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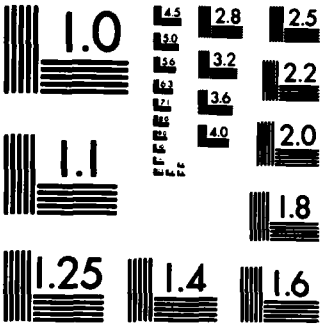

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The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Order statistics; Selection differential; Selection procedures; Bahadur representation; Outliers; Robust estimation; Nonparametric statistics; Sequential inference

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The research reported falls under the headings: (a) Order Statistics, (b) Nonparametric Statistics, (c) Sequential Inference.

(a) The second edition of a research monograph and text Order Statistics was published. Topics investigated include the following: Probability that a fuze will fire; moving order statistics; distribution theory and properties of the selection differential and related statistics; selection through an associated characteristic; Bahadur representation of

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20. Abstract Continued

sample quantiles; robust estimation in the presence of outliers.

(b) Properties of bootstrapping procedures were studied. A new approach to the Cramér-Smirnov test was developed.

(c) Sequential point estimation of the mean and properties of sequential estimation and testing were investigated.

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FINAL REPORT

1. INTRODUCTION

This final report covers two ARO contracts, namely DAAG 29-79-C-0083 for the period April 1, 1979 - June 30, 1982, and DAAG 29-82-K-0136 for the period July 1, 1982 - June 30, 1985. The title of both proposals is "Order Statistics and Nonparametric Statistics". The principal investigators were H. A. David and M. Ghosh. Dr. Ghosh left Iowa State University in 1982 but has continued to contribute to the research and has been supported by the contract on two short visits to the Statistical Laboratory.

2. SCIENTIFIC PERSONNEL PARTIALLY SUPPORTED, AND DEGREES AWARDED

*D. M. Andrews, Research Assistant

*H. A. David, Principal Investigator

J. K. Ghosh, Visiting Professor

M. Ghosh, Co-Principal Investigator and Visiting Professor

*I. H. Ha, Research Assistant

H. N. Nagaraja, Research Assistant Ph.D. (1980)

*M. P. Rogers, Research Assistant M.S. (1982)

W. R. Stephenson, Assistant Professor

S. Sukhatme, Associate Professor

I. Weissman, Visiting Associate Professor

W. B. Yeo, Research Assistant Ph.D. (1982)

*Currently active

3. SUMMARY OF RESEARCH FINDINGS

A. Order Statistics

1. General [A1; B3, 6, 7, 8, 16, 17, 18, 19; C1; D1]*. A second edition of Order Statistics (A1) was published by Wiley in 1981. Sections were added on order statistics for independent nonidentically distributed variates, on linear functions of order statistics, on concomitants of order statistics, and on testing for outliers from a regression model. Material on robust estimation was greatly expanded and much of the chapter on asymptotic theory was completely rewritten in light of important recent progress. The growth of the subject is reflected by an increase in the number of references from 700 in 1970 to 1000 in 1981.

   An entry on Order Statistics (D1) will appear in Vol. 6 of the Encyclopedia of Statistical Sciences.

   B7 resulted from a problem posed by an Army Scientist: A fuze will fire when at least n-1 (or n-2) of n detonators function within time span t. What is the probability of success? Other new aspects and uses of order statistics are investigated in B6 and B8. In particular, the theory of moving order statistics is developed in B8. These statistics are useful in quality control as bases for current measures of location and scale (e.g., moving medians and moving ranges).

2. Selection Problems [B20, 22, 23, 24, 30; D2]. H. N. Nagaraja undertook a thorough investigation of the large-sample distribution of the selection differential and related quantities (B22-24). If one selects the k largest observations in a sample, the difference between the average of these and the population mean (sample mean) is the selection differential (sample selection differential). In addition to the obvious importance of these statistics in helping to assess the effect of selection, they and their studentized versions are also relevant in tests for k outliers. Also studied was the induced selection differential which measures the effect of the above kind of selection on a related characteristic. For example, if the best k objects out of n are selected by some inexpensive method, how much better than the rest of the sample will they be when an expensive method of measurement is used?

   Related questions are addressed from quite a different point of view in B30 where small-sample results are obtained. In terms of the foregoing example the following question is answered: How many objects out of n should be selected on the basis of the inexpensive method to ensure with a probability greater than P (preassigned) that the k best objects as judged by the expensive

*Entries in square brackets refer to the list of publications in Section 4 of this report.

method, are included in the selected set? Tables for immediate implementation of the procedure were prepared.

3. Asymptotic Results [B15, 21, 26, 27, 29]. Asymptotic theory has already been alluded to in connection with the selection differential and occurs also more incidentally in some other references cited above. It plays a dominant role in the investigation of the useful but intricate Bahadur representation of sample quantiles under various conditions (B15, 26). Extreme-value theory is used in the study of records (B21) and life testing (B29).

4. Order Statistics under Non-Standard Conditions - Robustness to Outliers [B2, 4, 5]. In B4 a systematic study is begun of order statistics under non-standard conditions, such as non-identical distributions or dependence of the observations. The main applications are to reliability (lifetimes of systems of unlike components) and to robustness. Properties of robust estimators in the presence of outliers are studied in B2 and B5.

B. Nonparametric Statistics [B1, 13, 28; C2, 4; D3]

M. Ghosh co-authored B1 and B13 which deal with theoretical aspects of the popular new method of bootstrapping. He has also written an article on Rank Statistics and Limit Theorems (D3) for the Handbook of Nonparametric Inference.

In B28 S. Sukhatme provides a new approach to the Cramér-Smirnov goodness-of-fit statistic.

C. Sequential Inference [B9, 10, 11, 12, 14; C3]

M. Ghosh was sole author or senior author of all the listed papers which deal with sequential point estimation of the mean (B10-12) and properties of sequential estimation and testing (B9, 14).

4. PUBLICATIONS

A. Book Published.

1. David, H. A. (1981). Order Statistics, Second Edition. John Wiley and Sons: New York. xiii + 360 pages.

B. Papers Published

1. Athreya, K. B., Ghosh, M., Low, L. Y., and Sen, P. K. (1984). "Laws of Large Numbers for Bootstrapped U-Statistics", Journal of Statistical Planning and Inference 9, 185-194.

2. David, H. A. (1979). "Robust Estimation in the Presence of Outliers" in: R. L. Launer and G. N. Wilkinson, Robustness in Statistics, pp. 61-74. Academic Press, New York.

3. David, H. A. (1982). "Concomitants of Order Statistics" in: J. Tiago de Oliveira and B. Epstein (eds.) Some Recent Advances in Statistics, 89-100. Academic Press, London.

4. David, H. A. (1985). "Order Statistics under Non-Standard Conditions" in: P. K. Sen (ed.) Biostatistics: Statistics in Biomedical Public Health and Environmental Sciences, pp. 305-313. North Holland, Amsterdam.

5. David, H. A. and Ghosh, J. K. (1985). "The Effect of an Outlier on L-estimators of Location in Symmetric distributions", Biometrika 72, 216-218.

6. David, H. A. and Groeneveld, R. A. (1982). Measures of Local Variation in a Distribution: Expected Length of Spacings and Variances of Order Statistics", Biometrika 69, 227-32.

7. David, H. A. and Kinyon, L. C. (1983). "The Probability that out of n Events at least r($\geq$n-2) Occur Within Time Span t" in P. K. Sen (ed.), Contributions to Statistics, Essays in Honour of Norman L. Johnson, pp. 107-113. North Holland, Amsterdam.

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9. Ghosh, M. (1980). "Rate of Convergence to Normality for Random Means: Applications to Sequential Estimation", Sankhya A 231-240.

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15. Ghosh, M. and Sukhatme, S. (1981). "On Bahadur's Representation of Quantiles in Nonregular Cases", Communications in Statistics A **10**, 269-272.

16. Huang, J. S., Arnold, B. C., and Ghosh, M. (1979). "On Characterizations of the Uniform Distribution Based on Identically Distributed Spacings", Sankhya B **41**, 109-115.

17. Huang, J. S. and Ghosh, M. (1982). "A Note on Strong Unimodality of Order Statistics", Journal of the American Statistical Association **77**, 929-30.

18. Nagaraja, H. N. (1979). "Some Relations between Order Statistics Generated by Different Methods" Communications in Statistics - Simulation and Computation **B8**, 369-377.

19. Nagaraja, H. N. (1980). "Some Distributional Problems Connected with Order Statistics", Communications in Statistics A **9**, 935-942.

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21. Nagaraja, H. N. (1982). "Record Values and Extreme Value Distributions", Journal of Applied Probability **19**, 233-39.

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29. Weissman, I. (1980). "Estimation of Tail Parameters Under Type I Censoring", Communications in Statistics A **9**, 1165-1175.

30. Yeo, W. B. and David, H. A. (1984). "Selection Through an Associated Characteristic, with Applications to the Random Effects Model", Journal of the American Statistical Association **79**, 399-405.

C. Papers Accepted for Publication

1. David, H. A., "Order Statistics" and "Studentized Range", to appear in S. Kotz and N. L. Johnson (eds.), Encyclopedia of Statistical Sciences. Wiley, New York.

2. Ghosh, M., "Berry-Esseen Bounds for Functionals of U-Statistics", to appear in Sankhya A .

3. Ghosh, M. and Sen, P. K., "On Asymptotically Risk-Efficient Sequential Versions of Generalized U-Statistics", to appear in Sequential Analysis.

4. Stephenson, W. R. and Ghosh, M. "Two Sample Nonparametric Tests Based on Subsamples", to appear in Communications in Statistics, Theory and Methods.

D. Papers Submitted for Publication.

1. David, H. A., "Inequalities for Order Statistics Expressible as Sums", submitted to Annals of Statistical Mathematics, Tokyo.

2. David, H. A. and Ha, I. H., "Replacement of Parents by Meritorious Offspring in Within-Family Selection", submitted to Biometrics.

3. Ghosh, M., "Rank Statistics and Limit Theorems", submitted to Handbook of Nonparametric Inference edited by P. R. Krishnaiah and P. K. Sen.

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