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A CRITICAL LOOK AT MILITARY RECRUITMENT AND RETENTION POLICIES

William Lisowski

March 1985

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The original version of this study was prepared by the author, William Lisowski, as a dissertation in partial fulfillment of the requirements of the doctoral degree in policy analysis at The Rand Graduate Institute. It was approved by Dr. Lisowski's dissertation committee on November 14, 1984.

PREFACE

This paper provides a framework for analyzing military enlistment and reenlistment policies and incentives, looking beyond simple enlistee counts to measures which include the effects of differential retention, productivity, and costs. The methodology is applicable to other fields, such as teaching and occupations requiring lengthy training and/or apprenticeship, where entry is usually only at the most junior level.

This paper has been submitted and accepted as a doctoral dissertation in partial satisfaction of the requirements for the Ph.D. in The Rand Graduate Institute.

SUMMARY

Since the end of conscription, the military services have periodically found it difficult to attract and retain desired numbers of enlistees. Analyses of the many proposals for "solving" these problems have concentrated on the effects on recruiting and retention and on the monetary costs. Little attention has been paid to the less obvious costs to the services—costs in years of service, changes in experience levels of the force, and potential losses in productivity among the enlisted force.

Adequate assessment of manpower policies requires consideration of the patterns of enlistee losses from attrition and failure to reenlist. Also, variations in enlistee effectiveness and costs are as important as variations in retention in assessing enlistee worth. Effectiveness and costs differ not only among enlistees but also across time in the military careers of individual enlistees.

This dissertation provides methodologies which permit moving beyond simple counts of enlistments and reenlistments to measures of the short- and long-term costs and benefits accruing from policies designed to stimulate accessions and retention. They provide a common basis by which disparate measures—increases in enlistments under a set of incentives, bonus elasticities for reenlistments, and so forth—can be compared. The statistical nature of these techniques recognizes the randomness in the attrition and reenlistment behavior of individual enlistees.

The basis for these techniques is the retention function, which describes the (random) length of service of an individual enlistee. Policy analysis requires comparison of the aggregate effects of particular retention patterns on groups of enlistees. The retention function facilitates development of a rich set of measures for assessing long-term effects of manpower policies that might change enlistment and retention behavior. The short-term effects of such changes critically depend on the initial conditions (force composition, etc.) and are best assessed with standard aggregate force models; the retention function can provide parameter estimates needed by these models.

ACKNOWLEDGMENTS

As this research represents the author's doctoral dissertation at The Rand Graduate Institute, he would like to acknowledge the contributions of dissertation committee chairman Gus W. Haggstrom and committee members James R. Hosek and Albert P. Williams, without whose faith, hope, and charity the dissertation would never have been completed. Thanks are also due to Dean Charles Wolf, Jr., whose gentle prodding helped move (if not speed) it towards completion. Special thanks are in order for Molly McMullen, whose careful reading, perceptive questioning, and critical editing helped immeasurably in the transformation of the dissertation from its initial draft into its final form. Of course, any remaining errors, omissions, and obscurities must be the fault of the text processing and phototypesetting systems which the author used to produce this document, since he certainly knew what he intended to say.

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I. INTRODUCTION

Since the advent of the All-Volunteer Force (AVF), the military services have periodically found it difficult to attract and retain desired numbers of enlistees. This is especially true of enlistees in hard-to-fill occupational specialties such as combat arms and of "high-quality" enlistees (high school graduates who score above the median on the mental aptitude tests given potential enlistees). These difficulties stem from many sources. Military pay and allowances occasionally lag earnings in the private sector, and some benefits of military service have been materially reduced, such as the substitution of the Veterans' Educational Assistance Program (VEAP) for the GI Bill. Despite these long-term problems, the armed forces have enjoyed considerable recruiting success during the last two years, thanks to the reduced demand for young workers in the civilian sector during the recession. In addition to meeting their recruitment goals, the services have been able to raise their enlistment standards so that, in 1983, 90 percent of all enlistees had high school diplomas, up from 69 percent in 1980. With the quickening of the economy in 1984, however, the military is again experiencing difficulties meeting their recruiting objectives, and the long-term outlook is clouded by projections that the yearly number of male high school graduates will fall by about 9 percent between 1984 and 1990 (National Center for Education Statistics, 1984). To meet their recruiting goals over the next several years, the military will have to increase its "take" from this group in the face of greater competition from the civilian sector.

A number of responses to these problems have been suggested, including a return to the peacetime draft or some other form of mandatory service. Some seek across-the-board pay increases to stimulate enlistment and retention. Others suggest measures more readily targeted to populations of particular interest (high-quality enlistees in hard-to-fill specialties). Indicative of the services' attempts to find better recruiting strategies, several controlled experiments have been conducted since 1978 to test the attractiveness of various proposed enlistment incentives.

Analyses of the many proposals for "solving" the problems of the AVF have concentrated on the effects on recruiting and retention and on the monetary costs (Congressional Budget Office, 1980; U.S. Department of Defense, 1980). Little attention has been paid to the less obvious costs to the services—costs in years of service, changes in experience levels of the force, and potential losses in productivity among the enlisted force.

Besides force sizes and experience levels, there are other issues which need to be addressed in evaluating the effects of enlistment incentives. Changes in experience levels entail changes in compensation levels, because personnel in a more senior force are paid more on average. However, a more junior force, with higher turnover, has higher training costs, and productivity differences offset pay differences, to some degree.

These ideas can be illustrated with a series of examples which retain the salient features of more detailed manpower planning models but lend themselves to easy interpretation. The figures and examples are based on the author's analysis of the force structure implications of enlistment incentives offered during the Multiple Option Recruiting Experiment (Haggstrom et al., 1981). For purposes of illustration, let us suppose each year's cohort of enlistees is inducted simultaneously, and that the term of enlistment and reenlistment is three years.

If we graph, for a single cohort, the number of enlistees remaining in the service against time in years since induction, the resulting plot would resemble Figure 1. There are major drops in the cohort size at the reenlistment points. Between them, there are gradual, continuous declines corresponding to losses from attrition. This example will be referred to as the "reference case" below.

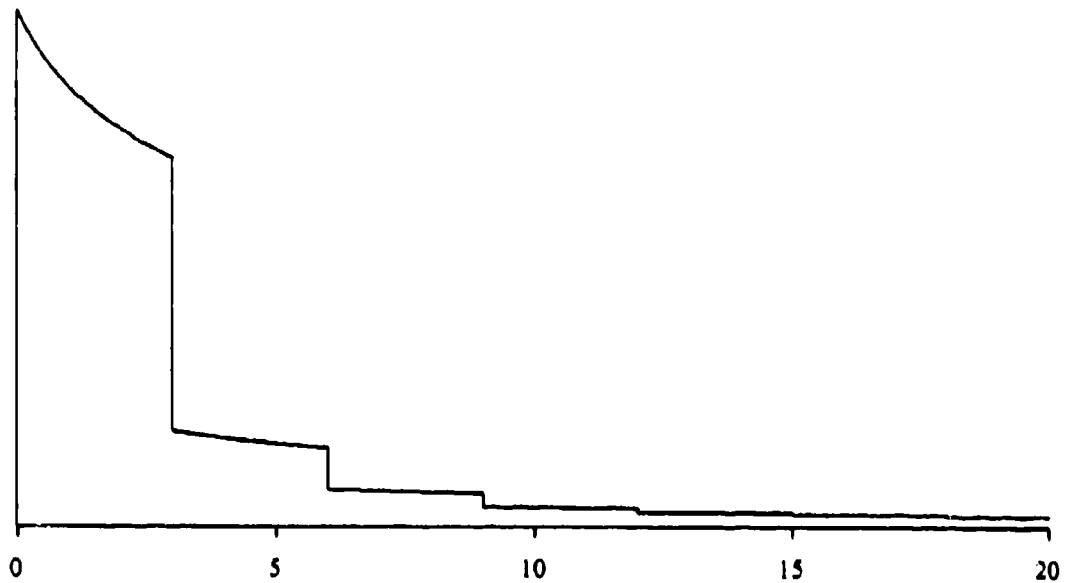


Fig. 1—Cohort size vs. time, reference case

Since the curve plots the number of enlistees against years of service, the area under the curve measures total enlistee-years of service from members of the cohort. For example, the area to the left of the first reenlistment point measures total service during the first term. Similarly, the remaining area measures total service in the second term and later.

Next, consider a second example, where the initial term of service is reduced to two years. Figure 2 compares the cohort size curve for this case with that for the reference case, under the assumption that the retention behavior of the two-year enlistees is the same as that for the three-year enlistees. Clearly, with the same initial cohort size, the total service from the second cohort is less than that for the reference case. In this example, the reduction is twenty-two percent.

As a third example, an increased initial size for the second cohort would compensate for the reduction in aggregate years of service. Figure 3 shows the same two curves as Figure 2, but with the size of the cohort of two-year enlistees increased uniformly so that the areas under the curves are identical. Notice the change in the distribution of experience levels: the cohort of two-year enlistees provides a greater portion of its service at lower levels of experience. Since salary increases with length of service, there is a reduction in costs, which is at least partially offset by the larger cohort size.

For a final example, we consider a cohort that is initially ten percent larger than the reference case, but has fewer reenlistments, so the cohorts are the same size in the second and succeeding terms. This reflects the likelihood that enlistees attracted by increased post-service educational benefits may be more likely to leave at the end of the first term to take advantage of these benefits, along with others who might have reenlisted in the absence of the benefits (Hosek, Fernandez, and Grissmer, 1984). Figure 4 shows this cohort size curve along with that for the reference case. Comparing the area under the curves, we see that although the initial cohort may have been ten percent larger, the total service increased by just seven percent.

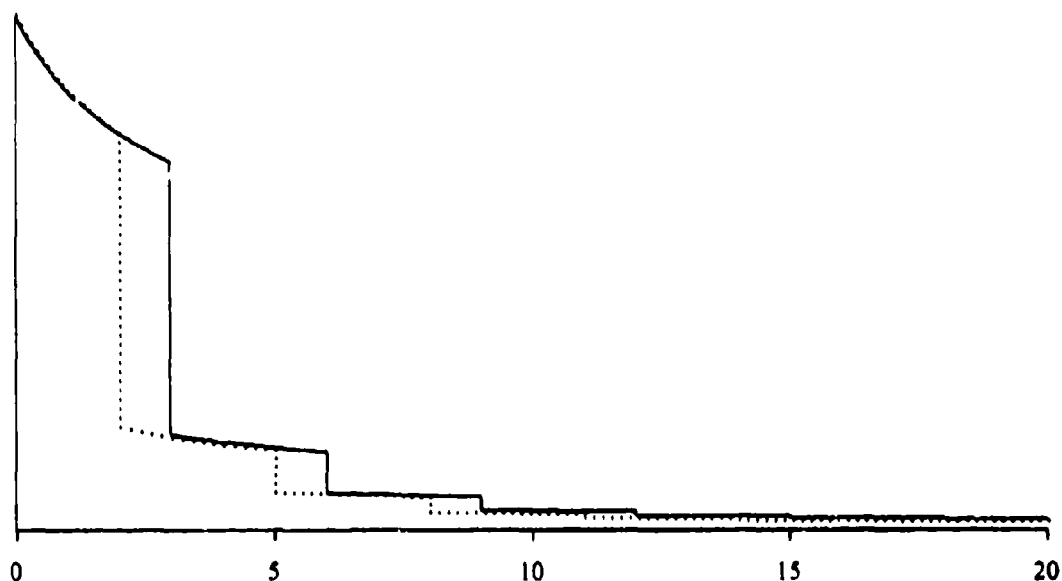


Fig. 2—Cohort size vs. time, two-year enlistments

Central to this dissertation is the observation that adequate assessment of manpower policies requires that we consider the patterns of enlistee losses from attrition and failure to reenlist. To the extent that these patterns differ for different types of enlistees or under different compensation schemes, enlistee counts are an inadequate measure of comparison. Our other central observation is that variations in enlistee effectiveness and costs are as important as variations in retention in assessing enlistee worth. Effectiveness and costs differ not only among enlistees but also across time in the military careers of individual enlistees.

While the examples illustrate these observations, they also reveal their own inadequacies. Cohorts are not homogeneous collections of enlistees with identical enlistment and reenlistment terms, propensities to reenlist, opportunities in the civilian sector, and so forth. Also, the measures used—cohort sizes and aggregate years of service—inadequately capture the differences in the nature of service provided.

This dissertation provides methodologies which permit moving beyond simple counts of enlistments and reenlistments to measures of the short- and long-term costs and benefits accruing from policies designed to stimulate accessions and retention. They provide a common basis by which disparate measures—increases in enlistments under a set of incentives, bonus elasticities for reenlistments, and so forth—can be compared. The statistical nature of these techniques recognizes the randomness in the attrition and reenlistment behavior of individual enlistees.

This methodology differs from other work on retention in the military which attempts to model individual attrition and reenlistment decisions (Warner, 1979; Chow and Polich, 1980; Gotz and McCall, 1980); instead, this work focuses on aggregate measures of cohorts that reflect force size, retention behavior, and composition. While it allows assessment of force-wide effects, it enlarges on models of force composition (Petruzzi, Broider, and Collins, 1980; Collins, Gass, and Rosendahl, 1983) by explicitly including a model of retention.

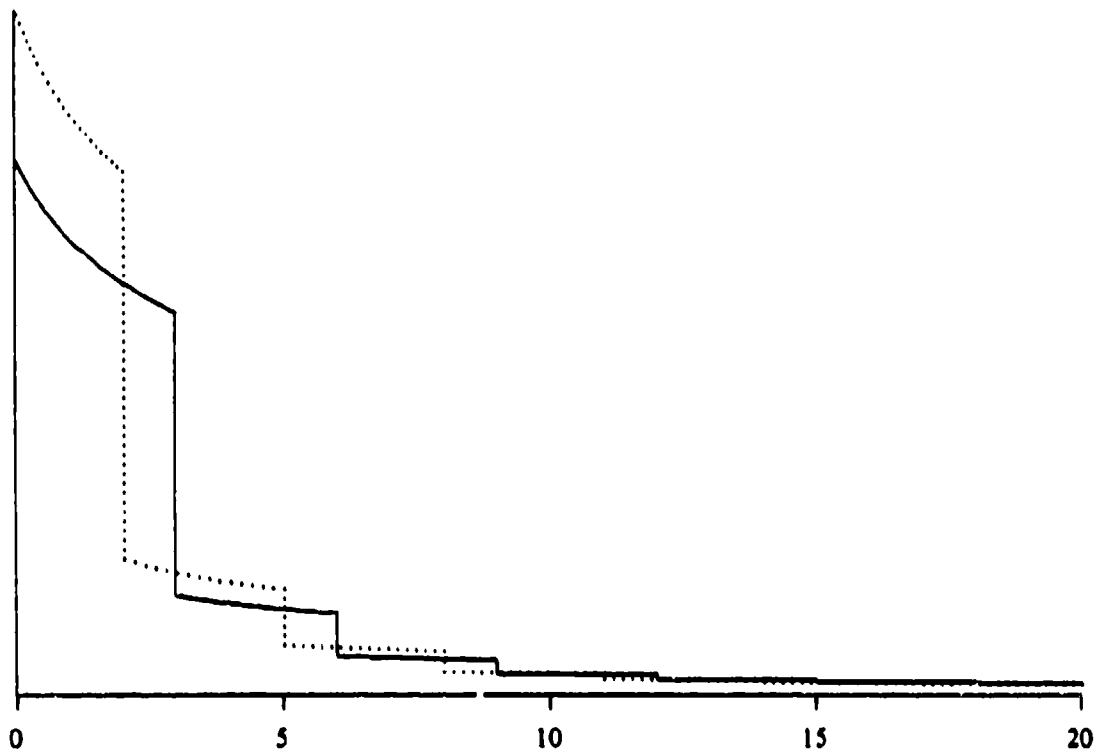


Fig. 3—Cohort size vs. time, two-year enlistments,
aggregate service identical to reference case

We lay the groundwork for this development by reviewing, in Section II, some features common to many aggregate manpower models. These models divide the enlisted force at any given time into groups of enlistees; over time enlistees move from one group to another and eventually leave the force, while others enter. The transition rates (at which enlistees move between groups) are the focal points of these models.

Section III introduces the retention function, which is the centerpiece of our methodology. The cohort size curves from our examples describe the length of service of all the members of the cohort; the retention function describes the (random) length of service of an individual enlistee. As such, it is analogous to the "survival function" in biostatistics and the "reliability function" in reliability theory. Although the retention function focuses on the individual enlistee, it ultimately improves our ability to examine force-wide effects of changes in individual behavior.

Section IV shows how the retention function can be used to measure the effects of varying patterns of enlistee retention. In our examples above, we measured the aggregate service of a cohort as the area under the cohort size curve. Similar results can be derived from the retention function. The area under the curve gives the expected length of service of an enlistee, and the retention function can be used to derive other, more interesting measures (e.g., expected compensation).

To make policy decisions we must compare not only these "individual effects" but also the "aggregate effects" of particular retention functions on groups of enlistees. Force size and

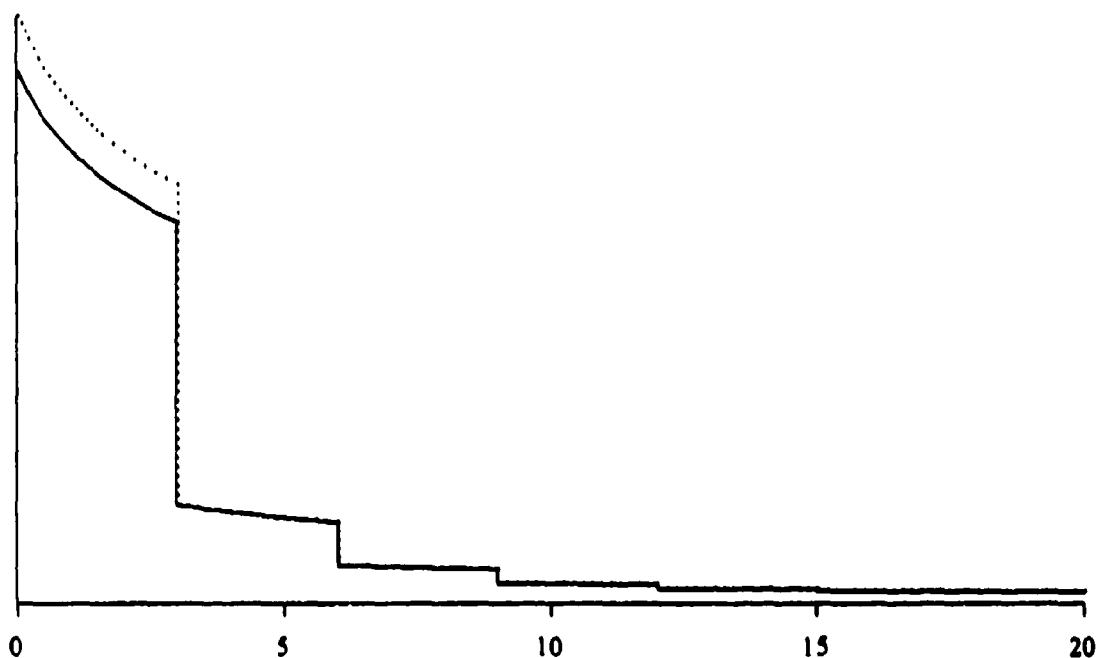


Fig. 4—Cohort size vs. time, increased enlistments with decreased first-term reenlistment

composition, experience levels, and so forth are the measures needed to judge the effectiveness of manpower policies. The retention function allows us to develop a rich set of measures for assessing long-term effects of manpower policies that might change enlistment and retention behavior. Over the long term, aggregate measures depend directly and indirectly on countless external factors—enlistments, military manpower policies, civilian labor and educational opportunities, and so forth. To even begin to be able to separate out the long-term effects of retention behavior, we utter the economist's benediction "*ceteris paribus*" and assume external factors are unchanging over time. This is not a critical restriction, as we do not intend that our models be used for precise forecasts of manpower trends decades into the future. Rather, they are used to compare alternatives under limited conditions, and perhaps check the sensitivity of the results to these conditions.

The short-term effects of such changes critically depend on the initial conditions (force composition, etc.) and are best assessed with aggregate force models of the type described in Section II. Doing so requires estimates of the transition rates between categories, and as suggested above, we show in Section IV how the retention function can provide them.

Section V provides an example of the application of these techniques, based on recent Army continuation data. This section is not a thorough policy analysis; it is intended only as a (relatively easily understood) numerical example. Because of this, simplifying assumptions are made which are artifacts of the example and should not be taken to be part of the methodology.

Section VI explores some of the problems of measuring enlistee effectiveness. We show how measures developed by the Enlisted Utilization Survey (Gay and Albrecht, 1979) can be used to assess force effectiveness under different retention patterns. Albrecht (1979) previously modeled

this data in the framework of the "production function" from the economic theory of the firm. Taking another tack, Haggstrom, Chow, and Gay (1984) applied the theory of "learning curves" from the psychology literature. We show how these approaches can be combined to model growth in effectiveness over the term of service, which can then be used with the retention function to measure aggregate effectiveness.

Finally, Section VII shows how we can apply these results to analyses of military manpower policy. Given suitable measures of costs and benefits, the methodology of Section IV allows us to assess expected costs and benefits. The effectiveness measures thus provide the remaining elements needed for assessment of long-term benefits.

II. MODELING AGGREGATE ENLISTEE RETENTION

In this section, we discuss the characteristics common to many aggregate models of the enlisted force. Doing so provides a framework for the methodology we develop in later sections, allowing us to relate it to current manpower models. Moreover, we show how that methodology can be used to improve the performance of the current models. We begin this section with a discussion of the general representation of the enlisted force as a system of stocks of enlistees and flows between these stocks. We then define the important concept of the continuation rate in terms of this representation, and describe briefly the problems involved in using past data to project future continuation rates for use in models of the enlisted force.

Stocks and Flows

Most military personnel planning models treat the enlisted force as "a system of stocks and flows," in the terminology of Bartholomew and Forbes (1979). To do this, the enlisted force is divided into categories based on attributes of interest, such as grade and length of service. At any time, the numbers of enlistees within categories are the *stocks* at that time. Over any interval of time, there are movements between categories; their magnitudes constitute the *flows* over the time period. The ratios of the flows to the initial stocks give the *flow rates* from category to category. Over time, the magnitudes of the stocks and flows may change; hence the dynamic nature of the system.

Flows are usually identified as stochastic or as deterministic, although the reality is usually somewhere between these two extremes. For example, when enlistees are categorized by grade, flows between grades are more or less controlled by military policy, and are largely deterministic. On the other hand, flows between the enlisted force and the civilian population (in both directions) result from thousands of individual choices and are in a large measure stochastic.

In this paper we focus our attention on issues involved in modeling retention, suppressing details involving grade progression, and thus treat flows as stochastic. This is of course a simplification, since reenlistment decisions can be affected by promotion opportunities and the like. However, we do not model the process whereby an enlistee makes his or her retention decisions, but instead treat the aggregate of these decisions as a random process.

Because our interest is in retention, and retention behavior varies with length of service, any categorization of enlistees includes length of service. Commonly, service is measured in full years, combining into a single category, for example, all service of at least four years, but not more than five years.

An arbitrary system of stocks and flows is subject to a pair of identities fundamental to modeling the system. First developing them in full generality, we introduce notation for describing stocks and flows. Let $n(i, t)$ be the stock in category i at time t . Let $M(i, j, t)$ be the flow (movement) from the stock in category i at time t into the stock in category j at time $t + 1$. Similarly, let $d(i, t)$ be the flow out of the system from the stock in category i at time t (departures), and $a(i, t)$ the flow into the stock in category i at time t from outside the system (accessions). The flows are nonnegative by definition; they are not in any sense net, since they are between successive times as well as categories. Now the stock in each category i at any time t is given by the equations

$$n(i, t) = \sum_j M(j, i, t-1) + a(i, t)$$

$$n(i, t) = \sum_j M(i, j, t) + d(i, t)$$

which amount to saying that everyone comes from somewhere and goes somewhere.

Deferring briefly consideration of any categorization beyond length of service, we obtain a particularly simple structure for these identities. Let category m consist of those enlistees with m full years of service, for $m = 0, 1, 2, \dots$. Note that when $k \neq m + 1$, $M(m, k, t) = 0$: in one year's time an enlistee gains one year of service or leaves the force. Also, since the bulk of recruitment consists of non-prior service enlistees, accessions are negligible for categories $m > 0$. Adopting the simplifying assumption that for $m > 0$, accessions $a(m, t) = 0$, the identities above become

$$n(m, t) = n(m-1, t-1) - d(m-1, t-1) \quad (II.1)$$

$$n(0, t) = a(0, t)$$

Continuation Rates

Military manpower planning models of the sort described above usually focus, not on flows, but rather on flow rates, which in this context are called *continuation rates*. We define the continuation rate as the fraction of those enlistees in the service at time t who do not depart from the service through time $t + 1$. Extending the notation of the previous section, $c(m, t)$, the continuation rate for enlistees with m years of service at time t , is given by

$$c(m, t) = \frac{n(m, t) - d(m, t)}{n(m, t)}$$

which, taken with (II.1) above, yields

$$n(m+1, t+1) = c(m, t) \times n(m, t) \quad (II.2)$$

Thus, the continuation rates serve as linkages between the stocks in successive periods.

We should note that other definitions of "continuation rate" are also found in the military manpower literature. For example, Warner (1979, p. 10) refers to continuation rates for enlistees over a given year of their service: from service anniversary to service anniversary. A typical continuation rate in his work would be the fraction of enlistees reaching their second anniversary (in some time frame) who remain in the service through their third anniversary. Our definition instead runs from calendar year to calendar year (or fiscal year to fiscal year). A typical continuation rate in this case is the fraction of enlistees at or beyond their second anniversary (but not yet to their third) at the start of a year who remain in the service at the end of the year. This definition conforms to that used in the Defense Manpower Data Center (1980) tabulations to which we apply our modeling effort.

Continuation rates are affected by enlistee characteristics and external factors. Typical enlistee characteristics of interest include ability, education, grade, and, as always, length of service. We continue to focus on retention and length of service, ignoring grade and grade progression. Enlistee characteristics other than length of service are considered to be unchanging over time. Thus, this year's high school graduate with six years of service came from last year's stock of graduates with five years of service, again disregarding the negligible returns to service. External factors include armed services compensation and manpower policies and general economic conditions. These are often subsumed into "time" ("continuation rates are decreasing over time ...") as there is a relative lack of information from controlled circumstances which would allow the disentanglement of the effects of the other external factors from each other. Recognizing this, and the use to which we put this model, we too consider

time as the sole external factor.

At this point we expand our notation to include enlistee characteristics. Let the vector x represent the fixed enlistee characteristics other than length of service, and let $N(m, x, t)$ be the stock of enlistees at time t with characteristics x and m years of service at that time. Then

$$C(m, x, t) = \frac{N(m+1, x, t+1)}{N(m, x, t)}$$

gives the continuation rate over the period $[t, t+1]$ for enlistees with characteristics x and m years of service at time t .

Although we have defined a continuation rate which can conceivably depend on a large number of enlistee characteristics, in typical military manpower planning models such as ASCAR (Petruzzi and Broider, 1980) this is not done. Instead, continuation rates are defined for relatively homogeneous groups of enlistees, formed by stratifying the enlistee population on the basis of their characteristics. In this case, the vector of enlistee characteristics can be simply an indicator of membership in a particular stratum. To assure that the bases for the rates are reasonably large, generally only a small number of characteristics can be used to define the strata. For any stratum, administrative records for a recent period yield observed values of the continuation rate over that period.

When there is variation in retention behavior for enlistees of differing characteristics, and this is not reflected in the stratifying variables, the observed continuation rates may not accurately project continuation rates under different force composition. For example, enlistees coming to a reenlistment point during the year have lower continuation rates than those with more than a year of service remaining. An increase in the proportion of two-year enlistees lowers observed continuation rates for groups of enlistees in their second year of service, while raising the rates for those in their third year. If stratification does not include "coming to a reenlistment point during the year," the change in enlistment patterns is not reflected in the projected continuation rates.

The dependence of continuation rates on time is often sidestepped. While past continuation rates have been observed, future continuation rates must be forecast from historical data. The simplest procedure is to assume that the rates do not change over the time frame of interest, perhaps by assuming that the external factors which influence continuation rates remain unchanged. For ASCAR, "[i]n most cases these factors represent historical rates obtained from Defense Manpower Data Center (DMDC); however, the analyst can replace the historical data to reflect new assumptions" (Petruzzi and Broider, 1980, p. 3-15, emphasis theirs).

Although the discussion in this section has treated continuation rates as though they are deterministic, we saw above that the flows on which they are based are stochastic. Thus, $N(m+1, x, t+1)$, the stock of enlistees at time $t+1$ with $m+1$ years of service and characteristics x , follows a binomial distribution with parameters $N(m, x, t)$ and $p(m, x, t)$. The parameter of interest is $p(m, x, t)$, the *expected continuation rate*, or the probability an individual selected at random from the $N(m, x, t)$ enlistees with characteristics x and m years of service at time t remains in the service at least one more year.

The link between the deterministic and stochastic models is that the observed continuation rate $C(m, x, t)$ is an estimate of the expected continuation rate $p(m, x, t)$, and if no further distributional assumptions are made, it is an unbiased estimate. However, adding a stochastic element to the observed continuation rates also adds uncertainty to models using them as estimates of future continuation rates. In this framework, we assume the expected continuation rate $p(m, x, t)$ does not change over time, so $p(m, x, t) = p(m, x, t_0)$ for $t > t_0$. The observed continuation rate $C(m, x, t_0)$ for the period $[t_0, t_0+1]$ is then an estimate of $p(m, x, t)$, the expected continuation rate over some future period $[t, t+1]$, and thus is a

forecast of $C(m, x, t)$, the continuation rate which will be observed over that period.

In the stochastic framework, each level of stratification of enlistee characteristics reduces the statistical precision of the observed continuation rates as estimates of the expected continuation rates. This is because the variance of the observed continuation rate $C(m, x, t_0)$ increases as the stratum size $N(m, x, t_0)$ decreases. Thus, a fine stratification which insures applicability of the estimates for different force compositions also reduces their individual precision.

Summary

In this section we have reviewed the basis of aggregate models of the enlisted force and shown how continuation rates form an important part of these models. In subsequent sections we further discuss the stochastic nature of enlistee service, developing a methodology for assessing it which can also improve our ability to project future continuation rates.

III. MODELING INDIVIDUAL ENLISTEE RETENTION

This section establishes the basis for a parametric model of enlistee retention. Earlier sections considered the behavior of enlistees in the aggregate. We now focus on the behavior of individual enlistees, exploring the stochastic nature of individual behavior, and developing a retention function to describe the (random) length of time an enlistee spends in the service. The retention function is related to the cohort size curve of Section I, but will prove a more useful concept to work with. In subsequent sections we show how this model of individual behavior can improve the model of aggregate behavior developed in Section II.

Length of Service

The stochastic flows and continuation rates of the preceding section arise as a consequence of the stochastic nature of individual enlistee behavior. Reenlistment and attrition decisions are made individually by the enlistees, in response to individual circumstances. The aggregation of these individual decisions yields the continuation rates, and the uncertainty in the individual decisions yields the uncertainty in the aggregate behavior.

To model individual behavior, we treat the total time an enlistee spends in the service as a random variable. Like a group's continuation rate, the time an individual enlistee spends in the service is affected by personal characteristics and external factors, but we defer discussion of these until later in this section.

Since an enlistee's length of service is random, we describe its distribution with the *retention function*, whose argument is length of service and whose value is the probability the enlistee serves that long or longer. Formally, if the random variable S is an enlistee's actual length of service, then we define the retention function by

$$R(s) = Pr\{S \geq s\} \quad (III.1)$$

for any length of service $s \geq 0$. Suppose, for example, that an enlistee has an initial enlistment term of three years, with three-year reenlistment terms thereafter. In this case, $R(3)$ is the probability the enlistee completes his or her initial term of service, and $R(6)$ is the probability the enlistee completes the second term. Also, $R(3) - R(6) = Pr\{3 \leq S < 6\}$ is the probability the enlistee completes at least the first term, but not the second. Finally, for this example, $R(6)/R(3) = Pr\{S \geq 6 | S \geq 3\}$ is the conditional probability that, if the enlistee completes the first term, then he or she will complete a second term as well.

A graph of the retention function will resemble Figure 5, where a retention function is plotted against length of service. The curve starts at 1 and declines monotonically to 0 as length of service increases. At the reenlistment points there are discontinuities in the retention function, corresponding to possible failure to reenlist. Between reenlistment points, there are gradual, continuous declines in the curve, corresponding to possible attrition from the service.

The resemblance of the plot of the retention function in Figure 3 to the cohort size curve of Figure 1 is not coincidental. Consider a cohort of N enlistees, each having the same retention function and reenlistment points. Then, for any length of service s , a randomly selected enlistee has probability $R(s)$ of remaining in the service that long or longer. The cohort size after s years of service is a binomial random variable whose expected value is $NR(s)$. Thus, the expected cohort size is proportional to the retention function of the enlistees in the cohort.

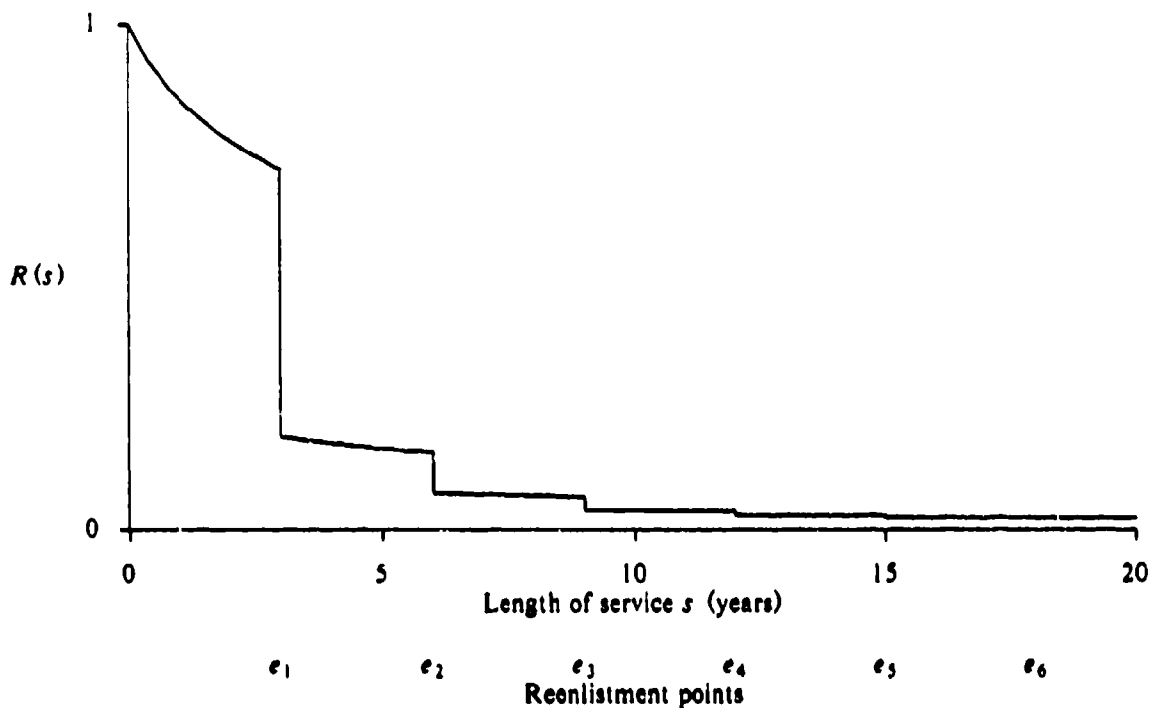


Fig. 5—Retention function vs. length of service

The Retention Function

We next define a parametric representation of the retention function. The form we use is not the only possibility. It has some theoretical justification and it fits the data at hand reasonably well. For these reasons, it is adequate for the expository purposes of this discussion. It is not, however, crucial to the development of the methodology that this particular representation of the retention function be used.

The retention function depends on the timing of the reenlistment points, which we treat as fixed for an individual enlistee, although different enlistees may have different reenlistment points. We denote the reenlistment points for an enlistee by e_1, e_2, \dots , as shown in Figure 5.

A retention function such as that in Figure 5 can be modeled as the product of a continuous function describing attrition and a step function, constant between reenlistment points, describing failure to reenlist. We write

$$R(s) = R_A(s) \times R_E(s) \tag{III.2a}$$

and discuss each factor separately below.

Our examination of attrition data leads us to posit that (1) attrition is decreasingly likely as the length of service increases and (2) the pattern of attrition losses changes substantially at the end of the first term of service. We have used a function of the form

$$R_A(s) = \begin{cases} (1 + \beta_1 s)^{-\alpha_1} & \text{if } 0 \leq s \leq e_1 \\ \kappa(1 + \beta_2 s)^{-\alpha_2} & \text{if } s > e_1 \end{cases} \quad (\text{III.2b})$$

to model the attrition process. This function consists of two segments—one for attrition during the first term, the other for the second and succeeding terms—with a common form but different parameters. The factor κ is chosen to ensure that $R_A(s)$ is continuous across the two segments. (Recall that discontinuities in the retention function correspond to reenlistment points.) Equating the two expressions in (III.2b) at $s = e_1$ and solving yields

$$\kappa = \frac{(1 + \beta_1 e_1)^{-\alpha_1}}{(1 + \beta_2 e_1)^{-\alpha_2}}. \quad (\text{III.2c})$$

In (III.2b), the factors of the form $(1 + \beta s)^{-\alpha}$ arise from considering length of service until attrition as following a Pareto Type II distribution. This distribution has been used elsewhere in manpower planning and reliability models (Bartholomew and Forbes, 1979, p. 49-50; Mann, Schafer, and Singpurwalla, 1974, p. 146-47). It has the property that the attrition rate declines as length of service increases. That is, of two enlistees, the one with the greater time in service will be the less likely to leave during any period in which neither enlistee has a reenlistment point. By allowing the parameters α and β to differ between the first and succeeding terms of service, we capture the change in attrition behavior which is observed to occur after the first term of service.

Reenlistment data show that reenlistment rates after the third and succeeding terms of service rise smoothly with increasing length of service. This continues until 20 years—the earliest point at which retirement benefits are available—after which point reenlistment drops sharply. Rather than model reenlistment rates beyond that point, we instead set the retention function $R(s) = 0$ for length of service $s > 20$, disregarding the small likelihood of service much beyond 20 years. We model the reenlistment process with a function of the form

$$R_R(s) = \begin{cases} 1 & \text{for } s \leq e_1 \\ \prod_{i=1}^{n-1} \rho_i & \text{for } e_{n-1} < s \leq e_n, s \leq 20 \\ 0 & \text{for } s > 20 \end{cases} \quad (\text{III.2d})$$

where each ρ_i is the expected reenlistment rate at the end of the i -th term of service, so $0 \leq \rho_i \leq 1$ for all i . After the third and succeeding terms, we assume that

$$\rho_i = (1 + \exp(-\gamma - \delta e_i))^{-1} \text{ for } i = 3, 4, \dots \quad (\text{III.2e})$$

This is a logistic function of e_i , the length of service at the time of reenlistment, rather than of the particular term of enlistment i . It is a smooth function which (for positive values of the parameter δ) increases asymptotically to 1 for increasing length of service e_i , again consistent with historical results.

The form for the retention function given in equations (III.2) ensures that it is characterized by the eight parameter values for any set of reenlistment points. That is, the retention functions of two otherwise identical enlistees facing different enlistment and reenlistment terms should have identical values for the parameters α_1 , β_1 , α_2 , β_2 , ρ_1 , ρ_2 , γ , and δ . Anticipating our applications in subsequent sections, this means that parameters estimated for one set of reenlistment points will apply for others. Although we present simple examples based on fixed three-year reenlistment terms, nothing in our methodology requires this. Also, data for enlistees with various enlistment and reenlistment terms can be combined to estimate these parameters.

The timing of reenlistment points is the only enlistee characteristic we treat explicitly. We handle the dependence of the retention function on other enlistee characteristics by allowing separate values of the parameters for each combination of enlistee characteristics. Such stratification is not the only possibility. With appropriate data, we could fit models which postulate a relationship between the parameter values for various combinations of enlistee characteristics. This would to some extent alleviate the imprecision inherent in estimating parameters for small strata, by allowing their estimates to be based in part on data from the larger strata.

We have not treated the dependence of the retention function on external factors. Data on retention under different external conditions (e.g., wage and unemployment rates in the civilian sector, reenlistment rates with and without bonuses) could allow models in which the parameters depend on these factors, at the simplest, by stratifying on external factors as well as enlistee characteristics. Alternatively, data on retention under different conditions may be lacking, but past experience in similar situations, theory, assumptions, guesswork, or whatever, may allow postulation of the nature of the effects, relative to conditions for which data does exist. For example, economic theory may predict the elasticities of reenlistment rates with respect to the size of a reenlistment bonus. Then given a retention function describing reenlistment in the absence of a bonus, the hypothesized elasticities could be used to adjust the reenlistment rate portion $R_E(s)$ of the retention function to reflect the effects of any particular bonus. Another example might be changes in retention stemming from changes in the term of the initial enlistment. Attrition under a two-year initial enlistment may be less likely than under a three-year enlistment, if enlistees are more likely to "tough it out" for the shorter period. Or, if the additional enlistees attracted by a two-year term are less motivated to serve in general, attrition may be more likely, and reenlistment less likely, than for three-year enlistees. If such possibilities are deemed important, then "length of initial commitment" is an external factor which should be considered in the retention function, by changing the attrition portion $R_A(s)$ of the retention function for three-year enlistees, as well as the first-term reenlistment rate ρ_1 .

Expanding our representation to include other enlistee characteristics and external factors would move our model towards a synthesis with other approaches to modeling enlistee retention (Warner, 1979; Chow and Polich, 1980; Gotz and McCall, 1980). These techniques involve "individual choice models" which describe the probability an enlistee reenlists, for example, as a function of his or her characteristics and external factors. They can be viewed as describing separate points on the retention function, which our model attempts to describe over its entire range.

Summary

There are two major results in this section. First, we have expounded a methodology, using the concept of the retention function, which allows us to parametrically model enlistee retention. To the degree that the model of the retention function reflects attrition and reenlistment patterns, estimates of future retention will be more accurate than would be the case without a model. Second, we have developed a particular parameterization which we use in the remainder of this work, realizing that although it is adequate for our purposes, other parameterizations are possible.

IV. MEASURING RETENTION'S EFFECTS

In this section, we explore use of the retention function as a tool to improve our understanding of the effects, both permanent and transitory, of changed force structure and patterns of attrition and reenlistment behavior. The retention function is shown to be of interest in its own right, providing derived measures which allow assessment of the long-term effects. Also, our model of the (individual) retention function yields a corresponding model of the (aggregate) continuation rates. These continuation rates can then be used to model the short-term effects: to show how, for example, the current force might evolve over time under varying conditions. Thus, the individual and aggregate models developed in the previous sections are shown to be related to each other.

Long-Term Effects

If a manpower system were to operate in unchanged circumstances for a number of years, one would expect that measures of its performance—force size, costs, effectiveness, etc.—would attain, after an initial period of some change, a certain degree of stability. Under these conditions of stability, we refer to the performance measures as the *long-term* effects of the policies and conditions governing the manpower system. In other circumstances, they are often referred to as "steady-state" results, but that has a technical connotation which we will avoid here.

The retention function is the key to evaluating long-term effects of certain manpower policies, as will be shown in the following paragraphs. Several useful measures of enlistee behavior can be calculated from the retention function, including expected length of service and the probability that a randomly selected enlistee will serve at least 20 years. Such simple measures are of limited interest for policy analysis, however. As indicated in Section I, variations in enlistee costs and effectiveness are crucial to assessing the effects of changes in force composition and retention.

We turn our attention to functions of length of service, such as total wages or total posttraining service, and develop a methodology which will utilize this information. Suppose $g(s)$ is such a function: for example, $g(s)$ might be total wages paid through s years of service. We call $g(s)$ a *cumulative response* (through s years of service). If $g(s)$ is differentiable, we call $g'(s)$ the *response* (at s years of service); it gives the rate of growth of $g(s)$. In particular, if $g(s)$ is total wages, $g'(s)$ is the wage rate (on a yearly basis) after s years of service.

Given such a cumulative response function, it is natural to consider evaluating it over an enlistee's entire term of service. If S is the length of service for a given enlistee, $g(S)$ gives the *career response* for that enlistee—e.g., total wages paid during the military career. Recalling that length of service S is random, the career response $g(S)$ is also random. A measure of interest is the *expected career response*, given by

$$E[g(S)] = \int_0^T g(s) dF_S(s) \quad (0.7)$$

where T is some length of time greater than the longest time that an enlistee can serve. We show in Appendix A.1 that when $g(s)$ is differentiable the expected career response is given by

$$E[g(S)] = \int_0^T R(s) g'(s) ds \quad (IV.1)$$

which is a simple Riemann integral.

A well-known corollary of this result is that the expected length of service is given by the area under the retention function. This follows from considering the cumulative response function $g(s) = s$, with a corresponding response function $g'(s) = 1$ (one year of service accumulated for every year in the service). Then the expected length of service is

$$E[S] = \int_0^T R(s) ds .$$

More generally, for a fixed initial length of service s_0 we can define $g(s, s_0)$ as 0 when $s < s_0$ and $s - s_0$ when $s \geq s_0$. For example, $g(s, 0.5)$ measures total service exclusive of the initial six months of basic training. This cumulative response function corresponds to the response function $g'(s) = 1$ for $s \geq s_0$ (one year of service accumulated for every year in the service after an initial s_0 years). For this function, the expected career response $E[g(S, s_0)] = \int_{s_0}^T R(s) ds$, so expected service beyond $s = s_0$ is the area under the retention function to the right of s_0 .

These measures allow us to evaluate the long-term effects of changes in retention behavior or differences in retention patterns across subpopulations of enlistees. For example, we can compare total service, or perhaps more importantly experienced service, from enlistees characterized by different retention functions—such as two-year vs. three-year initial enlistments, or with different first-term reenlistment rates.

The final step in modeling long-term effects is to combine the measures for individual enlistees into an assessment for the force as a whole. Doing so requires that we incorporate information about the numbers of enlistees with various characteristics.

To measure long-term aggregate effects, we consider a force of enlistees with certain characteristics. First, enlistees are not assumed to be homogeneous with respect to their retention functions. Instead, we assume there are a number of categories of enlistees, that enlistees do not move between categories over time, and that within each category the enlistees share a common retention function. Let $R_i(s)$ denote the retention function for enlistees in the i -th category. Also, let us assume that, within each category, accessions are assumed to follow a homogeneous Poisson process, with a mean annual rate of N_i for the i -th category. For a detailed discussion of the implications of this assumption, see Parzen (1962). Intuitively the Poisson assumption means that accessions occur completely randomly over time. The homogeneity assumption implies that the expected number of accessions over a time interval of fixed length is unchanging over time. This assumption can be relaxed, yielding results analogous to, but not as simple as, those described below. Under the above pair of assumptions, the aggregate measures (force composition, experience levels, training expenses, effectiveness, wage and benefits costs, etc.) we compute will all represent long-term (i.e., stable) effects of the assumed accession and retention behavior.

For such a force, the expected number of enlistees in category i at any point in time is the product of the category's accession rate N_i and the expected length of service of its members, $\int_0^T R_i(s) ds$. More generally, the expected number with more than s_0 years of experience is the product of the accession rate N_i and the expected amount of service beyond s_0 years, $\int_{s_0}^T R_i(s) ds$. Thus, for example, expected force size and career content are easily determined for each category i of enlistee, if the retention functions R_i are known.

These results follow from a more general theorem, discussed in detail in Appendix A.2. The statement of the theorem revolves around consideration of an aggregate response for the entire category, analogous to the response defined for individual enlistees above. Suppose $s_{ij}(t)$ is the length of service as of time t of each of the enlistees (indexed by j) in category i . Consider a cumulative response $g_i(s)$ (e.g., total wages) common to all enlistees in category i . To this corresponds the response $g_i'(s)$ (the yearly wage rate) for an enlistee in category i with s years

of service. We then define the *aggregate response* $G_i'(t)$ (the total yearly wage rate at time t for all members of category i) as $\sum_j g_i'(s_{ij}(t))$. The theorem in Appendix A.2 states that the expected aggregate response is the product of the accession rate and the expected career response:

$$E\{G_i'(t)\} = N_i E\{g(S)\} \quad (IV.2)$$

The results in the previous paragraph follow from consideration of a particular response function $g_i'(s, s_0) = 1$ for $s \geq s_0$.

This result, then, provides the technique needed to model the long-term aggregate effects of various accession and retention behaviors. For a variety of response functions—wages, training costs, post-service benefit entitlements, effectiveness—we can calculate the expected career response within each category using (IV.1), then the expected aggregate response for each category using (IV.2), and then sum these across all categories to obtain an expected aggregate response for the force as a whole.

We can also use these results to derive an expression for the turnover rate, a common measure of force stability. While Bartholomew and Forbes (1979) cite various definitions, the central concept is that of a ratio of losses to force size. For the long term, the expected number of enlistees in category i is unchanging, so losses must be balanced by accessions. We thus will define the long-term turnover rate for enlistees in category i as the ratio of expected annual accessions to the expected number of these enlistees. From the results above, this reduces to $1/\int_0^T R_i(s)ds$ which is the inverse of the expected length of service for enlistees in category i . Thus, a three-year expected length of service corresponds to a 33% turnover rate; six years, to 17%. Doubling the expected length of service halves the turnover rate, with concomitant implications for accessions and training costs. We should note that the turnover rate can be defined and derived more rigorously using results from renewal theory (Parzen, 1962), but the above is adequate for the purposes of this treatment.

Continuation Rates and the Retention Function

We next explore the relationship between our models of individual behavior, embodied in the retention function, and of aggregate behavior, characterized by stocks, flows, and continuation rates. It will now be shown that the retention function can be used to derive expected continuation rates for stocks of enlistees. Given the similarity between the two concepts, this result should come as no surprise. It is important, though, in that the stock and flow model is the basis for the assessment of short-term aggregate effects.

The expected continuation rate for a group of enlistees with a common retention function and common number of (full) years of service m can be expressed in terms of the retention function. We define $\bar{p}(s)$ as the probability an enlistee with s years of service remains in the service at least one more year, and note that

$$\bar{p}(s) = \frac{\text{Pr}\{S \geq s + 1\}}{\text{Pr}\{S \geq s\}}$$

This would be, by definition, the expected continuation rate for a group of such enlistees, except that we group enlistees by full years of service m rather than by exact length of service s . Using (III.1), this can be rewritten in terms of the common retention function $R(s)$ as

$$\bar{p}(s) = \frac{R(s+1)}{R(s)}$$

Next, recall from Section II that the expected continuation rate for a group of enlistees with m full years of service gives the probability an individual selected at random from the group will

remain in service at least one more year. It is thus a weighted average of the continuation probabilities $\beta(s)$ for values of length of service s between m and $m + 1$ years, with weights reflecting the relative frequencies of these lengths of service. However, there is little loss in making some simplification, such as assuming that the weighting is uniform over the interval $[m, m + 1)$ or, as we do below, assuming it to be concentrated in the middle of the interval at $m + 0.5$, so that

$$p(m) = \beta(m + 0.5) = \frac{R(m + 1.5)}{R(m + 0.5)}$$

We next incorporate enlistee characteristics and external factors directly into the expected continuation rate. Since the retention function depends on them, it is clear that the continuation probability $\beta(s)$ must be a function of enlistee characteristics x . In defining continuation rates we subsumed external factors into "time," so we do the same with the retention function. We designate this retention function by $R(s, x, t)$. It is interpreted as describing the (necessarily hypothetical) distribution of the length of service of an enlistee with characteristics x subject to unchanging external factors identical to those at time t over the entire term of service. Corresponding to this is the continuation probability

$$\beta(s, x, t) = \frac{R(s + 1, x, t)}{R(s, x, t)} \quad (IV.3)$$

which gives the probability an enlistee with characteristics x and s years of service at time t remains in the service one more year.

Combining the above, we can represent $p(m, x, t)$, which from Section II is the expected continuation rate for enlistees with characteristics x and m years of service, at time t , in terms of the retention function $R(s, x, t)$. Assuming that all enlistees with between m and $m + 1$ years of service have exactly $m + 0.5$ years leads to

$$p(m, x, t) = \beta(m + 0.5, x, t) = \frac{R(m + 1.5, x, t)}{R(m + 0.5, x, t)} \quad (IV.4)$$

Equations (A.1) of Appendix A.3 apply the representation of the retention function given in (III.2) to equation (IV.3) for the continuation probability. Combining (A.1) with (IV.4) then completes the process of parameterizing the expected continuation rates. The expected continuation rates $p(m, x, t)$ are thus a function of just eight parameters $\alpha_1, \beta_1, \rho_1, \rho_2, \alpha_2, \beta_2, \gamma,$ and δ , for a given x and t . Further, continuation rates at the end of the third and succeeding terms of service depend only on the last four of these parameters. We have augmented the relatively thin data on attrition and reenlistment among enlistees with this length of service by adding a parametric model which embodies our knowledge and assumptions about this process.

Short-Term Effects

We turn our attention next to modeling the short-term (transitory) effects caused by changes in force structure and retention. The long-term measures from above allow comparison of different retention patterns independent of the initial conditions, providing an assessment of the "ultimate" results of different policies. However, the feasibility of making a policy change depends not only on the long-term costs and benefits but also on the effects of the transition itself.

To assess these effects, we must take into account the initial conditions—force composition and retention patterns—as well as the ultimate conditions. To do so, we return to the stock and flow model of Section II. The expected continuation rates derived from the retention function provide the parameters of the model, so we can find simple expressions for the stocks in

succeeding years. Finally, given the year-by-year stocks, we need measures of the effects. These are provided by the response function in a natural fashion.

We begin modeling short-term effects by returning to the stylized force of enlistees from the early pages of this section. As before, we assume there are a number of enlistee categories, that an individual enlistee's category does not change over time, and that the retention function for the i -th category of enlistees is $R_i(s)$. From each such retention function, we can deduce the associated expected continuation rates $p_i(m)$ by the methodology of the preceding paragraphs.

Over the short term, we are concerned with year-to-year variations in stocks within each category resulting from varying initial stocks, accessions, and continuation rates. Similar to what was done in Section II, we let $N_i(m, t)$ be the number of enlistees in category i at time t with m years of service. Also, we let $A_i(t)$ be the number of non-prior service accessions into category i over the interval $[t, t + 1)$, and disregard the negligible accessions with prior service. Given our definition of the expected continuation rate $p_i(m)$, it follows that $N_i(m + 1, t + 1)$, conditional on $N_i(m, t)$, has a binomial distribution with parameters $N_i(m, t)$ and $p_i(m)$.

Although a common approach is to apply Markov models (Bartholomew and Forbes, 1979) in this situation, we will not do so. These models have two advantages. One is that familiar matrix notation and operations can be used to describe the process and provide an accounting system for keeping track of stocks and flows. Second, the steady-state theory for Markov processes is well developed. However, we have already derived steady-state characterizations for the effects of differing enlistee retention, and the notational and computational gains of the Markov model are no great advantage here, so we do not pursue this approach.

In light of the development above, we can write a set of simple expressions for the expected stocks in any year. In particular, the binomial conditional distribution of stocks yields

$$E\{N_i(m + 1, t + 1) | N_i(m, t)\} = p_i(m)N_i(m, t)$$

and thus the expected stocks in each category i of enlistee at successive times $t, t + 1, \dots$, are related to each other by the expected continuations rates:

$$E\{N_i(m + 1, t + 1)\} = p_i(m)E\{N_i(m, t)\} \quad (IV.5)$$

$$E\{N_i(0, t + 1)\} = E\{A_i(t)\}$$

for $t \geq t_0$ and $m \geq 0$.

With an expression for the expected stocks in hand, we need only determine expected effects on a year-by-year basis. Again, $g(s)$ is the cumulative response through s years of service. Note that if an enlistee has s_0 years of service at some point in time t , then over the following l years, the cumulative response will grow from $g(s_0)$ to $g(s_0 + l)$, a difference of $g(s_0 + l) - g(s_0)$, provided the enlistee remains in the service. For example, if $g(s)$ is total wages through s years, then $g(3) - g(2)$ is the wages paid during the third year of service. We call $g(s_0 + l) - g(s_0)$ the *incremental response* over l years, for an enlistee with s_0 years of service at the beginning of the period. Now, of the $N_i(m, t)$ enlistees in category i with m years of service at time t , $N_i(m + 1, t + 1)$ will remain in the service at time $t + 1$, and $N_i(m, t) - N_i(m + 1, t + 1)$ will have left. We again simplify by assuming that all enlistees with between m and $m + 1$ years of service at time t have precisely $m + 0.5$ years of service, and that all enlistees who leave the service between times t and $t + 1$ do so at $t + 0.5$. Then we have a full year's incremental response of $g(m + 1.5) - g(m + 0.5)$ for those who remained in the service, and a half year's response of $g(m + 1) - g(m + 0.5)$ for those who left during the year. Combining this with the expected stocks, and simplifying somewhat, we see that the aggregate incremental response, over the year $[t, t + 1)$ for an enlistee in category i with m years of service is

$$N_i(m, t)[g(m + 1) - g(m + 0.5)] + N_i(m + 1, t + 1)[g(m + 1.5) - g(m + 1)] .$$

The expected value of this is

$$N_i(m, t)[g(m + 1) - g(m + 0.5)] + p_i(m)[g(m + 1.5) - g(m + 1)] .$$

This provides the key to modeling the short-term aggregate effects of various accession and retention behaviors. Starting from a set of initial conditions, representing perhaps the "current force," repeated application of these relationships allows us to make year-by-year projections of expected stocks. We could write closed-form expressions for the expected stocks $E[N_i(m, t)]$ in general. In practice nothing would be gained by so doing, since we want to trace short-term effects on a year-by-year basis. From these projections of expected stocks, we can calculate aggregate incremental responses.

Summary

In this section, we brought together the models of individual and aggregate behavior from the previous two sections. The result is a tool with which we can improve our ability to evaluate long- and short-term consequences of differing retention behavior. In the next section, we shall demonstrate this theory on a concrete example, showing how this methodology can be applied to available data.

V. MODELING AN ENLISTED FORCE

This section consists of an illustrative example showing how the techniques of the preceding three sections can be applied in a realistic setting. We begin by developing a model of an enlisted force consisting of just two categories of personnel. Available tabulations of continuation data allow us to separately fit retention functions to the two populations and to compare their long-term characteristics. We then postulate a shift in the percentages of the force recruited from each population and track the short-term effects of the change.

A Stylized Enlisted Force

To develop a model of an enlisted force, we make a number of simplifications to avoid becoming encumbered in the details necessary to thoroughly characterize the enlisted force. Thus, there is no claim that this accurately represents any particular enlisted force. Instead, our objective is to succinctly show the uses of the techniques from the preceding sections.

Our force consists of Army male enlistees. The impetus for this dissertation was the author's examination of the force structure implications of a 1979 experiment testing incentives designed to attract high-quality recruits into hard-to-fill occupational specialties, many of which were closed to women (Haggstrom et al., 1981). To model the retention behavior of enlistees recruited under different incentives, that study used Defense Manpower Data Center (1980) tabulations of continuation data for Army enlisted personnel from fiscal year 1979 (FY79), which also provide the basic data for this work. Since data from this period contain only limited numbers of women, and fewer still with long service, this study, like the previous one, limits its scope to male enlistees.

We divide our force into two categories, based on educational attainment and mental aptitude, as measured by the tests required of all applicants. *High-quality* enlistees generally are those high-school graduates whose test scores placed them in the 50th percentile or above; others—those with lower scores or nongraduates—constitute *lower-quality* enlistees. Of those non-prior service Army males enlisting in calendar year 1979, roughly 17% were high-quality (Haggstrom et al., 1981). Besides the high-quality and lower-quality enlistee populations, there is a third group: those whose score disqualified them from enlisting. For example, in 1977 this amounted to about 20% of those taking the test (Berryman, Bell, and Lisowski, 1983). Thus, lower-quality should not be taken to mean "low-quality." Exploratory analysis showed that retention behavior differs between graduates and nongraduates. To control for these differences, the lower-quality category will be further subdivided into *HSG lower-quality* and *NHS lower-quality* enlistees (graduates and nongraduates, respectively).

Tables 1-4 give our basic continuation data for these two groups. The data do not permit us to distinguish enlistees by term of service, so some inferences will have to be drawn. We can compute the fraction of enlistees with any given number of years of service who face a reenlistment point during the fiscal year. Coupling this information with knowledge of Army enlistment and reenlistment policy leads to the plausible assumptions that enlistees with less than four years of service are in their first term; with at least four but less than eight years are in their second term; and that all others are in their third or later term. This is adequate for our purposes, as the parameterization we developed in Section III does not require knowledge of the precise term of service beyond the second.

We use the data in these tables to estimate the eight parameters of the retention function separately for high-quality and lower-quality enlistees, and also separately for the NHS and HSG lower-quality enlistees. It is a straightforward matter to write down the likelihood

Table 1

HIGH-QUALITY FISCAL YEAR 1979 CONTINUATION RATES:
ACTUAL VS. EXPECTED, WITH STANDARD DEVIATIONS

Length of Service (Full Years)	With Reenlistment Point During FY79				With no Reenlistment Point During FY79			
	Number	Continuation Rate			Number	Continuation Rate		
		Act.	Exp.	Std. Dev.		Act.	Exp.	Std. Dev.
0	-	-	-	-	31,196	0.881	0.883	0.002
1	-	-	-	-	36,712	0.920	0.917	0.001
2	26,898	0.222	0.240	0.003	18,869	0.936	0.936	0.002
3	13,873	0.278	0.244	0.004	12,305	0.942	0.947	0.002
4	1,696	0.443	0.444	0.012	14,371	0.946	0.943	0.002
5	3,688	0.433	0.448	0.008	12,375	0.954	0.952	0.002
6	5,214	0.427	0.451	0.007	11,128	0.957	0.958	0.002
7	3,550	0.506	0.453	0.008	7,436	0.958	0.963	0.002
8	1,362	0.606	0.588	0.013	5,021	0.962	0.967	0.003
9	1,104	0.668	0.661	0.014	4,935	0.968	0.970	0.002
10	1,159	0.719	0.727	0.013	5,622	0.969	0.973	0.002
11	931	0.772	0.783	0.014	3,846	0.972	0.975	0.003
12	636	0.788	0.830	0.015	3,144	0.974	0.977	0.003
13	414	0.872	0.867	0.017	2,290	0.977	0.978	0.003
14	359	0.894	0.896	0.016	2,285	0.982	0.980	0.003
15	214	0.949	0.919	0.019	2,320	0.984	0.981	0.003
16	309	0.955	0.936	0.014	2,561	0.989	0.982	0.003
17	372	0.952	0.949	0.011	2,684	0.990	0.983	0.002
18	423	0.972	0.959	0.010	2,599	0.987	0.984	0.002

function for the data observed, and to then use available software to maximize it numerically. The details of this are relegated to Appendix A.4.

The parameter estimates can then be used in equations (IV.3) and (A.1) to provide the estimates of the expected continuation rates given in Tables 1-4. The tables also provide estimates of the standard deviations of the continuation rates (for the number within each group at the start of the fiscal year).

Generally, the observed and expected continuation rates agree quite well. The exceptions occur for enlistees approaching a reenlistment point within the first eight years of service. This argues that our treatment of early reenlistment rates was too coarse. Other work, not reported here, indicates that enlistees who choose a four-year initial term are more likely than others to

Table 2

LOWER-QUALITY FISCAL YEAR 1979 CONTINUATION RATES:
ACTUAL VS. EXPECTED, WITH STANDARD DEVIATIONS

Length of Service (Full Years)	With Reenlistment Point During FY79				With no Reenlistment Point During FY79			
	Number	Continuation Rate			Number	Continuation Rate		
		Act.	Exp.	Std. Dev.		Act.	Exp.	Std. Dev.
0	-	-	-	-	67,723	0.846	0.842	0.001
1	-	-	-	-	75,748	0.875	0.884	0.001
2	44,677	0.282	0.292	0.002	19,572	0.923	0.909	0.002
3	8,637	0.352	0.298	0.005	13,554	0.936	0.925	0.002
4	1,894	0.514	0.495	0.011	14,832	0.947	0.947	0.002
5	3,474	0.482	0.499	0.008	10,829	0.953	0.955	0.002
6	3,938	0.484	0.502	0.008	9,705	0.957	0.962	0.002
7	3,257	0.533	0.505	0.009	6,817	0.963	0.966	0.002
8	1,437	0.657	0.638	0.013	5,135	0.969	0.970	0.002
9	1,042	0.698	0.700	0.014	4,899	0.969	0.973	0.002
10	1,106	0.756	0.755	0.013	5,672	0.977	0.975	0.002
11	1,080	0.792	0.802	0.012	4,148	0.974	0.977	0.002
12	852	0.795	0.841	0.013	4,091	0.978	0.979	0.002
13	660	0.876	0.873	0.013	3,489	0.982	0.981	0.002
14	561	0.902	0.899	0.013	3,778	0.986	0.982	0.002
15	464	0.914	0.919	0.013	4,237	0.987	0.983	0.002
16	606	0.946	0.935	0.010	5,523	0.990	0.984	0.002
17	778	0.965	0.948	0.008	5,182	0.990	0.985	0.002
18	897	0.965	0.958	0.007	5,251	0.989	0.986	0.002

reenlist. The longer enlistment is generally a requirement for fields with specialized training, and these enlistees may well be more predisposed to military service. Thus, perhaps the occupational specialty chosen by the enlistee should be included among the individual characteristics in a more detailed study of retention.

Evaluating Retention's Effects

To apply the methodology of Section IV to our enlisted force, the enlistment and reenlistment terms must be specified. Of course, the data used to estimate the parameters of the retention functions contained enlistees serving a variety of such terms. Since the form of our retention function separately estimates reenlistment rates, we can choose a simple form for

Table 3

HSG LOWER-QUALITY FISCAL YEAR 1979 CONTINUATION RATES:
ACTUAL VS. EXPECTED, WITH STANDARD DEVIATIONS

Length of Service (Full Years)	With Reenlistment Point During FY79				With no Reenlistment Point During FY79			
	Number	Continuation Rate			Number	Continuation Rate		
		Act.	Exp.	Std. Dev.		Act.	Exp.	Std. Dev.
0	42,387	0.883	0.884	0.002
1	41,770	0.919	0.918	0.001
2	24,096	0.305	0.317	0.003	14,101	0.939	0.937	0.002
3	7,715	0.356	0.322	0.005	10,898	0.945	0.948	0.002
4	1,575	0.537	0.513	0.013	12,198	0.953	0.950	0.002
5	3,004	0.499	0.518	0.009	9,517	0.958	0.959	0.002
6	3,533	0.500	0.521	0.008	8,879	0.962	0.964	0.002
7	2,898	0.554	0.523	0.009	6,413	0.964	0.969	0.002
8	1,356	0.668	0.651	0.013	4,879	0.970	0.972	0.002
9	963	0.720	0.710	0.015	4,654	0.970	0.975	0.002
10	1,047	0.760	0.762	0.013	5,386	0.978	0.977	0.002
11	1,031	0.792	0.807	0.012	3,969	0.976	0.979	0.002
12	809	0.796	0.844	0.013	3,936	0.979	0.981	0.002
13	624	0.878	0.875	0.013	3,355	0.982	0.982	0.002
14	529	0.898	0.900	0.013	3,625	0.986	0.983	0.002
15	441	0.916	0.920	0.013	4,093	0.987	0.984	0.002
16	582	0.945	0.936	0.010	5,327	0.989	0.985	0.002
17	754	0.966	0.948	0.008	4,977	0.990	0.986	0.002
18	861	0.967	0.958	0.007	5,030	0.989	0.987	0.002

reenlistment points without invalidating our parameter estimates. In particular, for purposes of this example the initial term and reenlistment terms will all be three years in length.

Table 5 compares the service characteristics of enlistees in the force. To begin with, the results for NHS and HSG lower-quality enlistees demonstrate that these two groups are dramatically different. By any of the measures shown, the NHS lower-quality enlistees provide less service than the HSG lower-quality enlistees.

We proceed to compare the two groups of greatest policy interest, high-quality and lower-quality, recognizing that the latter group's results depend critically on the mix of graduates and nongraduates, and that a comparison between the high-quality and HSG lower-quality groups might be more relevant to today's enlistment environment. High-quality and lower-quality

Table 4

NHS LOWER-QUALITY FISCAL YEAR 1979 CONTINUATION RATES:
ACTUAL VS. EXPECTED, WITH STANDARD DEVIATIONS

Length of Service (Full Years)	With Reenlistment Point During FY79				With no Reenlistment Point During FY79			
	Number	Continuation Rate			Number	Continuation Rate		
		Act.	Exp.	Std. Dev.		Act.	Exp.	Std. Dev.
0	-	-	-	-	25,336	0.784	0.779	0.003
1	-	-	-	-	33,978	0.822	0.830	0.002
2	20,581	0.254	0.256	0.003	5,471	0.883	0.861	0.005
3	922	0.315	0.264	0.015	2,656	0.897	0.883	0.006
4	319	0.401	0.364	0.027	2,634	0.921	0.913	0.005
5	470	0.374	0.369	0.022	1,312	0.919	0.927	0.007
6	405	0.346	0.373	0.024	826	0.910	0.937	0.008
7	359	0.368	0.376	0.026	404	0.936	0.945	0.011
8	81	0.469	0.440	0.055	256	0.961	0.951	0.014
9	79	0.430	0.539	0.056	245	0.963	0.956	0.013
10	59	0.678	0.634	0.063	286	0.951	0.960	0.012
11	49	0.776	0.718	0.064	179	0.939	0.963	0.014
12	43	0.767	0.788	0.062	155	0.961	0.966	0.015
13	36	0.833	0.842	0.061	134	0.978	0.968	0.015
14	32	0.969	0.882	0.057	153	0.987	0.970	0.014
15	23	0.870	0.912	0.059	144	0.986	0.972	0.014
16	24	0.958	0.933	0.051	196	0.995	0.974	0.011
17	24	0.958	0.947	0.046	205	0.995	0.975	0.011
18	36	0.917	0.958	0.034	221	0.991	0.976	0.010

enlistees both serve about 3.5 years on average. However, in the long run 66% of the lower-quality enlistees in the force will be in their first term, compared to 71% of the high-quality enlistees. Put another way, for the same numbers of enlistees, 17% more of the lower-quality group will be career (second term or later). This is despite substantially higher first term attrition for lower-quality enlistees (37% vs. 29%) since first-term reenlistment is also higher (32% vs. 26%). These differences may reflect greater civilian-sector opportunities for enlistees who do well on the mental aptitude test.

We turn next to evaluating the short-term effects of a change, comparing two hypothetical situations: a "base case" and an "incentive case." In the latter case, we assume that enlistment incentives increase the number of high-quality enlistees and that a decrease in lower-quality

Table 5

COMPARISON OF LONG-TERM SERVICE CHARACTERISTICS:
THREE-YEAR ENLISTMENT AND REENLISTMENT TERMS

Item	High-Quality	Lower-Quality		
		All	HSG	NHS
Expected years of service	3.47	3.51	3.99	2.69
Annual turnover percentage	28.8	28.5	25.0	37.2
Experience levels:				
percent under 2 years	50	47	43	56
percent under 3 years	71	66	62	77
percent under 4 years	76	71	68	82
percent under 10 years	92	90	88	96
First term				
attrition percentage	29	37	28	49
Reenlistment percentage:				
First term	26	32	34	30
Second term	47	52	54	40
Third term	61	66	67	46
Fourth term	80	82	82	75

keeps the total annual accession unchanged. We also assume that retention behavior is the same as in the base case for both groups. Starting at an initial point in time, the expected stocks in succeeding years are computed using equations (A.1), (IV.4) and (IV.5) with the retention function parameter estimates previously developed. Again, three-year enlistment and reenlistment terms are assumed, and the contributions of those in the service before the initial time, which will be the same for both cases, are ignored. Furthermore, we assume that the mix of HSG and NHS enlistees in the lower-quality category remains unchanged over time.

Table 6 shows the results of the comparison. The first year's recruiting accounts for 90,000 enlistees with under a year of service. (Since our enlistment counts are hypothetical, we assume that these counts of enlistments are net of attrition.) After a second year, attrition has reduced this to 77,010 and 78,240 in the base and incentive cases, respectively, and both cases have another batch of 90,000 new recruits. Continuing this process for seven years, spanning two reenlistment points, we compute cumulative total enlistees in the service at the end of each year under each case. The incentive case initially provides a greater number of enlistees, but the advantage diminishes over time as the reduced reenlistments among the larger number of high-quality enlistees offsets their reduced attrition. By the end of the seventh year, the incentive case has only 0.3% more enlistees (with six or fewer years of service) than does the base case.

Table 6

COMPARISON OF SHORT-TERM SERVICE CHARACTERISTICS:
DIFFERENT RECRUITING SITUATIONS,
THREE-YEAR ENLISTMENT AND REENLISTMENT TERMS

(Number of Enlistees)

Full Years of Service	Base Case			Incentive Case		
	High- Quality	Lower- Quality	Total	High- Quality	Lower- Quality	Total
0	30,000	60,000	90,000	60,000	30,000	90,000
1	26,490	50,520	77,010	52,980	25,260	78,240
2	24,291	44,660	68,951	48,583	22,330	70,913
3	5,830	13,041	18,871	11,660	6,520	18,180
4	5,521	12,063	17,584	11,042	6,031	17,073
5	5,206	11,423	16,629	10,412	5,712	16,124
6	2,332	5,700	8,032	4,665	2,850	7,515

Years After Start	Case		Percentage Incentive Above Base
	Base	Incentive	
1	90,000	90,000	0.0
2	167,010	168,240	0.7
3	235,961	239,153	1.4
4	254,832	257,333	1.0
5	272,416	274,406	0.7
6	289,045	290,530	0.5
7	297,077	298,045	0.3

Summary

In this section we have indicated how the methodology developed thus far can be applied to existing data, allowing evaluation of alternative force structures. This made apparent the need for better measures to adequately assess the effects of the mix of enlistee types, which will be addressed in the next section.

VI. MEASURING ENLISTEE EFFECTIVENESS

This section further investigates the problems of measuring and modeling the effectiveness of various force structures. The "service characteristics" presented in the previous section report different aspects of the enlisted force experience mix. Measures of expected length of service, career content, reenlistment rates, and so forth, reflect this concern with the experience level. But all of these measures simply reexpress various aspects of the retention function.

We aim to determine an "effectiveness function" giving "overall effectiveness" (however we might define that) as a function of length of service. Treating this as a response function and applying the methodology of Section IV then yields an overall measure of the "effectiveness" of an enlisted force.

We discuss the modeling of enlistee effectiveness from two points of view. One approach models effectiveness growth over time and is based on the parameterization and estimation of "learning curves" from the psychology literature. The other is the economist's approach, one based largely on methodology falling within the "theory of the firm." We conclude by combining the two approaches and show how the result can be applied to data from a survey of enlistee effectiveness.

Learning Curve Models

An enlistee's effectiveness depends on his or her experience. Career enlisted personnel are widely perceived to be more effective (more productive, better performers) than first-term personnel, all else being equal. Certainly, a new recruit is of limited utility during basic training and subsequent occupational specialty training, and training requires resources which could otherwise be put directly to productive use. Furthermore, the training process does not end with arrival at the first duty station. Explicitly or implicitly, the first few months on assignment constitute a period of "on-the-job training" as the recruit becomes acclimated to the work and the work environment. During this period, he or she will divert supervisory time from more directly productive use. As the recruit gains experience, supervisory requirements diminish, skills increase, and the recruit becomes an increasingly valuable member of the unit.

We postulate an *individual effectiveness function* which measures enlistee effectiveness as a function of experience. We let $v(s)$ represent this function, where s is, as always, the enlistee's length of service. We expect that as length of service s increases, effectiveness $v(s)$ will increase also, but only within some upper limit: enlistees should become more effective as they gain experience, although their effectiveness cannot increase indefinitely. Taylor and Vineberg (1971) choose a negative exponential form $v(s) = \alpha - \beta e^{-\gamma s}$ with little justification. Haggstrom, Chow, and Gay (1984) discuss the representation of $v(s)$ in the context of the literature on "learning curves." The form given above is equivalent to assuming that the growth in effectiveness is proportional to the difference between current effectiveness and its asymptotic maximum value. They compare a number of alternatives and choose the negative exponential specification for their work.

Given an individual effectiveness function, we can use it to describe various characteristics of enlistee behavior. Taylor and Vineberg determine the amount of training needed for an enlistee to reach acceptable levels of performance. Haggstrom, Chow, and Gay measure total effectiveness (which they call total productivity) over the initial years of service.

Economic Model:

Turning our attention to a more aggregate level, we first note that a unit's effectiveness depends on the resources it has available, which we classify as labor inputs (e.g., enlistees) and other inputs (materiel, facilities, etc.). Labor inputs in a given unit can be categorized by a number of factors: occupational specialty and experience are the two of greatest interest. Within a given specialty, one would expect to be able to substitute more experienced enlistees for less experienced ones, and vice versa, although not on a one-for-one basis. On the other hand, opportunity for substitution between specialties should be much less. Certainly, when they are narrowly defined, occupations with different titles may be very similar. But for broad enough occupational groups, substitution between groups should be more difficult than between different experience levels within the same group.

The thrust of the economic models of enlistee effectiveness is to describe the opportunity for, and effects of, substitution between enlistees of different experience levels, but in the same or similar occupational specialties. It will greatly simplify the exposition, with no loss of generality for the results, if we hereafter proceed as if the labor inputs to a unit consisted of enlistees from a single such group of occupations. Let the vector L give counts of enlistees in a unit with various levels of experience. For example, $L = (L_1, L_2)$ could be counts of first-term and career enlistees. Alternatively, $L = (L_0, L_1, \dots)$ could be counts of enlistees by number of years of service. Also, let the vector C measure the levels of all other resources available to the unit. Now the unit can obtain a given level of effectiveness with a variety of combinations L and C of labor and other inputs. It is natural to think of a functional relationship expressing this, and we let $V(L, C)$ represent this function. This is very much like the economist's production function, with output measured in "effectiveness" units.

The *marginal effectiveness* (ME) of any input tells how a change in the amount of that input changes the unit's effectiveness. In particular, the marginal effectiveness of enlistees in the i -th experience level is given by $ME_i = \partial V(L, C) / \partial L_i$. Thus ME can also vary with the composition of the labor and nonlabor inputs.

We postulate a *labor aggregation function* to measure substitutability among enlistees with varying experience. To do so, we assume unit effectiveness is given by $V(A(L), C)$ for appropriate functions $V(a, C)$ and $A(L)$. A unit's effectiveness V is a function of other inputs C and a single, aggregate measure a of the labor input. The labor aggregation function $A(L)$ entirely captures the contribution to effectiveness of any combination of experience levels, summarizing this as the single value $a = A(L)$ in units of "aggregate labor." However, our assumption that the labor inputs represented by L consist of enlistees in a single occupation group means that aggregate labor $A(L)$ combines only different experience levels and not different occupations.

In this framework, the *marginal aggregate labor* (MAL) for enlistees in an experience level tells how a change in the number of those enlistees changes the value of the labor aggregation function. The marginal aggregate labor of enlistees in the i -th experience level is given by $MAL_i = \partial A(L) / \partial L_i$; it is related to the marginal effectiveness for that level by $ME_i = MAL_i \times \partial V(a, C) / \partial a$. Hence, in general, marginal effectiveness ME and marginal aggregate labor MAL are not equivalent.

Often, the form for the labor aggregation function is selected from among the class of constant elasticity of substitution (CES) functions (Henderson and Quandt, 1971). These functions have two defining characteristics: they are homogeneous of degree one, and as their name implies, the elasticity of substitution σ is the same between all pairs of inputs and does not change with the input mix. The CES labor aggregation function is given by

$$A(L) = (\sum w_i L_i^{-\rho})^{-1/\rho}$$

$$\sum w_i = 1$$

$$\rho = (1 - \sigma)/\sigma .$$

Henderson and Quandt show that the special case $\sigma = 1$ (i.e., the limiting case as $\rho \rightarrow 0$) corresponds to the Cobb-Douglas function

$$A(L) = \prod L_i^{w_i} .$$

The case where the elasticity of substitution σ is infinite (i.e., the case $\rho = -1$) gives

$$A(L) = \sum w_i L_i .$$

In this case, the MAL for category i is a constant w_i , the MAL ratios between categories are constant, substitution can continue indefinitely, and aggregate labor is a linear function of the numbers at each experience level.

Labor aggregation functions for military personnel usually include two experience levels: first term and career. Jacquette and Nelson (1974) use both linear and Cobb-Douglas labor aggregation functions with five separate levels, one for every four years of service up to twenty years, paralleling (roughly) the reenlistment points. Gotz and Roll (1979) use a Cobb-Douglas first-term/career labor aggregation function. Albrecht (1979) uses a nested CES aggregation function, where the elasticities of substitution are estimated as a part of the model. He initially models a first-term aggregate of two levels, then uses this as an input to a first-term/career aggregate.

Finally, the chosen labor aggregation function can be used in an optimization model, either maximizing effectiveness or minimizing costs. Jacquette and Nelson maximize effectiveness subject to a budget constraint, with retention rates a function of pay rates, the control variables in the system. Gotz and Roll minimize cost, keeping effectiveness constant, using reenlistment bonuses to determine the first-term reenlistment rate. Albrecht also minimizes costs, treating as control variables either first term and career inventories, or first term inventory and career wages.

Enlistee Effectiveness Data

Before 1975, little empirical data existed on manpower effectiveness. This led to the use of proxy measures, such as career content, to measure effectiveness. Further, one of the major issues in military manpower—accession vs. retention—was cast in terms of the tradeoff between first-term and career enlistees, treating these as homogeneous categories.

The Enlisted Utilization Survey (EUS) was a large-scale survey, taken in 1975, which attempted to measure job performance of enlisted personnel (Gay, 1974). Rather than rely on measures such as job performance tests, the EUS instead solicited information from supervisors on the "net contribution to unit productivity" of first term enlistees under their supervision. Thus, instead of precise measures of various components of the enlistee's duties—some more important, some less—the EUS collected an overall assessment of effectiveness from those in a good position to measure it.

The EUS instrument asked the supervisors to evaluate enlistees' "net contribution to unit production" measured "relative to the average specialist with four years experience." Following Albrecht (1979), we interpret this as a ratio of two measures of marginal effectiveness: that of the enlistee and that of the average four-year specialist.

An ambiguity in the wording of the instrument leads to uncertainty regarding the reference value. Rating net productivity "relative to the average specialist with four years experience" is

subject to two interpretations. A respondent can take this as meaning "relative to the net productivity which you would expect of a typical specialist with four years experience, working in your unit." Alternatively, it can be taken as "relative to the average net productivity of all four-year specialists."

By interpreting the EUS data as marginal effectiveness ratios measured relative to a typical four-year specialist in the same unit, we can also interpret them as marginal aggregate labor ratios, as shown in Appendix A.5. This leads to considerable simplification in later modeling, since the latter ratios involve only labor inputs to a particular unit, and nonlabor inputs can be disregarded.

Of course, any summarization of effectiveness as a single measure is at best an oversimplification. The EUS data represent individuals' contributions in routine tasks on a day-to-day basis. Such measures miss the contribution of the enlistees to the general level of deterrence provided by the force. For example, consider that enlistees newly arrived at their initial duty station were often rated as having a zero or negative "net contribution" to unit production. By analogy, enlistees still in boot camp make no contribution to any unit's effectiveness. But, it is difficult to argue that a potential aggressor would be more deterred by our forces were all such enlistees removed from it. It is also the case that the appraisal of an enlistee's effectiveness may be affected by his or her role in the unit. Since careerists may assume leadership roles, their perceived effectiveness may be high, although it is likely that less senior enlistees could assume such roles successfully, were they given the opportunity. Finally, since the EUS data are based only on first-term enlistees, there is a certain amount of risk in extrapolating it beyond that point. The effectiveness function could assume unrealistically large values. However, the choice of a functional form with an upper limit reduces the likelihood of that occurring, and Appendix B of Haggstrom, Chow, and Gay (1984) shows that at the end of four years, the effectiveness function is usually within ten percent of its asymptotic maximum. More importantly, should enlistees' duties change substantially in later years of service, measures of first-term effectiveness could be irrelevant in assessing their contribution in the new tasks.

Thus, the EUS data as it stands is probably an inadequate basis for a thorough policy analysis. Other dimensions of effectiveness need to be accounted for. One possibility would be to seek some modification to the EUS data to incorporate the contribution of sheer bulk of enlistees to deterrence and to other aspects of armed forces (rather than work unit) effectiveness. Alternatively, optimization models might use "EUS effectiveness" as but one of several attributes of interest, adopting, for example, a "goal programming" methodology similar to that of the ASCAR model (Petruzzi and Broider, 1980).

Acknowledging these limitations, we proceed to use the EUS effectiveness measures as representative of the type of measure needed for a policy analysis of military manpower accession and retention.

A Hybrid Model

We next relate the economic and learning curve methodologies to each other, using as a framework the EUS data. From the preceding paragraphs it is clear that the two approaches have differing strengths and weaknesses. The learning curve methodology stresses the dependence of effectiveness on increased training and experience. The economic approach stresses the relationship of effectiveness to the other aspects of a unit's composition. In the same manner that the retention function methodology permits better estimates of continuation rates, incorporation of effectiveness growth models into the economic methodology should yield better analyses of labor substitutability.

Albrecht (1979) applies the economic approach to modeling the EUS data. He treats supervisor ratings as measures of relative MALs. That is, each observation gives the ratio of the enlistee's MAL relative to that of a four-year specialist. He uses this data, and data on the experience mix within each enlistee's work unit, to estimate a nested CES labor aggregation function. Although his unit of analysis is the work unit, his unit of observation is the enlistee. To avoid having to aggregate data into work units, he uses a two-stage technique to estimate a CES first-term labor aggregation function. Each MAL ratio is then adjusted to give a ratio of the enlistee's marginal "first-term aggregate" labor to that of a four-year specialist. These ratios are used to estimate a second CES labor aggregation function combining "first-term aggregate" labor and career labor. Once this is done, he uses the resulting nested aggregation function in two optimizing models to deduce the characteristics of cost minimizing first-term and career labor under various sets of assumptions.

Haggstrom, Chow, and Gay (1984) apply the learning curve approach to modeling the EUS data. The EUS requested supervisor ratings for each subject as of four points in time during the first four years of service, ranging from just after joining the unit to that (anticipated) at the end of four years' service. The authors fit learning curves to subpopulations of their data, first grouping enlistees by service, specialty, and type of training. They simply report their results, with a few summary and derived measures, leaving it to others to infer the implications of their work for personnel policies.

To combine the two approaches, we begin by postulating a labor aggregation function with an infinite elasticity of substitution between enlistees of different experience levels (as always, within the same occupation group). This yields a linear labor aggregation function of the form $A = \sum_j w_j = \sum_j w(s_j)$ where the j -th enlistee has s_j years of service. Jacquette and Nelson consider both a linear function and a Cobb-Douglas function, and Gotz and Roll only use a Cobb-Douglas function. Albrecht, on the other hand, estimates relevant elasticities of substitution as parameters to his labor aggregation model. He generally finds elasticity σ to be in excess of 1, but frequently "significantly different from infinity" (i.e., ρ significantly greater than -1). If elasticity is not infinite, a linear function is inappropriate. However, he fits a coarse function—one which partitions labor into only three experience categories, two for first-term enlistees and one for careerists. In Appendix A.6 we show that his procedure can bias the estimated elasticities away from infinity. Thus, it is not clear that a linear function is indeed inappropriate. Also, for relatively narrow changes in our input mix, a linear function should be a reasonable approximation. For all these reasons, we feel comfortable in choosing to use a linear function.

A consequence of our choice of a linear labor aggregation function is that the MALs do not depend on the input mix. That is, the MAL for enlistee j is $w(s_j)$, which depends only on the enlistee's length of service s_j . Now, from above, the EUS data provides MAL ratios, relative to that of "the average specialist with four years' experience." By choosing this latter value as our *numéraire* for measuring aggregate labor, we can interpret an EUS evaluation of an enlistee with s_j years of service as an observation of his or her MAL $w(s_j)$.

Because the MALs depend only on the enlistee's length of service, we use them as a measure of enlistee effectiveness, modeling them with an individual effectiveness function. Choosing the same functional form used by Taylor and Vinberg and by Haggstrom, Chow, and Gay, we represent the MAL by $w(s) = \alpha - \beta e^{-\gamma s}$. Our labor aggregation function then becomes $A = \sum_j (\alpha - \beta e^{-\gamma s_j})$.

Thus, by fitting learning curves to the EUS data in the fashion of Haggstrom, Chow, and Gay, we estimate the parameters of a labor aggregation function, for enlistees in a particular occupation group, analogous to that of Albrecht. The multiplicity of labor aggregation function

parameters $\{w(s_j)\}$ is reduced to the more manageable α , β , and γ from the learning curve.

Summary

This section explored some of the issues involved in modeling enlistee effectiveness. By way of example, we constructed a model of enlistee effectiveness with two major characteristics: it incorporates the notion of increasing effectiveness over time, and it fits into the framework of economic analysis of labor substitution. In the next section we show how such a model can be used in enlistment and retention cost-benefit analyses.

VII. SYNTHESIS, APPLICATIONS, AND CONCLUSIONS

In this final section, the models of retention and effectiveness are combined to provide a unified tool for evaluating the expected contribution of an enlistee to his or her unit's effectiveness. Generally, we do this by using the effectiveness function of Section VI as a response function in the methodology of Section IV. A series of examples shows how this methodology can be applied to military manpower analysis in a number of settings.

Retention and Effectiveness Together

The marginal aggregate labor function can be treated both as an effectiveness function, to be modeled with a learning curve model, and as a response function, to be combined with a retention function in a model of expected effectiveness for enlistees and expected aggregate labor for groups of enlistees. As in Section VI, we assume that enlistee substitutability within an occupational group can be described by a linear labor aggregation function $A = \sum_j w(s_j)$, where $w(s_j)$ is the MAL of an enlistee with s_j years of service. We further assume that $w(s_j)$ is an effectiveness function of the form $\alpha - \beta e^{-\gamma s}$, measuring effectiveness relative to that of our *numéraire* for labor, the average experienced specialist (with four years experience, to be precise) in the occupation group. Since $w(s)$ measures the enlistee's effectiveness at s years of service, the function $g(s) = \int_0^s w(\xi) d\xi$ measures the cumulative effectiveness over the initial s years of service. For example, $g(5) = 3$ implies that the enlistee's initial five years of service contribute as much to the unit's effectiveness as an average experienced specialist would over a three-year period. Thus $g(s)$ fulfills the role of a cumulative response function, and marginal aggregate labor $w(s) = g'(s)$ that of a response function, in the sense of Section IV.

We use the term *effectiveness-years* to denote both cumulative effectiveness and the units in which it is measured. One year of service by the average experienced enlistee (whose effectiveness is by definition 1) constitutes one effectiveness-year, as does 2 years service by an enlistee whose effectiveness is .5 over the period. This terminology is suggested by considering the integral defining $g(s)$ as measuring the area under a curve plotting our effectiveness measure $w(s)$ against years of service s . Note, though, that effectiveness-years are defined in terms of the effectiveness function for an individual occupation group. Thus, effectiveness-years are not comparable between occupation groups, since the *numéraire* for each group is different.

Applying the methodology of Section IV to the effectiveness (i.e., MAL) function allows us to evaluate expected aggregate labor for a group of enlistees. Specifically, an enlistee whose service is characterized by the retention function $R(s)$ and effectiveness function $g(s)$ will have *expected career effectiveness-years*

$$E[w(S)] = \int_0^T R(s)w(s)ds$$

using (IV.1) to evaluate the contribution over the enlistee's career. The aggregate labor of a group of such enlistees is given by $A = \sum_j w(s_j)$, which is an aggregate response in the sense of Section IV. Thus, we can apply (IV.2) to obtain the group's *expected aggregate labor* $E[A] = NE[g(S)]$ where N is the annual accession rate for such enlistees.

Let us illustrate this with a brief example, which will serve as the basis of later examples in this section. Consider Food Service Specialists drawn from our population of high-quality enlistees in the example from Section V. For this example, we assume that retention does not vary across specialties, and that for a given specialty, effectiveness does not vary by enlistee quality. While these assumptions may be debatable, they are reasonable for the purposes of this

example. We can then use, for high-quality Food Service Specialists, the retention function parameter estimates for all high-quality enlistees given in Appendix A.4, and the effectiveness function parameter estimates for all Food Service Specialists given in Appendix A.7, as taken from Haggstrom, Chow, and Gay (1984). From Section IV and the previous paragraphs, the high-quality Food Service Specialist yields 2.55 expected career effectiveness-years. Thus, although they serve varying lengths of time, on average each additional such enlistee provides about the same contribution to aggregate labor, from enlistment through separation, as the average experienced Food Service Specialist would in two years and seven months service. A group of 1000 such enlistees would be maintained by an annual accession of 288 recruits, based on Table 5 in Section V. Thus, the expected aggregate labor of this group would be $288 \times 2.55 = 734$. The 1000 enlistees of varying amounts of experience are equivalent to 734 average experienced specialists.

However, this analysis is subject to the caveats, given in Section VI, regarding effectiveness measurement. Regardless of the appropriateness of our particular effectiveness measure, though, the principles described remain valid.

Areas of Application

We next describe how the analytical framework of the preceding paragraphs can be applied to analyzing policy options. We discuss three potential applications: evaluating the draft, evaluating enlistment and reenlistment incentives, and determining optimal force structures.

Evaluating the draft. In periods of low unemployment, when many potential enlistees have more attractive civilian-sector opportunities, we can anticipate renewed calls for a return to conscription. As Cooper (1977) and others have shown, the draft implicitly imposes a tax on those serving at a wage rate lower than that at which an adequate number of enlistees would be forthcoming. Some portion of this "conscription tax" will constitute losses by those for whom the military benefits are less than they could obtain from some other activity. The remainder is "economic rent" representing the difference between the two wage rates. The conscription tax represents a transfer of income from the draft-age population to the general taxpayer. As such, it is both selective (only a small fraction of the population would ever be called to serve under current demographic conditions) and regressive (the draft-age population has lower income than the general taxpaying population). Nevertheless, it may be politically easier to impose these taxes on the draft-age population than on the general taxpayer.

To show how our framework can be applied to evaluating the likely effects of a renewed draft, we return to the Food Service Specialist example. If draftees displayed retention and effectiveness patterns similar to those observed in the enlistees, except for an initial term of two years rather than three, we would expect a career contribution of 1.94 effectiveness-years each, on average.

While we feel reasonably comfortable arguing for similar patterns in ability between enlistees and draftees, similarity of retention is not as tenable. Since draftees exhibit, at least initially, a markedly lesser taste for military service, we might expect, for example, lower first-term reenlistment rates. Since attrition is at least somewhat under military control, we might expect it to be reasonably unchanged.

We can determine a sort of "worst case" for the draftee's expected career effectiveness-years. To do this, we evaluate $\int_0^1 K(s)w(s)ds$, which gives expected effectiveness-years over the two-year first term of service. This follows the procedure described in Section IV. The value is 0.84 effectiveness-years for our example, or one-third the contribution of a three-year enlistee.

These results understate the benefits from conscription, to the extent that draftees are required to serve in the reserves following their tour of active duty. Tradeoffs between active

duty and reserve forces are well beyond the scope of this paper.

Evaluating enlistment and reenlistment incentives. An alternative to conscription is the stimulation of enlistment with various incentives to the enlistee. The Multiple Option Recruiting Experiment tested the response of potential enlistees to a variety of enlistment incentives (Haggstrom et al., 1981). The objective of the various options was to increase recruiting, especially among young "high-quality enlistees"—high school graduates with better-than-average mental ability. This was done with combinations of two-year initial enlistments, guaranteed in-service training, and increased postservice educational benefits. Availability of these incentives was generally restricted to high-quality enlistees, frequently in conjunction with other requirements such as service in a combat arms specialty or an initial tour in Europe.

Let us consider the effects of offering a two-year initial enlistment as an alternative to the usual three-year term. As before, in the context of our example we expect a career contribution of 1.84 effectiveness-years from an enlistee with a two-year initial term, if such enlistees display otherwise similar retention and effectiveness patterns to three-year enlistees.

A difficulty with such an incentive is that it will be available to those who would have enlisted even in its absence. Thus an increased number of enlistees will be offset, at least in part, by a lessened contribution from all enlistees. Given the choice between a commitment for two years and one for three years, with service alike in all other respects, we can assume for a worst case that all enlistees will choose the two-year option. Consider two groups of enlistees with identical patterns of effectiveness and retention, except that one group is manned by two-year enlistees, while the other by three-year enlistees. Using (IV.2), we can evaluate the expected aggregate labor for each group as the product of (a) the expected accession rate into the group and (b) the expected career effectiveness-years for an individual enlistee in that group. For the expected aggregate labor of the two groups to be the same, the ratio of the expected accession rates must be the inverse of the ratio of the values for expected career effectiveness-years. In our example, two-year enlistees must enlist at a rate of 2.55/1.84 times that for three-year enlistees. In other words, enlistments would have to increase by 39% under a two-year initial enlistment just to hold their expected aggregate labor unchanged.

An alternative to increasing enlistments is to increase retention of enlistees already in the service. Consider the expected effectiveness-years during the second term of an enlistee who completes a three-year first term and reenlists for three additional years. Evaluating (IV.1) as $\int_3 R(s)w(s)ds$ gives the expected effectiveness-years during the fourth through sixth years, but includes the effects of first-term attrition and failure to reenlist. Appendix A.1 can be easily adapted to show that dividing the above by the probability the enlistee completes the first term and reenlists gives the expected second-term effectiveness-years. Evaluating this yields 2.73 effectiveness-years. Thus, the expected effectiveness-years of a second-term enlistee exceeds even that of the entire career of a new two-year or three-year enlistee. This surprising result is a consequence of the second-term enlistee's greater effectiveness and the unlikelihood that the new enlistee will serve beyond the end of the first term.

Determining optimal force structure. Optimizing models of military manpower generally determine that combination of manpower inputs which maximizes effectiveness subject to budget and other limitations, or, symmetrically, minimizes costs subject to meeting required levels of effectiveness and other constraints. For example, Albrecht (1979) solves for the optimal mix of first-term and career labor, within an occupation group, minimizing costs subject to an effectiveness constraint.

A simple example demonstrates how our methodology can be used in an optimization framework. Consider the members of the enlisted force in a single occupation group. These enlistees may differ in a number of ways. They may be characterized by different effectiveness

functions. They may display differing retention behavior, described by different retention functions. Their costs—wages and other benefits—may vary as well.

Measuring manpower costs poses some new problems. Military compensation consists of a combination of pay, allowances, tax advantages, bonuses, postservice benefits, and other fringe benefits. To reflect the true costs of service, deferred costs of postservice benefits must be included in current compensation. Manpower costs must include a component of "expected postservice benefits" including pensions, veterans benefits, educational assistance, and so forth, which reflect variation in the amounts of benefits actually paid to different enlistees. For example, a reenlistment bonus is paid to eligible enlistees at the time of reenlistment, so the costs for these enlistees must include, at the reenlistment point, the amount of the bonus. Alternatively, if the eligible enlistees comprise a portion of a more general population, the costs for the latter population must include the amount of the bonus, multiplied by the fraction of the group which is eligible. Similarly, costs to the military of postservice benefits should be included on an accrual basis, taking into account the likelihood of use of the benefits, in a manner similar to the budgeting of pension costs, which takes into account variations in the likelihood of becoming vested and in life length after retiring.

Given a measure of manpower costs, the force optimization problem can be structured and solved on an occupation-by-occupation basis. We begin by dividing the enlistees into categories, where enlistees in a category share common effectiveness, cost, and retention functions. For each category k of enlistee, let N_k be the expected annual accession rate, and let their length of service be distributed as the random variable S_k with retention function $R_k(s)$. Further, let $w_k(s)$ give the effectiveness (i.e., MAL) for an enlistee with s years of service, and let $c_k(s)$ give the cost of an enlistee with s years of service, so $\int_0^s c_k(x)dx$ gives the total cost of an enlistee over the initial s years of service, including expected postservice benefits. We can treat both the w_k 's and c_k 's as response functions and proceed. Expected aggregate labor is given by $E[A_k] = N_k \int_0^{\infty} w_k(s)R_k(s)ds$ and expected aggregate costs by $E[C_k] = N_k \int_0^{\infty} c_k(s)R_k(s)ds$. Our decision variables are the N_k , the expected accession rates into each category, and these may be subject to some constraints such as $N_k \leq M_k$. Without loss of generality, we assume the categories are numbered in order of decreasing value of the ratio of expected benefits to expected costs, $E[A_k]/E[C_k]$. To minimize expected costs for a fixed level A of expected aggregate labor, we set $N_1 = M_1$ and compare $E[A_1]$ to A . If the former is larger, we reduce N_1 until $E[A_1] = A$ and stop, having found the solution. Otherwise, we leave $N_1 = M_1$ and compare M_2 to $A - E[A_1]$, continuing in that fashion until reaching the required level of expected benefits. Maximizing benefits for a given cost is done analogously.

This is a formal solution that may be unrealistic, given various constraints on supply, utilization, etc. It does not address how the minimal effectiveness levels or the budget constraints are to be set. However, the solution can suggest directions to pursue in increasing the cost-effectiveness of the force.

Conclusions

This dissertation demonstrated an integrated methodology for assessing the effects of military manpower policies which change the enlistment, attrition, and reenlistment behavior of the enlisted force. This methodology, based on the retention function, provides techniques for assessing both long-term and transitional effects of such changes. We have shown how the retention function methodology is related to the commonly-used measure of military retention, the continuation rate, and further have shown that standard tabulations of continuation data can be used to estimate the parameters of the retention function.

The retention function methodology, as it stands, does not integrate forecasts of changes in the recruiting and retention environment to allow it to be used for forecasting manpower trends any great distance into the future. However, exogenously determined effects of such changes on attrition and reenlistment can be incorporated into the retention function, and this expands its use into analysis of policies for which no corresponding retention data are available.

To demonstrate the retention function methodology required a measure of "benefits" from an enlistee, and we derived one such measure from other work on enlistee effectiveness. This measure, the effectiveness function, served well for illustrative purposes but probably is inadequate for more general policy analysis. Since the measures of costs and benefits are separate from the retention function as such, there are no impediments to incorporating more suitable alternative measures.

Although we have limited our discussion to military manpower topics, the methodology developed has potential applications in other manpower systems, especially those, such as teaching and occupations requiring lengthy training and/or apprenticeship, where entry into the system is usually only at the most junior level.

Appendix A

TECHNICAL APPENDIX

A.1 Derivation of Expected Career Response

Theorem. Let $F(s)$ be a distribution function for length of service S , with corresponding retention function $R(s)$. Let T be some finite time such that $F(T) = 1$. Let $g(s)$ be a differentiable function of length of service with $g(0) = 0$. Then $E[g(S)] = \int_0^T R(s)g'(s)ds$.

Proof. The proof follows from integration by parts.

$$\begin{aligned} E[g(S)] &= \int_0^T g(s)dF(s) \\ &= g(T)F(T) - g(0)F(0) - \int_0^T F(s)dg(s) \\ &= g(T) - \int_0^T F(s)g'(s)ds \\ &= \int_0^T (1 - F(s))g'(s)ds \\ &= \int_0^T R(s)g'(s)ds . \end{aligned}$$

A.2 Derivation of Expected Aggregate Response

For the following theorem and proof, we draw heavily on Parzen's (1962, pp. 144-59) development of the filtered Poisson process.

Definition. If $\{N(t), t \geq 0\}$ is a Poisson process with parameter ν , $\{Y_n\}$ a sequence of independent, identically distributed random variables, independent of $\{N(t), t \geq 0\}$, $w(t, \tau, y)$ a real-valued function of three variables, and $X(t) = \sum_{m=1}^{N(t)} w(t, \tau_m, y_m)$ where τ_m is the time of the m -th occurrence, then the stochastic process $\{X(t), t \geq 0\}$ is a *filtered Poisson process*.

Lemma. $E[X(t)] = \nu \int_0^t E[w(t, \tau, Y)]d\tau$.

Proof. See Parzen (1969, p. 147).

Let accessions into the force follow a Poisson process with expected accession rate ν , with accessions occurring at time $\tau_1 \leq \tau_2 \leq \dots$, and with $N(t)$ the total number of accessions through time t . Let the m -th accession have length of service S_m , and assume that service lengths are i.i.d. with distribution function $F(s)$ on $[0, T]$ and corresponding retention function $R(s)$. Let $g(s)$ be a cumulative response function in the sense of Section IV, and assume that g is differentiable. Thus $g'(s)$ is the corresponding response function, and $G'(t) = \sum g'(t - \tau_j)$ (with the sum taken over $\{j: \tau_j \leq t \leq \tau_j + S_j\}$) is the aggregate response function for the force at time t .

Theorem. Under the above conditions, for $t > T$ the expected aggregate response $E[G'(t)] = \nu E[g(S)]$, the product of the expected accession rate and the expected career response.

Proof. Let $w(t, \tau, S) = g'(t - \tau)$ when $\tau \leq t \leq \tau + S$, 0 otherwise. Then $G'(t) = \sum_{m=0}^{N(t)} w(t, \tau_m, S_m)$ and $E[G'(t)] = \nu \int_0^t E[w(t, \tau, S)]d\tau$ by the lemma. Now $w(t, \tau, S) = g'(t - \tau)$ when $S \geq t - \tau$, 0 otherwise, so $E[w(t, \tau, S)] = g'(t - \tau)Pr[S \geq t - \tau] = g'(t - \tau)R(t - \tau)$. Thus $E[G'(t)] = \nu \int_0^t g'(t - \tau)R(t - \tau)d\tau = \int_0^t g'(s)R(s)ds = \nu E[g(S)]$ when $t > T$.

A.3 Continuation Probability Expression

Substituting the representation of $R(s, x, t)$ from (III.2) into equation (IV.3) for $\beta(s, x, t)$ yields the following expressions. When the end of the first term is more than one year hence, $0 \leq s < e_1 - 1$ and

$$\beta(s, x, t) = \frac{(1 + \beta_1(s+1))^{-\alpha_1}}{(1 + \beta_1 s)^{-\alpha_1}} \quad (\text{A.1a})$$

which is the conditional probability of no attrition over the following year. With less than one year left in first term, $e_1 - 1 \leq s < e_1$, so that

$$\beta(s, x, t) = \frac{(1 + \beta_1 e_1)^{-\alpha_1}}{(1 + \beta_1 s)^{-\alpha_1}} \times \rho_1 \times \frac{(1 + \beta_2(s+1))^{-\alpha_2}}{(1 + \beta_1 e_1)^{-\alpha_2}} \quad (\text{A.1b})$$

which is the product of (a) the conditional probability of no attrition during the remainder of the first term, (b) the expected first term reenlistment rate, and (c) the conditional probability of no attrition after reenlistment to one year hence.

With more than one year left in the second term, $e_1 \leq s < e_2 - 1$ and

$$\beta(s, x, t) = \frac{(1 + \beta_2(s+1))^{-\alpha_2}}{(1 + \beta_2 s)^{-\alpha_2}} \quad (\text{A.1c})$$

analogous to (A.1a) above. With less than one year left in the second term, $e_2 - 1 \leq s < e_2$ and

$$\beta(s, x, t) = \frac{(1 + \beta_2(s+1))^{-\alpha_2}}{(1 + \beta_2 s)^{-\alpha_2}} \times \rho_2 \quad (\text{A.1d})$$

analogous to (A.1b) above, except the attrition process is the same before and after the second reenlistment point.

In general, for the third and succeeding terms, through 18 years of service, we have the following. With more than one year left in the term, $e_{i-1} \leq s < e_i - 1$, $s \leq 18$, $i \geq 3$, and

$$\beta(s, x, t) = \frac{(1 + \beta_2(s+1))^{-\alpha_2}}{(1 + \beta_2 s)^{-\alpha_2}} \quad (\text{A.1e})$$

as in (A.1c) above. With less than one year left in the term, $e_{i-1} \leq s < e_i$, $s \leq 18$, $i \geq 3$, and

$$\beta(s, x, t) = \frac{(1 + \beta_2(s+1))^{-\alpha_2}}{(1 + \beta_2 s)^{-\alpha_2}} \times (1 + \exp(-\gamma - \delta e_i))^{-1} \quad (\text{A.1f})$$

where the second factor is the expected reenlistment rate after e_i years of service. Finally, since we ignore the small possibility of service beyond 20 years, we have for $s \geq 19$

$$\beta(s, x, t) = 0 \quad (\text{A.1g})$$

A.4 Derivation and Maximization of Likelihood Function

Each of Tables 1-4 stratifies a portion of the enlisted population serving on October 1, 1978, into 38 categories based on years of service ($m = 0, \dots, 18$) and whether the end of a term of service did or did not occur during the subsequent year ($i = 1$ or 2 , respectively). For each combination of m and i two values were observed: N_{mi} , the number in the category on October 1, 1978, and K_{mi} , the number of those remaining in the service on October 1, 1979. Thus, each

of the ratios K_{mi}/N_{mi} constitutes an observed continuation rate whose expected value is given by the expected continuation rates of equations (A.1) and (IV.4). A complication arises since individuals have different terms of initial enlistment and reenlistment, so the reenlistment points e_1, e_2, \dots are not the same throughout the population. This motivated our assumption that enlistees with less than four years of service are in their first term, with at least four but less than eight years are in their second term, and all others are in their third or later term. This allows us to choose the appropriate expression from among (A.1a)-(A.1f).

Each K_{mi} constitutes an observation of a binomial random variable with parameters $N = N_{mi}$ and $p = p_{mi}$, the expected continuation rate for that category. The likelihood is given by

$$L = \prod_{m=0}^1 \prod_{i=1}^2 \binom{N_{mi}}{K_{mi}} p_{mi}^{K_{mi}} (1 - p_{mi})^{(N_{mi} - K_{mi})}$$

and thus the log-likelihood function is

$$\log L = C + \sum [K_{mi} \log p_{mi} + (N_{mi} - K_{mi}) \log(1 - p_{mi})]$$

Substituting the expressions for the expected continuation rates p_{mi} gives an expression for the log likelihood function in terms of the parameters $\alpha_1, \beta_1, \alpha_2, \beta_2, \rho_1, \rho_2, \gamma$ and δ . Numerical maximization techniques applied to the data of Tables 1-4 give the parameter estimates of Table A.1.

A.5 EUS Data as MAL Ratios

Consider a unit with labor inputs $L = \bar{L}$ and non-labor inputs $C = \bar{C}$. Suppose labor category 1 is the specialist with four years experience. Then the marginal effectiveness of a given labor category, relative to that of the first category, is

$$\left. \frac{\frac{\partial}{\partial L_1} E(A(L), C)}{\frac{\partial}{\partial L_1} E(A(L), C)} \right|_{L=L, C=C} = \left. \frac{\frac{\partial}{\partial a} E(a, C) \frac{\partial}{\partial L_1} A(L)}{\frac{\partial}{\partial a} E(a, C) \frac{\partial}{\partial L_1} A(L)} \right|_{L=L, C=C, a=A(L)}$$

$$= \left. \frac{\frac{\partial}{\partial L_1} A(L)}{\frac{\partial}{\partial L_1} A(L)} \right|_{L=L}$$

which is the ratio of the MAL for category 1 to that for specialists with four years experience.

A.6 Bias in Estimated Elasticities

The following example demonstrates how combining data for different enlistee categories can lead to a downward bias in perceived elasticities of substitution. We construct the example in such a fashion that it is independent of the method by which the data is combined and of the estimation technique.

Suppose we observe the following data similar to that used by Albrecht (1979). For units $k = 1, \dots, K$ and enlistee categories $c = 1, \dots, C$ we observe MAL_{kc} , the marginal contribution to the labor aggregate of enlistees in category c in unit k , and L_{kc} , the count of enlistees in the same category and work unit. Our MAL corresponds to Albrecht's MP, our L to his L , our categories to his first-term subgroups and to career labor, and our unit to his work unit. Suppose that MAL is constant across units for each category c , so $MAL_{kc} = MAL_c$ for

Table A.1

RETENTION FUNCTION PARAMETER ESTIMATES

Parameter	High-Quality	Lower-Quality		
		All	HSG	NHS
α_1	.2882	.4328	.2789	.7493
β_1	.7425	.6465	.7753	.4923
α_2	.3172	.2733	.2536	.4537
β_2	2.258	35320.	4483.	19390.
ρ_1	.2594	.3206	.3408	.2974
ρ_2	.4709	.5225	.5399	.3983
γ	-2.460	-1.929	-1.806	-3.830
δ	.3223	.2871	.2792	.4089

all k . In this case, an enlistee's MAL does not depend on the composition of his or her unit. Then $\log(\text{MAL}_{ki}/\text{MAL}_{kj})$ for each combination of i and j is constant across units, and the elasticity of substitution between categories i and j is infinite.

Suppose now we combine categories 2, \dots , C into a single category 0. To avoid having our argument depend on the technique used to combine data, we assume for this example that within each unit only one of the $L_{kc} > 0$ for $c = 2, \dots, C$. In each unit k let c_k be the category for which this occurs. Then it is clear that $L_{k0} = L_{kc_k}$ and $\text{MAL}_{k0} = \text{MAL}_{kc_k} = \text{MAL}_{c_k}$.

Next, we assume that in our data it so happens that the L_{kc} and MAL_{kc} are related, in that when $L_{kc} > 0$ it is inversely proportional to MAL_{kc} . That is, there are fewer of the (presumably more experienced) enlistees with higher MALs, a not-unlikely occurrence. Thus, we have

$$\begin{aligned} \log(L_{k0}/L_{k1}) &= \log(L_{kc_k}/L_{k1}) \\ &= \log(\text{MAL}_{k1}/\text{MAL}_{kc_k}) \\ &= -\log(\text{MAL}_{k0}/\text{MAL}_{k1}) \end{aligned}$$

and the elasticity of substitution between categories 0 and 1 appears to be 1, although this is entirely an artifact of the relationship between MAL and L in our data.

We have shown that combining categories of enlistees with differing MALs and with L related to MAL in the observed data can cause a downward bias in the apparent elasticity of substitution. Furthermore, Albrecht's scheme for choosing appropriate categories of first-term labor does not eliminate this possibility, and the apparent statistical significance of his tests for non-infinite elasticity may be a result of this.

A.7 Effectiveness Function Parameter Estimates

Haggstrom, Chow, and Gay (1984) fit a learning curve of the form

$$v(s) = \begin{cases} \alpha - \beta e^{-\gamma(s-s_0)} & s \geq s_0 \\ 0 & 0 \leq s \leq s_0 \end{cases}$$

where s_0 is the time spent in training, travel, and leave before arriving at the initial duty station. In the body of this work we omitted mention of training time to simplify the exposition; we have, though, included its effects in the manner shown above. The "non-productive" time of the training instructors is more correctly included as a cost rather than as lost effectiveness, reflecting the difference between the enlistee's role while in training and while assigned to a duty station.

Table A.2 repeats the parameter estimates from Table B-16 of Haggstrom, Chow, and Gay, for Army technical school graduate Food Service Specialists. It also contains the transformed values of these estimates to ones for our parameterization. These reflect their scaling of 100 as the value for the average specialist with four years' experience, which we scale as 1, and their scaling of time in months rather than years.

Table A.2

**LEARNING CURVE PARAMETER ESTIMATES:
ARMY TECHNICAL SCHOOL GRADUATE
FOOD SERVICE SPECIALISTS**

Parameter	Estimate	Transform
α	109.8	1.098
β	68.9	.689
γ	.046	.552
s_0	150 days	.411 years

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