skalafuris wrote DARPA on 3 February 1984 again expressing apprehensions about time synchronization in GPS. (This followed several letters by him to various USAF personnel.) As a result, on 27 February DARPA requested that JASON consider the issue in the 1984 Summer Study. This was done.

Our conclusions are (a preliminary report of these conclusions was sent in a letter to R. Cooper at DARPA in July 1984):

• The claims of Skalafuris, Cohen, Moses, and Rosenblum about GPS are completely invalid; the fundamentals of relativistic time synchronization on which the system is based are valid.

• Current operating procedures in GPS give time synchronization accurate to ~ 20 nsec, within current specifications. Simple improvements, requiring no new technology, could reduce the error to ~ 1 nsec. (These have nothing to do with relativistic effects.) Since needs for such higher precision do exist, such improvements should be implemented.

Meanwhile, as a result of a letter from Skalafuris to Nicholas Yannoni at USAF RADC (Hanscomb AFB, Massachusetts), an *ad hoc* committee has been formed by the Air Force Studies Board of the National Academy of Engineeringto look into the question. Professor Clifford Will of Washington University, one of the foremost authorities on experimental tests of relativity theory, is chairman of this Committee on Time Transfer in Satellite Systems. One of the authors of the present report (D. Eardley) is a member of this committee also. The report of this committee will be more comprehensive, and is expected to be released in late 1985.

2.0 RELATIVISTIC EFFECTS ON CLOCKS

Since Einstein it has been realized that time is not absolute and that it is generally not possible to synchronize a system of ideal clocks which are in relative motion (as shown in special relativity theory) or which lie in a spatially varying gravitational potential (as shown in general relativity theory).

2.1 Special Relativity

2.1.1 Time dilation A moving clock runs slow, as measured in the frame of reference of a stationary clock. This is known as "time dilation", and leads to the famous twin "paradox" of special relativity (this is not paradox but fact: A moving twin really is younger when he eventually comes back to rejoin the stationary twin). The magnitude of the effect is given in special relativity by the formula

$$\Delta s = \left(1 - v^2/c^2\right)^{\frac{1}{2}} \Delta t$$

where s is proper time as measured by the moving clock, and t is time in the reference frame of the stationary clock; v is clock velocity and c is the speed of light. For purposes of GPS, the second order approximation

$$\Delta s \approx \left(1 - \frac{v^2}{2c^2}\right) \Delta t$$

is entirely adequate. This result is often called the second order Doppler shift. These formulae are only valid in an inertial (*i.e.*, nonrotating) frame of reference. In a rotating frame the physics is exactly the same, but the expression of the results is somewhat different. The difference is important because the Earth rotates and satellites orbit about the Earth.

2.1.2 The Sagnac Effect In a rotating frame, second order Doppler shift causes time transfer to be nonintegrable — that is, if a clock is transferred (even very slowly) around a closed loop, its reading at the end of the transfer will lag that of a stationary clock, sitting at the beginning/ending point of the closed loop, by an amount

$$\Delta s pprox rac{2\omega A}{c^2}$$

where ω is the angular velocity of frame rotation, and A is the surface area of the loop, projected into a plane perpendicular to the axis of rotation. This is called the Sagnac effect¹. Here only terms up to second order in 1/c have been kept, which is again entirely adequate for GPS. The effect is nontrivial, and amounts to ~207 nsec for clock transfer around the equator of the rotating Earth, and 9.07 μ sec for clock transfer around the geosynchronous orbit (this last example is purely hypothetical — orbiting clocks have never been actually synchronized this way). Since modern atomic clocks are often accurate to within a few nsec/day, the Sagnac effect must be taken into account in comparing clocks at different places on the Earth and in space; and they are.^{2,3} Laser gyroscopes depend on this effect for measuring ω . Recently the effect has been directly measured and confirmed on a large scale by use of GPS satellites to transfer time all the way around the world and back to the starting point.^{4,5}

2.2 General Relativity — The Gravitational Redshift

According to general relativity theory, a gravitational potential well causes a clock to run slow. To second order, the effect is given by

$$\Delta s \approx \left(1 + \frac{\Phi}{c^2}\right) \Delta t$$

where Φ is gravitational potential, as measured relative to the fiducial clock.

2.3 Combining The Effects

If one needed the exact result in general relativity theory, then combining the above results is difficult, and indeed the separation of the various effects one from the other is not really well defined, although the clock rate itself, which is an observable, is of course well defined. However, we do not live on a neutron star or near a black hole, and the effects of special and general relativity are small enough for GPS purposes that the effects may simply be added together to more than adequate accuracy.

3.0 EFFECTS IN GPS

3.1 Relativistic Effects In GPS

The relative magnitude of time dilation and gravitational redshift effects for a GPS satellite in its 12 hr, near-circular orbit is ~ $4 \cdot 10^{-10}$, or about $\pm 10^4$ nsec/day, a very significant amount.

In GPS these relativistic effects are taken into $\operatorname{account}^{6,7}$ by two successive correction procedures: (1) Each satellite clock signal is offset by a relative amount $4.45 \cdot 10^{-10}$ by frequency mixing. This correction approximately, but not exactly, removes the relativisitic effects, *i.e.*, time dilation, Sagnac effect, and gravitational redshift, from the time signals as received by a GPS user. (2) Residual corrections are then made in software in each GPS user's receiver, based on correction parameters broadcast by each satellite. This residual correction accounts for all relativistic effects in GPS, down to the specified system accuracy. The correction parameters are updated from the ground once a day. In principle this is a completely adequate procedure according to relativity theory. Furthermore, many experiments have confirmed that the theory is correct to a higher degree of accuracy than is required in GPS.

A recent time transfer experiment^{4,5} using GPS satellites was able actually to test the standard Sagnac correction and checked the validity of the corrections to ± 5 nsec.

3.2 Other Effects

Other relativistic effects will come in only at accuracy levels of ≤ 1 cm (in position) or ≤ 0.03 nsec (in time transfer accuracy). These include: (1) Relativistic corrections to the satellite orbit; (2) Relativistic corrections to radio signal propagation (most significantly, Shapiro time delay); (3) Frame-dragging due to Earth rotation (Lense-Thirring effect).

In fact, a host of nonrelativistic effects, which we have not touched upon, are much more important. These include: (1) Ionospheric time delays (which are approximately cancelled out by a two-frequency technique in GPS; (2) Tropospheric propagation effects; (3) Clock drifts; (4) Unmodelled or inadequately modelled effects which alter the satellite ephemeris — including (4a) solar radiation pressure and (4b) transmitter recoil radiation pressure. Effect (2) seems likely in the long run to be the most recalcitrant for routine users of GPS, with errors of ~ 1 nsec. However, users able to carry out simple measurements of local tropospheric conditions can correct GPS readings down to ~ 0.1 nsec most of the time, using a model atmosphere calculation. Currently, system errors for time transfer are ~ 30 nsec, though levels of ~ 5 nsec are achievable under carefully chosen conditions.^{5,4} The principal source of error seems likely to be ephemeris errors, although formally the errors show up as clock errors in the current implementation of the Kalman filter. There are real needs for time transfer accurate to ~ 1 nsec, and GPS could fill them if it can be cleaned up a bit. This should not be hard, and we recommend that it be done. In particular, some independent, direct means of ranging to at least one GPS satellite (ground based radar, laser retroflector and event timer on satellite,...) should be implemented so that clock errors can be measured separately from ephemeris errors. This should be implementable with only trivial increase in complexity and no diminution of robustness.

4.0 THE CRITICISMS BY SKALAFURIS, COHEN, MOSES AND ROSENBLUM

Cohen and Skalafuris have furnished a proposal⁸ to DARPA which contains a detailed account of their criticisms of GPS. The principal points made in that proposal are:

- 1. Cohen and Moses⁹ showed in 1977 that a new effect exists in systems of rotating clocks, such that there would be a 9.07 μ sec error in global synchronization by means of clock transport of clocks in geosynchronous orbit.
- 2. Cohen, Moses and Rosenblum¹⁰ showed in 1983 that a similar but quantitatively distinct effect occurs for clocks synchronized by exchange of electromagnetic signals.
- 3. A time transfer experiment by Saburi *et al.*, ¹¹ confirms the 9.07 μ sec effect of Cohen and Moses⁹.
- 4. The paper of Ashby and Allan² on time transfer contains erroneous language as well as erroneous formulae. In particular, the important Equation (1) is wrong.

In addition, the proposal⁹ contains a number of significant statements which are erroneous. Among the more relevant of these are:

5. GPS satellite orbits are chosen so that time dilation and gravitational redshift cancel out.

6. Monitor control systems of GPS update each satellite's time signals to agree with UTC time, as maintained by the Naval Observatory.

Our comments and conclusions on these points are given below.

1. and 2. The effects treated by Cohen and Moses⁹, and those treated by Cohen, Moses and Rosenblum¹⁰, were not new at all. They are essentially equivalent to the Sagnac effect¹ in special relativity, well known to physicists for more than 50 years. A complete and general account of such effects, within the full context of general relativity theory, was for instance given in a standard textbook¹² in 1962.

The papers by Cohen and Moses⁹, and by Cohen, Moses and Rosenblum¹⁰, appear correct, but they contain nothing new for the purpose of time synchronization of satellite systems such as GPS.

The treatment of these effects for clock comparisons was reviewed by Ashby and Allan² in 1979; their formulas show in particular that clock transport synchronization and electromagnetic signal synchronization require different correction terms. The Bureau International Des Poids et Mesures has recognized these effects, and the proper corrections for them, in an official document³.

These effects are not uncontrollable errors; rather, they give completely computable corrections.

The treatment of these effects in GPS at present is entirely adequate in principle, to any achievable level of precision.

3. This point is completely wrong and involves an elementary misunderstanding of procedures for precise time transfer. Saburi *et al.*, ¹¹ found a ~9.439 μ sec offset between two distant atomic clocks. Offsets are accidents of history for clocks. There was no direct measurement of the Sagnac effect in this experiment; rather, the standard Sagnac correction was used without being tested in this particular experiment. Comparison of time transfer by means of electromagnetic signals bounced off a satellite in GEO, with time transfer by means of clock transfer on Earth surface, was however consistent with the usual Sagnac correction.

A more recent and more comprehensive time transfer experiment^{4,5} using GPS satellites was able actually to test the standard Sagnac correction and confirmed theory to ± 5 nsec.

4. We have independently derived Equation (1) and it is correct. While we have not checked the whole paper, we have read it all and reviewed important parts of it carefully, and we find no reason to believe that it is in any significant way erroneous.

It is true that the paper is not written in language similar to that of most textbooks in relativity theory, and somewhat different notation and terminology is used. Furthermore, some of the underlying assumptions and connections to fundamental theory are not spelled out as clearly as some theoretical physicists might hope. This is perhaps not surprising in that most such textbooks are at most tangentially concerned with real world measurements and systems, while the paper² is wholly so concerned. In our view the precise language or terminology is not very important; it is the results and formulae which are important. These would appear to be completely correct, for any time transer procedure on the Earth or in near Earth space, down to an accuracy level that any present or near term system might achieve. The most significant ignored terms are at the first post-Newtonian level of successive approximation, and have to do with the Shapiro relativistic time delay effect, which is of order 0.03 nsec here. For GPS, where system specifications are at the 30 nsec level, and where ultimate system capability with upgrades would be near 1 nsec, the paper² is completely adequate.

More recent papers (e.g., Ref. 15,16) have gone a good way toward spelling out in detail the connections of time transfer procedures to fundamental relativity theory. We

think that there is some need for further work along these lines, both original research and pedagogy. Skalafuris *et al.*, solicited comments on the paper of Ashby and Allan² from two respected relativity theorists, who have done distinguished work in mathematical relativity but who unfortunately have no research experience with experimental tests of relativity theory, actual measurements in relativity, or in particular with precise time transfer. These two relativists expressed doubts^{13,14} about the adequacy of the Ashby and Allan treatment although they did not point out any mistakes.

Many of their doubts can in fact be answered by appealing to the standard scientific literature on tests of relativity theory (see, e.g., Ref. 17). For instance, Seifert¹³ and Clarke¹⁴ both express doubt about the coordinate system used by Ashby and Allan; however, this coordinate system is just a special case of the PPN frame (cf. Ref. 17, Chap. 4) which is well justified and widely used in tests of relativity theory. Again, Clarke¹⁴ raises the question of "frame-dragging" (Lense-Thirring effect) due to the Earth's rotation as causing problems; however a short calculation shows that this effect is roughly 10^{12} times too small to be of any consequence for GPS. Indeed this effect has never been detected, although the NASA GP-B (Gravity Probe B) satellite is currently planned for launch several years hence, to attempt to detect it. As a result of these apparent confusions by highly respected researchers, we perceive some need for a pedagogical article that will explain time transfer and GPS, not in the practical terms in which they have been expounded up to now, but in the careful language of theoretical relativity.

Moreover, theory has been verified experimentally. The relativistic corrections for Sagnac effect have been recently measured in $GPS^{4,5}$ and agree with theory² to ± 5 nsec. This demonstrates that theory is correct to this level (which few physicists would doubt anyway), and also, quite importantly, serves as a consistency check that the actual implementation of the theoretical formulae in GPS is correct.

5. This is wrong⁶.

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6. Untrue. The system time of GPS is maintained separately from UTC, except for long term corrections to keep them from drifting more than 1000 nsec apart.

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