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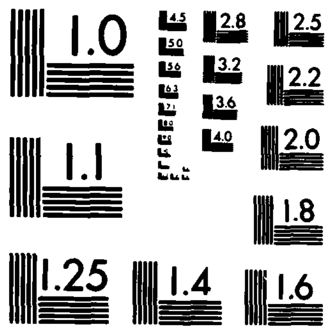

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A VARIANT OF SHANBHAG'S LEMMA ARISING OUT OF A MODIFIED RAO-RUBIN CONDITION

by

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A modified Rao-Rubin condition for damage models gives rise to a recurrence relation which is somewhat different from that considered by Shanbhag (1977). A complete solution to the new recurrence relation is obtained and its applications are indicated.

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Summary A modified Rao-Rubin condition for damage models gives rise to a recurrence relation which is somewhat different from that considered by Shanbhag (1977). A complete solution to the new recurrence relation is obtained and its applications are indicated.

Key Words: Damage models, Integrated Cauchy functional equation, Rao-Rubin condition, Shanbhag's lemma.

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1. INTRODUCTION

Let (X,Y) be a 2-vector of non-negative integer valued random variables such that $P(X \geq Y) = 1$ and the conditional distribution of Y given $X = n$ is binomial with index n and probability of success π. In such a case, the Rao-Rubin condition (see Rao and Rubin (1964))

$$P(Y = y) = P(Y = y | X = Y), \quad y = 0,1,\ldots \tag{1.1}$$

characterizes the distribution of X as Poisson. Shanbhag (1977) considered a more general convolution type conditional distribution of Y given X, in which case the condition (1.1) gives rise to the recurrence relation

$$v_m = \sum_{n=0}^{\infty} v_{m+n} w_n, \quad m = 0,1,\ldots . \tag{1.2}$$

He obtained a complete solution of (1.2) in the form

$$v_n = v_0 b^n, \quad \sum_{n=0}^{\infty} b^n w_n = 1 \tag{1.3}$$

provided $v_n \neq 0$ for some $n \geq 1$ and $w_1 > 0$. Shanbhag's lemma proved a useful tool in solving a variety of characterization problems (see Rao and Shanbhag (1984) and the references cited in the paper).

A question has been raised by Srivastava and Singh (1975) as to whether the Rao-Rubin result on the Poisson distribution holds under the modified Rao-Rubin condition

$$P(Y = y) = P(Y = y | X - Y = k), \quad k > 0, \ y = 0,1,\ldots . \tag{1.4}$$

Patil and Taillie (1979) showed that the Rao-Rubin result may not hold in such a case, but two conditions of the type (1.4) for k and $k+k_1$ where $k_1 > 0$ provide a unique characterization of the distribution of X. Shanbhag and Taillie (1979)

(1979) extended the result of Patil and Taillie to a more general conditional distribution of Y given X. It is seen that a single conditon of the type (1.4) leads to the recurrence relation

$$v_{m+k} = \sum_{n=0}^{\infty} v_{m+n} w_n, \quad m = 0,1,\ldots \tag{1.5}$$

where $k > 0$ is a fixed number. In such a case, the solution is not so simple. We obtain the complete solution to (1.5) and consider some applications.

The continuous analogues of the equations (1.2) and (1.5) have been considered by Lau and Rao (1982, 1984).

2. THE MAIN LEMMA

Let $\{(v_n, w_n): n = 0,1,\ldots\}$ be a sequence of vectors such that at least one $v_n \neq 0$ and $w_0 > 0$. Further let k be a positive integer such that the least common divisor of k and those n for which $w_n > 0$ be 1, and T be a non-negative matrix (in the sense of Senata (1973)) such taht the corresponding state space is $\{0,1,2,\ldots\}$ and the (i,j)-th element is

$$T_{ij} = \begin{cases} \delta_{ij} & \text{if } i,j = 0,\ldots,k-1 (\delta_{ii} = 1 \text{ and } \delta_{ij} = 0,\ i \neq j), \\ w_{j-i+k} & \text{if } i = k,k+1,\ldots \text{ and } j \geq i-k, \\ 0 & \text{otherwise.} \end{cases}$$

Define by $\{f_{i,r}: i \geq k,\ r = 0,\ldots,k-1\}$ the sequence of absorption measures corresponding to T and by $T_{ij}^{(n)}$, the n-step transition measure corresponding to the transition $i \to j$. Let

$$D = \{b: b > 0,\ b^k = \sum_{n=0}^{\infty} b^n w_n\}$$

LEMMA The sequence $\{(v_n, w_n): n=0,1,\ldots\}$ as defined above satisfies the recurrence equations

$$v_{m+k} = \sum_{n=0}^{\infty} w_n v_{n+m}, \quad m=0,1,\ldots \tag{2.1}$$

iff D is non-empty, $f_{m,0},\ldots,f_{m,k-1}$ are finite, and one of the following hold.

(i) D has only one point, $\sum_{n=0}^{\infty} (n-k)b^b w_n < 0$ for $b \in D$, and

$$v_m = f_{m,0}v_0 + \ldots + f_{m,k-1}v_{k-1}, \quad m=k,k+1\ldots .$$

(ii) D has only one point, $\sum_{n=0}^{\infty} (n-k)b^n w_n = 0$ for $b \in D$ and for some $c \geq 0$

$$v_m = f_{m,0}(v_0 - c.o.b^0.) + \ldots + f_{m,k-1}(v_{k-1} - c(k-1)b^{k-1}) + cmb^m,$$

$$m=k,k+1,\ldots,$$

with $b \in D$.

(iii) D contains two points and for some $c \geq 0$

$$v_m = f_{m,0}(v_0 - cb^0) + \ldots + f_{m,k-1}(v_{k-1} - cb^{k-1}) + cb^m, \quad m=k,k+1,\ldots,$$

with b as the larger of the two members of D.

Proof The 'if' part of the theorem follows by a straight-forward verification. We shall now establish the 'only if' part of the theorem. Let $V = (v_0, v_1, \ldots)'$. Clearly we have $V = TV$ and hence

$$V = T^n V, \quad n \geq 1. \tag{2.2}$$

Observe that $\{T_{ij}^{(n)}\}$ is an increasing sequence in n for each $i \geq k$ and $j=0,1,\ldots,k-1$. Consequently, $\lim_{n\to\infty} T_{ij}^{(n)}$ exists for each $i \geq k$ and $j=0,1,\ldots,k-1$. These are given by $f_{i,j}$ defined earlier. Then (2.2) implies that

$$v_m = f_{m,0}v_0 + \ldots + f_{m,k-1}v_{k-1} + \lim_{n\to\infty} \sum_{j=k}^{\infty} T_{mj}^{(n)} v_j, \quad m = k, k+1, \ldots \tag{2.3}$$

Since at least one $v_n \neq 0$, it is clear that there exists an $n_0 \geq k$ such that $v_{n_0} > 0$. Given any $j \in \{0,1,\ldots,k-1\}$, clearly (2.3) is valid with $\{v_m\}$ replaced by $\{v_m^{(j)}\}$ where

$$v_m^{(j)} = v_{m+n_0-j}, \quad m \geq 0 .$$

Consequently, it follows in view of (2.3) that $f_{m,j} < \infty$ for each $m \geq k$ and $j = 0,1,\ldots,k-1$. Replacing v_m in (2.3) by v_{m+r-k}, we get for $m = k$

$$v_r = f_{k,0}v_{r-k} + \ldots + f_{k,k-1}v_{r-1} + \lim_{n\to\infty} \sum_{j=k}^{\infty} T_{kj}^{(n)} v_{j+r-k}, \quad r \geq k. \tag{2.4}$$

Define

$$\xi_m^{(n)} = \sum_{j=k}^{\infty} T_{kj}^{(n)} v_{j+m-k}, \quad m = 0,1,2,\ldots$$

and $\xi_m = \lim_{n\to\infty} \xi_m^{(n)}$. In view of the dominated convergence theorem, the existence of $\lim_{n\to\infty} \xi_m^{(n)}$ for $m = 0,1,\ldots,k-1$ follows inductively from the fact that the sequence $\{\xi_m^{(n)}\}_{m=0}^{\infty}$ is a non-negative real sequence satisfying (2.1) for each n. By considering t to be the minimum of the states in the passage from k to j, we get

$$\begin{aligned}
\xi_m^{(n)} &= \sum_{j=k}^{\infty} T_{kj}^{(n)} v_{j+m-k} \\
&= \sum_{\ell=0}^{\infty} w_{\ell+k} \sum_{r=0}^{n-1} \sum_{t=k}^{\ell+k} {}_t f_{\ell+k,t}^{(r)} \sum_{j=t}^{\infty} T_{k,j-t+k}^{(n-r-1)} v_{j+m-k} \\
&= \sum_{\ell=0}^{\infty} w_{\ell+k} \sum_{r=0}^{n-1} \sum_{t=k}^{\ell+k} {}_t f_{\ell+k,t}^{(r)} \xi_{m+t-k}^{(n-r-1)},
\end{aligned} \tag{2.5}$$

where $t^{(r)}_{f_{\ell+k,t}}$ is the measure of the set that the first passage from $\ell+k$ to t avoiding states lower than t takes place at time r. The term following $w_{\ell+k}$ under the summation on the right hand side of (2.5) is bounded by $v_{\ell+m}$. Consequently, the dominated convergence theorem implies that

$$\xi_m = \sum_{\ell=0}^{\infty} w_{\ell+k} \sum_{r=0}^{\infty} \sum_{t=k}^{\ell+k} t^{(r)}_{f_{\ell+k,t}} \xi_{m+t-k}$$

$$= \sum_{t=k}^{\infty} \xi_{m+t-k} \sum_{\ell=t-k}^{\infty} w_{\ell+k} \sum_{r=0}^{\infty} t^{(r)}_{f_{\ell+k,t}} .$$

From Shanbhag's lemma, we can then conclude that

$$\xi_m = \alpha b^m, \quad m = 0,1,2,\ldots,$$

for some non-negative α and b. (For the case $w_\ell = 0$ for all $\ell \geq k+1$, the result is tirvial; otherwise this is a corollary of Shanbhag's lemma.) Using (2.4), it follows that

$$v_m = f_{k,0} v_{m-k} + \ldots + f_{k,k-1} v_{m-1} + \alpha\, b^m, \quad m \geq k. \tag{2.6}$$

If $\theta > 0$ such that

$$\theta^k = f_{k,0}\theta^0 + \ldots + f_{k,k-1}\theta^{k-1} , \tag{2.7}$$

then clearly by induction (in view of the fact that $f_{k,0},\ldots,f_{k,k-1}$ are respectively the first passage measures for the transition $m \to m-k+j$ avoiding the states lower than m until then for $j = 0,1,\ldots,k-1$).

$$\begin{aligned}\theta^m &= f_{k,0}\theta^{m-k} + \ldots + f_{k,k-1}\theta^{m-1} \\ &\vdots \\ &= f_{m,0}\theta^0 + \ldots + f_{m,k-1}\theta^{k-1}, \qquad m = k,k+1,\ldots,\end{aligned}$$

which implies $\theta \in D$. Thus D is non-empty. If $\alpha \neq 0$ and $b \neq 0$, then $b \in D$. If D is a singleton and $\alpha, b \neq 0$, then b satisfies (2.7) and there exists a $c > 0$ such that

$$ckb^k = c.0f_{k,0}b^0 + \ldots + c(k-1)f_{k,k-1}b^{k-1} + \alpha b^k.$$

Consequently (2.6) implies that

$$\begin{aligned} v_m - cmb^m &= f_{k,0}(v_{m-k} - c(m-k)b^{m-k}) + \ldots + f_{k,k-1}(v_{m-1} - c(m-1)b^{m-1}) \\ &\vdots \\ &= f_{m,0}(v_0 - c.o.b^0) + \ldots + f_{m,k-1}(v_{k-1} - c(k-1)b^{k-1}), \quad m \geq k, \end{aligned}$$

which, in turn, implies that $\{cmb^m\}$ satisfies (2.1) and hence in view of the fact that $b \in D$ yields that $\sum_{n=0}^{\infty}(n-k)b^n w_n = 0$. On the other hand, if D is a doubleton and $\alpha, b \neq 0$, then essentially an argument the same as the above implies that b cannot satisfy (2.7) since $\sum_{n=0}^{\infty}(n-k)b^n w_n = 0$ implies D to be a singleton. The b in question is a point of D and hence we have

$$b^k > f_{k,0}b^0 + \ldots + f_{k,k-1}b^{k-1} ,$$

which implies that there exists a $c \geq 0$ such that

$$cb^k = c.\ b^0 f_{k,0} + \ldots + cb^{k-1} f_{k,k-1} + \alpha b^k$$

or

$$\begin{aligned} v_m - cb^m &= f_{k,0}(v_{m-k} - cb^{m-k}) + \ldots + f_{k,k-1}(v_{m-1} - cb^{m-1}) \\ &\cdots \\ &= f_{m,0}(v_0 - cb^0) + \ldots + f_{m,k-1}(v_{k-1} - cb^{k-1}), \quad m \geq k. \end{aligned}$$

The result is now obvious. [Note that if D is a singleton and $b \in D$, then it is impossible that $\sum_{n=0}^{\infty}(n-k)b^n w_n = 0$.]

Recently Lau and Rao (1984) considered a continuous analogue of (2.1), which is a modified version of the integrated Cauchy functional equation discussed in Lau and Rao (1982), and obtained a general solution using a random walk approach.

Corollary If the sequence (2.1) is such that

$$\lambda v_{n+k} = v_{n+k_1}, \quad n = 0,1,\ldots,k,$$

for some integer $k_1 > k$ and some real λ, then

$$v_n = v_0 \lambda^{n/(k_1-k)}, \quad n = 0,1,\ldots .$$

Proof The result of the main lemma (in particular, the relation (2.6)) implies that

$$v_{n+k_1-k} = \lambda v_n \quad \text{for all } n \geq 0 \qquad (2.8)$$

with $\lambda > 0$. Reducing the system of equations in (2.6) to the one in Shanbhag's lemma by appropriate substitutions via (2.8) we arrive at the required result.

Note 1. It is also possible to prove the result of the corollary by using Perron-Frobenius theorem given in Seneta (1973, pp. 1-2). For details, the reader is referred to Alzaid (1983).

Note 2. Let $Y_{1,n} \leq \ldots \leq Y_{n,n}$ denote ordered boservations in a smaple of size n from a non-degenerate distribution F concentrated on non-negative integers. Arnold (1980) raised the question whether the independence of the r.v. $Y_{2,n}-Y_{1,n}$ and the event $\{Y_{1,n} = m\}$ for a fixed $m > 1$ implies the distribution F to be geometric. It is easily seen that this property leads to a recurrence relation of the type (2.1). Hence an application of the main lemma shows that the distribution is not necessarily goemetric. However, it can be shown that if the stated independence holds for two fixed values of m, then under some mild conditions, F is indeed geometric.

Note 3. In view of the result of our main lemma, it is seen that Theorem 4 of Krishnaji (1974) is not correct. This also follows from the counter example given by Patil and Taillie (1979). The error in Krishnaji's argument appears in the last sentence of the proof in which it is claimed that since $X = \Lambda\exp\{\Lambda(\theta-1)\}$ is degenerate, Λ has to be degenerate. We may however point out here that Krishnaji's theorem with the portion "G(t) is non-negative for all real t" in it replaced by "G(t) is infinitely divisible" is valid.

Note 4. The result of our main lemma could also be applied to obtain certain conclusions concerning stationary queue length distributions in $GI/E_k/1$ queueing systems and also in their modifications when the arrivals are in batches of random size. In particular, the result implies that the queue length distribution corresponding to such a system exists if andonly if the relative traffic intensity of the system is less than 1 and also that the stationary queue length distribution for the system GI/M/1 is geometric.

Note 5. Let (X,Y) be a random vector variable with non-negative integer valued components such that the conditional distribution of Y given X=x is given by

$$S(y|x) = \frac{a_y b_{x-y}}{c_x}, \quad y = 0,1,\ldots,x$$

for almost all x, where $\{c_n\}$ is the convolution of $\{a_n\}$ and $\{b_n\}$, $a_n > 0$ for all n, $b_0 > 1$ and $b_n \geq 0$, $n \geq 1$ with the least common divisor of n for which $b_n > 0$ equal to 1. Then, according to the corollary to the main lemma, the conditions

$$P(Y=y) = P(Y=y|X-Y=k_0), \quad y = 0,1,\ldots ,$$

$$P(Y=y \;\; X-Y=k_0) = P(Y=y \;\; X-Y=k_0+k_1), \quad y = 0,1,\ldots,k_0,$$

for some $0 \leq k_0 < k_0+k_1 < \infty$ with $P(X-Y=k_0) > 0$ and $P(X-Y=k_0+k_1) > 0$ is equivalent to the assertion that Y and X-Y are independent. This implies that

$$P(X=x) \propto c_x \theta^x, \quad x = 0,1,2,\ldots$$

for some $\theta > 0$.

The above result extends an unpublished characterization given by Shanghag and Taillie (1979) of a distribution $\{g_x\}$ of the type

$$g_x \propto c_x \theta^x, \quad x = 0,1,\ldots \qquad (2.9)$$

with θ as some positive constant, based on two modified Rao-Rubin conditions. Our lemma proves that there exist infinitely many distributions other than that of the form (3.9) for which one modified Rao-Rubin condition holds and implicitly identifies the class of such distributions.

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