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Mean Residual Life: Theory and Applications

by

Frank Guess and Frank Proschan

North Carolina State University and Florida State University

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Abstract

This is a chapter for the Handbook of Statistics, Volume 7, Quality Control and Reliability, edited by P. R. Krishnaiah. We survey the rich theory and important applications of the concept of mean residual life.

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1. Introduction and Summary.

The mean residual life (MRL) has been used as far back as the third century A.D. (cf. Deevey (1947) and Chiang (1968)). In the last two decades, however, reliabilists, statisticians, and others have shown intensified interest in the MRL and derived many useful results concerning it. Given that a unit is of age t , the remaining life after time t is random. The expected value of this random residual life is called the *mean residual life* at time t . Since the MRL is defined for each time t , we also speak of the MRL function. (See Section 2 for a more formal definition.)

The MRL function is like the density function, the moment generating function, or the characteristic function: for a distribution with a finite mean, the MRL completely determines the distribution via an inversion formula (e.g., see Cox (1962), Kotz and Shanbhag (1980), and Hall and Wellner (1981)). Hall and Wellner (1981) and Bhattacharjee (1982) derive necessary and sufficient conditions for an arbitrary function to be a MRL function. These authors recommend the use of the MRL as a helpful tool in model building.

Not only is the MRL used for parametric modeling but also for nonparametric modeling. Hall and Wellner (1981) discuss parametric uses of the MRL. Large nonparametric classes of life distributions such as decreasing mean residual life (DMRL) and new better than used in expectation (NBUE) have been defined using MRL. Barlow, Marshall, and Proschan (1963) note that the DMRL class is a natural one in reliability. Brown (1983) studies the problem of approximating increasing mean residual life (IMRL) distributions by exponential distributions. He mentions that certain IMRL distributions, "... arise naturally in a class of first passage time distributions for Markov processes, as first illuminated by Keilson." See Barlow and Proschan (1965) and Hollander and Proschan (1984) for further comments on the

nonparametric use of MRL.

A fascinating aspect about MRL is its tremendous range of applications. For example, Watson and Wells (1961) use MRL in studying burn-in. Kuo (1984) presents further references on MRL and burn-in in his Appendix 1, as well as a brief history on research in burn-in.

Actuaries apply MRL to setting rates and benefits for life insurance. In the biomedical setting researchers analyze survivorship studies by MRL. See Elandt-Johnson and Johnson (1980) and Gross and Clark (1975).

Morrison (1978) mentions IMRL distributions have been found useful as models in the social sciences for the lifetimes of wars and strikes. Bhattacharjee (1982) observes MRL functions occur naturally in other areas such as optimal disposal of an asset, renewal theory, dynamic programming, and branching processes.

In Section 2 we define more formally the MRL function and survey some of the key theory. In Section 3 we discuss further its wide range of applications.

2. Theory of mean residual life.

Let F be a life distribution (i.e., $F(t) = 0$ for $t < 0$) with a finite first moment. Let $\bar{F}(t) \equiv 1 - F(t)$. X is the random life with distribution F . The mean residual life function is defined as

$$(2.1) \quad \begin{aligned} m(t) &= E\{X - t | X > t\} && \text{for } \bar{F}(t) > 0 \\ &= 0 && \text{for } \bar{F}(t) = 0 \end{aligned}$$

for $t \geq 0$. Note that we can express $m(t) = \int_0^{\infty} \frac{\bar{F}(x+t)}{\bar{F}(t)} dx = \int_t^{\infty} \frac{\bar{F}(u)}{\bar{F}(t)} du$ when $\bar{F}(t) > 0$.

If F also has a density f we can write $m(t) = \int_t^{\infty} uf(u)du / \bar{F}(t) - t$.

Like the failure rate function (recall that it is defined as $r(t) = f(t)/\bar{F}(t)$ when $\bar{F}(t) > 0$), the MRL function is a conditional concept. Both functions are conditioned on survival to time t .

While the failure rate function at t provides information about a small interval after time t ("just after t ," see p. 10 Barlow and Proschan (1965)), the MRL function at t considers information about the whole interval after t ("all after t "). This intuition explains the difference between the two.

Note that it is possible for the MRL function to exist but for the failure rate function not to exist (e.g., consider the standard Cantor ternary function, see Chung (1974) p. 12). On the other hand, it is possible for the failure rate function to exist but the MRL function not to exist (e.g., consider modifying the Cauchy density to yield $f(t) = \frac{2}{\pi(1+t^2)}$ for $t \geq 0$). Both the MRL and the failure rate functions are needed in theory and in practice.

When m and r both exist the following relationship holds between the two:

$$(2.2) \quad m'(t) = m(t)r(t) - 1,$$

for m differentiable. See Watson and Wells (1961) for further comments on (2.2) and its uses.

If the failure rate is a constant (>0) the distribution is an exponential.

If the MRL is a constant (>0) the distribution is also an exponential.

Let $\mu = E(X)$. If $F(0) = 0$ then $m(0) = \mu$. If $F(0) > 0$ then $m(0) = \mu/\bar{F}(0) \neq \mu$. For simplicity in discussions and definitions in this section, we assume $F(0) = 0$. Let F be right continuous (not necessarily continuous). Knowledge of the MRL function completely determines the reliability function as follows:

$$(2.3) \quad \bar{F}(t) = \frac{m(0)}{m(t)} e^{-\int_0^t \frac{1}{m(u)} du} \quad \text{for } 0 \leq t < F^{-1}(1)$$

$$= 0 \quad \text{for } t \geq F^{-1}(1),$$

where $F^{-1}(1) \stackrel{\text{def}}{=} \sup\{t | F(t) < 1\}$.

Cox (1962) assigns as an exercise the demonstration that MRL determines the reliability. Meilijson (1972) gives an elegant, simple proof of (2.3). Kotz and Shanbhag (1980) derive a generalized inversion formula for distributions that are not necessarily life distributions. Hall and Wellner (1981) have an excellent discussion of (2.3) along with further references.

A natural question to ask is: what functions are MRL functions? A characterization is possible which answers this. By a function f being increasing (decreasing) we mean that $x \leq y$ implies $f(x) \leq (\geq) f(y)$.

Theorem 2.1. Consider the following conditions:

- i. $m: [0, \infty) \rightarrow [0, \infty)$.
- ii. $m(0) > 0$.
- iii. m is right continuous (not necessarily continuous).
- iv. $d(t) \stackrel{\text{def}}{=} m(t) + t$ is increasing on $[0, \infty)$.
- v. When there exists t_0 such that $m(t_0^-) \stackrel{\text{def}}{=} \lim_{t \rightarrow t_0^-} m(t) = 0$, then $m(t) = 0$ holds for $t \in [t_0, \infty)$. Otherwise, when there does not exist such a t_0 with

$$m(t_0^-) = 0, \text{ then } \int_0^{\infty} \frac{1}{m(u)} du = \infty \text{ holds.}$$

A function m satisfies i - v if and only if m is the MRL function of a nondegenerate at 0 life distribution.

See Hall and Wellner (1981) for a proof. See Bhattacharjee (1982) for another

characterization. Note that condition ii rules out the degenerate at 0 distribution. For iv note that $d(t)$ is simply the expected time of death (failure) given that a unit has survived to time t . Theorem 2.1 delineates which functions can serve as MRL functions, and hence, provides models for lifelengths.

We restate several bounds involving MRL from Hall and Wellner (1981). Recall $a^+ = a$ if $a \geq 0$, otherwise $a^+ = 0$.

Theorem 2.2. Let F be nondegenerate. Let $\mu_r = EX^r < \infty$ for $r > 1$.

- i. $m(t) \leq (F^{-1}(1) - t)^+$ for all t . Equality holds if and only if $F(t) = F((F^{-1}(1))^+)$ or 1.
- ii. $m(t) \leq (\mu/\bar{F}(t)) - t$ for all t . Equality holds if and only if $F(t) = 0$.
- iii. $m(t) < (\mu_r/\bar{F}(t))^{1/r} - t$ for all t .
- iv. $m(t) \geq (\mu - t)^+/\bar{F}(t)$ for $t < F^{-1}(1)$. Equality holds if and only if $F(t) = 0$.
- v. $m(t) > [\mu - F(t)(\mu_r/F(t))^{1/r}]/\bar{F}(t) - t$ for $t < F^{-1}(1)$.
- vi. $m(t) \geq (\mu - t)^+$ for all t . Equality holds if and only if $F(t) = 0$ or 1.

Various nonparametric classes of life distributions have been defined using MRL. (Recall, for simplicity we assume $F(0) = 0$ and the mean is finite for these definitions.)

Definition 2.3. DMRL. A life distribution F has decreasing mean residual life if its MRL m is a decreasing function.

Definition 2.4. NBUE. A life distribution F is new better than used in expectation if $m(0) \geq m(t)$ for all $t \geq 0$.

Definition 2.5. DMRL. A life distribution F has increasing then decreasing mean residual life if there exist $\tau \geq 0$ such that m is increasing on $[0, \tau)$ and decreasing on $[\tau, \infty)$.

Each of these classes above has an obvious dual class associated with it, i.e., increasing mean residual life, new worse than used in expectation (NWUE), and decreasing then increasing mean residual life (DIMRL), respectively.

The DMRL class models aging that is adverse (e.g., wearing occurs). Barlow, Marshall, and Proschan (1963) note that the DMRL class is a natural one in reliability. See also Barlow and Proschan (1965). The older a DMRL unit is, the shorter is the remaining life on the average. Chen, Hollander and Langberg (1983) contains an excellent discussion of the uses of the DMRL class.

Burn-in procedures are needed for units with IMRL. E.g., integrated circuits have been observed empirically to have decreasing failure rates; and thus they satisfy the less restrictive condition of IMRL. Investigating job mobility, social scientists refer to IMRL as inertia. See Morrison (1978) for example. Brown (1983) studies approximating IMRL distributions by exponentials. He comments that certain IMRL distributions, "... arise naturally in a class of first passage time distributions for Markov processes, as first illuminated by Keilson."

Note that DMRL implies NBUE. The NBUE class is a broader and less restrictive class. Hall and Wellner (1981) show for NBUE distributions that the coefficient of variation $\sigma/\mu \leq 1$, where $\sigma^2 = \text{Var}(X)$. They also comment on the use of NBUE in renewal theory. Bhattacharjee (1984b) discusses a new notion, age-smoothness, and its relation to NBUE for choosing life distribution models for equipment subject to eventual wear. Note that burn-in is appropriate for NWUE units.

For relationships of DMRL, IMRL, NBUE, and NWUE with other classes used in

reliability see the survey paper Hollander and Proschan (1984).

The DMRL class models aging that is initially beneficial, then adverse. Situations where it is reasonable to postulate an DMRL model include:

- i Length of time employees stay with certain companies: An employee with a company for four years has more time and career invested in the company than an employee of only two months. The MRL of the four-year employee is likely to be longer than the MRL of the two-month employee. After this initial DMRL (this is called "inertia" by social scientists), the processes of aging and retirement yield a DMRL period.
- ii Life lengths of humans: High infant mortality explains the initial DMRL. Deterioration and aging explain the later DMRL stage.

See Guess (1984) and Guess, Hollander, and Proschan (1983) for further examples and discussion. Bhattacharjee (1983) comments that Gertsbakh and Kordonskiy (1969) graph the MRL function of a lognormal distribution that has a "bath-tub" shaped MRL (i.e., DMRL).

Hall and Wellner (1981) characterize distributions with MRL's that have linear segments. They use this characterization as a tool for choosing parametric models. Morrison (1978) investigates linearly DMRL. He states and proves that if F is a mixture of exponential then F has linearly DMRL if and only if the mixing distribution, say G , is a gamma. Howell (1984) studies and lists other references on linearly DMRL.

In renewal theory MRL arises naturally also. For a renewal process with underlying distribution F , let $\bar{G}(t) = \frac{\int_0^t \bar{F}(u) du}{t}$. G is the limiting distribution of both

the forward and the backward recurrence times. See Cox (1962) for more details. Also if the renewal process is in equilibrium then G is the exact distribution of the recurrence times. $\bar{G}(t) = (m(t)\bar{F}(t))/\mu$. The failure rate of G , r_G , is inversely related to the MRL of F , m_F . I.e., $r_G(t) = 1/m_F(t)$. Note, however, that $r_F(t) = 1/m_F(t)$ is usually the case. See Hall and Wellner (1981), Rolski (1975), Meilijson (1972), and Watson and Wells (1961) for related discussions.

Kotz and Shanbhag (1980) establish a stability result concerning convergence of an arbitrary sequence of MRL functions to a limiting MRL function. (See also Bhattacharjee (1982)). They show an analogous stability result for hazard measures. (When the failure rate for F exists and v_F is F 's hazard measure, then $v_F(B) = \int_B r_F(t)dt$ for B a Borel set.) Their results imply that MRL functions can provide more stable and reliable information than hazard measures when assessing noncontinuous distributions from data.

In a multivariate setting, Lee (1985) shows the effect of dependence by total positivity on MRL functions.

3. Applications of mean residual life.

A mean is easy to calculate and explain to a person not necessarily skilled in statistics. To calculate the empirical MRL function, one does not need calculus. Details of computing the empirical MRL follow.

Let X_1, X_2, \dots, X_n be a random sample from F . For simpler initial notation, we assume first no ties. Later we allow for ties. Order the observations as

$$(3.1) \quad X_{1n} < X_{2n} < \dots < X_{nn}.$$

Let $X_{0n} = 0$. The empirical MRL function is defined as

$$(3.2) \quad m_n(t) = \frac{\sum_{i=k+1}^n (X_{in} - t)}{n-k} \quad \text{for } t \in [X_{kn}, X_{(k+1)n}),$$

and $k=0, 1, \dots, n-1$. $m_n(t) = 0$ for $t \geq X_{nn}$.

Note that (3.2) is simply

$$(3.3) \quad m_n(t) = \frac{\text{Total time on test observed after } t}{\text{Number of units observed after } t}.$$

The empirical MRL function at 0, $m_n(0) = \bar{X}_n \stackrel{\text{def}}{=} (\sum_{i=1}^n X_i)/n$, is just the usual sample

mean when no unit fails at time 0. If a unit fails at 0 then $m_n(0) > \bar{X}_n$.

If ties exist let

$$(3.4) \quad 0 = \bar{x}_{0l} < \bar{x}_{1l} < \bar{x}_{2l} < \dots < \bar{x}_{ll}$$

be the distinct ordered times of failure,

$$(3.5) \quad n_i = \text{number of observed failures at time } \bar{x}_{il}, \quad s_i = n - \sum_{j=0}^i n_j$$

for $i=0, 1, \dots, l < n$. Note that $n_i = 0$, $i=1, \dots, l$, while $n_0 = 0$ is allowed.

$$(3.6) \quad m_n(t) = \frac{\sum_{i=k+1}^l n_i (\bar{x}_{il} - t)}{s_k} \quad \text{for } t \in [\bar{x}_{kl}, \bar{x}_{(k+1)l}),$$

$$m_n(t) = 0 \quad \text{for } t \geq \bar{x}_{ll}$$

for $k=0, 1, \dots, l-1$. Note that (3.6) is simply notation for (3.3).

We illustrate in the following example.

Example 3.1. Bjerkedal (1960) studies the lifelengths of guinea pigs injected with different amounts of tubercle bacilli. Guinea pigs are known to have a high susceptibility to human tuberculosis, which is one reason for choosing this species.

We describe the only study (M) in which animals in a single cage are under the same regimen. The regimen number is the common log of the number of bacillary units in .5 ml of the challenge solution, e.g., regimen 4.3 corresponds to 2.2×10^4 bacillary units per .5 ml ($\log_{10}(2.2 \times 10^4) = 4.342$). Table 3.1 presents the data from regimen 5.5 and the empirical MRL.

Graphs of MRL provide useful information not only for data analysis but also for presentations. Commenting on fatigue longevity and on preventive maintenance, Gertsbakh and Kordonskiy (1969) recommend the MRL function as another helpful tool in such analyses. They graph the MRL for different distributions (e.g., Weibull, lognormal, and gamma). Hall and Wellner (1979) graph the empirical MRL for Bjerkedal's (1960) regimen 4.3 and regimen 6.6 data. Bryson and Siddiqui (1969) illustrate the graphical use of the empirical MRL on survival data from chronic granulocytic leukemia patients. Using the standard Kaplan-Meier estimator (e.g., see Lawless (1982), Nelson (1982), or Miller (1980)), Chen, Hollander, and Langberg (1983) graph the empirical MRL analogue for censored lifetime data.

Gertsbakh and Kordonskiy (1969) note that estimation of MRL is more stable than estimation of the failure rate. Statistical properties of estimated means are better than those of estimated derivatives (which enter into failure rates).

Yang (1978) shows that the empirical MRL is uniformly strongly consistent. She establishes that m_n , suitably standardized, converges weakly to a Gaussian process. Hall and Wellner (1979) require less restrictive conditions to apply these results. They derive and illustrate the use of simultaneous confidence bands for m . Yang (1978) comments that for $t > 0$, $m_n(t)$ is a slightly biased estimator. Specifically, $E(m_n(t)) = m(t)(1 - F^n(t))$. Note, however, that $\lim_{n \rightarrow \infty} E(m_n(t)) = m(t)$. Thus, for larger samples $m_n(t)$ is practically unbiased. See also Gertsbakh and Kordonskiy (1969).

TABLE 3.1.

Empirical mean residual life in days at the unique times of death for the 72 guinea pigs under regimen 5.5.
(We include the empirical MRL at time 0 also.)

<u>Number of Ties</u>	<u>Time of Death</u>	<u>Empirical MRL</u>
n_i	\bar{x}_{in}	$\frac{m}{n}(\bar{x}_{in})$
0	0	141.85
1	43	100.24
1	45	99.64
1	53	92.97
2	56	92.66
1	57	93.05
1	58	93.46
1	66	86.80
1	67	87.16
1	73	82.47
1	74	82.80
1	79	79.10
2	80	80.79
3	81	84.15
1	82	84.69
2	83	86.90
1	84	87.59
1	88	85.26
1	89	85.98
2	91	87.55
2	92	90.40
1	97	87.34
2	99	89.40
2	100	92.53
1	101	94.18
3	102	100.94
1	103	102.80
1	104	104.79
1	107	104.88
1	108	107.13
1	109	109.55
1	113	109.07
1	114	111.79
1	118	111.64
1	121	112.67

Table 3.1 (Continued)

<u>Number of Ties</u>	<u>Time of Death</u>	<u>Empirical MRL</u>
$\underline{n_i}$	$\underline{\bar{X}_{in}}$	$\underline{m_n(\bar{X}_{in})}$
1	123	114.92
1	126	116.40
1	128	119.17
1	137	114.96
1	138	119.14
1	139	123.76
1	144	124.70
1	145	130.21
1	147	135.33
1	156	133.76
1	162	135.75
1	174	132.07
1	178	137.14
1	179	146.62
1	184	153.42
1	191	159.73
1	198	168.00
1	211	172.22
1	214	190.38
1	243	184.43
1	249	208.17
1	329	153.80
1	380	128.50
1	403	140.67
1	511	49.00
1	522	76.00
1	598	0.00

Yang (1977) studies estimation of the MRL function when the data are randomly censored. Joe and Proschan (1981) develop isotonized estimators of MRL and of the life distribution under the assumption of DMRL. Both the complete data model and the randomly censored data model are treated.

For parametric modeling Hall and Wellner (1981) use the empirical MRL plot. They observe that the empirical MRL function is a helpful addition to other life data techniques, such as total time on test plots, empirical (cumulative) failure rate functions, etc. The MRL plot detects certain aspects of the distribution more readily than other techniques. See Hall and Wellner (1981), Hall and Wellner (1979), and Gertsbakh and Kordonskiy (1969) for further comments.

When a parametric approach seems inadvisable, the MRL function can still be used as a nonparametric tool. Broad classes defined in terms of MRL allow a more flexible approach while still incorporating preliminary information. For example, to describe a wear process, a DMRL is appropriate. When newly developed components are initially produced, many may fail early (such early failure is called infant mortality and this early stage is called the debugging stage). Another subgroup tends to last longer. Depending on information about this latter subgroup, we suggest IMRL (e.g., lifelengths of integrated circuits) or IDMRL (e.g., more complicated systems where there are infant mortality, useful life, and wear out stages).

Objective tests exist for these and other classes defined in terms of MRL. E.g., see Hollander and Proschan (1984) and Guess, Hollander, and Proschan (1983).

To describe "burn-in" the MRL is a natural function to use. Kuo's (1984) Appendix 1 presents an excellent brief introduction to burn-in problems and applications of MRL.

Actuaries apply MRL to setting rates and benefits for life insurance. In the biomedical setting researchers analyze survivorship studies by MRL. For example,

see Elandt-Johnson and Johnson (1980) and Gross and Clark (1975).

Social scientists use IMRL for studies on job mobility, length of wars, duration of strikes, etc. See Morrison (1978).

In economics MRL arises also. Bhattacharjee and Krishnaji (1981) present applications of MRL for investigating landholding. Bhattacharjee (1984a) uses NBUE for developing optimal inventory policies for perishable items with random shelf life and variable supply.

Bhattacharjee (1982) observes MRL functions occur naturally in other areas such as optimal disposal of an asset, renewal theory, dynamic programming, and branching processes.

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ABSTRACT (Continue on reverse side if necessary and identify by block number)

This is a chapter for the Handbook of Statistics, Volume 7, Quality Control and Reliability, edited by P. R. Krishnaiah. We survey the rich theory and important applications of the concept of mean residual life.

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19. ABSTRACT (Continue on reverse if necessary and identify by block number) In the last two decades, reliability, statisticians, and others have shown intensified interest in the mean residual life (MRL) and derived many useful results concerning it. Given that a unit is of age t, the remaining life after time t is random. The expected value of this random residual life is called the mean residual life at time t. Since the MRL is defined for each time t, we also speak of the MRL function. The MRL function is like the density function the moment generating function, or the characteristic function: for a distribution with a finite mean, the MRL completely determines the distribution via an inversion formula. Not only is the MRL used for parametric modeling but also for nonparametric modeling. Large non-parametric classes of life distributions such as decreasing mean residual life (DMRL) and new better than used in expectation (NBUE) have been defined using MRL. In this paper we define the MRL function formally and survey some of the key theory. We also discuss further its wide range of applications, is also discussed.			
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