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REVISION 2

AIRCRAFT FLIGHT PERFORMANCE METHODS

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HIGH SPEED AERO PERFORMANCE BRANCH
FLIGHT MECHANICS DIVISION

JULY 1985

TECHNICAL REPORT AFFDL-TR-75-89
FINAL REPORT FOR PERIOD JUNE 1974 – JUNE 1975

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This report addresses the problems associated with the derivation of aircraft performance characteristics. The emphasis is on segments which are a part of a mission profile. In particular, the following areas are studied: takeoff, climb, cruise, descent and glide, turning, and landing. The generalized approach to performance estimates is first presented. Next, appropriate solutions are derived. Last, sensitivity relationships are developed. The relationships determine changes in the performance characteristics as a result of an aircraft configuration change. This permits rapid evaluation of the performance variations for any segment of a mission profile. Additional Keywords:

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FOREWORD

This report was prepared by Dr. Linn Earl Miller and Philip G. Koch of the High Speed Aero Performance Branch, Flight Mechanics Division of the Air Force Flight Dynamics Laboratory (AFFDL/FXG), Wright-Patterson Air Force Base, Ohio. This research was accomplished under Project 1366, "Aeroperformance and Aeroheating Technology," Task 136602, "Performance Analysis of Military Flight Vehicle," and Job Order Number 13660218, "Analysis Techniques for Military Flight Vehicle Performance." The effort reported in this report was conducted during the period of June 1974 through June 1975.

Special acknowledgment is extended to Frances Jarnagin for typing the manuscript, Brett Benson for the sketches, and Judy Baker for the cover artistry.

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LIST OF TABLES

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LIST OF SYMBOLS

a	coefficient in fuel flow rate equation, speed of sound (feet/second), or acceleration (feet/second/second)
$\bar{a}_1$, $\bar{a}_2$, $\bar{a}_3$	average acceleration (feet/second/second)
A, A_1, A_2, A_3	coefficient in atmospheric pressure-density relation or coefficient in ground roll relation
B, B_1, B_2, B_3	coefficient in atmospheric pressure-density relation or coefficient in ground roll relation
$A(M)$, $B(M)$	Mach number functions
b	coefficient in fuel flow rate equation
C	proportionality factor or coefficient in the ground roll relation
C_D	aerodynamic total drag coefficient
C_{D_G}	aerodynamic drag coefficient in ground roll
C_{D_0}, $C_{D_{MIN}}$	aerodynamic zero lift and minimum drag coefficients
C_L, C_L'	aerodynamic lift coefficient and C_L at $C_{D_{MIN}}$
C_{L_G}	aerodynamic lift coefficient in ground roll
C_{L_α}	lift curve slope
D	aerodynamic drag (pounds)
F_a	external force
$f(T)$	atmospheric temperature ratio
$F(M)$	Mach number function
F_f	frictional force (pounds)
F_N	single engine net installed thrust (pounds)
g	acceleration of gravity (feet/second/second)
$G(M)$	Mach number function
H_{MAX}	maximum altitude (feet)

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LIST OF SYMBOLS (CONTINUED)

H_{OB}	obstacle height (feet)
h, h_1	altitude (feet)
K	aerodynamic induced drag factor
L	aerodynamic lift (pounds)
m	mass (slugs)
M	Mach number
M_c	corner Mach number
n	load factor
N	engine throttle setting
p	atmospheric pressure (pounds/square foot)
q	dynamic pressure (pounds/square foot)
R	gas constant climb variable, or radius of circular arc
R/C	rate of climb (feet/minute)
R_F	range factor (nautical miles)
R_γ, R_σ	radius of turn (feet)
S	aerodynamic reference area (square feet), sensitivity parameters, or runway distance (feet)
S_A	landing distance to clear obstacle
SFC	specific fuel consumption (pounds/hour/pound)
S_F, s_1	transition distance (feet)
S_G	ground run or ground roll (feet)
t	time (seconds)
t_c	time to climb (seconds)
T	thrust (pounds) or temperature
u	transformed speed (feet/second)
V	speed (feet/second)

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LIST OF SYMBOLS (CONTINUED)

$V_{CH}, V_{CH_1}, V_{CH_2}$	aerodynamic or engine speed change (feet/second)
V_{n^*}	speed for maximum sustained level flight load factor (feet/second)
V_R	reference speed (feet/second)
$V_{R^*_\sigma}$	speed for minimum sustained level flight turning radius (feet/second)
V_s	sinking speed (feet/second)
V_w	wind speed (feet/second)
$V_{\dot{\sigma}^*}$	speed for maximum sustained level flight turning rate (feet/second)
V_{ϕ^*}	speed for maximum sustained level flight bank angle (feet/second)
$\bar{V}_1, \bar{V}_2$	speeds corresponding to average acceleration (feet/second)
W	weight (pounds)
W_c	fuel to climb (pounds)
W_F	fuel flow rate (pounds/hour)
W_f	final weight (pounds)
X	range (nautical miles)
X(1)	time to climb variable
X(2)	fuel to climb variable
X(3)	speed variable
x,y,z	rectangular Cartesian coordinates
Z	climb variable

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GREEK SYMBOLS

α	angle of attack
β	atmospheric density coefficient (feet^{-1})
γ	flight path angle
δ	atmospheric pressure ratio
Δ	change in a performance variable
λ	constant
μ	coefficient of friction
ρ	atmospheric density (slugs/cubic foot)
σ	atmospheric density ratio or heading angle
τ	endurance time variable (seconds)
ϕ	bank angle or runway slope

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SUBSCRIPTS

APP	approach
i	initial
f	final
LO	lift off
MAX	maximum value
o	sea level
R	reference value
TD	touchdown

SUPERSCRIPTS

*	optimal value
(˙)	time derivative of ( )
‾	average value

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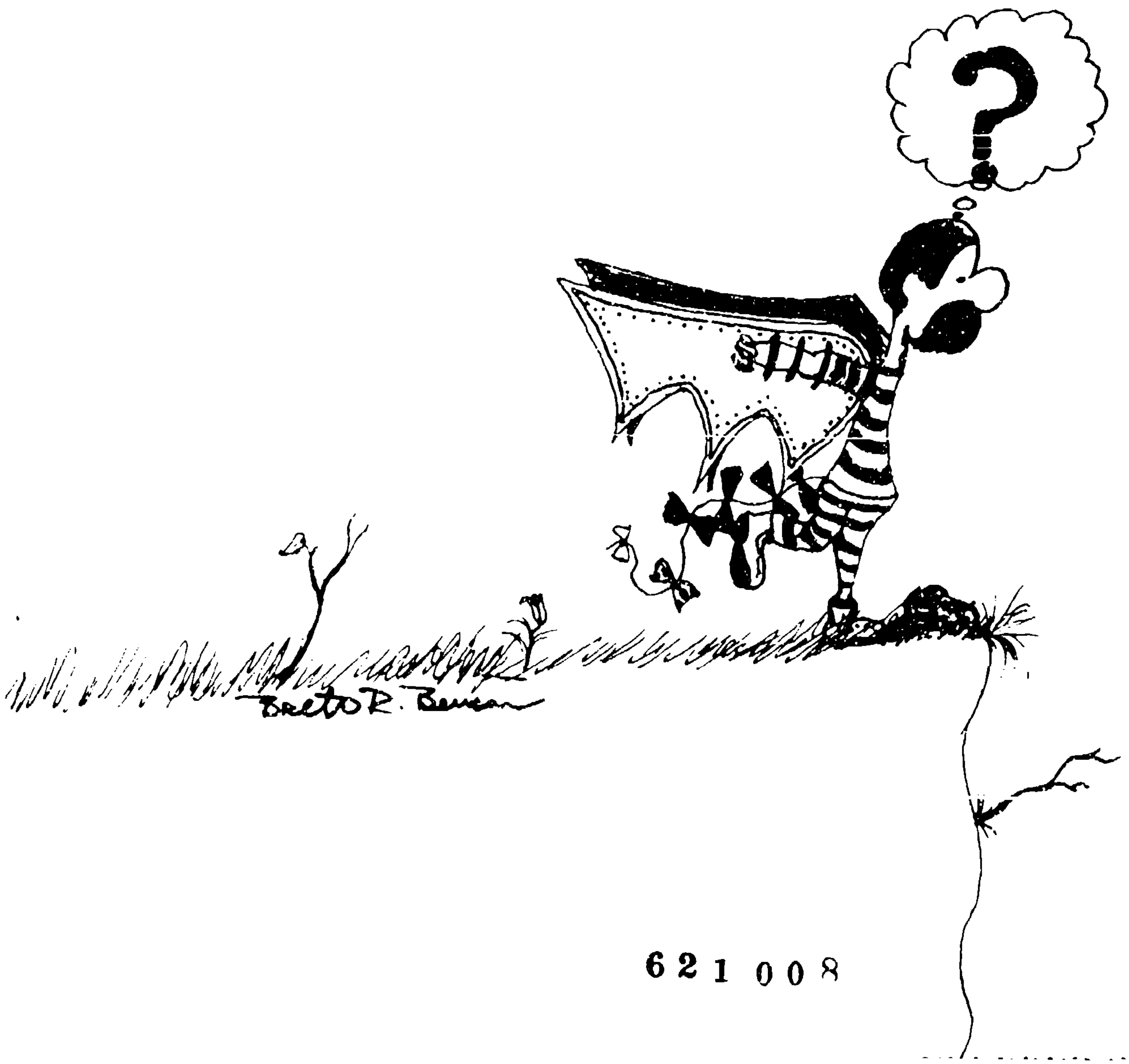
SECTION I
INTRODUCTION



SECTION I

INTRODUCTION

One of the better known sources for aircraft performance estimates is the text by Perkins and Hage, "Airplane Performance Stability and Control." Since the introduction of their text in 1949 there have appeared several other publications directed at predicting performance. By and large, these efforts were concerned with evaluating the performance characteristics of segments which, when added together, define an aircraft mission profile. These segments include takeoff, climb, cruise, acceleration, deceleration, descent and glide, turning, and landing.

Today digital computer programs exist which analyze the segments given the aircraft configuration definition. This definition consists of the aerodynamic and engine performance characteristics, the internal and external fuel load, and the payload. The computer programs determine the fuel used, the time, and the distance covered in each segment of the profile. If the aircraft configuration or mission is changed, then in general the program must be rerun from the start in order to establish the different mission profile. Generally the time required to do this is available. Unfortunately this is not always the case and often is not the economical approach, particularly if the change is small. Thus it is desirable to have available techniques or data which account for the changes in the profile or configuration.

This effort has as one of its objectives the derivation of sensitivity analysis which predicts first order changes in performance due to small changes in the configuration. Configuration changes could be an aerodynamic change, such as the addition of external stores, or a modified wing, or any other external modification. Inlet modifications or engine changes represent installed engine performance changes. Weight changes result from the previous changes, different payloads, and different fuel loads. Thus the ability to rapidly and accurately account for small configuration changes seems to be a desirable goal. Unfortunately, techniques or data for accomplishing this are lacking.

Sensitivity results are an end product and are derived as follows. Each segment of a mission profile, the climb, cruise, etc., is studied separately. The mathematical formulation of the segment is the first step. This includes the definition and assumptions which are necessary for formulating the problem, and is referred to as generalized performance. The second step is the derivation of approximate analytical solutions. Differentiation of these analytical solutions determines the sensitivity relations. Substitution of the configuration characteristics into the sensitivity relations then provides the desired sensitivity results.

It should be clear at this point that we are talking primarily about point performance problems. In other words, we are discussing the local properties of the trajectory for a given mission segment. We may integrate over a given segment with respect to time or some other selected independent variable, but the result should not be construed as an optimal solution between the initial and the final mission points. We attempt to determine the best solutions for each segment of a profile but clearly they are suboptimal for the reason

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that the total mission may not be optimized. A global optimal solution is that solution which optimizes the performance function over the whole mission. Examples are the maximization of the total range for a specified amount of available fuel or the minimization of the fuel required to fly between two specified points. These and similar problems are beyond the scope of this effort.

Organization of this report is by section, takeoff, climb, cruise, descent or glide, turning, and landing performance. In each section generalized performance is presented first. Next, approximate analytical solutions are determined. Finally, sensitivity parameters are derived. Graphical solutions are presented for each problem studied.

The performance estimates in general are based upon a standard day. Those non-standard day problems which are studied are identified in the text. For the analytical results an exponential atmosphere is often assumed. This corresponds to an exponential relation between atmospheric density and altitude.

In subsequent sections, reference will be made to the troposphere and stratosphere. Both are atmospheric layers above the earth's surface. The first layer is the troposphere. Across this layer the atmospheric temperature varies with altitude. Initially, in the second layer (the stratosphere) the temperature is constant. This layer extends to approximately 66,000 feet. The altitude at which the troposphere and stratosphere meet is called the tropopause and corresponds approximately to 36,089 feet for a 1962 standard atmosphere. Atmospheric data for a standard day are presented in the Appendix. For performance analysis the primary atmospheric variables are the density and the speed of sound. For a non-standard day the corrections for these variables are as follows.

It will be assumed that the pressure altitude is the same for both a standard and non-standard day. The temperature will be different, however; consequently both the density, ρ, and speed of sound, a, will be different. Subscripts S and O will denote a standard-day, and a standard-day sea-level value, respectively. From the equation of state

$$\frac{P}{P_S} = \frac{\rho}{\rho_S}\frac{T}{T_S} \tag{1-1}$$

In terms of standard day sea level density Equation 1-1 can be rewritten as

$$\frac{\rho}{\rho_{SO}} = \frac{\rho}{\rho_S}\frac{\rho_S}{\rho_{SO}}$$

$$= \sigma_S/f(T) \tag{1-2}$$

where

$$f(T) = T/T_S \tag{1-3}$$

In Equation 1-2, σ_S is the standard day density ratio which can be extracted directly from the tables in the Appendix. The non-dimensional temperature ratio f(T) is presented in Figure 1-1 as a function of altitude and ΔT, where ΔT is the difference in temperature between a standard and non-standard day. The interpolation is linear for any value of ΔT. The speed of sound is proportional to the square root of the absolute temperature, thus

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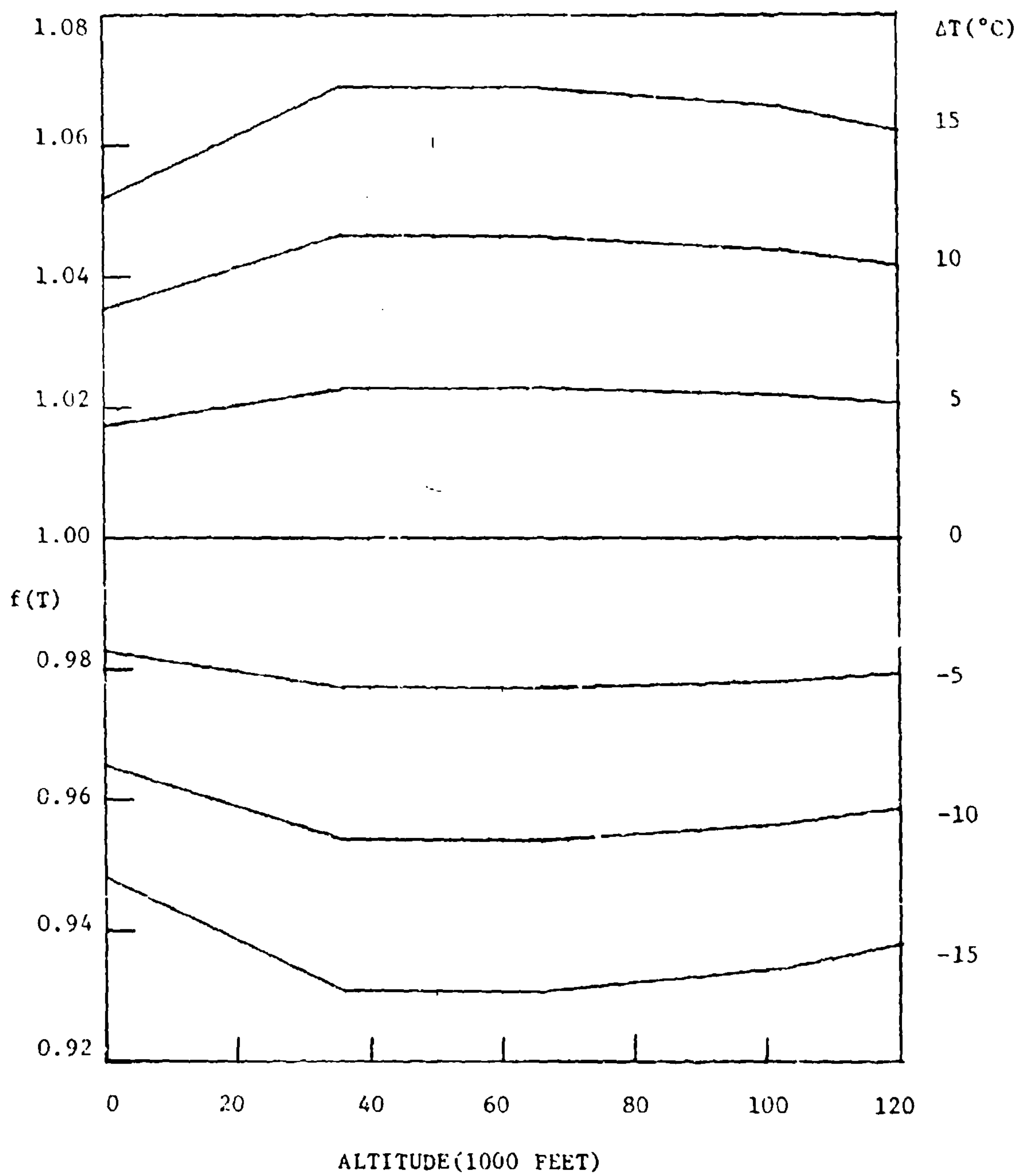

Figure 1-1 Non-Standard Day Temperature Ratio

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$$\frac{a}{a_s} = \sqrt{\frac{T}{T_s}}$$

$$= \sqrt{f(T)} \qquad (1-4)$$

The standard day speed of sound, a_s, is obtained from the tables in the Appendix. Thus the density and speed of sound for a non-standard day can be derived from standard day atmospheric tables and the correction f(T).

Numerical solutions to all problems except climb performance were derived by means of a scientific calculator. Any electronic slide rule capable of performing chain operations is adequate for determining the numerical solutions.

The first section will address problems relative to the estimation of takeoff performance.

SECTION II
TAKE-OFF PERFORMANCE

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SECTION II

TAKEOFF PERFORMANCE

Problem Definition and Assumptions

In today's age of larger and larger military and commercial aircraft, the available takeoff field length becomes one of the key design requirements. For a given design mission the takeoff field length requirement can directly impact the design of an aircraft. This is particularly true for the bomber and cargo carrying aircraft where high payload and long range are major mission requirements. For the smaller high thrust to weight ratio fighter and interceptor aircraft, the takeoff field length requirement does not usually impact the design. The high thrust-to-weight ratio will produce relatively short field lengths. However, in the preliminary design of any aircraft the designers must first consider takeoff requirements for the design mission.

An example of how the takeoff requirements can affect the design of an aircraft is illustrated by today's military and commercial aircraft. Their sophisticated wing flap systems are a result of the takeoff requirements (also landing requirements which will be covered in Section VII). The engine is usually sized to meet field length requirements for the large commercial and military aircraft. (Note: This may not be true in the case of a supersonic bomber or fighter where the engine size may be determined by climb or cruise requirements.) This results in designing increased weight and complexity into the aircraft to handle the first three to five minutes of the mission. Certainly then, the takeoff requirements are a very critical parameter in mission planning and preliminary design of an aircraft.

The purpose of this section is to provide information and background so that the preliminary design analyst can determine field length capability of a given configuration. The takeoff problem talked about here will be the conventional ground run followed by transition to a given obstacle height. The obstacle height is 50 feet for military aircraft and 35 feet for civilian aircraft. The takeoff field length for any aircraft is the sum of the ground roll distance to lift-off and the transition distance to the obstacle height. This is illustrated in Figure 2-1. Airport and runway conditions which affect the field length are temperature, altitude, wind, and runway slope. For preliminary design work it is assumed there is no wind or runway slope. Runway slope and wind components are taken into consideration for operational and flight handbooks.

Methodology

The ground run will be considered first in the takeoff analysis problem. This is the distance from aircraft brake release to lift-off speed. The lift-off speed is based on the attainable C_L (lift coefficient) at rotation of the aircraft. Determination of the C_L at lift-off will be discussed later in this section.

The derivation of the equations of motion for the ground roll portion will now be made. Figure 2-2 illustrates the external forces acting on the aircraft during the ground roll. These forces are lift, drag, thrust, weight, and ground friction.

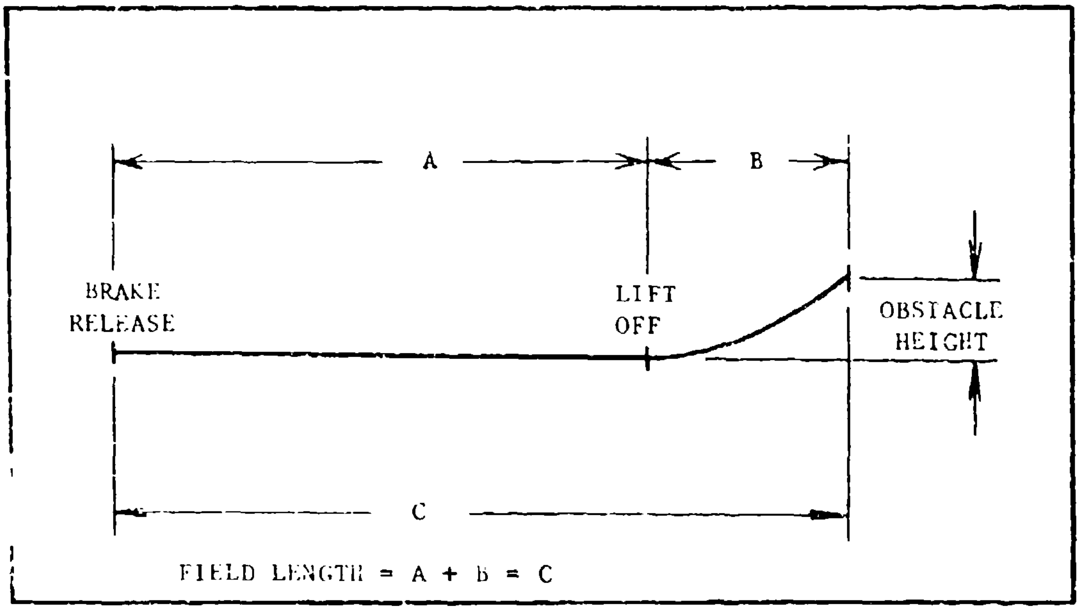

Figure 2-1 Definition of Take-off Field Length

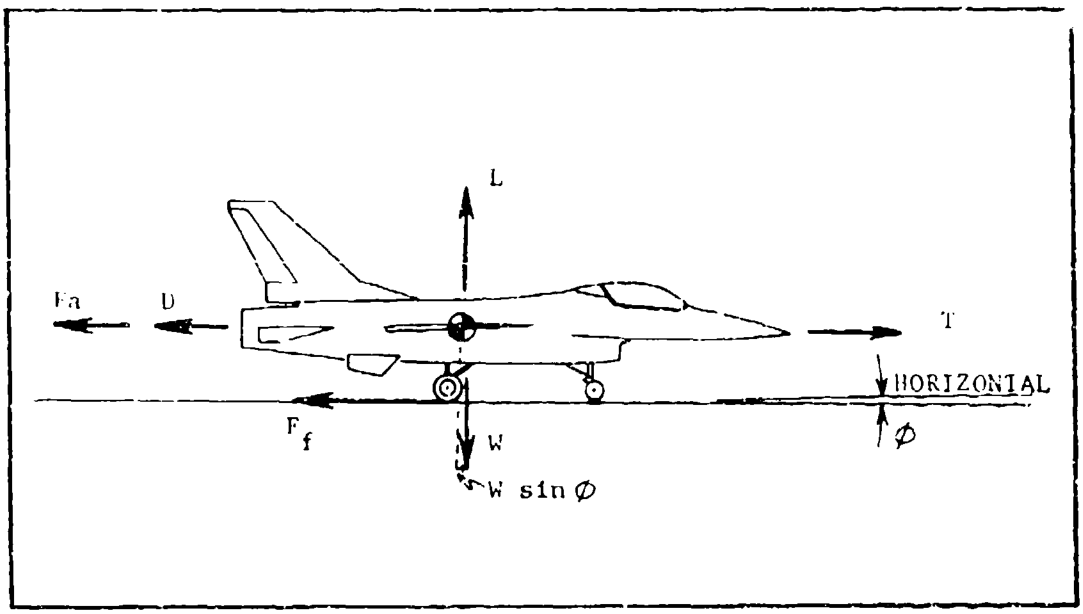

Figure 2-2 Forces Acting on the Aircraft For The Ground Run

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The velocity (V) and acceleration (a) at any given time is defined as:

$$V = \frac{dS}{dt}, \quad a = \frac{dV}{dt} \tag{2-1}$$

$$dS = V\,dt \quad \text{and} \quad dt = \frac{dV}{a}$$

Therefore

$$dS = V\frac{dV}{a} \tag{2-2}$$

where S is the distance traveled.

Integrating Equation 2-2 gives

$$S_G = \int_{V_w}^{V_{LO}} \frac{(V-V_w)dV}{a} \tag{2-3}$$

where

S_G = ground run distance

V_{LO} = lift-off speed

V_w = wind speed

The ground run distance can now be calculated by determining the acceleration of the aircraft. This can be done by summing the forces acting on the aircraft. The frictional force F_f is

$$F_f = \mu N = \mu(W-L)$$

where μ is the coefficient of friction between the runway surface and the tires. The external fo e F_a is

$$F_a = ma = \frac{W}{g}a \tag{2-4}$$

Summing the forces acting on the aircraft and substituting them in Equation 2-4 gives

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$$T - D - \mu(W-L) - W \sin \phi = \frac{W}{g} a$$

where a negative ϕ is a downhill slope and a positive ϕ is an uphill slope. We assumed earlier that the slope would be zero for preliminary design, therefore,

$$W \sin \phi = 0$$

and,

$$T - D - \mu(W-L) = \frac{W}{g} a$$

Solving for the acceleration gives

$$a = \frac{g}{W}\left[(T - \mu W) - (D - \mu L)\right] \tag{2-5}$$

The lift and drag forces are defined as follows

$$L = C_L\, q\, S \quad \text{and} \quad D = C_D\, q\, S$$

Substituting the lift and drag into Equation 2-5 gives:

$$a = \frac{g}{W}\left[(T - \mu W) - (C_D - \mu C_L)\, S q\right] \tag{2-6}$$

and substituting Equation 2-6 into Equation 2-3 gives:

$$S_G = \int_{V_w}^{V_{LO}} \frac{W}{g} \frac{(V - V_w)\, dV}{\left[(T-\mu W) - (C_D-\mu C_L)Sq\right]} \tag{2-7}$$

This equation can now be integrated from the initial velocity (V_w) to the lift-off speed (V_{LO}) to give the ground roll distance. The lift-off speed is determined from the following equation by knowing the lift-off C_L.

$$V_{LO} = \sqrt{\frac{2W}{C_{L_{LO}} S\, \rho}} \tag{2-8}$$

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The lift-off C_L can be derived from wind tunnel data, analytical predictions, or flight test data. The angle of attack to produce the lift-off C_L should be calculated with adequate ground clearance to clear the tail and the landing gear struts in the static compressed position. In determining the lift-off C_L a check must be made to determine if the C_L falls within the required constraints. For example, MIL-C-5011A states that the lift-off C_L must not be greater than that for 110% of power-off stall speed. It also states that distances to clear the obstacle shall be based on 120% of the power-off stall speed. In these two cases the appropriate lift coefficients are defined as:

$$C_{L_{110\%\ V_{STALL}}} = \frac{C_{L_{STALL}}}{1.21}$$

$$C_{L_{120\%\ V_{STALL}}} = \frac{C_{L_{STALL}}}{1.44}$$

Figure 2-3 illustrates the relationship of the ground run C_L, lift-off C_L, C_L limits, and the C_L stall. If the geometry limited lift-off C_L falls below the C_L for 110 and 120% V_{STALL}, it would be used for the lift-off C_L (as shown in Figure 2-3). Should the geometry limited lift-off C_L fall above the C_L for 110 and 120% V_{STALL} the appropriate C_L for lift-off and transition to the obstacle height must be used.

It should be noted the wing in proximity to the ground has an effect on the C_L vs. α curve as shown in Figure 2-3. This is commonly known as ground effect. If the analyst has good wind tunnel data available for ground effect they may want to include it in takeoff or landing analysis.

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However, the ground effect is a function of the altitude of the wing above the ground and previous analysis shows that this effect deteriorates very rapidly. For wing heights of one-half wing span the C_L vs. α is very close to that of free air. For high wing aircraft the ground effect could be very small. For preliminary design the ground effect is often ignored.

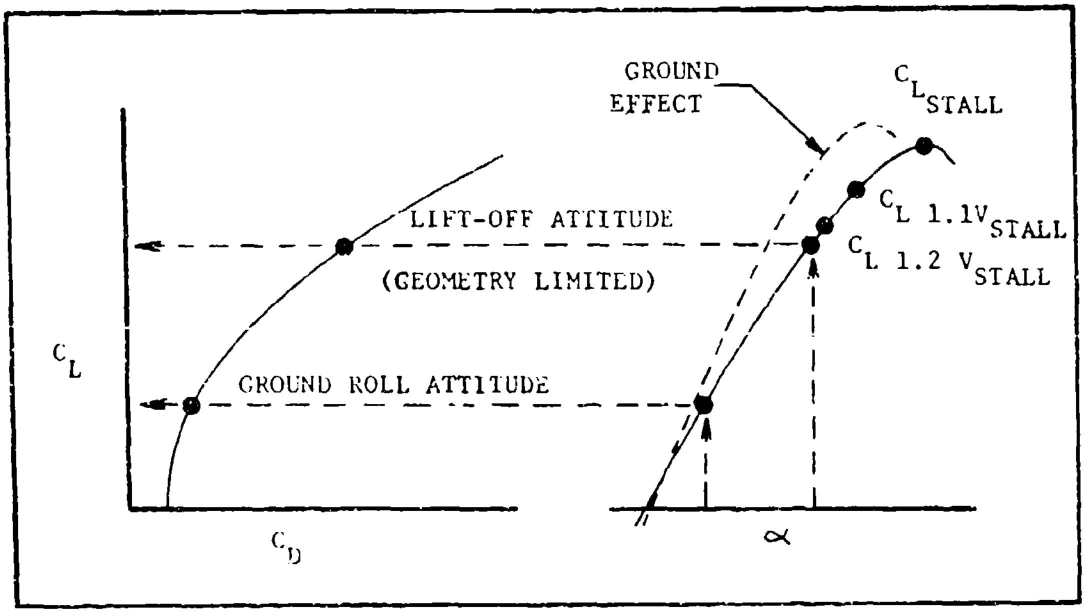

Figure 2-3 Angle of Attack and C_L Relation for Ground Run and Lift-Off

In general, it is necessary to solve S_G by numerical integration. However, for the purposes of preliminary design an analytic expression can be derived from Equation 2-7. By assuming that $C_D = C_{D_G}$, $C_L = C_{L_G}$, and the thrust are all constant, Equation 2-7 may be written

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$$S_G = \frac{1}{g} \int_{V_w}^{V_{LO}} \frac{(V - V_w)\, dV}{\frac{T}{w} - \mu - \frac{\rho S V^2}{2w} (C_{D_G} - \mu C_{L_G})} \qquad (2\text{-}9)$$

$$= A \int_{V_w}^{V_{LO}} \frac{(V - V_w)\, dV}{C^2 - V^2}$$

where

$$A = 1 / \frac{g\rho S}{2w} (C_{D_G} - \mu C_{L_G})$$

$$C^2 = \frac{T/w - \mu}{\frac{\rho S}{2w} (C_{D_G} - \mu C_{L_G})}$$

Rewriting the integral in Equation 2-9 in partial fractions gives

$$S_G = A \int_{V_w}^{V_{LO}} \left(\frac{C - V_w}{2C} \cdot \frac{1}{C-V} - \frac{C + V_w}{2C} \cdot \frac{1}{C+V} \right) dV \qquad (2\text{-}10)$$

Integrating Equation 2-10 gives

$$S_G = \frac{A}{2C} (C-V_w)\, \ell n \frac{C-V_w}{C-V_{LO}} + \frac{A}{2C} (C+V_w)\, \ell n \frac{C+V_w}{C+V_{LO}} \qquad (2\text{-}11)$$

By assuming the no wind condition, the ground roll distance reduces to

$$S_G = \frac{A}{2}\, \ell n \frac{C^2}{C^2 - {V_{LO}}^2} \qquad (2\text{-}12)$$

By using Equations 2-8 and the definitions for A and C^2, Equation 2-12 can be solved and presented in generalized terms. Figures 2-4, 2-5, and 2-6 present the lift-off speed (V_{LO}) and ground roll distance (S_G) in terms of $W/C_{L_G}S$, altitude, $W/S(C_{D_G} - \mu C_{L_G})$, and T/W. These data are for standard day conditions.

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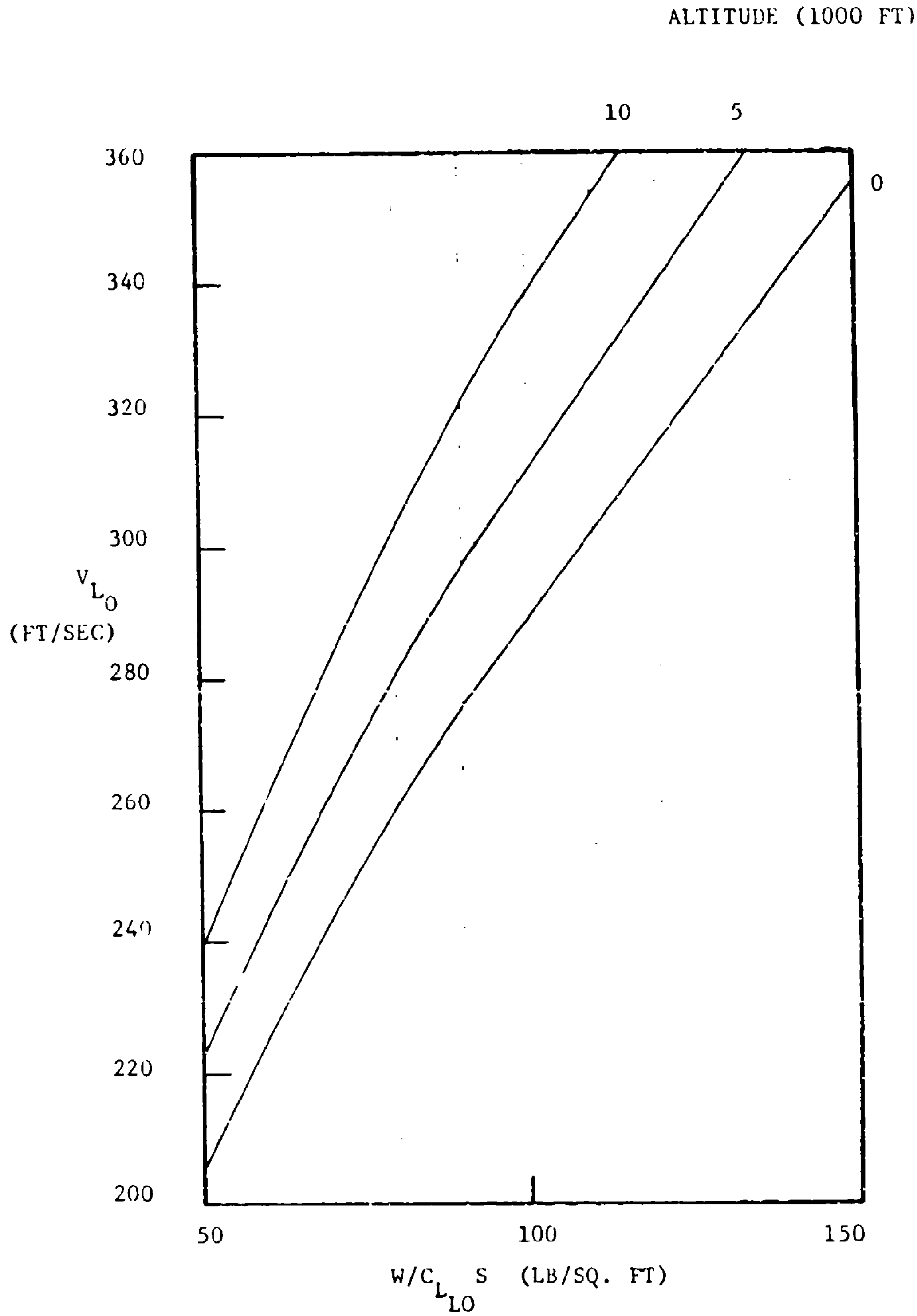

Figure 2-4 Lift-Off Speed

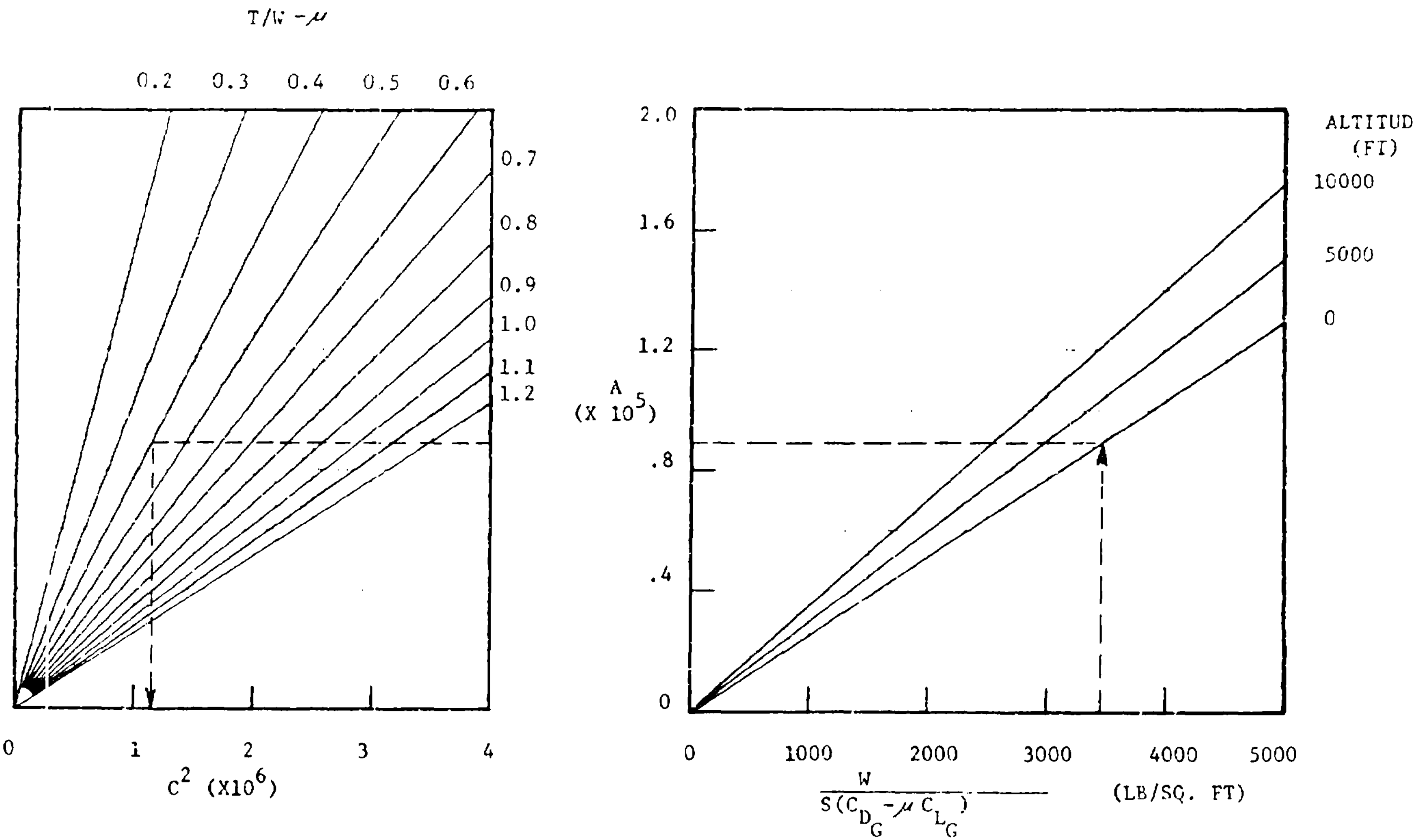

Figure 2-5 Take-Off Parameters for Ground Run

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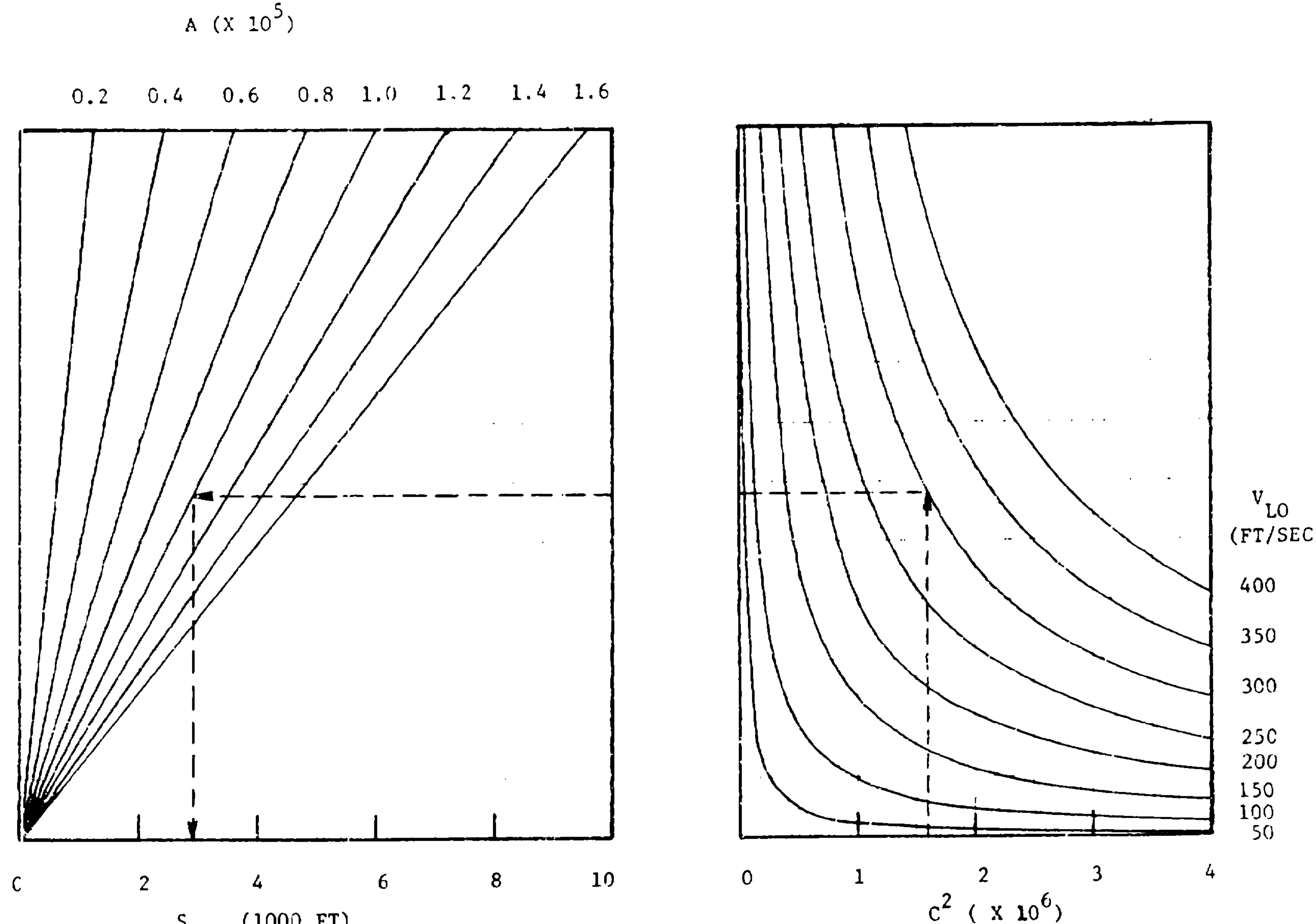

Figure 2-6 Generalized Take-Off Ground Run Distance

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Another approach to determining the ground run is based upon the assumption of average acceleration. This leads to a simple form for the ground run. In Equation 2-6 we see that if the aerodynamic coefficients are constant and the thrust fairly constant, the acceleration varies as the speed squared. This is illustrated in Figure 2-7. The average acceleration will be at $(a_1 + a_2)/2$. The velocity at this point will then be the average velocity for the ground roll. Assuming the no-wind condition ($V_w = 0$), the velocity increment (ΔV) is then equal to V_{LO}. The average velocity squared is then

$$\bar{V}^2 = V_{LO}^2/2$$

and solving for the average velocity gives

$$\bar{V} = \sqrt{\frac{V_{LO}^2}{2}} = 0.707\ V_{LO} \qquad (2\text{-}13)$$

Knowing V_{LO}, we can compute the ground run distance in one step by using average values. Equation 2-3 can be integrated from $V_w = 0$ to V_{LO} to give Equation 2-14.

$$S_G = \frac{1}{2\bar{a}} V_{LO}^2 \qquad (2\text{-}14)$$

where,

$$\bar{a} = g/w \left[(T - \mu w) - (C_D - \mu C_L)\ Sq\right] \qquad (2\text{-}15)$$

and q is evaluated at $\bar{V}$.

Substitution of A and C^2 gives:

$$S_G = \frac{A}{2} \cdot \frac{V_{LO}^2}{C^2 - \frac{1}{2}V_{LO}^2}$$

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The difference between this solution and Equation 2-12 increases as V_{LO} approaches C. For example, for an extreme case such as the KC-135, which has a low thrust to weight ratio and high lift-off speed, the difference is less than one-half of one percent. The parameters T and q are evaluated at $\bar{V}$. The parameters S, W, C_D, C_L, and μ are characteristics of the aircraft.

The transition from lift-off to obstacle height is the second phase of the takeoff run and is included in the field length calculation. This phase is basically a problem of determining the time required to clear the obstacle from the lift-off point. Because the obstacle height is small, the transition times are very small (1 to 6 seconds depending on climb capability), which allows it to be calculated very easily in one step by assuming average values for speed and load factor.

The time and distance to clear the obstacle can be determined by examining the forces acting on the aircraft as illustrated in Figure 2-8. Summing the forces parallel to the flight path gives:

$$T-D-W \sin \gamma - \frac{W}{g}\left(\frac{dV}{dt}\right) = 0 \tag{2-16}$$

Summing the forces perpendicular to the flight path gives:

$$L - \frac{W}{g}\left(\frac{d\gamma}{dt}\right) V - W \cos \gamma = 0 \tag{2-17}$$

Solving Equations 2-16 and 2-17 for $\dot{V}$ and $\dot{\gamma}$ gives

$$\dot{V} = g \left(\frac{T}{W} - \frac{D}{W} - \sin \gamma \right) \tag{2-18}$$

and,

$$\dot{\gamma} = \frac{g}{V} \left(\frac{L}{W} - \cos \gamma \right) \tag{2-19}$$

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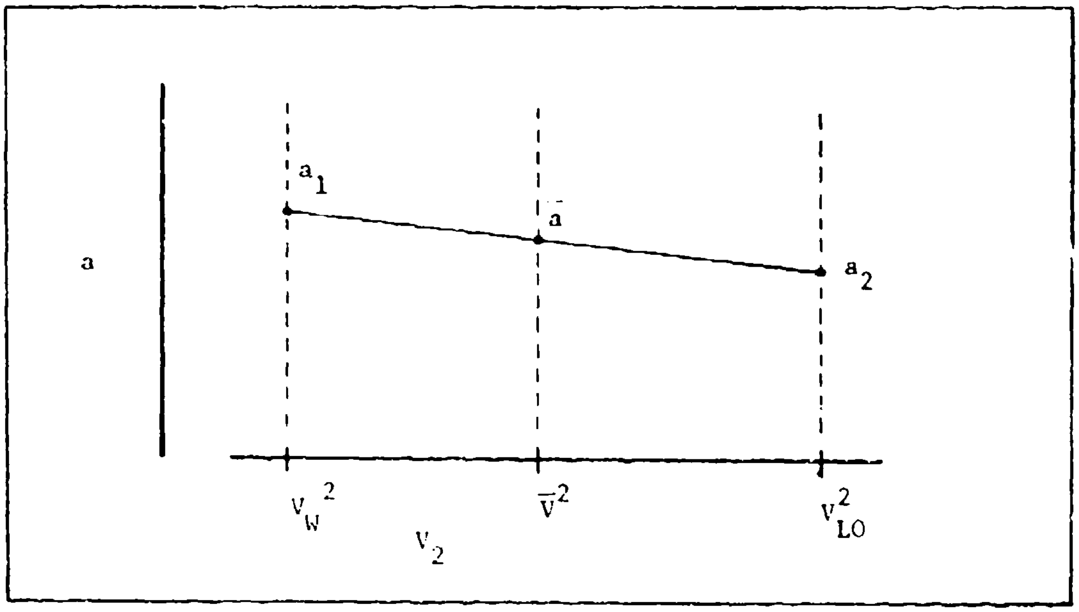

Figure 2-7 Acceleration vs. Velocity Squared

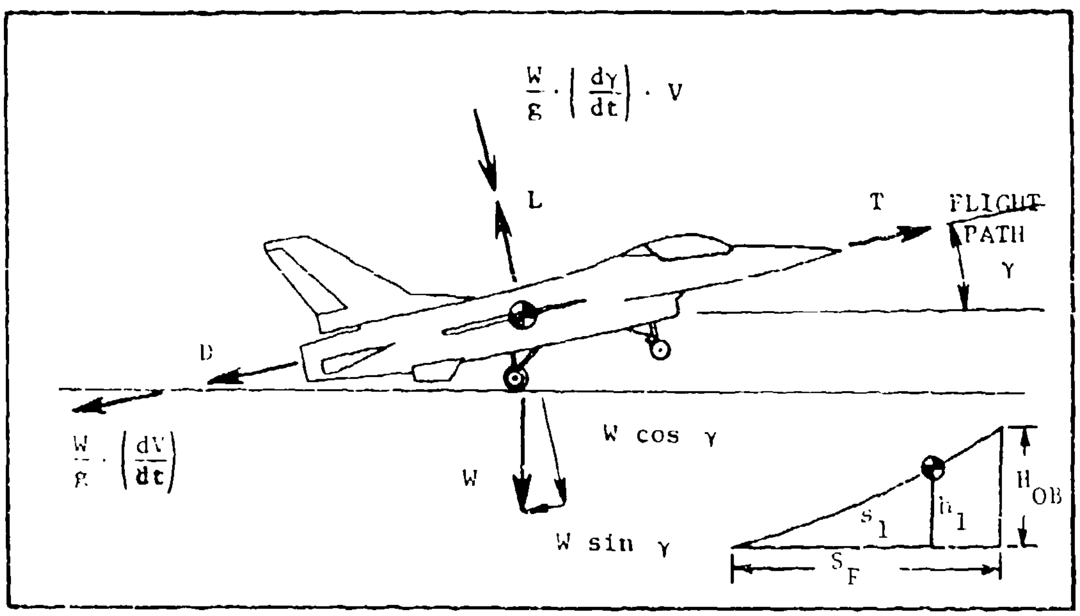

Figure 2-8 Forces Acting on the Aircraft For The Flare

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where $\dot{V}$ and $\dot{\gamma}$ are $\frac{dV}{dt}$ and $\frac{d\gamma}{dt}$ respectively. Assuming the speed is essentially constant and γ is small, integrating Equation 2-19 gives

$$\gamma = \frac{g}{V}(n-1)t \tag{2-20}$$

where n is defined as the load factor and is equal to L/W. For a constant speed Equation 2-18 may be written

$$0 = g\left(\frac{T}{W} - \frac{D}{W} - \sin\gamma\right)$$

and solving for γ gives

$$\gamma = \sin^{-1}\left(\frac{T-D}{W}\right)$$

This γ would be the maximum flight path angle attainable during the transition with no speed loss and n = 1.0. Therefore,

$$\gamma_{MAX} = \sin^{-1}\left(\frac{T-D}{W}\right)$$

or,

$$\gamma_{MAX} = \sin^{-1}\left(\frac{T}{W} - \frac{1}{L/D}\right) \tag{2-21}$$

where n = L/W = 1.0. Referring to the inset of Figure 2-8, if the obstacle is reached before γ_{MAX}, which is usually the case, then

$$h_1 = H_{OB} = \int_0^{t_1} V \sin\gamma \, dt = \int_0^{t_1} V\gamma \, dt$$

for small γ's. Substituting Equation 2-20 and integrating gives

$$h_1 = 1/2\ g\ (n-1)\ t_1^{\,2} \tag{2-22}$$

The distance covered is

$$s_1 = S_F = \int_0^{t_1} V \cos\gamma \, dt$$

therefore,

$$s_1 = Vt$$

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The time to clear the obstacle is solved from Equation 2-22

$$t_1 = \sqrt{\frac{2H_{OB}}{g(n-1)}} \tag{2-23}$$

and the distance is

$$S_F = V \sqrt{\frac{2H_{OB}}{g(n-1)}} \tag{2-24}$$

If γ_{MAX} for load factor of 1.0 is reached before the obstacle, γ_{MAX} is flown from h_1 to H_{OB}. Since

$$\tan \gamma_{MAX} = \frac{H_{OB} - h_1}{S_F - s_1}$$

$$S_F = s_1 + (H_{OB} - h_1) \cot \gamma_{MAX} \tag{2-25}$$

where

$$t_1 = \frac{\gamma_{MAX} V}{g(n-1)}$$

$$h_1 = 1/2\ g\ (n-1)\ t_1^2$$

$$s_1 = Vt_1$$

and n is the load factor during the flare to γ_{MAX}.

The total field length can then be derived by summing the distances calculated in Equations 2-24 or 2-25 with those in 2-12 or 2-14.

Applications and Sensitivity Analysis

For preliminary design analysis it is convenient to have generalized takeoff performance in the form of a chart or graph. This offers the engineer an aid for visualizing sensitivity to parameters. Takeoff field lengths can then be adjusted as the configuration is altered. This section will demonstrate by example graphs which provide the engineer with the capability to calculate takeoff field length. The graphs include multi-engine aircraft with critical field lengths based on an engine failure.

In Equation 2-15 we see that the average acceleration is a function of T/W, S/W, q, C_D, C_L, and μ. We can let T/W, S/W, and q be variables and we can then make assumptions about μ, C_L, and C_D which are representative of the aircraft configuration under analysis. For example, a typical configuration might have the following characteristics which would apply to many variations of that configuration: (1) The ground run rolling μ is widely accepted to be 0.025, which corresponds to rubber tires on asphalt or concrete. For other applications the rolling μ for hard turf, short grass, long grass, and soft ground are 0.04, 0.05, 0.10, and 0.10 to 0.30, respectively. (2) The value for C_L ground run is usually very small and can be assumed to be zero. (3) The ground run drag coefficient is 300 counts (C_{D_G} = 0.0300), which is typical for large aircraft. For data other than assumed here, reference should be made to Figures 2-5 and 2-6 to determine field length variations.

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With the above assumptions and Equation 2-14 we can construct a chart (Figure 2-9) of ground roll distance as a function of T/W, S/W, and V_{LO}. This chart is very accurate when used for preliminary design. The parameter sensitivity represented in the chart can be applied to many configuration variations. The lift-off speed (V_{LO}) is determined from Equation 2-8 corrected to knots and the parameters T/W and S/W are characteristics of the particular configuration. This information used along with the chart will then provide all of the engine takeoff ground run distance.

In the case of large multi-engine aircraft, military and civilian regulations require that critical field lengths be calculated to include an engine failure. The engine failure point or critical engine speed is such that the distance to abort and stop the aircraft is equal to the distance to continue the takeoff with an engine out. An engine failure prior to the critical engine speed would result in an aborted takeoff although an engine failure after the critical engine speed (V_1) has been reached would result in a committed takeoff. Determination of the V_1 speed is done analytically and is an iterative process. A V_1 speed is selected and the aircraft accelerated to that speed. At this point an engine is cut and the takeoff run is continued to the obstacle and the field length calculated. A second calculation is made from the V_1 speed (engine failure point) to determine the distance required to make a maximum-effort stop. The two distances are then compared and the V_1 speed is adjusted until the distances are equal. The resulting distance is then the critical field length. An engine failure before this V_1 speed will result in a shorter

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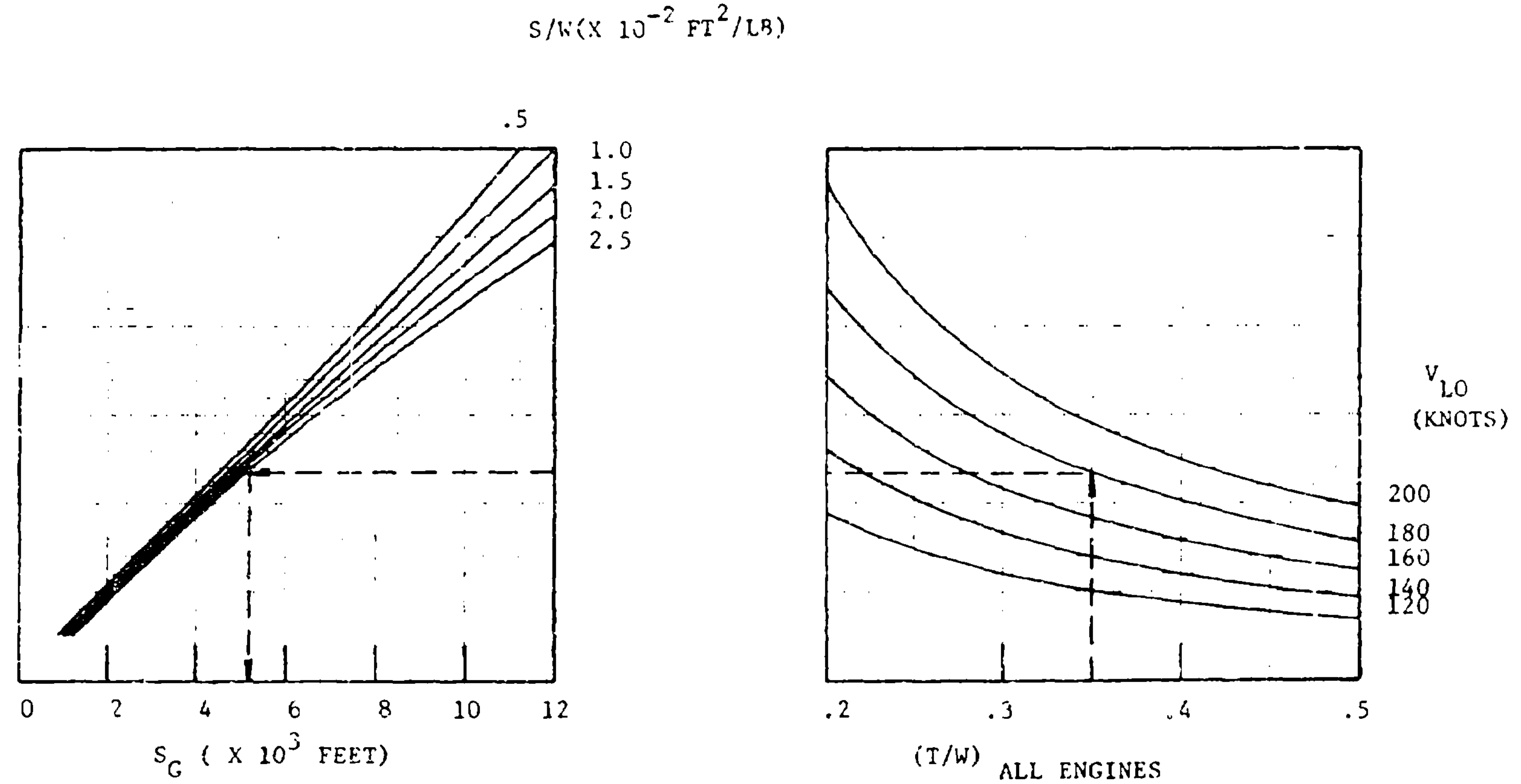

Figure 2-9 Ground Run Distance For All Engine Takeoff

stopping distance and a failure at a higher speed will result in a shorter "engine out go" distance. Figure 2-10 shows the relationship of the speeds to distances for the takeoff run. The "maximum-effort stop" distance is a function of many variables. These include engine spin down time, pilot reaction time, effective braking coefficient, and energy absorbing capability of the brakes. For preliminary design it is usually assumed that the braking system will have the capability to perform a maximum effort stop at maximum design gross weight. The other variables are a function of the particular ground rules which apply to the type of aircraft being used and are later verified in flight test.

Assuming the critical engine speed occurs at 75% lift-off speed, we can construct, for example, a chart identical to Figure 2-9 that reflects an engine failure at the critical engine speed. Figures 2-11 and 2-12 then give ground roll distance for a 25 and 50% loss of power, respectively. To calculate the engine-out ground-roll distance for a four-engine airplane Figure 2-11 would be used, and Figure 2-12 for a two-engine airplane. The engine-out ground-roll distance for a three-engine airplane would mean a 33-1/3% power loss for an engine failure. For this case it will be necessary to interpolate between Figures 2-11 and 2-12.

The determination of the transition distance for preliminary design work can be made using Equation 2-24. A chart can be constructed of transition distance vs. speed for various load factors. For most aircraft a constant speed and load factor can be assumed

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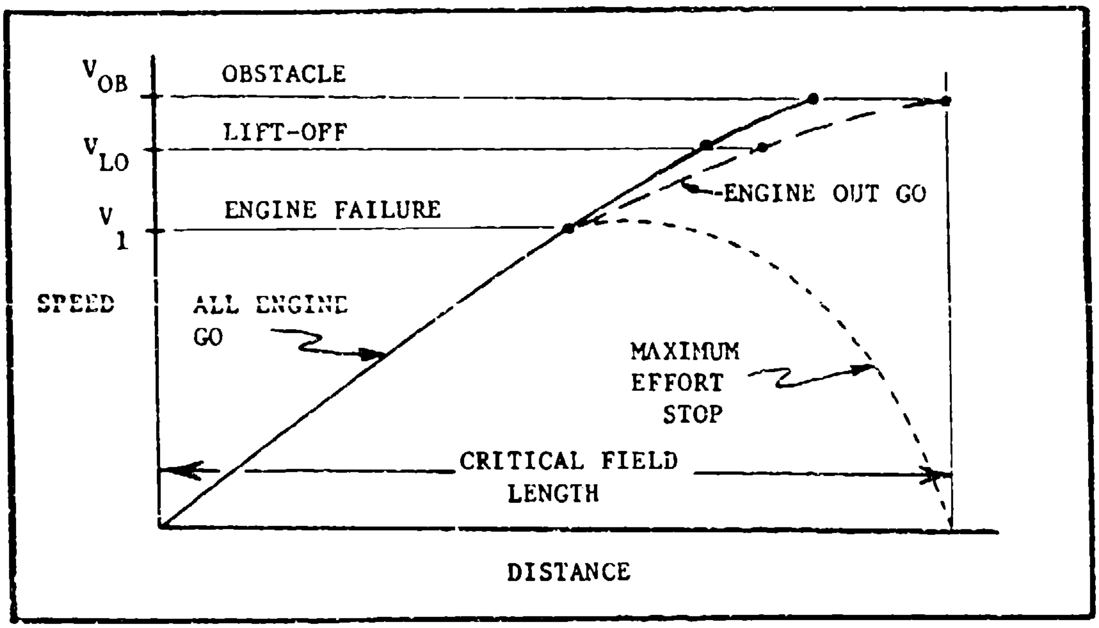

Figure 2-10 Critical Field Length

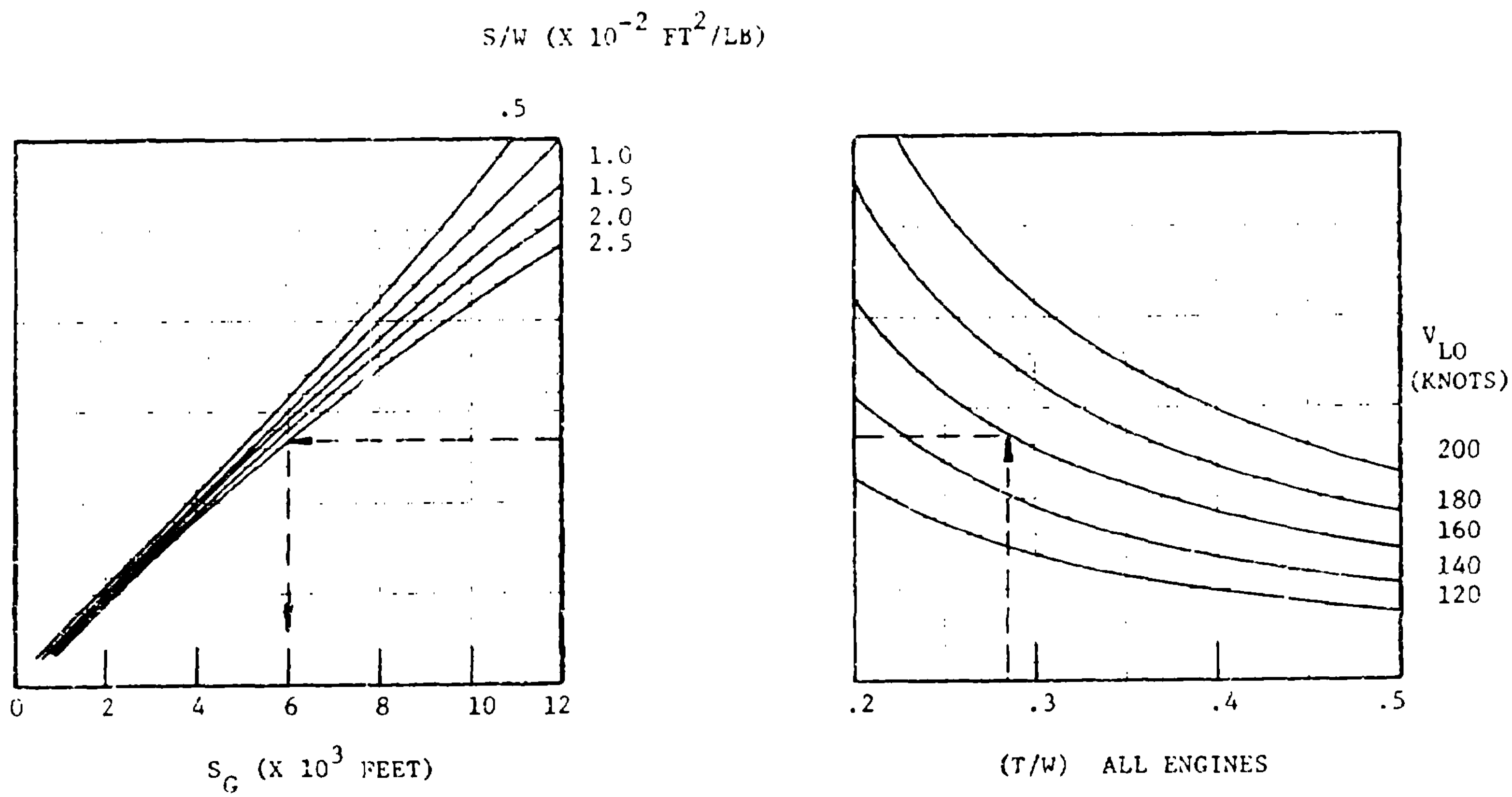

Figure 2-11 Ground Run Distance for 25% Power Loss At Critical Engine Speed

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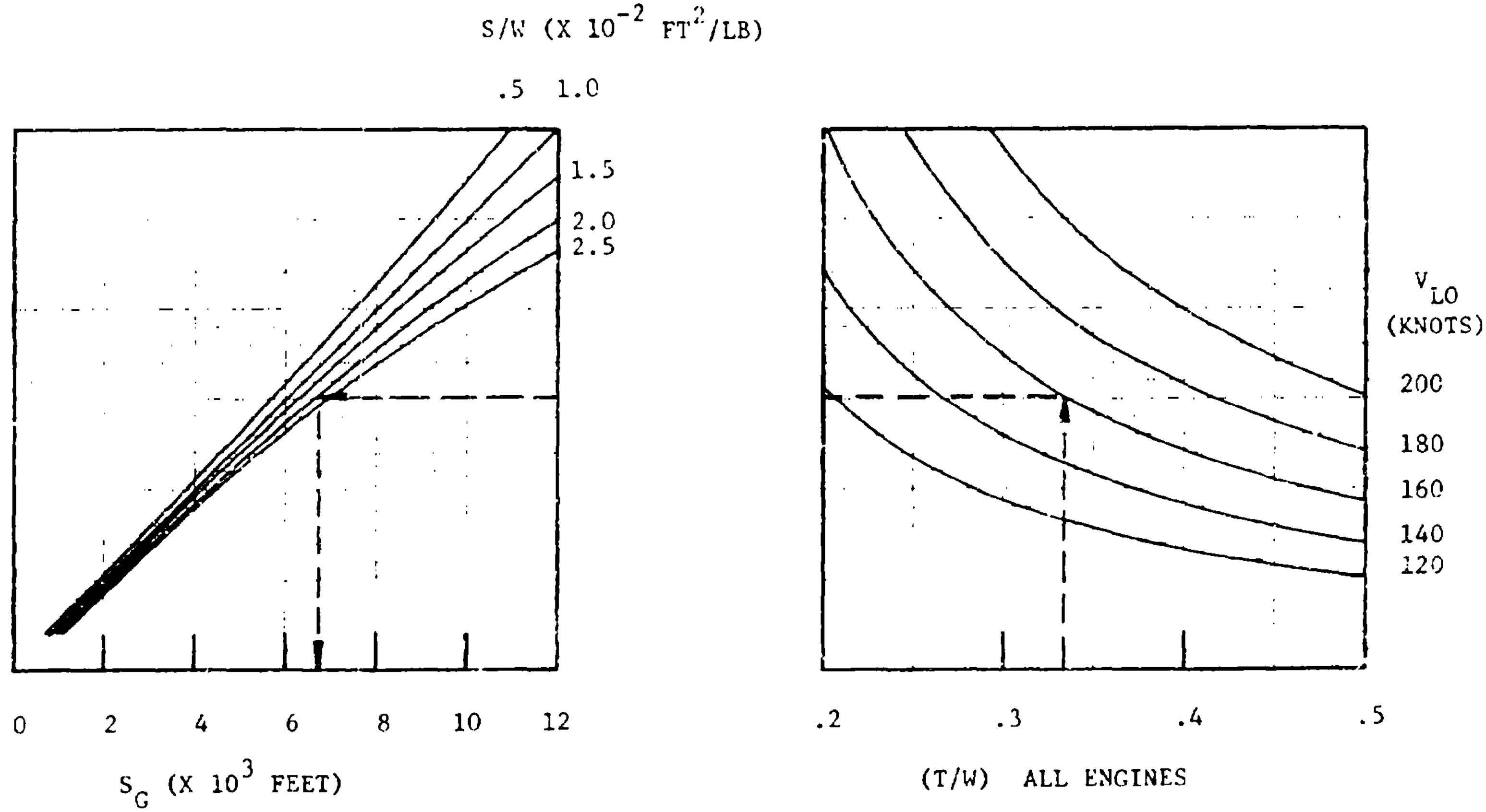

Figure 2-12 Ground Run Distance For 50% Power Loss At Critical Engine Speed

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throughout the transition to the obstacle height. Figure 2-13 gives the transition distance to clear a 50 foot obstacle which is the obstacle height for field length calculations for military aircraft. The flight path angle at the obstacle in Figure 2-13 is calculated from Equation 2-20. This flight path angle should be checked against Figure 2-14, derived from Equation 2-21, which presents the maximum flight path angle attainable for the configuration being analyzed. If the flight path angle at the obstacle is equal to or less than the maximum flight path angle in Figure 2-14, then the transition distance in Figure 2-13 applies. Otherwise, Equation 2-25 will be used to calculate the transition distance.

The speed used in the transition should reflect the applicable ground rules. For example, MIL-C-5011A requires that speed used to calculate flare distance be at least 120% of the power-off stall speed. This speed can be derived from the relationship

$$V = \left(\frac{2\,W}{1.2\,C_{L_{STALL}}\,\rho S} \right)^{1/2}$$

For other ground rules it may be applicable to use an average of lift-off and obstacle speed or to assume lift-off speed throughout the transition.

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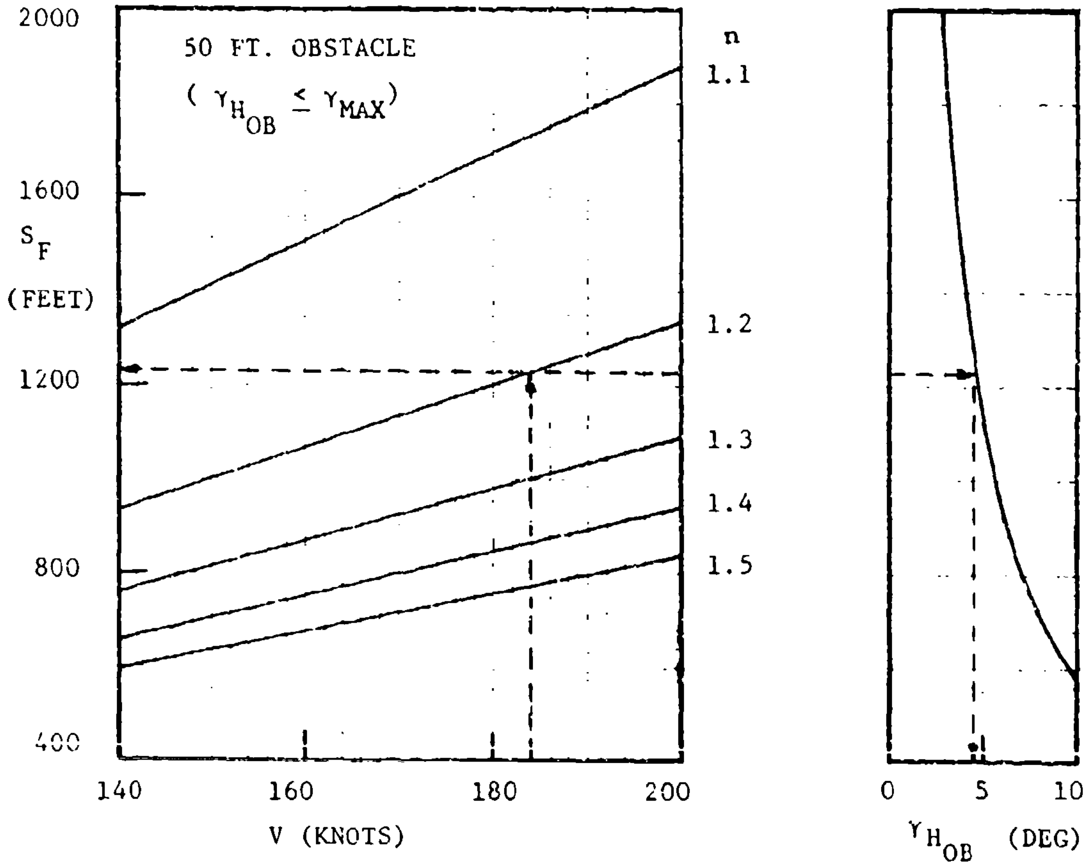

Figure 2-13 Take-Off Transition Distance

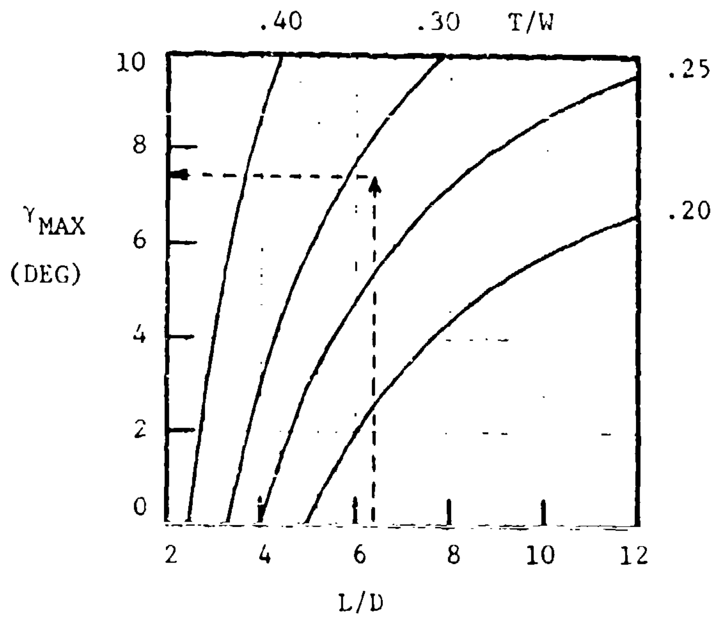

Figure 2-14 Maximum Flight Path Angle (n=1.0)

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Once the field length requirement for a given configuration has been established the variation of the ground roll to small parameter changes is of interest. This is important for first-order effects in preliminary design work. The analyst then has a good insight into the sensitivity of various parameters. By considering Equation 2-14 for this part of the analysis, the effect of the parameters T/W, W/SC_{L_G}, $W/S\ (C_{D_G} - \mu C_{L_G})$, and altitude (h) can be determined. Logrithmic differentiation of Equation 2-14 yields

$$\frac{dS_G}{S_G} = \frac{dV_{LO}^2}{V_{LO}^2} - \frac{d\bar{a}}{\bar{a}} \tag{2-26}$$

where dS_G, dV_{LO}^2, and $d\bar{a}$ are changes in S_G, V_{LO}^2 and $\bar{a}$, respectively. From the definition of V_{LO}, Equation 2-8

$$\frac{dV_{LO}^2}{V_{LO}^2} = \frac{dW/SC_{L_G}}{W/SC_{L_G}} - \frac{d\rho}{\rho} \tag{2-27}$$

For an exponential atmosphere, the density ρ is related to altitude in the following approximate way,

$$\rho = \rho_o\, e^{-\beta h}$$

where ρ_o is the sea-level density and β is approximately 1/30,500. Substitution of the expression for ρ into Equation 2-27 gives

$$\frac{dV_{LO}^2}{V_{LO}^2} = \frac{dW/SC_{L_G}}{W/SC_{L_G}} + \beta dh \tag{2-28}$$

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Consequently, by increasing W/SC_{L_G} or altitude the lift-off speed would be increased. Differentiation of Equation 2-15 and the knowledge that $q = \frac{1}{2}\rho V^2$ gives

$$d\bar{a} = g\left[\frac{T}{W}\frac{d(T/W)}{T/W} + \frac{g}{4}\frac{V_{LO}^{2}}{W/S(C_{D_G}-\mu C_{L_G})}\,dh\right.$$

$$\left. - \frac{\rho}{4}\frac{V_{LO}^{2}}{W/S(C_{D_G}-\mu C_{L_G})}\frac{dV_{LO}^{2}}{V_{LO}^{2}} + \frac{\rho}{4}\frac{V_{LO}^{2}}{W/S(C_{D_G}-\mu)}\frac{d\dfrac{W}{S(C_{D_G}-\mu C_{L_G})}}{\dfrac{W}{S(C_{D_G}-\mu C_{L_G})}}\right] \tag{2-29}$$

Substitution of Equations 2-28 and 2-29 into Equation 2-26 gives

$$\frac{dS_G}{S_G} = S_{T/W}\frac{d\ T/W}{T/W} + S_{W/SC_{L_G}}\frac{dW/SC_{L_G}}{W/SC_{L_G}}$$

$$+ S_{\frac{W}{S(C_{D_G}-\mu C_{L_G})}}\frac{d\dfrac{W}{S(C_{D_G}-\mu C_{L_G})}}{\dfrac{W}{S(C_{D_G}-\mu C_{L_G})}} + S_h dh \tag{2-30}$$

where $S_{T/W}$, etc. are sensitivity parameters defined in Table 2-1.

In the form presented in Equation 2-30, any change in W/S, C_{D_G}, μC_{L_G} or C_{L_G} is accounted for. If the external configuration is fixed, the aerodynamics should not change. In this event the appropriate form is

$$\frac{dS_G}{S_G} = S_{T/W}\frac{d\ T/W}{T/W} + S_{W/S}\frac{d\ W/S}{W/S} + S_h dh \tag{2-31}$$

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TABLE 2-1

GROUND RUN SENSITIVITY PARAMETERS

Parameter	Sensitivity Parameter
T/W	$-\frac{g}{\bar{a}}\frac{T}{W}$
W/SC_{L_G}	$1+\frac{1}{4}\frac{\rho g}{\bar{a}}\frac{V_{LO}^2}{W/S(C_{D_G}-\mu C_{L_G})}$
$W/S(C_{D_G}-\mu C_{L_G})$	$-\frac{1}{4}\frac{\rho g}{\bar{a}}\frac{V_{LO}^2}{W/S(C_{D_G}-\mu C_{L_G})}$
h	$\beta\left[1+\frac{1}{4}\frac{\rho g}{\bar{a}}\frac{V_{LO}^2}{W/S\,(C_{D_G}-\mu C_{L_G})}\right]$

Substitution of the average speed into Equation 2-15 gives

$$\bar{a} = g\left(\frac{T}{W}-\mu-\frac{(C_{D_G}-\mu C_{L_G})}{2\,C_{L_{LO}}}\right)$$

where $\bar{V} = \sqrt{\frac{V_{LO}^2}{2}}$

Consequently, Equation 2-31 can be written

$$\frac{d\,S_G}{S_G} = -\frac{g}{\bar{a}}\frac{T}{W}\cdot\frac{d\,T/W}{T/W}+\beta dh+\frac{d\,W/S}{W/S} \qquad (2\text{-}32)$$

An increase in T/W results in a decrease in S_G. An increase in h or W/S increases S_G.

Summary of the Takeoff Performance Problem

Methods were derived to calculate the takeoff field length including the transition to clear the obstacle. These methods include the capability of using variable accelerations or assuming constant accelerations. Application to representative configurations shows the ground roll distance to be very sensitive to lift-off speed and thrust to weight ratio and less sensitive to wing loading. Transition analysis shows the distance to be sensitive to lift-to-drag ratio, thrust-to-weight ratio, and independent of speed for a given load factor. Sensitivity parameters are provided for the ground roll to account for configuration changes.

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SECTION III
CLIMB PERFORMANCE

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SECTION III

CLIMB PERFORMANCE

Problem Definition and Assumptions

For static climb performance the problems of interest are the maximum flight path angle, maximum climb rate, and minimum fuel consumption for a given altitude increase. From an integral performance standpoint, items of interest are those associated with minimum fuel to climb and minimum time to climb. These integral performance problems will be discussed in this section.

It will be assumed that the trajectory remains in the same vertical plane; that is, there is no turning. The equations of motion and kinematic equations are

$$\dot{V} = g\left(\frac{T}{W} - \frac{D}{W} - \sin\gamma\right) \tag{3-1}$$

$$\dot{\gamma} = \frac{g}{V}\left(\frac{L}{W} + \frac{T}{W}\sin\alpha - \cos\gamma\right) \tag{3-2}$$

$$\dot{X} = V\cos\gamma \tag{3-3}$$

$$\dot{h} = V\sin\gamma \tag{3-4}$$

$$\dot{W} = -SFC \cdot T \tag{3-5}$$

where V is the speed, γ is the flight path angle, T is the engine thrust, D is the drag, n is the load factor, SFC is the specific fuel consumption, α is the angle-of-attack, and g is the acceleration of gravity. The load factor is related to the lift L and T through

$$n = \frac{L}{W} + \frac{T}{W}\sin\alpha \tag{3-6}$$

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For high angles of attack and high thrust to weight ratios the second term on the right can be significant. The drag is dependent upon the speed, altitude, weight, and the load factor, or aerodynamic angle of attack.

It will be convenient to treat the altitude as the independent variable. Consequently, time becomes a dependent variable. The first step is the elimination of sin γ. From Equations 3-1 and 3-4

$$\frac{dV}{dh} = \frac{g}{V \sin \gamma} \left(\frac{T}{W} - \frac{D}{W} - \sin \gamma \right)$$

Solving for sin γ gives

$$\sin \gamma = \frac{\frac{T}{W} - \frac{D}{W}}{1 + \frac{V}{g}\frac{dV}{dh}}$$

The transformations of Equations 3-3, 3-5, and 3-4 become

$$\frac{dx}{dh} = \frac{\sqrt{1 - \left(\frac{\frac{T}{W} - \frac{D}{W}}{1 + \frac{V}{g}\frac{dV}{dh}} \right)^2}}{\frac{T}{W} - \frac{D}{W}} \left(1 + \frac{V}{g}\frac{dV}{dh}\right)$$

$$\frac{dW}{dh} = - \frac{SFC \cdot T}{V \left(\frac{T}{W} - \frac{D}{W}\right)} \left(1 + \frac{V}{g}\frac{dV}{dh}\right)$$

$$\frac{dt}{dh} = \frac{1 + \frac{V}{g}\frac{dV}{dh}}{V \left(\frac{T}{W} - \frac{D}{W}\right)}$$

Given the speed as a function of the altitude, the thrust, and the aerodynamic drag, it is straightforward as far as determining the numerical solutions for the distance, fuel, and time in climb. Unfortunately, the problems of interest concern the derivation of the speed schedule for optimizing distance, fuel, or time in climb. Thus, the speed schedule is unknown a priori. There are two ways for circumventing this difficulty. The first is to formulate the problem as a calculus of variations problem and determine the optimal climb schedule. The alternate approach is to make assumptions relative to the climb schedule and go from there. The latter will be the approach taken here.

The first assumption is that the change in speed with altitude is such that acceleration correction satisfies

$$\frac{V}{g}\frac{dV}{dh} << 1$$

If the climb speed is constant, then this relation is certainly satisfied. On the other hand, if the speed goes from 800 feet per second at sea level to 1000 feet per second at 30,000 feet, the acceleration correction is approximately 0.2. Consequently, in some problems the correction may be significant.

If the acceleration correction is negligible, then it follows that

$$\sin \gamma = \frac{T-D}{W} \tag{3-7}$$

$$n = \cos \gamma \tag{3-8}$$

Recall that the drag is dependent upon the speed, altitude, weight, and the load factor or angle-of-attack. In general, an iterative

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approach is required for evaluating the performance since the drag is related to the load factor which is dependent upon the flight path angle through Equation 3-8 and the flight path angle is related to the drag through Equation 3-7.

The iterative approach is as follows. For a given throttle setting, the thrust is known in terms of the speed and altitude. First assume that the flight path angle is approximately zero. Then from Equation 3-8 the load factor is approximately one. For a given altitude and speed the drag is computed and the flight path angle recomputed from Equation 3-7. A new load factor is recomputed and the process repeated until the change in γ is negligible. When this occurs the flight path angle is computed from Equation 3-7 and the rate of climb, R/C, is determined from Equation 3-4

$$R/C = \dot{h} = V \sin \gamma = \frac{V\ (T-D)}{W} \tag{3-9}$$

The fuel consumption per unit altitude change, dW/dh, is determined from Equations 3-4, 3-5, and 3-7

$$\frac{dW}{dh} = - \frac{SFC \cdot T}{V \sin \gamma} = - \frac{SFC \cdot T \cdot W}{V(T-D)} \tag{3-10}$$

From Equations 3-7, 3-9, and 3-10 the maximum flight path angle, maximum rate of climb, and minimum fuel consumption per unit altitude increase can be determined.

The flight path angle for a given throttle setting and weight is determined from data like that in Figure 3-1, where the excess thrust per unit weight $\frac{T-D}{W}$ is plotted versus speed and altitude.

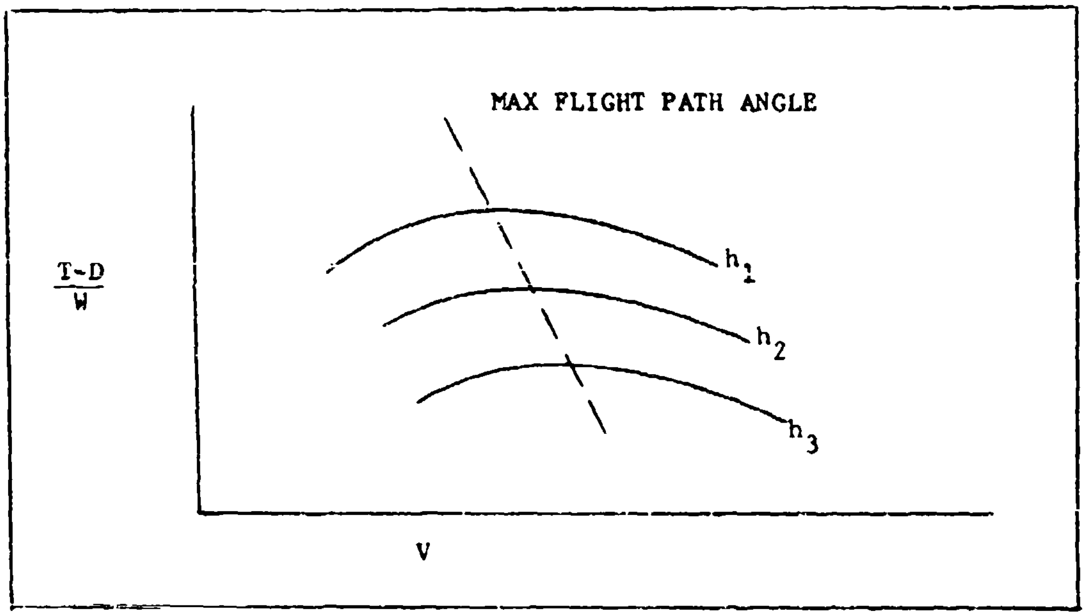

Figure 3-1 Excess Thrust and Maximum Flight Path Angle

The maximum flight path angle at a given altitude is determined from the maximum value of $\frac{T-D}{W}$. Consequently, the speed is identified as a function of altitude for the steepest climb path.

Maximum rate of climb is determined from excess power $V(T-D)/W$. Rate of climb at a given altitude is presented in Figure 3-2. In addition to determining maximum rate of climb at a specified altitude, maximum flight path angle can also be determined from Figure 3-2. A line through the origin has slope

$$\frac{R/C}{V}$$

But this is sin γ; therefore the tangent through the origin to a rate of climb curve at a given altitude determines the maximum flight path angle. Also, Figure 3-2 provides both the fastest climb and the maximum flight path angle.

Fuel consumption per unit altitude increase is presented in Figure 3-3. The locus of points through the minimum values of $W_f/(R/C)$ at each altitude defines the climb schedule for minimum fuel.

Approximate Climb Solutions

Under suitable assumptions, approximate analytic solutions can be derived for the time, fuel, and distance covered in climb. The time, t_c, and fuel to climb, W_c, between altitudes h_o and h_f are obtained from the following integrals

$$t_c = \int_{h_o}^{h_f} \frac{dh}{\dot{h}} \tag{3-11}$$

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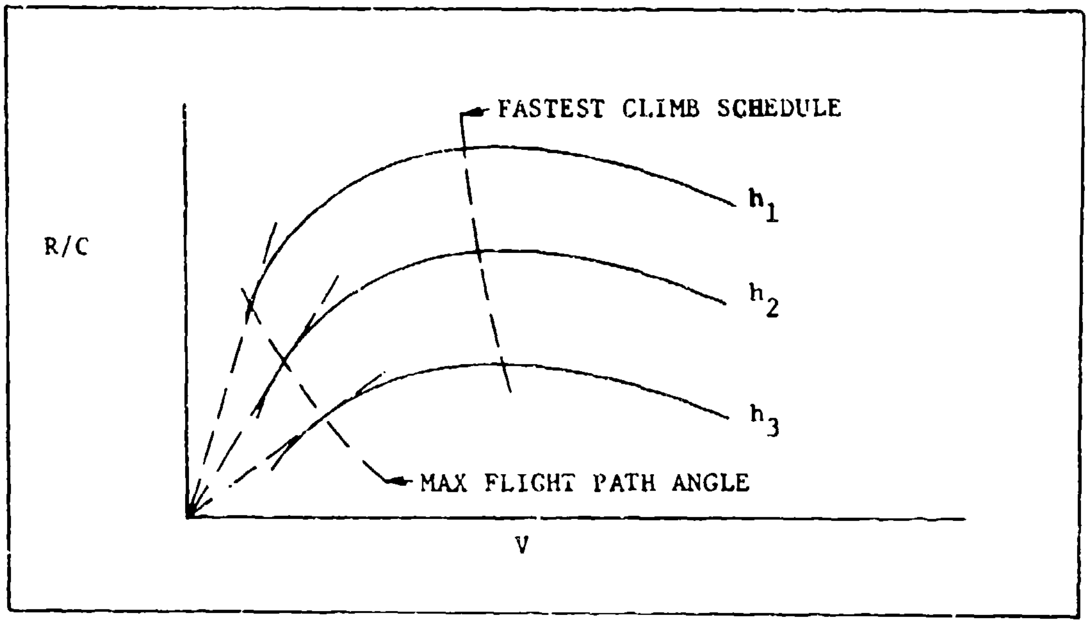

Figure 3-2 Rate of Climb

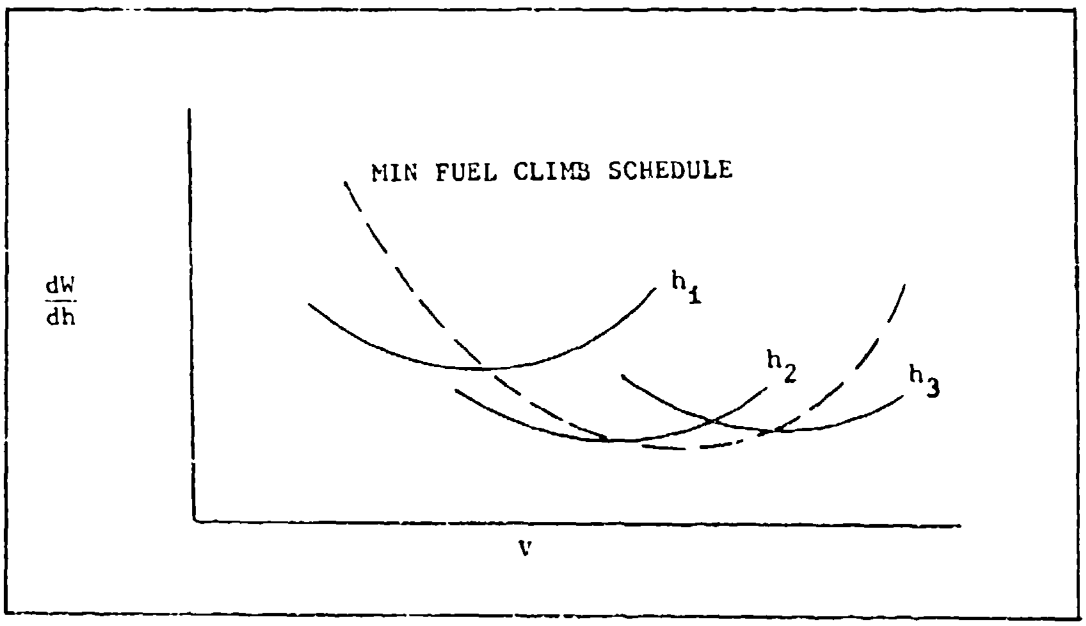

Figure 3-3 Fuel Consumption Per Unit Altitude Change

$$W_c = -\int_{h_o}^{h_f} \frac{\dot{W}}{\dot{h}}\, dh \tag{3-12}$$

where $\dot{h}$ and $\dot{W}$ are defined by Equations 3-4 and 3-5. In light of Equations 3-1 and 3-2 the solution for minimum time and minimum fuel is a calculus of variations problem. If, however, assumptions about $\dot{V}$ and $\dot{\gamma}$ are introduced, then the problems reduce to ordinary calculus problems. The first assumption is that γ is small, hence Equation 3-2 gives

$$n = 1$$

If the speed change is negligible, then $\dot{V}$ is approximately zero and Equation 3-1 reduces to

$$\sin\gamma = \frac{T-D}{W} \tag{3-13}$$

Thus Equation 3-4 becomes

$$\dot{h} = \frac{V}{W}(T-D) \tag{3-14}$$

Equations 3-11 and 3-12 become

$$t_c = \int_{h_o}^{h_f} \frac{W}{V(T-D)}\, dh \tag{3-15}$$

$$W_c = \int_{h_o}^{h_f} \frac{W \cdot SFC \cdot T}{V(T-D)}\, dh \tag{3-16}$$

If no other constraints hold, then minimization of t_c and W_c results if

$$\frac{\partial}{\partial V} \frac{W}{V(T-D)} = 0$$

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$$\frac{\partial}{\partial V} \frac{W \cdot SFC \cdot T}{V(T-D)} = 0$$

Assuming that T and SFC change negligibly with speed and the change in the weight is also negligible, then the necessary condition is

$$\frac{\partial}{\partial V} \left[V(T-D)\right] = 0 \tag{3-17}$$

It should be noted that even though the necessary condition for time and fuel to climb are the same, the two solutions for the optimal speed may be different if different throttle settings are employed. From an optimal throttle control standpoint the settings would be different. For the problems here, the throttle control is assumed to be specified.

Expansion of Equation 3-17 gives

$$T - D + V \frac{\partial}{\partial V} (T-D) = 0$$

For a parabolic drag polar with constant aerodynamic coefficients, this becomes

$$T - \frac{1}{2} \rho S V^2 C_{D_o} - \frac{2KW^2}{\rho S V^2} + V\left(-\rho S V C_{D_o} + \frac{4KW^2}{\rho S V^3}\right) = 0$$

or

$$T - \frac{3}{2} \rho S V^2 C_{D_o} + \frac{2KW^2}{\rho S V^2} = 0$$

The solution for the best climb speed is therefore

$$V = \left\{ \frac{W}{3\rho S C_{D_o}} \left[\frac{T}{W} + \sqrt{\left(\frac{T}{W}\right)^2 + \frac{3}{(L/D)^2_{MAX}}} \right] \right\}^{\frac{1}{2}} \tag{3-18}$$

Substitution of the best climb speed into Equations 3-15 and 3-16, after rearranging, gives

$$t_c = \left(\frac{3\rho_o}{W/SC_{D_o}}\right)^{\frac{1}{2}} \int_{h_o}^{h_f} \frac{R\sigma^{\frac{1}{2}}}{Z}\, dh \qquad (3\text{-}19)$$

$$\frac{W_c}{W} = SFC \left(\frac{3\rho_o}{W/SC_{D_o}}\right)^{\frac{1}{2}} \int_{h_o}^{h_f} \frac{T}{W} \cdot \frac{R\sigma^{\frac{1}{2}}}{Z}\, dh \qquad (3\text{-}20)$$

where

$$R = \left[\frac{T}{W} + \sqrt{\left(\frac{T}{W}\right)^2 + \frac{3}{(L/D)^2_{MAX}}}\ \right]^{\frac{1}{2}} \qquad (3\text{-}21)$$

$$Z = \frac{2}{3}\left(\frac{T}{W}\right)^2 + \frac{2}{3}\left(\frac{T}{W}\right)\sqrt{\left(\frac{T}{W}\right)^2 + \frac{3}{(L/D)^2_{MAX}}} - \frac{2}{(L/D)^2_{MAX}} \qquad (3\text{-}22)$$

and σ is the density ratio. The horizontal range X is obtained from Equations 3-3, 3-4, and 3-14

$$X = \int_{h_o}^{h_f} \frac{dh}{\sin\gamma} = \int_{h_o}^{h_f} \frac{dh}{\frac{T}{W} - \frac{D}{W}}$$

$$= \int_{h_o}^{h_f} \frac{R^2}{Z}\, dh \qquad (3\text{-}23)$$

The solution of Equations 3-19, 3-20, and 3-23 are limited by

$$h_f \leq H_{MAX}$$

where H_{MAX} is the altitude where the thrust and drag are equal.

Substitution of Equation 3-18 into T = D yields

$$\frac{T}{W} \left(\frac{L}{D} \right)_{MAX} = 1 \tag{3-24}$$

It will be assumed that T/W varies according to the density ratio in the following way

$$\frac{T}{W} = \left(\frac{T}{W} \right)_o \sigma^y \tag{3-25}$$

where $(T/W)_o$ is the sea level thrust to weight ratio. The exponent y is an empirical constant which varies between 0.8 and 1.0 in the troposphere and is approximately 1.0 above the tropopause. Hereafter y is taken to be one. Substitution of Equation 3-25 into Equation 3-24 and solving for σ gives

$$\sigma = \frac{1}{\left(\frac{T}{W} \right)_o \left(\frac{L}{D} \right)_{MAX}} \tag{3-26}$$

The solution of Equation 3-26 identifies the maximum altitude, H_{MAX}.

Numerical integration is required to determine the solutions for t_c, W_c/W, and X. Let functions X(1) and X(2) be defined as follows

$$X(1) = (3\rho_o)^{\frac{1}{2}} \, 10^{-5} \int_o^{h_f} \frac{R\sigma}{Z}^{\frac{1}{2}} dh$$

$$X(2) = (3\rho_o)^{\frac{1}{2}} \, 10^{-4} \int_o^{h_f} \frac{T}{W} \cdot \frac{R\sigma}{Z}^{\frac{1}{2}} dh$$

The powers of ten are introduced to normalize X(1) and X(2) to the order of one. Clearly X(1) and X(2) are functions of T/W, $(L/D)_{MAX}$, and h. The solutions for t_c and W_c/W are therefore

$$t_c = \frac{X(1)10^5}{(W/C_{D_o}S)^{\frac{1}{2}}}$$

$$\frac{W_c}{W} = \frac{X(2)10^4}{(W/C_{D_o}S)^{\frac{1}{2}}} \cdot SFC$$

The solutions for X(1) and t_c are presented in Figures 3-4 and 3-5. In Figure 3-5, the time to climb is from sea level. In Figures 3-6 and 3-7 are presented X(2) and W_c/W. Figure 3-8 presents the distance covered in climb.

From Equations 3-18 and 3-21, it follows that

$$V = X(3)\sqrt{\frac{W}{C_{D_o}S}}$$

where

$$X(3) = \frac{R}{\sqrt{3\rho}}$$

X(3) and the best climb speed are presented in Figures 3-9 and 3-10.

As an example consider the F4C with the following characteristics climbing from sea level to 30,000 feet.

W = 50,000 pounds

S = 530 square feet

C_{D_o} = 0.0125

MAX L/D = 11.56

T_o = 38,000 pounds (maximum power)

SFC = 2.45 pounds/hour/pound

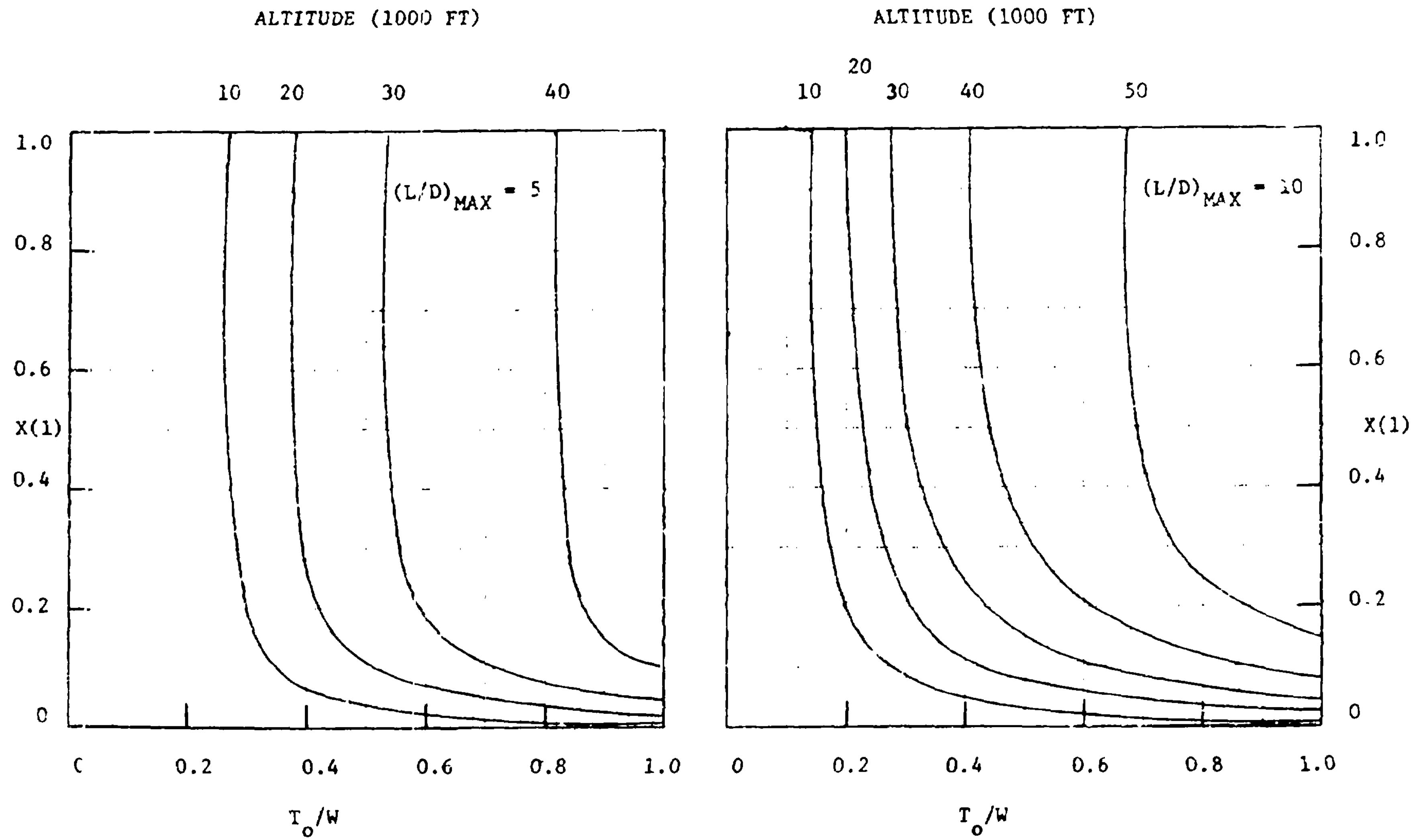

Figure 3-4 Time to Climb Variable

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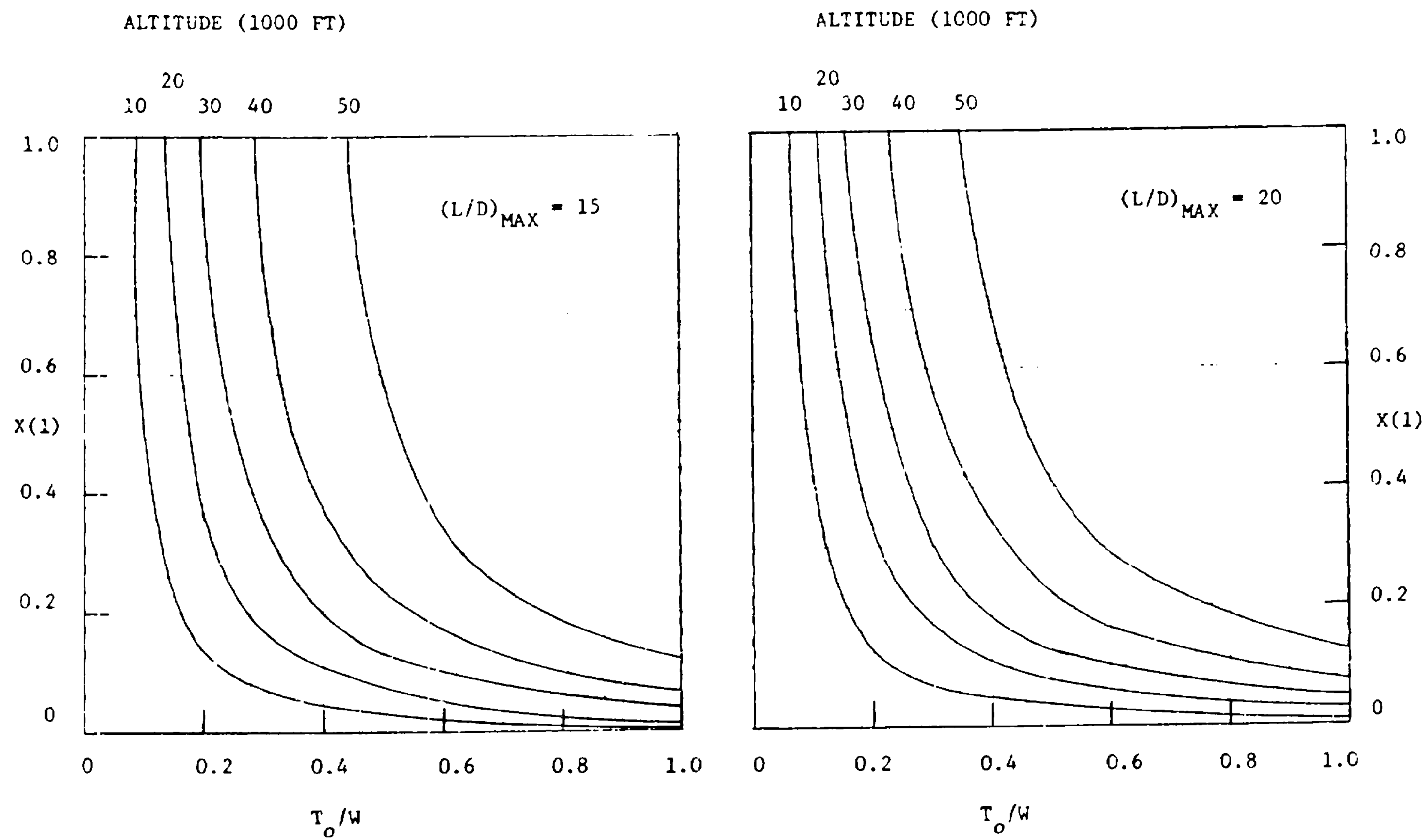

Figure 3-4 Time to Climb Variable

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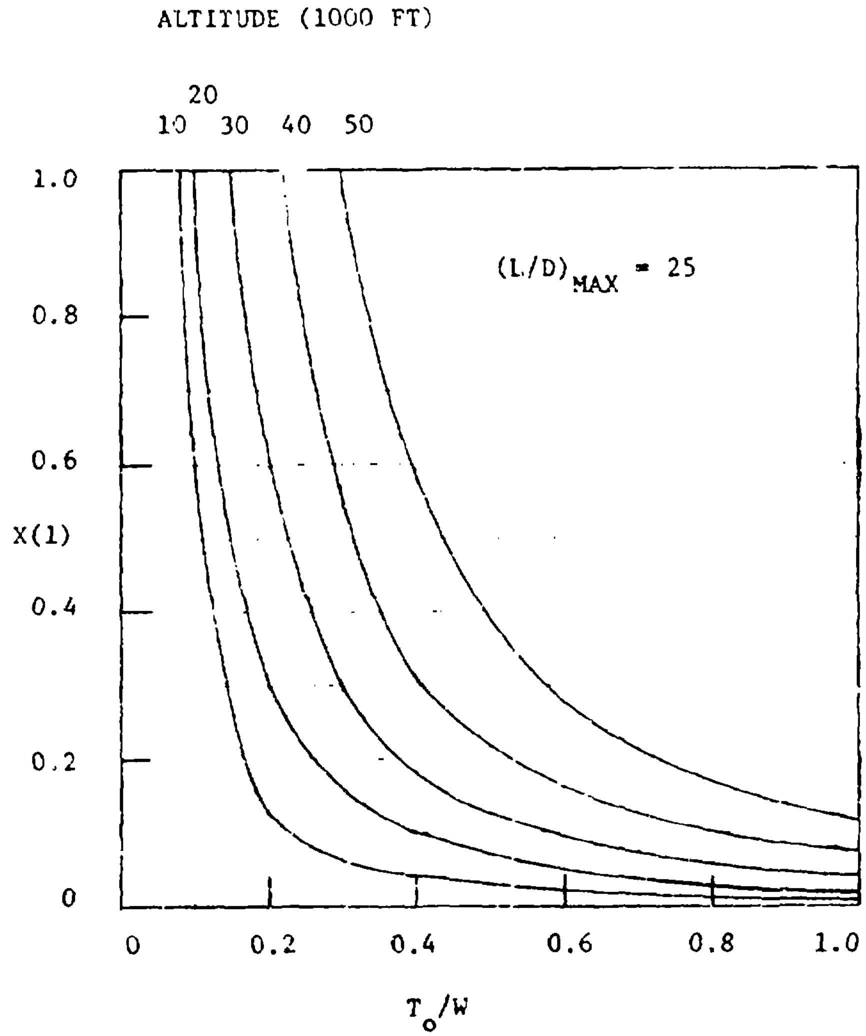

Figure 3-4 Time to Climb Variable

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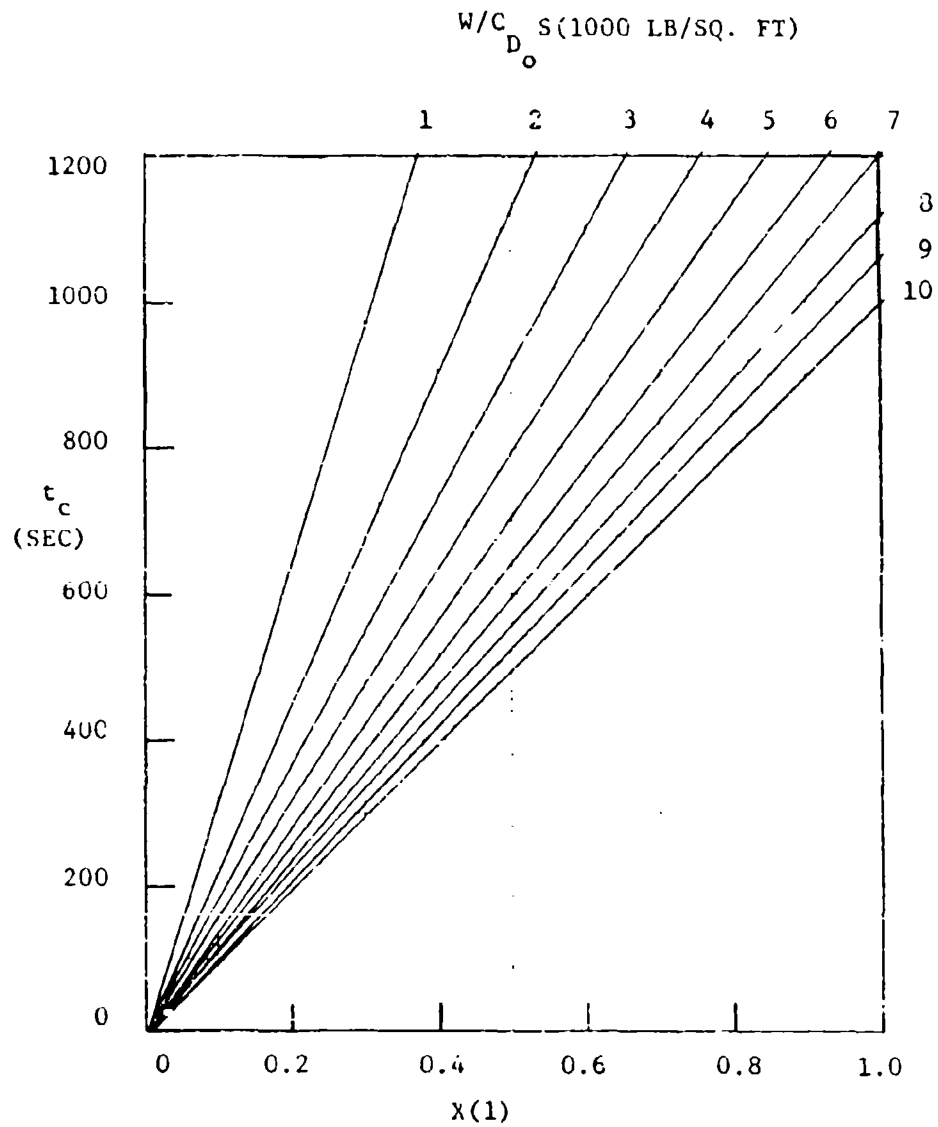

Figure 3-5 Time to Climb from Sea Level

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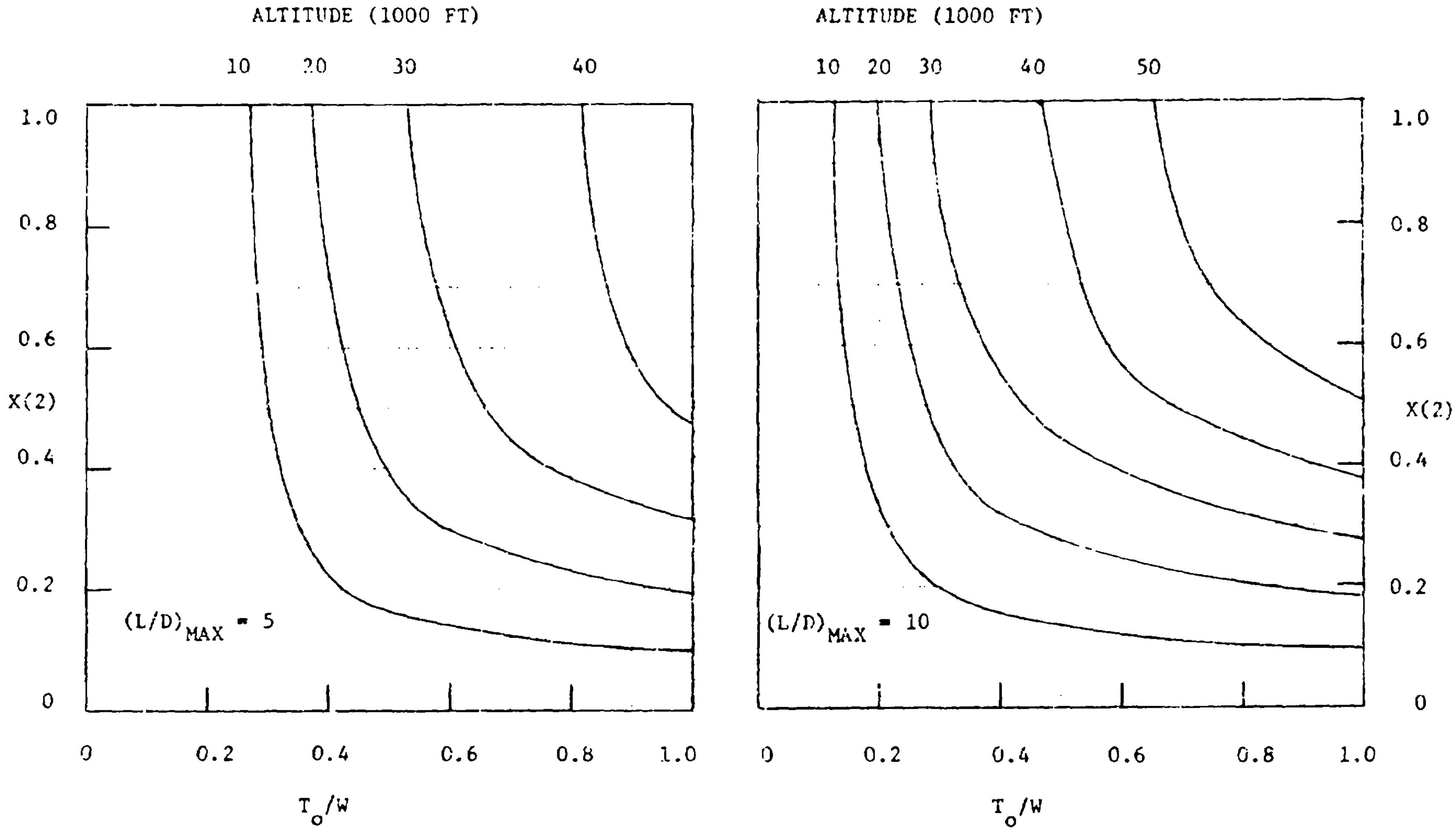

Figure 3-6 Fuel to Climb Variable

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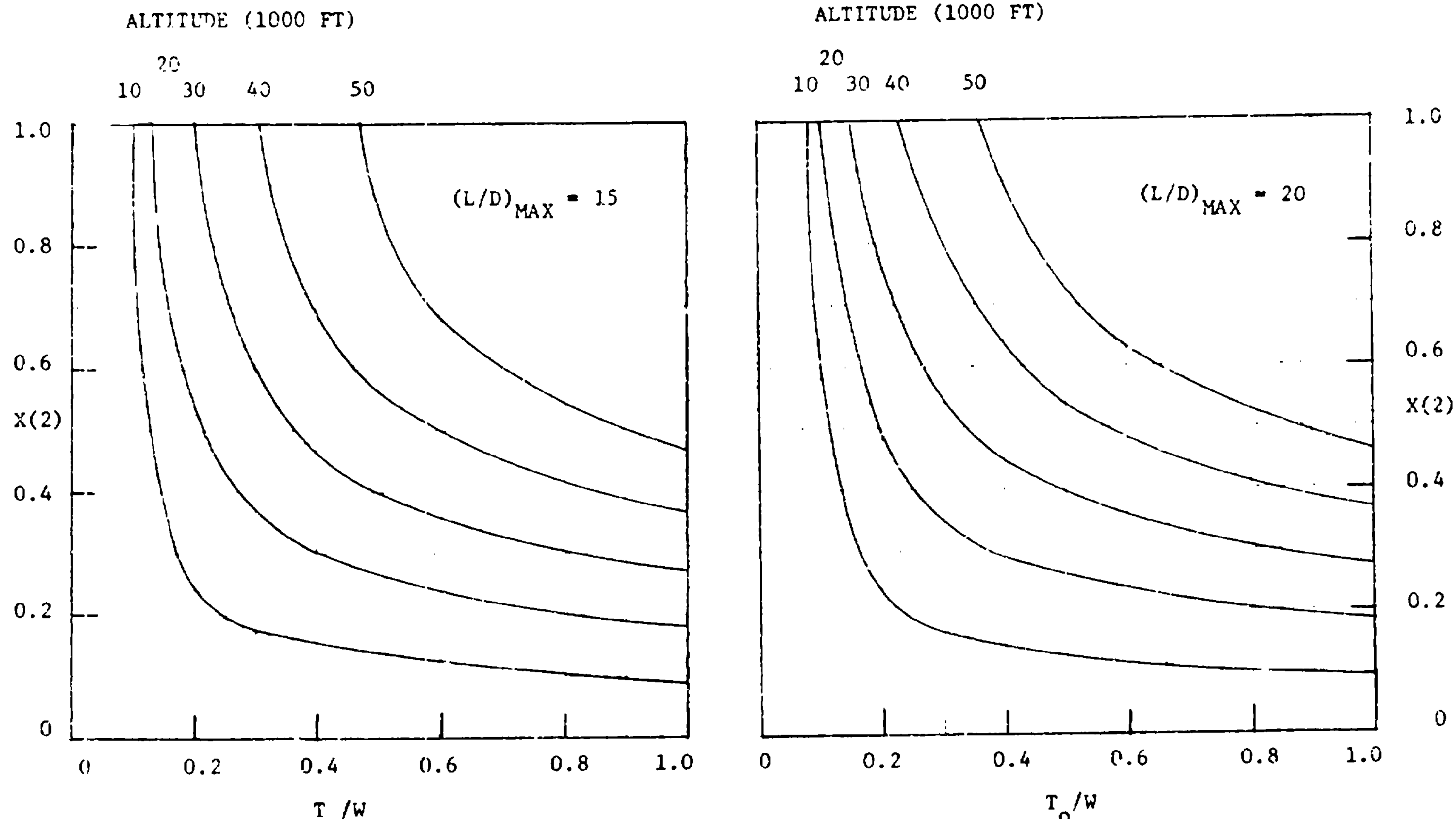

Figure 3-6 Fuel to Climb Variable

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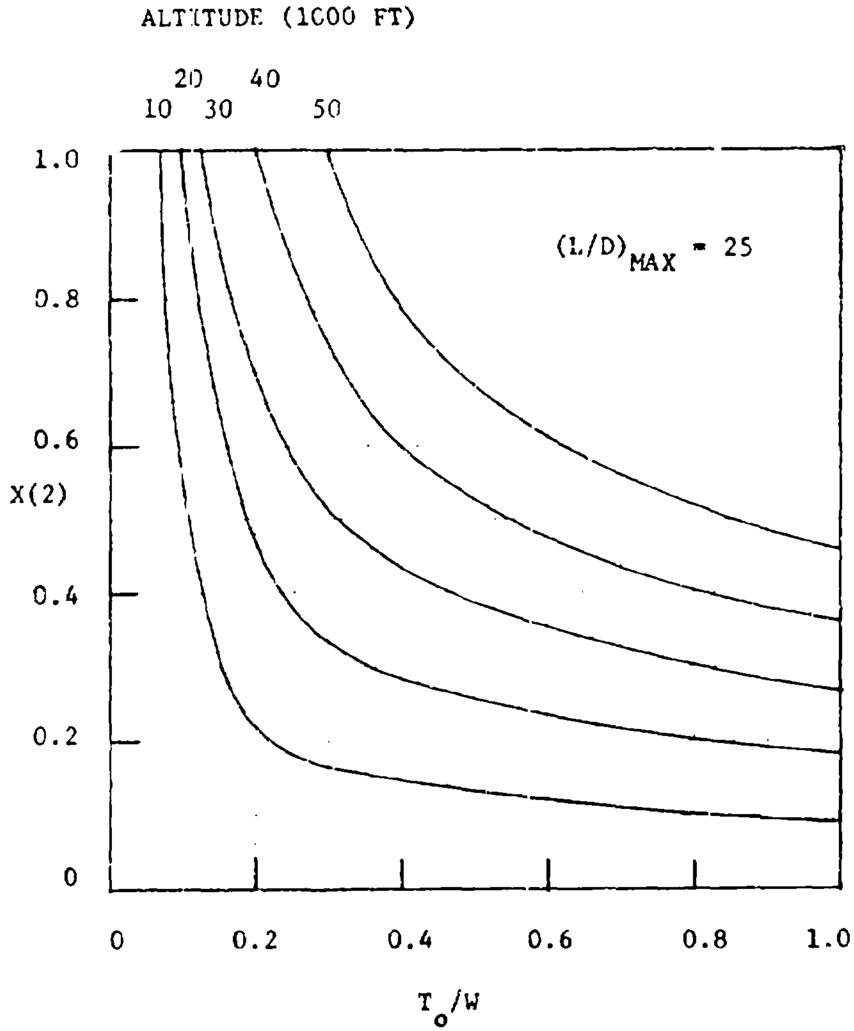

Figure 3-6 Fuel to Climb Variable

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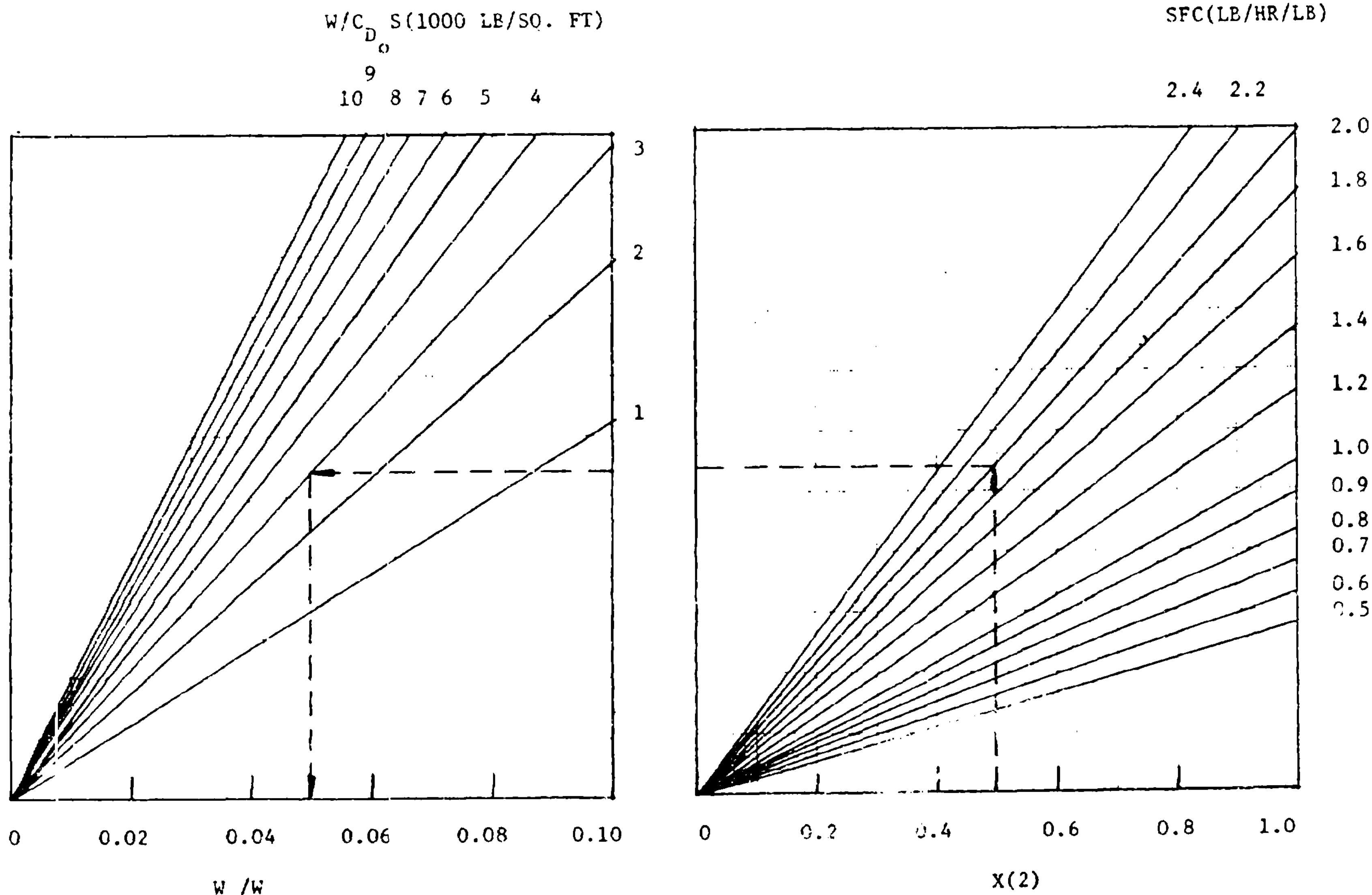

Figure 3-7 Fuel Fraction to Climb from Sea Level

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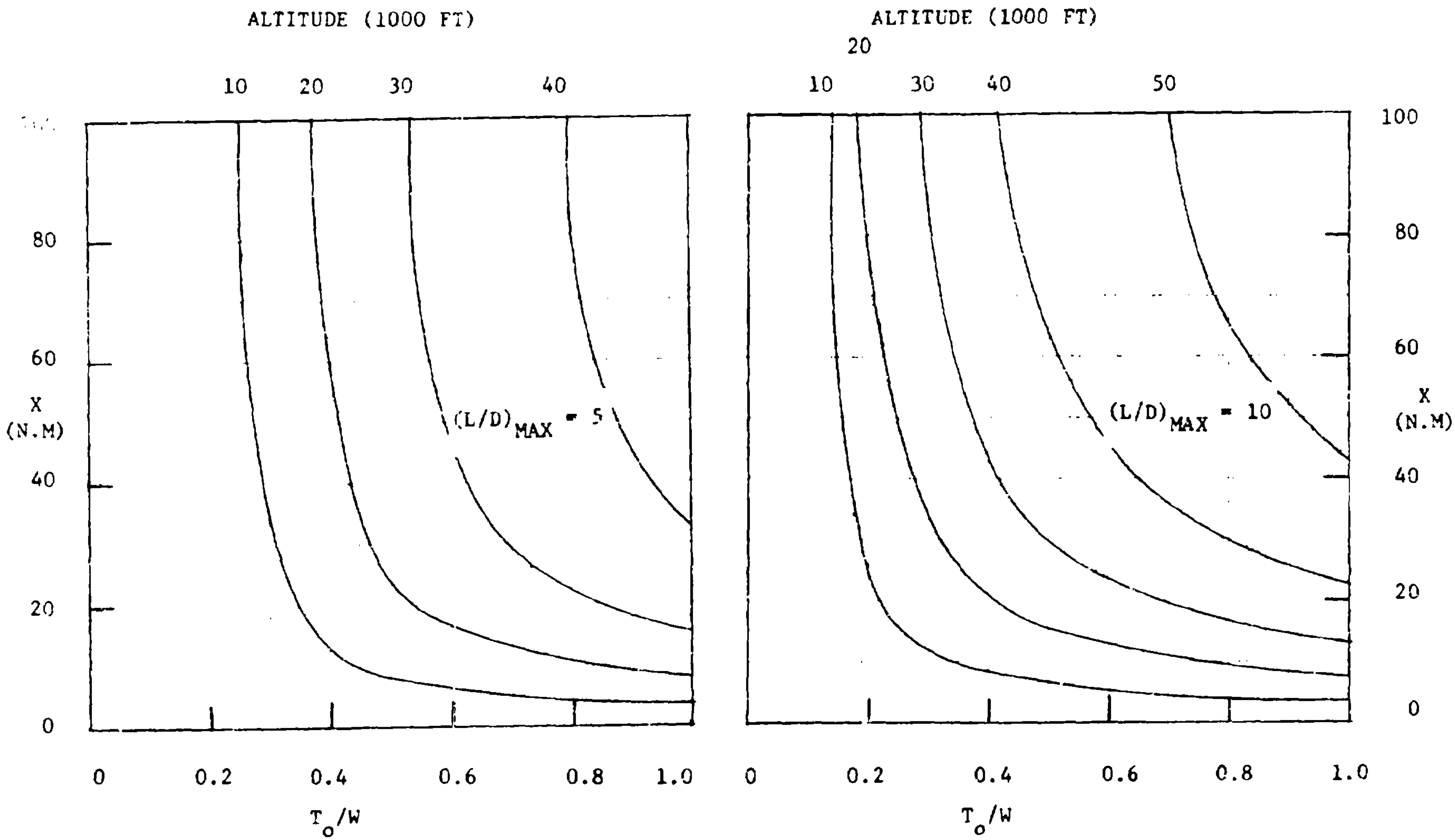

Figure 3-8 Distance in Climb from Sea Level

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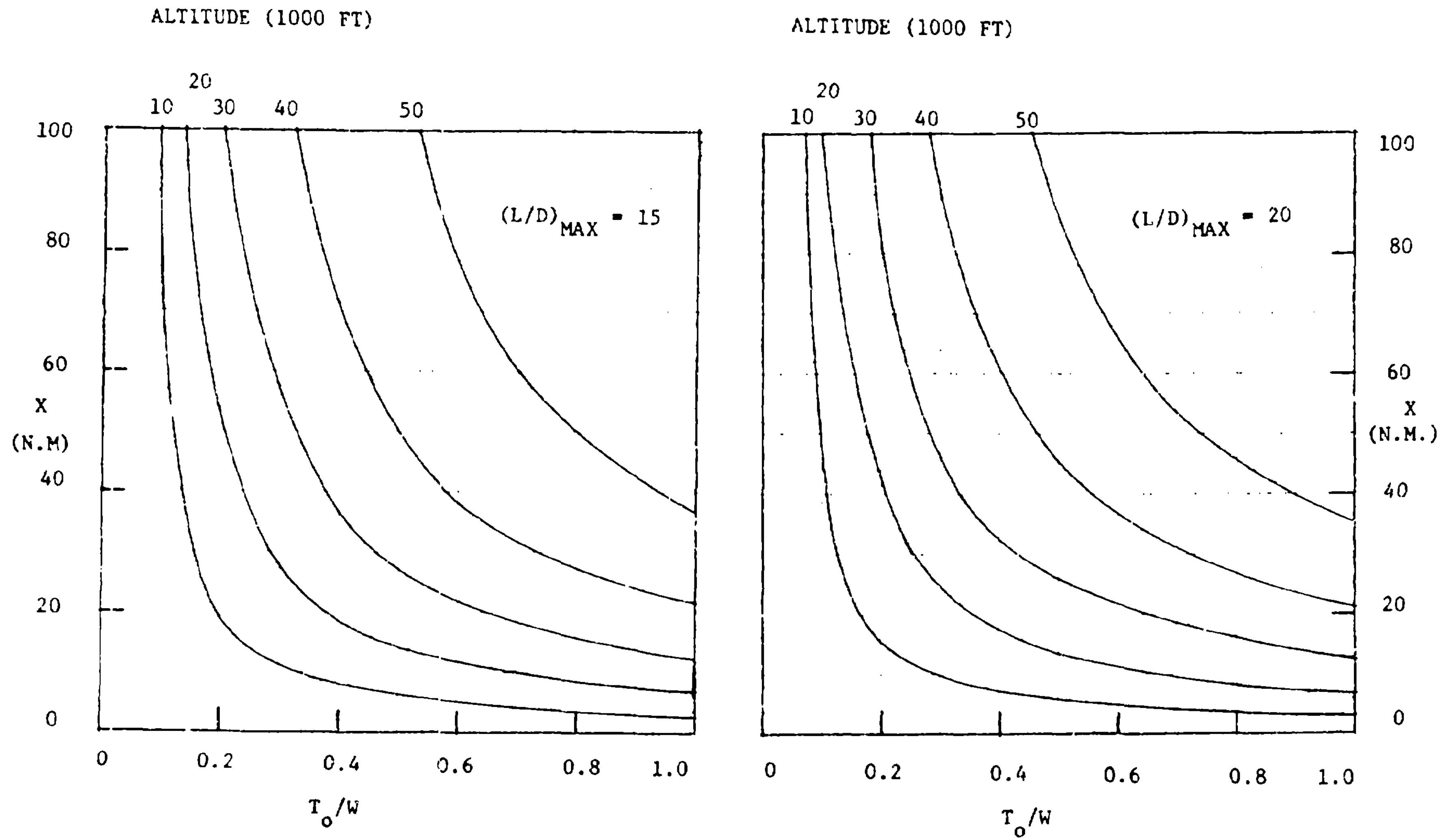

Figure 3-8 Distance in Climb from Sea Level

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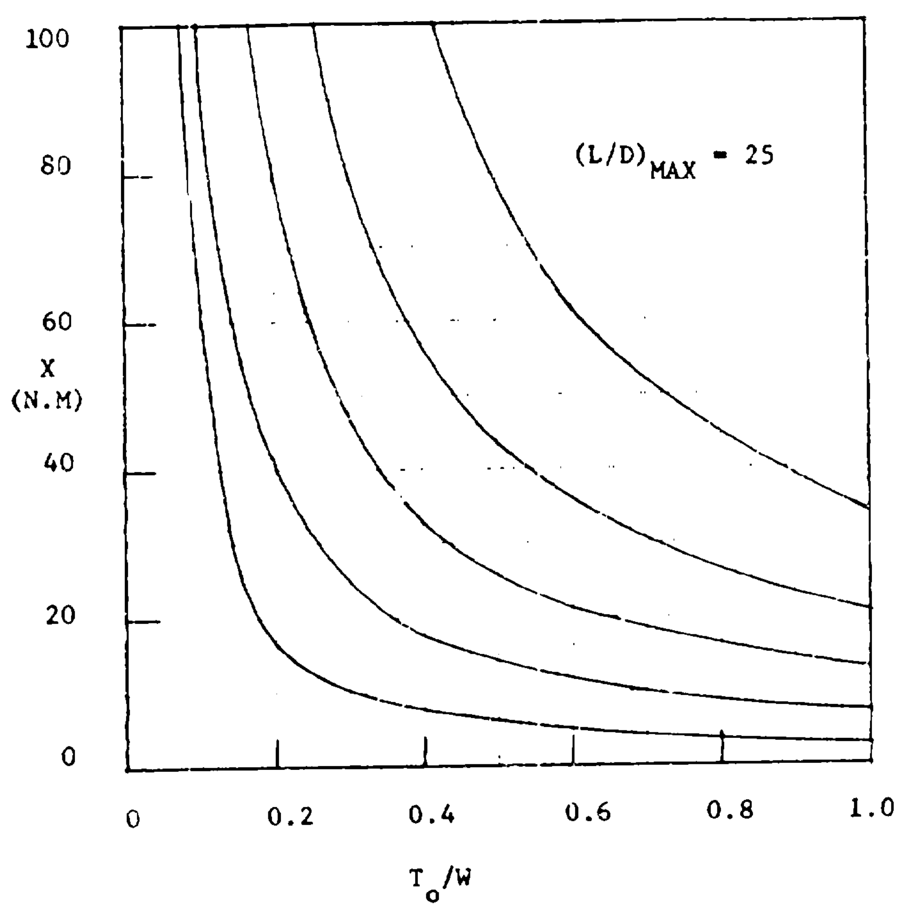

Figure 3-8 Distance in Climb from Sea Level

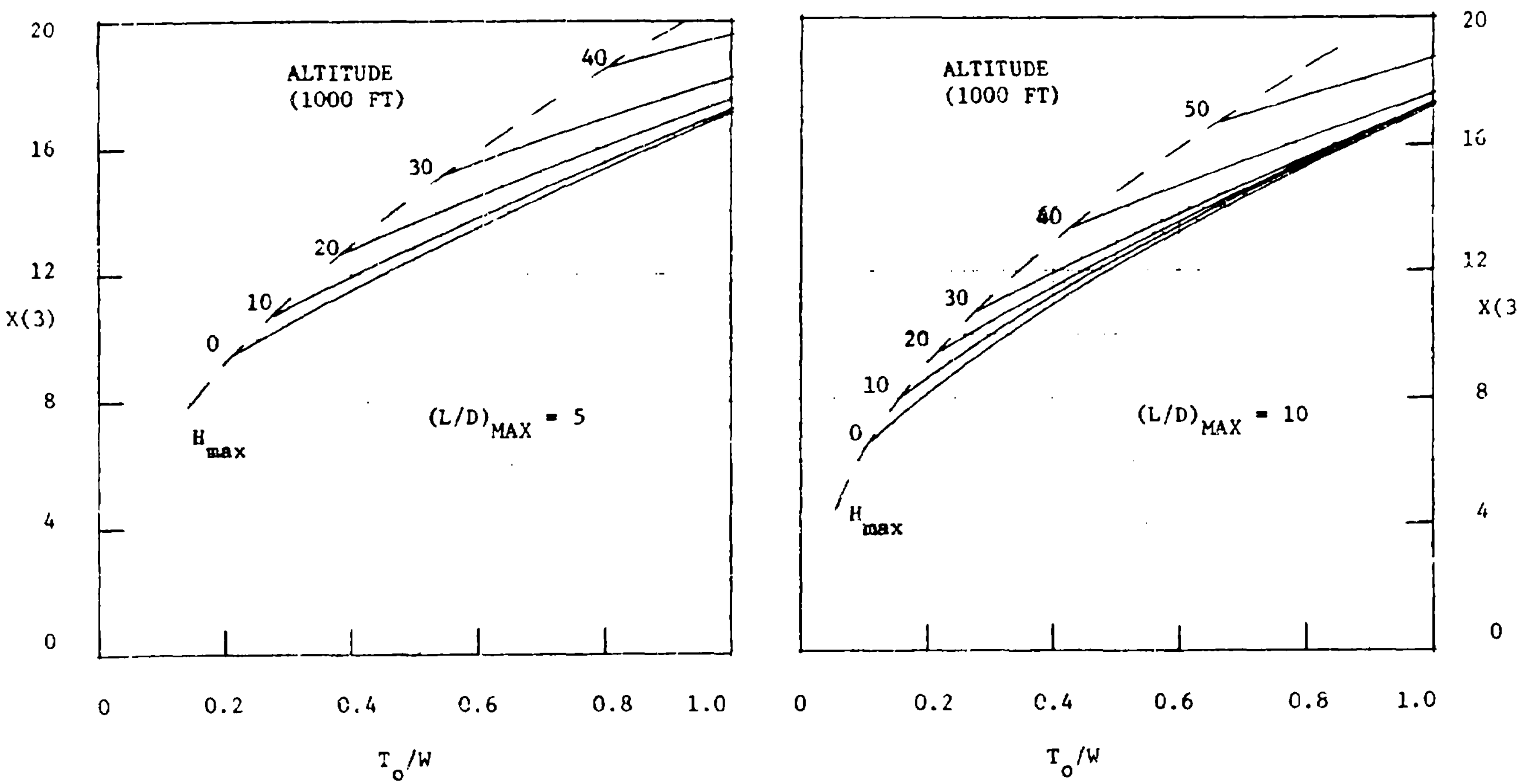

Figure 3-9 Climb Speed Variable

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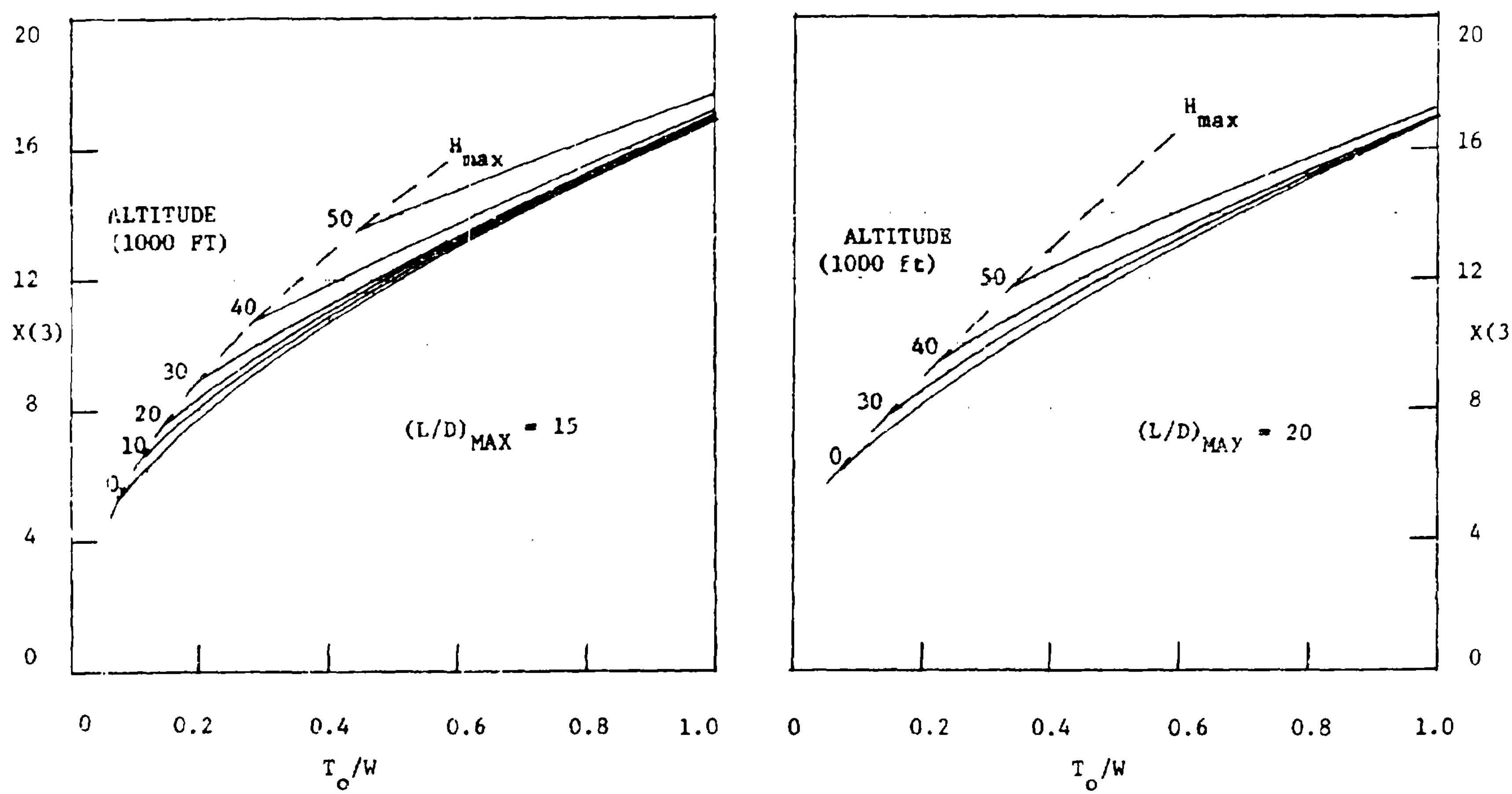

Figure 3-9 Climb Speed Variable

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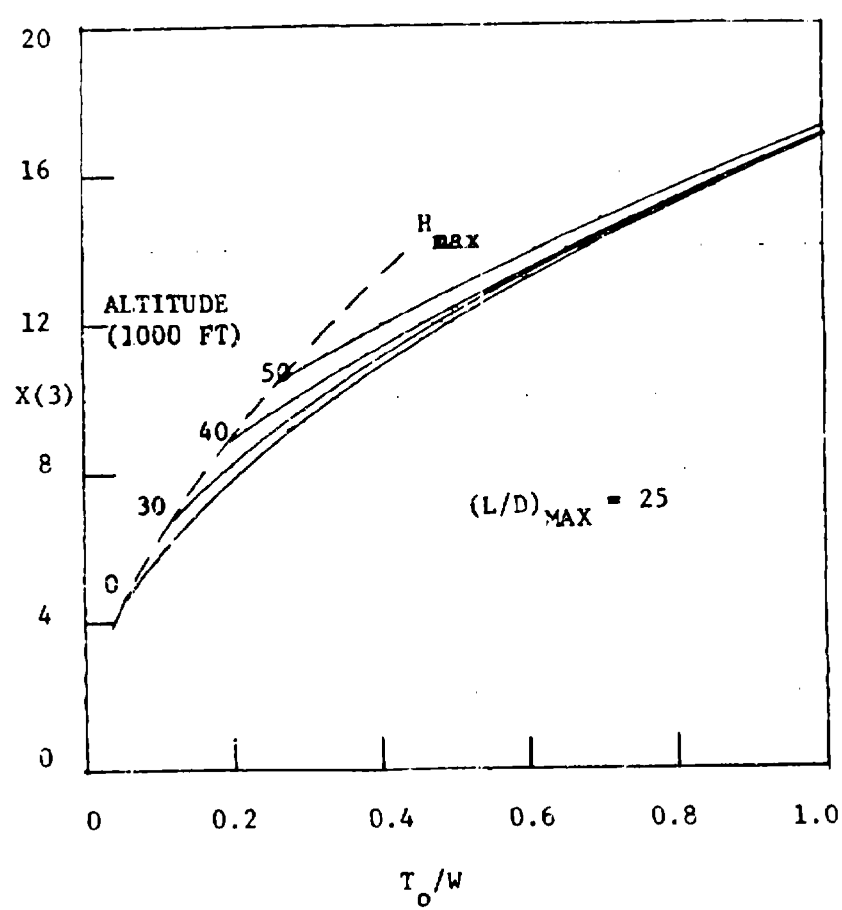

Figure 3-9 Climb Speed Variable

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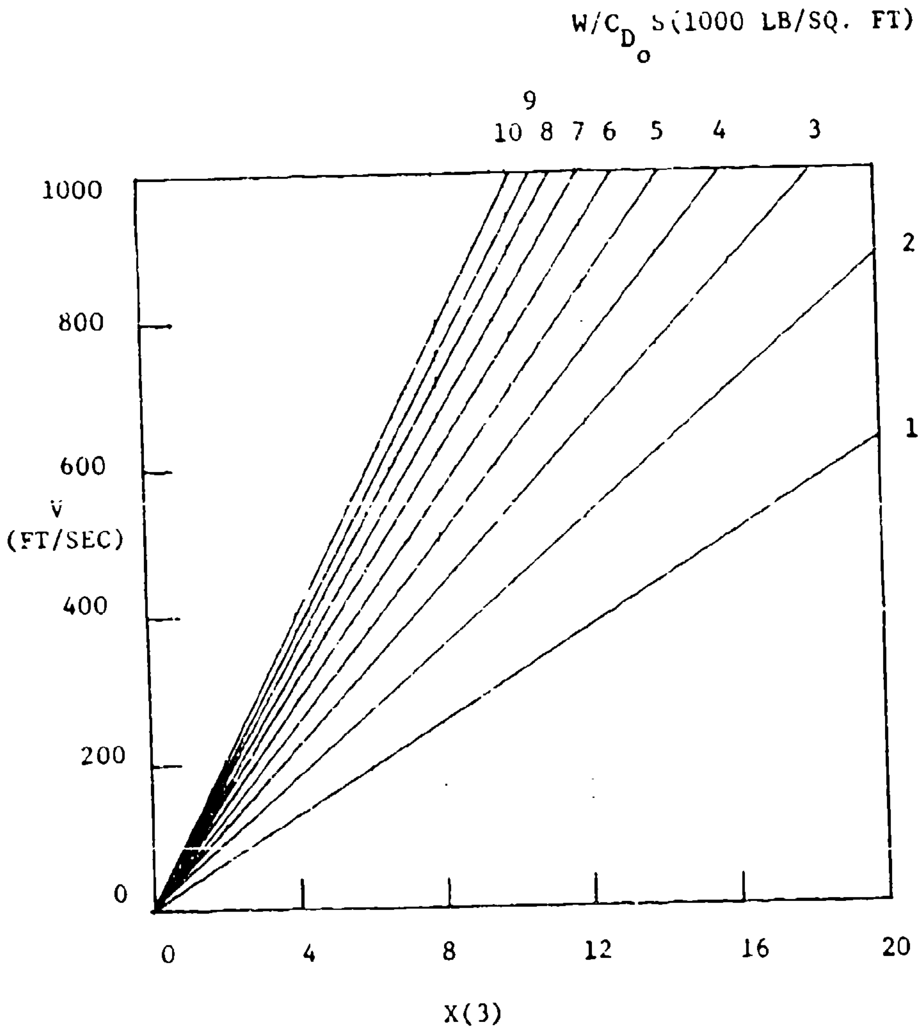

Figure 3-10 Best Climb Speed

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The approximate solutions are

$$t_c = 85 \text{ seconds}$$

$$W_c = 1260 \text{ pounds}$$

$$X = 17 \text{ nautical miles}$$

The F4C performance substantiation data gives approximately

$$t_c = 72 \text{ seconds}$$

$$W_c = 1290 \text{ pounds}$$

$$X = 12 \text{ nautical miles}$$

The errors are 18% in time to climb, 2% in fuel to climb, and 42% in the range. The differences in the time and range are significant. From a total mission standpoint, however, the differences will be small. As an example, for an F4C combat air patrol mission, the total mission time and radius are 1.39 hours and 250 nautical miles. Based upon these numbers the errors are negligible.

Acceleration Correction

The approximate climb solutions were based upon the assumption that the acceleration, $\dot{V}$, was zero. Based upon the best climb speed schedules depicted in Figures 3-9 and 3-10, this assumption requires further examination. From Equations 3-1 and 3-4

$$\frac{dV}{dh} = \frac{g}{V \sin\gamma}\left(\frac{T}{W} - \frac{D}{W} - \sin\gamma\right)$$

Solving for sin γ gives

$$\sin\gamma = \frac{\frac{T}{W} - \frac{D}{W}}{1 + \frac{V}{g}\frac{dV}{dh}} \qquad (3\text{-}27)$$

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Equations 3-11, 3-12, and 3-23 become, upon substitution of Equation 3-27,

$$t_c = \int_{h_o}^{h_f} \frac{\left(1 + \frac{V}{g}\frac{dV}{dh}\right) dh}{V\left(\frac{T}{W} - \frac{D}{W}\right)}$$

$$\frac{W_c}{W} = \int_{h_o}^{h_f} \frac{\frac{T}{W} \cdot SFC \cdot \left(1 + \frac{V}{g}\frac{dV}{dh}\right) dh}{V\left(\frac{T}{W} - \frac{D}{W}\right)}$$

$$X = \int_{h_o}^{h_f} \frac{\left(1 + \frac{V}{g}\frac{dV}{dh}\right) dh}{\frac{T}{W} - \frac{D}{W}}$$

The solutions reduce to the previous solutions if

$$\frac{dV}{dh} = 0$$

They reduce approximately to the previous results if

$$\frac{V}{g}\frac{dV}{dh} \ll 1$$

We can check if this is the case for the speed defined in Equation 3-18. If we assume an exponential atmosphere, i.e., the density varies exponentially with altitude, then

$$\sigma = \sigma_R \, e^{-\beta h} \tag{3-28}$$

where σ_R and β are coefficients defined as follows

Atmospheric Layer	σ_R	β (Feet^{-1})
Troposphere	1.0	1/30500
Stratosphere	1.712	1/20600

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Substitution of Equation 3-28 into Equation 3-18 and differentiating gives

$$\frac{V}{g}\frac{dV}{dh} = \frac{\beta \dfrac{W}{SC_{D_o}}}{2\left(\frac{L}{D}\right)^2_{MAX} g\rho_o \sqrt{\left(\frac{T_o}{W}\right)^2 \sigma^2 + \dfrac{3}{(L/D)^2_{MAX}}}} \qquad (3\text{-}29)$$

Based upon the maximum altitude defined by Equation 3-26

$$\sigma \geq \frac{1}{\frac{T_o}{W} \cdot \left(\frac{L}{D}\right)_{MAX}}$$

Thus

$$\frac{V}{g}\frac{dV}{dh} \leq \frac{\beta \dfrac{W}{SC_{D_o}}}{4\left(\frac{L}{D}\right)_{MAX} g\rho_o} \qquad (3\text{-}30)$$

The error is largest at the maximum altitude. For the F4C data examined earlier, the right side of relation (3-30) yields

$$\frac{\beta \dfrac{W}{SC_{D_o}}}{4\left(\frac{L}{D}\right)_{MAX} g\rho_o} = 0.07$$

The maximum error $\frac{V}{g}\frac{dV}{dh}$ is presented in Figure 3-11 as a function of $W/C_{D_o}S$ and maximum L/D. If maximum L/D is greater than 15, then the error is less than 0.1.

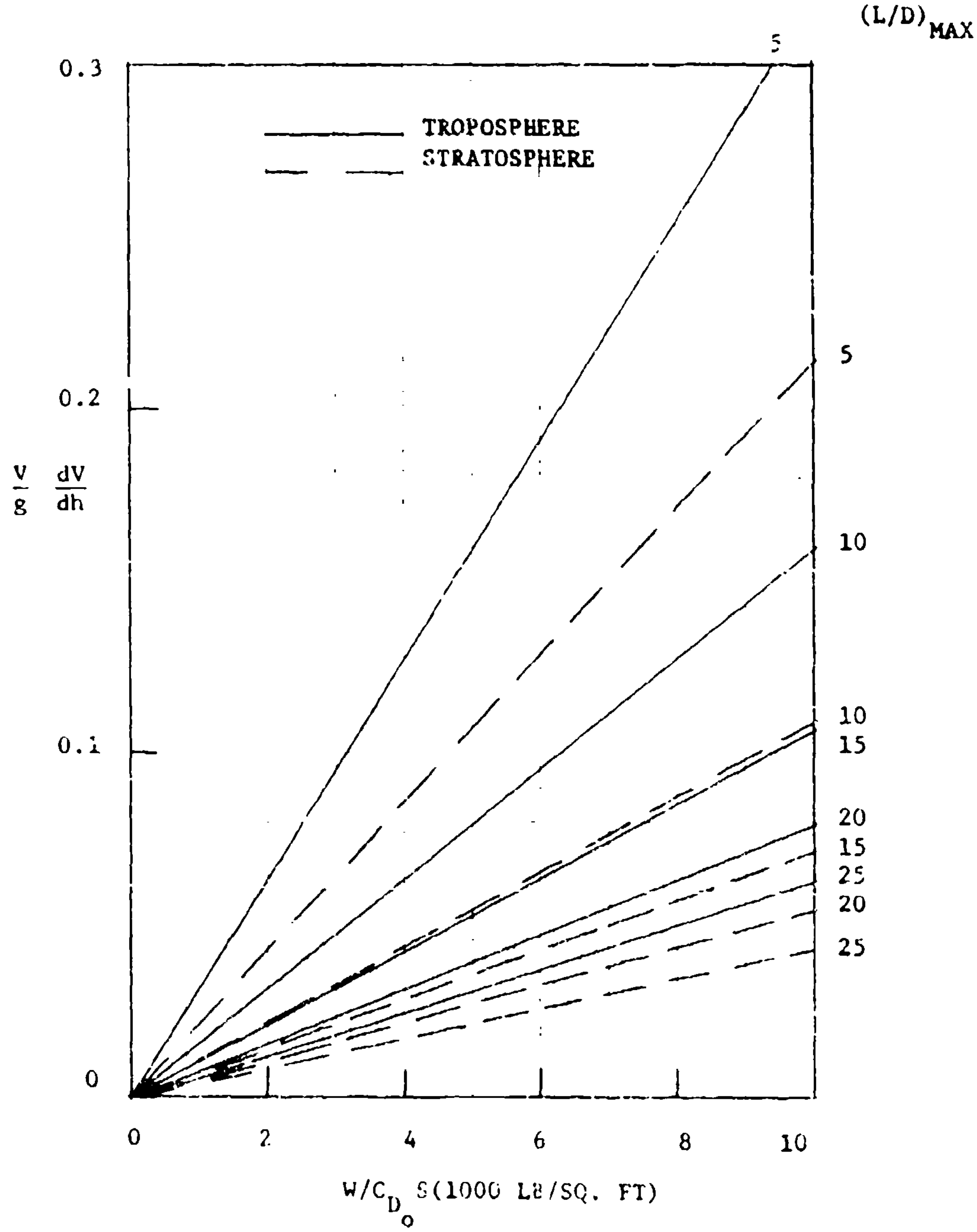

Figure 3-11 Acceleration Factor

Sensitivity Analysis

The sensitivity of time to climb, fuel to climb, and distance covered in climb have the following form

$$\Delta = S_{W/C_{D_o}S} \frac{d(W/C_{D_o}S)}{W/C_{D_o}S} + S_{(\frac{L}{D})_{MAX}} \frac{d(L/D)_{MAX}}{(L/D)_{MAX}}$$

$$+ S_{(T/W)_o} \frac{d(T/W)_o}{(T/W)_o}$$

where Δ is the change in a performance variable, namely dt_c/t_c, dW_c/W_c, or dX/X. $S_{W/C_{D_o}S}$, $S_{(L/D)_{MAX}}$, and $S_{(T/W)_o}$ are the sensitivity parameters. These parameters are defined in Table 3-1. Fuel to climb is directly proportional to changes in SFC.

The sensitivity parameters for $(L/D)_{MAX}$ and $(T/W)_o$ are all negative. Consequently, an increase in either or both results in a decrease in t_c, W_c, and X. In addition, the change in any variable is more sensitive to changes in $(T/W)_o$ rather than $(L/D)_{MAX}$. The biggest impact in a change in $(L/D)_{MAX}$ is in the maximum altitude H_{MAX}. In Table 3-2, the sensitivity parameters $S_{(L/D)_{MAX}}$ and $S_{(T/W)_o}$ are presented for selected values of $(L/D)_{MAX}$, $(T/W)_o$, and the altitude. The data show that as the product $(\frac{L}{D})_{MAX}$ $(\frac{T}{W})_o$ increases, the sensitivity parameters increase (decrease in absolute magnitude). The sensitivity parameter $S_{(T/W)_o}$ asymptotically approaches -1.5, -0.5, and -1.0 for increasing values of $(L/D)_{MAX}$ $(T/W)_o$. $S_{(L/D)_{MAX}}$ asymptotically approaches zero.

TABLE 3-1

SENSITIVITY PARAMETERS FOR CLIMB PERFORMANCE

Variable	$S_{W/C_{D_o}S}$	$S_{(L/D)_{MAX}}$	$S_{(T/W)_o}$
t_c	-1/2	$\frac{(L/D)_{MAX}}{X(1)} \frac{\partial X(1)}{\partial (L/D)_{MAX}}$	$\frac{T_o/W}{X(1)} \frac{\partial X(1)}{\partial (T_o/W)}$
W_c	-1/2	$\frac{(L/D)_{MAX}}{X(2)} \frac{\partial X(2)}{\partial (L/D)_{MAX}}$	$\frac{T_o/W}{X(2)} \frac{\partial X(2)}{\partial (T_o/W)}$
X	0	$(L/D)_{MAX} \frac{\partial X}{\partial (L/D)_{MAX}}$	$T_o/W \frac{\partial X}{\partial (T_o/W)}$

TABLE 3-2

SENSITIVITY PARAMETERS

Variable - $S_{(T/W)_o}$

$(L/D)_{MAX}$	$(T/W)_o$	Altitude	t_c	W_c	X
5	0.5	25,000	-4.37	-2.96	-4.12
5	1.0	40,000	-2.74	-1.40	-2.41
15	0.5	25,000	-1.61	-0.60	-1.15
15	1.0	40,000	-1.57	-0.55	-1.09
25	0.5	25,000	-1.54	-0.54	-1.05
25	1.0	40,000	-1.52	-0.52	-1.03

Variable - $S_{(L/D)_{MAX}}$

$(L/D)_{MAX}$	$(T/W)_o$	Altitude	t_c	W_c	X
5	0.5	25,000	-2.86	-2.46	-3.11
5	1.0	40,000	-1.24	-0.90	-1.41
15	0.5	25,000	-0.11	-0.10	-0.15
15	1.0	40,000	-0.07	-0.05	-0.09
25	0.5	25,000	-0.04	-0.04	-0.05
25	1.0	40,000	-0.02	-0.02	-0.03

Summary of the Climb Performance Problems

Approximate time, fuel, and range solutions were derived for trajectories where the acceleration was negligible. Sensitivity analysis demonstrated that changes in sea level thrust to weight ratio resulted in bigger changes in performance relative to the changes in maximum lift-to-drag ratio.

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SECTION IV

CRUISE PERFORMANCE

SECTION IV

CRUISE PERFORMANCE

Problem Definition and Assumptions

In general, cruise performance is characterized by flight in the vertical plane at small flight path angles with small changes in the speed and altitude. Typical cruise performance problems involve the derivation of the solutions for maximum range, maximum endurance, and level flight trajectories. The solution to these problems defines the necessary conditions for the aerodynamic and engine controls. The definition of the controls provides the information for integrating the differential equations of motion and kinematic relations.

Small changes in the speed imply negligible longitudinal acceleration. This assumption along with small changes in the flight path angle results in the thrust, T, equalling the drag, D.

$$T = D \tag{4-1}$$

The thrust is a function of the Mach number, M, altitude, h, and engine throttle setting, N. The drag is a function of Mach number, altitude, and aerodynamic lift coefficient or angle-of-attack. For the problems to be addressed here, the lift coefficient will be the aerodynamic control. The time-rate-of-change of the range, X, is

$$\dot{X} = V \cos \gamma$$

which, for small flight path angles, may be approximated by

$$\dot{X} = V \tag{4-2}$$

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The time-rate-of-change of the altitude, h, satisfies

$$\dot{h} = V \sin \gamma$$

which can be approximated by

$$\dot{h} = V\gamma \tag{4-3}$$

The assumption of small flight path angles implies negligible normal acceleration. Thus, for trajectories in the vertical plane, the weight, W, and aerodynamic lift, L, are equal or

$$L = W \tag{4-4}$$

The aerodynamic lift is a function of Mach number, altitude, and lift coefficient. The final differential equation is the time-rate-of-change of the weight,

$$\dot{W} = -T \cdot SFC \tag{4-5}$$

where SFC is the specific fuel consumption. The aerodynamic drag and lift are defined by

$$D = \tfrac{1}{2} \rho S V^2 C_D \tag{4-6}$$

$$L = \tfrac{1}{2} \rho S V^2 C_L \tag{4-7}$$

where C_D is the aerodynamic drag coefficient, ρ is the atmospheric density, and S is the reference area for the aerodynamic coefficients. The aerodynamic drag coefficient is a function of C_L and Mach number, M,

$$C_D = f_1(M, C_L) \tag{4-8}$$

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The thrust and specific fuel consumption are functions of Mach number, altitude, and throttle setting, N,

$$T = f_2(M,h,N) \qquad (4\text{-}9)$$

$$SFC = f_3(M,h,N) \qquad (4\text{-}10)$$

Equations 4-1 through 4-10 provide the relations necessary for determining cruise performance subject to specified trajectory characteristics, such as best cruise, best endurance, or constant altitude-constant speed trajectories.

As an example, if altitude and weight are specified, then C_L can be determined from Equations 4-4 and 4-7 as a function of V, h, and W. From Equation 4-8, C_D is computed as a function of V, h, and W. Next, D is computed from Equation 4-6 as a function of V, h, and W. Equations 4-1 and 4-9 establish the engine control. From Equations 4-10 and 4-5, SFC and $\dot{W}$ are computed as functions of V, h, and W. The flight path angle can be determined from Equation 4-3 if h is specified as a function of time.

It is convenient at this point to introduce a transformation whereby the weight is treated as the independent variable. Consequently, the time becomes a dependent variable. Dividing Equations 4-2 and 4-3 by Equation 4-5 gives

$$\frac{dX}{dW} = -\frac{V}{T \cdot SFC}$$

$$\frac{dh}{dW} = -\frac{V\gamma}{T \cdot SFC}$$

Substituting Equation 4-1 yields

$$\frac{dX}{dW} = -\frac{V}{D \cdot SFC} \tag{4-11}$$

$$\frac{dh}{dW} = -\frac{V\gamma}{D \cdot SFC} \tag{4-12}$$

The transformation of Equation 4-5 is

$$\frac{dt}{dW} = -\frac{1}{T \cdot SFC}$$

which becomes upon substitution of Equation 4-1

$$\frac{dt}{dW} = -\frac{1}{D \cdot SFC} \tag{4-13}$$

Equations 4-11 through 4-13, 4-4, and 4-6 through 4-10 constitute the formulation of the cruise performance problem. We now turn our attention to four specific problems, namely best cruise performance, best endurance, constant speed, and constant altitude trajectories. The constant altitude-constant speed case is representative of a low altitude bomber penetration. The constant speed trajectory is representative of a supersonic cruise like the B70 or Supersonic Transport. The constant altitude case is representative of the control maintained by the Federal Aviation Agency during peacetime operations. Each of these examples will be studied in detail and appropriate approximations introduced whenever possible in order to achieve analytical results.

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Best Cruise Performance

In general, the goal for this problem is to determine the speed, altitude, and the aerodynamic and engine control for maximum range covered for given fuel available for cruise. Thus, the weight is the appropriate independent variable. If Equation 4-11 is maximized at each point on the trajectory, then the integration of Equation 4-11 yields the maximum range. The resulting trajectory is valid if the aerodynamic and engine controls can be realized along the trajectory.

Rewriting Equation 4-11 and substituting Equation 4-4 gives

$$\frac{dX}{dW} = -\frac{V(L/D)}{SFC} \cdot \frac{1}{W} \tag{4-14}$$

The range factor, R_F, is defined as

$$R_F = \frac{V(L/D)}{SFC} \tag{4-15}$$

Maximum range, therefore, corresponds to maximum range factor. The general technique for solving the maximum range factor is as follows. For a given weight and altitude, C_L as a function of V, h, and W is determined from Equations 4-4 and 4-7. C_D as a function of V, h, and W is determined from Equation 4-8. This gives L/D since according to Equations 4-6 and 4-7

$$\frac{L}{D} = \frac{C_L}{C_D} \tag{4-16}$$

L/D versus V for fixed values of W and h is displayed in Figure 4-1. For fixed V and W, L/D versus h looks like that in Figure 4-2.

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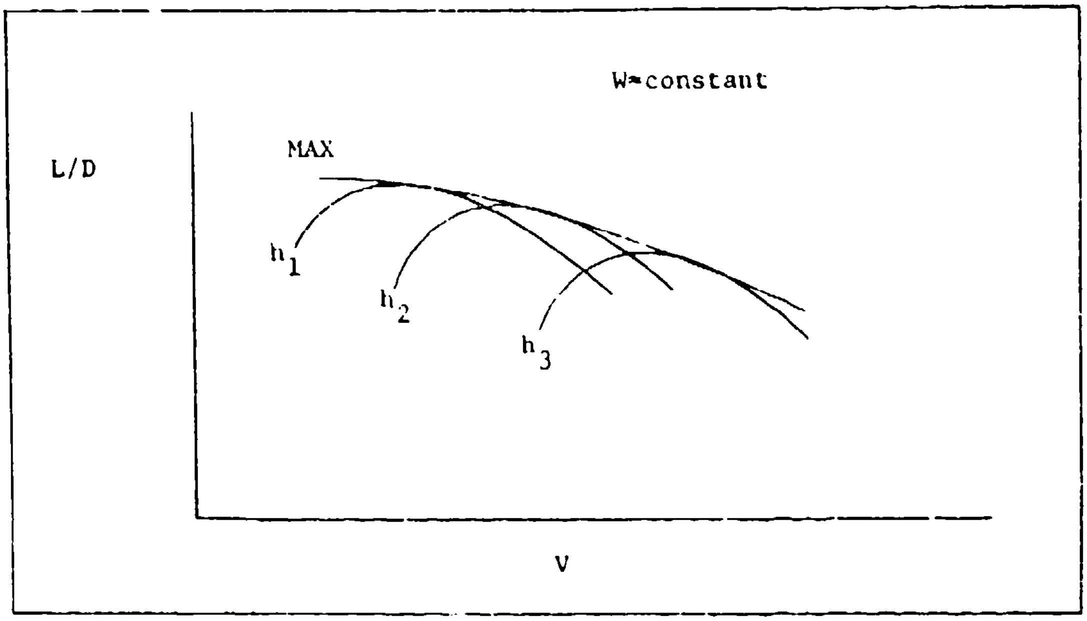

Figure 4-1 L/D Versus V

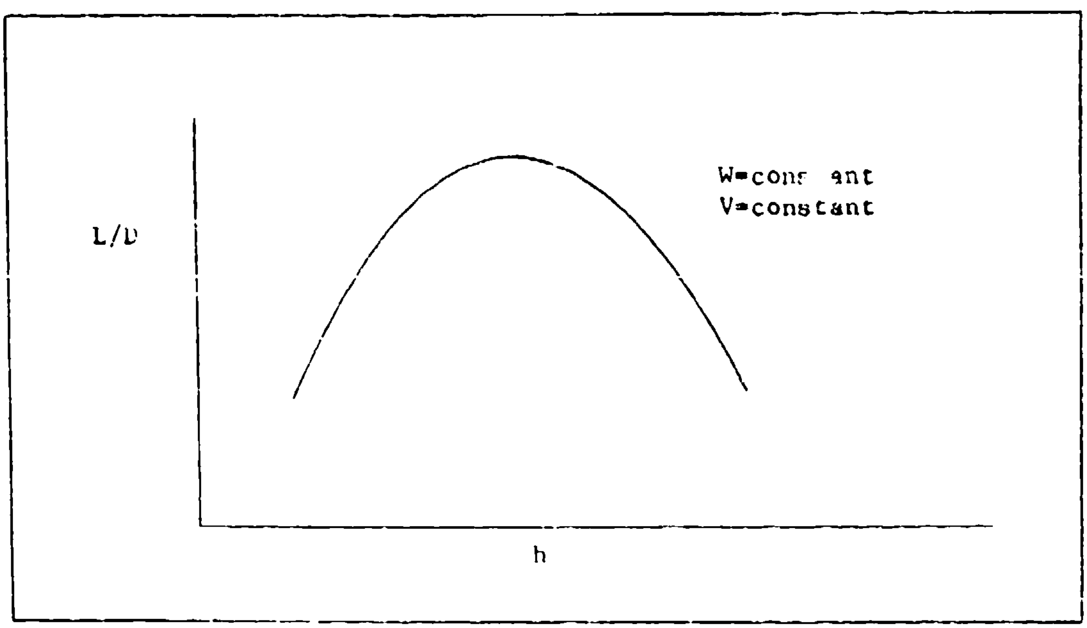

Figure 4-2 L/D Versus h

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For a given speed and weight, L/D increases with increasing altitude until the altitude for maximum L/D is reached. At higher altitudes than this altitude, L/D then continuously decreases.

Since C_D has been determined, then D is determined from Equation 4-6 which identifies T from Equation 4-1. From Equation 4-9 N is determined and SFC is then obtained from Equation 4-10. SFC versus V for fixed W and h is presented in Figure 4-3. The combination of Figures 4-1 and 4-3 along with the definition of the range factor as defined by Equation 4-15 gives R_F as a function of V, h, and W as demonstrated in Figure 4-4. From data like that in Figure 4-4, best cruise speed, best cruise altitude, and R_F are determined for the given weight W. If the process is repeated for different weights, then R_F as a function of W looks like that in Figure 4-5. In Figure 4-5, W_i and W_f are the initial and final weights during cruise. Consequently, the cruise range as a function of W_i and W_f is determined from Figure 4-5 and Equation 4-14

$$X = -\int_{W_i}^{W_f} R_F \frac{dW}{W} \quad (4\text{-}17)$$

If R_F is approximately constant, then Equation 4-17 reduces to

$$X = \frac{V(L/D)}{SFC} \ln \frac{W_i}{W_f}$$

$$= R_F \ln \frac{W_i}{W_f} \quad (4\text{-}18)$$

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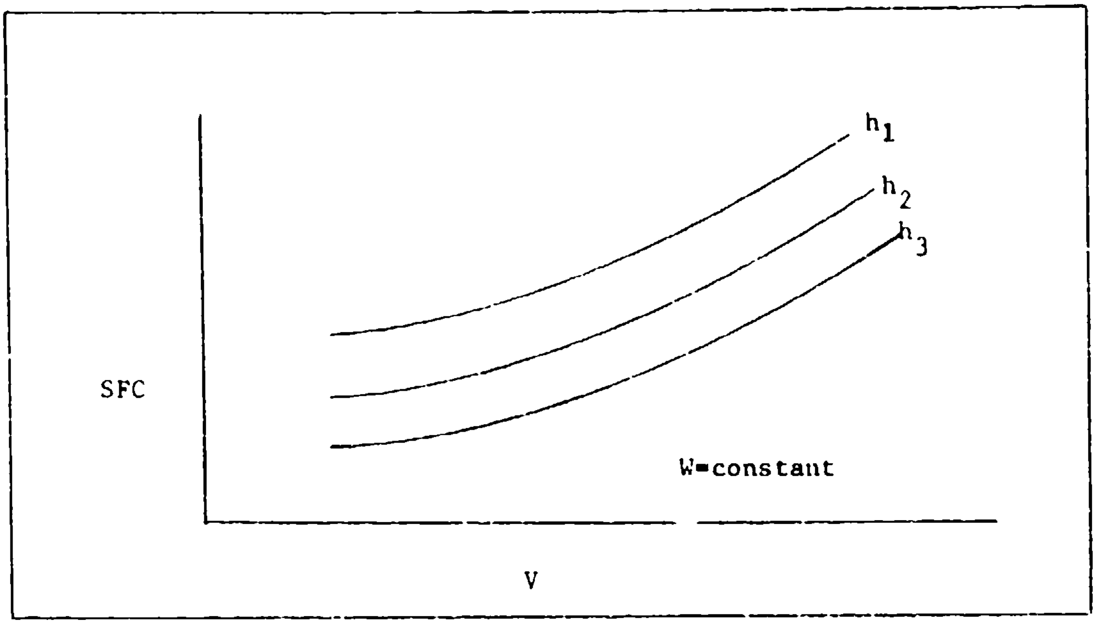

Figure 4-3 SFC Versus V

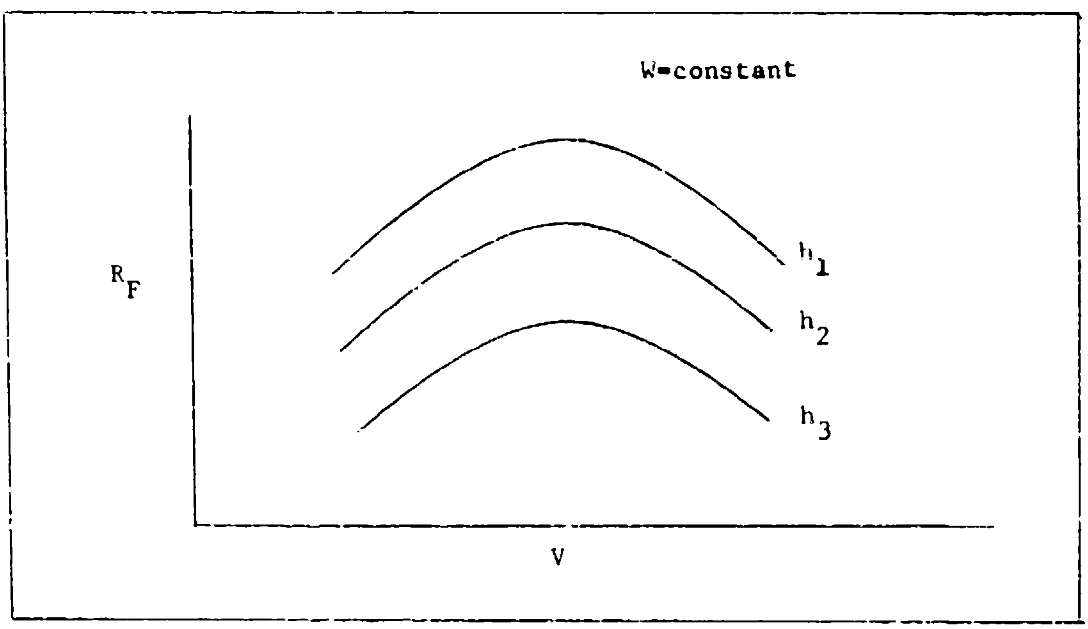

Figure 4-4 R_T Versus V

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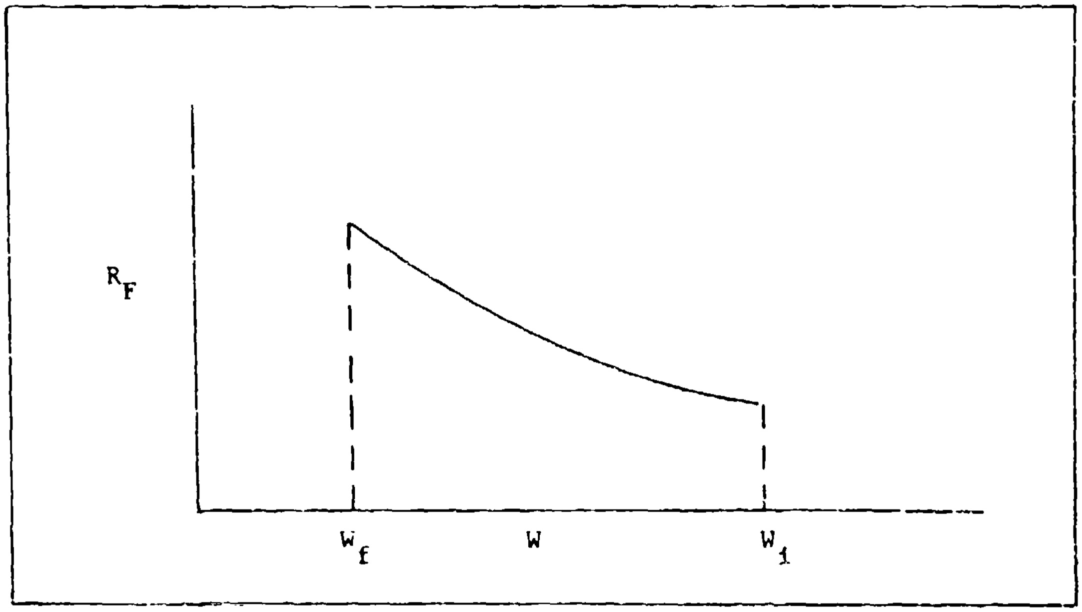

Figure 4-5 R_F Versus W

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The aforementioned technique is clearly a numerical approach to solving the best cruise performance. An analytical solution is desirable if it can be determined and if it is sufficiently accurate. For certain assumptions an analytical solution can be determined. The assumptions which result in an analytical solution are as follows:

1. $$C_D = C_{D_{MIN}} + K(C_L - C_L')^2 \qquad (4\text{-}19)$$

This assumption corresponds to a parabolic drag polar shifted by C_L', the C_L for minimum drag coefficient ($C_{D_{MIN}}$). A parabolic polar is a fairly good approximation over a limited range of C_L values for most drag polars. The coefficients C_{D_0} and K are Mach number dependent.

2. $$SFC = f(M) \qquad (4\text{-}20)$$

This approximation assumes that the SFC is independent of altitude. Furthermore, it is assumed that the value of SFC is the minimum value. The rationale for this assumption is that if the aircraft aerodynamics and engine are properly matched, then the throttle setting for cruise should be in the neighborhood of minimum SFC. Also, from Equation 4-14, minimum SFC is desired for best cruise. If this assumption holds, then the engine throttle setting, N, is a variable which does not need to be determined.

The analytical approach will be to determine the cruise speed and altitude which maximizes range factor for a given weight. Hence the optimal solution is the solution of

$$\max_{h,V} \frac{V}{D \cdot SFC} \qquad (4\text{-}21)$$

In light of the assumption that the aerodynamic coefficients and SFC are functions of Mach number only, we will treat the Mach number and the density ratio as the independant variables. The Mach number, M, is defined by

$$M = \frac{V}{a} \tag{4-22}$$

where a is the speed of sound. The speed of sound is related to absolute temperature, T, in the following way

$$a = C\sqrt{T} \tag{4-23}$$

where C is a constant proportionality factor. From the gas equation, P, ρ, and T are related by

$$P = \rho RT \tag{4-24}$$

where R is the gas constant. From Figure 4-6, P can be approximately related to ρ according to

$$\frac{P}{P_o} = B\left(\frac{\rho}{\rho_o}\right)^A$$

$$= B\,\sigma^A \tag{4-25}$$

where subscript o implies sea level values and $\sigma = \rho/\rho_o$ is the density ratio. For a standard day, from Figure 4-6, A and B are as follows

ATMOSPHERIC LAYER	A	B
Troposphere	1.235	1.0
Stratosphere	1.0	0.752

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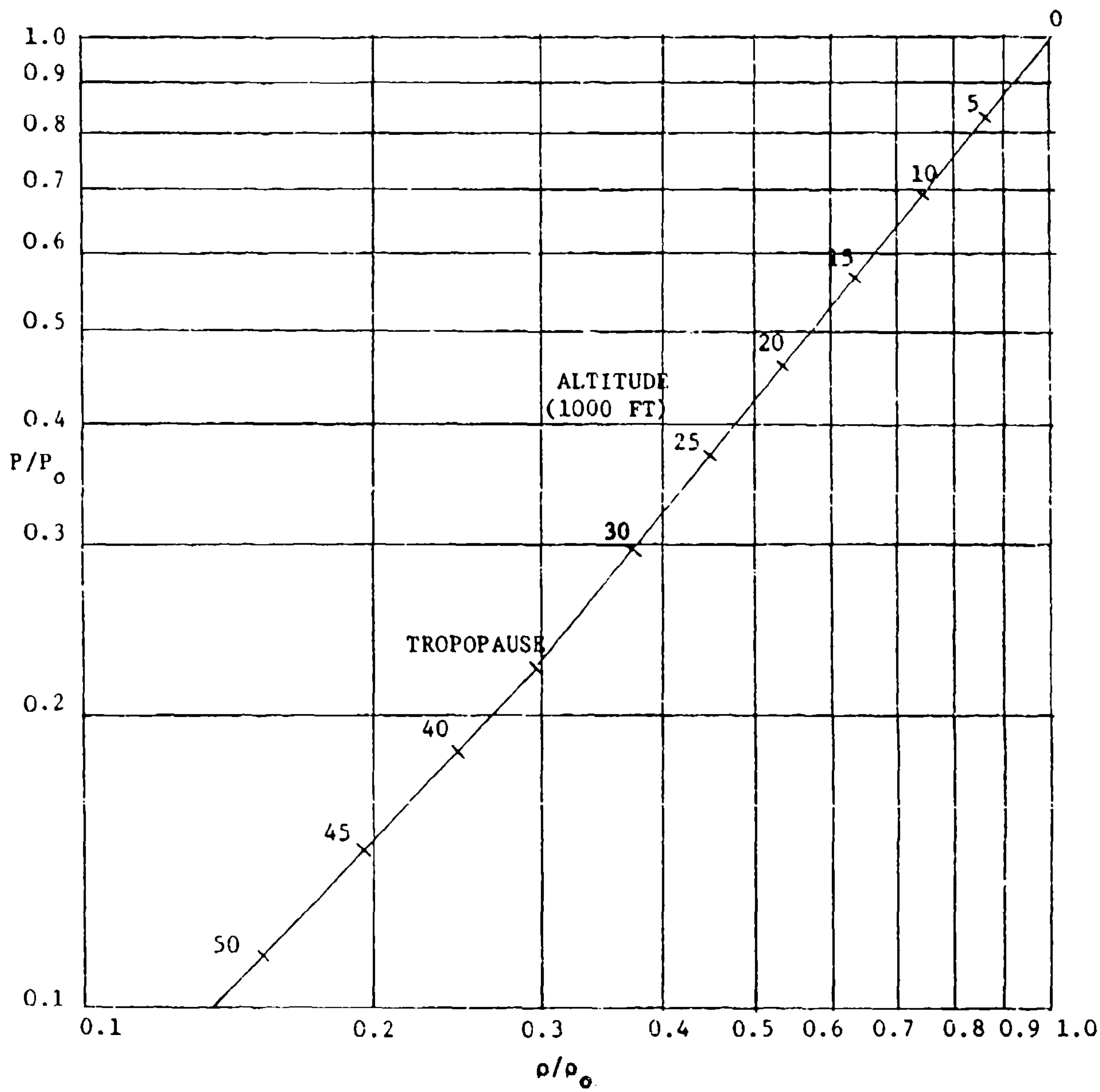

Figure 4-6 Pressure Ratio Versus Density Ratio for a Standard Day

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Substitution of Equations 4-24 and 4-25 into Equation 4-23 gives

$$\rho a^2 = \frac{C^2 P_o B}{R} \sigma^A \tag{4-26}$$

The dynamic pressure, q, is defined as

$$q = \frac{1}{2} \rho V^2$$

Substituting Equations 4-22 and 4-26 gives

$$q = \frac{1}{2} \rho M^2 a^2$$

$$= \frac{1}{2} \frac{C^2 P_o B}{R} M^2 \sigma^A \tag{4-27}$$

$$= \frac{1}{2} \lambda M^2 \sigma^A$$

where λ is a constant defined as

$$\lambda = \frac{1}{R} C^2 P_o B = a_o^2 \rho_o B \tag{4-28}$$

The equation for the drag can now be formulated as a function of M and σ. Substituting Equations 4-19, 4-4, and 4-7 into Equation 4-6 gives

$$D = qS C_D$$

$$= \frac{1}{2} \lambda M^2 \sigma^A S \left[C_{D_{MIN}} + K \left(\frac{2W}{\lambda M^2 \sigma^A S} - C_L' \right)^2\right] \tag{4-29}$$

We can now determine the optimal cruise Mach number and density ratio for maximizing the range factor.

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The necessary conditions for maximum range factor are

$$\frac{\partial}{\partial \sigma} \left(\frac{Ma}{DSFC} \right) = 0 \qquad (4\text{-}30)$$

$$\frac{\partial}{\partial M} \left(\frac{Ma}{DSFC} \right) = 0 \qquad (4\text{-}31)$$

Substituting Equation 4-26 into 4-30 gives

$$\frac{\partial}{\partial \sigma} \left(\frac{M\sigma^{\frac{1}{2}(A-1)}}{DSFC} \right) = 0$$

Expanding this equation yields

$$\frac{1}{2}(A-1)\frac{1}{\sigma} - \frac{1}{D}\frac{\partial D}{\partial \sigma} = 0 \qquad (4\text{-}32)$$

From Equation 4-29

$$\frac{1}{D}\frac{\partial D}{\partial \sigma} = \frac{A}{\sigma} - \frac{2AK}{C_D} \left(\frac{2W}{\lambda M^2 \sigma^{A+1} S} \right) \left(\frac{2W}{\lambda M^2 \sigma^A S} - C_L' \right) \qquad (4\text{-}33)$$

Substitution into Equation 4-32 gives

$$\frac{A+1}{2} C_D - 2AK \left(\frac{2W}{\lambda M^2 \sigma^A S} \right) \left(\frac{2W}{\lambda M^2 \sigma^A S} - C_L' \right) = 0$$

Substituting C_D and

$$C_{D_o} = C_{D_{MIN}} + K C_L'^2$$

gives

$$(A+1) C_{D_o} \sigma^{2A} + K \left[(1-3A) \left(\frac{2W}{\lambda M^2 S} \right)^2 + 2(A-1) \left(\frac{2W}{\lambda M^2 S} \right) C_L' \sigma^A \right] = 0$$

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This equation can be solved for σ as a function of M; the result is

$$\sigma = \left\{ \frac{2W}{(A+1)\lambda M^2 SC_{D_o}} \left[-(A-1)KC_L' + \sqrt{(A-1)^2K^2C_L'^2 + (A+1)(3A-1)KC_{D_o}} \right] \right\}^{1/A}$$

(4-34)

Rewriting gives

$$\frac{W}{\sigma^A} = \frac{1}{2}(A+1)\lambda M^2 SC_{D_o} \div \left[-(A-1)KC_L' + \sqrt{(A-1)^2K^2C_L'^2 + (A+1)(3A-1)KC_{D_o}} \right]$$

Substituting Equations 4-25 and 4-28 gives

$$\frac{W}{\delta} = \frac{1}{2}(A+1)\rho_o a_o^2 M^2 SC_{D_o} \div \left[-(A-1)KC_L' + \sqrt{(A-1)^2K^2C_L'^2 + (A+1)(3A-1)KC_{D_o}} \right]$$

where δ is the atmospheric pressure ratio P/P_o. Consequently, W/δ is constant if K, C_L, M, and C_{D_o} are constant.

Substituting Equation 4-34 into Equation 4-29 and expanding gives

$$\frac{D}{W} = \frac{4A}{A+1} \frac{(A-1)K^2C_L'^2 + (A+1)KC_{D_o} - KC_L'\left[(A-1)^2K^2C_L'^2 + (A+1)(3A-1)KC_{D_o}\right]^{\frac{1}{2}}}{-(A-1)KC_L' + \left[(A-1)^2K^2C_L'^2 + (A+1)(3A-1)KC_{D_o}\right]^{\frac{1}{2}}}$$

The reciprocal is the cruise lift to drag ratio

$$\frac{L}{D} = \left(\frac{A+1}{4A}\right) \frac{-(A-1)KC_L' + \left[(A-1)^2K^2C_L'^2 + (A+1)(3A-1)KC_{D_o}\right]^{\frac{1}{2}}}{(A-1)K^2C_L'^2 + (A+1)KC_{D_o} - KC_L'\left[(A-1)^2K^2C_L'^2 + (A+1)(3A-1)KC_{D_o}\right]^{\frac{1}{2}}}$$

(4-35)

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It is easily shown that the maximum L/D for Equation 4-19 is

$$\left(\frac{L}{D}\right)_{MAX} = \frac{1}{2\left(\sqrt{KC_{D_o}} - KC_L'\right)} \qquad (4\text{-}36)$$

There are two special cases of interest for Equation 4-35. The first is A = 1; substitution gives

$$\frac{L}{D} = \frac{1}{2}\,\frac{2\sqrt{KC_{D_o}}}{2KC_{D_o} - 2KC_L'\sqrt{KC_{D_o}}}$$

$$= \frac{1}{2\left(\sqrt{KC_{D_o}} - KC_L'\right)}$$

This is identical to Equation 4-36, thus cruise is at maximum L/D in the stratosphere. If $C_L' = 0$, Equation 4-35 reduces to

$$\frac{L}{D} = \frac{A+1}{4A}\,\frac{\sqrt{(A+1)(3A-1)KC_{D_o}}}{(A+1)KC_{D_o}}$$

$$= \frac{1}{4A}\sqrt{\frac{(A+1)(3A-1)}{KC_{D_o}}}$$

Substituting Equation 4-36 gives

$$\frac{L}{D} = \left(\frac{L}{D}\right)_{MAX} \cdot \frac{1}{2A}\sqrt{(A+1)(3A-1)} \qquad (4\text{-}37)$$

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If A = 1.235, Equation 4-37 becomes

$$\frac{L}{D} = 0.996 \left(\frac{L}{D}\right)_{MAX}$$

If $C_L' = 0$, then cruise in the troposphere is approximately at maximum L/D. If C_L' is not zero, the conclusion is still the same. Substitution of the aerodynamics into Equation 4-35 and A = 1.235 shows that the cruise L/D is within one percent of maximum L/D at least for systems like the F4C and KC135.

Best cruise Mach number, M*, is determined from optimization of the range factor

$$R_F = \frac{V(L/D)}{SFC}$$

$$= \frac{Ma(L/D)_{MAX}}{SFC}$$

The necessary condition is therefore

$$F(M) = \frac{1}{M} + \frac{1}{(L/D)_{MAX}} \frac{d(L/D)_{MAX}}{dM} - \frac{1}{SFC} \frac{dSFC}{dM} = 0 \qquad (4\text{-}38)$$

The range factor becomes

$$R_F = \frac{V(L/D)}{SFC} \qquad (4\text{-}39)$$

$$= \frac{a_0 B^{-\frac{1}{2}} M (L/D)_{MAX}}{SFC} \quad \text{in the stratosphere } (A=1, B=0.752)$$

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$$R_F = \frac{\sqrt{B}}{(1.689)^{\frac{A-1}{A}}} \left(\frac{A+1}{4}\right)^{\frac{3A+1}{4A}} \frac{(3A-1)^{\frac{A-1}{4A}}}{A(\rho_o)^{\frac{A-1}{2A}}} \left(\frac{K}{C_{D_o}}\right)^{\frac{A-1}{4A}}$$

$$\cdot \frac{(Ma_o)^{\frac{1}{A}}}{SFC} \left(\frac{W}{S}\right)^{\frac{A-1}{2A}} \left[1 - \frac{(A-1)KC_L'}{\sqrt{(3A-1)(A+1)KC_{D_o}}}\right]^{\frac{3A-1}{2A}}$$

$$\div \left(\sqrt{\frac{A+1}{3A-1} KC_{D_o}} - KC_L'\right) \quad \text{in the troposphere } A=1.235, \; B=1.0$$

Substitution into Equation 4-17 and integrating gives optimal cruise range

$$X = R_F \ln \frac{W_i}{W_f} \qquad \text{in the stratosphere} \qquad (4\text{-}40)$$

$$= \frac{2A}{A-1} R_{F_i} \left[1 - \left(\frac{W_i}{W_f}\right)^{\frac{1-A}{2A}}\right] \quad \text{in the troposphere}$$

Note that in the stratosphere R_F is constant whereas it varies with W in the troposphere. Consequently for the latter case R_{F_i} in Figure 4-9 is the initial value of the range factor.

The approach for determining best cruise performance via the approximate analytical approach is as follows:

1. Fit parabolic polar to aerodynamic data according to Equation 4-19. Expansion gives

$$C_D = C_{D_o} - 2KC_L'C_L + KC_L'^2$$

One way of determining the coefficients C_{D_o}, K, and C_L' is to select a given number of data points C_D and C_L and then minimize the sum of the differences between the theoretical and experimental values.

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Assume N values of C_L are selected, $C_L(i)$. Then C_{D_o}, K, and C_L are determined from the simultaneous solution of the following equations

$$C_{D_o} \sum_{i=1}^{N} 1 - 2KC_L' \sum_{i=1}^{N} C_L(i) + K \sum_{i=1}^{N} C_L^2(i) = \sum_{i=1}^{N} C_{D_{EXP}}(i)$$

$$C_{D_o} \sum_{i=1}^{N} C_L(i) - 2KC_L' \sum_{i=1}^{N} C_L^2(i) + K \sum_{i=1}^{N} C_L^3(i) = \sum_{i=1}^{N} C_{D_{EXP}}(i) C_L(i)$$

$$C_{D_o} \sum_{i=1}^{N} C_L^2(i) - 2KC_L' \sum_{i=1}^{N} C_L^3(i) + K \sum_{i=1}^{N} C_L^4(i) = \sum_{i=1}^{N} C_{D_{EXP}}(i) C_L^2(i)$$

Maximum L/D is then determined as a function of Mach number from Equation 4-36.

2. From engine performance determine minimum SFC as a function of Mach number.

3. Substitute maximum L/D, SFC, and M into Equation 4-38. The alternative is to compute and plot R_F as a function of Mach number.

4. Substitute M* and the corresponding values for C_{D_o}, K, and C_L' into Equation 4-34 and solve for σ as a function of W.

5. From atmospheric tables determine best cruise altitude as a function of the weight.

6. Determine the cruise range from Equation 4-40.

For maximum range, the cruise altitude, range factor, and cruise range can easily be obtained from Figures 4-7 through 4-9 once M* is determined.

As an example, imagine that a certain aircraft yields M* = 0.8 as the solution to Equation 4-37. The parabolic fit to the drag polar gives for M = 0.8, C_{D_o} = 0.015, C_L' = 0, and K = 0.107. The wing loading is W/S = 60 pounds per square foot. The specific fuel

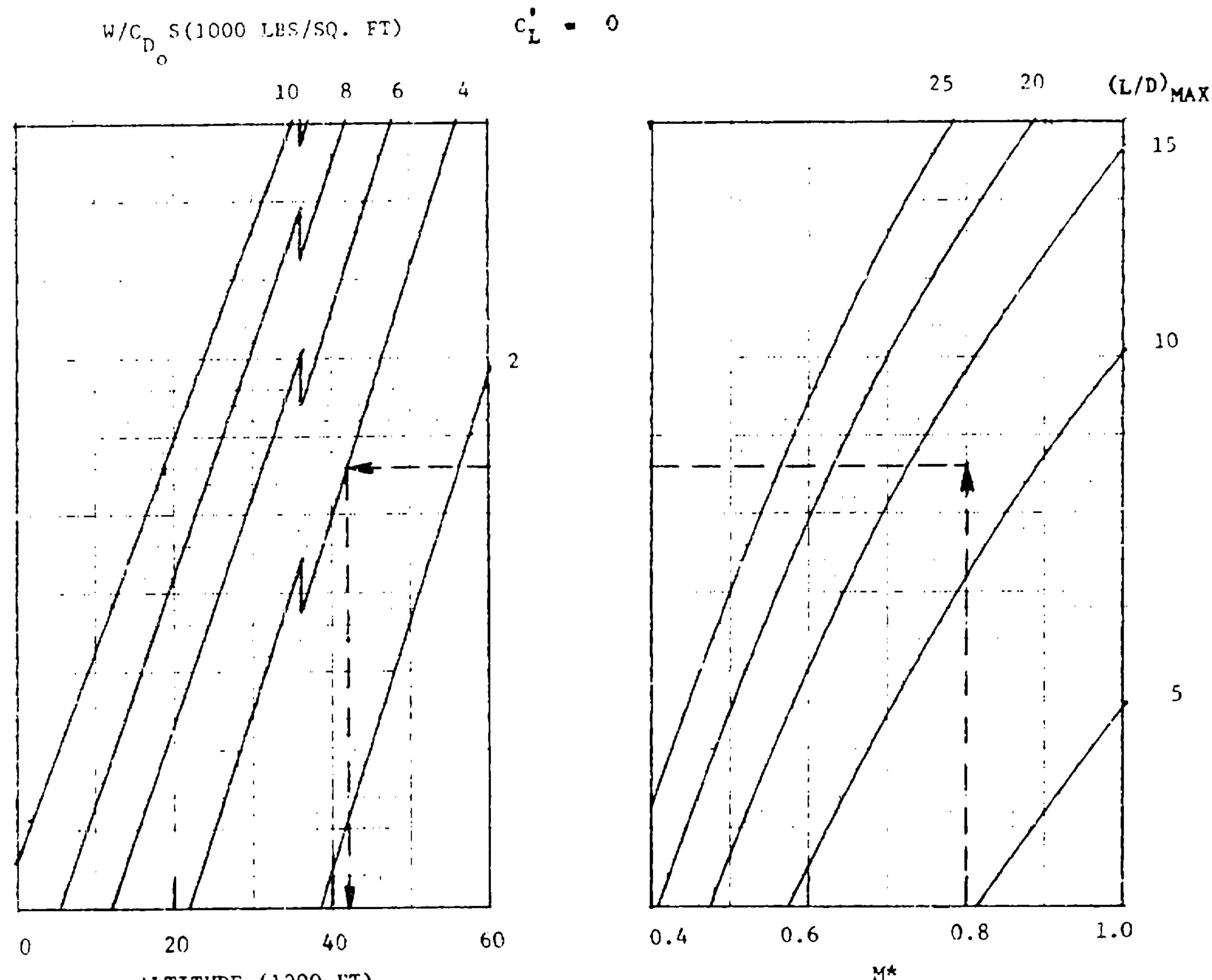

Figure 4-7 Cruise Altitude for Best Cruise Range

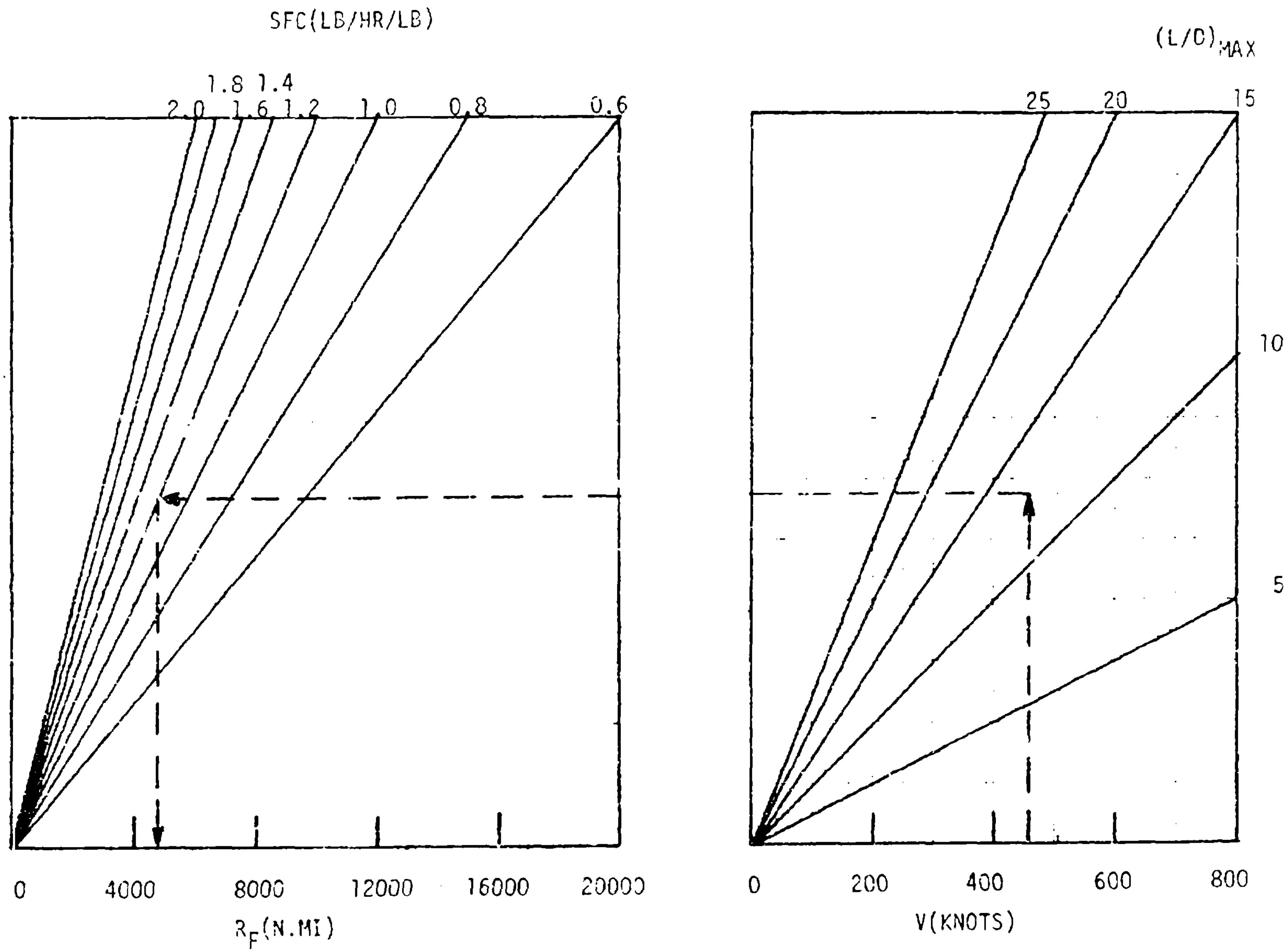

Figure 4-8 Best Cruise Range Factor

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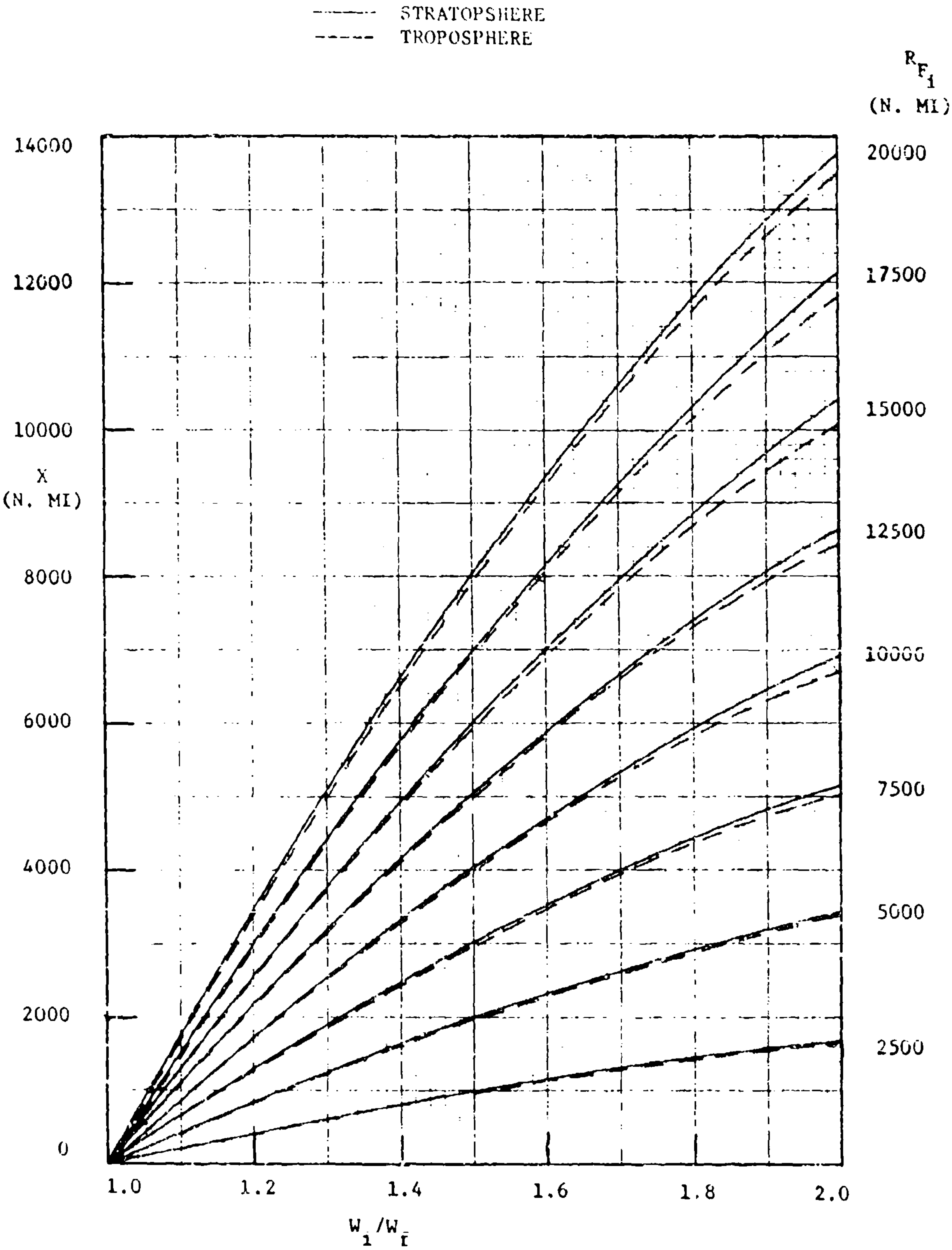

Figure 4-9 Best Cruise Range

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consumption is SFC = 1.2 pounds per hour per pound and W_i/W_f = 1.5. These data then lead to $(L/D)_{MAX}$ = 12.5 and $W/C_{D_o}S$ = 4000. From Figure 4-7, M* = 0.8, $(L/D)_{MAX}$ = 12.5, and $W/C_{D_o}S$ = 4000 gives approximately 42,000 feet for the best cruise altitude. In the stratosphere M* = 0.8 gives V = 460 knots. From Figure 4-8, V = 460 knots, $(L/D)_{MAX}$ = 12.5, and SFC = 1.2 gives R_F = 4800 nautical miles. From Figure 4-9, the cruise range is X = 1940 nautical miles.

We next turn our attention to the sensitivity of the cruise altitude and range factor to changes in the aerodynamic and engine performance characteristics.

Best Cruise Performance Sensitivity Analysis

Rewriting Equation 4-34 in logarithmic form and then forming the total differential gives approximately

$$\frac{d\sigma}{\sigma} = \frac{1}{2A}\frac{dK}{K} - \frac{1}{2A}\frac{dC_{D_o}}{C_{D_o}} + \frac{1}{A}\frac{d(W/S)}{W/S} - \frac{2}{A}\frac{dM}{M} \qquad (4\text{-}41)$$

Now σ is related approximately to the altitude in the following way

$$\sigma = \sigma_R\, e^{-\beta h} \qquad (4\text{-}42)$$

where σ_R and β are defined as follows:

ATMOSPHERIC LAYER	σ_R	β (Feet^{-1})
Troposphere	1.0	1/30500
Stratosphere	1.712	1/20600

Differentiating Equation 4-42 and combining with Equation 4-41 gives

$$dh = -\frac{1}{\beta}\frac{d\sigma}{\sigma}$$

$$= S_K\frac{dK}{K} + S_{C_{D_o}}\frac{dC_{D_o}}{C_{D_o}} + S_{W/S}\frac{d(W/S)}{W/S} + S_M\frac{dM}{M} \qquad (4\text{-}43)$$

where S_K, $S_{C_{D_0}}$, and S_M are sensitivity parameters. Note that an increase in K gives a decrease in h whereas an increase in C_{D_0} gives an increase in h. The solution for C_L at max L/D is

$$C_L = \sqrt{\frac{C_{D_0}}{K}}$$

Also, from L = W

$$C_L = \frac{W}{\frac{1}{2}\rho S V^2}$$

Assume W and V are constant. Then an increase in C_{D_0} gives an increase in C_L, which requires a decrease in ρ, which results in an increase in h. An increase in K gives a decrease in C_L, which requires an increase in ρ, which corresponds to a decrease in h.

Expanding Equation 4-39 in logarithmic form and differentiating gives approximately

$$\frac{dR_F}{R_F} = \frac{dM}{M} - \frac{dSFC}{SFC} + \frac{d(L/D)_{MAX}}{(L/D)_{MAX}}$$

$$= S_M \frac{dM}{M} + S_{SFC} \frac{dSFC}{SFC} + S_{C_{D_0}} \frac{dC_{D_0}}{C_{D_0}} + S_K \frac{dK}{K} + S_{C_L'} \frac{dC_L'}{C_L'} \quad (4-44)$$

Equations 4-43 and 4-44 form the basis for the sensitivity analysis.

There are at least two ways to examine the sensitivity equations. The first is to vary one parameter at a time while holding all other parameters constant. This is like a design change. The other way is to vary the parameters simultaneously like C_{D_0}, K, C_L', and SFC as functions of M. This could represent an operational change.

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Table 4-1 presents the "sensitivity parameters to one parameter" variations. The way that Table 4-1 is used is as follows. Assume stratospheric flight, A = 1 and β = 1/20,600. For a 10% increase in C_{D_o} alone, the change in cruise altitude is

$$dh = \frac{20,600}{2(1)}(0.1)$$

$$= 1,030 \text{ feet}$$

An increase in 10% in K gives a decrease of 1,030 feet. Assume that the aerodynamic coefficients are

$$K = 0.2$$

$$C_{D_o} = 0.02$$

$$C_L' = 0.05$$

The sensitivity coefficients are -0.406, -0.594, and 0.188 for K, C_{D_o}, and C_L' respectively. The biggest improvement in R_F results from an decrease in C_{D_o}.

TABLE 4-1

SENSITIVITY TO ONE PARAMETER VARIATIONS

Variable	S_K	$S_{C_{D_o}}$	S_M	S_{SFC}	$S_{W/S}$	$S_{C_L'}$
h	$-\frac{1}{2A\beta}$	$\frac{1}{2A\beta}$	$\frac{2}{A\beta}$	0	$-\frac{1}{A\beta}$	0
R_F	$\frac{2KC_L' - \sqrt{KC_{D_o}}}{2(\sqrt{KC_{D_o}} - KC_L')}$	$\frac{-\sqrt{KC_{D_o}}}{2(\sqrt{KC_{D_o}} - KC_L')}$	1	-1	0	$\frac{KC_L'}{\sqrt{KC_{D_o}} - KC_L'}$

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When C_{D_0}, K, and SFC are functions of M, Equation 4-43 becomes

$$dh = \frac{M}{2A\beta}\left(-\frac{1}{K}\frac{\partial K}{\partial M} + \frac{1}{C_{D_0}}\frac{\partial C_{D_0}}{\partial M} + \frac{4}{M}\right)\frac{dM}{M} - \frac{1}{A\beta}\frac{d(W/S)}{W/S} \tag{4-45}$$

$$= S_M \frac{dM}{M} + S_{W/S}\frac{d(W/S)}{W/S}$$

Equation 4-44 becomes

$$\frac{dR_F}{R_F} = \frac{dM}{M} - \frac{1}{SFC}\frac{\partial SFC}{\partial M}dM + \frac{1}{(L/D)_{MAX}}\frac{\partial (L/D)_{MAX}}{\partial M}dM$$

$$= \left[1 - \frac{M}{SFC}\frac{\partial SFC}{\partial M} + \frac{M}{(L/D)_{MAX}}\frac{\partial (L/D)_{MAX}}{\partial M}\right]\frac{dM}{M} \tag{4-46}$$

$$= S_M \frac{dM}{M}$$

The multiplier for dM/M in the latter equation is a factor in Equation 4-38. Thus, when Equation 4-38 is satisfied, $S_M = 0$ in dR_F/R_F. Table 4-2 contains the sensitivity parameters.

TABLE 4-2

SENSITIVITY TO MACH NUMBER VARIATIONS

Variable	S_M	$S_{W/S}$
h	$\frac{M}{2A\beta}\left(-\frac{1}{K}\frac{\partial K}{\partial M} + \frac{1}{C_{D_0}}\frac{\partial C_{D_0}}{\partial M} + 4\right)$	$-\frac{1}{A\beta}$
R_F	$1 - M\left[\frac{1}{SFC}\frac{\partial SFC}{\partial M} + \frac{1}{(L/D)_{MAX}}\frac{\partial (L/D)_{MAX}}{\partial M}\right]$	0

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S_M in Equation 4-46 is illustrated in Figure 4-10. In Figure 4-10 when $M = M^*$, $S_M = 0$. Below M^*, $dM < 0$ and $S_M > 0$, thus $dR_F < 0$. Above M^*, $dM > 0$ and $S_M < 0$, thus $dR_F < 0$. Consequently, R_F decreases as the Mach number moves away from M^*.

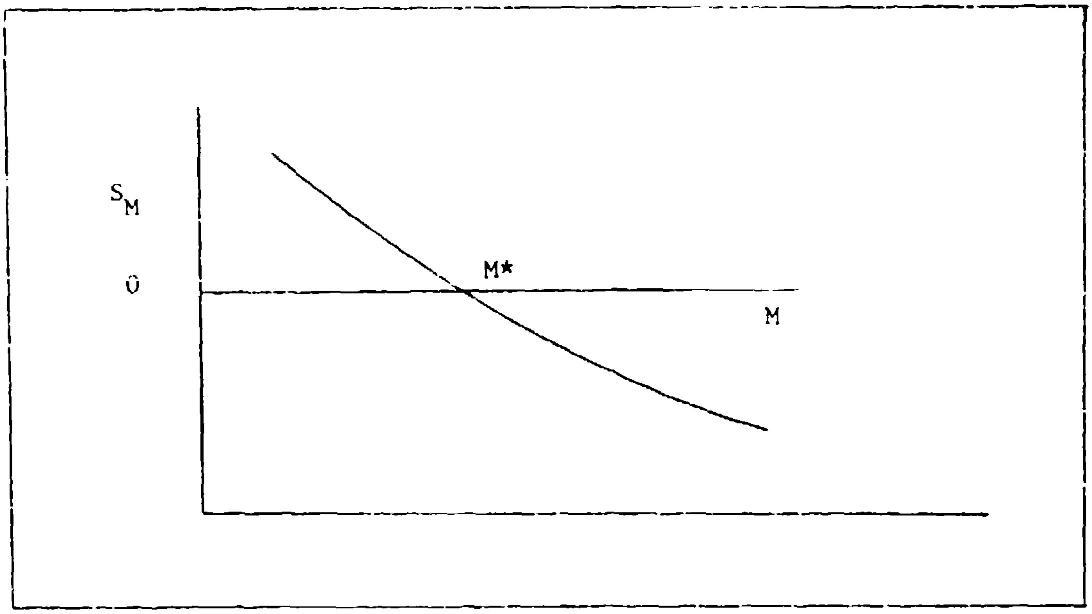

Figure 4-10 Sensitivity of Range Factor to Variations in Mach Number

We next turn our attention to an application of the analytical results derived thus far.

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Application of the Analytical Optimal Cruise Solution

The aircraft which will be considered is the F4C. Two configurations will be considered. The first configuration carries external stores and the second configuration is clean. These configurations are representative of an outbound and return leg of a military mission.

The first step is to fit parabolic polars to the aerodynamic data. This is illustrated in Figure 4-11 for M = 0.8. This should be done for all Mach numbers of interest. In Figure 4-11, the higher drag configuration is the stores configuration. The theoretical polars were derived from the minimization of the sum of the errors between the experimental data and a theoretical parabolic form. For this example C_L' was not zero, consequently $C_{D_{MIN}}$ and C_{D_0} were not the same.

SFC versus fraction of military thrust is presented in Figure 4-12 for three different Mach numbers and altitudes. As can be observed from these data, SFC is relatively insensitive to altitude changes, hence the assumption that SFC is a function of Mach number alone is satisfactory.

For this example, maximum R_F is solved for best cruise Mach number. The results of the best cruise performance are summarized in Table 4-3. The outbound cruise leg corresponds to the stores configuration. The return leg corresponds to the clean configuration. Two assumptions were examined, namely A = 1 which is representative of a stratospheric atmosphere and A = 1.235 which corresponds to a tropospheric atmosphere.

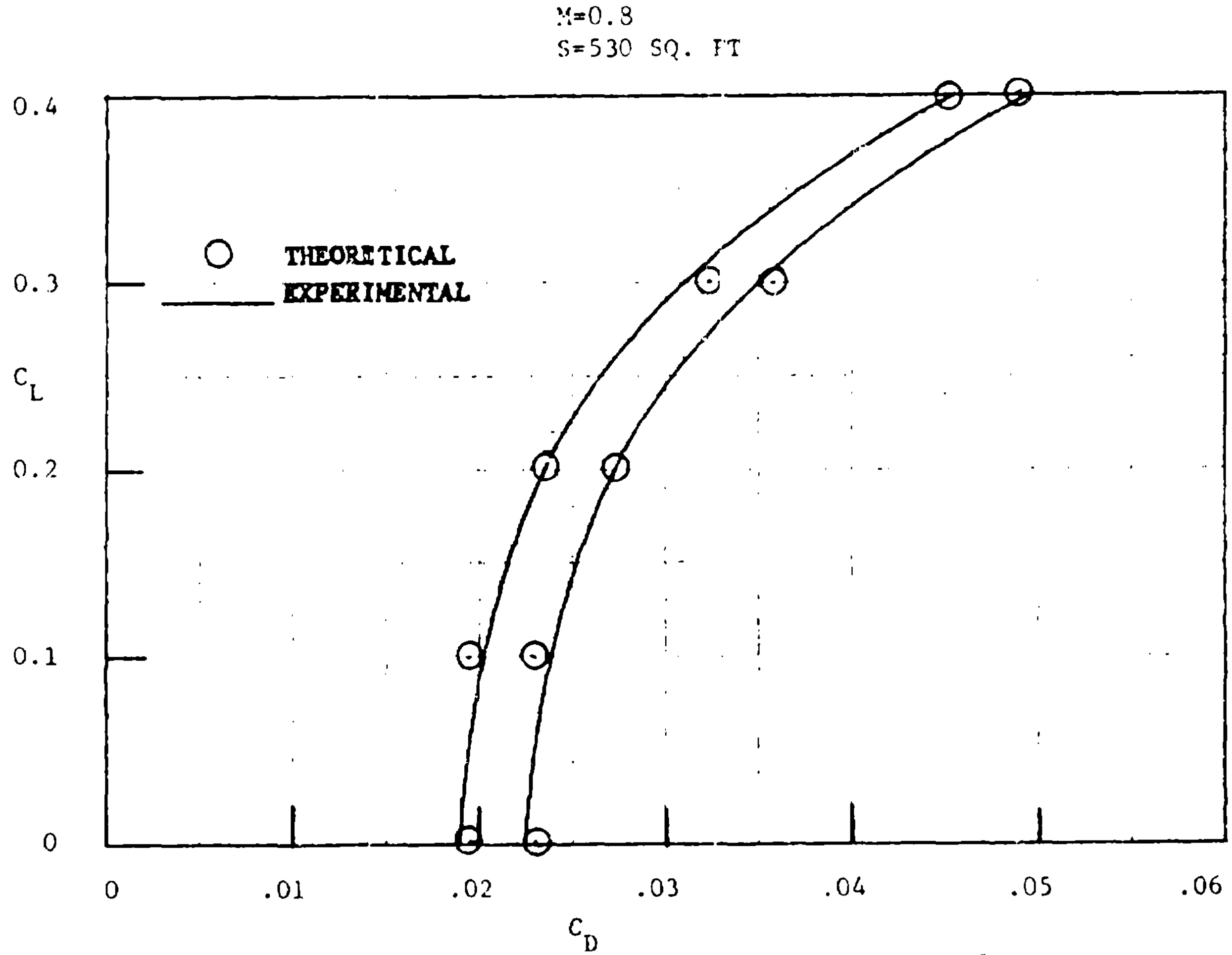

Figure 4-11 Actual and Approximate Aerodynamic Polars

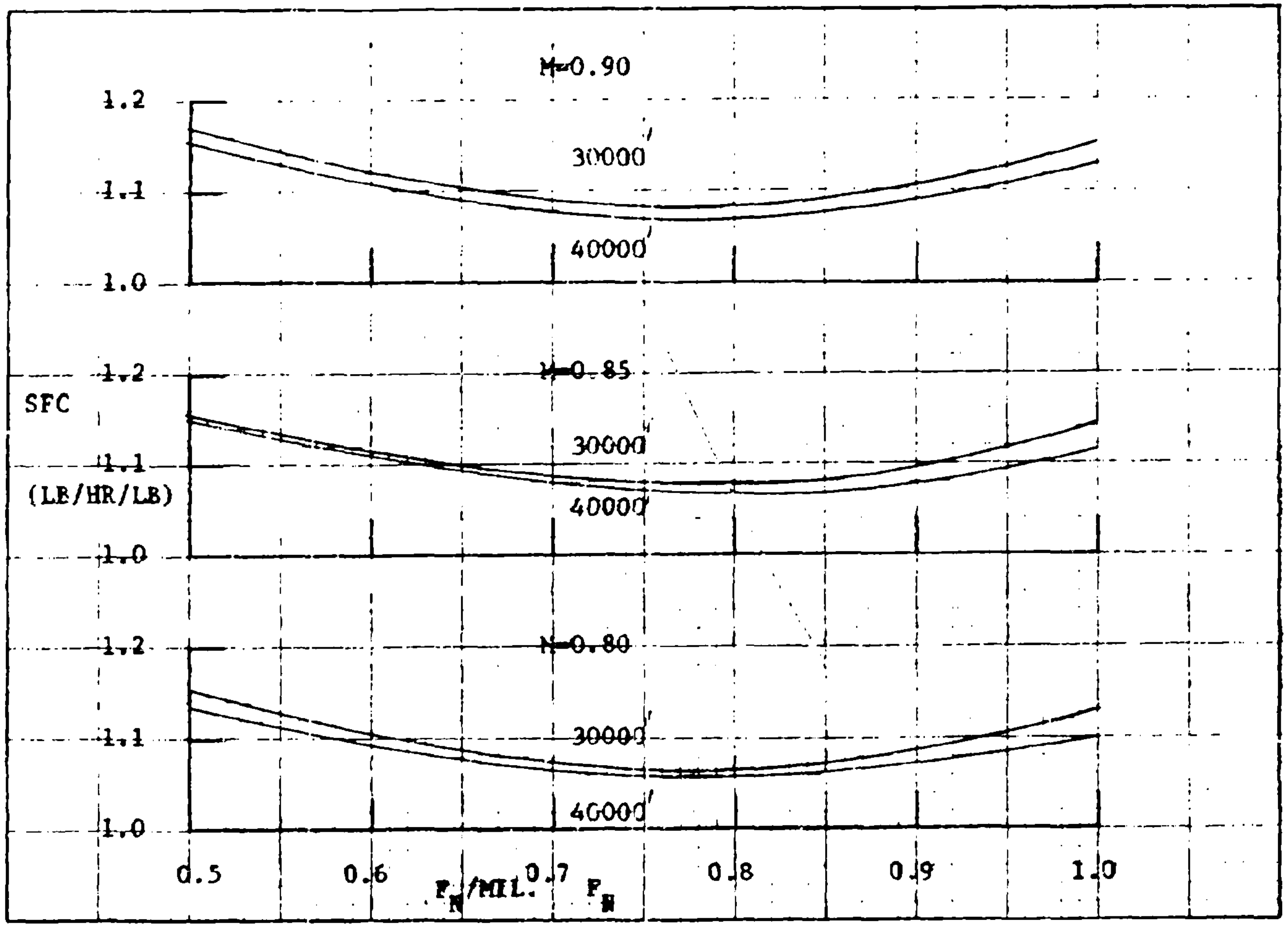

Figure 4-12 F4C Specific Fuel Consumption

TABLE 4-3

COMPARISON OF THE ANALYTICAL AND F4C CRUISE PERFORMANCE

Outbound Cruise Leg	Estimated	Generalized	Error (%)
W_i (pounds)	47779	47779	0
W_f (pounds)	46878	46878	0
$M_i = M_f$	0.85	0.86	1.2
h_i (A = 1.235) (feet)	31000	31300	1.0
h_i (A = 1.0) (feet)	31900	31300	1.9
h_f (A = 1.235) (feet)	31400	31600	0.6
h_f (A = 1.0) (feet)	32400	31600	2.5
Average V/TSFC (n.miles/pound)	0.0792	0.0810	2.2
X (nautical miles)	71	73	2.7
Return Cruise Leg			
W_i (pounds)	37406	37406	0
W_f (pounds)	33888	33888	0
$M_i = M_f$	0.85	0.87	2.3
h_i (A = 1.235) (feet)	34500	36600	5.7
h_i (A = 1.0) (feet)	36200	36600	1.1
h_f (A = 1.235) (feet)	36500	38800	5.9
h_f (A = 1.0) (feet)	38200	38800	1.6
Average V/TSFC (n.miles/pound)	0.1131	0.1131	0
X (nautical miles)	398	398	0

For the outbound leg the cruise was in the troposphere. The solution for best cruise altitude was in best agreement with the generalized data for $A = 1.235$ which corresponds to the troposphere. Best cruise Mach number was determined from the maximization of $M(L/D)_{MAX}$. The errors in cruise range and specific range factor V/W_f were less three percent.

For the return leg the beginning of the cruise was in the troposphere and the end was in the stratosphere. The stratospheric atmosphere gave the best results for best cruise altitude. The errors in cruise range and specific range factor were negligible.

The sensitivity parameter S_M in the dR_F equation is presented in Figure 4-13 for the clean configuration and $A = 1.0$. The sensitivity parameter is nearly linear with Mach number. The range factor variation with respect to Mach number is nonlinear, however. For example, a 1% change in Mach number gives a -0.12% decrease in R_F. A 5% change in M results in approximately a 3% decrease in R_F.

The next problem to be addressed is the determination of the best altitude and speed for maximum endurance.

Best Endurance Performance

The difference between this problem and the previous one is that Equation 4-13 rather than Equation 4-11 is to be optimized. Examination of Equations 4-1 and 4-13 shows that maximization of endurance time corresponds to minimization of fuel flow rate. The graphical

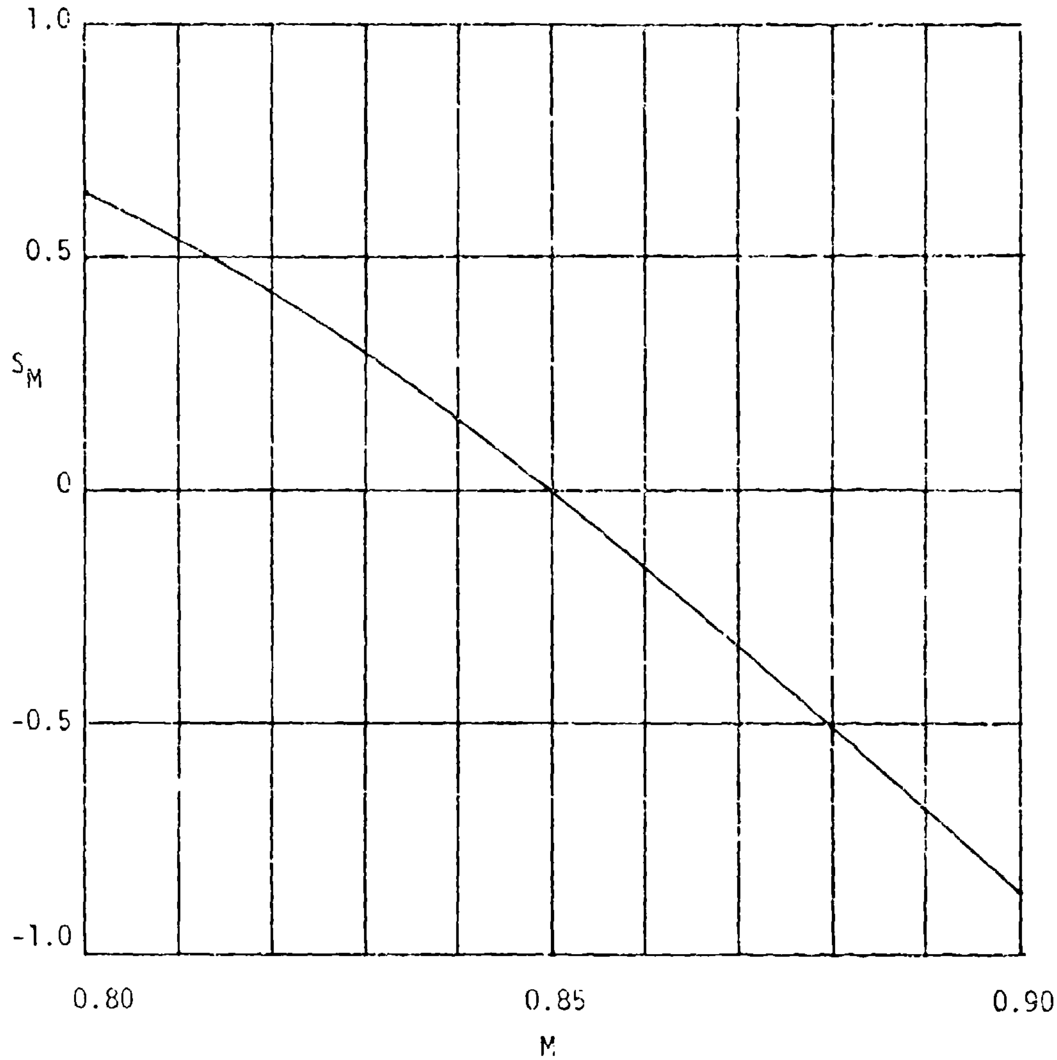

Figure 4-13 F4C Range Factor Sensitivity

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method for determining best endurance is straightforward. For a selected weight, both altitude and speed are varied until the minimum fuel flow condition is met. This approach is illustrated in Figure 4-14.

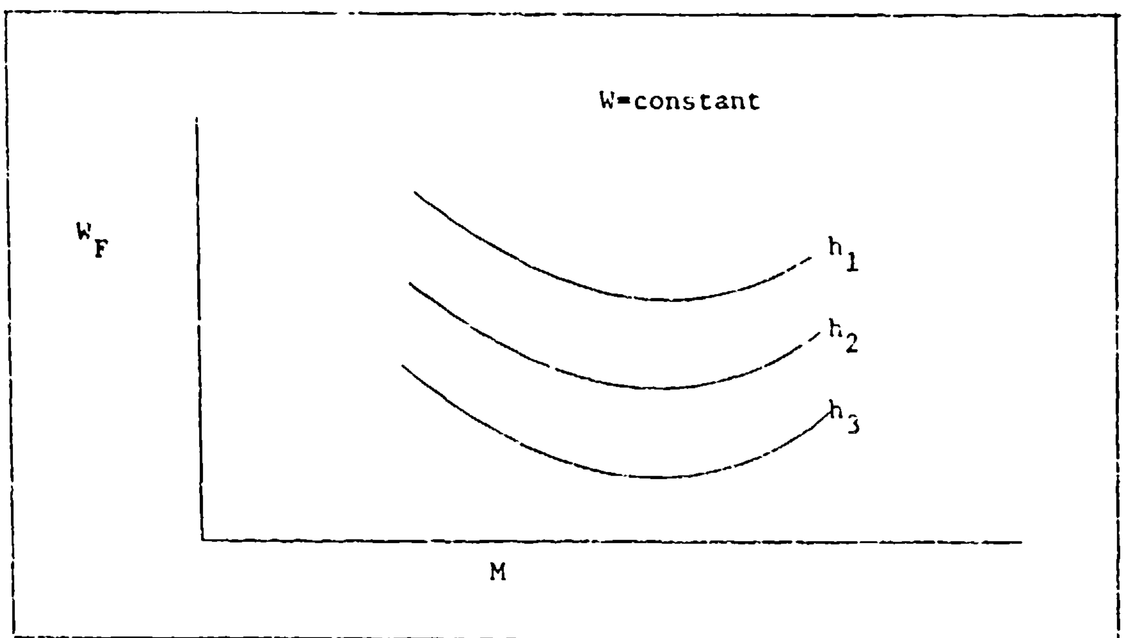

Figure 4-14 W_F Versus M

For each altitude there is a speed which yields minimum fuel flow rate. The plot of minimum W_F and the corresponding best cruise speed for each altitude then identifies best cruise speed and altitude for maximum endurance and the minimum fuel flow rate. This situation is presented in Figure 4-15.

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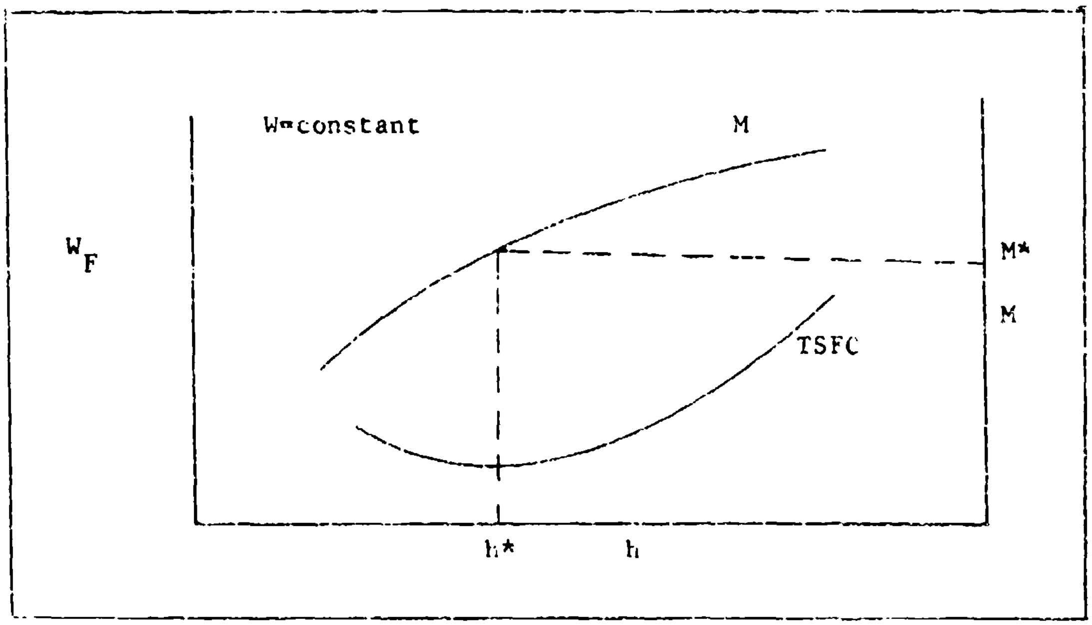

Figure 4-15 W_F and M Versus h

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In Figure 4-15, h* and M* are the cruise altitude and Mach number for maximum endurance. For each weight, minimum W_F is therefore easily determined, the result being like that in Figure 4-16. The best endurance solution is obtained by integration of the reciprocal of the TSFC, that is

$$t = -\int_{W_i}^{W_f} \frac{1}{W_F} dW$$

Under certain assumptions an approximate semi-analytic solution can be determined for the best endurance trajectory. We now turn our attention to this problem.

An Approximate Solution to the Maximum Endurance Problem

Three assumptions are introduced, one of which is the same as the previous problem. The assumptions are as follows.

1. The aerodynamic drag polar can be approximated by a parabolic polar, i.e.,

$$C_D = C_{D_o} + KC_L^2$$

2. The relationship between W_F and thrust is linear. (This does not imply that SFC is constant.) Thus

$$W_F = aT + b$$

where a and b are constants. Since thrust and drag are equal, minimum TSFC corresponds to minimum drag.

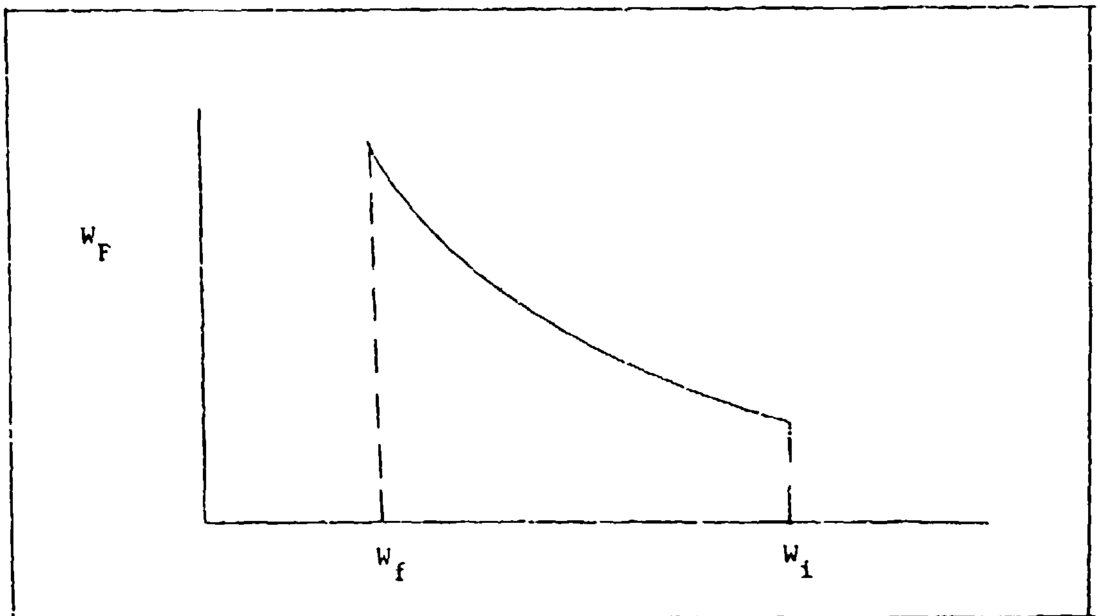

Figure 4-16 Minimum W_F Versus W

3. The aerodynamic coefficients C_{D_o} and K are constant. The justification for this is that analyses of maximum endurance for existing aircraft have occurred at speeds where the aerodynamic coefficients were constant.

In light of these assumptions, maximum endurance corresponds to minimum drag and consequently max L/D. Since lift and weight are equal, the speed and altitude are these values which minimize the drag. Since

$$D = \frac{1}{2}\rho SV^2 C_{D_o} + \frac{2KW^2}{\rho SV^2}$$

minimization with respect to V gives the necessary condition

$$\rho SVC_{D_o} - \frac{4KW^2}{\rho SV^3} = 0 \qquad (4\text{-}47)$$

Minimization with respect to the density gives

$$\frac{1}{2} SV^2 C_{D_o} - \frac{2KW^2}{\rho^2 SV^2} = 0 \qquad (4\text{-}48)$$

Equations 4-47 and 4-48 are not independent, hence a dilemma results; two unknowns and one independent equation exist.

The best that can be done under these circumstances is to let either altitude or speed be an independent variable and the other be a dependent variable. Let h or ρ be the independent variable and V the dependent variable. From either Equation 4-47 or 4-48, best cruise speed V* is

$$V^* = \left(\frac{4W^2K}{\rho^2 S^2 C_{D_o}} \right)^{1/4} \qquad (4\text{-}49)$$

Clearly V^* increases with increasing altitude. For a given weight, Equation 4-49 defines the relation between V^* and altitude. The final step is to substitute h and V^* into the actual engine performance and calculate TSFC. The optimum altitude, h^*, is that altitude which minimizes TSFC. This then identifies the altitude and speed for maximum endurance.

Substitution of Equation 4-49 into the drag relation gives

$$D = 2W\sqrt{KC_{D_0}}$$

Thus, in light of the second assumption

$$\frac{dt}{dW} = -\frac{1}{TSFC}$$

$$= -\frac{1}{aD+b}$$

$$= -\frac{1}{2a\,W\sqrt{KC_{D_0}}+b} \qquad (4\text{-}50)$$

This equation is analytically integrable, thus the endurance, t, is

$$t = \frac{1}{2a\sqrt{KC_{D_0}}}\ln\frac{2aW_i\sqrt{KC_{D_0}}+b}{2aW_f\sqrt{KC_{D_0}}+b} \qquad (4\text{-}51)$$

Equation 4-51 then defines the approximate semi-analytical solution for maximum endurance. We next turn our attention to the sensitivity parameters relative to endurance.

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Sensitivity Parameters for the Endurance Problem

In light of Equation 4-50, the only parameters which can change the endurance time are K and C_{D_o} since W is the independent variable. Endurance is sensitive to changes in the fuel flow rate, W_F. Differentiating

$$W_F = 2aW\sqrt{KC_{D_o}} + b$$

gives

$$\frac{dW_F}{W_F} = \frac{aW\sqrt{KC_{D_o}}}{2aW\sqrt{KC_{D_o}} + b}\left(\frac{dK}{K} + \frac{dC_{D_o}}{C_{D_o}}\right)$$

$$= S_K\frac{dK}{K} + S_{C_{D_o}}\frac{dC_{D_o}}{C_{D_o}}$$

The sensitivity parameters are therefore

$$S_K = S_{C_{D_o}} = \frac{aW\sqrt{KC_{D_o}}}{2aW\sqrt{KC_{D_o}} + b} \tag{4-52}$$

Application of the Maximum Endurance Solution

Again the F4C will be studied. Only the clean configuration will be considered this time. The relation between $W_f/2$ and $F_N/2$ (the McDonnell data is for single engine performance) is presented in Figure 4-17. F_N is the net installed thrust, and W_f is the fuel flow rate. The upper and lower bounds represent the difference in the data at Mach numbers of 0.6 and 0.7 and altitudes of 25,000, 30,000 and

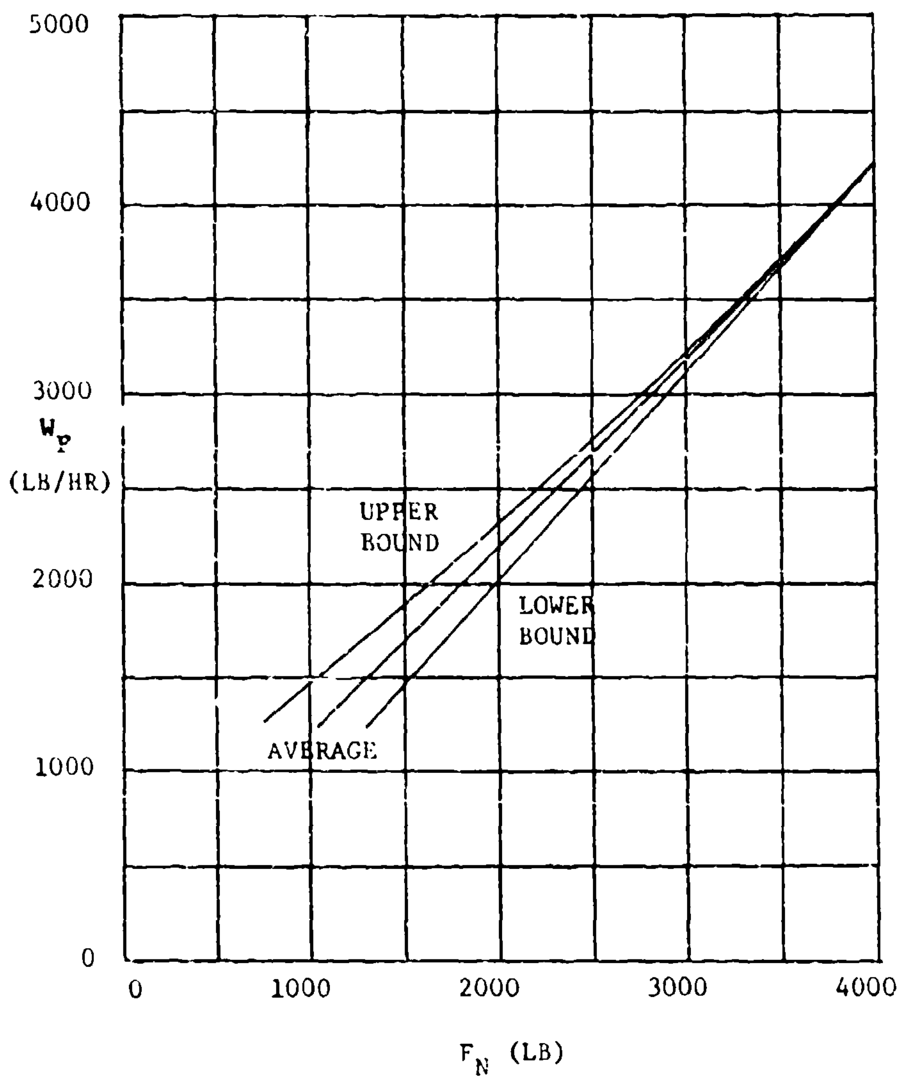

Figure 4-17 F4C Single Engine Fuel Flow Versus Thrust

35,000 feet. The linear fit through the average values gives

$$W_F = 473.4 + 0.983F_N$$

Below M = 0.7 the aerodynamic coefficients are constant. Assume W = 30,000 pounds. The solution of Equation 4-49 is presented in Figure 4-18. The agreement with the MAC data was very good - the biggest error was 2.6% at sea level. Given the speed and altitude, as in Figure 4-18, then the drag can be computed as a function of altitude. Using the MAC engine performance then determines W_f, which is presented in Figure 4-19. The altitude for maximum endurance at W = 30,000 pounds is approximately 30,000 feet. The MAC result was 32,800 feet, which represents an 8.5% error.

The sensitivity parameters for W = 30,000 pounds are

$$S_K = S_{C_{D_o}} = 0.428$$

Consequently, a 1% change in either C_{D_o} or K results in a 0.428% change in W_f.

The next problem to be studied is the constant-altitude trajectory.

Constant Altitude Cruise Trajectory

The objective here is the evaluation of the best cruise speed and the penalties associated with cruising at speeds other than the speed and altitude for maximum cruise range. As in the problem for best cruise range performance, the objective is to maximize the range factor. The difference here is that only the speed is free. The general approach for determining best R_F is to determine L/D as in

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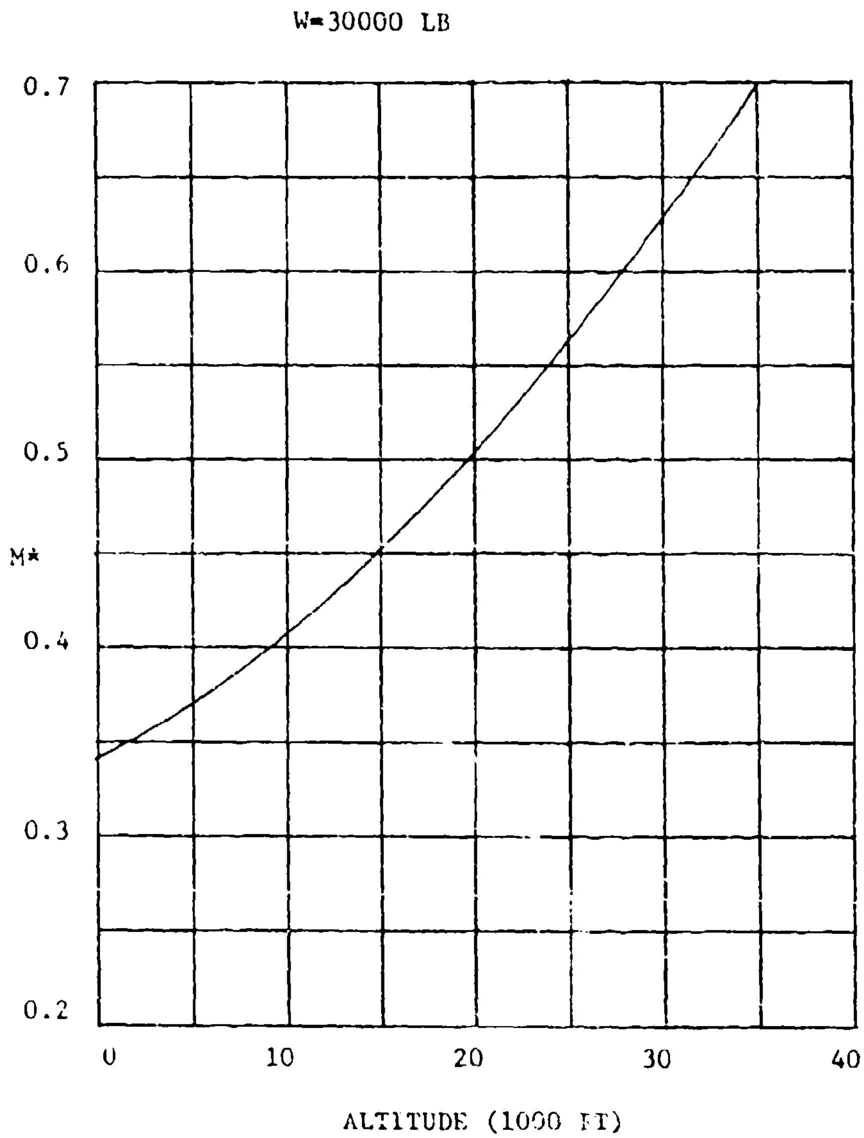

Figure 4-18 F4C Best Endurance Speed

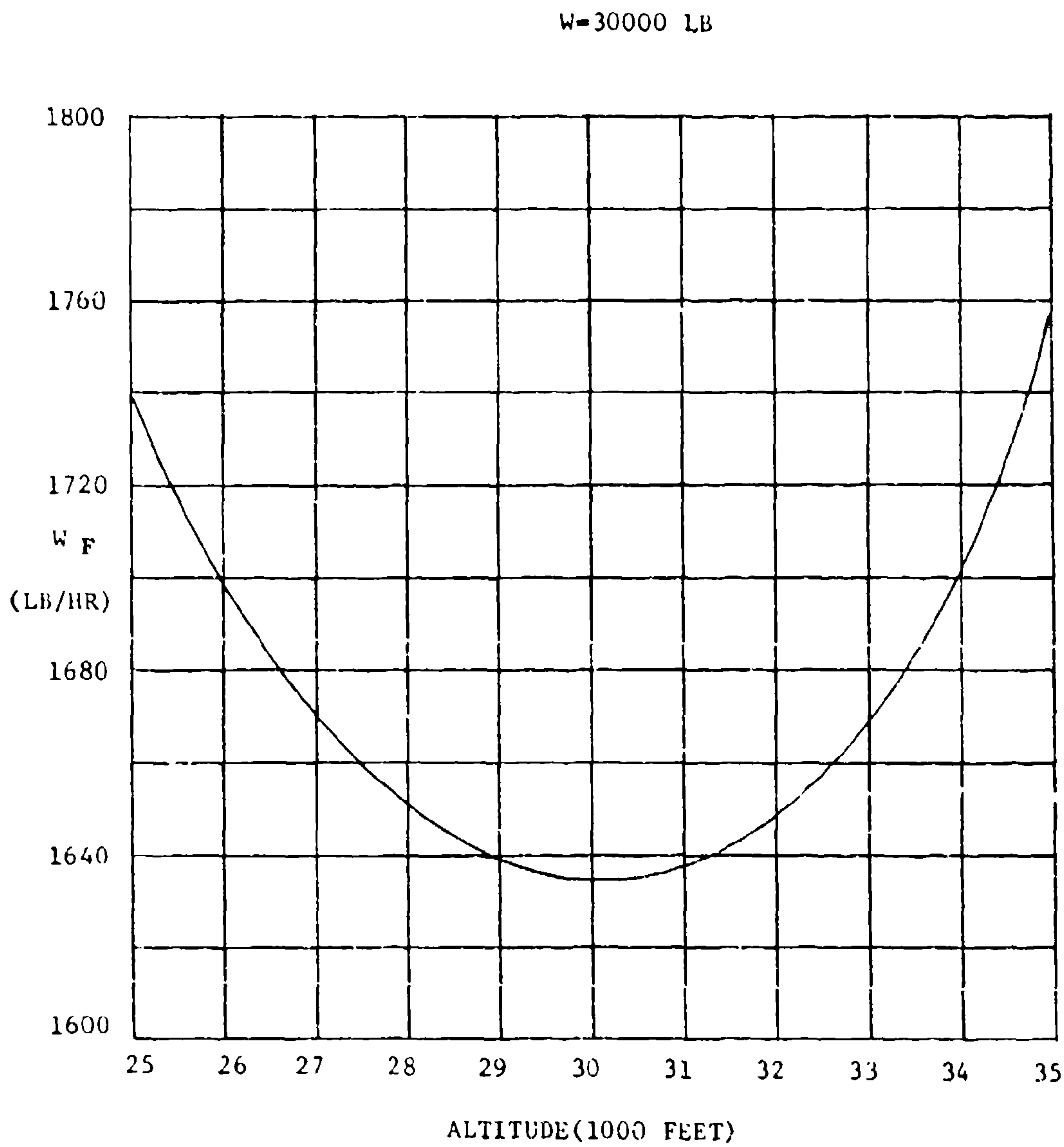

Figure 4-19 F4C Single Engine Fuel Flow

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Figure 4-1 for the given weight and altitude, SFC as in Figure 4-3, and then R_F as in Figure 4-4.

The analytical work was accomplished earlier. The necessary condition is Equation 4-31. Expanding gives Equation 4-38. Substituting the parabolic polar then gives the necessary condition for the cruise Mach number, namely

$$
\begin{aligned}
-\frac{M}{SFC}\frac{\partial SFC}{\partial M} - \frac{\lambda S M^2 \sigma^A C_{D_o}}{2D}\left(1 + \frac{M}{C_{D_o}}\frac{\partial C_{D_o}}{\partial M}\right) \\
+ \frac{2KW^2}{\lambda S M^2 \sigma^A D}\left(3 - \frac{M}{K}\frac{\partial K}{\partial M}\right) = 0
\end{aligned}
\tag{4-53}
$$

The solution of Equation 4-53 yields best cruise Mach number for the specified altitude and weight. From this the speed and drag can be computed and then R_F determined. The details are as follows. From Equation 4-29

$$
D = \tfrac{1}{2}\lambda S M^2 \sigma^A C_{D_o} + \frac{2KW^2}{\lambda S M^2 \sigma^A}
$$

Since L = W and V = Ma, the range factor is

$$
R_F = \frac{MaW}{SFC\left(\frac{1}{2}\lambda S M^2 \sigma^A C_{D_o} + \frac{2KW^2}{\lambda S M^2 \sigma^A}\right)} \tag{4-54}
$$

Note that Equation 4-53 gives best cruise Mach number, M*, as a function of the weight. Furthermore, as the weight decreases, M* also decreases. The rationale for this conclusion is as follows.

Let

$$A(M) = \frac{\lambda^2 S^2 M^4 \sigma^2 A C_{D_o}}{4KW^2}\left(1 + \frac{M}{C_{D_o}}\frac{\partial C_{D_o}}{\partial M} + \frac{M}{SFC}\frac{\partial SFC}{\partial M}\right)$$

$$B(M) = 3 - \frac{M}{K}\frac{\partial K}{\partial M} - \frac{M}{SFC}\frac{\partial SFC}{\partial M}$$

The necessary condition for best cruise speed is

$$A(M^*) = B(M^*)$$

In Figure 4-20, A(M) and B(M) are illustrated for different weights W_1 and W_2 where W_2 is greater than W_1. It can be seen that $M_2^* > M_1^*$.

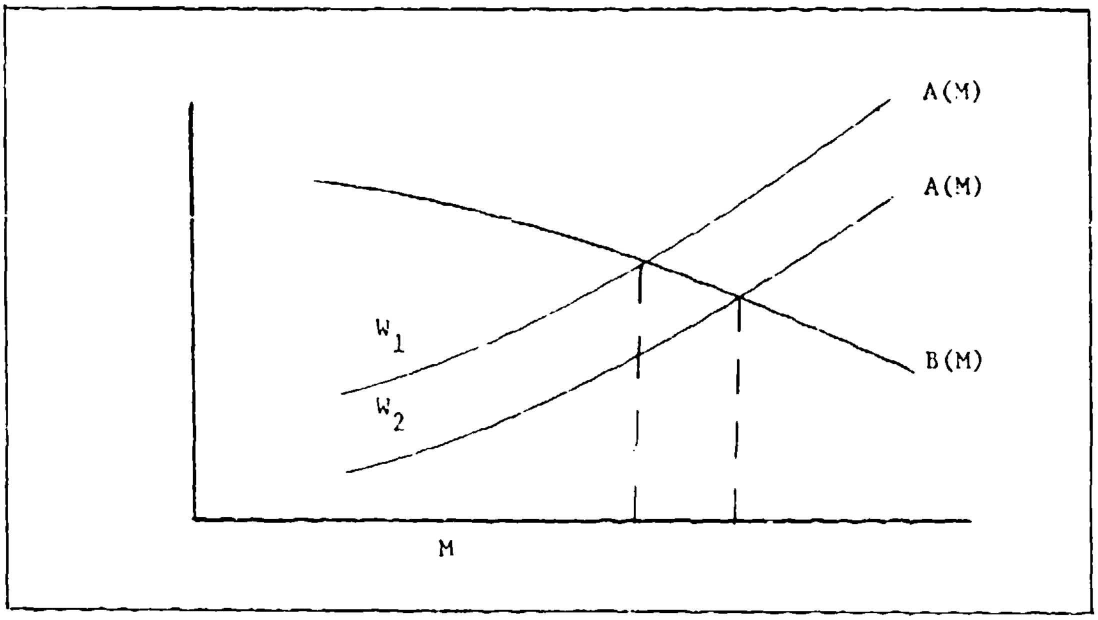

Figure 4-20 Best Cruise Mach Number as a Function of Weight

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Since the solution of Equation 4-53 yields M^* as a function of weight, integration of R_F/W in Equation 4-17 can be accomplished only by numerical integration. If, however, the change in M^* with W is small then Equation 4-17 can be analytically integrated. Based upon this assumption, Equations 4-54 and 4-17 give

$$X = \frac{Ma}{SFC\sqrt{KC_{D_o}}}\left(\tan^{-1}\frac{2KW_i}{\lambda SM^2\sigma^{A}C_{D_o}} - \tan^{-1}\frac{2KW_f}{\lambda SM^2\sigma^{A}C_{D_o}}\right) \qquad (4\text{-}55)$$

We next address the sensitivity of the range factor to changes in the parameters.

<u>Sensitivity Parameters for the Constant Altitude Problem</u>

The general form for the total differential of R_F is

$$\frac{dR_F}{R_F} = S_M\frac{dM}{M} + S_{C_{D_o}}\frac{dC_{D_o}}{C_{D_o}} + S_K\frac{dK}{K} + S_{SFC}\frac{dSFC}{SFC} + S_{W/S}\frac{d(W/S)}{W/S}$$

If the parameters C_{D_o} and K and the variable M are treated as independent, then the sensitivity parameters are as follows.

$$S_M = \frac{1}{D}\left(\frac{6KW^2}{\lambda SM^2\sigma^{A}} - \frac{1}{2}\lambda SM^2\sigma^{A}C_{D_o}\right)$$

$$S_{C_{D_o}} = -\frac{\lambda SM^2\sigma^{A}C_{D_o}}{2D}$$

$$S_K = -\frac{2W^2K}{\lambda SM^2\sigma^{A}D}$$

$$S_{SFC} = -1$$

$$S_{W/S} = \frac{1}{D}\left(\frac{1}{2}\lambda S M^2 \sigma A C_{D_o} - \frac{2KW^2}{\lambda M^2 \sigma A S}\right)$$

If, on the other hand, C_{D_o}, K, and SFC are functions of M, then the sensitivity parameter is S_M where

$$S_M = -\frac{M}{SFC}\frac{\partial SFC}{\partial M} - \frac{\lambda S M^2 \sigma A C_{D_o}}{2D}\left(1 + \frac{M}{C_{D_o}}\frac{\partial C_{D_o}}{\partial M}\right) + \frac{2KW^2}{\lambda S M^2 \sigma A D}\left(3 - \frac{M}{K}\frac{\partial K}{\partial M}\right) \qquad (4\text{-}56)$$

S_M is the left side of Equation 4-53. Thus $S_M = 0$ at the optimal cruise speed. Furthermore, S_M as a function of speed looks like that illustrated in Figure 4-21.

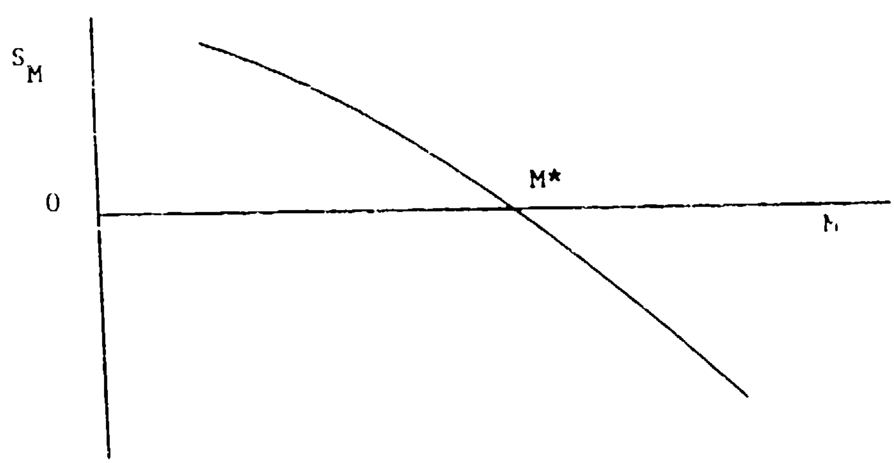

Figure 4-21 Range Factor Sensitivity to Variations in Mach Number

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In Figure 4-21, M* is the best cruise Mach number. Cruise at speeds other than at M* results in a decrease in R_F since S_M dM/M is negative on either side of M*.

Application of the Constant Altitude Cruise Solution

The F4C, clean configuration, will be considered again. Assume that the cruise altitude is 36,000 feet. This altitude falls between the initial and final cruise altitudes of the return leg defined in the problem on optimal cruise range performance. The solutions for M* according to Equation 4-53, L/D, and R_F are as follows:

			h=const	Best Cruise Climb
W	M*	L/D	R_F	R_F
33,000	0.861	8.65	4010	4052
38,000	0.876	8.77	4043	4052

Consequently, the range factor variation for constant altitude cruise relative to best cruise climb is 0.2 to 1.0%. In addition, the change in R_F due to changes in W is negligible; therefore, R_F for the F4C at 36,000 feet altitude can be taken as constant.

For the sensitivity parameter S_M, when SFC, C_{D_0}, and K are functions of M, Figure 4-22 shows that S_M varies linearly with M. The sensitivity of the range factor to changes in the weight is negligible. For a 1% change in cruise Mach number, the range factor decreases by approximately 0.2%. For a 5% change in the cruise Mach number, the range factor changes by approximately 4.0%.

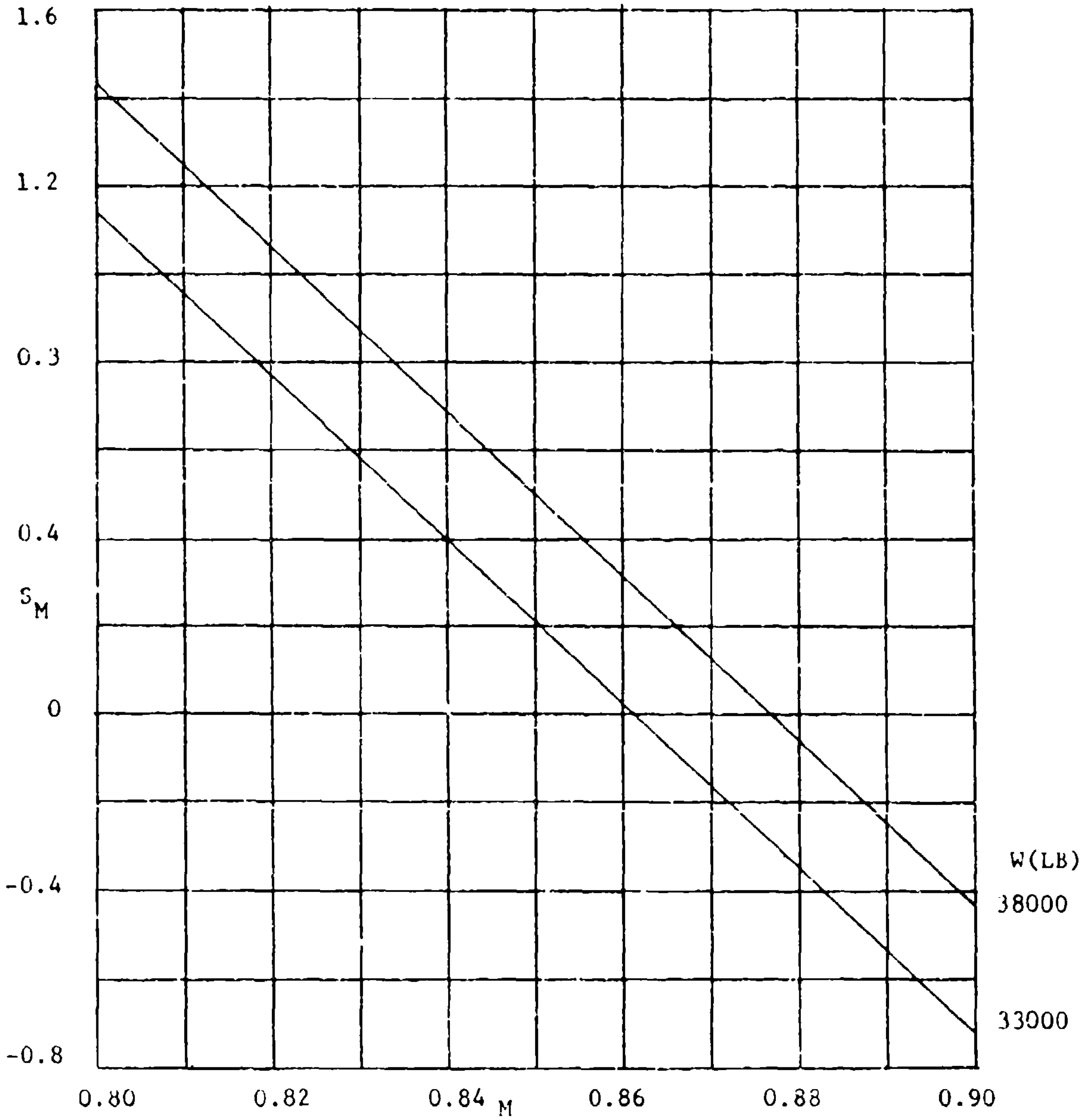

Figure 4-22 F4C Range Factor Sensitivity

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Constant Speed Cruise Performance

Given the cruise speed or Mach number, the problem is to determine the cruise altitude for a given weight which maximizes the range factor. The general approach is to first determine L/D as in Figure 4-2 for the given weight and cruise speed. SFC is determined from data as in Figure 4-3 and finally R_F is determined from Figure 4-4.

The analytical approach follows. The necessary condition for best cruise altitude is defined by Equation 4-30.

$$\frac{\partial}{\partial \sigma} R_F = \frac{\partial}{\partial \sigma} \left(\frac{MaW}{D \cdot SFC} \right) = 0 \qquad (4\text{-}30)$$

There exists a constraint which must be considered in order to obtain a valid solution. It is necessary that the solution of Equation 4-30 result in the drag equal to or less than the maximum thrust available. For a parabolic polar, expansion of Equation 4-30 gives

$$\frac{1}{a}\frac{\partial a}{\partial \sigma} - \frac{1}{D}\frac{\partial D}{\partial \sigma} - \frac{1}{SFC}\frac{\partial SFC}{\partial \sigma} = 0$$

Substitution for a and D gives

$$\frac{1}{2}\,(A-1)\,\frac{1}{\sigma} - \frac{A}{D\sigma}\left(\frac{1}{2}\,\lambda S M^2 \sigma^A C_{D_0} - \frac{2KW^2}{\lambda S M^2 \sigma^A} \right) \qquad (4\text{-}57)$$

$$- \frac{1}{SFC}\frac{\partial SFC}{\partial \sigma} = 0$$

Equation 4-57 is the general relationship which must be solved for σ. The solution for σ substituted into Equation 4-29 gives the drag. This must then be compared with the maximum available thrust. If the

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drag exceeds the maximum thrust, then the correct solution for σ is determined from the equality of the drag and maximum thrust.

There is a special case for which an analytical solution for σ can be determined. If cruise is in the stratosphere, $A = 1$. If in addition SFC is a function of Mach number alone, then Equation 4-57 has for the solution

$$\sigma = \frac{2W}{\lambda SM^2} \sqrt{\frac{K}{C_{D_o}}} \tag{4-58}$$

which agrees with Equation 4-34 when $A = 1$. The solution of Equation 4-58 corresponds to maximum L/D ratio for the specified cruise Mach number.

Equation 4-58 when substituted into Equation 4-29 gives

$$D = 2W\sqrt{KC_{D_o}} \tag{4-59}$$

If this value exceeds maximum thrust, then the solution must be modified. In the stratosphere it can generally be assumed that

$$\frac{MAX\ T}{\sigma} = \text{constant}$$

Equating maximum thrust and drag gives

$$\frac{MAX\ T}{\sigma} = \frac{1}{\sigma}\left(\tfrac{1}{2}\lambda SM^2\sigma\, C_{D_o} + \frac{2KW^2}{\lambda SM^2\sigma} \right) \tag{4-60}$$

The solution for σ is therefore

$$\sigma = \left(\frac{2KW^2}{\lambda SM^2}\right)^{\frac{1}{2}} \left(\frac{MAXT}{\sigma} - \frac{1}{2}\lambda SM^2 C_{D_o} \right)^{-\frac{1}{2}} \tag{4-61}$$

Substitution of σ into D gives

$$D = \frac{W}{M}\left(\frac{MAXT}{\sigma}\right)\sqrt{\frac{2K}{\lambda S}}\left(\frac{MAXT}{\sigma} - \frac{1}{2}\lambda S M^2 C_{D_o}\right)^{-\frac{1}{2}} \tag{4-62}$$

Cruise L/D is therefore

$$\frac{L}{D} = \frac{M}{\left(\frac{MAXT}{\sigma}\right)}\left[\frac{\lambda S}{2K}\left(\frac{MAXT}{\sigma} - \frac{1}{2}\lambda S M^2 C_{D_o}\right)\right]^{\frac{1}{2}} \tag{4-63}$$

Thus, when the drag corresponding to MAX L/D exceeds the max thrust available, the altitude is determined from Equation 4-61 and the cruise L/D is defined by Equation 4-63. Note that L/D is constant unless the aerodynamic coefficients or max T/σ changes with altitude.

The range factor reduces to, for constant speed, SFC(M), and stratospheric cruise,

$$R_F = \frac{V(L/D)_{MAX}}{SFC} \qquad \text{if } MAXT \geq \frac{W}{MAX\ (L/D)}$$

$$= \frac{VM}{\left(\frac{MAXT}{\sigma}\right)\cdot SFC}\left[\frac{\lambda S}{2K}\left(\frac{MAXT}{\sigma} - \frac{1}{2}\lambda S M^2 C_{D_o}\right)\right]^{\frac{1}{2}} \tag{4-64}$$

$$\text{if } MAXT < \frac{W}{MAX(L/D)}$$

Consider the sensitivity of altitude and range factor to aerodynamic parameters and engine performance.

Sensitivity Parameters for the Constant Speed Trajectory

There are two situations which must be considered, thrust greater than or equal to the drag, and thrust less than the drag. If the

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thrust is greater, then Equations 4-58 and 4-59 hold when it is assumed that SFC is independent of altitude and cruise is in the stratosphere. The total differentials of Equations 4-57 and 4-42 give

$$dh = \frac{1}{2\beta}\left(\frac{dC_{D_o}}{C_{D_o}} - \frac{dK}{K}\right) = S_{C_{D_o}}\frac{dC_{D_o}}{C_{D_o}} + S_K\frac{dK}{K} \tag{4-65}$$

An increase in C_{D_o} (K) gives an increase (decrease) in h. The explanation for this was presented in the sensitivity analysis of the first problem on best cruise performance. If the maximum thrust is less than the drag corresponding to maximum L/D, then Equations 4-42 and 4-61 give

$$dh = -\frac{1}{2\beta}\frac{dK}{K} + \frac{1}{2\beta}\frac{d\left(\frac{MAXT}{\sigma}\right) - \frac{1}{2}\lambda SM^2 dC_{D_o}}{\frac{MAXT}{\sigma} - \frac{1}{2}\lambda SM^2 C_{D_o}}$$

$$= S_K\frac{dK}{K} + S_{C_{D_o}}\frac{dC_{D_o}}{C_{D_o}} + S_{T/\sigma}\frac{d\left(\frac{TMAX}{\sigma}\right)}{\frac{TMAX}{\sigma}}$$

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The total differential of R_F for constant V and W becomes

$$\frac{dR_F}{R_F} = -\frac{1}{2}\left(\frac{dK}{K} + \frac{dC_{D_o}}{C_{D_o}}\right) - \frac{dSFC}{SFC} \qquad \text{if } MAXT \geq \frac{W}{MAX(L/D)}$$

$$= -\frac{1}{2}\frac{dK}{K} - \frac{dSFC}{SFC} + \frac{1}{2}\,\frac{\left[d\left(\frac{MAXT}{\sigma}\right) - \frac{1}{2}\lambda S M^2 dC_{D_o}\right]}{\frac{MAXT}{\sigma} - \frac{1}{2}\lambda S M^2 C_{D_o}}$$

$$\text{if } MAXT < \frac{W}{MAX(L/D)}$$

$$= S_K \frac{dK}{K} + S_{C_{D_o}} \frac{dC_{D_o}}{C_{D_o}} + S_{SFC} \frac{dSFC}{SFC} + S_{T/\sigma} \frac{d\left(\frac{MAXT}{\sigma}\right)}{\frac{MAXT}{\sigma}}$$

The sensitivity parameters are tabulated in Table 4-4.

An Application of the Constant Speed Cruise Solution

We will consider the supersonic cruise performance of the Supersonic Transport (SST). The cruise is in an atmospheric environment which is 8^0 above that for a standard day at the same pressure altitude. As a consequence, in the stratosphere the density-pressure relation must be adjusted to the following

$$\frac{P}{P_o} = 0.7542\sigma \qquad \rho_o = \text{non standard sea level density}$$

The cruise Mach number is 2.62. A parabolic approximation to the drag polar at M = 2.62 gives C_{D_o} = 0.0084 and K = 0.497. The reference area is 7700 square feet. Minimum SFC shows negligible variation with

TABLE 4-4

SENSITIVITY PARAMETERS FOR CONSTANT SPEED CRUISE PERFORMANCE

$$MAXT \geq \frac{W}{MAX(L/D)}$$

Variable	S_K	$S_{C_{D_o}}$	S_{SFC}	$S_{T/\sigma}$
h	$-1/2\beta$	$1/2\beta$	0	0
R_F	$-1/2$	$-1/2$	-1	0

$$MAXT \leq \frac{W}{MAX(L/D)}$$

Variable	S_K	$S_{C_{D_o}}$	S_{SFC}	$S_{T/\sigma}$
h	$-1/2\beta$	$\dfrac{-\frac{1}{4\beta}\lambda SM^2 C_{D_o}}{\frac{MAXT}{\sigma} - \frac{1}{2}\lambda SM^2 C_{D_o}}$	0	$\dfrac{\frac{1}{2\beta}\frac{MAXT}{\sigma}}{\frac{MAXT}{\sigma} - \frac{1}{2}\lambda SM^2 C_{D_o}}$
R_F	$-1/2$	$\dfrac{-\frac{1}{4}\lambda SM^2 C_{D_o}}{\frac{MAXT}{\sigma} - \frac{1}{2}\lambda SM^2 C_{D_o}}$	-1	$\dfrac{\frac{1}{2}\frac{MAXT}{\sigma}}{\frac{MAXT}{\sigma} - \frac{1}{2}\lambda SM^2 C_{D_o}}$

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altitude between 55,000 and 65,800 feet. The ratio of maximum thrust to density ratio shows negligible variation with altitude and is taken as

$$\frac{MAXT}{\sigma} = 880,000$$

The maximum L/D solution for specified weights follows:

W	σ	h	D/σ	L/D
644,370	0.0839	62,200	992,000	7.74
467,880	0.0609	68,800	992,000	7.74

Since D/σ exceeds $MAXT/\sigma$, cruise can not be at maximum L/D. The solution for equal drag and maximum available thrust is required and is:

W	σ	h	D/σ	L/D
644,370	0.0954	59,600	880,000	7.68
467,880	0.0704	66,000	880,000	7.55

The range factor is 7555 nautical miles using an average SFC of 1.527. The comparison of the predicted results and the Boeing data are presented in Table 4-5.

Relative to the sensitivity analysis, a 10% increase in K gives an altitude decrease of 990 feet ($1/\beta$ = 19,842 feet). A 10% increase in C_{D_0} gives an altitude decrease of 1170 feet. A 10% increase in $MAXT/\sigma$ results in an altitude increase of 2160 feet. Relative to

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range factor, a 10% increase in K, C_{D_0}, and MAXT/σ results in a 5% decrease, 5.9% decrease, and a 10.9% increase, respectively.

TABLE 4-5

SUMMARY OF CONSTANT SPEED CRUISE PERFORMANCE

	Estimated	Boeing	Error(%)
W_i (pounds)	644,370	644,370	0
$M_i = M_f$	2.62	2.62	0
h_i (feet)	59,600	59,500	0.2
L/D_i	7.68	7.58	1.3
R_{F_i} (nautical miles)	7,705	7,586	1.6
W_f (pounds)	467,880	467,880	0
h_f (feet)	66,000	65,600	0.6
L/D_f	7.58	7.43	1.6
R_{F_f} (nautical miles)	7,555	7,398	2.1
X (nautical miles)	2,442	2,414	1.2

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Summary of the Cruise Performance Problems

Four problems were studied in this chapter: optimal cruise performance, best cruise altitude for a specified speed, best cruise speed for a specified altitude, and maximum endurance. In addition to presenting the general approach for determining the graphical solution to cruise performance problems, approximate analytical solutions were developed. In general, the difference between the analytical and generalized results was less than 3%. The biggest error was in the estimation of the cruise altitude for the F4C assuming the cruise was in the troposphere. When the pressure/density relation for the stratosphere was employed, the error was significantly reduced.

Sensitivity parameters were derived for the four problems. The utility of these parameters is that first-order variations in the aerodynamic and engine performance characteristics were identified and their effect on cruise performance was assessed.

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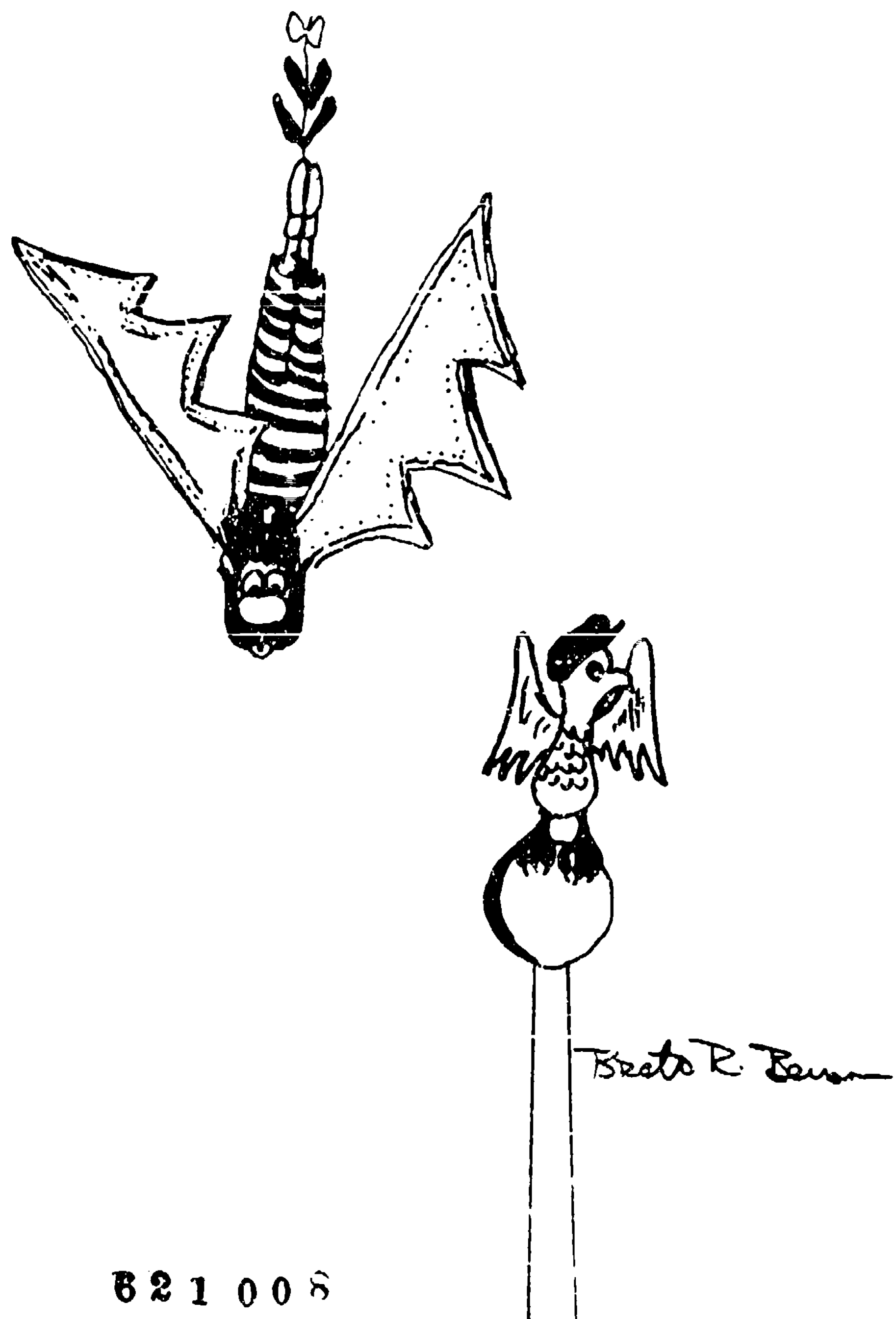
SECTION V

DESCENT AND GLIDE PERFORMANCE



SECTION V

DESCENT AND GLIDE PERFORMANCE

Problem Definition and Assumptions

There are several problems of interest relative to descent and glide performance. The distinction between descent and glide is that the former may have engine power-on, whereas the latter corresponds strictly to engine power-off. Specific problems considered in this section are maximum glide flight path angle, minimum glide sinking speed, maximum range for specified altitude drop, maximum endurance for specified speed change, constant angle-of-attack, and constant L/D glide trajectories. It will be assumed throughout this section that the flight path angle is small. Since

$$x = V \cos \gamma$$

$$h = V \sin \gamma$$

the approximate equations are

$$x = V \qquad (5\text{-}1)$$

$$h = V\gamma \qquad (5\text{-}2)$$

It will also be assumed that

$$L = W \qquad (5\text{-}3)$$

The change in the weight will assumed to be negligible, thus

$$W = 0$$

The acceleration is

$$\dot{V} = \frac{g}{W}(T - D) - g\gamma \qquad (5\text{-}4)$$

For constant speed, Equation 5-4 can be solved for γ, i.e.,

$$\gamma = \frac{1}{W}(T - D) \qquad (5\text{-}5)$$

A further assumption is that the aerodynamic polar can be approximated by a parabolic polar

$$C_D = C_{D_o} + KC_L^{\,2} \qquad (5\text{-}6)$$

Equation 5-6 will be employed in the analytical analysis but it is unnecessary to the general approach for solving the descent and glide performance problems.

The previous equations form the set of equations to be used in solving the descent and glide performance problems. Each problem will be studied separately and additional assumptions introduced whenever necessary.

Maximum Glide Path Angle

The thrust is zero for this problem. For a given speed and weight, the approach for determining the maximum glide path angle is straightforward. Refer to Figure 5-1. The speed V* for maximum glide flight path angle corresponds to maximum -D or equivalently, minimum D. This, substituted into Equation 5-5, gives the maximum glide flight path angle. Thus the solution can easily be obtained by plotting -D versus V, selecting maximum -D (or minimum D), and substituting into Equation 5-5 to determine maximum γ.

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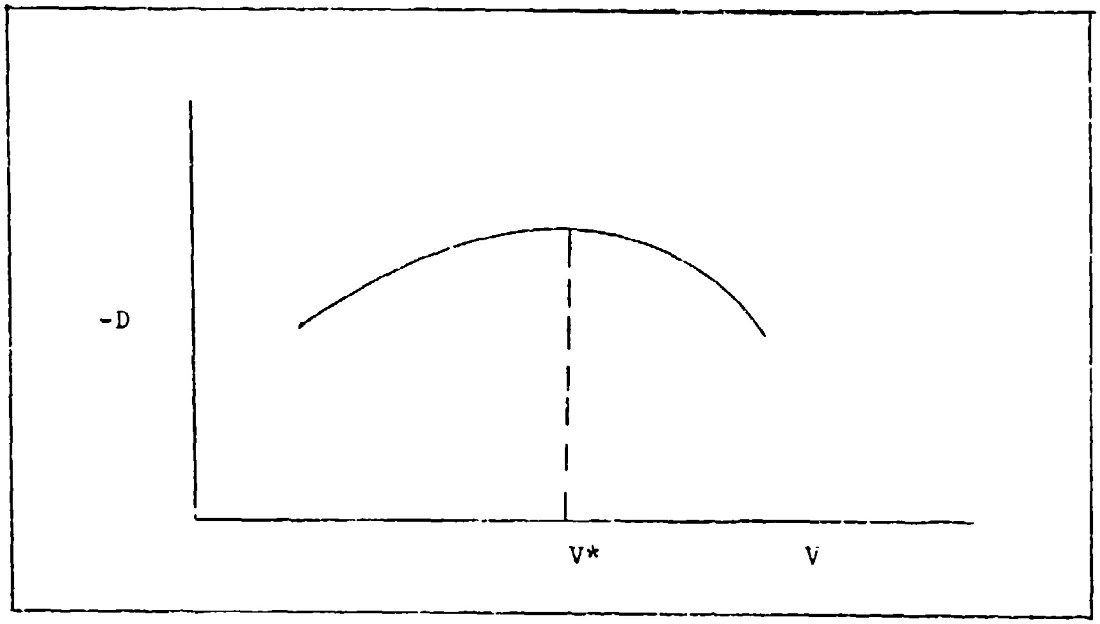

Figure 5-1 -D Versus V

An approximate analytical solution can also be obtained. It will be assumed that the speed is constant as are the aerodynamic coefficients in Equation 5-6. From Equation 5-5 for $T = 0$

$$\gamma = -\frac{D}{W} \tag{5-7}$$

thus maximum γ corresponds to minimum drag. Substitution of Equations 5-3 and 5-6 into the drag equation gives

$$D = \tfrac{1}{2}\rho S V^2 C_{D_o} + \frac{2KW^2}{\rho S V^2} \tag{5-8}$$

The maximum glide flight path angle corresponds to the speed for minimum drag. Differentiating Equation 5-8 with respect to V and solving for the best glide speed V* for maximum γ gives

$$V^* = \left(\frac{KW^2}{\rho^2 S^2 C_{D_o}} \right)^{\frac{1}{4}} \tag{5-9}$$

Consequently, for maximum glide path angle, the trajectory corresponds to constant dynamic pressure, q, defined by

$$q = \tfrac{1}{2} \rho V^2 = \left(\frac{KW^2}{S^2 C_{D_o}} \right)^{\frac{1}{2}}$$

Substitution of Equation 5-9 into Equation 5-8 gives

$$D = 2W \sqrt{KC_{D_o}}$$

Substitution into Equation 5-7 gives the maximum glide path angle γ^*

$$\gamma^* = \text{MAX } \gamma = -2 \sqrt{KC_{D_o}} \tag{5-10}$$

Since

$$\left(\frac{L}{D} \right)_{MAX} = \frac{1}{2 \sqrt{KC_{D_o}}}$$

Equation 5-10 becomes

$$\gamma^* = -\frac{1}{(L/D)_{MAX}} \tag{5-11}$$

Thus maximum glide path angle requires maximum L/D flight.

The solution for the Mach number for maximum glide path angle is presented in Figure 5-2. Maximum flight path angle is presented in Figure 5-3. As an example, assume

$W/C_{D_o}S$ = 6000 pounds per square foot

h = 20,000 feet

$(L/D)_{MAX}$ = 15

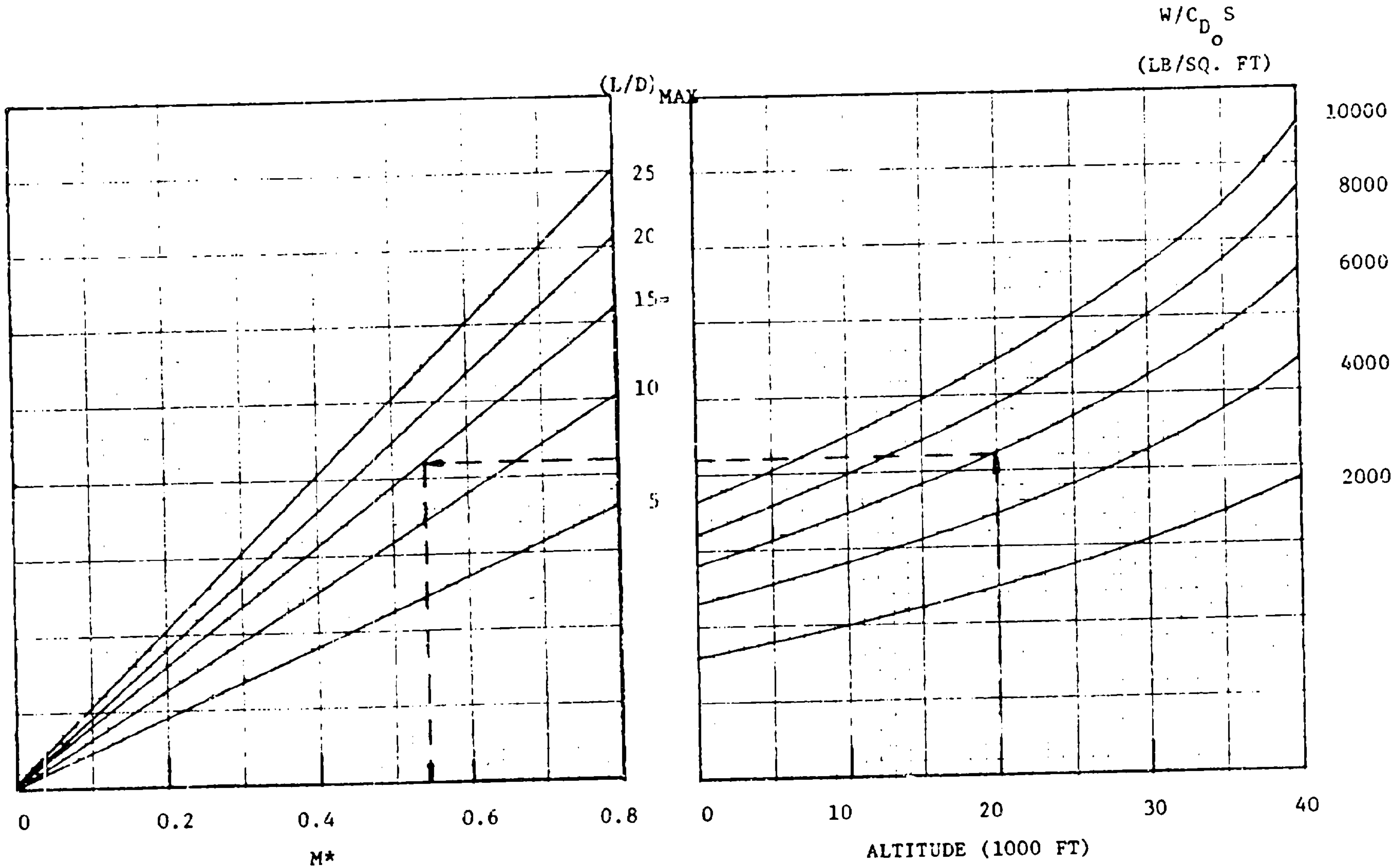

Figure 5-2 Mach Number for Maximum Glide Path Angle and Maximum Range

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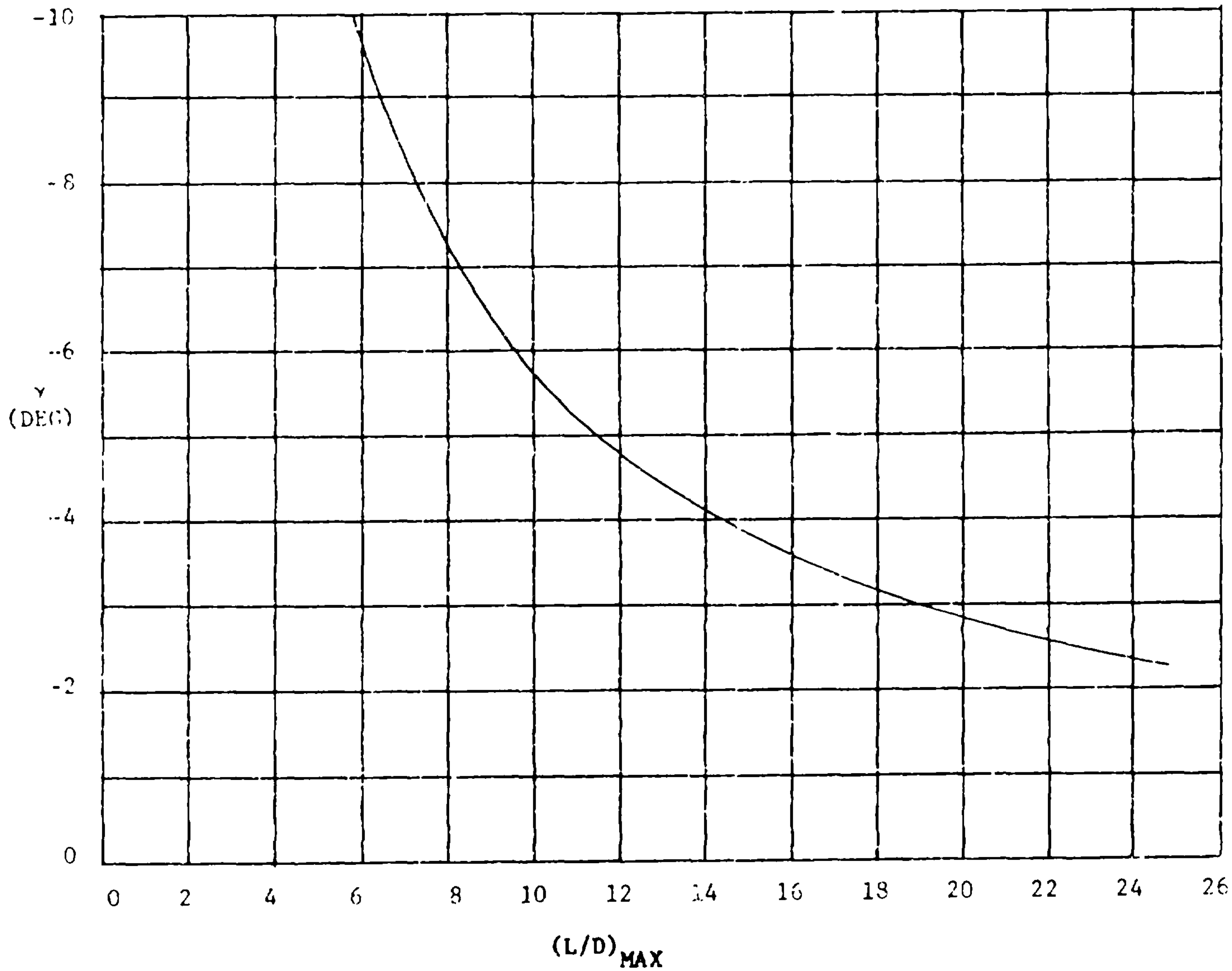

Figure 5-3 Maximum Glide Path Angle

From Figure 5-2

$$M^* = 0.542$$

From Figure 5-3

$$\gamma^* = -3.8^{\circ}$$

Relative to the sensitivity analysis, the significant parameters are C_{D_o}, K, and W/S. The variation of V* to changes in these parameters is

$$\frac{dV^*}{V^*} = \frac{1}{4}\left(\frac{dK}{K} - \frac{dC_{D_o}}{C_{D_o}}\right) + \frac{1}{2}\,\frac{d(W/S)}{W/S}$$

$$= S_K \frac{dK}{K} + S_{C_{D_o}} \frac{dC_{D_o}}{C_{D_o}} + S_{W/S} \frac{d(W/S)}{W/S}$$

where

$$S_K = -S_{C_{D_o}} = \frac{1}{4}$$

$$S_{W/S} = \frac{1}{2}$$

Consequently, V* is twice as sensitive to changes in W/S as it is to changes in K and C_{D_o}. Furthermore, an increase in K or W/S results in an increase in V* while an increase in C_{D_o} decreases V*.

The sensitivity of maximum flight path angle to variations in the parameter is

$$d\gamma^* = -(KC_{D_o})^{\frac{1}{2}}\left(\frac{dC_{D_o}}{C_{D_o}} + \frac{dK}{K}\right)$$

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Substituting maximum L/D gives

$$\frac{d\gamma^*}{\gamma^*} = \frac{1}{2} \left(\frac{dC_{D_o}}{C_{D_o}} + \frac{dK}{K} \right)$$

Thus, the change in γ^* is the same for both K and C_{D_o} and is proportional to γ^*.

As an example of the application of the maximum glide path angle solution, consider the F4C without external stores and engine power-off. Neglecting the difference in the drag due to the engine-off condition gives

$$C_{D_o} = 0.0157$$

$$K = 0.145$$

The variation of best glide Mach number, M*, is presented in Figure 5-4. The maximum glide path angle is

$$\gamma^* = -5.5 \text{ degrees}$$

A 10% increase in either C_{D_o} or K results in a change in γ^* of

$$d\gamma^* = -0.275 \text{ degrees}$$

which is equivalent to a 5% change in γ^*.

The next problem to be addressed is the trajectory for minimum glide sinking speed.

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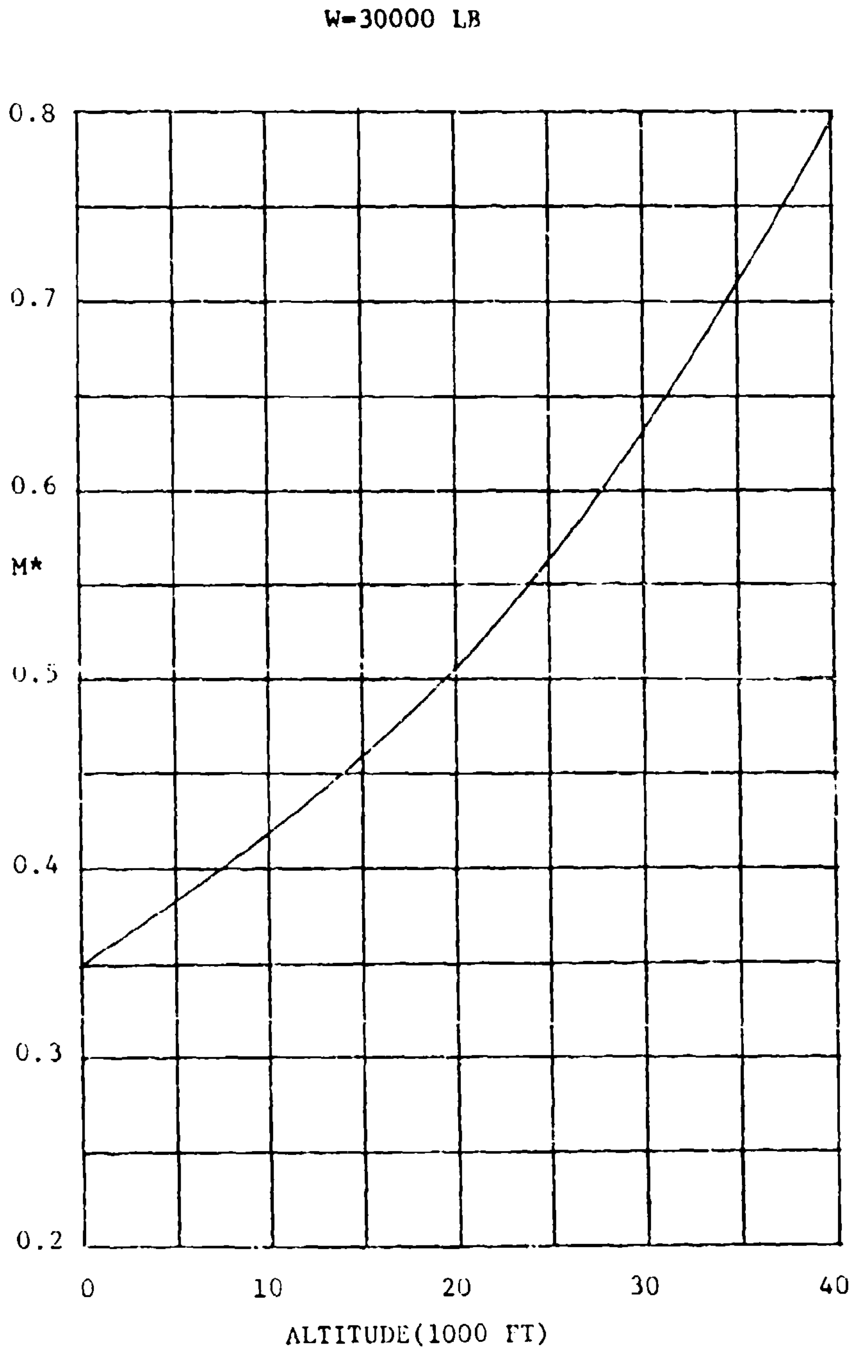

Figure 5-4 F4C Mach Number for Maximum Glide Angle

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Minimum Glide Sinking Speed Trajectory

Sinking speed V_s is the negative of $\dot{h}$. Hence, minimum sinking speed is the same as maximum $\dot{h}$. From Equations 5-2, 5-5, and $V_s = -\dot{h}$

$$V_s = V \frac{D}{W} \tag{5-12}$$

In Figure 5-5, V_s as a function of V is presented.

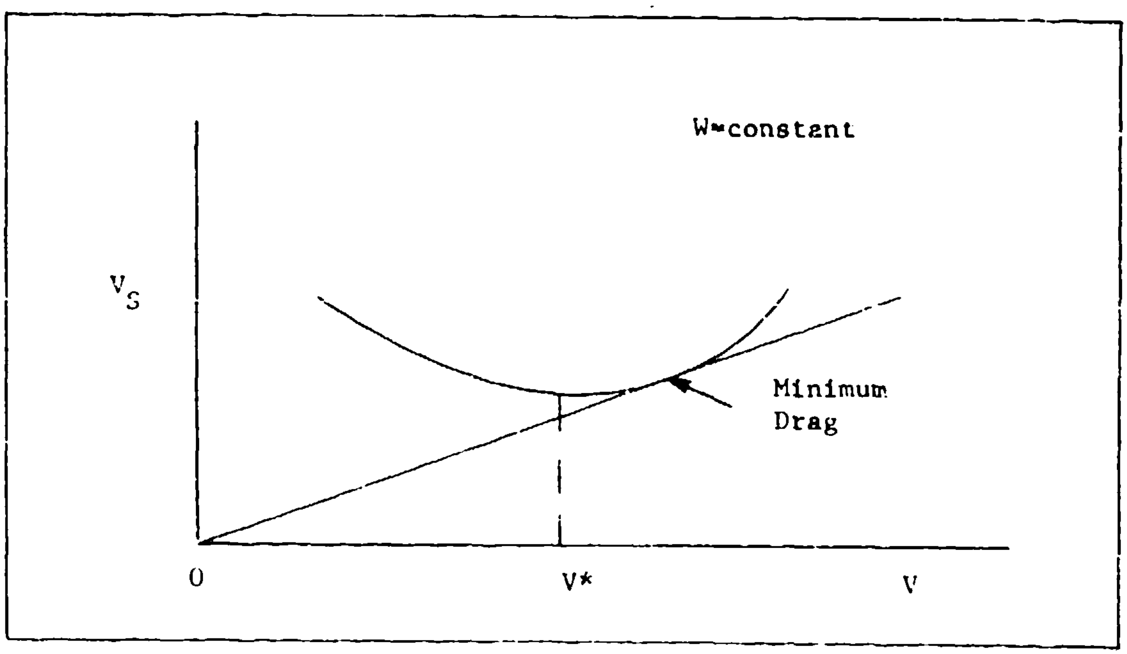

Figure 5-5 V_s Versus V

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The minimum sinking speed is read directly from plots like that in the previous figure. Figure 5-5 can also be used to determine the maximum flight path which was studied earlier. A line drawn tangent to the V_s curve and through the origin defines the maximum glide path angle. Since

$$\frac{V_s}{V} = \frac{D}{W} = -\gamma$$

maximum γ corresponds to minimum V_s/V which is the tangent to the curve. Consequently, V_s versus V has more utility than D versus V since the former gives solutions for both minimum sinking speed and maximum glide path angle.

An approximate analytical solution can be determined for minimum sinking speed trajectory. For constant W, the necessary condition for minimum V_s in Equation 5-12 is

$$\frac{\partial}{\partial V}(VD) = 0$$

As in the previous problem, assume a parabolic drag polar. Substituting the polar into the drag equation and then differentiating Equation 5-12 gives the solution for the speed V* for minimum sinking speed

$$V^* = \left(\frac{4KW^2}{3\rho^2 S^2 C_{D_o}}\right)^{\frac{1}{4}} \tag{5-13}$$

Thus, the trajectory corresponds to constant dynamic pressure defined by

$$q = \frac{1}{2}\rho V^2 = \left(\frac{KW^2}{3S^2 C_{D_o}}\right)^{\frac{1}{2}}$$

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The speed defined by Equation 5-13 is less than the speed for minimum drag. Thus the flight is at a speed which is on the back side of the drag curve. This is an unstable situation since a disturbance in the speed will result in the speed diverging from V* if no aerodynamic corrections are applied.

Substitution of V* into D gives

$$D = 4W \left(\frac{1}{3} K C_{D_0} \right)^{\frac{1}{2}}$$

Thus

$$\frac{L}{D} = \frac{1}{4} \sqrt{\frac{3}{K C_{D_0}}} = \frac{\sqrt{3}}{2} \left(\frac{L}{D} \right)_{MAX}$$

Relative to the previous problem for maximum flight path angle, the minimum sinking speed trajectory is slower by 24% and the dynamic pressure is lower by 73%. Glide L/D is 42% of maximum L/D. Substitution of V* into Equation 5-12 gives the minimum sinking speed

$$V_s^* = \frac{2}{(27)^{\frac{1}{4}}} \; \frac{1}{(L/D)_{MAX}^{3/2}} \; \sqrt{\frac{W}{\rho S C_{D_0}}} \qquad (5\text{-}14)$$

The relationship for best Mach number, M*, and minimum sinking speed is presented in Figures 5-6 and 5-7 as functions of the parameters in the problem. As an example, assume

$$h = 20{,}000 \text{ feet}$$

$$W/C_{D_0}S = 6000 \text{ pounds/square foot}$$

$$(L/D)_{MAX} = 15$$

Figure 5-6 gives approximately

$$M^* = 0.412$$

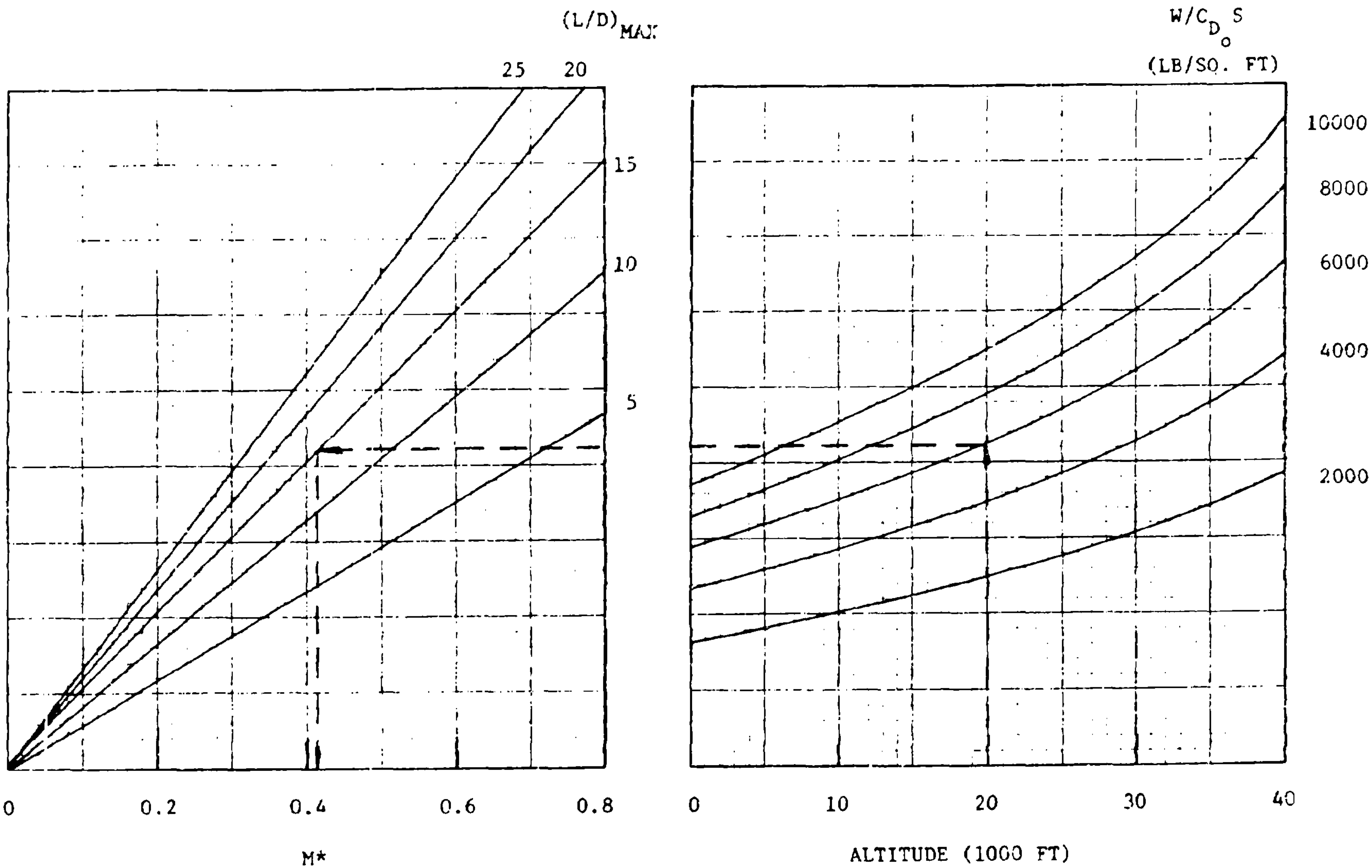

Figure 5-6 Glide Path Mach Number for Minimum Sinking Speed and Maximum Endurance

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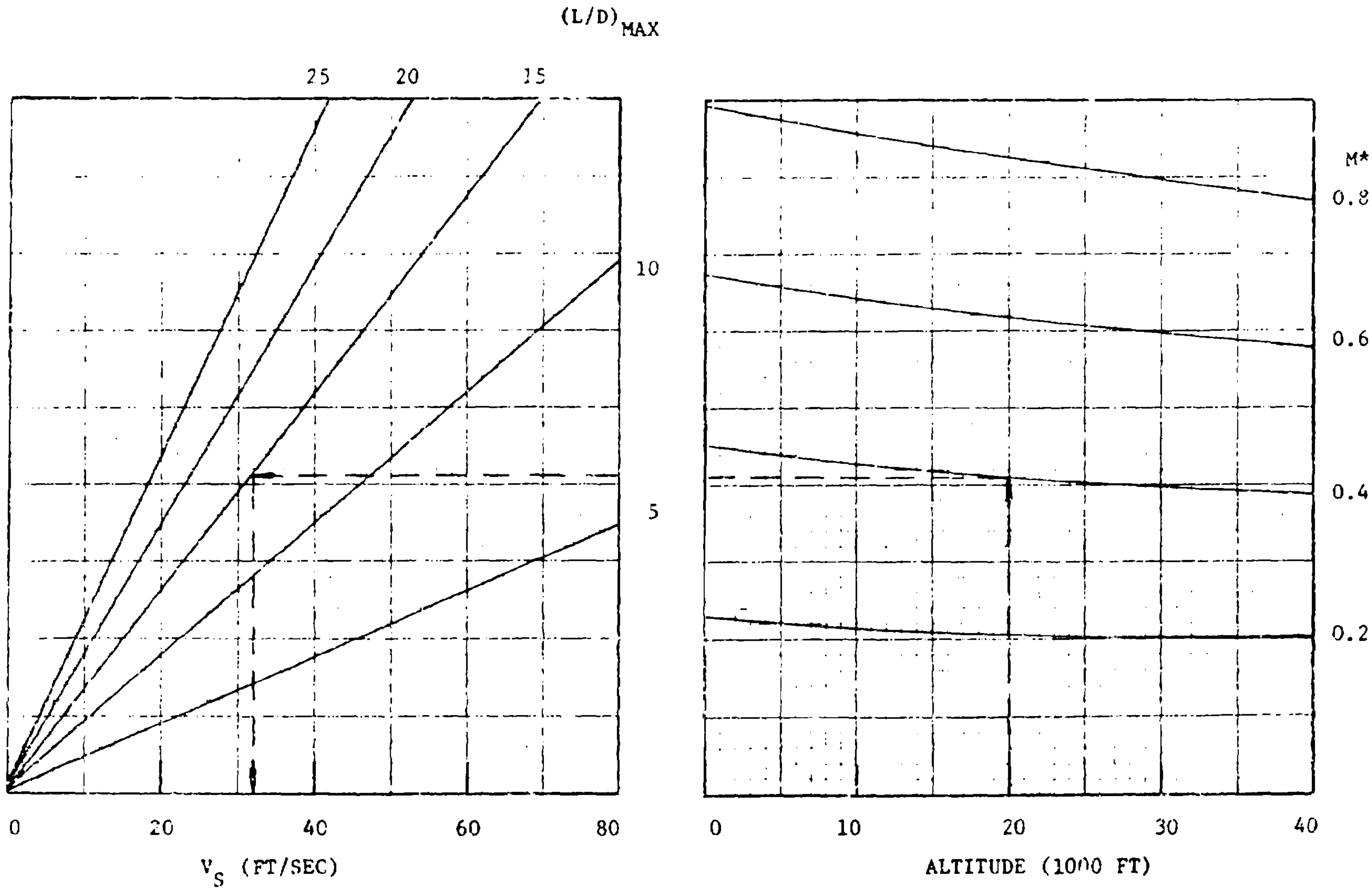

Figure 5-7 Minimum Sinking Speed

From Figure 5-7

$$V_s = 32 \text{ feet/second.}$$

The sensitivity of V* to changes in the parameters is

$$\frac{dV^*}{V^*} = \frac{1}{2}\frac{d(W/S)}{W/S} + \frac{1}{4}\frac{dK}{K} - \frac{1}{4}\frac{dC_{D_o}}{C_{D_o}}$$

$$= S_K\frac{dK}{K} + S_{C_{D_o}}\frac{dC_{D_o}}{C_{D_o}} + S_{W/S}\frac{d(W/S)}{W/S}$$

where

$$S_K = -S_{C_{D_o}} = \frac{1}{4}$$

$$S_{W/S} = \frac{1}{2}$$

which is the same as for the maximum flight path angle. The sensitivity of minimum sinking speed to variations in the parameters is

$$\frac{dV_s^*}{V_s^*} = \frac{1}{2}\frac{d(W/S)}{W/S} + \frac{1}{4}\frac{dC_{D_o}}{C_{D_o}} + \frac{3}{4}\frac{dK}{K}$$

$$= S_{W/S}\frac{d(W/S)}{W/S} + S_{C_{D_o}}\frac{dC_{D_o}}{C_{D_o}} + S_K\frac{dK}{K}$$

where

$$S_{W/S} = \frac{1}{2}$$

$$S_{C_{D_o}} = \frac{1}{4}$$

$$S_K = \frac{3}{4}$$

For equal changes in the parameters, K produces the largest change in V_s^*.

Let us consider again the F4C for application of the previous results. The aerodynamic characteristics and the weight are the same as those in the previous problem. The Mach number for minimum sinking speed, M^*, and minimum sinking speed V_s^* are presented in Figure 5-8. A 10% change in K, C_{D_0}, or W/S results in a 2.5% increase, a 2.5% decrease, and a 5% increase in V^*, respectively. The corresponding changes in V_s^* are 7.5% increase, 2.5% increase, and 5.0% increase.

Maximum Range for Specified Altitude Drop

For problems relative to a specified altitude drop it is convenient to treat the altitude h as the independent variable rather than time. Combining Equations 5-1 and 5-2 gives

$$\frac{dX}{dh} = \frac{1}{\gamma} \qquad (5\text{-}15)$$

Equation 5-4 transforms to

$$V \frac{dV}{dh} = \frac{g}{W\gamma} (T-D) - g \qquad (5\text{-}16)$$

If the change in V is negligible, Equation 5-16 reduces to

$$\gamma = \frac{T-D}{W} \qquad (5\text{-}17)$$

Substitution into Equation 5-15 gives

$$\frac{dX}{dh} = \frac{W}{T-D} \qquad (5\text{-}18)$$

If Equation 5-18 is optimal everywhere along the trajectory, then its integral yields the maximum range for a specified altitude drop.

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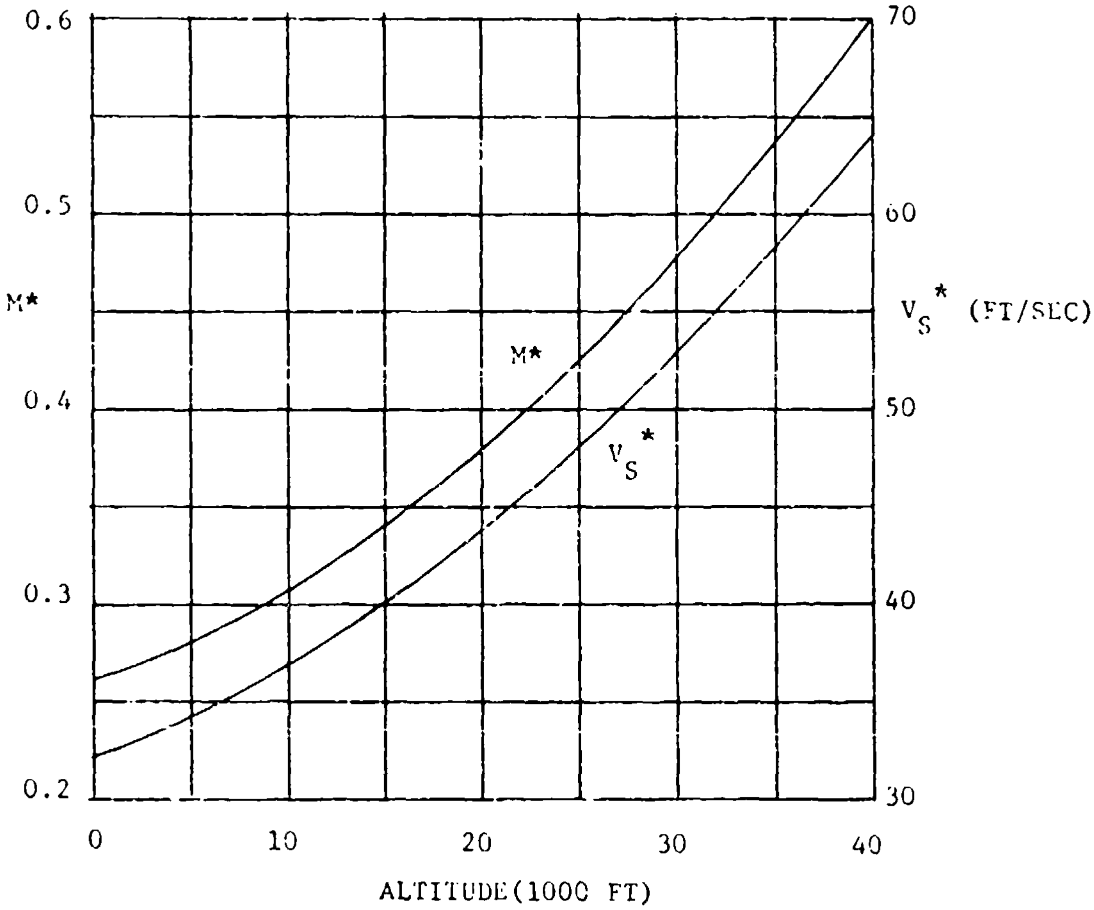

Figure 5-8 F4C Mach Number and Minimum Sinking Speed

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For negligible change in the weight and thrust, maximum range results from minimum drag. One approach to solving for the speed is the same as that used for determining maximum flight path angle. Another approach is illustrated in Figure 5-9.

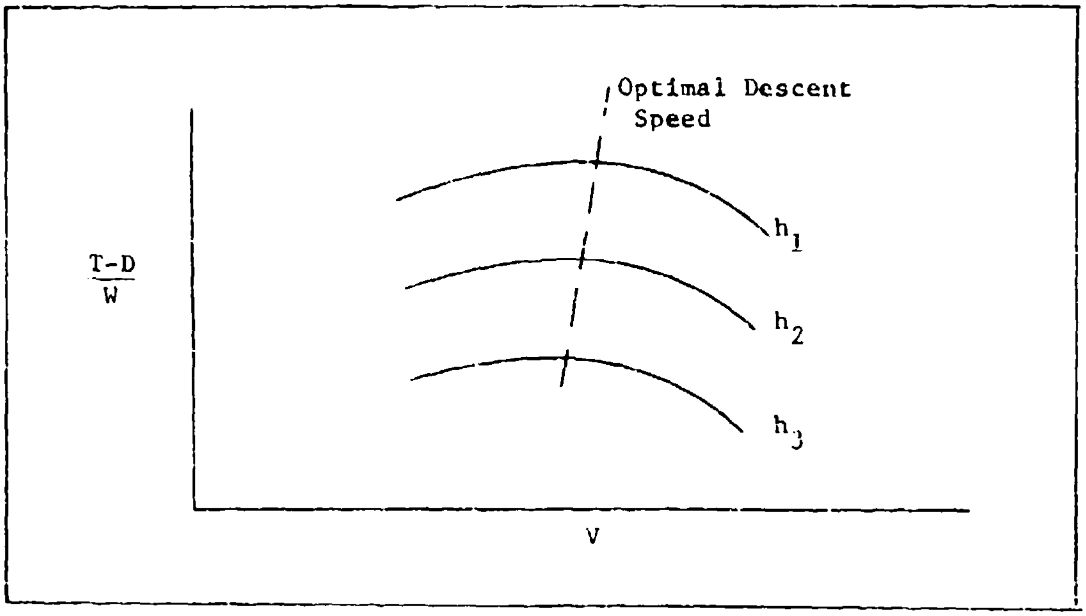

Figure 5-9 (T-D)/W Versus V

For selected altitudes, (T-D)/W is determined as a function of V. The optimal descent schedule is the locus of speeds corresponding to the maximum value of (T-D)/W. Substitution of maximum (T-D)/W into Equation 5-18 and integrating gives the maximum range.

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An approximate analytical solution can be derived for this problem. For a parabolic drag polar

$$\gamma = \frac{T}{W} - 2\sqrt{KC_{D_o}} = \frac{T}{W} - \frac{1}{(L/D)_{MAX}}$$

Substitution into Equation 5-15 gives

$$\frac{dX}{dh} = \frac{1}{\frac{T}{W} - \frac{1}{(L/D)_{MAX}}}$$

Integrating gives the maximum range solution X*

$$X^* = (h_f - h_i)\left[\frac{T}{W} - \frac{1}{(L/D)_{MAX}}\right]^{-1} \qquad (5\text{-}19)$$

where h_f and h_i are the final and initial altitudes. The speed for maximum range is presented in Figure 5-2. This follows from the observation that the speed for maximum flight path angle and maximum range are the same - namely, the speed corresponding to minimum drag. The solution for maximum range is presented in Figure 5-10. As an example assume

$$(L/D)_{MAX} = 15$$

$$T/W = 0.02$$

$$h_i - h_f = 5000 \text{ feet}$$

Figure 5-10 gives

$$X^* = 17.6 \text{ nautical miles}$$

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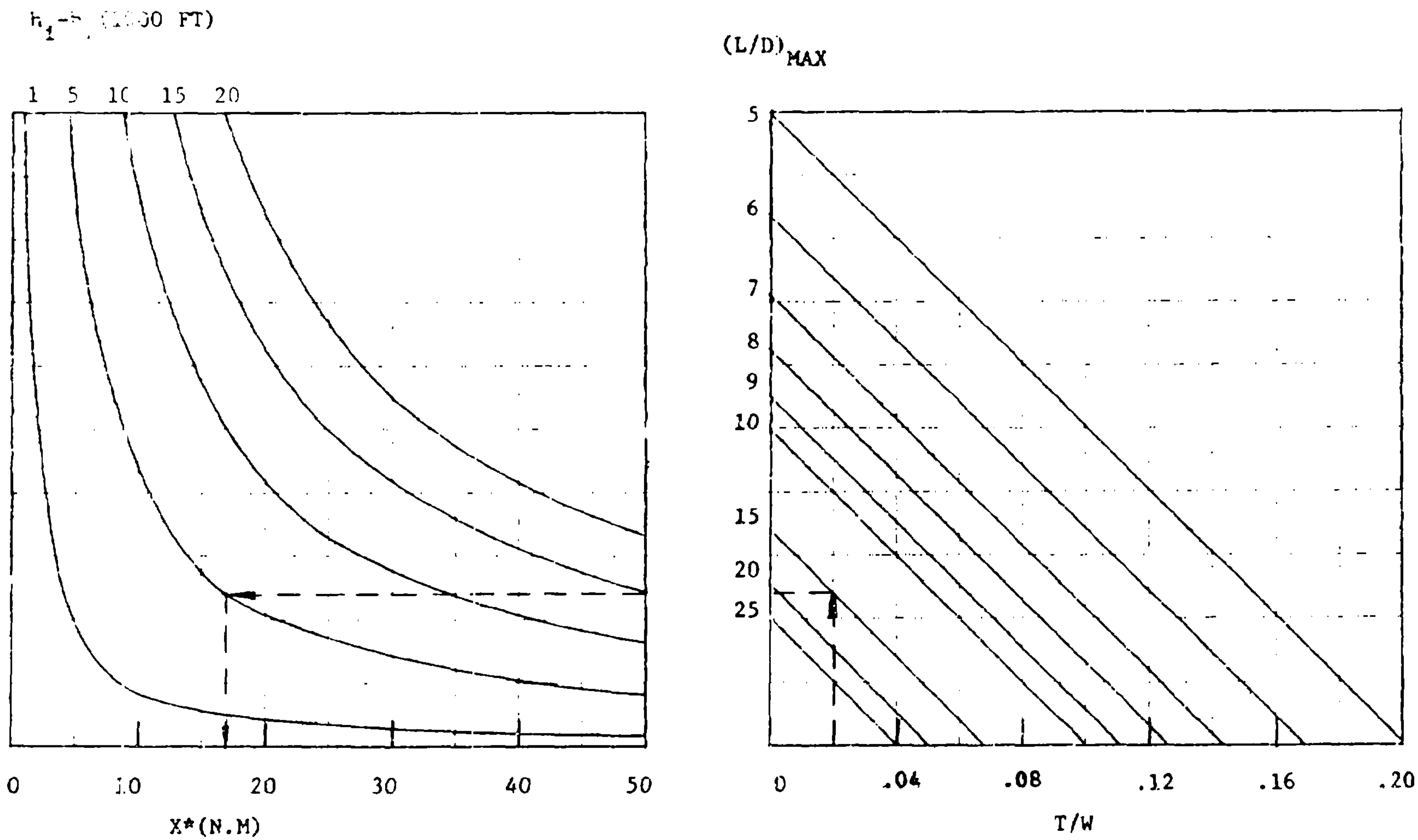

Figure 5-10 Maximum Descent Range

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The sensitivity of the best descent speed to changes in the parameters is the same as that for the maximum flight path angle. The sensitivity of maximum X, X*, to variations in the aerodynamic and engine performance characteristics is derived from Equation 5-19

$$\frac{dX^*}{X^*} = -\left[\frac{T}{W} - 2\sqrt{KC_{D_o}} \right] \left[d\left(\frac{T}{W}\right) - \sqrt{KC_{D_o}} \left(\frac{dC_{D_o}}{C_{D_o}} + \frac{dK}{K} \right) \right]$$

$$= S_{T/W} \frac{d(T/W)}{T/W} + S_K \frac{dK}{K} + S_{C_{D_o}} \frac{dC_{D_o}}{C_{D_o}}$$

where

$$S_{T/W} = -\frac{\frac{T}{W}}{\frac{T}{W} - \frac{1}{(L/D)_{MAX}}}$$

$$S_K = S_{C_{D_o}} = \frac{\frac{1}{2(L/D)_{MAX}}}{\frac{T}{W} - \frac{1}{(L/D)_{MAX}}}$$

For descending flight the previous analysis is restricted to situations where

$$\frac{T}{W} < \frac{1}{(L/D)_{MAX}}$$

It follows that

$$S_{T/W} > 0$$

$$S_K = S_{C_{D_o}} < 0$$

Consequently, an increase in thrust gives an increase in X*. An increase in K or C_{D_o} results in a decrease in X*.

As an example, consider an F4C aircraft operating at idle engine power, starting at 20,000 feet, and ending at 10,000 feet. The weight is 30,000 pounds. The aerodynamic coefficients are the same as the previous F4 problems. Idle engine power thrust is 500 pounds. The maximum range is

$$X^* = 20.9 \text{ nautical miles}$$

The sensitivity parameters for range variations are

$$S_{T/W} = 0.537$$

$$S_K = S_{C_{D_o}} = -0.768$$

Consequently, for the same percentage variation, an improvement in C_{D_o} or K produces a bigger change in X* than the change produced by d(T/W).

Maximum Endurance for a Specified Altitude Drop

As in the previous problem, it is convenient to treat altitude as the independent variable. Equation 5-2 transforms to

$$\frac{dt}{dh} = \frac{1}{V\gamma} \tag{5-20}$$

If Equation 5-20 is optimal everywhere along the trajectory, then integration gives maximum endurance. If the change in the speed is negligible, then γ is defined by Equation 5-17. Substitution into Equation 5-20 gives

$$\frac{dt}{dh} = \frac{W}{V(T-D)} \tag{5-21}$$

For negligible changes in the weight and thrust, maximum endurance corresponds to minimum VD. This is the same as the minimum sinking speed trajectory. The general approach is like that depicted in Figure 5-5 for determining the best speed V*. Substituting V* into Equation 5-21 and integrating gives maximum endurance.

An analytical solution can be derived when the assumption of constant speed, thrust, and weight, and a parabolic aerodynamic polar drag with constant coefficients is introduced. It was derived for the minimum sinking speed trajectory that

$$V^* = \left(\frac{4KW^2}{3C_{D_o}\rho^2 S^2} \right)^{\frac{1}{4}} \tag{5-13}$$

Substitution into D gives

$$D = 4W \sqrt{\frac{1}{3} KC_{D_o}}$$

which is independent of the altitude. Substituting D and V* into Equation 5-21 gives

$$\frac{dt}{dh} = \frac{\left(\frac{3C_{D_o} S^2}{4KW^2} \right)^{\frac{1}{4}}}{\frac{T}{W} - 4\sqrt{\frac{1}{3}KC_{D_o}}} \rho^{\frac{1}{2}} \tag{5-22}$$

The density is related to the altitude by the following approximate equation (Equation 4-42 in Section IV)

$$\rho = \rho_R e^{-\beta h} \tag{5-23}$$

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Substituting Equation 5-23 into Equation 5-22 and integrating gives maximum endurance, t*

$$t^* = \frac{2}{\beta}\left(\rho_i^{\frac{1}{2}} - \rho_f^{\frac{1}{2}}\right)\frac{\left(\frac{3C_{D_o}S^2}{4KW^2}\right)^{\frac{1}{4}}}{\frac{T}{W} - 4\sqrt{\frac{1}{3}KC_{D_o}}}$$

$$= \frac{2}{\beta}\left(\frac{1}{V_i^*} - \frac{1}{V_f^*}\right)\left[\frac{T}{W} - \frac{2}{\sqrt{3}\,(L/D)_{MAX}}\right]^{-1} \qquad (5\text{-}24)$$

The solution for best Mach number is presented in Figure 5-6. Maximum endurance is obtained from Figure 5-11. In Figure 5-11, τ is defined as

$$\tau = -\frac{2}{\beta V}\left[\frac{T}{W} - \frac{2}{\sqrt{3}\,(L/D)_{MAX}}\right]^{-1} \qquad (5\text{-}25)$$

Due to the presence of β in Equation 5-25, two different tropopauses are identified in Figure 5-11. The curve which yields the higher value of τ corresponds to the top of the troposphere, whereas the other is the bottom of the stratosphere. For trajectories which cross the tropopause, the endurance time t* is made up of the time to reach the tropopause (lower curve) plus the increment from the higher curve to the final altitude. As an example, assume that the index number is 6, h_i = 50,000 feet, and h_f = 30,000 feet. From Equations 5-24 and 5-25, it follows that the maximum endurance solution is

$$t^* = \tau_f - \tau_i$$

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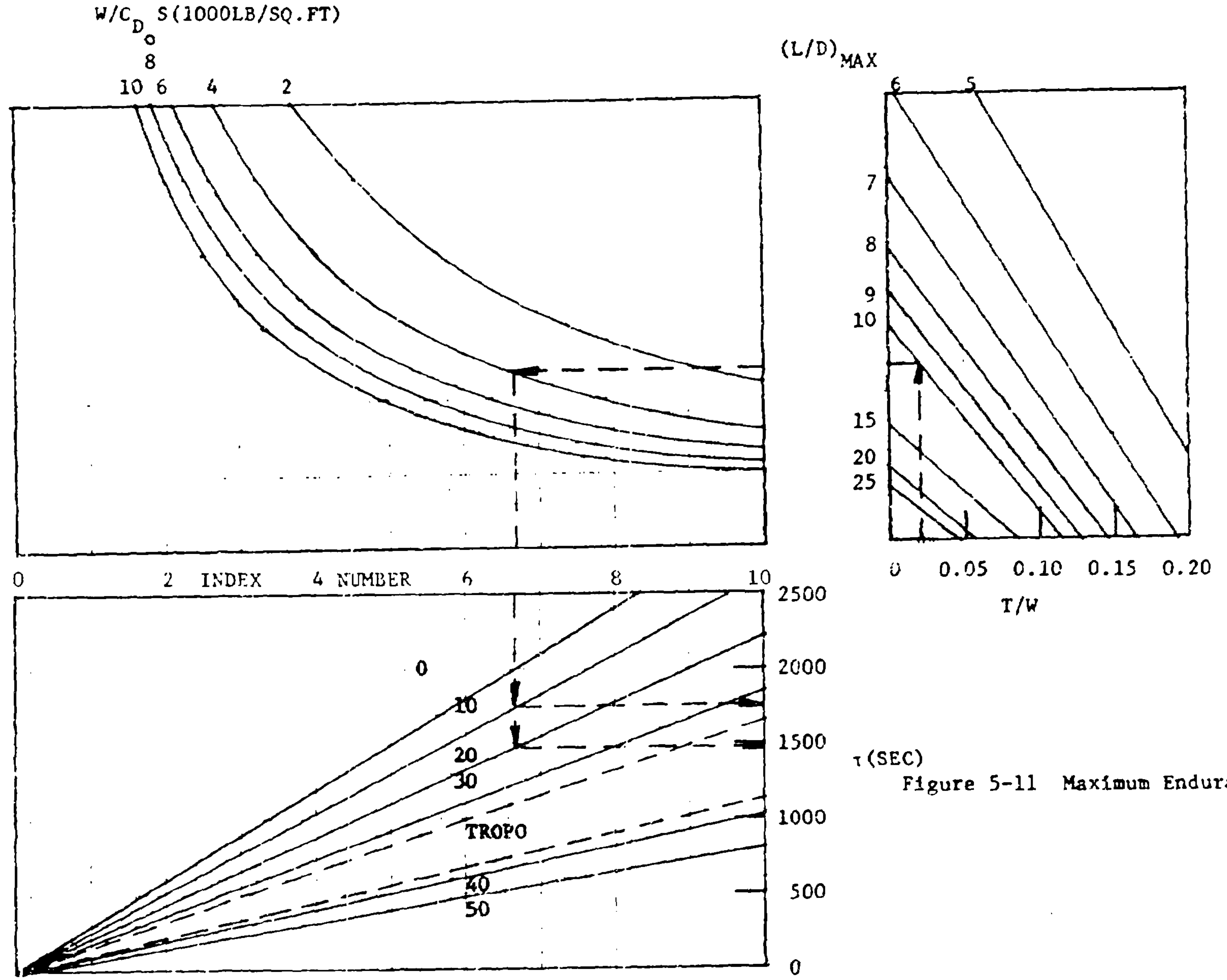

Figure 5-11 Maximum Endurance

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From Figure 5-11, the time to descend from 50,000 feet to the tropopause is

$$670 - 480 = 190 \text{ seconds}$$

The time to descend from the tropopause to 30,000 feet is

$$1120 - 990 = 130 \text{ seconds}$$

The endurance is therefore

$$190 + 130 = 320 \text{ seconds}$$

As an example of the application of Figure 5-11, consider the following characteristics

$$W/C_{D_0}S = 4000 \text{ pounds/square foot}$$

$$T/W = 0.02$$

$$(L/D)_{MAX} = 10$$

$$h_i = 20{,}000 \text{ feet}$$

$$h_f = 10{,}000 \text{ feet}$$

The following results are obtained from Figure 5-11

$$T_i = 1500 \text{ seconds}$$

$$T_f = 1760 \text{ seconds}$$

$$t^* = 260 \text{ seconds}$$

Equation 5-25 gives

$$T_i = 1495 \text{ seconds}$$

$$T_f = 1761 \text{ seconds}$$

$$t^* = 266 \text{ seconds}$$

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The sensitivity of the change in the endurance to variations in the parameters can be determined from Equation 5-25. Differentiating this equation gives

$$\frac{d\tau}{\tau} = -\frac{dV}{V} - \frac{T/W}{\frac{T}{W} - \frac{2}{\sqrt{3}\,(L/D)_{MAX}}} \cdot \frac{d(T/W)}{T/W}$$

$$+ \frac{1}{\sqrt{3}\,(L/D)_{MAX}\left[\frac{T}{W} - \frac{2}{\sqrt{3}\,(L/D)_{MAX}}\right]}\left(\frac{dC_{D_o}}{C_{D_o}} + \frac{dK}{K}\right) \tag{5-26}$$

For descending flight the denominator is

$$\frac{T}{W} - \frac{2}{\sqrt{3}\,(L/D)_{MAX}} < 0$$

Hence, any speed other than V* reduces the endurance time. An increase (decrease) in T/W increases (decreases) endurance. An increase (decrease) in either C_{D_o} or K decreases (increases) endurance. Substitution of V* from Equation 5-13 into Equation 5-26 gives

$$\frac{d\tau}{\tau} = -\frac{T/W}{\frac{T}{W} - \frac{2}{\sqrt{3}\,(L/D)_{MAX}}}\,\frac{d(T/W)}{T/W}$$

$$+ \left\{\frac{1}{\sqrt{3}\,(L/D)_{MAX}\left[\frac{T}{W} - \frac{2}{\sqrt{3}\,(L/D)_{MAX}}\right]} - \frac{1}{4}\right\}\frac{dC_{D_o}}{C_{D_o}} \tag{5-27}$$

$$+ \left\{\frac{1}{\sqrt{3}\,(L/D)_{MAX}\left[\frac{T}{W} - \frac{2}{\sqrt{3}\,(L/D)_{MAX}}\right]} + \frac{1}{4}\right\}\frac{dK}{K}$$

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For the application consider the F4C, the characteristics for which were defined earlier. Assume that

$$h_i = 20{,}000 \text{ feet}$$

$$h_f = 10{,}000 \text{ feet}$$

The solutions for the endurance and the sensitivity parameters are

$$V_i^* = 396.4 \text{ feet/second}$$

$$V_f^* = 336.5 \text{ feet/second}$$

$$t^* = 292 \text{ seconds}$$

$$S_{T/W} = 0.178$$

$$S_{C_{D_o}} = -0.839$$

$$S_K = -0.339$$

The biggest improvement in endurance is by a decrease in C_{D_o}.

Maximum Range for Specified Speed Change

This problem differs from one of the previous problems, in that the speed rather than the altitude is specified. Problems that fall into this category are supersonic decelerations. Assuming that the flight path angle is approximately zero leads to the following combination of Equations 5-1 and 5-4

$$\frac{dx}{dV} = \frac{V}{\frac{g}{W}(T-D)} \tag{5-28}$$

The independent variable is V. The problem is the determination of the best altitude as a function of speed. If Equation 5-28 is optimal everywhere along the best trajectory, then integration gives maximum range.

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In general both T and D are functions of altitude. For descending flight,

$$T - D < 0$$

Since V decreases with increasing time, it is convenient to introduce the following linear transformation

$$u = V_i - V$$

Now $u \geq 0$, thus Equation 5-28 transforms to

$$\frac{dx}{du} = \frac{V}{\frac{g}{W}(D-T)} \tag{5-29}$$

Consequently, for maximum range, the altitude as a function of V or u is evaluated such that Equation 5-29 is maximum. This corresponds to minimum (D-T)/V. The general approach is to determine the speed for a given altitude, weight, and engine throttle setting which minimizes (D-T)/V. The situation is depicted in Figure 5-12. The optimal speed V* for decelerating flight is obtained by drawing a line through the origin and tangent to the (D-T) curve. The relationship between altitude and speed is obtained by varying the altitude and applying Figure 5-12.

An approximate analytical solution can be obtained as follows. Neglecting changes in T, W, and the aerodynamic coefficients gives the necessary condition for V*

$$\frac{\partial}{\partial V}\left(\frac{D}{V}\right) = 0$$

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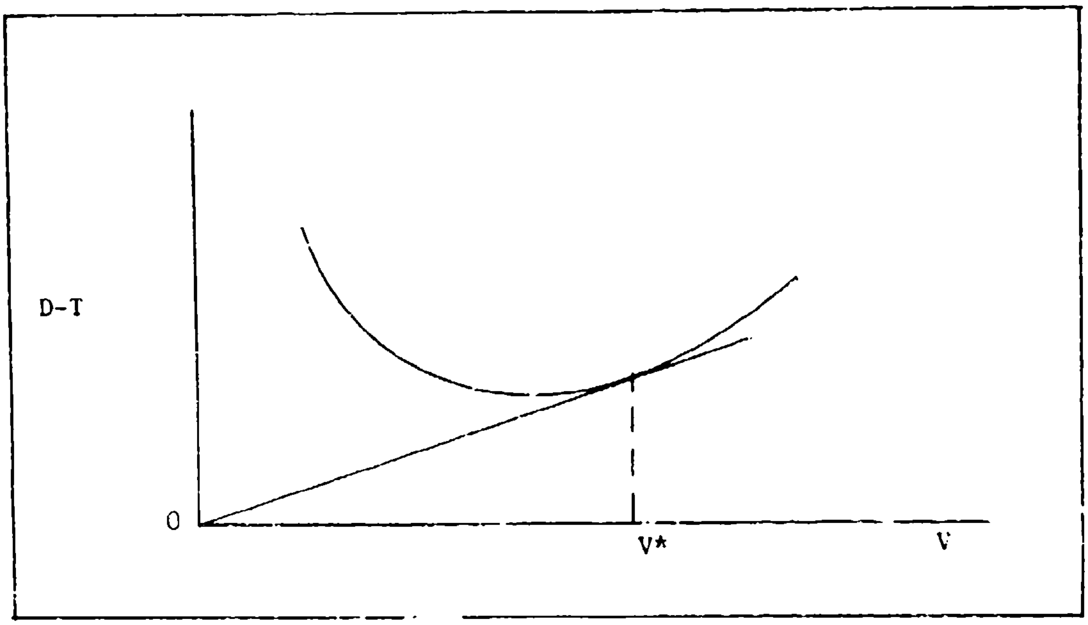

Figure 5-12 Optimal Speed for Maximum Range

Substitution of a parabolic polar gives

$$\rho = \frac{\sqrt{3}\; W}{C_{D_0} S V^2 \; (L/D)_{MAX}}$$

Substitution into the drag equation yields

$$\frac{L}{D} = \frac{\sqrt{3}}{2} \left(\frac{L}{D} \right)_{MAX}$$

Thus the trajectory corresponds to approximately 87% of maximum lift-to-drag ratio. It was shown earlier that this also corresponds to the trajectory for minimum glide sinking speed.

Equation 5-28 becomes, for maximum range,

$$\frac{dX}{dV} = \frac{V}{g\left(\frac{T}{W} - \frac{2}{\sqrt{3}\,(L/D)_{MAX}}\right)}$$

Using average values of T and $(L/D)_{MAX}$ over the speed range from V_i to V_f gives the following solution for maximum range

$$X^* = \frac{V_i^2 - V_f^2}{2g\left(\frac{2}{\sqrt{3}\,(L/D)_{MAX}} - \frac{T}{W}\right)}$$

V_i and V_f are the initial and final speeds, respectively. The solution for X(V), defined by

$$X(V) = \frac{V^2}{2g\left(\frac{2}{\sqrt{3}\,(L/D)_{MAX}} - \frac{T}{W}\right)} \qquad (5\text{-}30)$$

is presented in Figure 5-13. The optimal deceleration range, X*, is obtained from

$$X^* = X(V_i) - X(V_f)$$

As an example, consider the following characteristics

$T/W = 0.05$

$(L/D)_{MAX} = 8$

V_i = 2000 feet per second

V_f = 1000 feet per second

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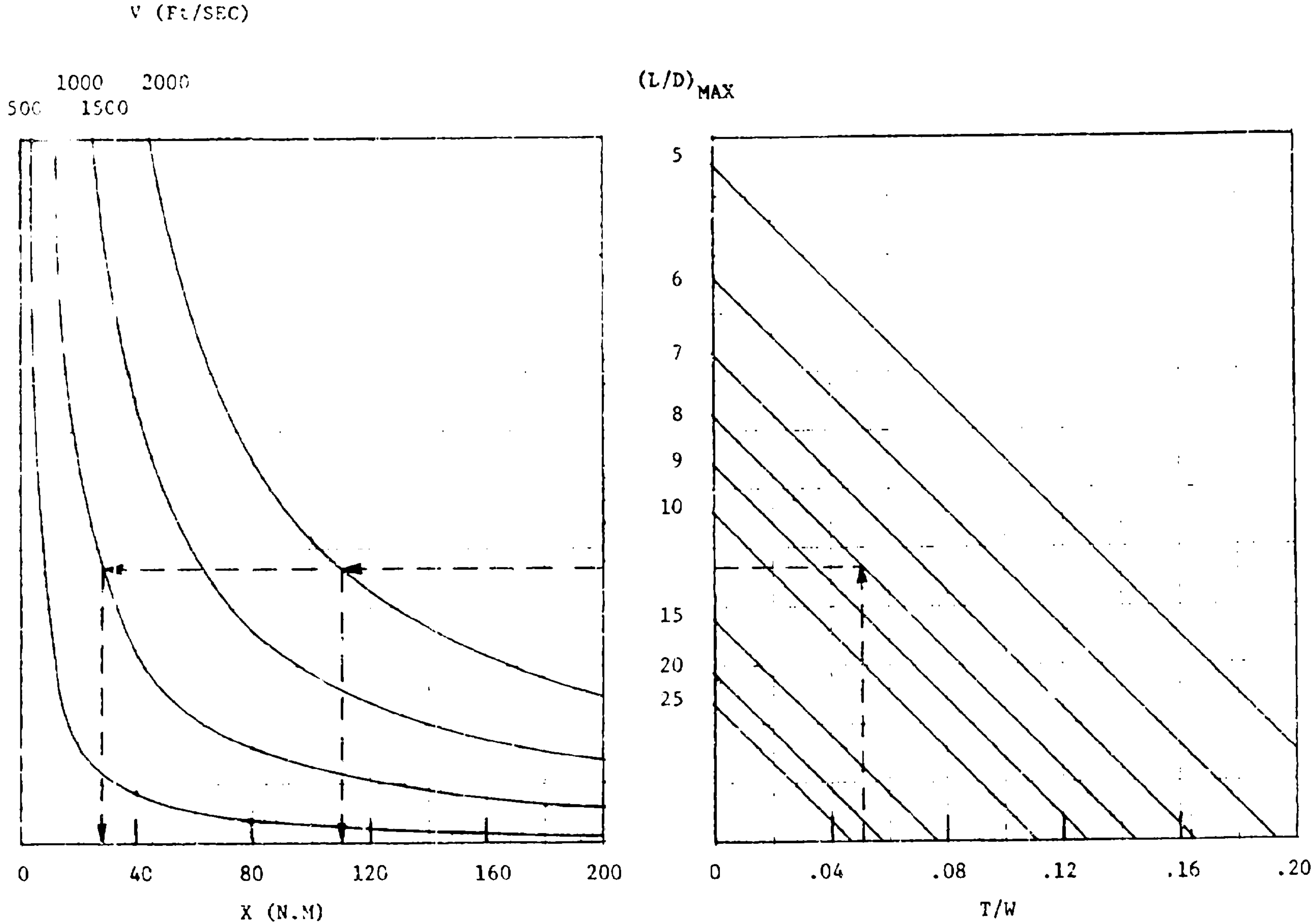

Figure 5-13 Maximum Deceleration Range

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From Figure 5-13

$$X(V_l) = 109 \text{ nautical miles}$$

$$X(V_f) = 28 \text{ nautical miles}$$

Therefore

$$X^* = 81 \text{ nautical miles}$$

The sensitivity of the maximum range to changes in the parameters is

$$\frac{dX^*}{X^*} = \frac{1}{\dfrac{2}{\sqrt{3}\,(L/D)_{MAX}} - \dfrac{T}{W}} \left[- \frac{1}{\sqrt{3}\,(L/D)_{MAX}} \left(\frac{dK}{K} + \frac{dC_{D_o}}{C_{D_o}} \right) + \frac{T}{W} \frac{d(T/W)}{T/W} \right]$$

$$= S_K \frac{dK}{K} + S_{C_{D_o}} \frac{dC_{D_o}}{C_{D_o}} + S_{T/W} \frac{d(T/W)}{T/W}$$

where S_K, $S_{C_{D_o}}$, and $S_{T/W}$ are the sensitivity parameters. For the previous example, the sensitivity parameters are

$$S_K = S_{C_{D_o}} = -0.765$$

$$S_{T/W} = 0.530$$

Consequently, a decrease of 1% in K or C_{D_o} results in a bigger improvement in X than a 1% increase in T/W would achieve.

The significant point about this problem is that the descent and deceleration should follow a trajectory close to 87% of maximum lift-to-drag ratio.

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Constant Angle-of-Attack Endurance Performance

The problem of interest in this section is the decelerating performance of a supersonic glider. It no longer suffices to assume that the change in the speed is negligible. The purpose is to develop an approximate relationship between endurance and the parameters in the problem.

The general approach involves the integration of the longitudinal and normal accelerations and the differential equation for the time-rate-of-change of altitude. Given the initial conditions and the controls, it is then straightforward as far as determining a trajectory. The sensitivity of the trajectory characteristics to changes in the parameters can be determined by rerunning a trajectory computer program. Unfortunately, it may be difficult to determine a relationship between a trajectory performance variable and the aircraft characteristics. Thus an approximate formulation is sought if it can lead to the desired but unknown relationship. We turn our attention to such a derivation.

The governing equation is

$$\dot{V} = -g\, D/W \tag{5-31}$$

Engine power-off is certainly appropriate here. The assumption of negligible γ relative to D/W does not generally hold everywhere along the trajectory. Thus the following analysis is restricted to portions of a trajectory where D/W is much greater than γ.

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The specific problem to be addressed here is the evaluation of the endurance at or above a given speed where the aerodynamic control corresponds to a constant angle-of-attack. It will be assumed that the aerodynamic drag polar is parabolic and the lift coefficient varies linearly with angle-of-attack. For constant weight, Equation 5-31 becomes

$$\frac{dV}{dt} = -\frac{\rho g S V^2}{2W}\left(C_{D_o} + K C_{L_\alpha}^2 \alpha^2\right) \tag{5-32}$$

where C_{L_α} is the lift curve slope and α is the aerodynamic angle-of-attack. Since V is the independent variable and t the dependent variable, Equation 5-32 becomes

$$\frac{dt}{dV} = -\frac{2W}{\rho g S V^2 \left(C_{D_o} + K C_{L_\alpha}^2 \alpha^2\right)} \tag{5-33}$$

If the density is known in terms of V and likewise the aerodynamic coefficients, then Equation 5-33 can at least be integrated numerically. To derive an analytic solution for endurance, it will be assumed that average values can be selected for ρ, C_{D_o}, K, and C_L. It should be clear that care must be exercised whenever this is done. Equation 5-33 is integrable, the solution for the maximum endurance above a specified speed V_f is t*

$$t^* = \frac{2W\left(\frac{1}{V_f} - \frac{1}{V_i}\right)}{\bar{\rho} g S \left(\bar{C}_{D_o} + \bar{K}\,\bar{C}_{L_\alpha}^2 \alpha^2\right)} \tag{5-34}$$

where a bar over a parameter or variable denotes average value and V_i is the initial speed.

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Rewriting Equation 5-34 gives

$$t^* = \frac{2\left(\frac{1}{V_f} - \frac{1}{V_i}\right)}{\bar{\rho}\, g} \left(\frac{W}{\bar{C}_D S}\right) \qquad (5\text{-}35)$$

where

$$\bar{C}_D = \bar{C}_{D_o} + \bar{K}\, \bar{C}_{L_\alpha}^2\, \alpha^2$$

Equation 5-35 shows that for constant angle-of-attack t is linear with respect to wing loading W/S. Consequently, the variation in endurance time is equal to the change in wing loading. Let

$$t(V) = \frac{2}{V\bar{\rho} g} \left(\frac{W}{\bar{C}_D S}\right)$$

Maximum endurance is therefore

$$t^* = t(V_f) - t(V_i)$$

The solution for t(V) is presented in Figure 5-14. Note that the scale on t(V) is logarithmic.

As an example, the endurance according to Equation 5-35 was compared with the result obtained from the integration of the differential equations of motion. The initial Mach number was 4.4 and the final Mach number was 3.0. The altitude varied between 97,000 and 108,000 feet. The angle-of-attack was 12 degrees. The density corresponding to the average of these two altitudes was used. The average values for C_{D_o}, K, and C_{L_α} were 0.034, 1.164, and 0.015, respectively.

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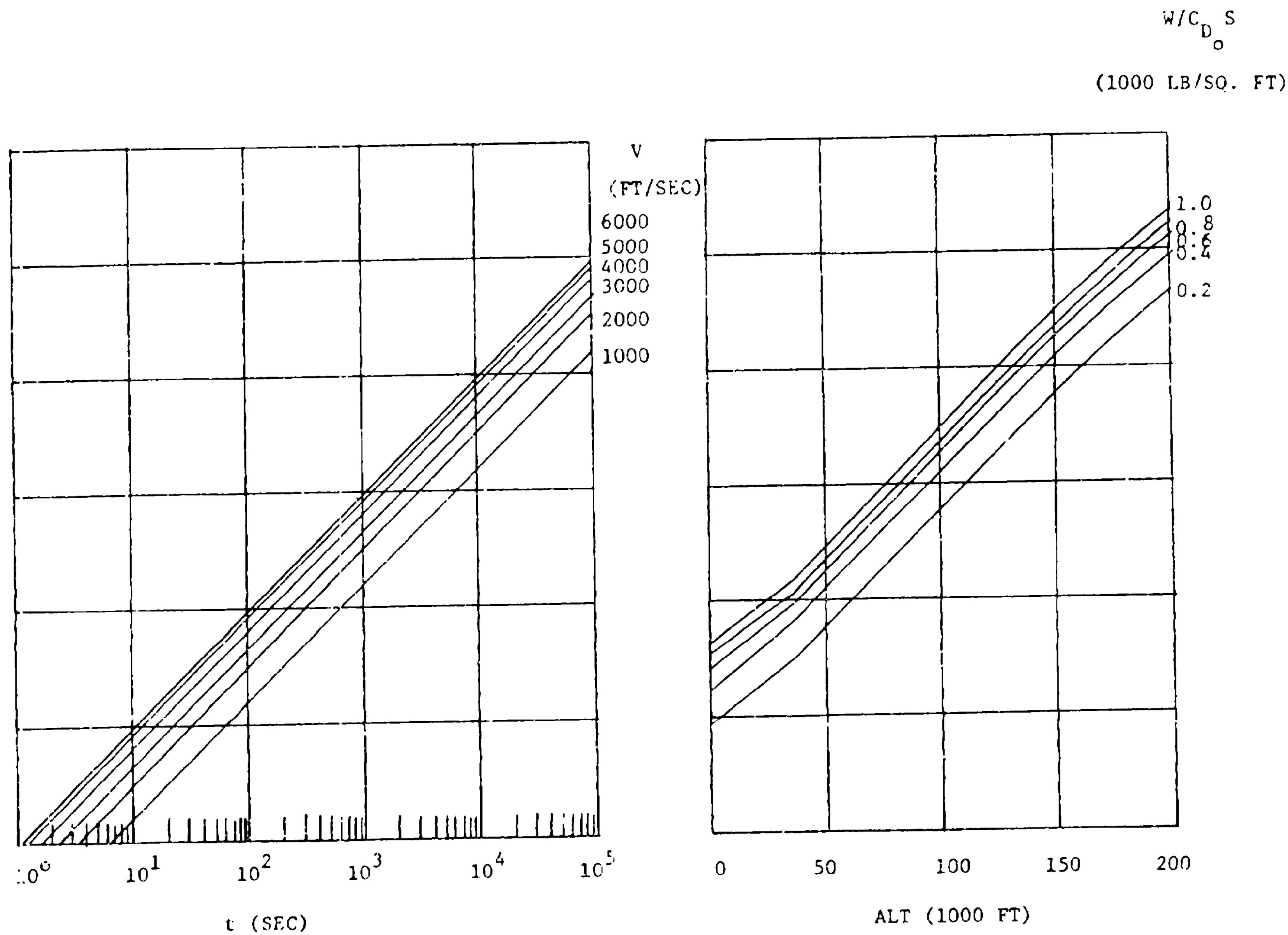

Figure 5-14 Maximum Endurance

The wing loading was 30 pounds per square foot. Equation 5-35 resulted in an endurance of 96 seconds, whereas the integration of the equations of motion gave 85 seconds. Thus, Equation 5-35 gives a representative answer for endurance.

The sensitivity of the endurance to variations in the parameters is

$$\frac{dt^*}{t^*} = \frac{d(W/S)}{W/S} - \frac{d\bar{C}_D}{\bar{C}_D}$$

Consequently, the endurance changes in direct proportion to the increase in W/S ($\bar{C}_D$).

For the last problem, the glide is at constant L/D rather than constant angle-of-attack.

Constant L/D Endurance Performance

The problem here is the same as the previous problem except that the aerodynamic control corresponds to constant L/D ratio rather than aerodynamic angle of attack. The general approach is the same as before, the exception being in the control.

For the approximate formulation, Equation 5-31 holds. For constant L/D and L = W, Equation 5-31 becomes

$$\frac{dV}{dt} = -g\,\frac{D}{L}$$

or

$$\frac{dt}{dV} = -\frac{1}{g}\left(\frac{L}{D}\right)$$

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Integrating gives endurance

$$t = \frac{1}{g}\frac{L}{D}(V_i - V_f) \tag{5-36}$$

Equation 5-36 shows that endurance varies linearly with L/D. Thus the change in endurance is equal to the change in L/D.

For an example, the characteristics of the previous problem result in a L/D ratio of 2.51. The endurance at or above a Mach number equal to 3.0 is 108 seconds. It should be pointed out that a comparison with 85 seconds (the integration of the equations of motion) does not apply since this solution corresponded to a constant angle-of-attack control.

Summary of the Descent and Glide Performance Problems

Seven problems were studied in this section. Approximate analytical solutions were derived for all of the problems. For those problems where the aerodynamic control was free, it was established that the optimal trajectories correspond to either maximum lift-to-drag ratio or the aerodynamic control which corresponds to minimum sinking speed. In addition, the sensitivity of the performance was determined in terms of changes in the aircraft parameters.

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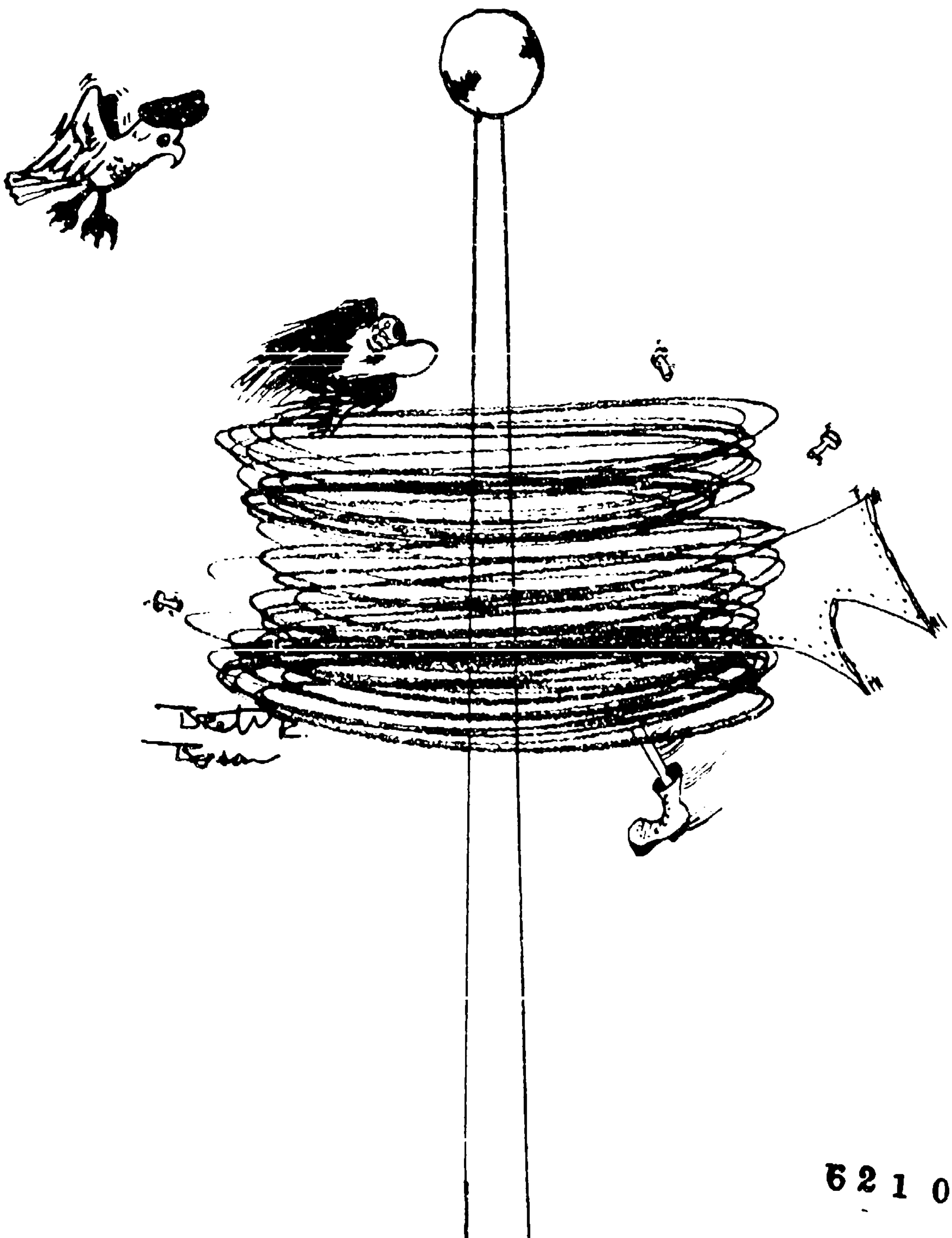
SECTION VI
TURNING PERFORMANCE



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SECTION VI

TURNING PERFORMANCE

Problem Definition and Assumptions

Although turning performance is generally studied in a horizontal plane, it is of equal importance for both ascending and descending flight. For example, heading changes after takeoff are representative of ascending turns. Altitude changes during holding patterns above airports involve descending turns. In addition, heading changes for final approach to landings are descending turns.

For a point mass assumption, i.e., the mass is concentrated at the center of mass, six differential equations hold. These equations involve the speed V, the flight path angle γ, the heading angle σ, and three position coordinates. The six state variables are presented in Figure 6-1 where point P is the center of mass.

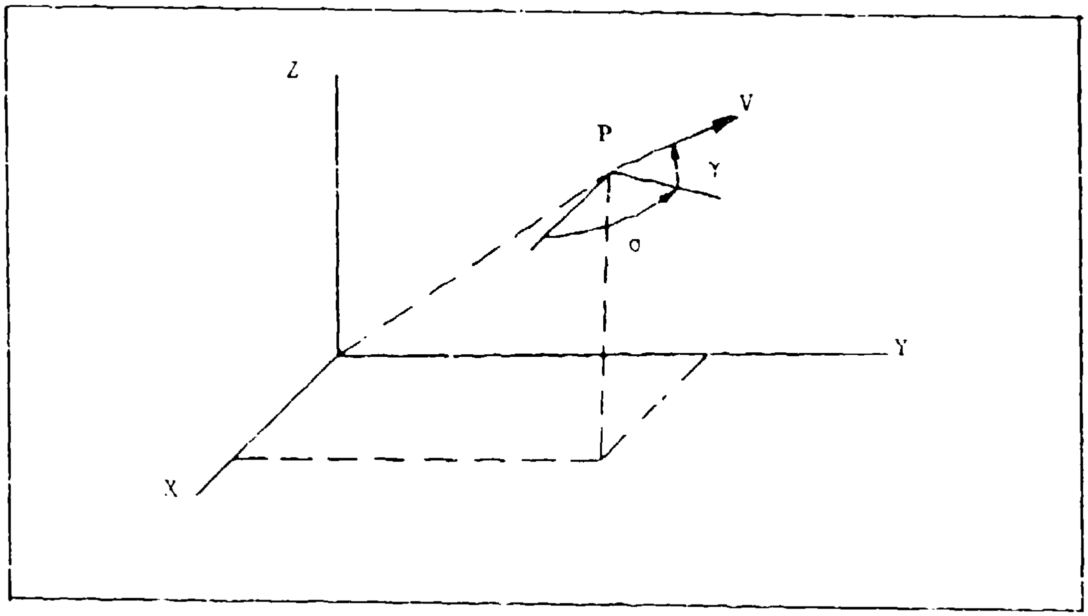

Figure 6-1 State Variables

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The differential equations are as follows:

$$\dot{V} = g\left(\frac{T}{W} - \frac{D}{W} - \sin\gamma\right) \qquad (6\text{-}1)$$

$$\dot{\sigma} = \frac{g}{V}\left(\frac{L}{W} + \frac{T}{W}\sin\alpha\right)\frac{\sin\phi}{\cos\gamma} \qquad (6\text{-}2)$$

$$\dot{\gamma} = \frac{g}{V}\left[\left(\frac{L}{W} + \frac{T}{W}\sin\alpha\right)\cos\phi - \cos\gamma\right] \qquad (6\text{-}3)$$

$$\dot{x} = V\cos\gamma\cos\sigma \qquad (6\text{-}4)$$

$$\dot{y} = V\cos\gamma\sin\sigma \qquad (6\text{-}5)$$

$$\dot{z} = V\sin\gamma \qquad (6\text{-}6)$$

The load factor n is defined as

$$n = \frac{L}{W} + \frac{T}{W}\sin\alpha$$

Substitution into Equations 6-2 and 6-3 gives

$$\dot{\sigma} = \frac{g}{V}\frac{n\sin\phi}{\cos\gamma} \qquad (6\text{-}7)$$

$$\dot{\gamma} = \frac{g}{V}(n\cos\phi - \cos\gamma) \qquad (6\text{-}8)$$

An additional variable of interest is the radius of turn. In a horizontal plane the radius of turn is R_σ where

$$R_\sigma = \frac{V}{|\dot{\sigma}|} \qquad (6\text{-}9)$$

and $|\dot{\sigma}|$ is the absolute value of $\dot{\sigma}$. In the vertical plane the radius of turn is R_γ where

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$$R_\gamma = \frac{V}{|\dot{\gamma}|} \tag{6-10}$$

and $|\dot{\gamma}|$ is the absolute value of $\dot{\gamma}$.

It is easily shown that at the same speed and maximum allowable load factor a faster turning rate can be achieved in the vertical plane than in the horizontal plane. As a consequence, R_σ is greater than R_γ. In the vertical plane if $\phi = \pi$ the lift and weight component are additive, hence

$$|\dot{\gamma}|_{MAX} = \frac{g}{V} (n_{MAX} + \cos \gamma)$$

In the horizontal plane

$$\gamma = \dot{\gamma} = 0 \tag{6-11}$$

and it follows that

$$\cos \phi = 1/n_{MAX} \tag{6-12}$$

Substitution into Equation 6-7 gives

$$|\dot{\sigma}|_{MAX} = \frac{g}{V} \sqrt{n_{MAX}^2 - 1} \tag{6-13}$$

Since

$$n_{MAX} + \cos \gamma \geq n_{MAX} > \sqrt{n_{MAX}^2 - 1}$$

it follows that at the same speed

$$|\dot{\gamma}|_{MAX} > |\dot{\sigma}|_{MAX}$$

Therefore from Equations 6-9 and 6-10

$$R_\sigma > R_\gamma$$

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Optimal Turning in the Horizontal Plane

Turning capability is constrained by the maximum allowable load factor. Maximum load factor as a function of Mach number is illustrated in Figure 6-2.

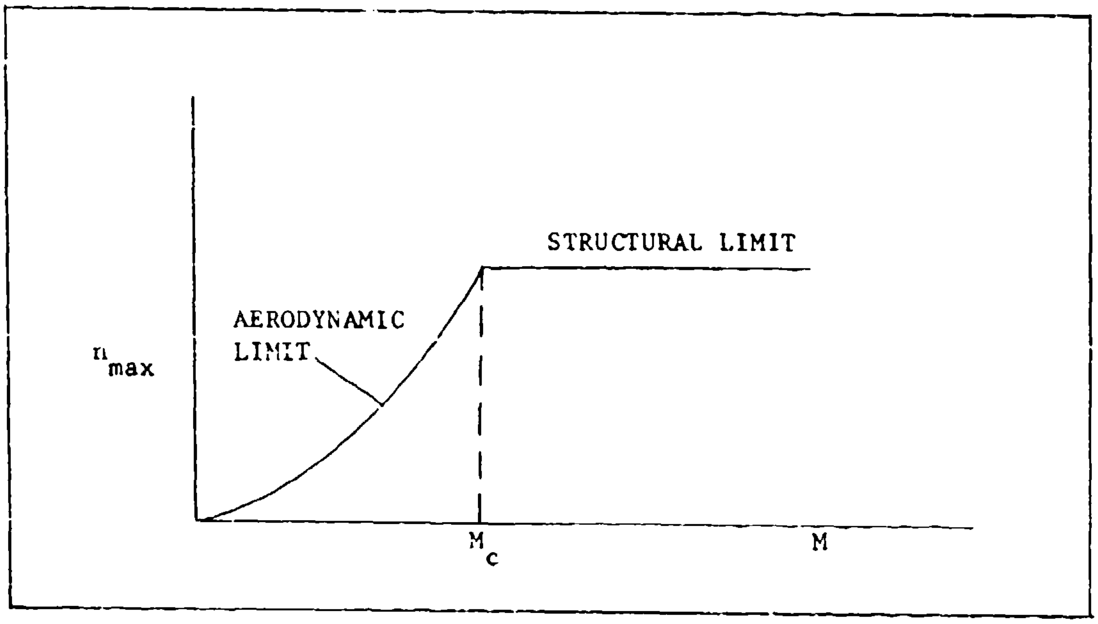

Figure 6-2 Maximum Allowable Load Factor

M_c corresponds to the corner speed and is defined as the intersection between the aerodynamic limit and the structural limit. For level flight performance, M_c and the structural load factor limit yield minimum R_t, maximum $|\dot{\sigma}|$, and maximum bank angle. For level flight turning performance Equations 6-11 through 6-13 hold. Thus, above M_c the radical in equation 6-13 is constant but V increases; thus

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maximum $|\dot{\sigma}|$ decreases along the structural limit. Along the aerodynamic limit n_{MAX} is related to maximum lift coefficient $C_{L_{MAX}}$ by

$$n_{MAX} = \frac{\rho S V^2 C_{L_{MAX}}}{2W} \tag{6-14}$$

Substitution into Equation 6-13 gives

$$|\dot{\sigma}|_{MAX} = \frac{g}{V} \sqrt{\left(\frac{\rho S V^2 C_{L_{MAX}}}{2W} \right)^2 - 1} \tag{6-15}$$

Clearly n_{MAX} must be greater than or equal to one for real $\dot{\sigma}$. Consequently, along the aerodynamic limit $|\dot{\sigma}|_{MAX}$ increases with increasing V and the maximum value occurs at the corner speed. Thus, $|\dot{\sigma}|$ is maximum at the corner speed. From Equations 6-9 and 6-13

$$R_{\sigma} = \frac{V^2}{g \sqrt{n_{MAX}^2 - 1}} \tag{6-16}$$

Along the structural limit R_{σ} is minimum at the corner speed. Along the aerodynamic limit Equations 6-14 and 6-16 give

$$R_{\sigma} = \frac{V^2}{g \sqrt{\left(\frac{\rho S V^2 C_{L_{MAX}}}{2W} \right)^2 - 1}} \tag{6-17}$$

Along the aerodynamic limit R_{σ} decreases with increasing V, hence the corner speed yields minimum R_{σ}. Finally maximum bank angle, ϕ, occurs along the structural limit. This conclusion follows directly from Equation 6-12.

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The next four problems concern sustained level flight turning performance. This infers that the speed, the flight path angle, and the turning performance variables - n, R_σ, ϕ, and $\dot{\sigma}$ do not change with time. The engine throttle setting is also fixed. First, the general case will be considered and then the special case of a parabolic drag polar will be addressed. Since the speed is constant and $\gamma = 0$, Equation 6-1 requires that

$$T = D \tag{6-18}$$

Equation 6-3 requires that

$$\cos \phi = 1/n \tag{6-19}$$

Equation 6-2 becomes upon substitution of Equation 6-19

$$\dot{\sigma} = \frac{g}{V}\sqrt{n^2-1} \tag{6-20}$$

The radius of turn is

$$R_\sigma = \frac{V}{\dot{\sigma}} \tag{6-21}$$

The drag is defined as

$$D = \frac{1}{2}\rho S V^2 C_D \tag{6-22}$$

The drag coefficient C_D is a function of altitude, Mach number, and lift coefficient, C_L. The lift coefficient is related to n through

$$n = \frac{\rho S V^2 C_L}{2W} \tag{6-23}$$

Given the speed and altitude, C_D is obtained from Equations 6-18 and 6-22. C_L is determined from the relation between C_L, C_D, M, and altitude. Finally, n is derived from Equation 6-23. Thus n as a function of V for T = D is like that illustrated in Figure 6-3. Superimposed on Figure 6-3 are the aerodynamic and structural limits.

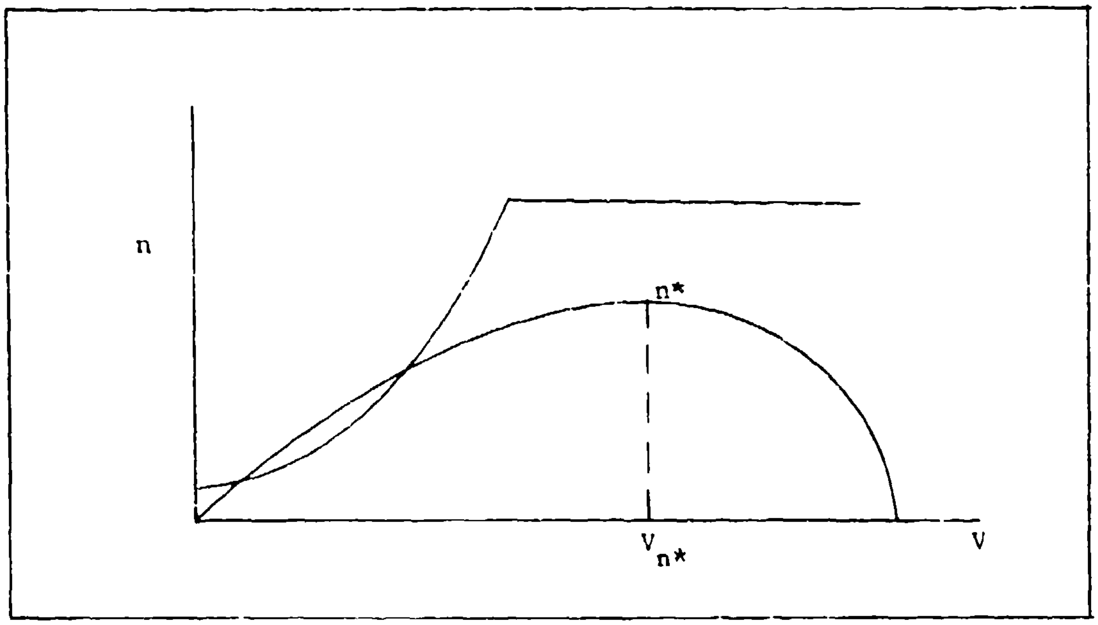

Figure 6-3 Sustained Level Flight Load Factor

Data like that in Figure 6-3 identifies n* and the speed for optimal n, V_{n*}. Maximum bank angle ϕ^* is obtained from Equation 6-19

$$\phi^* = \cos^{-1}(1/n^*)$$

Relative to turning rate, substitution of the relation between n and V into Equation 6-20 gives $\dot{\sigma}$ as a function of V. The result is presented in Figure 6-4.

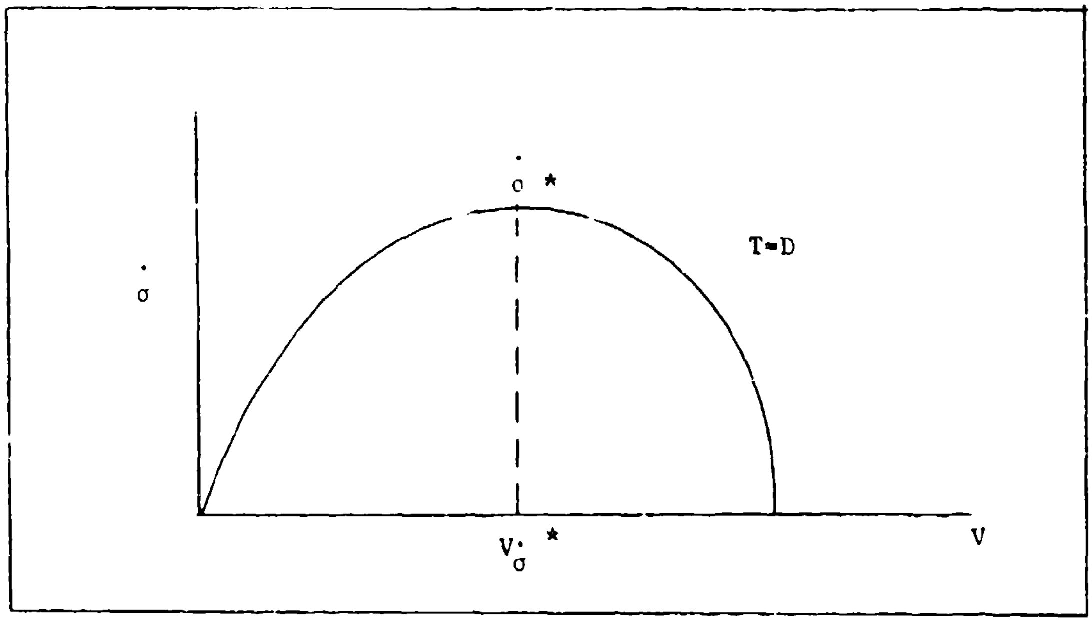

Figure 6-4 Sustained Level Flight Turning Rate

The global maximum sustained value for the turning rate, $\dot{\sigma}^*$, and the corresponding optimal speed $V_{\dot{\sigma}^*}$, are indicated. This solution is correct if the value of n corresponding to $\dot{\sigma}^*$ is interior. If not, $\dot{\sigma}^*$ is constrained by the maximum allowable load factor.

Substitution of Equation 6-20 into Equation 6-21 gives

$$R_{\sigma} = \frac{V^2}{g\sqrt{n^2 - 1}} \qquad (6\text{-}24)$$

The relation between n and V when substituted into Equation 6-24 gives data like that in Figure 6-5. As before, the value of n corresponding to minimum R_{σ} must be compared with the constraints on maximum allowable load factor. Presentations like the data in Figures 6-3 through 6-5 along with the maximum allowable load factor identify optimal values for the load factor, bank angle, turning rate, and turning radius.

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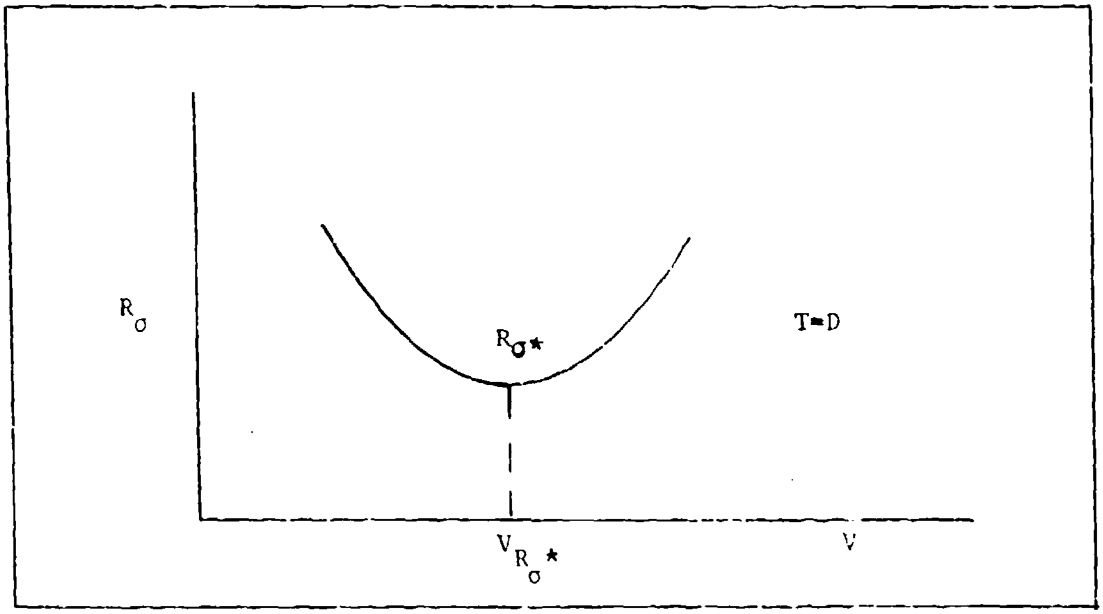

Figure 6-5 Sustained Level Flight Turning Radius

We now turn our attention to the special case of a parabolic aerodynamic polar. The aerodynamic relation is

$$C_D = C_{D_0} + K\, C_L^{\,2}$$

where C_{D_0} and K are assumed to be constant. Substituting Equations 6-18 and 6-23 gives

$$n = \frac{V}{W}\sqrt{\frac{\rho S}{2K}\left(T - \frac{1}{2}\rho S V^2 C_{D_0}\right)} \qquad (6\text{-}25)$$

Substitution into Equation 6-20 gives

$$\dot{\sigma} = \frac{g}{V}\sqrt{\frac{\rho S V^2}{2KW^2}\left(T - \frac{1}{2}\rho S V^2 C_{D_0}\right) - 1} \qquad (6\text{-}26)$$

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The relation for R_σ is shown in Equation 6-21, and can be rewritten thus

$$R_\sigma = V^2 \Big/ g\sqrt{\frac{\rho S V^2}{2KW^2}\left(T - \frac{1}{2}\rho S V^2 C_{D_o}\right) - 1} \qquad (6\text{-}27)$$

Equations 6-25 through 6-27 and

$$\cos \phi = 1/n$$

define the relationships between the turning performance variables, the speed, the altitude, and the aircraft characteristics. Hereafter, it will be assumed that the aerodynamic coefficients and the thrust are independent of speed. The problems which will be addressed are the derivation of the speeds which maximize n and $\dot{\sigma}$ and minimize R_σ.

It is easily shown that the speed which maximizes n, V_{n*}, is

$$V_{n*} = \sqrt{\frac{T}{\rho S C_{D_o}}} \qquad (6\text{-}28)$$

The global maximum value of n, n*, is

$$n^* = \frac{T}{W}\left(\frac{L}{D}\right)_{MAX} \qquad (6\text{-}29)$$

Whether or not this is a feasible solution is dependent upon whether or not V_{n*} and n* are interior to the aerodynamic and structural limits. It should be pointed out that there is an alternate way for deriving Equation 6-29. From the definition of n and T = D, it follows that

$$n = \frac{L}{W} = \frac{L}{D}\frac{T}{W}$$

Thus the maximum value of n is

$$n^* = \max_V \left(\frac{L}{D} \frac{T}{W} \right)$$

which reduces to Equation 6-29 if the thrust is independent of speed. The relation between n*, L/D, and T/W is presented in Figure 6-6. Maximum bank angle is determined directly from

$$\cos \phi^* = 1/n^*$$

and is presented in Figure 6-7.

The speed which maximizes $\dot{\sigma}$, $V_{\dot{\sigma}^*}$ is

$$V_{\dot{\sigma}^*} = \left(\frac{4KW^2}{\rho^2 S^2 C_{D_o}} \right)^{\frac{1}{4}} \qquad (6\text{-}30)$$

which is recognized as the speed for maximum L/D. Substitution into Equation 6-26 gives the global maximum value for $\dot{\sigma}$, $\dot{\sigma}^*$

$$\dot{\sigma}^* = g \sqrt{2 \frac{\rho S C_{D_o}}{W} \left(\frac{L}{D} \right)_{MAX} \left[\left(\frac{L}{D} \right)_{MAX} \frac{T}{W} - 1 \right]} \qquad (6\text{-}31)$$

Maximum sustained turning rate is presented in Figure 6-8.

For minimum R_σ, R_σ^*, the optimal speed, $V_{R_\sigma^*}$ is

$$V_{R_\sigma^*} = 2W \sqrt{\frac{K}{\rho S T}} \qquad (6\text{-}32)$$

The relation for R_σ^* is therefore

$$R_\sigma^* = W/g\, \rho C_{D_o} S \left(\frac{L}{D} \right)_{MAX} \sqrt{\left(\frac{T}{W} \right)^2 \left(\frac{L}{D} \right)_{MAX}^2 - 1} \qquad (6\text{-}33)$$

Minimum radius of turn is presented in Figure 6-9.

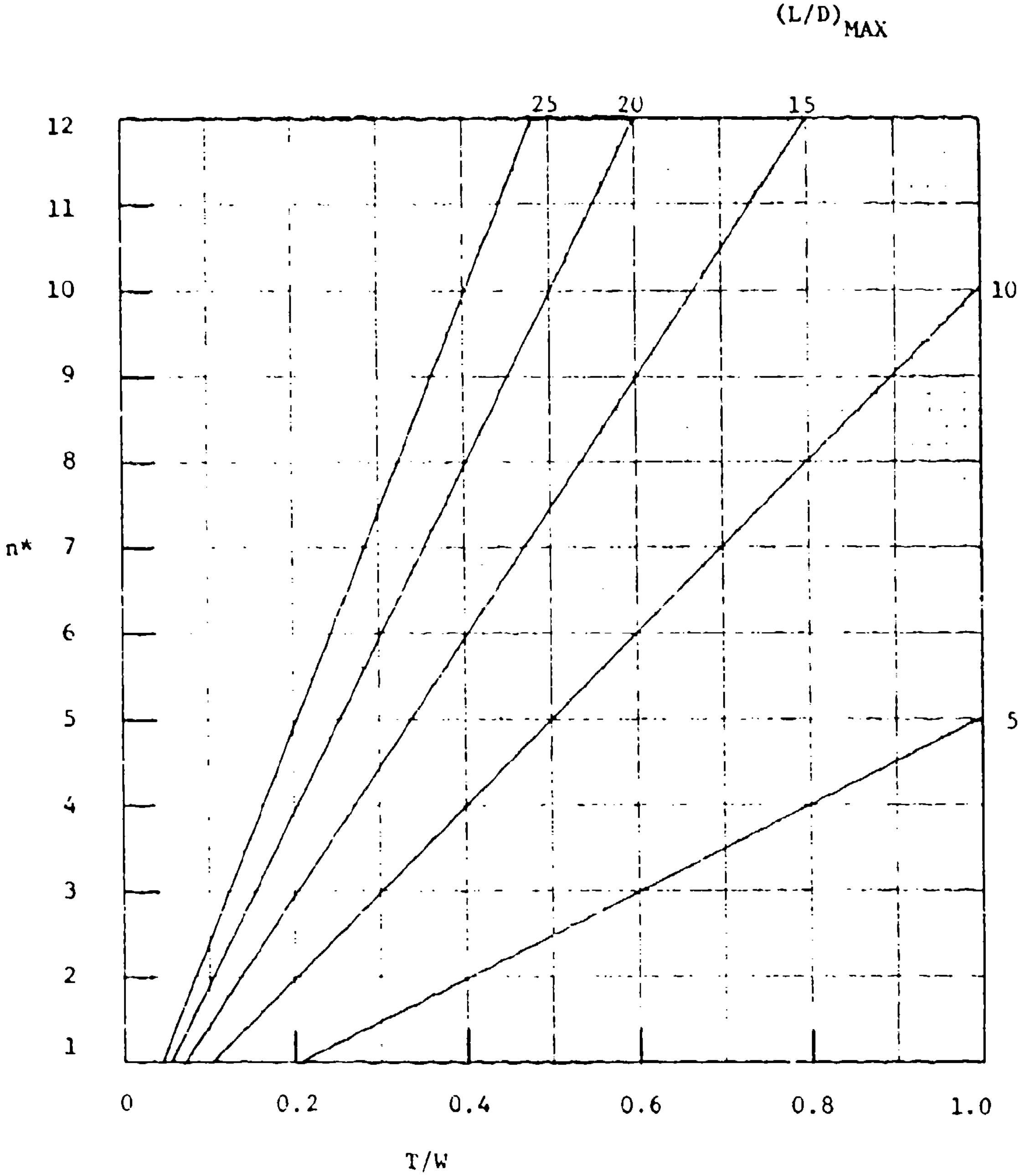

Figure 6-6 Maximum Sustained Load Factor for Level Flight

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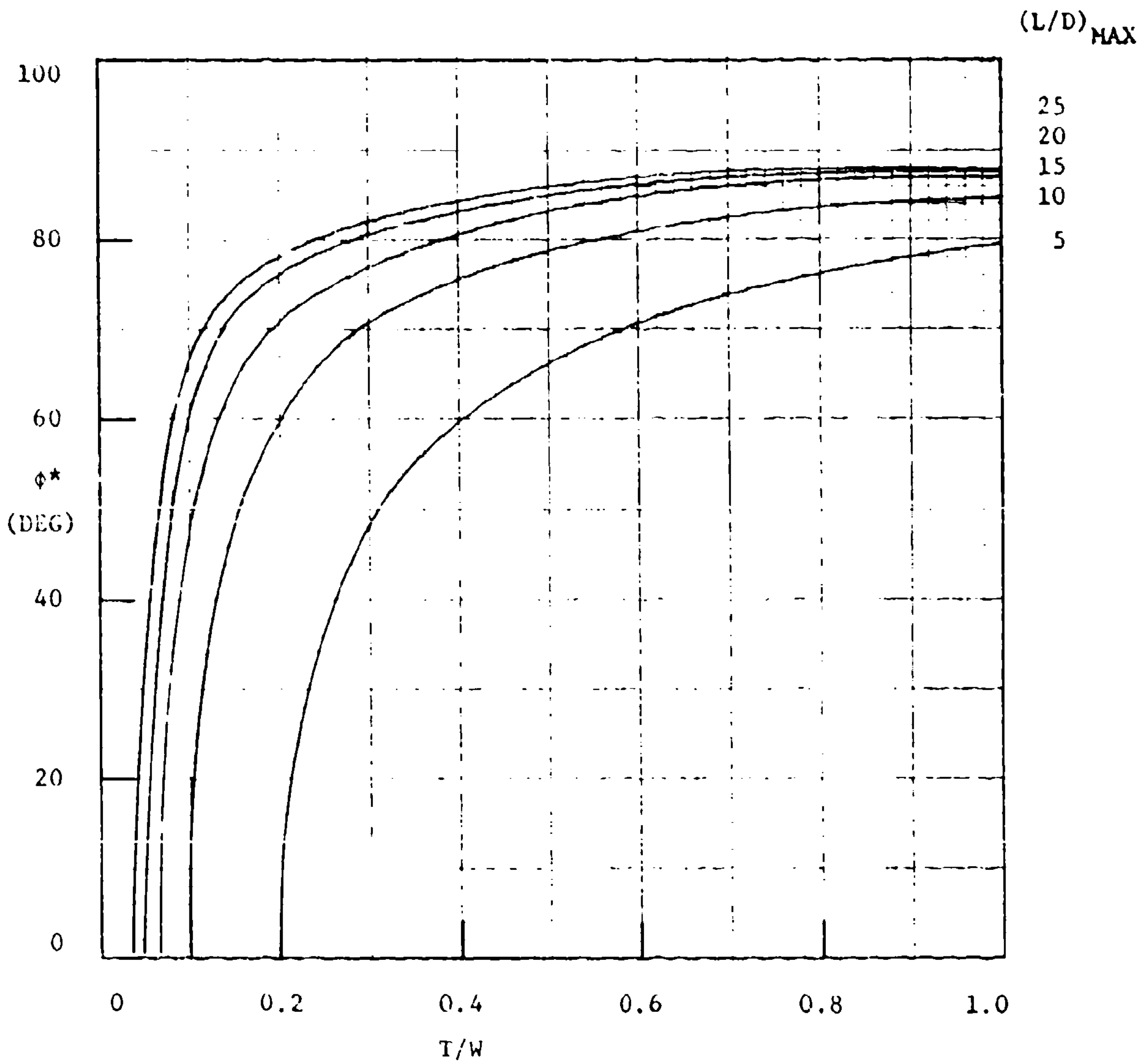

Figure 6-7 Maximum Sustained Bank Angle for Level Flight

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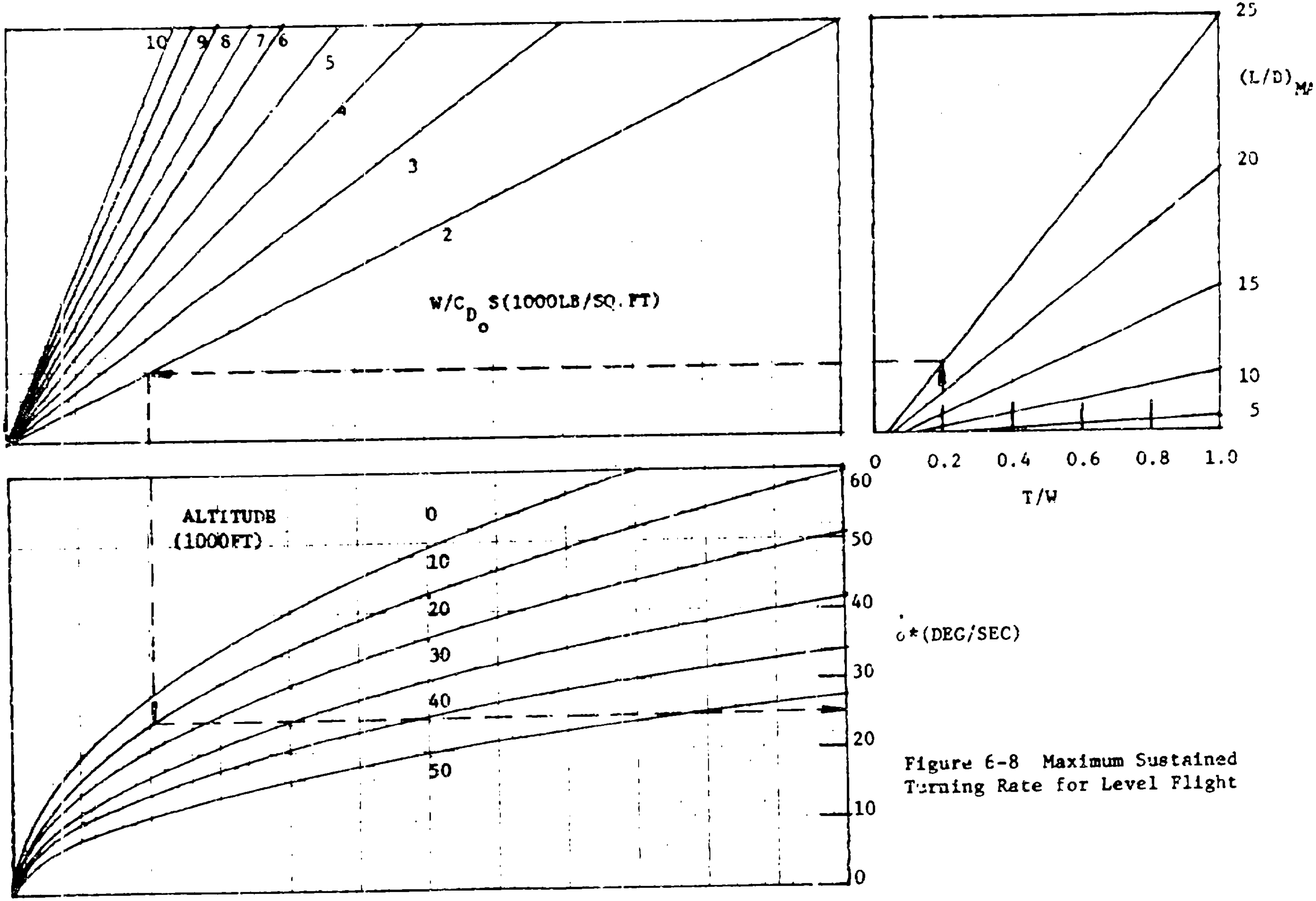

Figure 6-8 Maximum Sustained Turning Rate for Level Flight

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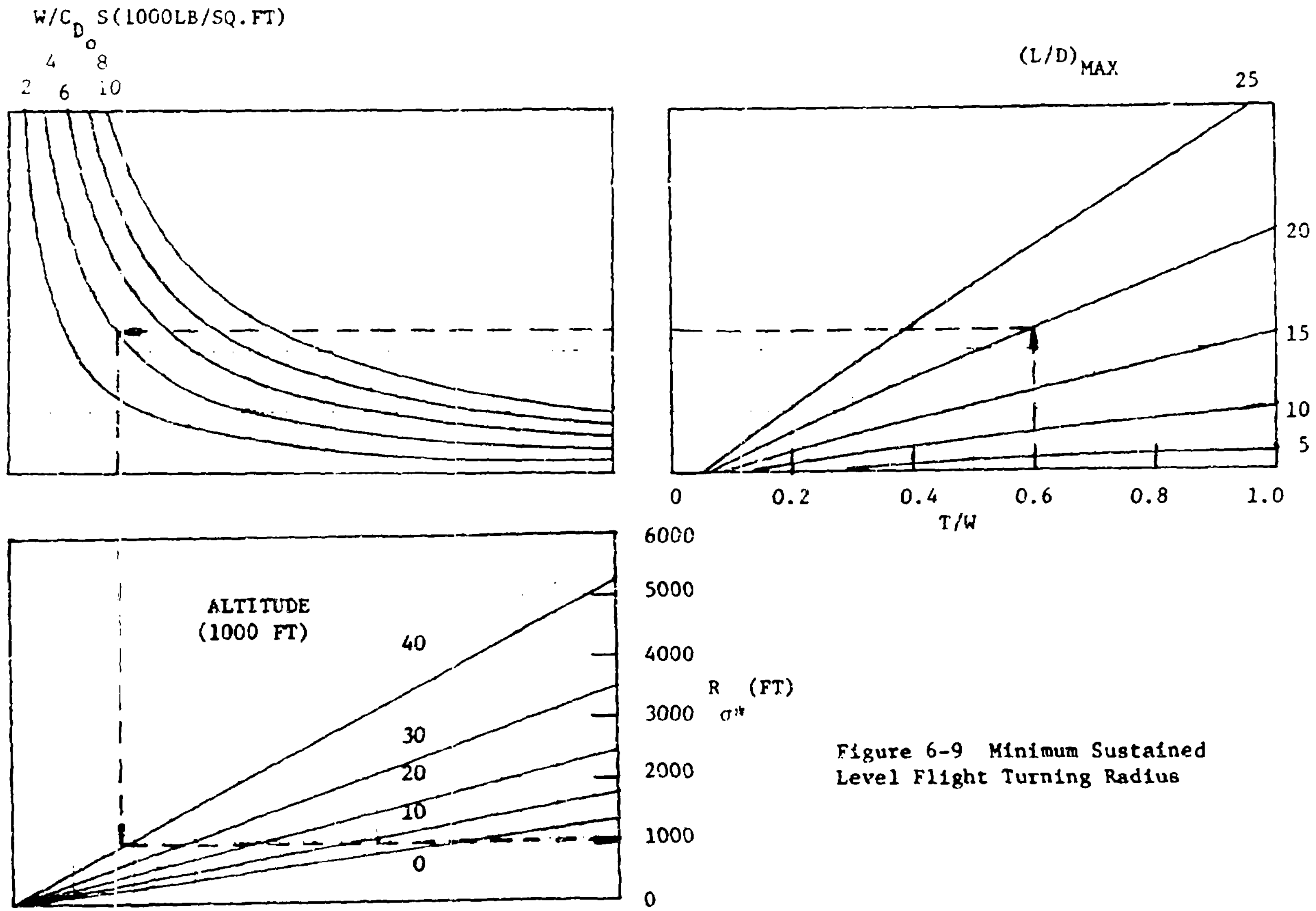

Figure 6-9 Minimum Sustained Level Flight Turning Radius

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Relative to the optimal speeds derived previously, it can be easily shown that at the same altitude

$$V_{n^*} = V_{\phi^*} \geq V_{\dot{\sigma}^*} \geq V_{R_\sigma^*} \qquad (6\text{-}34)$$

The equality follows from

$$\cos \phi^* = 1/n^*$$

From Equations 6-28 and 6-30

$$\left(\frac{V_{n^*}}{V_{\dot{\sigma}^*}} \right)^2 = \frac{T/\rho S C_{D_o}}{\frac{2W}{\rho S} \sqrt{\frac{K}{C_{D_o}}}}$$

$$= \frac{T}{W} \left(\frac{L}{D} \right)_{MAX}$$

From Equation 6-29 and $n^* \geq 1$ for level flight turning, it follows that

$$V_{n^*} \geq V_{\dot{\sigma}^*}$$

From Equations 6-30 and 6-32

$$\left(\frac{V_{\dot{\sigma}^*}}{V_{R_\sigma}^*} \right)^2 = \frac{\frac{2W}{\rho S} \sqrt{\frac{K}{C_{D_o}}}}{4W^2 \cdot \frac{K}{\rho S T}} = \frac{T}{W} \left(\frac{L}{D} \right)_{MAX} \geq 1$$

Therefore

$$V_{\dot{\sigma}^*} \geq V_{R_\sigma^*}$$

In Equation 6-34 strict equality holds if $n^* = 1$. This corresponds to the subsonic ceiling.

Sensitivity Analysis

The sensitivity of the turning performance variables to changes in the aircraft characteristics is presented in Table 6-1 where

$$\Delta = S_{T/W}\frac{dT/W}{T/W} + S_{L/D}\frac{dL/D}{L/D} + S_h\frac{dh}{h} + S_{W/C_{D_o}S}\frac{dW/C_{D_o}S}{W/C_{D_o}S}$$

and Δ is the relative change in the turning performance variable (e.g., dn^*/n^*). The sensitivity parameters $S_{T/W}$ and $S_{L/D}$ for the bank angle, turning rate, and radius of turn are presented in Figures 6-10 through 6-12. The conclusion to be drawn from these data is that an increase in L/D relative to an equal increase in T/W results in an equal change in n^* or ϕ^*, an increase in $\dot{\sigma}^*$, and a decrease in R_{σ}^*. Thus, for sustained level flight turning performance the biggest improvements result from an increase in maximum L/D ratio.

Level Flight Accelerating and Decelerating Turning Performance

We present here only the general theory of nonsteady state level flight turning performance. The analytical derivation of the trajectory is beyond the scope of this effort and will be presented in Volume II. Refer to Figure 6-13 which is the same as Figure 6-3. Interior to the maximum allowable load factor are the two regions $T < D$ and $T > D$. The curve $T = D$ was examined earlier and is the constant speed situation.

There are three situations which require examination. They all depend upon the initial speed, load factor, and bank angle. From Equations 6-1 and 6-3 for level flight

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TABLE 6-1

SUSTAINED LEVEL FLIGHT TURNING PERFORMANCE SENSITIVITY PARAMETERS

Variable	$S_{T/W}$	$S_{L/D}$	S_h	$S_{W/C_{D_o}S}$
n^*	1	1	0	0
ϕ^*	$1/\sqrt{n^{*2}-1}\ \sec^{-1} n^*$	$S_{T/W}$	0	0
$\dot{\sigma}^*$	$\frac{n^*}{2(n^*-1)}$	$\frac{2n^*-1}{2(n^*-1)}$	$-\frac{1}{2}\beta h$	$-\frac{1}{2}$
R_σ^*	$\frac{-n^{*2}}{n^{*2}-1}$	$-\frac{2n^{*2}-1}{n^{*2}-1}$	βh	1

$$n^* = \frac{T}{W}\left(\frac{L}{D}\right)_{MAX}$$

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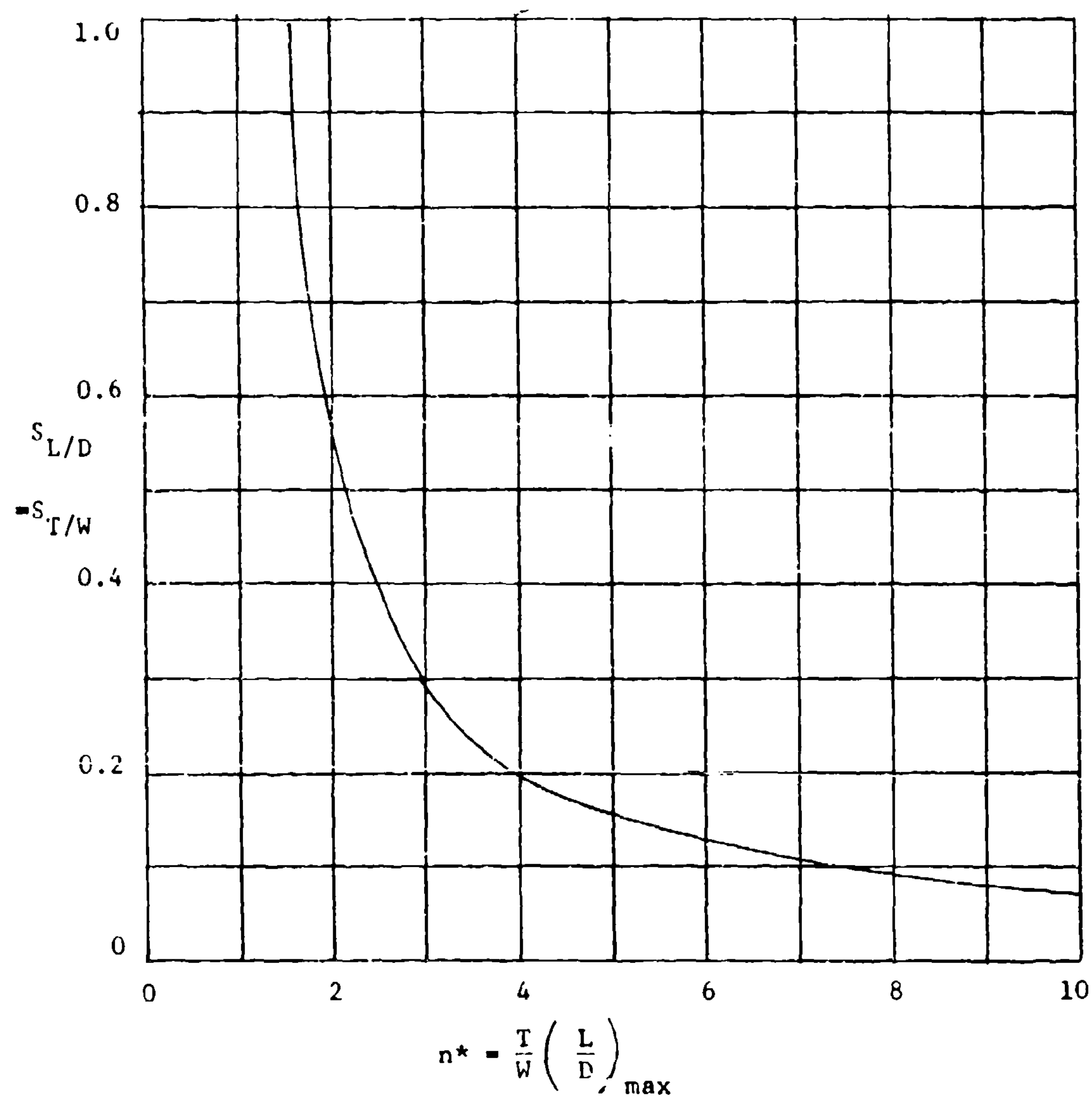

Figure 6-10 Bank Angle Sensitivity Parameters

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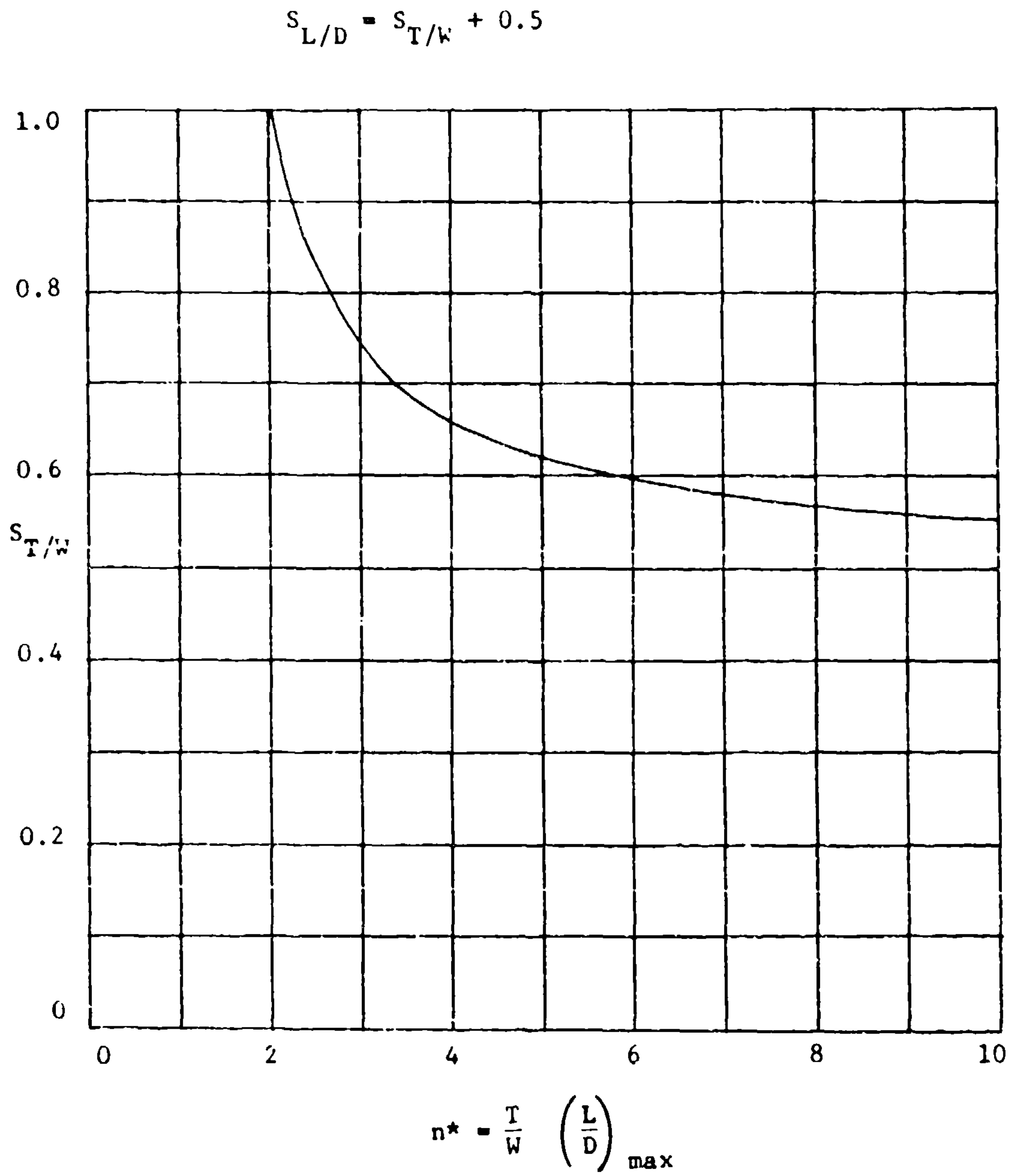

Figure 6-11 Turning Rate Sensitivity Parameters

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$S_{L/D} = S_{T/W} - 1$

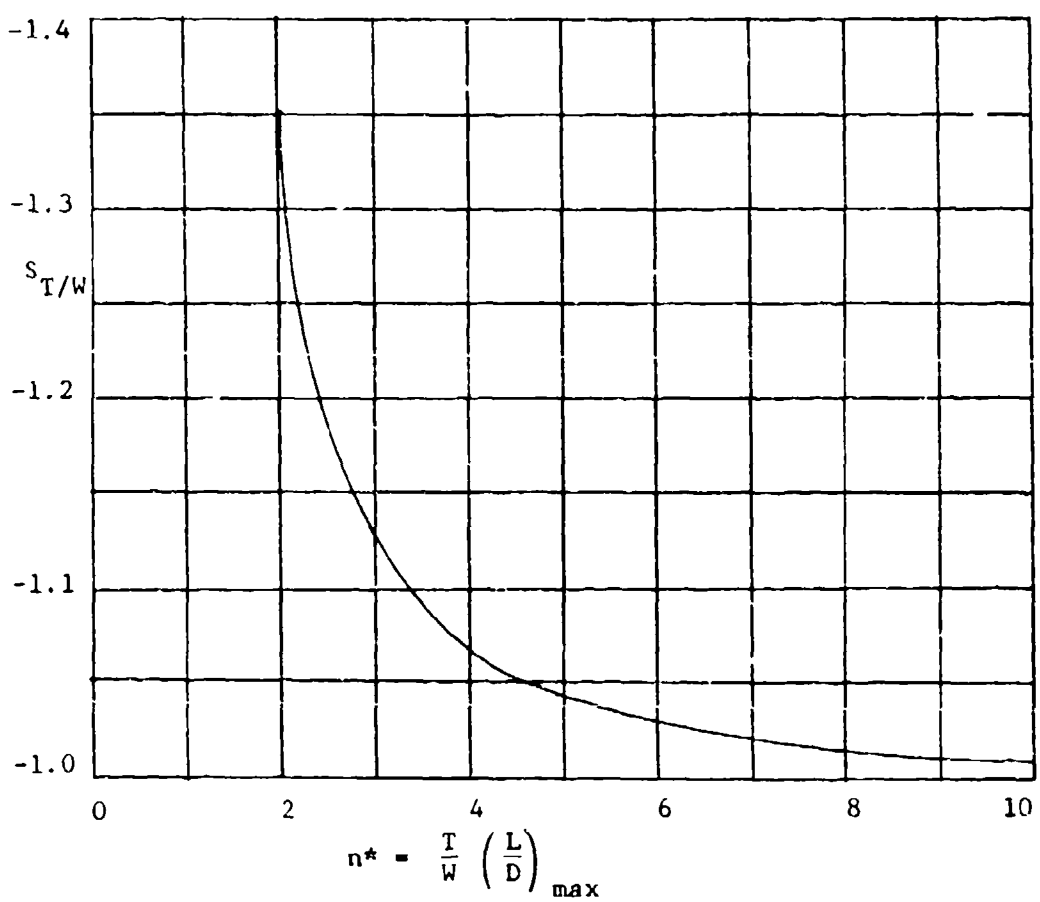

Figure 6-12 Turning Radius Sensitivity Parameters

$$\dot{V} = g\left(\frac{T}{W} - \frac{D}{W}\right) \qquad (6\text{-}35)$$

$$n \cos \phi = 1 \qquad (6\text{-}36)$$

Consider first the situation where the initial point corresponds to point A in Figure 6-13.

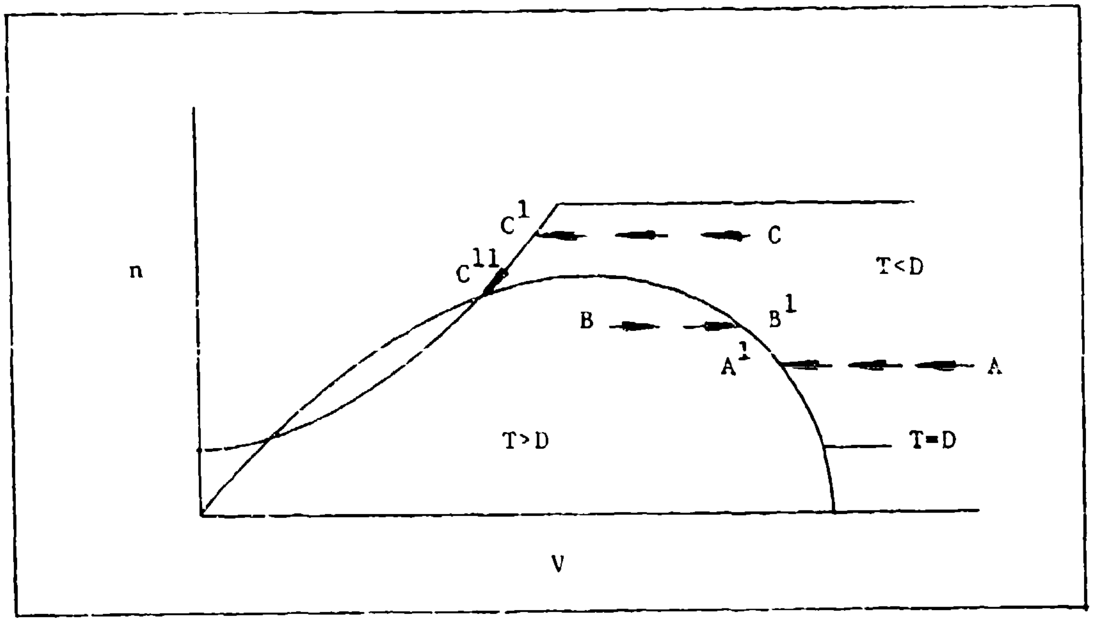

Figure 6-13 Accelerating and Decelerating Turning Performance

Since $T < D$, $\dot{V} < 0$ from Equation 6-35. Consequently, the speed decreases until the curve $T = D$ is reached at point A'. Point A' is a stable point in that a perturbation in the speed will result in the speed returning to the point A'. Also, along A - A' n and ϕ remain constant.

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If point B represents the initial state, then $T > D$ and $\dot{V} > 0$. Therefore, the speed increases until the curve $T = D$ is reached. The point B' is also stable. The load factor and bank angle are constant along B - B'.

When point C represents the initial state, the trajectory corresponds to the path C - C' - C". Since $T < D$ at point C, $\dot{V} < 0$ and the speed decreases at constant ϕ and n until point C' is reached. Since point C' is on the aerodynamic limit and $T < D$, the speed decreases along the aerodynamic limit until point C" is reached. On the aerodynamic limit both ϕ and n decrease as V decreases. Point C" is unstable since a perturbation in the speed causes the speed to move away from C". If the perturbation is positive, $T > D$ and the speed increases until the curve $T = D$ is reached. If the perturbation is negative the speed decreases with a stall occurring.

Thus, Figure 6-13 illustrates the sequence of events for level flight turning maneuvers.

Turning Performance for Constant Speed and Flight Path

The analysis here has application to both ascending and descending flight. Since the speed and flight path angle are constant, Equations 6-1 and 6-3 give

$$T = D + W \sin \gamma \tag{6-37}$$

$$n \cos \phi = \cos \gamma \tag{6-38}$$

Consequently, the thrust control maintains constant speed while load factor and bank angle maintain constant flight path angle. The thrust

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for ascending and descending flight is different due to the difference in the sign for the flight path angle.

Since the flight path angle and speed are constant, from Equations 6-2 and 6-37 the heading angular rate is also constant

$$\dot{\sigma} = \frac{g}{V} \frac{n \sin \phi}{\cos \gamma} \tag{6-39}$$

Consequently, Equations 6-39, 6-4, 6-5, and 6-6 are integrable

$$\sigma = \sigma_0 + \frac{gt}{V} \frac{n \sin \phi}{\cos \gamma}$$

$$x = x_0 + \frac{V}{\dot{\sigma}} \cos \gamma \ (\sin \sigma - \sin \sigma_0)$$

$$y = y_0 + \frac{V}{\dot{\sigma}} \cos \gamma \ (\cos \sigma_0 - \cos \sigma)$$

$$z = z_0 + Vt \sin \gamma$$

where subscript zero implies initial value. Clearly, the trajectory is a helix. The radius of the trace of the trajectory in the x-y plane, R_σ, is

$$R_\sigma = \frac{V}{\dot{\sigma}} \cos \gamma$$

This reduces to the radius of turn Equation 6-9 for horizontal turns.

By eliminating the load factor by means of Equation 6-38, the following equations result

$$n = \frac{\cos \gamma}{\cos \phi}$$

$$R_\sigma = \frac{V^2}{g} \cos \gamma \cot \phi$$

$$\Delta t = \frac{V}{g} \Delta\sigma \cot \phi$$

$$\Delta h = \Delta t \cdot V \sin \gamma$$

where Δt, $\Delta\sigma$, Δh are the changes in the time, heading angle, and altitude during the turn. The variables are presented in Figures 6-14 through 6-17. In Figure 6-17, the sign for γ and Δh is negative for descending turns.

The sensitivity of the turning performance variables to changes in γ, ϕ, and V are presented in Table 6-2 where

$$\Delta = S_\gamma \, d\gamma + S_\phi \, d\phi + S_V \frac{dV}{V} + S_{\Delta\sigma} \frac{d(\Delta\sigma)}{\Delta\sigma}$$

and Δ is the change in a performance variable (eq. dn/n).

TABLE 6-2

ASCENDING AND DESCENDING TURNING PERFORMANCE SENSITIVITY PARAMETERS

Variable	S_γ	S_ϕ	S_V	$S_{\Delta\sigma}$
n	$-\tan \gamma$	$\tan \phi$	0	0
R_σ	$-\tan \gamma$	$-\csc^2 \phi$	2	0
Δt	0	$-\csc^2 \phi$	1	1
Δh	$\cot \gamma$	$-\csc^2 \phi$	2	1

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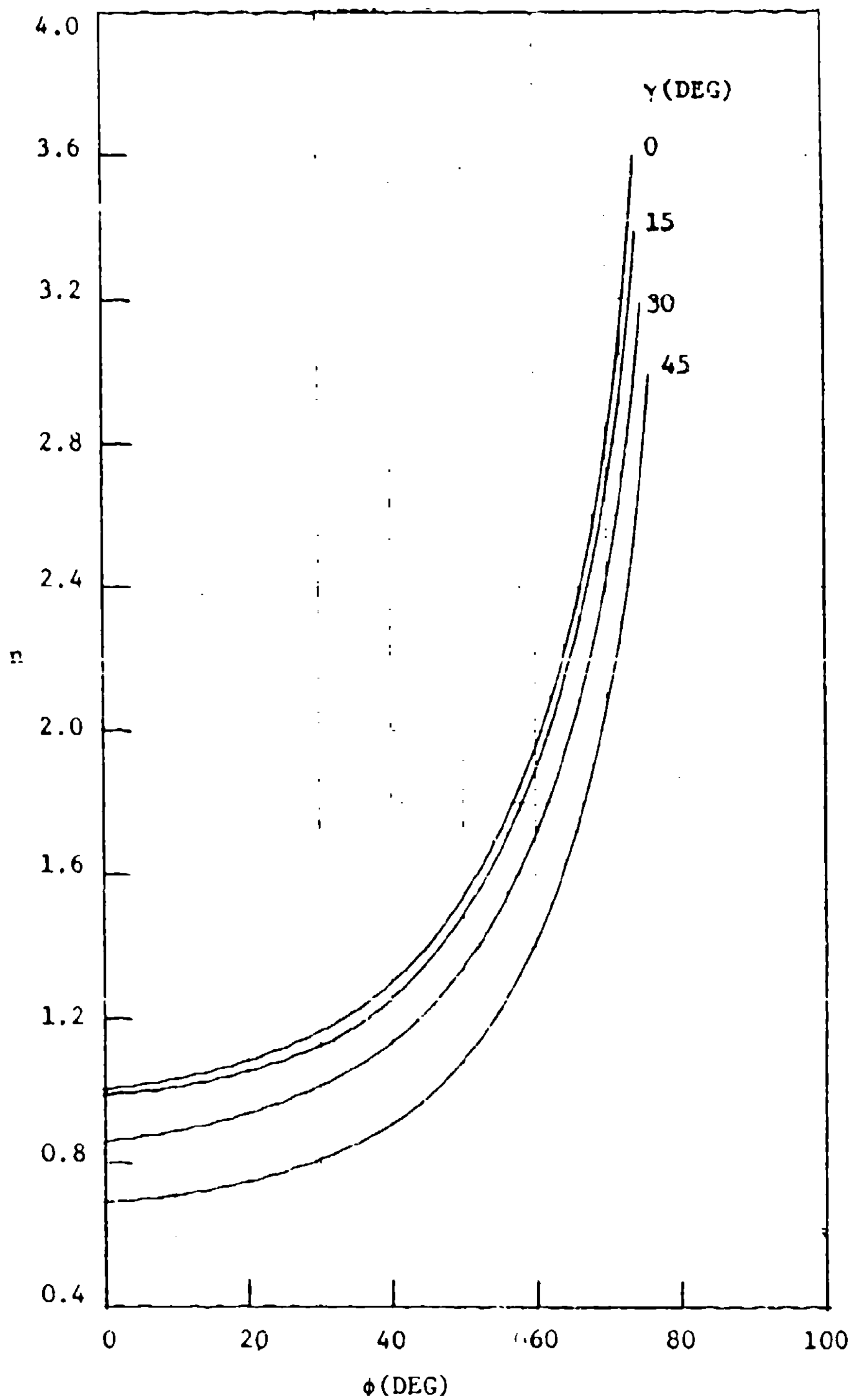

Figure 6-14 Load Factor for Ascending or Descending Turns

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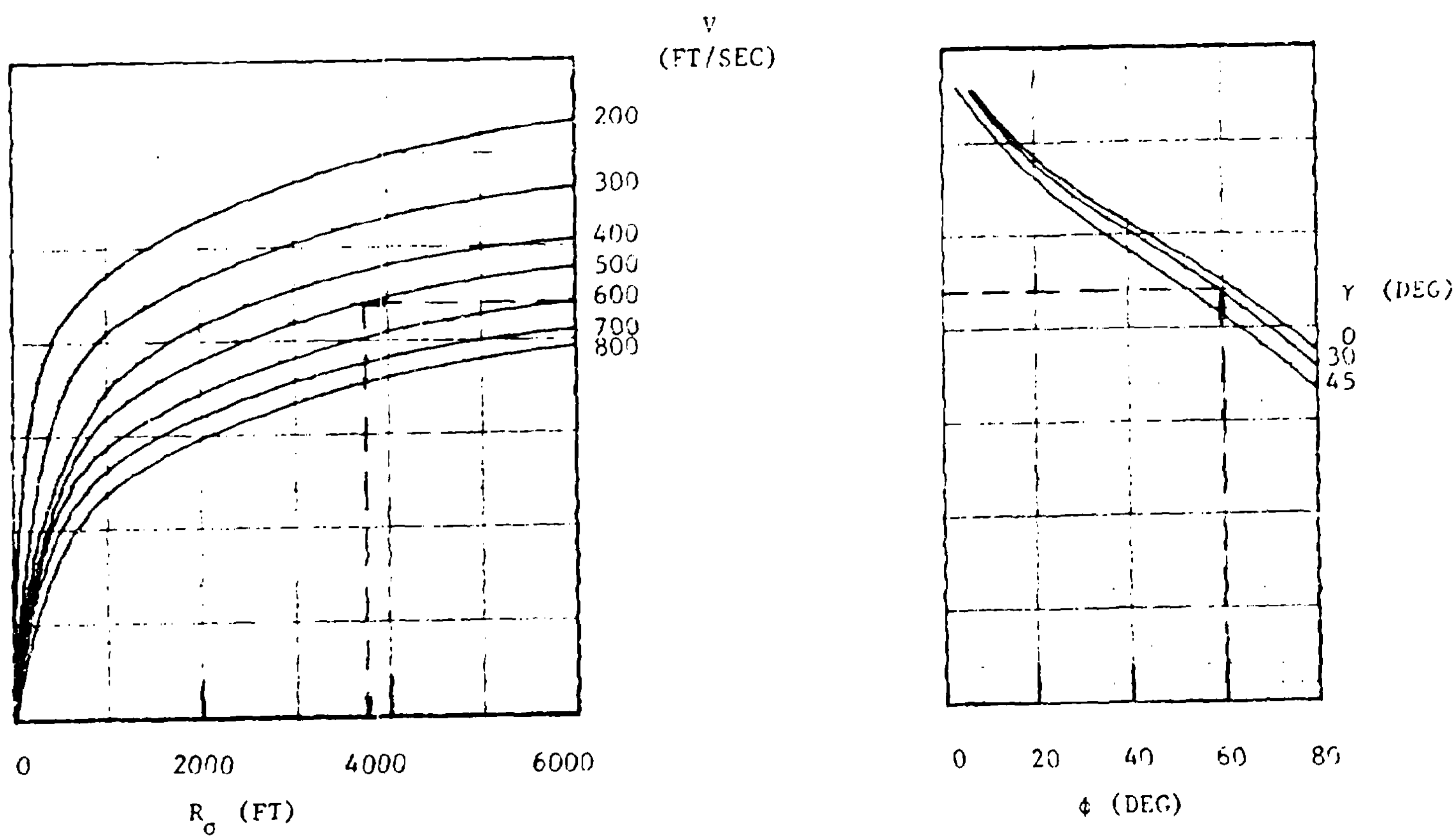

Figure 6-15 Radius of Turn in the Horizontal Plane

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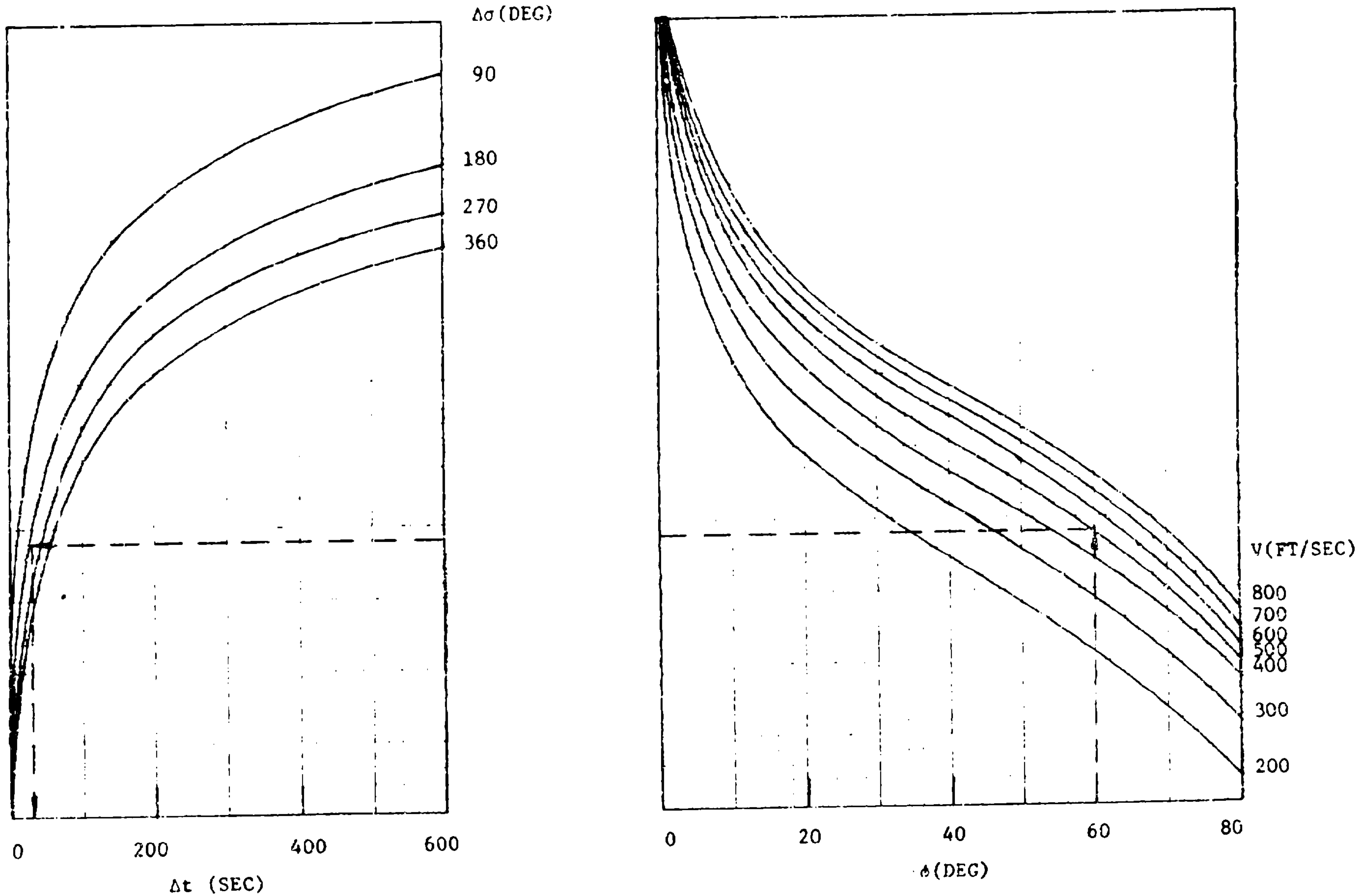

Figure 6-16 Time Required to Perform a Turn

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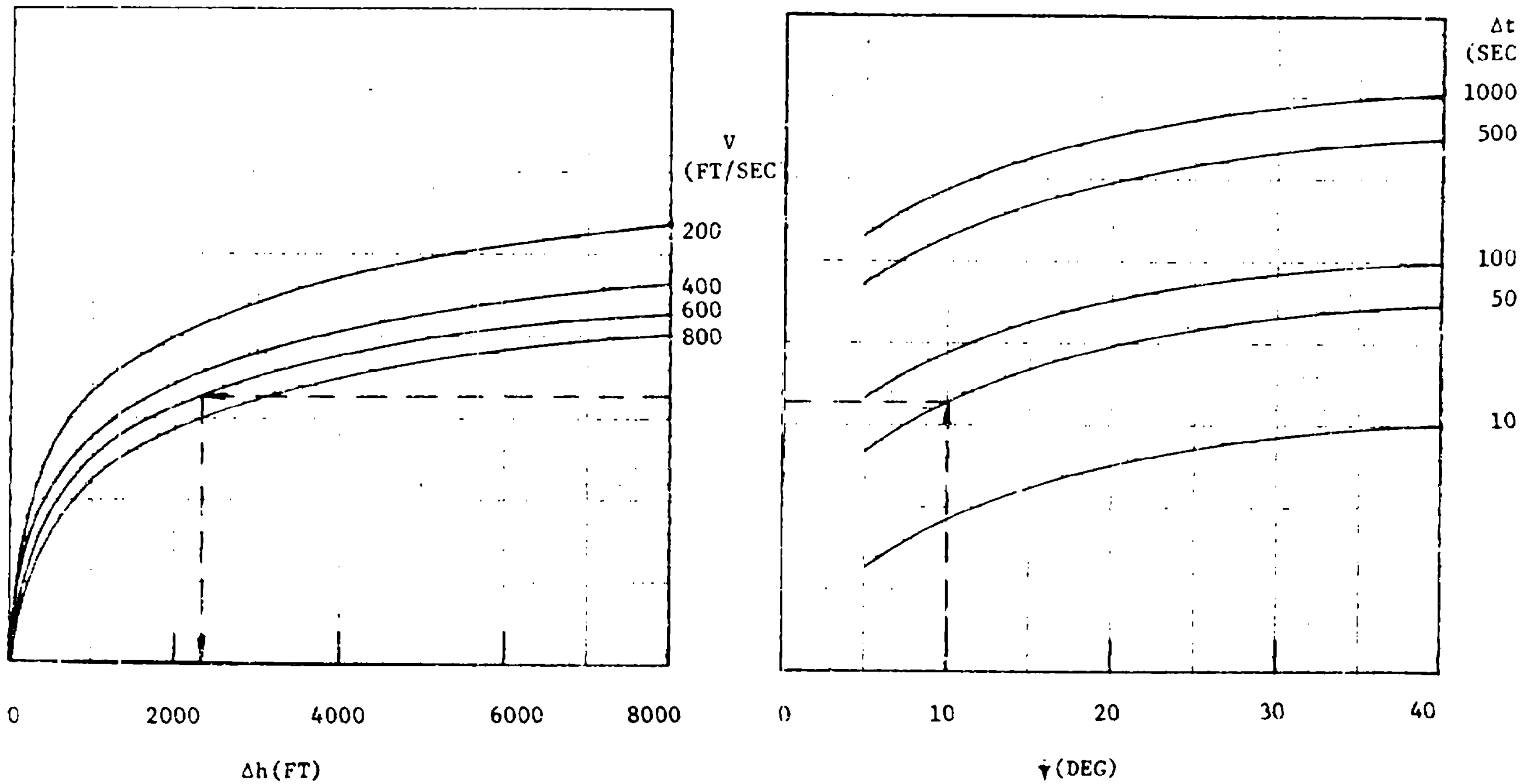

Figure 6-17 Altitude Change During an Ascending or Descending Turn

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Summary of the Turning Performance Problems

Analytical solutions were derived for constant speed and constant flight path angle trajectories. In addition, sensitivity parameters were determined for these problems. The situation wherein acceleration or deceleration occurs during a horizontal turn was discussed but no solution was derived.

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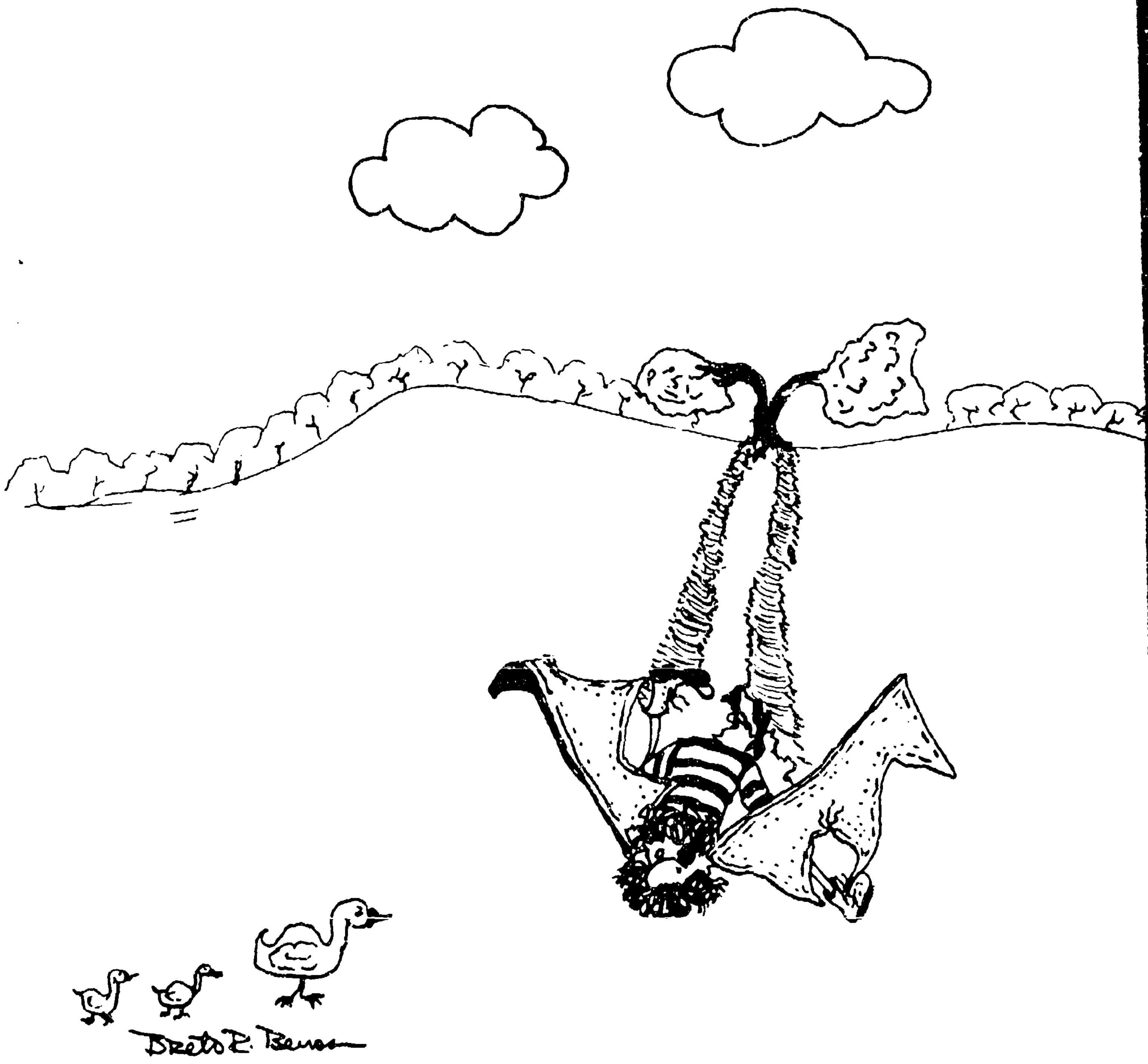

SECTION VII

LANDING PERFORMANCE

SECTION VII

LANDING PERFORMANCE

Problem Definition and Assumptions

As was the case with takeoff performance, the design of an aircraft can be influenced greatly by the requirement to land the aircraft in a given distance. In many of the aircraft today the field length required to land the aircraft approaches the field length required for takeoff. The two main factors which influence the landing field length are the speed at which the aircraft approaches and the braking capability of the aircraft once it is on the ground. The lower the approach speed and the higher the retarding force on the ground the shorter will be the field length required. This has led to the larger and faster aircraft being designed with sophisticated flap and spoiler systems.

The determination of the landing field length is handled in much the same way as the takeoff. The landing is made in two phases - the approach over the obstacle with a flare to touchdown and the ground roll to a full stop. The field length is defined as the distance from the obstacle height to a full stop on the runway. Figure 7-1 illustrates the definition of the landing field length.

The calculation of the landing flare and ground roll is made with the assumption that there is no wind and the aircraft is in a steady state approach toward the obstacle. Military and civilian ground rules for landing field length requirements usually specify that the calculated field length be increased by a constant percentage to account for

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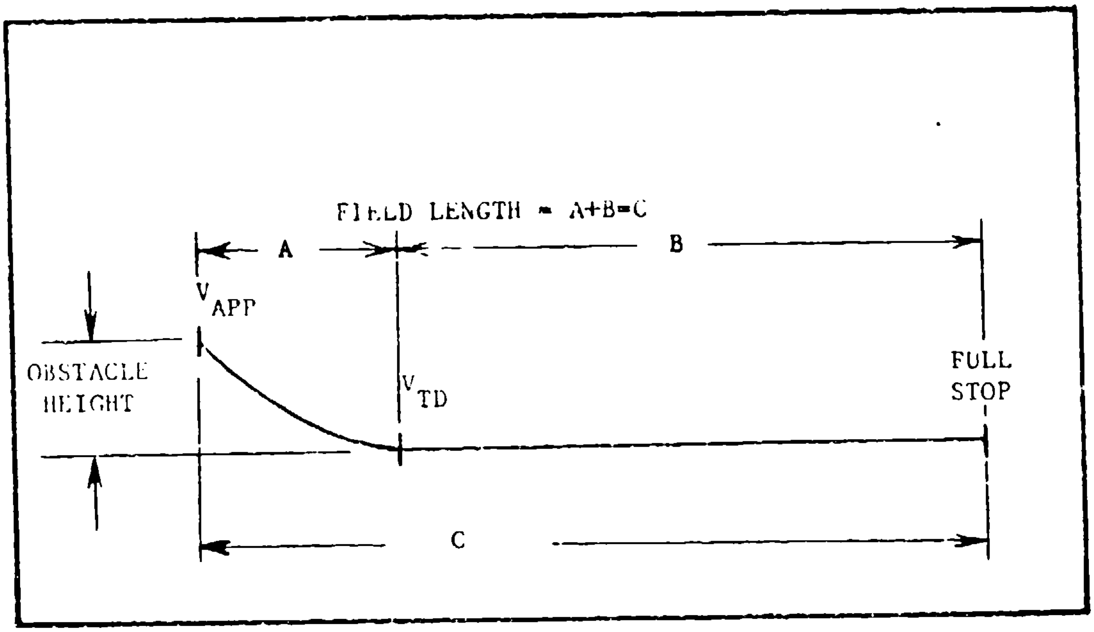

Figure 7-1 Definition of Landing Field Length

variations in runway conditions, aircraft braking, efficiency, and pilot technique. For example, the Federal Aviation Agency landing field length requirement is defined as the calculated landing distance to a full stop over a 50-foot obstacle multiplied by the constant 1.667. In determining landing field length requirements the engineer must be knowledgeable of the appropriate regulations which apply to his aircraft.

Methodology

The calculation of the landing distance will be handled in two phases. The phases are the distance for the approach over the obstacle with the flare and the ground roll distance.

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The approach and flare will be the first to be considered. The distance covered in this portion is the distance from the given obstacle height to the point of touchdown. The approach is made at a steady state glide slope at an approach speed determined by the approach C_L. The equation for approach speed is:

$$V_{APP} = \sqrt{\frac{2W/S}{\rho C_{L_{APP}}}} \tag{7-1}$$

The approach C_L is determined from aerodynamic characteristics of the aircraft in the landing configuration and established under military or civilian ground rules. For example, MIL-C-5011A specifies that the approach speed shall be at least 120% of the power-off stall speed for the landing configuration. For this case the approach C_L would be

$$C_{L_{APP}} = \frac{C_{L_{STALL}}}{1.44} \tag{7-2}$$

The aircraft attitude during approach should be such that the main gear will be the first point of contact with the runway. If the angle of attack at the approach C_L is such that the aircraft tail will strike the ground, then the C_L should be reduced to allow ground clearance. To be conservative the ground clearance is usually checked with the struts compressed.

Equation 7-1 would then give the speed at the obstacle height to begin calculation of the distance to touchdown. This distance can be determined by examining Figure 7-2.

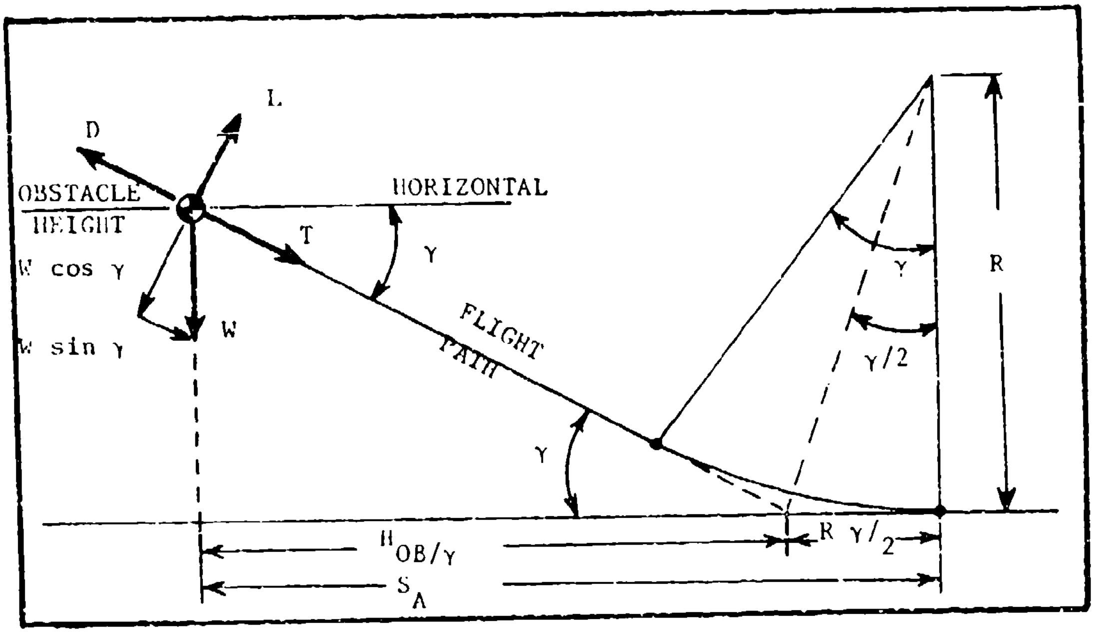

Figure 7-2 Approach and Flare Distance for Landing

The distance (S_A) from the obstacle to touchdown is defined as:

$$S_A = \frac{H_{OB}}{\tan \gamma} + R \tan \gamma/2 \tag{7-3}$$

The glide path angle (γ) is usually very small. Therefore, we can say that the $\tan \gamma \approx \sin \gamma \approx \gamma$ radians. The distance from the obstacle to touchdown is then

$$S_A = \frac{H_{OB}}{\gamma} + R\,\gamma/2 \tag{7-4}$$

If we assume an obstacle height of 50 feet which is standard for military and civilian ground rules, we can write Equation 7-4 to be

$$S_A = \frac{50}{\gamma} + R\,\gamma/2 \tag{7-5}$$

To solve Equation 7-5 we must determine expressions for γ and R in terms of known aircraft parameters and forces.

For the steady state approach the sum of the forces parallel to the flight path is

$$D = T + W \sin \gamma \qquad (7\text{-}6)$$

and the sum of the forces perpendicular to the flight path is

$$L = W \cos \gamma \qquad (7\text{-}7)$$

Solving Equation 7-6 for sin γ and Equation 7-7 for cos γ and dividing gives

$$\frac{\sin \gamma}{\cos \gamma} = \frac{D - T}{L} = \tan \gamma \qquad (7\text{-}8)$$

For small angles and steady state conditions $\tan \gamma \approx \gamma$ radians and $L = W$, therefore

$$\gamma = \frac{1}{L/D} - T/W \qquad (7\text{-}9)$$

The acceleration normal to the flight path needed to flare is attained by rotating the aircraft to a slightly higher C_L defined as C_L'. The lift force is then

$$L' = C_L' \frac{1}{2} \rho V^2 S \qquad (7\text{-}10)$$

The force normal to the flight path is

$$F_N = L' - W \cos \gamma = L' - W \qquad (7\text{-}11)$$

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During the steady state approach and glide the lift force is

$$L = C_{L_{APP}} \frac{1}{2} \rho V^2 S \tag{7-12}$$

and can be assumed equal to the weight. If we set Equation 7-12 equal to the weight and divide Equation 7-10 by 7-12 the result is

$$\frac{L'}{W} = \frac{C_L'}{C_{L_{APP}}}$$

or

$$L' = \frac{C_L'}{C_{L_{APP}}} W \tag{7-13}$$

This assumes that the velocity (V) is essentially constant. Substituting Equation 7-13 into Equation 7-11 the result is

$$F_N = W \frac{C_L'}{C_{L_{APP}}} - W = W \left(\frac{C_L'}{C_{L_{APP}}} - 1 \right)$$

but

$$n = L'/W = C_L'/C_L$$

where n is defined as the load factor. Therefore,

$$F_N = W (n-1) \tag{7-14}$$

The acceleration normal to the flight path for the flare is

$$a_N = V \frac{d\gamma}{dt} \tag{7-15}$$

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and from the relation of angular velocity to the corresponding linear velocity

$$V = R \frac{d\gamma}{dt}$$

therefore,

$$\frac{d\gamma}{dt} = \frac{V}{R} \tag{7-16}$$

Substituting Equation 7-16 into 7-15 gives

$$a_n = \frac{V^2}{R} \tag{7-17}$$

The normal force to the flight path is

$$F_N = ma = \frac{W}{g} a_N \tag{7-18}$$

Substituting Equation 7-14 and 7-17 into Equation 7-18 gives

$$W\,(n-1) = \frac{W}{g} \frac{V^2}{R}$$

Solving for R gives

$$R = \frac{V^2}{g\,(n-1)} \tag{7-19}$$

From Equation 7-10

$$V^2 = \frac{L'}{S} \frac{2}{\rho C_{L'}}$$

and

$$L' = n\,W$$

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Therefore,

$$R = \frac{2nW}{\rho S C_L' g\ (n-1)} \tag{7-20}$$

Substituting Equation 7-20 and 7-9 into Equation 7-5 gives

$$S_A = \frac{50}{\frac{1}{L/D} - \frac{T}{W}} + \frac{\left(\frac{1}{L/D} - \frac{T}{W}\right)}{2} \times \frac{2nW}{\rho S C_L' g\ (n-1)}$$

but,

$$C_L' = n\, C_{L_{APP}}$$

Therefore, the distance to clear a 50-foot obstacle in terms of aircraft parameters and approach conditions is

$$S_A = \frac{50}{\left(\frac{1}{L/D} - \frac{T}{W}\right)} + \frac{\frac{W}{S}\left(\frac{1}{L/D} - \frac{T}{W}\right)}{\rho C_{L_{APP}} g\ (n-1)} \tag{7-21}$$

The stopping distance after touchdown (S_G) can be determined in the same way as the takeoff in Section II. To be conservative we can assume that the touchdown speed is equal to the approach speed and that the derotation is instantaneous. The velocity (V) and acceleration (a) at any time are defined as

$$V = \frac{dS}{dt} \quad \text{and} \quad a = \frac{dV}{dt}$$

or

$$dS = V\,dt \quad \text{and} \quad dt = \frac{dV}{a}$$

Therefore,

$$dS = V\frac{dV}{a} \tag{7-22}$$

Assuming the no-wind condition, the ground stopping distance is defined as

$$S_G = \int_{V_{TD}}^{0} \frac{V\,dV}{a} \tag{7-23}$$

where V_{TD} = touchdown speed.

Figure 7-3 shows the forces acting on the vehicle during the braking portion of the ground roll.

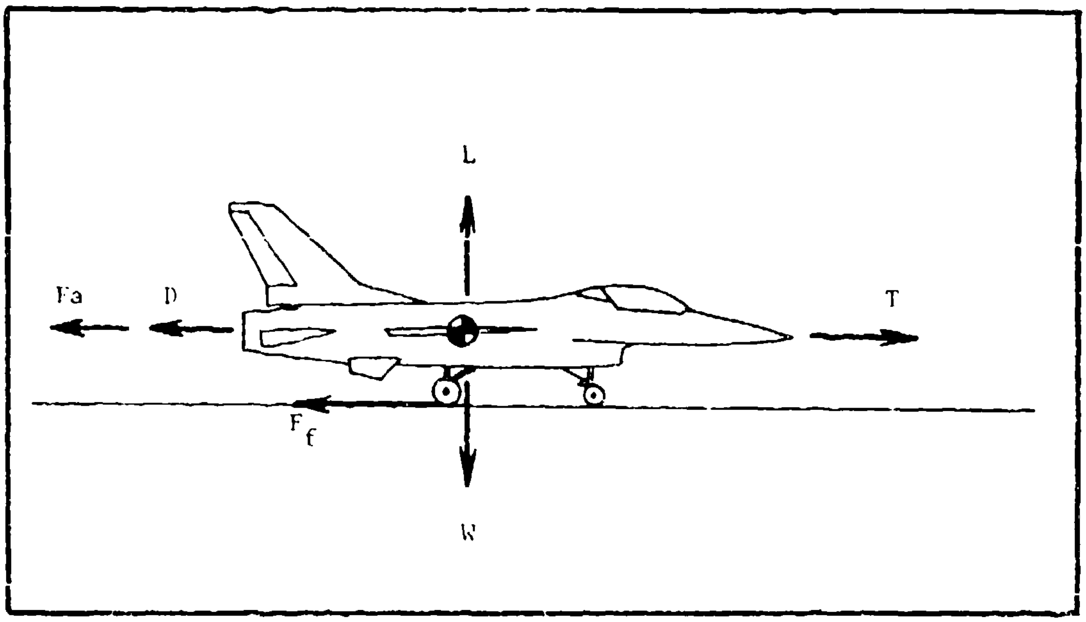

Figure 7-3 Forces Acting on the Aircraft for the Ground Roll

To solve Equation 7-23 an expression for acceleration (a) in terms of aircraft parameters must be determined. The stopping frictional force (F_f) between the runway and the tires is defined as

$$F_f = \mu N = \mu (W - L) \tag{7-24}$$

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The external force acting on the vehicle is

$$F_a = ma = \frac{W}{g} a \qquad (7\text{-}25)$$

The remaining forces are lift (L), drag (D), thrust (T), and weight (W). By summing the forces acting on the aircraft parallel to the direction of motion the result is

$$T - D - \mu (W - L) = \frac{W}{g} a$$

Solving for (a) gives

$$a = g \left[\left(\frac{T}{W} - \mu \right) - \left(\frac{D}{W} - \mu \frac{L}{W} \right) \right]$$

Substituting the definition for lift (L) and drag (D) which are

$$L = C_L qS \qquad \text{and} \qquad D = C_D qS$$

the expression for (a) becomes

$$a = g \left[\left(\frac{T}{W} - \mu \right) - q \frac{S}{W} (C_D - \mu C_L) \right] \qquad (7\text{-}26)$$

Substituting Equation 7-26 into 7-23 gives the expression for the ground roll distance (S_G).

$$S_G = \int_{V_{TD}}^{o} \frac{V\, dV}{g \left[\left(\frac{T}{W} - \mu \right) - q \frac{S}{W} (C_D - \mu C_L) \right]} \qquad (7\text{-}27)$$

Equation 7-27 can now be solved by numerical integration to give the ground-roll distance. However, for the purposes of preliminary design, assumptions can be made to give an analytic expression for

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the grand roll distance. Those assumptions are that $C_D = C_{D_G}$, $C_L = C_{L_G}$, and the thrust are all constant.

Substituting the expression for q, Equation 7-27 may be written

$$S_G = \int_{V_{TD}}^{o} \frac{V\,d\,V}{g\left[\left(\frac{T}{W} - \mu\right) - \frac{\rho S V^2}{2W}\left(C_D - \mu\, C_L\right)\right]} \tag{7-28}$$

Equation 7-28 can then be reduced to the form

$$S_G = A \int_{V_{TD}}^{o} \frac{V\,d\,V}{-B^2 - V^2} \tag{7-29}$$

where B^2 and A are defined as

$$B^2 = \frac{\mu - \frac{T}{W}}{\frac{\rho S}{2W}\left(C_{D_G} - \mu\, C_{L_G}\right)} = -C^2 \tag{7-30}$$

$$A = \frac{1}{\frac{g\rho S}{2W}\left(C_{D_G} - \mu\, C_{L_G}\right)} \tag{7-31}$$

C^2 is the parameter used in the takeoff calculation.

Equation 7-29 can be written

$$S_G = A \int_{o}^{V_{TD}} \frac{V\,d\,V}{B^2 + V^2}$$

and integrating gives the analytical expression

$$S_G = \frac{A}{2} \ln \frac{B^2 + V_{TD}^{\;2}}{B^2} \tag{7-32}$$

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Assuming that $V_{TD} = V_{AP}$, Equations 7-1, 7-30, and 7-31 can be substituted into Equation 7-32 to give a generalized solution for the ground roll. The results of this procedure are presented in Figures 7-4, 7-5, and 7-6. Figure 7-4 presents the approach speed as a function of known aircraft parameters and altitude. Figures 7-5 and 7-6 give the ground roll in terms of known aircraft parameters and altitude. In Figure 7-6 V_{TD} is equal to V_R.

Another approach for determining the ground roll is based upon the assumption of average acceleration. This leads to a simple form for the ground roll that can be solved very quickly. In Equation 7-26 we can see that if the aerodynamic coefficients, thrust, and μ are fairly constant, the acceleration will vary with the speed squared. This is illustrated in Figure 7-7.

The average acceleration $(\bar{a})$ will then be $(a_1 + a_2)/2$ and the average speed squared is defined as being at this point. Writing this term in equation form we get

$$\bar{V}^2 = \frac{0 + V_{TD}^2}{2} = \frac{V_{TD}^2}{2}$$

Solving for $\bar{V}$ gives

$$\bar{V} = \sqrt{\frac{V_{TD}^2}{2}} = 0.707\ V_{TD} \qquad (7\text{-}33)$$

Equation 7-26 can then be written as an average constant acceleration

$$\bar{a} = g\left[\left(\frac{T}{W} - \mu\right) - q\frac{S}{W}\left(C_D - \mu C_L\right)\right] \qquad (7\text{-}34)$$

where q and T are evaluated at $\bar{V}$.

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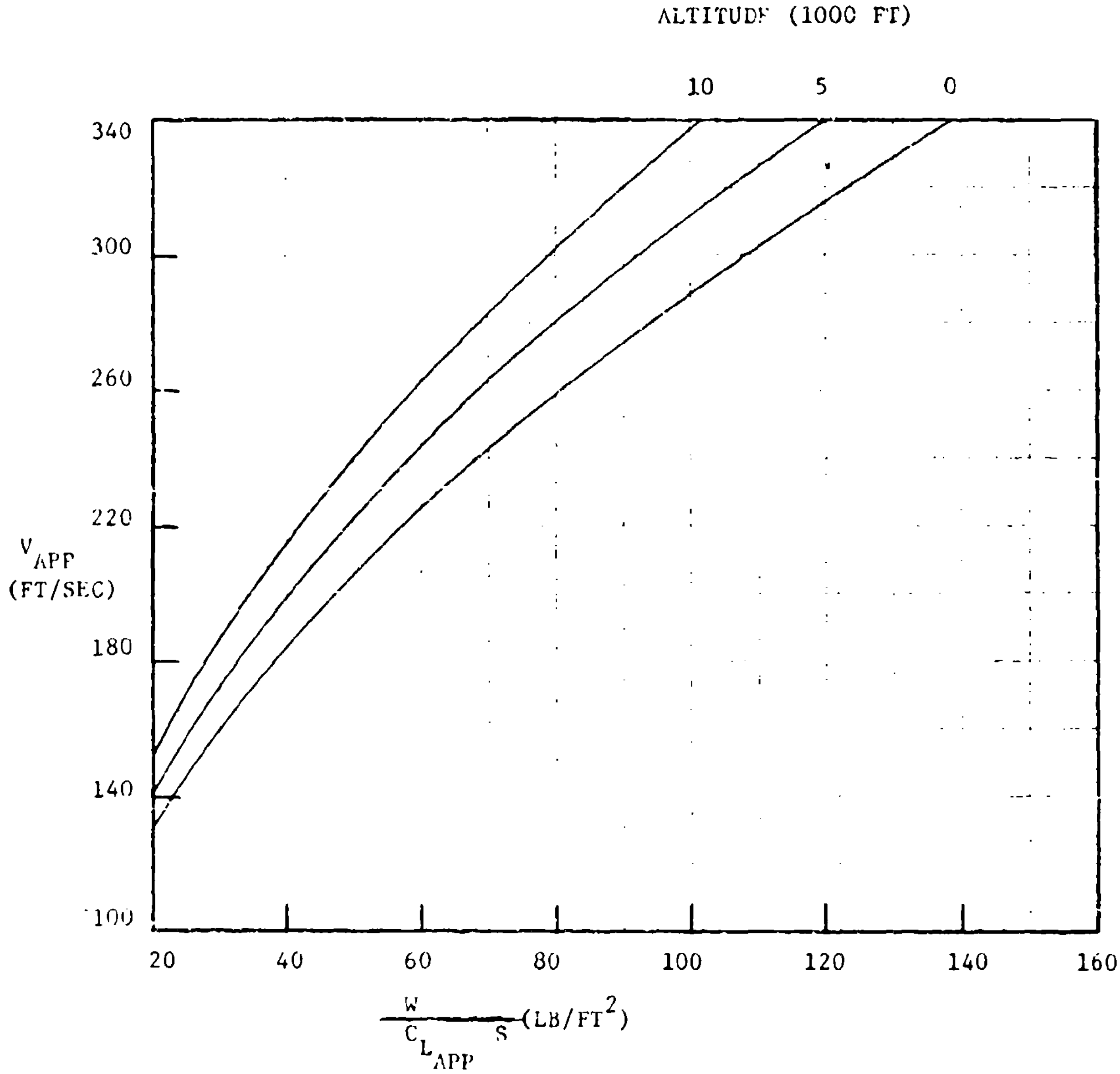

Figure 7-4 Approach Speed

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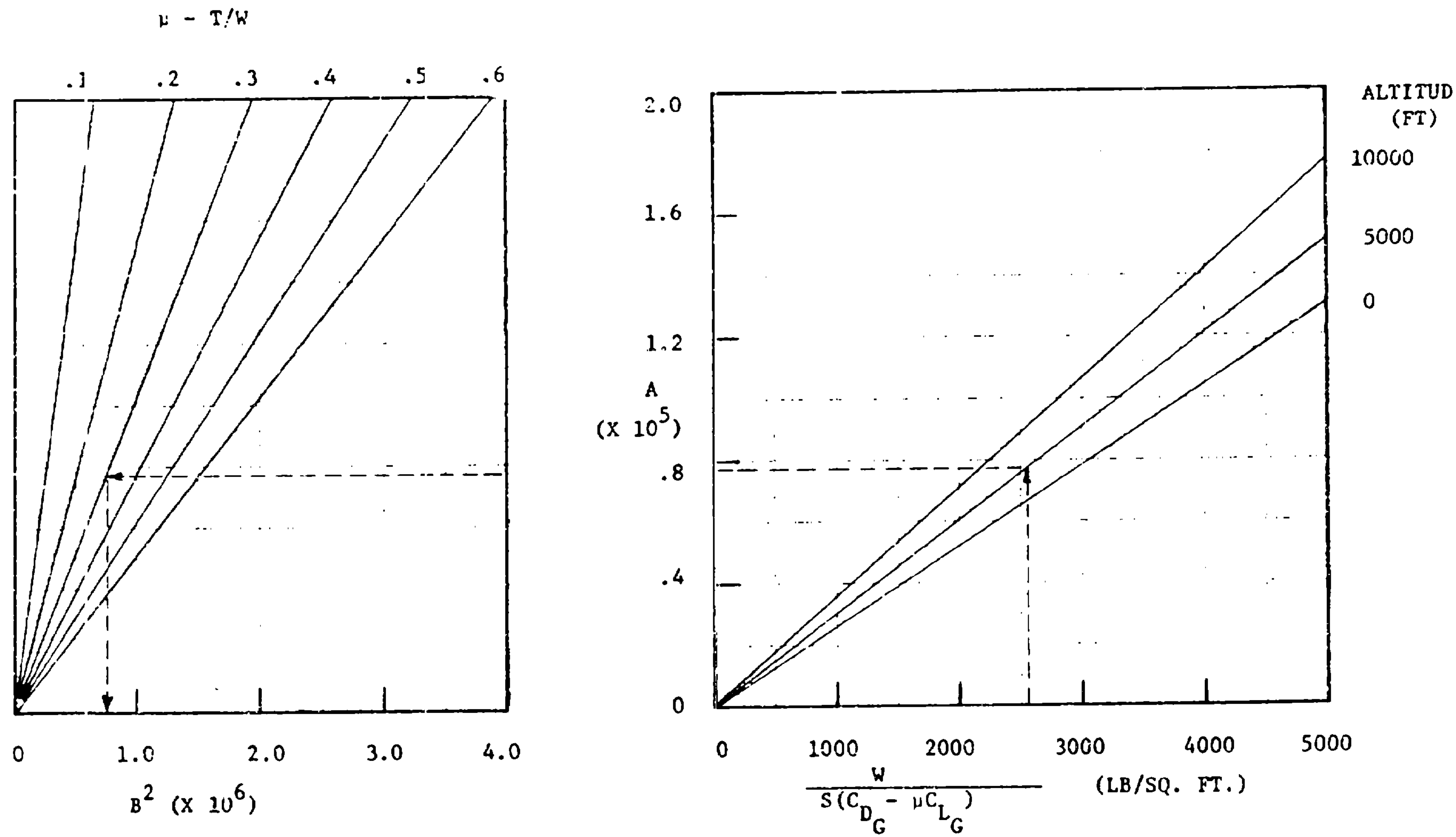

Figure 7-5 Landing Parameters for Ground Roll

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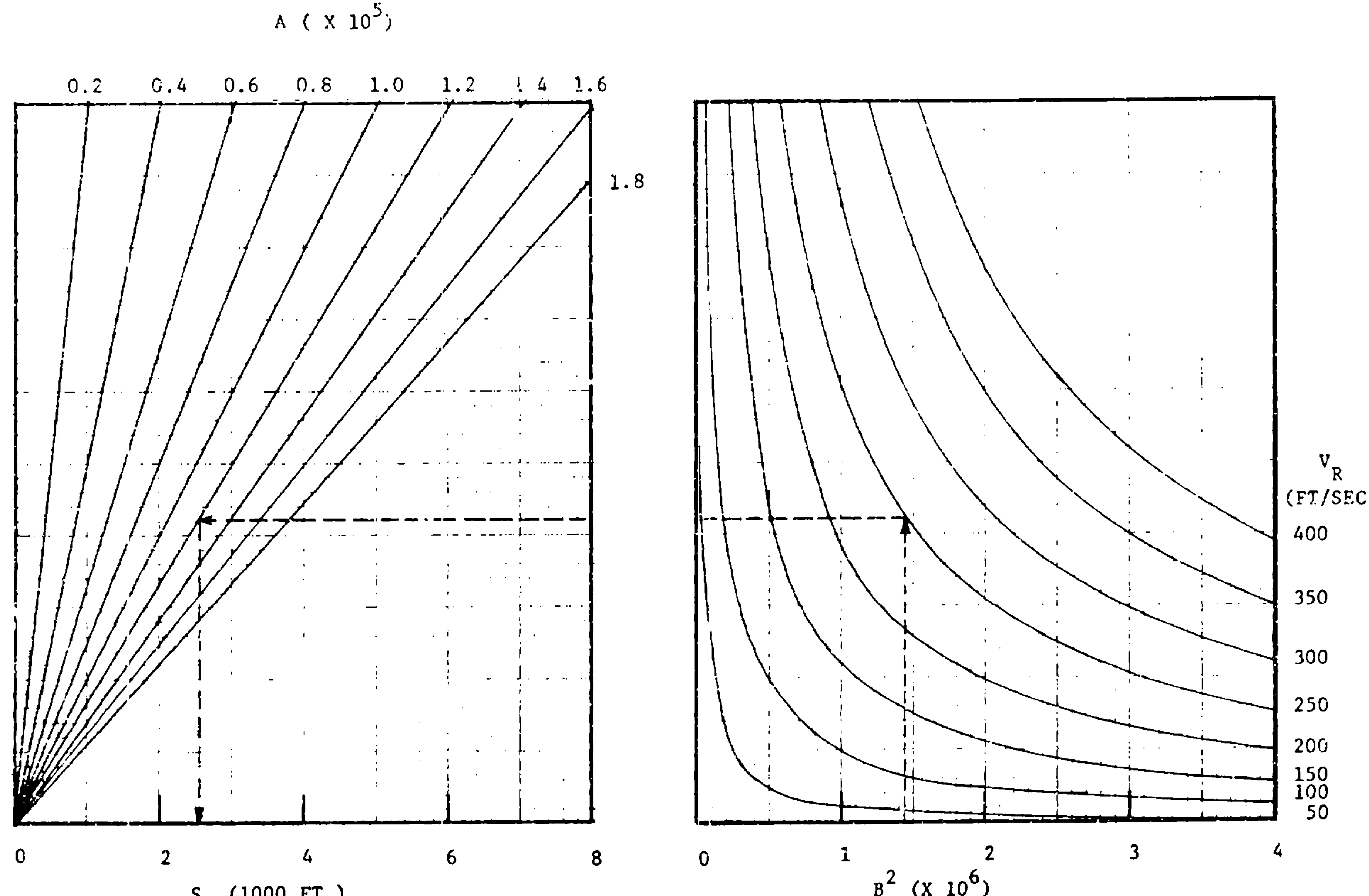

Figure 7-6 Generalized Landing Ground Roll Distance

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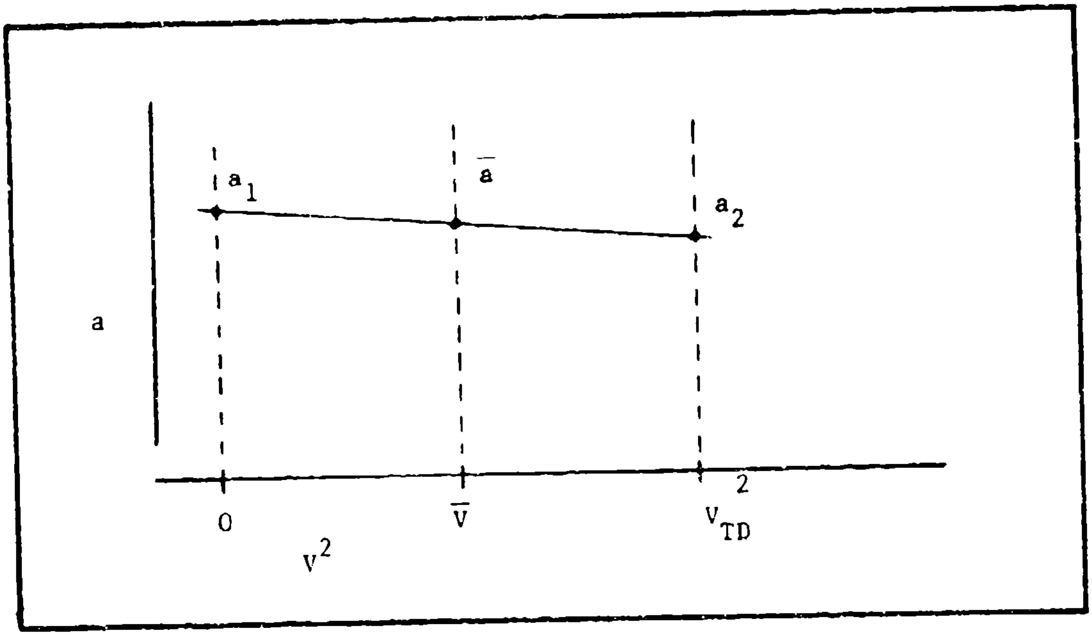

Figure 7-7 Acceleration Vs. Velocity Squared

With a constant acceleration Equation 7-23 can be integrated and written

$$S_G = -\frac{1}{2\,\bar{a}} V_{TD}^2 \qquad (7\text{-}35)$$

Written in terms of B^2 and A, as defined in Equations 7-30 and 7-31, respectively, Equation 7-35 would be

$$S_G = \frac{A \cdot V_{TD}^2}{2\,(B^2 + V_{TD}^2/2)} \qquad (7\text{-}36)$$

The results obtained in Equations 7-35 and 7-36 are in excellent agreement with those obtained in Equation 7-32.

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Situations where drag devices, such as drag chutes, flaps, spoilers, and thrust reversers are employed can also be handled using Figure 7-6. These problems are handled as follows. If drag chutes are used, they are released at some time after touchdown. Assuming average accelerations gives the following equation

$$S_G = \int_{V_{TD}}^{V_{CH}} \frac{V\,dV}{\bar{a}_1} + \int_{V_{CH}}^{o} \frac{V\,dV}{\bar{a}_2}$$

where $\bar{a}_1$, $\bar{a}_2$ are average accelerations over each interval and V_{CH} is the speed at which the chute is deployed. Chute deployment is assumed to be instantaneous. The speeds at which $\bar{a}_1$ and $\bar{a}_2$ are evaluated are

$$\bar{V}_1 = \sqrt{\frac{V_{TD}^2 + V_{CH}^2}{2}}$$

$$\bar{V}_2 = \sqrt{\frac{V_{CH}^2}{2}}$$

Integrating gives

$$S_G = \frac{1}{\bar{a}_1}\left(\frac{V_{CH}^2 - V_{TD}^2}{2}\right) - \frac{1}{\bar{a}_2}\,\frac{V_{CH}^2}{2}$$

Substituting A and B, appropriately defined over each interval, and rearranging the resulting equation gives

$$S_G = \frac{A_1}{2}\left(\frac{V_{TD}^2 + V_{CH}^2}{B_1^2 + \frac{V_{CH}^2 + V_{TD}^2}{2}} - \frac{2V_{CH}^2}{B_1^2 + \frac{V_{CH}^2 + V_{TD}^2}{2}}\right) + \frac{A_2}{2}\,\frac{V_{CH}^2}{B_2^2 + \frac{V_{CH}^2}{2}} \qquad (7\text{-}37)$$

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For this example B_1 and B_2 would be the same. A_1 and A_2 differ by the increase in the chute drag. Comparison shows that the first and third terms are like Equation 7-36. Consequently, Figure 7-6 can be used directly where

$$V_R = \sqrt{2}\,\bar{V}_1 = \sqrt{V_{CH}^2 + V_{TD}^2} \qquad \text{for the first part}$$

$$V_R = \sqrt{2}\,\bar{V}_2 = V_{CH} \qquad \text{for the third part}$$

The second term is different from Equation 7-36 since V_{TD} does not appear in the numerator. But for practical problems V_{TD} can be dropped from the second term since its contribution is negligible. Thus the second term is similar to Equation 7-36 except for the factor two. The contribution from this term to S_G can be obtained from Figure 7-6 with $V_R = V_{CH}$ but it must be multiplied by two. As an example, assume the following values

$$B_1^2 = B_2^2 = 2.1 \times 10^6$$

$$A_1 = 1.2 \times 10^5$$

$$A_2 = 0.36 \times 10^5$$

$$V_{TD} = 300 \text{ feet/second}$$

$$V_{CH} = 250 \text{ feet/second}$$

Let subscripts 1, 2, 3 refer to the first, second, and third parts in S_G. Now

$$V_R = \sqrt{1.525 \times 10^5} \qquad \text{for the first part}$$

$$V_R = 250 \qquad \text{for the second and third parts}$$

From Figure 7-6

$$S_{G_1} = 4200 \text{ feet}$$

$$S_{G_2} = 1700 \text{ feet}$$

$$S_{G_3} = 500 \text{ feet}$$

Substitution into Equation 7-37 gives

$$S_G = 4200 - 2(1700) + 500$$
$$= 1300 \text{ feet}$$

With no drag chute, the ground run would be based upon $V_R = 300$. The ground run for this case is 2300 feet.

For thrust reversers the situation is different from the previous case. As an example, the thrust reverser is engaged after touchdown at speed V_{CH_1} and disengaged at speed V_{CH_2} before zero speed is reached. This is to prevent foreign injection into the inlet. The relation for ground run is as follows:

$$S_G = \int_{V_{TD}}^{V_{CH_1}} \frac{VdV}{\bar{a}_1} + \int_{V_{CH_1}}^{V_{CH_2}} \frac{VdV}{\bar{a}_2} + \int_{V_{CH_2}}^{o} \frac{VdV}{\bar{a}_3}$$

Integrating, as before, gives

$$S_G = \frac{1}{\bar{a}_1}\left(\frac{V_{CH_1}^2 - V_{TD}^2}{2}\right) + \frac{1}{\bar{a}_2}\left(\frac{V_{CH_2}^2 - V_{CH_1}^2}{2}\right) - \frac{V_{CH_2}^2}{2\bar{a}_3}$$

where a_1, a_2, a_3 are also computed in the same way as in the previous example.

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Substituting A and B gives

$$S_G = \frac{A_1}{2}\left(\frac{V_{TD}^2 - V_{CH_1}^2}{B_1^2 + \frac{V_{CH_1}^2 + V_{TD}^2}{2}}\right) + \frac{A_2}{2}\left(\frac{V_{CH_1}^2 - V_{CH_2}^2}{B_2^2 + \frac{V_{CH_1}^2 + V_{CH_2}^2}{2}}\right)$$

$$+ \frac{A_3}{2}\left(\frac{V_{CH_2}^2}{B_3^2 + \frac{V_{CH_2}^2}{2}}\right)$$

Rearranging gives

$$S_G = \frac{A_1}{2}\left(\frac{V_{TD}^2 + V_{CH_1}^2}{B_1^2 + \frac{V_{CH_1}^2 + V_{TD}^2}{2}} - \frac{2V_{CH_1}^2}{B_1^2 + \frac{V_{CH_1}^2 + V_{TD}^2}{2}}\right)$$

$$+ \frac{A_2}{2}\left(\frac{V_{CH_1}^2 + V_{CH_2}^2}{B_2^2 + \frac{V_{CH_1}^2 + V_{CH_2}^2}{2}} - \frac{2V_{CH_2}^2}{B_2^2 + \frac{V_{CH_1}^2 + V_{CH_2}^2}{2}}\right)$$

$$+ \frac{A_3}{2}\left(\frac{V_{CH_2}^2}{B_3 + \frac{V_{CH_2}^2}{2}}\right)$$

For thrust reversers, A_1, A_2, and A_3 are the same. Figure 7-6 can be used to determine an approximate solution for S_G. Breaking S_G into five parts and applying the following table

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i	A	B	V_R	S_G
1	A_1	B_1	$\sqrt{V_{TD}^2 + V_{CH_1}^2}$	S_{G_1}
2	A_1	B_1	V_{CH_1}	S_{G_2}
3	A_2	B_2	$\sqrt{V_{CH_1}^2 + V_{CH_2}^2}$	S_{G_3}
4	A_2	B_2	V_{CH_2}	S_{G_4}
5	A_3	B_3	V_{CH_2}	S_{G_5}

and then combining the results gives S_G

$$S_G = S_{G_1} - 2S_{G_2} + S_{G_3} - 2S_{G_4} + S_{G_5}$$

As an example, assume the following values

$$A_1 = A_2 = A_3 = 1.2 \times 10^5$$

$$B_1^2 = B_3^2 = 2.1 \times 10^6$$

$$B_2^2 = 4.0 \times 10^6 \text{ (approximately T/W = - 0.3)}$$

$$V_{TD} = 300 \text{ feet/second}$$

$$V_{CH_1} = 200 \text{ feet/second}$$

$$V_{CH_2} = 100 \text{ feet/second}$$

From Figure 7-6

$$S_{G_1} = 4200 \text{ feet}$$

$$S_{G_2} = 1700 \text{ feet}$$

$$S_{G_3} = 1000 \text{ feet}$$

$$S_{G_4} = 50 \text{ feet}$$

$$S_{G_5} = 100 \text{ feet}$$

The ground roll is therefore

$$S_G = 4200 - 2(1700) + 1000 - 2(50) + 100 = 1800 \text{ feet}$$

Application and Sensitivity Analysis

In preliminary design work it is desirable to have a generalized chart available which will predict landing characteristics. This chart is usually tailored to a specific set of ground rules and some knowledge of the configuration. For example, MIL-C-5011A states that landing distances to clear a 50-foot obstacle are based on speeds that are at least 120% of the power-off stall speed, and that ground-roll distances shall be for hard-surface runways having a braking coefficient (μ) of 0.30. This defines the basic ground rules for the calculation of the landing requirement. Additional assumptions, such as selecting standard-day sea-level conditions and a nominal load factor (n) of 1.10 for the flare, will give the necessary information to calculate landing distance.

The approach and flare distance is determined from Equation 7-21. This equation can be written as a function of approach flight path angle (γ) by substitution of Equation 7-9 into Equation 7-21. The result would be

$$S_A = \frac{50}{\gamma} + \frac{\frac{W}{S}\gamma}{\rho\, C_{L_{APP}} g(n-1)} \qquad (7\text{-}38)$$

A generalized chart can then be constructed as a function of γ, W/S, and $C_{L_{APP}}$ for a given density (ρ) and load factor (n) condition. Figure 7-8 gives the results for a sea level standard day and a load factor of 1.10. Knowing the $C_{L_{APP}}$ and the desired approach glide slope, the distance to clear a 50-foot obstacle can then be determined. This figure also shows the sensitivity to wing loading (W/S), glide slope (γ), and approach C_L for the given conditions.

The ground-roll distance can be determined from Figures 7-5 and 7-6 or a generalized chart can be constructed using the average acceleration method and Equations 7-34 and 7-35. The results of the average method are shown in Figure 7-9 for a braking coefficient of 0.3, a C_{D_G} of 0.03, and a C_{L_G} of 0.0. This figure gives the engineer the visibility for variations in approach speed and ground thrust-to-weight ratio. As was the case for takeoff, the ground-roll distance is less sensitive to wing loading. The summation of the results derived from Figures 7-8 and 7-9 will give the landing field length requirements with no tolerance applied. The appropriate tolerance would then be applied, depending on the regulations under which the aircraft is being designed.

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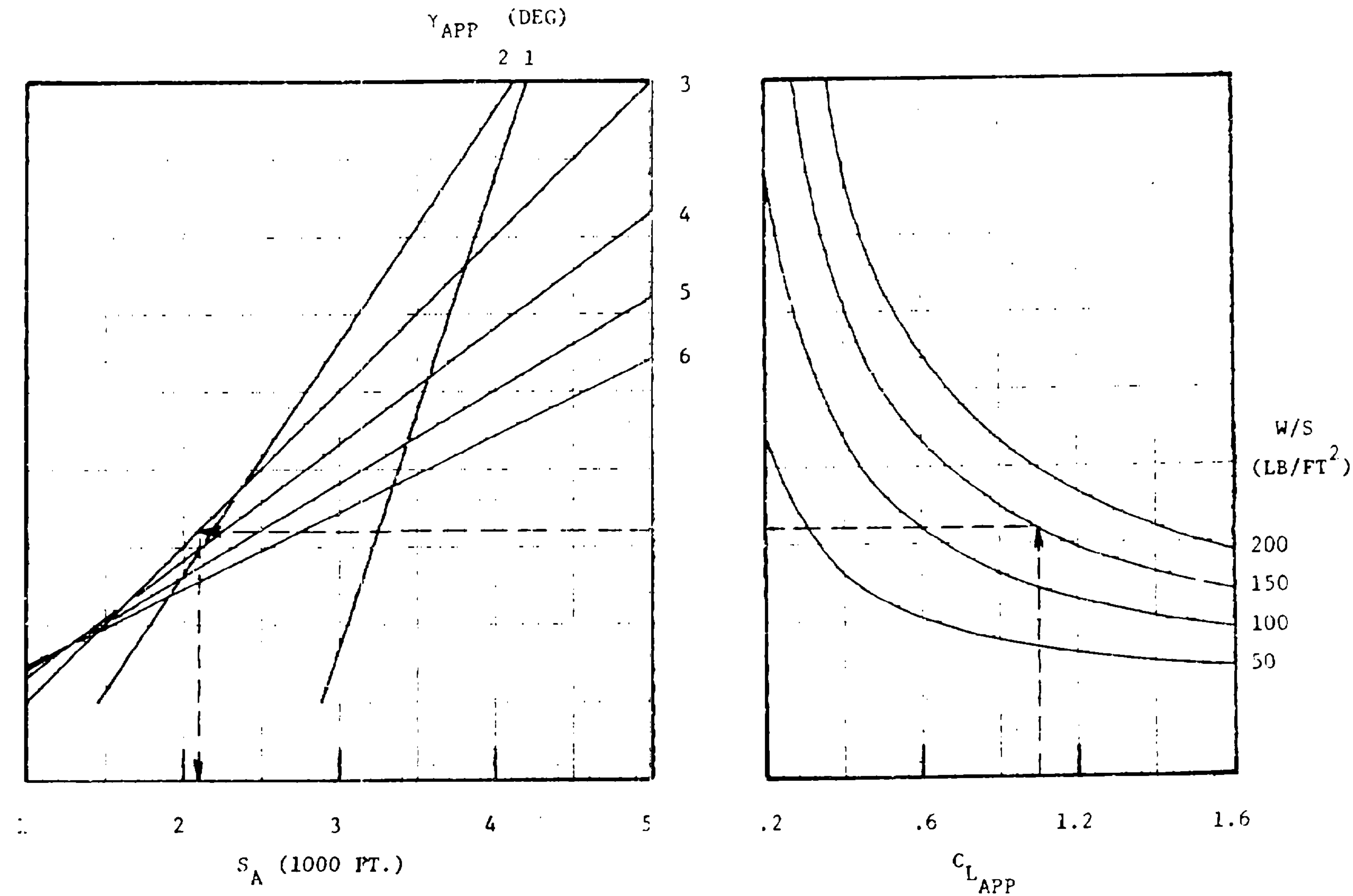

Figure 7-8 Landing Flare Distance

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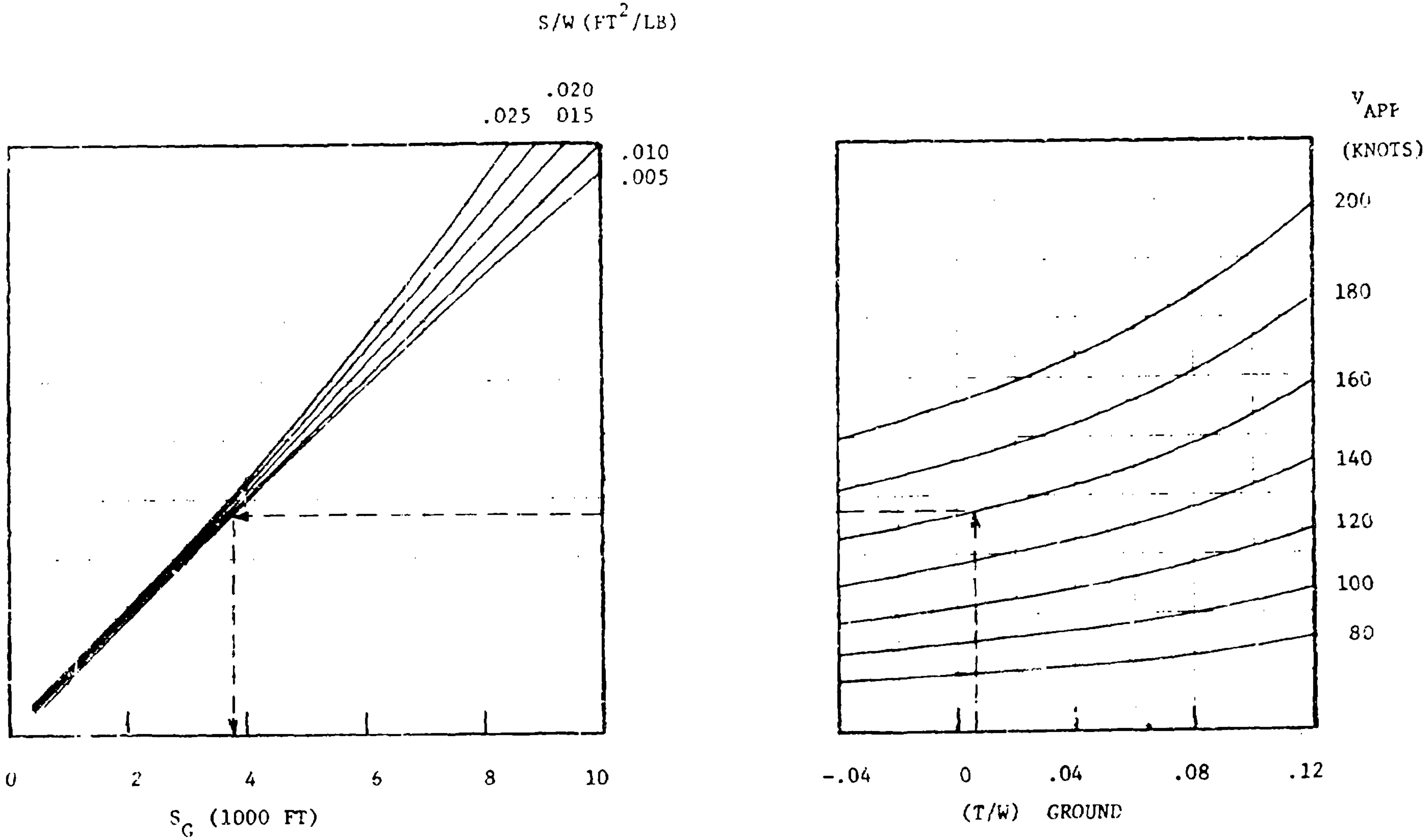

Figure 7-9 Landing Ground Roll Distance, μ = 0.3

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Summary of Land Performance Problem

Methods were derived to solve landing-field-length distances including approach to clear an obstacle, flare, and ground-roll stopping distance. These methods provide the engineer the means to quickly evaluate the landing requirement based on design parameters. Sensitivity to parameter variations is quickly visualized with the development of appropriate charts.

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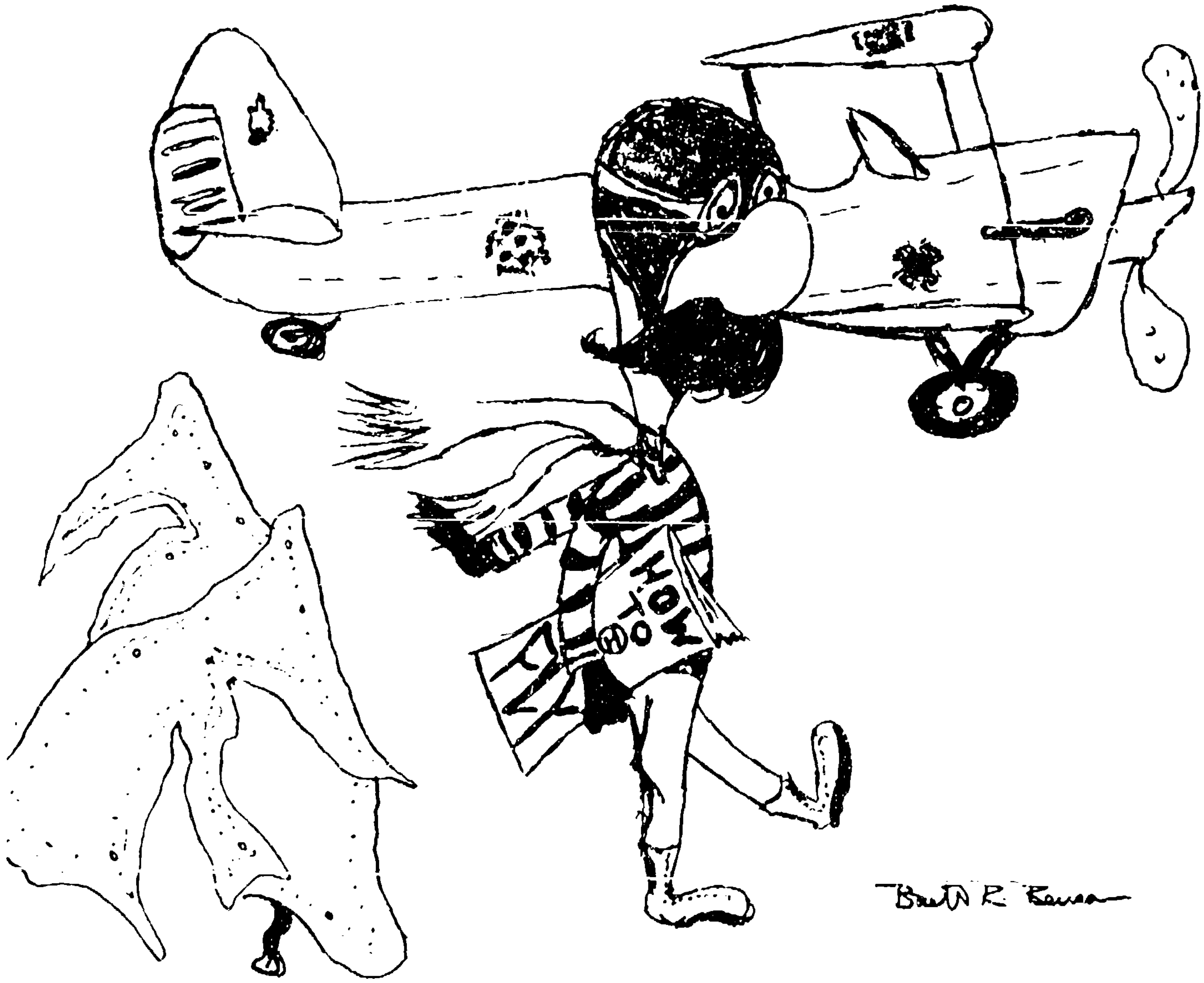
SECTION VIII
AN EXAMPLE



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SECTION VIII

AN EXAMPLE

As an example of the utility of the methods derived herein, an F4C combat air patrol (CAP) mission will be examined. The mission is defined in Figure 8-1. The segments which can be handled are segments B, C, H, and I. Segments A, E, and F are configuration dependent and generally are not suitable for evaluation by generalized handbook techniques. Segment G involves only a weight change. Endurance, segment D, can be handled if the altitude or speed are specified.

Three distinct external configurations must be considered. For Segment B and part of C, the F4 is equipped with missiles and two different types of external fuel tanks. During these portions of the mission, fuel is used from one of the tanks - when empty the tank is dropped. During the remainder of segment C the fuel is used from the other tank. For the return cruise, segment H, the configuration is clean - there are no external fuel tanks or missiles.

The first step is the evaluation of the trajectory along segment B which is the climb portion. The starting altitude and weight are known. The end weight and altitude are unknown and will be determined in conjunction with the initial cruise solution. The best climb speed is determined from Equation 3-18.

$$V = \left\{ \frac{W}{3\rho S C_{D_0}} \left[\frac{T}{W} + \sqrt{\left(\frac{T}{W}\right)^2 + \frac{3}{(L/D)^2_{MAX}}} \right] \right\}^{\frac{1}{2}} \qquad (3\text{-}18)$$

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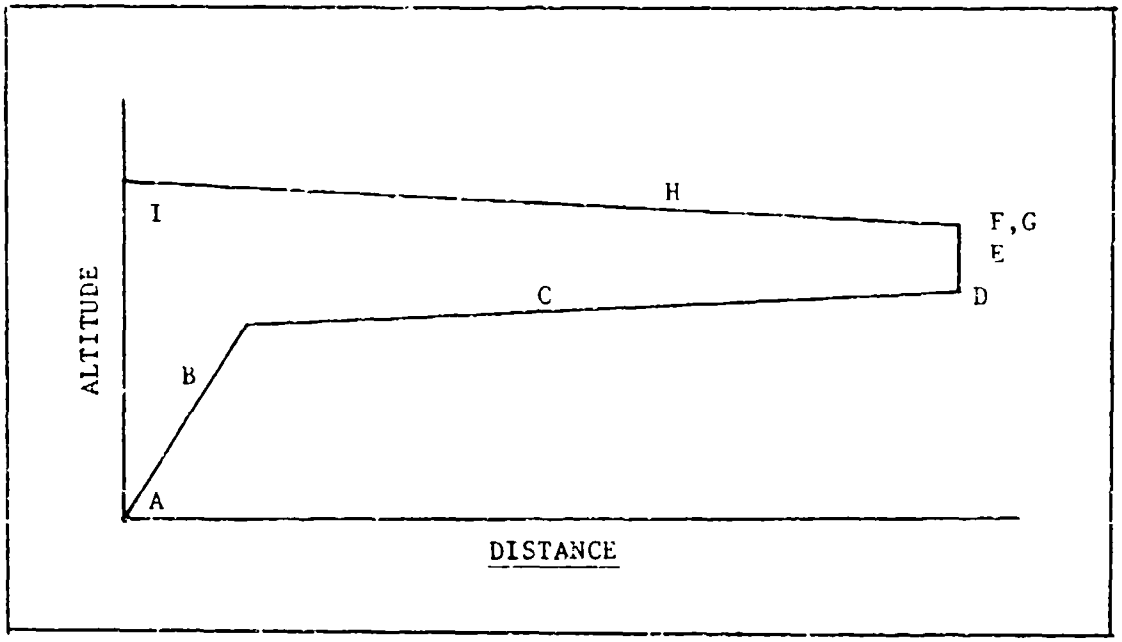

A. Warm-up, take-off and accelerate: 5 min. with normal power plus 1 min. with afterburner.

B. Climb on course to optimum cruise altitude with military power.

C. Cruise out at optimum cruise altitude and speed for long range.

D. Patrol at speed and altitude for maximum endurance.

E. Climb to acceleration altitude with maximum power.

F. Accelerate with maximum power at acceleration altitude from cruise speed to M = 1.5 and remain at this speed and altitude for 2 min. at maximum power.

G. Expend missiles.

H. Cruise in at optimum cruise altitude and speed for long range.

I. Reserves: 20 min. at speed for maximum endurance at sea level (2 engines operating) plus 5% of the initial fuel load.

Figure 8-1

F-4C
COMBAT AIR PATROL MISSION DEFINITION

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Unfortunately, both sides of Equation 3-18 are functions of Mach number. Therefore, an iterative approach is required. The substitution of C_{D_0}, $(L/D)_{MAX}$, and sea level military thrust as functions of Mach number into Equation 3-18 gives approximately M = 0.6 at sea level. This yields $(L/D)_{MAX}$ equal to 8.87 and T_0/W equal to 0.357. This combination of $(L/D)_{MAX}$ and T_0/W yields a maximum altitude, H_{MAX}, of approximately 35,000 feet. At this altitude, Equation 3-18 gives M equal to 0.83. To compensate for the variation in Mach number with altitude, we select the average of the Mach numbers at sea level and H_{MAX}. As a consequence, the sea level thrust-to-weight ratio becomes 0.365. This represents a 2.2% change in T_0/W. SFC, C_{D_0}, and $(L/D)_{MAX}$ are 1.12 pounds/hour/pound, 0.0227, and 8.87, respectively. The solutions for time, (t_C), fuel (W_C), and range (X) in climb are presented in Figure 8-2. The data were obtained from Figures 3-4 through 3-8. The end-of-climb weight will be determined when the initial cruise trajectory is obtained.

For the cruise trajectory, the first step is the solution for the best cruise Mach number. This can be accomplished by substituting $(L/D)_{MAX}$ and minimum SFC into Equation 4-38. Since SFC is nearly constant for the F4C during cruise, the approach will be to maximize $M(L/D)_{MAX}$. Thus best cruise Mach number is the solution of

$$\text{MAX } M \cdot (L/D)_{MAX} \tag{8-1}$$

Best cruise Mach number for the three configurations is approximately 0.85. The best fit of a parabolic polar form to the experimental

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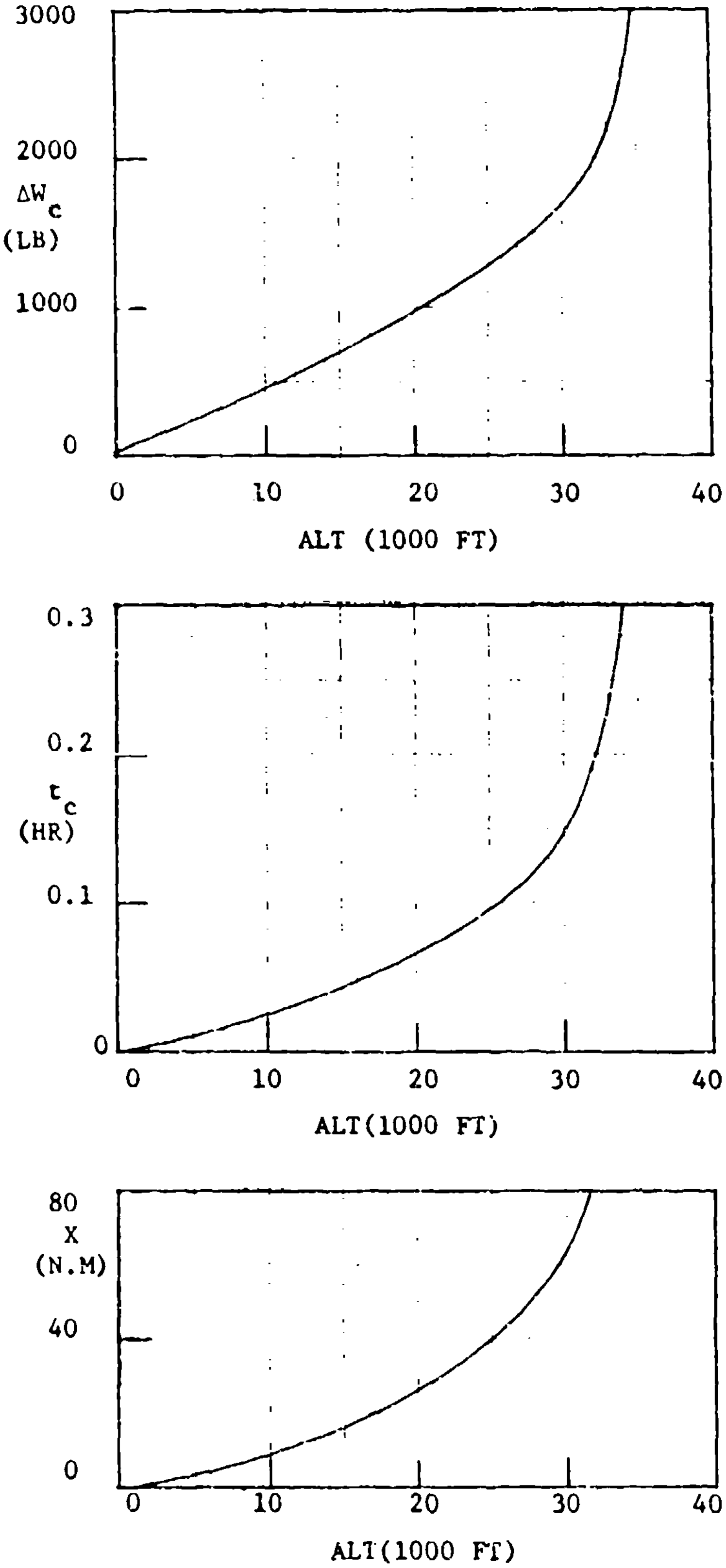

Figure 8-2 F4C Climb Performance

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data gives the following theoretical aerodynamic coefficients:

Configuration	C_{D_0}	K	C_L'	$(L/D)_{MAX}$
A	.0250	.231	.056	7.92
B	.0232	.231	.056	8.28
C	.0204	.231	.056	8.97

The reason for no change in K and C_L' is that the experimental data for configurations A and B were obtained by the addition of constant drag increments to the data for the clean configuration.

The cruise segments require iteration since either or both final and initial weights are unknown. For the initial cruise segment, M = 0.85 and the aerodynamic data are substituted into Equation 4-34. The resulting solution for cruise weight as a function of altitude is presented in Figure 8-4. The intersection of the initial climb and cruise trajectories defines the initial cruise weight, 49,750 pounds. The final weight for the first outbound leg, 48,967 pounds, occurs when the first external fuel tank is empty. The SFC is 1.08 pounds/hour/pound. The initial range factor from Equation 4-39 is 3636 nautical miles. The cruise range from Equation 4-40 is 56.1 nautical miles. The calculations are based upon A = 1.235.

At this point we can make a comparison between the predictions and the generalized F4C data for the initial climb and cruise segments.

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CONFIGURATION	DESCRIPTION
A	First outbound cruise
B	Second outbound cruise
C	Return cruise

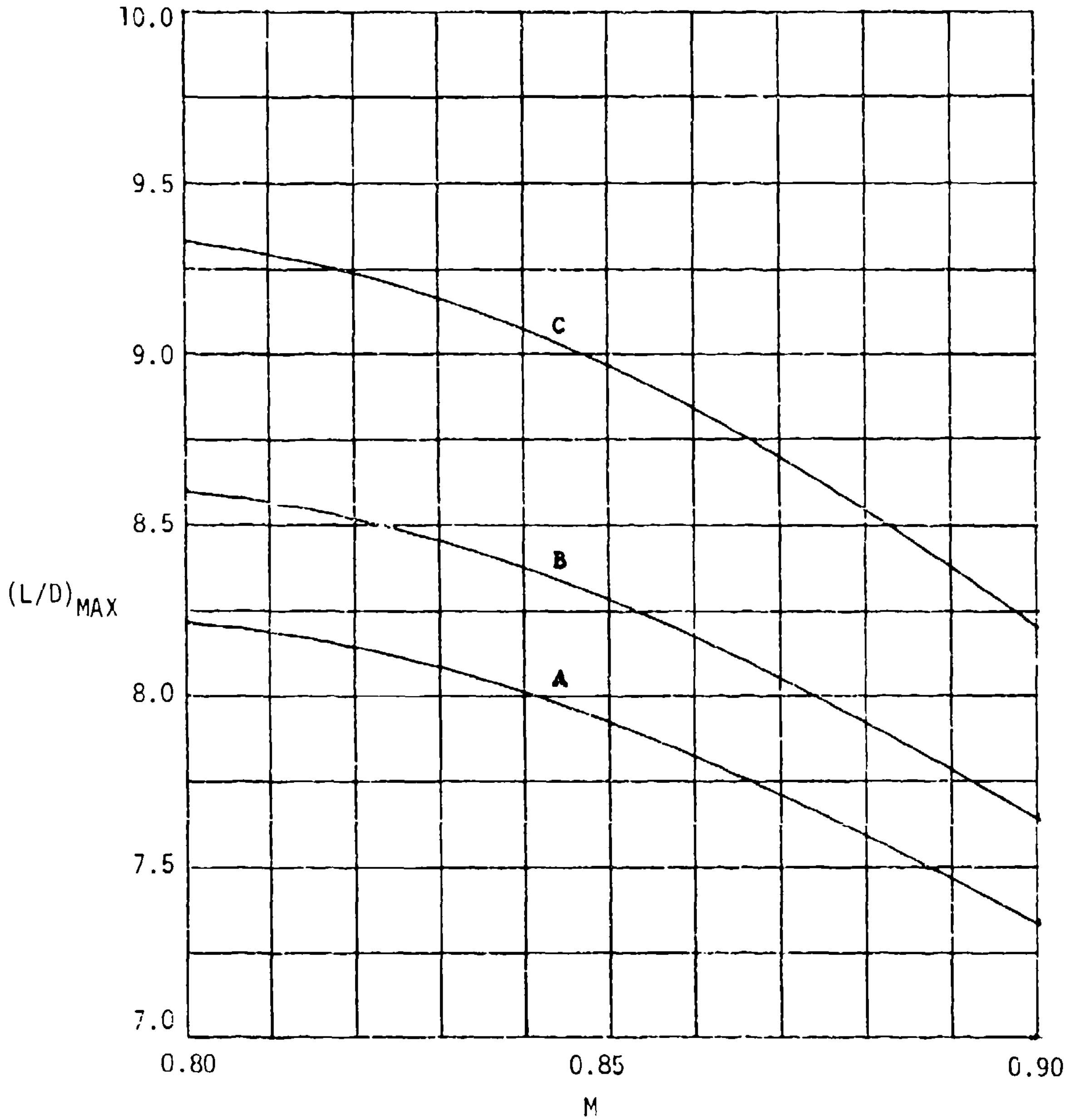

Figure 8-3 Maximum L/D

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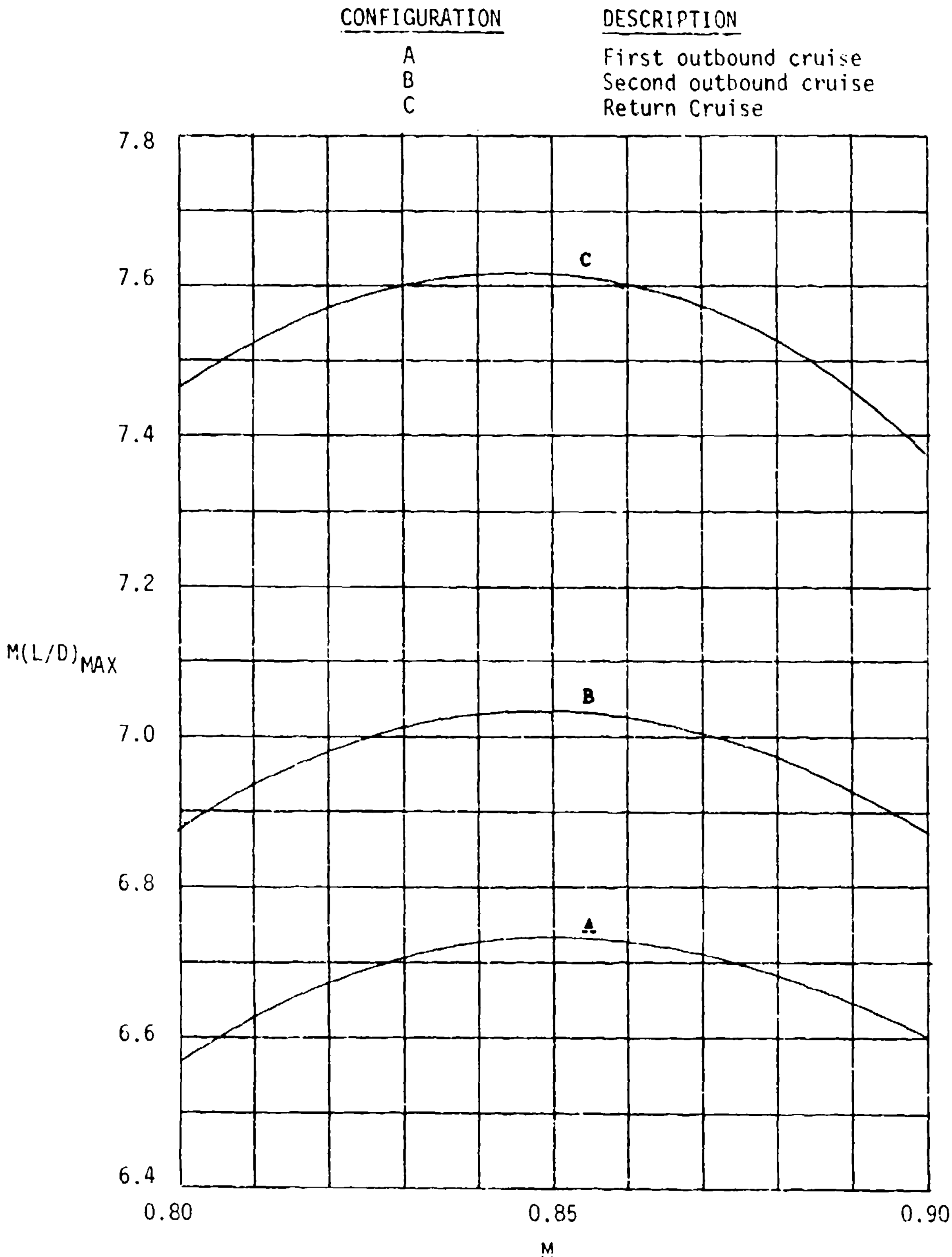

Figure 8-4 $M(L/D)_{MAX}$

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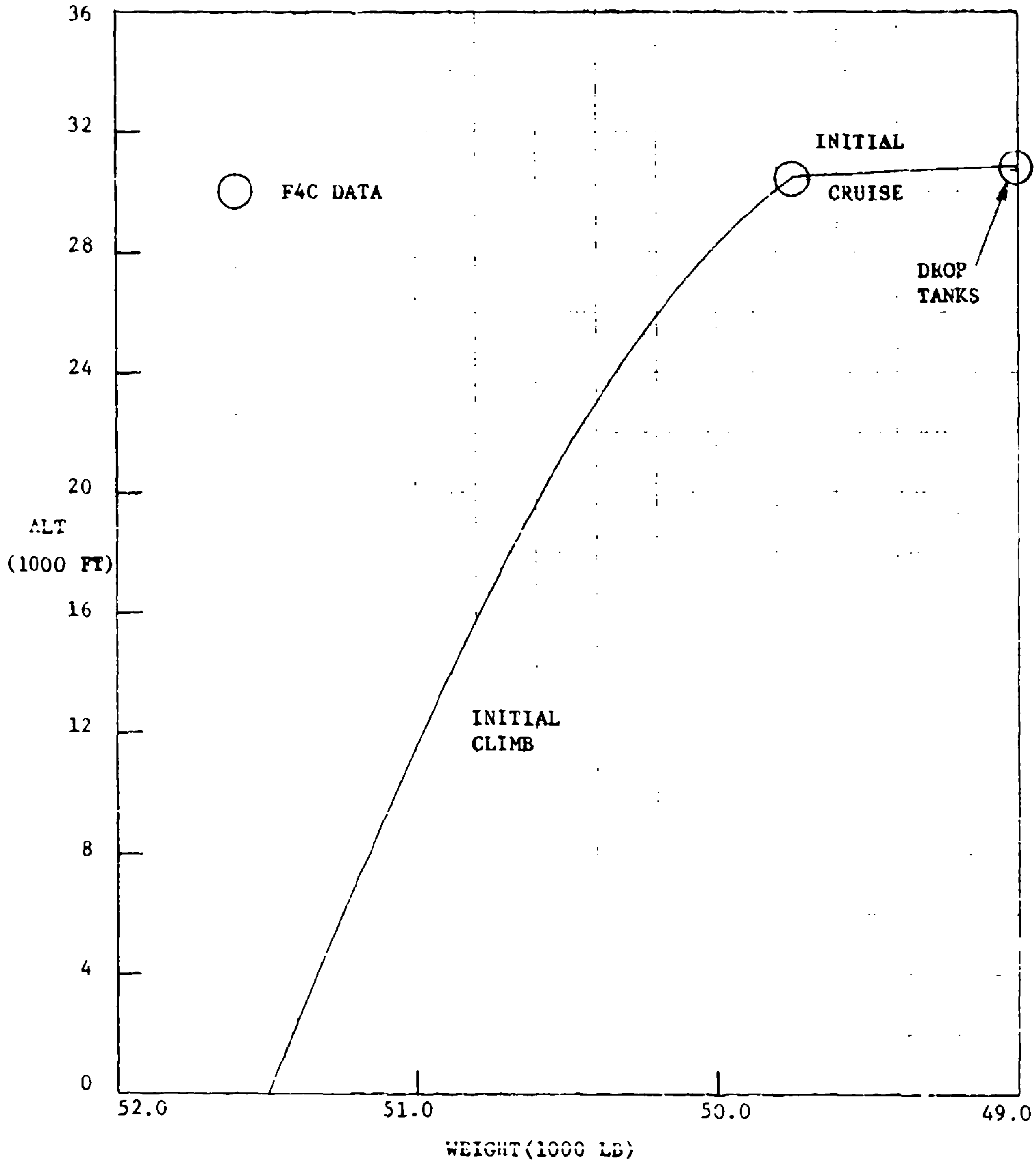

Figure 8-5 Initial Climb and Cruise Trajectories

	Estimated	F4C Data
Initial Climb:		
Initial/Final Weights	51,500/49,750	51,500/49,757
Final altitude	30,400	30,400
Time (hour)	0.150	0.123
Distance (nautical miles)	67	58
First Outbound Cruise Leg:		
Initial/Final Weights	49,750/48,987	49,757/48,987
Initial/Final Altitudes	30,500/30,900	30,400/30,700
Cruise Mach number	0.850	0.859
Distance	56	55
Time	0.112	0.110

F4C cruise data are based upon the larger of the speeds for 99% of maximum nautical mile per pound. A factor of 5% is also included in the SFC.

The initial weight for the second outbound leg is 48,307 pounds. The final weight is obtained from the F4C data and is 46,569 pounds. Best cruise Mach number is 0.85. Maximum L/D is 8.28 and the SFC is 1.08. The cruise performance is obtained in the same way as before. The comparison of the predictions and the F4C data follows.

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	Estimated	F4C Data
Second Outbound Leg:		
Initial/Final Weights	48,307/46,569	48,357/46,569
Initial/Final Altitudes	30,450/31,200	30,700/32,000
Cruise Mach number	0.85	0.86
Distance	139	136
Time	0.280	0.271

For the return cruise, the initial and final weights are obtained from the F4C data. Best cruise Mach number from Figure 8-4 is 0.85. Maximum L/D is 8.97 and SFC is 1.08. A is one. The comparison between the estimated predictions and the F4C data are as follows.

	Estimated	F4C Data
Final Cruise Leg:		
Initial/Final Weights	34,320/32,193	34,320/32,193
Initial/Final Altitudes	38,000/39,300	38,000/39,750
Cruise Mach number	0.85	0.88
Distance	259	250
Time	0.531	0.498

In addition to the climb and cruise portions, the available data can also provide an estimate of the endurance at a given altitude. Therefore, we will consider segment I of the CAP mission. Five percent of the initial fuel is 1,076 pounds. The empty weight plus this reserve is 30,597. From Equation 4-49 the Mach number for best endurance is 0.36. The thrust required is derived from

$$T = \frac{W}{(L/D)} = 2,985 \text{ pounds}$$

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From the engine performance data, TSFC is 4620 pounds/hour. Thus, for 20 minutes' operation, the fuel allowance is 1540 pounds. The F4C gives 1595 pounds.

The previous example demonstrates that the methods developed here can provide a near accurate estimate of specific segments of a mission profile.

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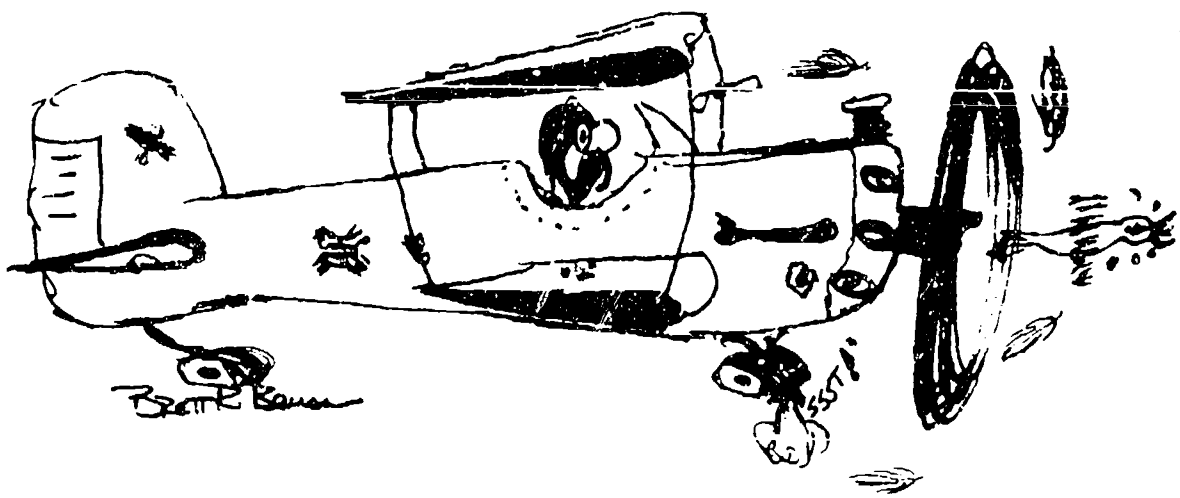
APPENDIX
1962 STANDARD ATMOSPHERE



Reference Sea Level Values:

Density (RHO) = 0.0023769 slugs/cubic foot

Pressure = 2116.2 pounds/square foot

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1962 U.S. STANDARD ATMOSPHERE

PRESSURE ALT (FT)	TEMPERATURE (DEG.CENTIGRADE)	SIGMA (RHO/S.L.RHO)	DELTA (PRESS/S.L.PRESS)	SPEED OF SOUND (FT/SEC)
0.	.14900E+02	.10000E+01	.10000E+01	.11165E+04
100.	.14702E+02	.99713E+00	.99639E+00	.11161E+04
200.	.14504E+02	.99426E+00	.99279E+00	.11157E+04
300.	.14306E+02	.99123E+00	.98921E+00	.11153E+04
400.	.14108E+02	.98835E+00	.98563E+00	.11149E+04
500.	.13909E+02	.98545E+00	.98206E+00	.11145E+04
600.	.13801E+02	.98255E+00	.97851E+00	.11141E+04
700.	.13613E+02	.97969E+00	.97496E+00	.11138E+04
800.	.13403E+02	.97680E+00	.97143E+00	.11134E+04
900.	.13207E+02	.97393E+00	.96790E+00	.11130E+04
1000.	.13009E+02	.97107E+00	.96439E+00	.11125E+04
1100.	.12811E+02	.96821E+00	.96083E+00	.11122E+04
1200.	.12613E+02	.96535E+00	.95739E+00	.11118E+04
1300.	.12415E+02	.96251E+00	.95391E+00	.11114E+04
1400.	.12217E+02	.95967E+00	.95044E+00	.11111E+04
1500.	.12019E+02	.95684E+00	.94697E+00	.11107E+04
1600.	.11820E+02	.95402E+00	.94352E+00	.11103E+04
1700.	.11622E+02	.95120E+00	.94008E+00	.11099E+04
1800.	.11424E+02	.94839E+00	.93665E+00	.11095E+04
1900.	.11226E+02	.94558E+00	.93323E+00	.11091E+04
2000.	.11028E+02	.94279E+00	.92982E+00	.11087E+04
2100.	.10830E+02	.93999E+00	.92641E+00	.11084E+04
2200.	.10632E+02	.93720E+00	.92302E+00	.11080E+04
2300.	.10434E+02	.93442E+00	.91964E+00	.11075E+04
2400.	.10236E+02	.93164E+00	.91627E+00	.11072E+04
2500.	.10038E+02	.92887E+00	.91291E+00	.11068E+04
2600.	.98395E+01	.92611E+00	.90956E+00	.11064E+04
2700.	.96417E+01	.92335E+00	.90622E+00	.11060E+04
2800.	.94440E+01	.92061E+00	.90289E+00	.11057E+04
2900.	.92453E+01	.91787E+00	.89957E+00	.11053E+04
3000.	.90473E+01	.91513E+00	.89625E+00	.11049E+04
3100.	.88492E+01	.91240E+00	.89295E+00	.11045E+04
3200.	.86511E+01	.90967E+00	.88965E+00	.11041E+04
3300.	.84531E+01	.90695E+00	.88638E+00	.11037E+04
3400.	.82550E+01	.90425E+00	.88311E+00	.11033E+04
3500.	.80570E+01	.90154E+00	.87985E+00	.11029E+04
3600.	.78589E+01	.89884E+00	.87560E+00	.11025E+04
3700.	.76609E+01	.89615E+00	.87335E+00	.11022E+04
3800.	.74628E+01	.89346E+00	.87012E+00	.11018E+04
3900.	.72648E+01	.89078E+00	.86690E+00	.11014E+04
4000.	.70667E+01	.88811E+00	.86369E+00	.11010E+04
4100.	.68687E+01	.88544E+00	.86048E+00	.11006E+04
4200.	.66706E+01	.88278E+00	.85729E+00	.11002E+04
4300.	.64725E+01	.88012E+00	.85411E+00	.10998E+04
4400.	.62745E+01	.87747E+00	.85093E+00	.10994E+04
4500.	.60764E+01	.87483E+00	.84777E+00	.10990E+04
4600.	.58784E+01	.87219E+00	.84461E+00	.10987E+04
4700.	.56803E+01	.86955E+00	.84147E+00	.10983E+04
4800.	.54824E+01	.86693E+00	.83833E+00	.10979E+04
4900.	.52844E+01	.86431E+00	.83520E+00	.10975E+04
5000.	.50864E+01	.86170E+00	.83208E+00	.10971E+04

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1962 U.S. STANDARD ATMOSPHERE

PRESSURE ALT (FT)	TEMPERATURE (DEG.CENTIGRADE)	SIGMA (RHO/S.L.RHO)	DELTA (PRESS/S.L.PRESS)	SPEED OF SOUND (FT/SEC)
5100.	.48884E+01	.85909E+00	.82898E+00	.10967E+04
5200.	.45903E+01	.85649E+00	.82588E+00	.10963E+04
5300.	.44923E+01	.85390E+00	.82279E+00	.10959E+04
5400.	.42943E+01	.85131E+00	.81971E+00	.10955E+04
5500.	.40963E+01	.84873E+00	.81664E+00	.10951E+04
5600.	.38983E+01	.84615E+00	.81358E+00	.10948E+04
5700.	.37002E+01	.84358E+00	.81053E+00	.10944E+04
5800.	.35022E+01	.84102E+00	.80749E+00	.10940E+04
5900.	.33042E+01	.83846E+00	.80445E+00	.10936E+04
6000.	.31062E+01	.83590E+00	.80143E+00	.10932E+04
6100.	.29082E+01	.83335E+00	.79842E+00	.10928E+04
6200.	.27102E+01	.83082E+00	.79541E+00	.10924E+04
6300.	.25122E+01	.82828E+00	.79241E+00	.10920E+04
6400.	.23142E+01	.82575E+00	.78943E+00	.10916E+04
6500.	.21162E+01	.82323E+00	.78645E+00	.10912E+04
6600.	.19182E+01	.82071E+00	.78348E+00	.10908E+04
6700.	.17202E+01	.81820E+00	.78052E+00	.10904E+04
6800.	.15222E+01	.81570E+00	.77757E+00	.10900E+04
6900.	.13242E+01	.81320E+00	.77463E+00	.10897E+04
7000.	.11263E+01	.81070E+00	.77170E+00	.10893E+04
7100.	.92827E+00	.80822E+00	.76878E+00	.10889E+04
7200.	.73028E+00	.80573E+00	.76586E+00	.10885E+04
7300.	.53230E+00	.80326E+00	.76296E+00	.10881E+04
7400.	.33432E+00	.80079E+00	.76006E+00	.10877E+04
7500.	.13634E+00	.79832E+00	.75717E+00	.10873E+04
7600.	-.61639E-01	.79587E+00	.75429E+00	.10869E+04
7700.	-.25961E+00	.79341E+00	.75142E+00	.10865E+04
7800.	-.45759E+00	.79097E+00	.74856E+00	.10861E+04
7900.	-.65556E+00	.78853E+00	.74571E+00	.10857E+04
8000.	-.85352E+00	.78609E+00	.74287E+00	.10853E+04
8100.	-.10515E+01	.78366E+00	.74003E+00	.10849E+04
8200.	-.12495E+01	.78124E+00	.73721E+00	.10845E+04
8300.	-.14474E+01	.77882E+00	.73439E+00	.10841E+04
8400.	-.16454E+01	.77641E+00	.73159E+00	.10837E+04
8500.	-.18433E+01	.77400E+00	.72879E+00	.10833E+04
8600.	-.20413E+01	.77160E+00	.72600E+00	.10830E+04
8700.	-.22393E+01	.76921E+00	.72321E+00	.10826E+04
8800.	-.24372E+01	.76682E+00	.72044E+00	.10822E+04
8900.	-.26352E+01	.76443E+00	.71768E+00	.10818E+04
9000.	-.28331E+01	.76206E+00	.71492E+00	.10814E+04
9100.	-.30311E+01	.75969E+00	.71217E+00	.10810E+04
9200.	-.32290E+01	.75732E+00	.70943E+00	.10806E+04
9300.	-.34269E+01	.75495E+00	.70670E+00	.10802E+04
9400.	-.36249E+01	.75260E+00	.70398E+00	.10798E+04
9500.	-.38228E+01	.75025E+00	.70127E+00	.10794E+04
9600.	-.40208E+01	.74791E+00	.69857E+00	.10790E+04
9700.	-.42187E+01	.74557E+00	.69587E+00	.10786E+04
9800.	-.44166E+01	.74324E+00	.69318E+00	.10782E+04
9900.	-.46146E+01	.74091E+00	.69050E+00	.10778E+04
10000.	-.48125E+01	.73859E+00	.68783E+00	.10774E+04

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1962 U.S. STANDARD ATMOSPHERE

PRESSURE ALT (FT)	TEMPERATURE (DEG.CENTIGRADE)	SIGMA (RHO/S.L.RHO)	DELTA (PRESS/S.L.PRESS)	SPEED OF SOUND (FT/SEC)
10100.	-.50104E+01	.73627E+00	.68517E+00	.10770E+04
10200.	-.52084E+01	.73395E+00	.68252E+00	.10766E+04
10300.	-.54063E+01	.73165E+00	.67987E+00	.10762E+04
10400.	-.56042E+01	.72935E+00	.67723E+00	.10758E+04
10500.	-.58021E+01	.72707E+00	.67460E+00	.10754E+04
10600.	-.60001E+01	.72478E+00	.67198E+00	.10750E+04
10700.	-.61980E+01	.72250E+00	.66937E+00	.10746E+04
10800.	-.63959E+01	.72022E+00	.66677E+00	.10742E+04
10900.	-.65938E+01	.71795E+00	.66417E+00	.10738E+04
11000.	-.67917E+01	.71568E+00	.66158E+00	.10734E+04
11100.	-.69895E+01	.71342E+00	.65900E+00	.10730E+04
11200.	-.71875E+01	.71117E+00	.65643E+00	.10726E+04
11300.	-.73854E+01	.70892E+00	.65387E+00	.10722E+04
11400.	-.75833E+01	.70668E+00	.65131E+00	.10718E+04
11500.	-.77812E+01	.70444E+00	.64877E+00	.10714E+04
11600.	-.79791E+01	.70220E+00	.64623E+00	.10710E+04
11700.	-.81770E+01	.69998E+00	.64370E+00	.10706E+04
11800.	-.83749E+01	.69775E+00	.64118E+00	.10702E+04
11900.	-.85728E+01	.69554E+00	.63866E+00	.10698E+04
12000.	-.87707E+01	.69333E+00	.63615E+00	.10694E+04
12100.	-.89686E+01	.69112E+00	.63366E+00	.10690E+04
12200.	-.91665E+01	.68892E+00	.63117E+00	.10686E+04
12300.	-.93644E+01	.68672E+00	.62868E+00	.10682E+04
12400.	-.95623E+01	.68453E+00	.62621E+00	.10678E+04
12500.	-.97602E+01	.68235E+00	.62374E+00	.10674E+04
12600.	-.99581E+01	.68017E+00	.62128E+00	.10670E+04
12700.	-.10156E+02	.67800E+00	.61883E+00	.10666E+04
12800.	-.10354E+02	.67583E+00	.61639E+00	.10662E+04
12900.	-.10552E+02	.67367E+00	.61395E+00	.10658E+04
13000.	-.10750E+02	.67151E+00	.61153E+00	.10654E+04
13100.	-.10947E+02	.66935E+00	.60911E+00	.10650E+04
13200.	-.11147E+02	.66721E+00	.60669E+00	.10646E+04
13300.	-.11343E+02	.66507E+00	.60429E+00	.10642E+04
13400.	-.11541E+02	.66293E+00	.60189E+00	.10638E+04
13500.	-.11739E+02	.66080E+00	.59950E+00	.10634E+04
13600.	-.11937E+02	.65867E+00	.59712E+00	.10630E+04
13700.	-.12135E+02	.65655E+00	.59475E+00	.10626E+04
13800.	-.12332E+02	.65444E+00	.59233E+00	.10622E+04
13900.	-.12530E+02	.65233E+00	.59003E+00	.10618E+04
14000.	-.12728E+02	.65022E+00	.58768E+00	.10614E+04
14100.	-.12926E+02	.64812E+00	.58533E+00	.10610E+04
14200.	-.13124E+02	.64603E+00	.58300E+00	.10606E+04
14300.	-.13322E+02	.64394E+00	.58067E+00	.10602E+04
14400.	-.13520E+02	.64185E+00	.57835E+00	.10598E+04
14500.	-.13717E+02	.63978E+00	.57604E+00	.10594E+04
14600.	-.13915E+02	.63770E+00	.57373E+00	.10590E+04
14700.	-.14113E+02	.63563E+00	.57144E+00	.10585E+04
14800.	-.14311E+02	.63357E+00	.56915E+00	.10582E+04
14900.	-.14509E+02	.63151E+00	.56685E+00	.10578E+04
15000.	-.14707E+02	.62945E+00	.56459E+00	.10574E+04

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1962 U.S. STANDARD ATMOSPHERE

PRESSURE ALT (FT)	TEMPERATURE (DEG.CENTIGRADE)	SIGMA (RHO/S.L.RHO)	DELTA (PRESS/S.L.PRESS)	SPEED OF SOUND (FT/SEC)
15100.	-.14904E+02	.62741E+00	.56232E+00	.10570E+04
15200.	-.15102E+02	.62537E+00	.56006E+00	.10565E+04
15300.	-.15300E+02	.62333E+00	.55781E+00	.10561E+04
15400.	-.15498E+02	.62130E+00	.55556E+00	.10557E+04
15500.	-.15696E+02	.61927E+00	.55332E+00	.10553E+04
15600.	-.15894E+02	.61725E+00	.55109E+00	.10549E+04
15700.	-.16091E+02	.61523E+00	.54887E+00	.10545E+04
15800.	-.16289E+02	.61322E+00	.54665E+00	.10541E+04
15900.	-.16487E+02	.61121E+00	.54444E+00	.10537E+04
16000.	-.16685E+02	.60921E+00	.54224E+00	.10533E+04
16100.	-.16883E+02	.60721E+00	.54005E+00	.10529E+04
16200.	-.17081E+02	.60522E+00	.53786E+00	.10525E+04
16300.	-.17278E+02	.60323E+00	.53568E+00	.10521E+04
16400.	-.17476E+02	.60125E+00	.53351E+00	.10517E+04
16500.	-.17674E+02	.59927E+00	.53134E+00	.10513E+04
16600.	-.17872E+02	.59730E+00	.52918E+00	.10509E+04
16700.	-.18070E+02	.59533E+00	.52703E+00	.10505E+04
16800.	-.18267E+02	.59337E+00	.52489E+00	.10500E+04
16900.	-.18465E+02	.59141E+00	.52275E+00	.10496E+04
17000.	-.18663E+02	.58945E+00	.52062E+00	.10492E+04
17100.	-.18861E+02	.58751E+00	.51850E+00	.10488E+04
17200.	-.19059E+02	.58557E+00	.51638E+00	.10484E+04
17300.	-.19256E+02	.58363E+00	.51427E+00	.10480E+04
17400.	-.19454E+02	.58170E+00	.51217E+00	.10476E+04
17500.	-.19652E+02	.57977E+00	.51007E+00	.10472E+04
17600.	-.19850E+02	.57785E+00	.50798E+00	.10468E+04
17700.	-.20048E+02	.57593E+00	.50590E+00	.10464E+04
17800.	-.20245E+02	.57402E+00	.50383E+00	.10460E+04
17900.	-.20443E+02	.57211E+00	.50176E+00	.10456E+04
18000.	-.20641E+02	.57021E+00	.49970E+00	.10451E+04
18100.	-.20839E+02	.56831E+00	.49765E+00	.10447E+04
18200.	-.21036E+02	.56642E+00	.49560E+00	.10443E+04
18300.	-.21234E+02	.56453E+00	.49356E+00	.10439E+04
18400.	-.21432E+02	.56265E+00	.49153E+00	.10435E+04
18500.	-.21630E+02	.56077E+00	.48950E+00	.10431E+04
18600.	-.21827E+02	.55889E+00	.48748E+00	.10427E+04
18700.	-.22025E+02	.55702E+00	.48547E+00	.10423E+04
18800.	-.22223E+02	.55515E+00	.48346E+00	.10419E+04
18900.	-.22421E+02	.55330E+00	.48146E+00	.10415E+04
19000.	-.22619E+02	.55145E+00	.47947E+00	.10410E+04
19100.	-.22816E+02	.54959E+00	.47749E+00	.10406E+04
19200.	-.23014E+02	.54775E+00	.47551E+00	.10402E+04
19300.	-.23212E+02	.54591E+00	.47353E+00	.10398E+04
19400.	-.23410E+02	.54407E+00	.47157E+00	.10394E+04
19500.	-.23607E+02	.54224E+00	.46961E+00	.10390E+04
19600.	-.23805E+02	.54042E+00	.46766E+00	.10386E+04
19700.	-.24003E+02	.53859E+00	.46571E+00	.10382E+04
19800.	-.24201E+02	.53678E+00	.46377E+00	.10378E+04
19900.	-.24399E+02	.53497E+00	.46184E+00	.10373E+04
20000.	-.24596E+02	.53316E+00	.45991E+00	.10369E+04

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1962 U.S. STANDARD ATMOSPHERE

PRESSURE ALT (FT)	TEMPERATURE (DEG.CENTIGRADE)	SIGMA (RHO/S.L.RHO)	DELTA (PRESS/S.L.PRESS)	SPEED OF SOUND (FT/SEC)
20100.	-.24794E+02	.53135E+00	.45799E+00	.10365E+04
20200.	-.24992E+02	.52955E+00	.45608E+00	.10361E+04
20300.	-.25189E+02	.52775E+00	.45417E+00	.10357E+04
20400.	-.25387E+02	.52598E+00	.45227E+00	.10353E+04
20500.	-.25585E+02	.52419E+00	.45039E+00	.10349E+04
20600.	-.25782E+02	.52241E+00	.44849E+00	.10345E+04
20700.	-.25980E+02	.52064E+00	.44661E+00	.10340E+04
20800.	-.26178E+02	.51887E+00	.44474E+00	.10336E+04
20900.	-.26375E+02	.51710E+00	.44287E+00	.10332E+04
21000.	-.26573E+02	.51534E+00	.44101E+00	.10328E+04
21100.	-.26771E+02	.51359E+00	.43915E+00	.10324E+04
21200.	-.26969E+02	.51183E+00	.43730E+00	.10320E+04
21300.	-.27167E+02	.51009E+00	.43546E+00	.10316E+04
21400.	-.27364E+02	.50834E+00	.43362E+00	.10311E+04
21500.	-.27562E+02	.50660E+00	.43179E+00	.10307E+04
21600.	-.27760E+02	.50487E+00	.42997E+00	.10303E+04
21700.	-.27957E+02	.50314E+00	.42815E+00	.10299E+04
21800.	-.28155E+02	.50142E+00	.42634E+00	.10295E+04
21900.	-.28353E+02	.49970E+00	.42454E+00	.10291E+04
22000.	-.28550E+02	.49799E+00	.42274E+00	.10286E+04
22100.	-.28748E+02	.49627E+00	.42094E+00	.10282E+04
22200.	-.28946E+02	.49457E+00	.41916E+00	.10278E+04
22300.	-.29144E+02	.49286E+00	.41738E+00	.10274E+04
22400.	-.29341E+02	.49117E+00	.41560E+00	.10270E+04
22500.	-.29539E+02	.48947E+00	.41383E+00	.10266E+04
22600.	-.29737E+02	.48779E+00	.41207E+00	.10262E+04
22700.	-.29934E+02	.48610E+00	.41032E+00	.10257E+04
22800.	-.30132E+02	.48442E+00	.40857E+00	.10253E+04
22900.	-.30330E+02	.48275E+00	.40682E+00	.10249E+04
23000.	-.30527E+02	.48108E+00	.40509E+00	.10245E+04
23100.	-.30725E+02	.47941E+00	.40335E+00	.10241E+04
23200.	-.30923E+02	.47775E+00	.40163E+00	.10236E+04
23300.	-.31120E+02	.47609E+00	.39991E+00	.10232E+04
23400.	-.31318E+02	.47444E+00	.39819E+00	.10228E+04
23500.	-.31516E+02	.47279E+00	.39649E+00	.10224E+04
23600.	-.31713E+02	.47115E+00	.39479E+00	.10220E+04
23700.	-.31911E+02	.46951E+00	.39309E+00	.10216E+04
23800.	-.32109E+02	.46787E+00	.39140E+00	.10211E+04
23900.	-.32306E+02	.46624E+00	.38972E+00	.10207E+04
24000.	-.32504E+02	.46462E+00	.38804E+00	.10203E+04
24100.	-.32702E+02	.46300E+00	.38637E+00	.10199E+04
24200.	-.32899E+02	.46138E+00	.38470E+00	.10195E+04
24300.	-.33097E+02	.45977E+00	.38304E+00	.10190E+04
24400.	-.33295E+02	.45816E+00	.38139E+00	.10186E+04
24500.	-.33492E+02	.45655E+00	.37973E+00	.10182E+04
24600.	-.33690E+02	.45495E+00	.37809E+00	.10178E+04
24700.	-.33888E+02	.45335E+00	.37645E+00	.10174E+04
24800.	-.34085E+02	.45176E+00	.37482E+00	.10169E+04
24900.	-.34283E+02	.45018E+00	.37320E+00	.10165E+04
25000.	-.34481E+02	.44859E+00	.37158E+00	.10161E+04

1962 U.S. STANDARD ATMOSPHERE

PRESSURE ALT (FT)	TEMPERATURE (DEG.CENTIGRADE)	SIGMA (RHO/S.L.RHO)	DELTA (PRESS/S.L.PRESS)	SPEED OF SOUND (FT/SEC)
25100.	-.34678E+02	.44701E+00	.36995E+00	.10157E+04
25200.	-.34875E+02	.44544E+00	.36835E+00	.10153E+04
25300.	-.35074E+02	.44387E+00	.36675E+00	.10149E+04
25400.	-.35271E+02	.44230E+00	.36515E+00	.10144E+04
25500.	-.35469E+02	.44074E+00	.36356E+00	.10140E+04
25600.	-.35667E+02	.43918E+00	.36198E+00	.10136E+04
25700.	-.35864E+02	.43763E+00	.36040E+00	.10132E+04
25800.	-.36062E+02	.43608E+00	.35882E+00	.10127E+04
25900.	-.36259E+02	.43454E+00	.35725E+00	.10123E+04
26000.	-.36457E+02	.43300E+00	.35569E+00	.10119E+04
26100.	-.36655E+02	.43146E+00	.35413E+00	.10115E+04
26200.	-.36852E+02	.42993E+00	.35258E+00	.10110E+04
26300.	-.37050E+02	.42840E+00	.35103E+00	.10106E+04
26400.	-.37248E+02	.42688E+00	.34949E+00	.10102E+04
26500.	-.37445E+02	.42536E+00	.34795E+00	.10098E+04
26600.	-.37643E+02	.42384E+00	.34642E+00	.10093E+04
26700.	-.37840E+02	.42233E+00	.34490E+00	.10089E+04
26800.	-.38038E+02	.42082E+00	.34338E+00	.10085E+04
26900.	-.38236E+02	.41932E+00	.34185E+00	.10081E+04
27000.	-.38433E+02	.41782E+00	.34036E+00	.10077E+04
27100.	-.38631E+02	.41632E+00	.33885E+00	.10072E+04
27200.	-.38829E+02	.41483E+00	.33735E+00	.10068E+04
27300.	-.39026E+02	.41335E+00	.33586E+00	.10064E+04
27400.	-.39224E+02	.41185E+00	.33437E+00	.10060E+04
27500.	-.39421E+02	.41039E+00	.33289E+00	.10055E+04
27600.	-.39619E+02	.40891E+00	.33142E+00	.10051E+04
27700.	-.39816E+02	.40744E+00	.32995E+00	.10047E+04
27800.	-.40014E+02	.40597E+00	.32848E+00	.10043E+04
27900.	-.40212E+02	.40451E+00	.32702E+00	.10038E+04
28000.	-.40409E+02	.40305E+00	.32556E+00	.10034E+04
28100.	-.40607E+02	.40160E+00	.32411E+00	.10030E+04
28200.	-.40804E+02	.40015E+00	.32267E+00	.10026E+04
28300.	-.41002E+02	.39870E+00	.32123E+00	.10021E+04
28400.	-.41200E+02	.39725E+00	.31980E+00	.10017E+04
28500.	-.41397E+02	.39582E+00	.31837E+00	.10013E+04
28600.	-.41595E+02	.39439E+00	.31694E+00	.10009E+04
28700.	-.41792E+02	.39296E+00	.31552E+00	.10004E+04
28800.	-.41990E+02	.39153E+00	.31411E+00	.99999E+03
28900.	-.42187E+02	.39011E+00	.31270E+00	.99956E+03
29000.	-.42385E+02	.38869E+00	.31130E+00	.99914E+03
29100.	-.42583E+02	.38728E+00	.30990E+00	.99871E+03
29200.	-.42780E+02	.38587E+00	.30851E+00	.99828E+03
29300.	-.42978E+02	.38445E+00	.30712E+00	.99785E+03
29400.	-.43175E+02	.38305E+00	.30574E+00	.99742E+03
29500.	-.43373E+02	.38165E+00	.30436E+00	.99699E+03
29600.	-.43571E+02	.38027E+00	.30299E+00	.99657E+03
29700.	-.43768E+02	.37888E+00	.30162E+00	.99614E+03
29800.	-.43966E+02	.37749E+00	.30025E+00	.99571E+03
29900.	-.44163E+02	.37611E+00	.29890E+00	.99528E+03
30000.	-.44361E+02	.37473E+00	.29754E+00	.99485E+03

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1962 U.S. STANDARD ATMOSPHERE

PRESSURE ALT (FT)	TEMPRATURE (DEG.CENTIGRADE)	SIGMA (RHO/S.L.RHO)	DELTA (PRESS/S.L.PRESS)	SPEED OF SOUND (FT/SEC)
30100.	-.44559E+02	.37335E+00	.29620E+00	.99442E+03
30200.	-.44756E+02	.37198E+00	.29485E+00	.99399E+03
30300.	-.44953E+02	.37061E+00	.29352E+00	.99356E+03
30400.	-.45151E+02	.36925E+00	.29218E+00	.99313E+03
30500.	-.45348E+02	.36789E+00	.29085E+00	.99270E+03
30600.	-.45546E+02	.36654E+00	.28953E+00	.99227E+03
30700.	-.45743E+02	.36519E+00	.28821E+00	.99184E+03
30800.	-.45941E+02	.36384E+00	.28690E+00	.99141E+03
30900.	-.46138E+02	.36249E+00	.28559E+00	.99098E+03
31000.	-.46335E+02	.36115E+00	.28429E+00	.99055E+03
31100.	-.46534E+02	.35981E+00	.28299E+00	.99011E+03
31200.	-.46731E+02	.35848E+00	.28170E+00	.98968E+03
31300.	-.46929E+02	.35715E+00	.28041E+00	.98925E+03
31400.	-.47126E+02	.35583E+00	.27912E+00	.98882E+03
31500.	-.47324E+02	.35451E+00	.27784E+00	.98839E+03
31600.	-.47521E+02	.35319E+00	.27657E+00	.98795E+03
31700.	-.47719E+02	.35187E+00	.27530E+00	.98752E+03
31800.	-.47916E+02	.35056E+00	.27403E+00	.98709E+03
31900.	-.48114E+02	.34925E+00	.27277E+00	.98665E+03
32000.	-.48311E+02	.34795E+00	.27152E+00	.98622E+03
32100.	-.48509E+02	.34665E+00	.27025E+00	.98579E+03
32200.	-.48706E+02	.34535E+00	.26902E+00	.98536E+03
32300.	-.48904E+02	.34407E+00	.26778E+00	.98492E+03
32400.	-.49101E+02	.34278E+00	.26654E+00	.98449E+03
32500.	-.49299E+02	.34150E+00	.26531E+00	.98405E+03
32600.	-.49496E+02	.34022E+00	.26408E+00	.98362E+03
32700.	-.49694E+02	.33894E+00	.26286E+00	.98319E+03
32800.	-.49891E+02	.33767E+00	.26164E+00	.98275E+03
32900.	-.50089E+02	.33640E+00	.26042E+00	.98232E+03
33000.	-.50286E+02	.33513E+00	.25921E+00	.98188E+03
33100.	-.50484E+02	.33387E+00	.25801E+00	.98145E+03
33200.	-.50681E+02	.33261E+00	.25681E+00	.98101E+03
33300.	-.50879E+02	.33135E+00	.25561E+00	.98058E+03
33400.	-.51076E+02	.33011E+00	.25442E+00	.98014E+03
33500.	-.51274E+02	.32885E+00	.25323E+00	.97970E+03
33600.	-.51471E+02	.32761E+00	.25205E+00	.97927E+03
33700.	-.51669E+02	.32637E+00	.25087E+00	.97883E+03
33800.	-.51866E+02	.32514E+00	.24970E+00	.97840E+03
33900.	-.52064E+02	.32391E+00	.24853E+00	.97795E+03
34000.	-.52261E+02	.32267E+00	.24737E+00	.97752E+03
34100.	-.52459E+02	.32145E+00	.24621E+00	.97709E+03
34200.	-.52656E+02	.32023E+00	.24505E+00	.97665E+03
34300.	-.52854E+02	.31901E+00	.24390E+00	.97621E+03
34400.	-.53051E+02	.31779E+00	.24275E+00	.97577E+03
34500.	-.53249E+02	.31658E+00	.24161E+00	.97534E+03
34600.	-.53446E+02	.31537E+00	.24047E+00	.97490E+03
34700.	-.53643E+02	.31417E+00	.23934E+00	.97446E+03
34800.	-.53841E+02	.31297E+00	.23821E+00	.97402E+03
34900.	-.54039E+02	.31177E+00	.23708E+00	.97358E+03
35000.	-.54236E+02	.31058E+00	.23596E+00	.97314E+03

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1962 U.S. STANDARD ATMOSPHERE

PRESSURE ALT (FT)	TEMPERATURE (DEG.CENTIGRADE)	SIGMA (RHO/S.L.RHO)	DELTA (PRESS/S.L.PRESS)	SPEED OF SOUND (FT/SEC)
35100.	-.54433E+02	.30939E+00	.23485E+00	.97270E+03
35200.	-.54631E+02	.30820E+00	.23373E+00	.97227E+03
35300.	-.54829E+02	.30702E+00	.23263E+00	.97183E+03
35400.	-.55026E+02	.30584E+00	.23152E+00	.97139E+03
35500.	-.55223E+02	.30466E+00	.23042E+00	.97095E+03
35600.	-.55421E+02	.30349E+00	.22933E+00	.97051E+03
35700.	-.55619E+02	.30232E+00	.22824E+00	.97007E+03
35800.	-.55816E+02	.30115E+00	.22715E+00	.96963E+03
35900.	-.56013E+02	.29999E+00	.22607E+00	.96919E+03
36000.	-.56210E+02	.29883E+00	.22499E+00	.96875E+03
36100.	-.56408E+02	.29767E+00	.22392E+00	.96830E+03
36200.	-.56510E+02	.29633E+00	.22285E+00	.96808E+03
36300.	-.56510E+02	.29497E+00	.22179E+00	.96808E+03
36400.	-.56510E+02	.29357E+00	.22072E+00	.96808E+03
36500.	-.56510E+02	.29216E+00	.21967E+00	.96808E+03
36600.	-.56510E+02	.29077E+00	.21862E+00	.96808E+03
36700.	-.56510E+02	.28938E+00	.21757E+00	.96808E+03
36800.	-.56510E+02	.28799E+00	.21653E+00	.96808E+03
36900.	-.56510E+02	.28662E+00	.21550E+00	.96808E+03
37000.	-.56510E+02	.28525E+00	.21447E+00	.96808E+03
37100.	-.56510E+02	.28389E+00	.21344E+00	.96808E+03
37200.	-.56510E+02	.28253E+00	.21242E+00	.96808E+03
37300.	-.56510E+02	.28118E+00	.21141E+00	.96808E+03
37400.	-.56510E+02	.27984E+00	.21040E+00	.96808E+03
37500.	-.56510E+02	.27851E+00	.20939E+00	.96808E+03
37600.	-.56510E+02	.27717E+00	.20839E+00	.96808E+03
37700.	-.56510E+02	.27584E+00	.20740E+00	.96808E+03
37800.	-.56510E+02	.27453E+00	.20641E+00	.96808E+03
37900.	-.56510E+02	.27322E+00	.20542E+00	.96808E+03
38000.	-.56510E+02	.27191E+00	.20444E+00	.96808E+03
38100.	-.56510E+02	.27061E+00	.20346E+00	.96808E+03
38200.	-.56510E+02	.26932E+00	.20249E+00	.96808E+03
38300.	-.56510E+02	.26803E+00	.20152E+00	.96808E+03
38400.	-.56510E+02	.26675E+00	.20056E+00	.96808E+03
38500.	-.56510E+02	.26548E+00	.19960E+00	.96808E+03
38600.	-.56510E+02	.26421E+00	.19865E+00	.96808E+03
38700.	-.56510E+02	.26295E+00	.19770E+00	.96808E+03
38800.	-.56510E+02	.26169E+00	.19676E+00	.96808E+03
38900.	-.56510E+02	.26044E+00	.19582E+00	.96808E+03
39000.	-.56510E+02	.25920E+00	.19488E+00	.96808E+03
39100.	-.56510E+02	.25796E+00	.19395E+00	.96808E+03
39200.	-.56510E+02	.25673E+00	.19302E+00	.96808E+03
39300.	-.56510E+02	.25550E+00	.19210E+00	.96808E+03
39400.	-.56510E+02	.25428E+00	.19118E+00	.96808E+03
39500.	-.56510E+02	.25306E+00	.19027E+00	.96808E+03
39600.	-.56510E+02	.25185E+00	.18936E+00	.96808E+03
39700.	-.56510E+02	.25065E+00	.18846E+00	.96808E+03
39800.	-.56510E+02	.24945E+00	.18756E+00	.96808E+03
39900.	-.56510E+02	.24825E+00	.18666E+00	.96808E+03
40000.	-.56510E+02	.24708E+00	.18577E+00	.96808E+03

1962 U.S. STANDARD ATMOSPHERE

PRESSURE ALT (FT)	TEMPERATURE (DEG.CENTIGRADE)	SIGMA (RHO/S.L.RHO)	DELTA (PRESS/S.L.PRESS)	SPEED OF SOUND (FT/SEC)
40100.	-.56510E+02	.24593E+00	.18488E+00	.95808E+03
40200.	-.56510E+02	.24472E+00	.18400E+00	.95808E+03
40300.	-.56510E+02	.24355E+00	.18312E+00	.95808E+03
40400.	-.56510E+02	.24239E+00	.18225E+00	.95808E+03
40500.	-.56510E+02	.24123E+00	.18137E+00	.95808E+03
40600.	-.56510E+02	.24008E+00	.18051E+00	.95808E+03
40700.	-.56510E+02	.23893E+00	.17965E+00	.95808E+03
40800.	-.56510E+02	.23779E+00	.17879E+00	.95808E+03
40900.	-.56510E+02	.23665E+00	.17793E+00	.95808E+03
41000.	-.56510E+02	.23553E+00	.17708E+00	.95808E+03
41100.	-.56510E+02	.23440E+00	.17624E+00	.95808E+03
41200.	-.56510E+02	.23328E+00	.17540E+00	.95808E+03
41300.	-.56510E+02	.23217E+00	.17456E+00	.95808E+03
41400.	-.56510E+02	.23105E+00	.17373E+00	.95808E+03
41500.	-.56510E+02	.22995E+00	.17290E+00	.95808E+03
41600.	-.56510E+02	.22885E+00	.17207E+00	.95808E+03
41700.	-.56510E+02	.22775E+00	.17125E+00	.95808E+03
41800.	-.56510E+02	.22666E+00	.17043E+00	.95808E+03
41900.	-.56510E+02	.22559E+00	.16962E+00	.95808E+03
42000.	-.56510E+02	.22452E+00	.16881E+00	.95808E+03
42100.	-.56510E+02	.22345E+00	.16800E+00	.95808E+03
42200.	-.56510E+02	.22239E+00	.16720E+00	.95808E+03
42300.	-.56510E+02	.22132E+00	.16640E+00	.95808E+03
42400.	-.56510E+02	.22025E+00	.16561E+00	.95808E+03
42500.	-.56510E+02	.21921E+00	.16481E+00	.95808E+03
42600.	-.56510E+02	.21815E+00	.16403E+00	.95808E+03
42700.	-.56510E+02	.21712E+00	.16324E+00	.95808E+03
42800.	-.56510E+02	.21608E+00	.16246E+00	.95808E+03
42900.	-.56510E+02	.21505E+00	.16169E+00	.95808E+03
43000.	-.56510E+02	.21402E+00	.16092E+00	.95808E+03
43100.	-.56510E+02	.21300E+00	.16015E+00	.95808E+03
43200.	-.56510E+02	.21198E+00	.15938E+00	.95808E+03
43300.	-.56510E+02	.21097E+00	.15862E+00	.95808E+03
43400.	-.56510E+02	.20996E+00	.15787E+00	.95808E+03
43500.	-.56510E+02	.20895E+00	.15711E+00	.95808E+03
43600.	-.56510E+02	.20795E+00	.15636E+00	.95808E+03
43700.	-.56510E+02	.20697E+00	.15561E+00	.95808E+03
43800.	-.56510E+02	.20598E+00	.15487E+00	.95808E+03
43900.	-.56510E+02	.20500E+00	.15413E+00	.95808E+03
44000.	-.56510E+02	.20402E+00	.15340E+00	.95808E+03
44100.	-.56510E+02	.20305E+00	.15266E+00	.95808E+03
44200.	-.56510E+02	.20209E+00	.15193E+00	.95808E+03
44300.	-.56510E+02	.20111E+00	.15121E+00	.95808E+03
44400.	-.56510E+02	.20015E+00	.15049E+00	.95808E+03
44500.	-.56510E+02	.19920E+00	.14977E+00	.95808E+03
44600.	-.56510E+02	.19825E+00	.14905E+00	.95808E+03
44700.	-.56510E+02	.19730E+00	.14834E+00	.95808E+03
44800.	-.56510E+02	.19636E+00	.14763E+00	.95808E+03
44900.	-.56510E+02	.19542E+00	.14693E+00	.95808E+03
45000.	-.56510E+02	.19449E+00	.14623E+00	.95808E+03

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1962 U.S. STANDARD ATMOSPHERE

PRESSURE ALT (FT)	TEMPERATURE (DEG.CENTIGRADE)	SIGMA (RHO/S.L.RHO)	DELTA (PRESS/S.L.PRESS)	SPEED OF SOUND (FT/SEC)
45100.	-.56510E+02	.19355E+00	.14553E+00	.95808E+03
45200.	-.56510E+02	.19263E+00	.14483E+00	.95808E+03
45300.	-.56510E+02	.19171E+00	.14414E+00	.95808E+03
45400.	-.56510E+02	.19080E+00	.14346E+00	.95808E+03
45500.	-.56510E+02	.18989E+00	.14277E+00	.95808E+03
45600.	-.56510E+02	.18899E+00	.14209E+00	.95808E+03
45700.	-.56510E+02	.18808E+00	.14141E+00	.95808E+03
45800.	-.56510E+02	.18713E+00	.14074E+00	.95808E+03
45900.	-.56510E+02	.18620E+00	.14006E+00	.95808E+03
46000.	-.56510E+02	.18540E+00	.13939E+00	.95808E+03
46100.	-.56510E+02	.18451E+00	.13873E+00	.95808E+03
46200.	-.56510E+02	.18363E+00	.13807E+00	.95808E+03
46300.	-.56510E+02	.18275E+00	.13741E+00	.95808E+03
46400.	-.56510E+02	.18189E+00	.13675E+00	.95808E+03
46500.	-.56510E+02	.18102E+00	.13610E+00	.95808E+03
46600.	-.56510E+02	.18015E+00	.13545E+00	.95808E+03
46700.	-.56510E+02	.17929E+00	.13480E+00	.95808E+03
46800.	-.56510E+02	.17844E+00	.13416E+00	.95808E+03
46900.	-.56510E+02	.17758E+00	.13352E+00	.95808E+03
47000.	-.56510E+02	.17674E+00	.13288E+00	.95808E+03
47100.	-.56510E+02	.17589E+00	.13225E+00	.95808E+03
47200.	-.56510E+02	.17505E+00	.13162E+00	.95808E+03
47300.	-.56510E+02	.17422E+00	.13099E+00	.95808E+03
47400.	-.56510E+02	.17339E+00	.13036E+00	.95808E+03
47500.	-.56510E+02	.17256E+00	.12974E+00	.95808E+03
47600.	-.56510E+02	.17173E+00	.12912E+00	.95808E+03
47700.	-.56510E+02	.17091E+00	.12851E+00	.95808E+03
47800.	-.56510E+02	.17010E+00	.12783E+00	.95808E+03
47900.	-.56510E+02	.16929E+00	.12728E+00	.95808E+03
48000.	-.56510E+02	.16849E+00	.12667E+00	.95808E+03
48100.	-.56510E+02	.16768E+00	.12607E+00	.95808E+03
48200.	-.56510E+02	.16687E+00	.12547E+00	.95808E+03
48300.	-.56510E+02	.16608E+00	.12487E+00	.95808E+03
48400.	-.56510E+02	.16529E+00	.12427E+00	.95808E+03
48500.	-.56510E+02	.16450E+00	.12368E+00	.95808E+03
48600.	-.56510E+02	.16371E+00	.12309E+00	.95808E+03
48700.	-.56510E+02	.16293E+00	.12250E+00	.95808E+03
48800.	-.56510E+02	.16215E+00	.12192E+00	.95808E+03
48900.	-.56510E+02	.16138E+00	.12134E+00	.95808E+03
49000.	-.56510E+02	.16061E+00	.12076E+00	.95808E+03
49100.	-.56510E+02	.15984E+00	.12019E+00	.95808E+03
49200.	-.56510E+02	.15908E+00	.11961E+00	.95808E+03
49300.	-.56510E+02	.15832E+00	.11904E+00	.95808E+03
49400.	-.56510E+02	.15756E+00	.11847E+00	.95808E+03
49500.	-.56510E+02	.15681E+00	.11790E+00	.95808E+03
49600.	-.56510E+02	.15606E+00	.11734E+00	.95808E+03
49700.	-.56510E+02	.15532E+00	.11678E+00	.95808E+03
49800.	-.56510E+02	.15458E+00	.11622E+00	.95808E+03
49900.	-.56510E+02	.15384E+00	.11567E+00	.95808E+03
50000.	-.56510E+02	.15311E+00	.11512E+00	.95808E+03

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1962 U.S. STANDARD ATMOSPHERE

PRESSURE ALT (FT)	TEMPERATURE (DEG.CENTIGRADE)	SIGMA (RHO/S.L.RHO)	DELTA (PRESS/S.L.PRESS)	SPEED OF SOUND (FT/SEC)
50100.	-.56510E+02	.15238E+00	.11457E+00	.95808E+03
50200.	-.56510E+02	.15165E+00	.11402E+00	.95808E+03
50300.	-.56510E+02	.15093E+00	.11349E+00	.95808E+03
50400.	-.56510E+02	.15021E+00	.11295E+00	.95808E+03
50500.	-.56510E+02	.14949E+00	.11240E+00	.95808E+03
50600.	-.56510E+02	.14878E+00	.11186E+00	.95808E+03
50700.	-.56510E+02	.14807E+00	.11133E+00	.95808E+03
50800.	-.56510E+02	.14736E+00	.11079E+00	.95808E+03
50900.	-.56510E+02	.14665E+00	.11027E+00	.95808E+03
51000.	-.56510E+02	.14595E+00	.10974E+00	.95808E+03
51100.	-.56510E+02	.14525E+00	.10922E+00	.95808E+03
51200.	-.56510E+02	.14457E+00	.10869E+00	.95808E+03
51300.	-.56510E+02	.14388E+00	.10818E+00	.95808E+03
51400.	-.56510E+02	.14319E+00	.10766E+00	.95808E+03
51500.	-.56510E+02	.14251E+00	.10715E+00	.95808E+03
51600.	-.56510E+02	.14183E+00	.10663E+00	.95808E+03
51700.	-.56510E+02	.14115E+00	.10613E+00	.95808E+03
51800.	-.56510E+02	.14048E+00	.10562E+00	.95808E+03
51900.	-.56510E+02	.13981E+00	.10512E+00	.95808E+03
52000.	-.56510E+02	.13914E+00	.10461E+00	.95808E+03
52100.	-.56510E+02	.13848E+00	.10412E+00	.95808E+03
52200.	-.56510E+02	.13782E+00	.10362E+00	.95808E+03
52300.	-.56510E+02	.13715E+00	.10312E+00	.95808E+03
52400.	-.56510E+02	.13650E+00	.10263E+00	.95808E+03
52500.	-.56510E+02	.13585E+00	.10214E+00	.95808E+03
52600.	-.56510E+02	.13520E+00	.10166E+00	.95808E+03
52700.	-.56510E+02	.13455E+00	.10117E+00	.95808E+03
52800.	-.56510E+02	.13392E+00	.10069E+00	.95808E+03
52900.	-.56510E+02	.13328E+00	.10021E+00	.95808E+03
53000.	-.56510E+02	.13264E+00	.99729E-01	.95808E+03
53100.	-.56510E+02	.13201E+00	.99253E-01	.95808E+03
53200.	-.56510E+02	.13138E+00	.98780E-01	.95808E+03
53300.	-.56510E+02	.13075E+00	.98309E-01	.95808E+03
53400.	-.56510E+02	.13013E+00	.97840E-01	.95808E+03
53500.	-.56510E+02	.12951E+00	.97373E-01	.95808E+03
53600.	-.56510E+02	.12889E+00	.96908E-01	.95808E+03
53700.	-.56510E+02	.12828E+00	.96446E-01	.95808E+03
53800.	-.56510E+02	.12765E+00	.95985E-01	.95808E+03
53900.	-.56510E+02	.12705E+00	.95528E-01	.95808E+03
54000.	-.56510E+02	.12645E+00	.95072E-01	.95808E+03
54100.	-.56510E+02	.12585E+00	.94619E-01	.95808E+03
54200.	-.56510E+02	.12525E+00	.94168E-01	.95808E+03
54300.	-.56510E+02	.12465E+00	.93718E-01	.95808E+03
54400.	-.56510E+02	.12405E+00	.93271E-01	.95808E+03
54500.	-.56510E+02	.12346E+00	.92827E-01	.95808E+03
54600.	-.56510E+02	.12287E+00	.92384E-01	.95808E+03
54700.	-.56510E+02	.12229E+00	.91943E-01	.95808E+03
54800.	-.56510E+02	.12170E+00	.91505E-01	.95808E+03
54900.	-.56510E+02	.12112E+00	.91068E-01	.95808E+03
55000.	-.56510E+02	.12055E+00	.90634E-01	.95808E+03

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1962 U.S. STANDARD ATMOSPHERE

PRESSURE ALT (FT)	TEMPERATURE (DEG.CENTIGRADE)	SIGMA (RHO/S.L.RHO)	DELTA (PRESS/S.L.PRESS)	SPEED OF SOUND (FT/SEC)
55100.	-.56510E+02	.11997E+00	.90201E-01	.95808E+03
55200.	-.56510E+02	.11940E+00	.89771E-01	.95808E+03
55300.	-.56510E+02	.11883E+00	.89343E-01	.95808E+03
55400.	-.56510E+02	.11825E+00	.88917E-01	.95808E+03
55500.	-.56510E+02	.11770E+00	.88493E-01	.95808E+03
55600.	-.56510E+02	.11714E+00	.88071E-01	.95808E+03
55700.	-.56510E+02	.11658E+00	.87651E-01	.95808E+03
55800.	-.56510E+02	.11602E+00	.87233E-01	.95808E+03
55900.	-.56510E+02	.11547E+00	.86817E-01	.95808E+03
56000.	-.56510E+02	.11492E+00	.86403E-01	.95808E+03
56100.	-.56510E+02	.11437E+00	.85991E-01	.95808E+03
56200.	-.56510E+02	.11382E+00	.855[illegible]	.95808E+03
56300.	-.56510E+02	.11328E+00	.85172E-01	.95808E+03
56400.	-.56510E+02	.11274E+00	.84765E-01	.95808E+03
56500.	-.56510E+02	.11220E+00	.84362E-01	.95808E+03
56600.	-.56510E+02	.11167E+00	.83960E-01	.95808E+03
56700.	-.56510E+02	.11114E+00	.83559E-01	.95808E+03
56800.	-.56510E+02	.11061E+00	.83161E-01	.95808E+03
56900.	-.56510E+02	.11008E+00	.82764E-01	.95808E+03
57000.	-.56510E+02	.10955E+00	.82369E-01	.95808E+03
57100.	-.56510E+02	.10903E+00	.81977E-01	.95808E+03
57200.	-.56510E+02	.10851E+00	.81586E-01	.95808E+03
57300.	-.56510E+02	.10799E+00	.81197E-01	.95808E+03
57400.	-.56510E+02	.10748E+00	.80809E-01	.95808E+03
57500.	-.56510E+02	.10697E+00	.80424E-01	.95808E+03
57600.	-.56510E+02	.10645E+00	.80041E-01	.95808E+03
57700.	-.56510E+02	.10595E+00	.79659E-01	.95808E+03
57800.	-.56510E+02	.10544E+00	.79279E-01	.95808E+03
57900.	-.56510E+02	.10494E+00	.78901E-01	.95808E+03
58000.	-.56510E+02	.10444E+00	.78525E-01	.95808E+03
58100.	-.56510E+02	.10394E+00	.78150E-01	.95808E+03
58200.	-.56510E+02	.10345E+00	.77778E-01	.95808E+03
58300.	-.56510E+02	.10295E+00	.77407E-01	.95808E+03
58400.	-.56510E+02	.10246E+00	.77038E-01	.95808E+03
58500.	-.56510E+02	.10197E+00	.76670E-01	.95808E+03
58600.	-.56510E+02	.10149E+00	.76305E-01	.95808E+03
58700.	-.56510E+02	.10100E+00	.75941E-01	.95808E+03
58800.	-.56510E+02	.10052E+00	.75579E-01	.95808E+03
58900.	-.56510E+02	.10004E+00	.75219E-01	.95808E+03
59000.	-.56510E+02	.99565E-01	.74860E-01	.95808E+03
59100.	-.56510E+02	.99091E-01	.74503E-01	.95808E+03
59200.	-.56510E+02	.98618E-01	.74148E-01	.95808E+03
59300.	-.56510E+02	.98149E-01	.73794E-01	.95808E+03
59400.	-.56510E+02	.97683F-01	.73442E-01	.95808E+03
59500.	-.56510E+02	.97215E-01	.73092E-01	.95808E+03
59600.	-.56510E+02	.96751E-01	.72744E-01	.95808E+03
59700.	-.56510E+02	.96290E-01	.72397E-01	.95808E+03
59800.	-.56510E+02	.95831E-01	.72052E-01	.95808E+03
59900.	-.56510E+02	.95374E-01	.71709E-01	.95808E+03
60000.	-.56510E+02	.94919E-01	.71367E-01	.95808E+03

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1962 U.S. STANDARD ATMOSPHERE

PRESSURE ALT (FT)	TEMPERATURE (DEG.CENTIGRADE)	SIGMA (RHO/S.L.RHO)	DELTA (PRESS/S.L.PRESS)	SPEED OF SOUND (FT/SEC)
60100.	-.56510E+02	.94467E-01	.71026E-01	.95808E+03
60200.	-.56510E+02	.94015E-01	.70689E-01	.95808E+03
60300.	-.56510E+02	.93568E-01	.70351E-01	.95808E+03
60400.	-.56510E+02	.93122E-01	.70015E-01	.95808E+03
60500.	-.56510E+02	.92678E-01	.69682E-01	.95808E+03
60600.	-.56510E+02	.92235E-01	.69349E-01	.95808E+03
60700.	-.56510E+02	.91797E-01	.69019E-01	.95808E+03
60800.	-.56510E+02	.91359E-01	.68690E-01	.95808E+03
60900.	-.56510E+02	.90924E-01	.68362E-01	.95808E+03
61000.	-.56510E+02	.90490E-01	.68036E-01	.95808E+03
61100.	-.56510E+02	.90059E-01	.67712E-01	.95808E+03
61200.	-.56510E+02	.89629E-01	.67389E-01	.95808E+03
61300.	-.56510E+02	.89202E-01	.67068E-01	.95808E+03
61400.	-.56510E+02	.88777E-01	.66749E-01	.95808E+03
61500.	-.56510E+02	.88354E-01	.66430E-01	.95808E+03
61600.	-.56510E+02	.87933E-01	.66114E-01	.95808E+03
61700.	-.56510E+02	.87514E-01	.65798E-01	.95808E+03
61800.	-.56510E+02	.87095E-01	.65485E-01	.95808E+03
61900.	-.56510E+02	.86691E-01	.65173E-01	.95808E+03
62000.	-.56510E+02	.86264E-01	.64862E-01	.95808E+03
62100.	-.56510E+02	.85857E-01	.64553E-01	.95808E+03
62200.	-.56510E+02	.85449E-01	.64245E-01	.95808E+03
62300.	-.56510E+02	.85040E-01	.63939E-01	.95808E+03
62400.	-.56510E+02	.84635E-01	.63634E-01	.95808E+03
62500.	-.56510E+02	.84232E-01	.63331E-01	.95808E+03
62600.	-.56510E+02	.83830E-01	.63029E-01	.95808E+03
62700.	-.56510E+02	.83431E-01	.62729E-01	.95808E+03
62800.	-.56510E+02	.83033E-01	.62430E-01	.95808E+03
62900.	-.56510E+02	.82637E-01	.62132E-01	.95808E+03
63000.	-.56510E+02	.82243E-01	.61836E-01	.95808E+03
63100.	-.56510E+02	.81851E-01	.61541E-01	.95808E+03
63200.	-.56510E+02	.81461E-01	.61248E-01	.95808E+03
63300.	-.56510E+02	.81073E-01	.60956E-01	.95808E+03
63400.	-.56510E+02	.80687E-01	.60666E-01	.95808E+03
63500.	-.56510E+02	.80302E-01	.60376E-01	.95808E+03
63600.	-.56510E+02	.79919E-01	.60089E-01	.95808E+03
63700.	-.56510E+02	.79539E-01	.59802E-01	.95808E+03
63800.	-.56510E+02	.79160E-01	.59517E-01	.95808E+03
63900.	-.56510E+02	.78782E-01	.59234E-01	.95808E+03
64000.	-.56510E+02	.78407E-01	.58951E-01	.95808E+03
64100.	-.56510E+02	.78033E-01	.58670E-01	.95808E+03
64200.	-.56510E+02	.77661E-01	.58391E-01	.95808E+03
64300.	-.56510E+02	.77291E-01	.58113E-01	.95808E+03
64400.	-.56510E+02	.76923E-01	.57836E-01	.95808E+03
64500.	-.56510E+02	.76556E-01	.57560E-01	.95808E+03
64600.	-.56510E+02	.76192E-01	.57285E-01	.95808E+03
64700.	-.56510E+02	.75828E-01	.57013E-01	.95808E+03
64800.	-.56510E+02	.75467E-01	.56741E-01	.95808E+03
64900.	-.56510E+02	.75107E-01	.56471E-01	.95808E+03
65000.	-.56510E+02	.74750E-01	.56202E-01	.95808E+03

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1962 U.S. STANDARD ATMOSPHERE

PRESSURE ALT (FT)	TEMPERATURE (DEG.CENTIGRADE)	SIGMA (RHO/S.L.RHO)	DELTA (PRESS/S.L.PRESS)	SPEED OF SOUND (FT/SEC)
65100.	-.56510E+02	.74393E-01	.55934E-01	.96808E+03
65200.	-.56510E+02	.74039E-01	.55657E-01	.96808E+03
65300.	-.56510E+02	.73686E-01	.55402E-01	.96808E+03
65400.	-.56510E+02	.73335E-01	.55138E-01	.96808E+03
65500.	-.56510E+02	.72985E-01	.54875E-01	.96808E+03
65600.	-.56510E+02	.72638E-01	.54614E-01	.96808E+03
65700.	-.56510E+02	.72292E-01	.54354E-01	.96808E+03
65800.	-.56510E+02	.71947E-01	.54095E-01	.96808E+03
65900.	-.56487E+02	.71597E-01	.53837E-01	.96813E+03
66000.	-.56457E+02	.71246E-01	.53580E-01	.96819E+03
66100.	-.56426E+02	.70895E-01	.53325E-01	.96826E+03
66200.	-.56396E+02	.70540E-01	.53071E-01	.96833E+03
66300.	-.56365E+02	.70203E-01	.52819E-01	.96840E+03
66400.	-.56335E+02	.69859E-01	.52567E-01	.96847E+03
66500.	-.56305E+02	.69517E-01	.52317E-01	.96853E+03
66600.	-.56275E+02	.69176E-01	.52069E-01	.96860E+03
66700.	-.56245E+02	.68837E-01	.51820E-01	.96867E+03
66800.	-.56214E+02	.68500E-01	.51573E-01	.96874E+03
66900.	-.56184E+02	.68165E-01	.51328E-01	.96880E+03
67000.	-.56154E+02	.67831E-01	.51084E-01	.96887E+03
67100.	-.56124E+02	.67499E-01	.50841E-01	.96894E+03
67200.	-.56093E+02	.67169E-01	.50599E-01	.96901E+03
67300.	-.56063E+02	.66840E-01	.50359E-01	.96907E+03
67400.	-.56033E+02	.66513E-01	.50119E-01	.96914E+03
67500.	-.56002E+02	.66188E-01	.49881E-01	.96921E+03
67600.	-.55972E+02	.65864E-01	.49644E-01	.96928E+03
67700.	-.55942E+02	.65542E-01	.49408E-01	.96934E+03
67800.	-.55912E+02	.65221E-01	.49173E-01	.96941E+03
67900.	-.55881E+02	.64903E-01	.48940E-01	.96948E+03
68000.	-.55851E+02	.64585E-01	.48707E-01	.96955E+03
68100.	-.55821E+02	.64270E-01	.48476E-01	.96961E+03
68200.	-.55790E+02	.63956E-01	.48246E-01	.96968E+03
68300.	-.55760E+02	.63643E-01	.48017E-01	.96975E+03
68400.	-.55730E+02	.63332E-01	.47789E-01	.96982E+03
68500.	-.55700E+02	.63023E-01	.47562E-01	.96988E+03
68600.	-.55669E+02	.62715E-01	.47336E-01	.96995E+03
68700.	-.55639E+02	.62409E-01	.47112E-01	.97002E+03
68800.	-.55609E+02	.62104E-01	.46888E-01	.97009E+03
68900.	-.55579E+02	.61801E-01	.46665E-01	.97015E+03
69000.	-.55548E+02	.61499E-01	.46444E-01	.97022E+03
69100.	-.55518E+02	.61199E-01	.46224E-01	.97029E+03
69200.	-.55488E+02	.60900E-01	.46005E-01	.97036E+03
69300.	-.55457E+02	.60603E-01	.45787E-01	.97042E+03
69400.	-.55427E+02	.60309E-01	.45570E-01	.97049E+03
69500.	-.55397E+02	.60013E-01	.45354E-01	.97056E+03
69600.	-.55366E+02	.59721E-01	.45139E-01	.97063E+03
69700.	-.55336E+02	.59430E-01	.44925E-01	.97069E+03
69800.	-.55306E+02	.59140E-01	.44712E-01	.97075E+03
69900.	-.55276E+02	.58852E-01	.44501E-01	.97083E+03
70000.	-.55245E+02	.58565E-01	.44290E-01	.97090E+03

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1962 U.S. STANDARD ATMOSPHERE

PRESSURE ALT (FT)	TEMPERATURE (DEG.CENTIGRADE)	SIGMA (RHO/S.L.RHO)	DELTA (PRESS/S.L.PRESS)	SPEED OF SOUND (FT/SEC)
70100.	-.55213E+02	.58279E-01	.44080E-01	.97095E+03
70200.	-.55185E+02	.57995E-01	.43871E-01	.97103E+03
70300.	-.55155E+02	.57713E-01	.43664E-01	.97110E+03
70400.	-.55124E+02	.57432E-01	.43457E-01	.97117E+03
70500.	-.55094E+02	.57152E-01	.43251E-01	.97123E+03
70600.	-.55064E+02	.56874E-01	.43047E-01	.97130E+03
70700.	-.55033E+02	.56597E-01	.42843E-01	.97137E+03
70800.	-.55003E+02	.56321E-01	.42640E-01	.97144E+03
70900.	-.54973E+02	.56047E-01	.42439E-01	.97150E+03
71000.	-.54943E+02	.55774E-01	.42238E-01	.97157E+03
71100.	-.54912E+02	.55503E-01	.42038E-01	.97164E+03
71200.	-.54882E+02	.55233E-01	.41840E-01	.97171E+03
71300.	-.54852E+02	.54964E-01	.41642E-01	.97177E+03
71400.	-.54822E+02	.54697E-01	.41445E-01	.97184E+03
71500.	-.54791E+02	.54431E-01	.41249E-01	.97191E+03
71600.	-.54761E+02	.54165E-01	.41054E-01	.97198E+03
71700.	-.54731E+02	.53903E-01	.40861E-01	.97204E+03
71800.	-.54700E+02	.53641E-01	.40668E-01	.97211E+03
71900.	-.54670E+02	.53380E-01	.40476E-01	.97218E+03
72000.	-.54640E+02	.53121E-01	.40284E-01	.97224E+03
72100.	-.54610E+02	.52863E-01	.40094E-01	.97231E+03
72200.	-.54579E+02	.52606E-01	.39905E-01	.97238E+03
72300.	-.54549E+02	.52350E-01	.39717E-01	.97245E+03
72400.	-.54519E+02	.52095E-01	.39529E-01	.97251E+03
72500.	-.54489E+02	.51843E-01	.39343E-01	.97258E+03
72600.	-.54458E+02	.51591E-01	.39157E-01	.97265E+03
72700.	-.54428E+02	.51341E-01	.38972E-01	.97272E+03
72800.	-.54398E+02	.51092E-01	.38789E-01	.97278E+03
72900.	-.54367E+02	.50844E-01	.38606E-01	.97285E+03
73000.	-.54337E+02	.50597E-01	.38424E-01	.97292E+03
73100.	-.54307E+02	.50352E-01	.38243E-01	.97299E+03
73200.	-.54277E+02	.50107E-01	.38062E-01	.97305E+03
73300.	-.54246E+02	.49864E-01	.37883E-01	.97312E+03
73400.	-.54216E+02	.49622E-01	.37704E-01	.97319E+03
73500.	-.54186E+02	.49382E-01	.37527E-01	.97325E+03
73600.	-.54155E+02	.49142E-01	.37350E-01	.97332E+03
73700.	-.54125E+02	.48904E-01	.37174E-01	.97339E+03
73800.	-.54095E+02	.48667E-01	.36999E-01	.97346E+03
73900.	-.54065E+02	.48431E-01	.36825E-01	.97352E+03
74000.	-.54035E+02	.48197E-01	.36652E-01	.97359E+03
74100.	-.54004E+02	.47963E-01	.36479E-01	.97366E+03
74200.	-.53974E+02	.47731E-01	.36307E-01	.97373E+03
74300.	-.53944E+02	.47500E-01	.36136E-01	.97379E+03
74400.	-.53913E+02	.47270E-01	.35966E-01	.97386E+03
74500.	-.53883E+02	.47041E-01	.35797E-01	.97393E+03
74600.	-.53853E+02	.46813E-01	.35629E-01	.97399E+03
74700.	-.53823E+02	.46587E-01	.35461E-01	.97406E+03
74800.	-.53792E+02	.46361E-01	.35295E-01	.97413E+03
74900.	-.53762E+02	.46137E-01	.35129E-01	.97420E+03
75000.	-.53732E+02	.45914E-01	.34963E-01	.97426E+03

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1962 U.S. STANDARD ATMOSPHERE

PRESSURE ALT (FT)	TEMPERATURE (DEG.CENTIGRADE)	SIGMA (RHO/S.L.RHO)	DELTA (PRESS/S.L.PRESS)	SPEED OF SOUND (FT/SEC)
75100.	-.53702E+02	.45691E-01	.34799E-01	.97433E+03
75200.	-.53671E+02	.45470E-01	.34636E-01	.97440E+03
75300.	-.53641E+02	.45251E-01	.34473E-01	.97446E+03
75400.	-.53611E+02	.45032E-01	.34311E-01	.97453E+03
75500.	-.53581E+02	.44814E-01	.34150E-01	.97460E+03
75600.	-.53550E+02	.44597E-01	.33989E-01	.97467E+03
75700.	-.53520E+02	.44382E-01	.33830E-01	.97473E+03
75800.	-.53490E+02	.44167E-01	.33671E-01	.97480E+03
75900.	-.53460E+02	.43954E-01	.33513E-01	.97487E+03
76000.	-.53429E+02	.43742E-01	.33356E-01	.97493E+03
76100.	-.53399E+02	.43530E-01	.33199E-01	.97500E+03
76200.	-.53369E+02	.43320E-01	.33043E-01	.97507E+03
76300.	-.53339E+02	.43111E-01	.32888E-01	.97514E+03
76400.	-.53308E+02	.42903E-01	.32734E-01	.97520E+03
76500.	-.53278E+02	.42695E-01	.32580E-01	.97527E+03
76600.	-.53248E+02	.42490E-01	.32428E-01	.97534E+03
76700.	-.53218E+02	.42285E-01	.32275E-01	.97540E+03
76800.	-.53187E+02	.42081E-01	.32124E-01	.97547E+03
76900.	-.53157E+02	.41878E-01	.31974E-01	.97554E+03
77000.	-.53127E+02	.41676E-01	.31824E-01	.97561E+03
77100.	-.53096E+02	.41475E-01	.31675E-01	.97567E+03
77200.	-.53066E+02	.41275E-01	.31526E-01	.97574E+03
77300.	-.53036E+02	.41075E-01	.31378E-01	.97581E+03
77400.	-.53005E+02	.40877E-01	.31231E-01	.97587E+03
77500.	-.52975E+02	.40680E-01	.31085E-01	.97594E+03
77600.	-.52945E+02	.40484E-01	.30940E-01	.97601E+03
77700.	-.52915E+02	.40289E-01	.30795E-01	.97607E+03
77800.	-.52885E+02	.40095E-01	.30651E-01	.97614E+03
77900.	-.52854E+02	.39902E-01	.30507E-01	.97621E+03
78000.	-.52824E+02	.39710E-01	.30364E-01	.97628E+03
78100.	-.52794E+02	.39519E-01	.30222E-01	.97634E+03
78200.	-.52764E+02	.39328E-01	.30081E-01	.97641E+03
78300.	-.52733E+02	.39139E-01	.29940E-01	.97648E+03
78400.	-.52703E+02	.38950E-01	.29800E-01	.97654E+03
78500.	-.52673E+02	.38763E-01	.29661E-01	.97661E+03
78600.	-.52643E+02	.38576E-01	.29522E-01	.97668E+03
78700.	-.52612E+02	.38391E-01	.29384E-01	.97674E+03
78800.	-.52582E+02	.38206E-01	.29247E-01	.97681E+03
78900.	-.52552E+02	.38022E-01	.29110E-01	.97688E+03
79000.	-.52522E+02	.37839E-01	.28974E-01	.97695E+03
79100.	-.52491E+02	.37657E-01	.28839E-01	.97701E+03
79200.	-.52461E+02	.37475E-01	.28704E-01	.97708E+03
79300.	-.52431E+02	.37295E-01	.28570E-01	.97715E+03
79400.	-.52401E+02	.37117E-01	.28436E-01	.97721E+03
79500.	-.52370E+02	.36939E-01	.28303E-01	.97728E+03
79600.	-.52340E+02	.36761E-01	.28171E-01	.97735E+03
79700.	-.52310E+02	.36584E-01	.28040E-01	.97741E+03
79800.	-.52280E+02	.36409E-01	.27909E-01	.97748E+03
79900.	-.52249E+02	.36234E-01	.27779E-01	.97755E+03
80000.	-.52219E+02	.36060E-01	.27649E-01	.97762E+03

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1962 U.S. STANDARD ATMOSPHERE

PRESSURE ALT (FT)	TEMPERATURE (DEG.CENTIGRADE)	SIGMA (RHO/S.L.RHO)	DELTA (PRESS/S.L.PRESS)	SPEED OF SOUND (FT/SEC)
80100.	-.52189E+02	.35887E-01	.27520E-01	.97769E+03
80200.	-.52159E+02	.35714E-01	.27392E-01	.97775E+03
80300.	-.52129E+02	.35543E-01	.27264E-01	.97782E+03
80400.	-.52099E+02	.35372E-01	.27137E-01	.97788E+03
80500.	-.52068E+02	.35202E-01	.27010E-01	.97795E+03
80600.	-.52038E+02	.35034E-01	.26884E-01	.97802E+03
80700.	-.52007E+02	.34866E-01	.26759E-01	.97808E+03
80800.	-.51977E+02	.34698E-01	.26634E-01	.97815E+03
80900.	-.51947E+02	.34532E-01	.26510E-01	.97822E+03
81000.	-.51917E+02	.34366E-01	.26387E-01	.97828E+03
81100.	-.51886E+02	.34201E-01	.26264E-01	.97835E+03
81200.	-.51856E+02	.34037E-01	.26141E-01	.97842E+03
81300.	-.51826E+02	.33874E-01	.26020E-01	.97848E+03
81400.	-.51796E+02	.33712E-01	.25898E-01	.97855E+03
81500.	-.51765E+02	.33550E-01	.25778E-01	.97862E+03
81600.	-.51735E+02	.33390E-01	.25658E-01	.97869E+03
81700.	-.51705E+02	.33230E-01	.25538E-01	.97875E+03
81800.	-.51675E+02	.33070E-01	.25419E-01	.97882E+03
81900.	-.51645E+02	.32912E-01	.25301E-01	.97889E+03
82000.	-.51614E+02	.32754E-01	.25183E-01	.97895E+03
82100.	-.51584E+02	.32598E-01	.25066E-01	.97902E+03
82200.	-.51554E+02	.32442E-01	.24950E-01	.97909E+03
82300.	-.51524E+02	.32286E-01	.24834E-01	.97915E+03
82400.	-.51493E+02	.32132E-01	.24718E-01	.97922E+03
82500.	-.51463E+02	.31979E-01	.24603E-01	.97929E+03
82600.	-.51433E+02	.31826E-01	.24489E-01	.97935E+03
82700.	-.51403E+02	.31673E-01	.24375E-01	.97942E+03
82800.	-.51372E+02	.31521E-01	.24262E-01	.97949E+03
82900.	-.51342E+02	.31371E-01	.24149E-01	.97955E+03
83000.	-.51312E+02	.31221E-01	.24037E-01	.97962E+03
83100.	-.51282E+02	.31071E-01	.23925E-01	.97969E+03
83200.	-.51251E+02	.30923E-01	.23814E-01	.97975E+03
83300.	-.51221E+02	.30775E-01	.23703E-01	.97982E+03
83400.	-.51191E+02	.30628E-01	.23593E-01	.97989E+03
83500.	-.51161E+02	.30481E-01	.23484E-01	.97995E+03
83600.	-.51130E+02	.30335E-01	.23375E-01	.98002E+03
83700.	-.51100E+02	.30191E-01	.23266E-01	.98009E+03
83800.	-.51070E+02	.30047E-01	.23158E-01	.98015E+03
83900.	-.51040E+02	.29903E-01	.23051E-01	.98022E+03
84000.	-.51010E+02	.29760E-01	.22944E-01	.98029E+03
84100.	-.50979E+02	.29619E-01	.22838E-01	.98035E+03
84200.	-.50949E+02	.29477E-01	.22732E-01	.98042E+03
84300.	-.50919E+02	.29336E-01	.22626E-01	.98049E+03
84400.	-.50889E+02	.29196E-01	.22521E-01	.98055E+03
84500.	-.50858E+02	.29057E-01	.22417E-01	.98062E+03
84600.	-.50828E+02	.28918E-01	.22313E-01	.98069E+03
84700.	-.50798E+02	.28780E-01	.22210E-01	.98075E+03
84800.	-.50768E+02	.28643E-01	.22107E-01	.98082E+03
84900.	-.50737E+02	.28507E-01	.22004E-01	.98089E+03
85000.	-.50707E+02	.28371E-01	.21902E-01	.98095E+03

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1962 U.S. STANDARD ATMOSPHERE

PRESSURE ALT (FT)	TEMPERATURE (DEG.CENTIGRADE)	SIGMA (RHO/S.L.RHO)	DELTA (PRESS/S.L.PRESS)	SPEED OF SOUND (FT/SEC)
85100.	-.50677E+02	.28235E-01	.21801E-01	.98102E+03
85200.	-.50647E+02	.28101E-01	.21700E-01	.98109E+03
85300.	-.50615E+02	.27967E-01	.21599E-01	.98115E+03
85400.	-.50585E+02	.27834E-01	.21499E-01	.98122E+03
85500.	-.50555E+02	.27701E-01	.21400E-01	.98129E+03
85600.	-.50525E+02	.27569E-01	.21301E-01	.98135E+03
85700.	-.50495E+02	.27438E-01	.21202E-01	.98142E+03
85800.	-.50465E+02	.27307E-01	.21104E-01	.98149E+03
85900.	-.50435E+02	.27177E-01	.21006E-01	.98155E+03
86000.	-.50405E+02	.27048E-01	.20909E-01	.98162E+03
86100.	-.50375E+02	.26919E-01	.20813E-01	.98169E+03
86200.	-.50344E+02	.26791E-01	.20716E-01	.98175E+03
86300.	-.50314E+02	.26663E-01	.20621E-01	.98182E+03
86400.	-.50284E+02	.26536E-01	.20525E-01	.98189E+03
86500.	-.50254E+02	.26410E-01	.20430E-01	.98195E+03
86600.	-.50223E+02	.26285E-01	.20336E-01	.98202E+03
86700.	-.50193E+02	.26160E-01	.20242E-01	.98209E+03
86800.	-.50163E+02	.26035E-01	.20148E-01	.98215E+03
86900.	-.50133E+02	.25911E-01	.20055E-01	.98222E+03
87000.	-.50103E+02	.25788E-01	.19963E-01	.98229E+03
87100.	-.50072E+02	.25666E-01	.19870E-01	.98235E+03
87200.	-.50042E+02	.25544E-01	.19779E-01	.98242E+03
87300.	-.50012E+02	.25422E-01	.19687E-01	.98249E+03
87400.	-.49982E+02	.25301E-01	.19597E-01	.98255E+03
87500.	-.49951E+02	.25181E-01	.19506E-01	.98262E+03
87600.	-.49921E+02	.25062E-01	.19416E-01	.98269E+03
87700.	-.49891E+02	.24943E-01	.19326E-01	.98275E+03
87800.	-.49861E+02	.24824E-01	.19237E-01	.98282E+03
87900.	-.49831E+02	.24706E-01	.19149E-01	.98289E+03
88000.	-.49800E+02	.24589E-01	.19060E-01	.98295E+03
88100.	-.49770E+02	.24472E-01	.18972E-01	.98302E+03
88200.	-.49740E+02	.24356E-01	.18885E-01	.98309E+03
88300.	-.49710E+02	.24241E-01	.18798E-01	.98315E+03
88400.	-.49679E+02	.24126E-01	.18711E-01	.98322E+03
88500.	-.49649E+02	.24011E-01	.18625E-01	.98328E+03
88600.	-.49619E+02	.23897E-01	.18539E-01	.98335E+03
88700.	-.49589E+02	.23784E-01	.18454E-01	.98342E+03
88800.	-.49559E+02	.23671E-01	.18369E-01	.98348E+03
88900.	-.49528E+02	.23559E-01	.18284E-01	.98355E+03
89000.	-.49498E+02	.23447E-01	.18200E-01	.98362E+03
89100.	-.49468E+02	.23336E-01	.18116E-01	.98368E+03
89200.	-.49438E+02	.23225E-01	.18033E-01	.98375E+03
89300.	-.49407E+02	.23115E-01	.17949E-01	.98382E+03
89400.	-.49377E+02	.23006E-01	.17867E-01	.98388E+03
89500.	-.49347E+02	.22897E-01	.17785E-01	.98395E+03
89600.	-.49317E+02	.22789E-01	.17703E-01	.98402E+03
89700.	-.49287E+02	.22681E-01	.17621E-01	.98409E+03
89800.	-.49256E+02	.22573E-01	.17540E-01	.98415E+03
89900.	-.49226E+02	.22466E-01	.17460E-01	.98421E+03
90000.	-.49196E+02	.22360E-01	.17379E-01	.98428E+03

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1962 U.S. STANDARD ATMOSPHERE

PRESSURE ALT (FT)	TEMPERATURE (DEG.CENTIGRADE)	SIGMA (RHO/S.L.RHO)	DELTA (PRESS/S.L.PRESS)	SPEED OF SOUND (FT/SEC)
90100.	-.49165E+02	.22254E-01	.17299E-01	.98435E+03
90200.	-.49135E+02	.22149E-01	.17220E-01	.98441E+03
90300.	-.49105E+02	.22044E-01	.17141E-01	.98448E+03
90400.	-.49075E+02	.21940E-01	.17062E-01	.98455E+03
90500.	-.49045E+02	.21836E-01	.16983E-01	.98461E+03
90600.	-.49015E+02	.21733E-01	.16905E-01	.98468E+03
90700.	-.48984E+02	.21630E-01	.16828E-01	.98475E+03
90800.	-.48954E+02	.21528E-01	.16750E-01	.98481E+03
90900.	-.48924E+02	.21426E-01	.16673E-01	.98488E+03
91000.	-.48894E+02	.21325E-01	.16597E-01	.98494E+03
91100.	-.48863E+02	.21224E-01	.16521E-01	.98501E+03
91200.	-.48833E+02	.21124E-01	.16445E-01	.98508E+03
91300.	-.48803E+02	.21024E-01	.16369E-01	.98514E+03
91400.	-.48773E+02	.20924E-01	.16294E-01	.98521E+03
91500.	-.48743E+02	.20826E-01	.16219E-01	.98528E+03
91600.	-.48712E+02	.20727E-01	.16145E-01	.98534E+03
91700.	-.48682E+02	.20629E-01	.16071E-01	.98541E+03
91800.	-.48652E+02	.20532E-01	.15997E-01	.98548E+03
91900.	-.48622E+02	.20435E-01	.15924E-01	.98554E+03
92000.	-.48592E+02	.20339E-01	.15851E-01	.98561E+03
92100.	-.48561E+02	.20243E-01	.15779E-01	.98567E+03
92200.	-.48531E+02	.20147E-01	.15705E-01	.98574E+03
92300.	-.48501E+02	.20052E-01	.15634E-01	.98581E+03
92400.	-.48471E+02	.19958E-01	.15562E-01	.98587E+03
92500.	-.48440E+02	.19863E-01	.15491E-01	.98594E+03
92600.	-.48410E+02	.19770E-01	.15420E-01	.98601E+03
92700.	-.48380E+02	.19675E-01	.15349E-01	.98607E+03
92800.	-.48350E+02	.19584E-01	.15279E-01	.98614E+03
92900.	-.48320E+02	.19491E-01	.15209E-01	.98620E+03
93000.	-.48289E+02	.19400E-01	.15139E-01	.98627E+03
93100.	-.48259E+02	.19308E-01	.15070E-01	.98634E+03
93200.	-.48229E+02	.19217E-01	.15001E-01	.98640E+03
93300.	-.48199E+02	.19127E-01	.14932E-01	.98647E+03
93400.	-.48169E+02	.19037E-01	.14864E-01	.98654E+03
93500.	-.48138E+02	.18947E-01	.14795E-01	.98660E+03
93600.	-.48108E+02	.18858E-01	.14728E-01	.98667E+03
93700.	-.48078E+02	.18769E-01	.14661E-01	.98673E+03
93800.	-.48048E+02	.18680E-01	.14594E-01	.98680E+03
93900.	-.48018E+02	.18593E-01	.14527E-01	.98687E+03
94000.	-.47987E+02	.18505E-01	.14461E-01	.98693E+03
94100.	-.47957E+02	.18418E-01	.14395E-01	.98700E+03
94200.	-.47927E+02	.18331E-01	.14329E-01	.98707E+03
94300.	-.47897E+02	.18245E-01	.14263E-01	.98713E+03
94400.	-.47867E+02	.18159E-01	.14198E-01	.98720E+03
94500.	-.47836E+02	.18074E-01	.14133E-01	.98726E+03
94600.	-.47806E+02	.17989E-01	.14069E-01	.98733E+03
94700.	-.47776E+02	.17904E-01	.14004E-01	.98740E+03
94800.	-.47746E+02	.17820E-01	.13940E-01	.98746E+03
94900.	-.47716E+02	.17735E-01	.13877E-01	.98753E+03
95000.	-.47685E+02	.17653E-01	.13813E-01	.98759E+03

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1962 U.S. STANDARD ATMOSPHERE

PRESSURE ALT (FT)	TEMPERATURE (DEG.CENTIGRADE)	SIGMA (RHO/S.L.RHO)	DELTA (PRESS/S.L.PRESS)	SPEED OF SOUND (FT/SEC)
95100.	-.47655E+02	.17570E-01	.13750E-01	.98766E+03
95200.	-.47625E+02	.17487E-01	.13687E-01	.98773E+03
95300.	-.47593E+02	.17405E-01	.13625E-01	.98779E+03
95400.	-.47564E+02	.17324E-01	.13563E-01	.98786E+03
95500.	-.47534E+02	.17242E-01	.13501E-01	.98793E+03
95600.	-.47504E+02	.17161E-01	.13439E-01	.98799E+03
95700.	-.47474E+02	.17081E-01	.13378E-01	.98806E+03
95800.	-.47444E+02	.17001E-01	.13317E-01	.98812E+03
95900.	-.47413E+02	.16921E-01	.13256E-01	.98819E+03
96000.	-.47383E+02	.16841E-01	.13196E-01	.98826E+03
96100.	-.47353E+02	.16762E-01	.13136E-01	.98832E+03
96200.	-.47323E+02	.16684E-01	.13076E-01	.98839E+03
96300.	-.47293E+02	.16605E-01	.13016E-01	.98845E+03
96400.	-.47262E+02	.16528E-01	.12957E-01	.98852E+03
96500.	-.47232E+02	.16450E-01	.12898E-01	.98859E+03
96600.	-.47202E+02	.16373E-01	.12839E-01	.98865E+03
96700.	-.47172E+02	.16296E-01	.12781E-01	.98872E+03
96800.	-.47142E+02	.16220E-01	.12722E-01	.98878E+03
96900.	-.47111E+02	.16144E-01	.12664E-01	.98885E+03
97000.	-.47081E+02	.16068E-01	.12607E-01	.98892E+03
97100.	-.47051E+02	.15993E-01	.12549E-01	.98898E+03
97200.	-.47021E+02	.15918E-01	.12492E-01	.98905E+03
97300.	-.46991E+02	.15843E-01	.12435E-01	.98912E+03
97400.	-.46960E+02	.15769E-01	.12379E-01	.98918E+03
97500.	-.46930E+02	.15695E-01	.12322E-01	.98925E+03
97600.	-.46900E+02	.15622E-01	.12266E-01	.98931E+03
97700.	-.46870E+02	.15549E-01	.12211E-01	.98938E+03
97800.	-.46840E+02	.15476E-01	.12155E-01	.98945E+03
97900.	-.46810E+02	.15403E-01	.12100E-01	.98951E+03
98000.	-.46779E+02	.15331E-01	.12045E-01	.98958E+03
98100.	-.46749E+02	.15260E-01	.11990E-01	.98964E+03
98200.	-.46719E+02	.15189E-01	.11936E-01	.98971E+03
98300.	-.46689E+02	.15117E-01	.11881E-01	.98978E+03
98400.	-.46659E+02	.15046E-01	.11827E-01	.98984E+03
98500.	-.46629E+02	.14976E-01	.11774E-01	.98991E+03
98600.	-.46598E+02	.14906E-01	.11720E-01	.98997E+03
98700.	-.46568E+02	.14836E-01	.11667E-01	.99004E+03
98800.	-.46538E+02	.14767E-01	.11614E-01	.99010E+03
98900.	-.46508E+02	.14698E-01	.11561E-01	.99017E+03
99000.	-.46477E+02	.14629E-01	.11509E-01	.99024E+03
99100.	-.46447E+02	.14561E-01	.11456E-01	.99030E+03
99200.	-.46417E+02	.14493E-01	.11404E-01	.99037E+03
99300.	-.46387E+02	.14425E-01	.11353E-01	.99043E+03
99400.	-.46357E+02	.14358E-01	.11301E-01	.99050E+03
99500.	-.46326E+02	.14291E-01	.11250E-01	.99057E+03
99600.	-.46296E+02	.14224E-01	.11199E-01	.99063E+03
99700.	-.46266E+02	.14158E-01	.11148E-01	.99070E+03
99800.	-.46236E+02	.14092E-01	.11097E-01	.99076E+03
99900.	-.46205E+02	.14026E-01	.11047E-01	.99083E+03
100000.	-.46175E+02	.13960E-01	.10997E-01	.99090E+03

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1962 U.S. STANDARD ATMOSPHERE

PRESSURE ALT (FT)	TEMPERATURE (DEG.CENTIGRADE)	SIGMA (RHO/S.L.RHO)	DELTA (PRESS/S.L.PRESS)	SPEED OF SOUND (FT/SEC)
100100.	-.45145E+02	.13895E-01	.10947E-01	.99095E+03
100200.	-.45115E+02	.13830E-01	.10898E-01	.99103E+03
100300.	-.45085E+02	.13766E-01	.10848E-01	.99109E+03
100400.	-.45055E+02	.13702E-01	.10799E-01	.99116E+03
100500.	-.45025E+02	.13638E-01	.10750E-01	.99123E+03
100600.	-.45994E+02	.13574E-01	.10701E-01	.99129E+03
100700.	-.45964E+02	.13511E-01	.10653E-01	.99136E+03
100800.	-.45934E+02	.13449E-01	[illegible]	.99142E+03
100900.	-.45904E+02	.13386E-01	.10557E-01	.99149E+03
101000.	-.45874E+02	.13323E-01	.10509E-01	.99155E+03
101100.	-.45843E+02	.13261E-01	.10461E-01	.99162E+03
101200.	-.45813E+02	.13199E-01	.10414E-01	.99169E+03
101300.	-.45783E+02	.13138E-01	.10367E-01	.99175E+03
101400.	-.45753E+02	.13076E-01	.10320E-01	.99182E+03
101500.	-.45723E+02	.13015E-01	.10273E-01	.99188E+03
101600.	-.45692E+02	.12955E-01	.10227E-01	.99195E+03
101700.	-.45662E+02	.12895E-01	.10181E-01	.99202E+03
101800.	-.45632E+02	.12835E-01	.10134E-01	.99208E+03
101900.	-.45602E+02	.12775E-01	.10089E-01	.99215E+03
102000.	-.45572E+02	.12716E-01	.10043E-01	.99221E+03
102100.	-.45542E+02	.12656E-01	.99975E-02	.99228E+03
102200.	-.45511E+02	.12597E-01	.99525E-02	.99234E+03
102300.	-.45481E+02	.12539E-01	.99079E-02	.99241E+03
102400.	-.45451E+02	.12481E-01	.98627E-02	.99248E+03
102500.	-.45421E+02	.12423E-01	.98182E-02	.99254E+03
102600.	-.45391E+02	.12365E-01	.97738E-02	.99261E+03
102700.	-.45360E+02	.12307E-01	.97297E-02	.99267E+03
102800.	-.45330E+02	.12250E-01	.96857E-02	.99274E+03
102900.	-.45300E+02	.12193E-01	.96420E-02	.99280E+03
103000.	-.45270E+02	.12137E-01	.95985E-02	.99287E+03
103100.	-.45240E+02	.12080E-01	.95551E-02	.99294E+03
103200.	-.45210E+02	.12024E-01	.95120E-02	.99300E+03
103300.	-.45179E+02	.11968E-01	.94691E-02	.99307E+03
103400.	-.45149E+02	.11913E-01	.94264E-02	.99313E+03
103500.	-.45119E+02	.11857E-01	.93839E-02	.99320E+03
103600.	-.45089E+02	.11802E-01	.93415E-02	.99327E+03
103700.	-.45059E+02	.11747E-01	.92994E-02	.99333E+03
103800.	-.45028E+02	.11693E-01	.92574E-02	.99340E+03
103900.	-.44998E+02	.11639E-01	.92157E-02	.99346E+03
104000.	-.44968E+02	.11585E-01	.91742E-02	.99353E+03
104100.	-.44938E+02	.11531E-01	.91328E-02	.99359E+03
104200.	-.44908E+02	.11477E-01	.90916E-02	.99366E+03
104300.	-.44878E+02	.11424E-01	.90507E-02	.99373E+03
104400.	-.44847E+02	.11371E-01	.90099E-02	.99379E+03
104500.	-.44817E+02	.11319E-01	.89593E-02	.99386E+03
104600.	-.44787E+02	.11266E-01	.89289E-02	.99392E+03
104700.	-.44757E+02	.11214E-01	.88887E-02	.99399E+03
104800.	-.44727E+02	.11162E-01	.88487E-02	.99405E+03
104900.	-.44696E+02	.11110E-01	.88089E-02	.99412E+03
105000.	-.44666E+02	.11059E-01	.87532E-02	.99418E+03

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1962 U.S. STANDARD ATMOSPHERE

PRESSURE ALT (FT)	TEMPERATURE (DEG.CENTIGRADE)	SIGMA (RHO/S.L.RHO)	DELTA (PRESS/S.L.PRESS)	SPEED OF SOUND (FT/SEC)
105100.	-.44636E+02	.11007E-01	.87297E-02	.99425E+03
105200.	-.44605E+02	.10955E-01	.86904E-02	.99432E+03
105300.	-.44575E+02	.10905E-01	.86513E-02	.99438E+03
105400.	-.44546E+02	.10855E-01	.86124E-02	.99445E+03
105500.	-.44515E+02	.10805E-01	.85736E-02	.99451E+03
105600.	-.44441E+02	.10753E-01	.85350E-02	.99468E+03
105700.	-.44355E+02	.10701E-01	.84967E-02	.99486E+03
105800.	-.44272E+02	.10648E-01	.84585E-02	.99504E+03
105900.	-.44187E+02	.10597E-01	.84205E-02	.99523E+03
106000.	-.44103E+02	.10545E-01	.83827E-02	.99541E+03
106100.	-.44018E+02	.10494E-01	.83450E-02	.99559E+03
106200.	-.43934E+02	.10443E-01	.83076E-02	.99578E+03
106300.	-.43849E+02	.10392E-01	.82703E-02	.99596E+03
106400.	-.43765E+02	.10342E-01	.82332E-02	.99614E+03
106500.	-.43680E+02	.10292E-01	.81963E-02	.99633E+03
106600.	-.43595E+02	.10242E-01	.81596E-02	.99651E+03
106700.	-.43511E+02	.10192E-01	.81231E-02	.99669E+03
106800.	-.43427E+02	.10143E-01	.80867E-02	.99688E+03
106900.	-.43343E+02	.10094E-01	.80505E-02	.99706E+03
107000.	-.43258E+02	.10045E-01	.80145E-02	.99724E+03
107100.	-.43174E+02	.99964E-02	.79786E-02	.99743E+03
107200.	-.43089E+02	.99481E-02	.79430E-02	.99761E+03
107300.	-.43005E+02	.99000E-02	.79075E-02	.99779E+03
107400.	-.42920E+02	.98522E-02	.78721E-02	.99798E+03
107500.	-.42836E+02	.98046E-02	.78370E-02	.99816E+03
107600.	-.42751E+02	.97572E-02	.78020E-02	.99834E+03
107700.	-.42667E+02	.97101E-02	.77672E-02	.99853E+03
107800.	-.42582E+02	.96633E-02	.77325E-02	.99871E+03
107900.	-.42498E+02	.96167E-02	.76981E-02	.99889E+03
108000.	-.42413E+02	.95703E-02	.76638E-02	.99907E+03
108100.	-.42329E+02	.95242E-02	.76296E-02	.99926E+03
108200.	-.42244E+02	.94783E-02	.75956E-02	.99944E+03
108300.	-.42160E+02	.94326E-02	.75618E-02	.99962E+03
108400.	-.42075E+02	.93872E-02	.75282E-02	.99981E+03
108500.	-.41991E+02	.93420E-02	.74947E-02	.99999E+03
108600.	-.41907E+02	.92971E-02	.74613E-02	.10002E+04
108700.	-.41822E+02	.92524E-02	.74282E-02	.10004E+04
108800.	-.41738E+02	.92079E-02	.73952E-02	.10005E+04
108900.	-.41653E+02	.91637E-02	.73623E-02	.10007E+04
109000.	-.41569E+02	.91195E-02	.73296E-02	.10009E+04
109100.	-.41484E+02	.90759E-02	.72971E-02	.10011E+04
109200.	-.41400E+02	.90323E-02	.72647E-02	.10013E+04
109300.	-.41315E+02	.89889E-02	.72325E-02	.10014E+04
109400.	-.41231E+02	.89458E-02	.72004E-02	.10016E+04
109500.	-.41146E+02	.89029E-02	.71685E-02	.10018E+04
109600.	-.41062E+02	.88603E-02	.71367E-02	.10020E+04
109700.	-.40978E+02	.88179E-02	.71051E-02	.10022E+04
109800.	-.40893E+02	.87756E-02	.70737E-02	.10024E+04
109900.	-.40809E+02	.87335E-02	.70424E-02	.10025E+04
110000.	-.40724E+02	.86918E-02	.70112E-02	.10027E+04

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1962 U.S. STANDARD ATMOSPHERE

PRESSURE ALT (FT)	TEMPERATURE (DEG.CENTIGRADE)	SIGMA (RHO/S.L.RHO)	DELTA (PRESS/S.L.PRESS)	SPEED OF SOUND (FT/SEC)
110100.	-.40640E+02	.86502E-02	.69802E-02	.10029E+04
110200.	-.40555E+02	.86088E-02	.69493E-02	.10031E+04
110300.	-.40471E+02	.85677E-02	.69186E-02	.10033E+04
110400.	-.40386E+02	.85267E-02	.68881E-02	.10035E+04
110500.	-.40302E+02	.84860E-02	.68577E-02	.10036E+04
110600.	-.40217E+02	.84455E-02	.68274E-02	.10038E+04
110700.	-.40133E+02	.84052E-02	.67973E-02	.10040E+04
110800.	-.40049E+02	.83651E-02	.67673E-02	.10042E+04
110900.	-.39964E+02	.83252E-02	.67374E-02	.10044E+04
111000.	-.39880E+02	.82855E-02	.67077E-02	.10045E+04
111100.	-.39795E+02	.82460E-02	.66782E-02	.10047E+04
111200.	-.39711E+02	.82067E-02	.66488E-02	.10049E+04
111300.	-.39626E+02	.81675E-02	.66195E-02	.10051E+04
111400.	-.39542E+02	.81287E-02	.65904E-02	.10053E+04
111500.	-.39457E+02	.80900E-02	.65614E-02	.10055E+04
111600.	-.39373E+02	.80515E-02	.65325E-02	.10056E+04
111700.	-.39289E+02	.80132E-02	.65038E-02	.10058E+04
111800.	-.39204E+02	.79751E-02	.64752E-02	.10060E+04
111900.	-.39120E+02	.79373E-02	.64468E-02	.10062E+04
112000.	-.39035E+02	.78995E-02	.64185E-02	.10064E+04
112100.	-.38951E+02	.78620E-02	.63903E-02	.10065E+04
112200.	-.38866E+02	.78247E-02	.63622E-02	.10067E+04
112300.	-.38782E+02	.77875E-02	.63343E-02	.10069E+04
112400.	-.38698E+02	.77505E-02	.63066E-02	.10071E+04
112500.	-.38613E+02	.77139E-02	.62789E-02	.10073E+04
112600.	-.38529E+02	.76773E-02	.62514E-02	.10074E+04
112700.	-.38444E+02	.76410E-02	.62240E-02	.10076E+04
112800.	-.38360E+02	.76048E-02	.61968E-02	.10078E+04
112900.	-.38275E+02	.75688E-02	.61696E-02	.10080E+04
113000.	-.38191E+02	.75329E-02	.61427E-02	.10082E+04
113100.	-.38107E+02	.74973E-02	.61158E-02	.10084E+04
113200.	-.38022E+02	.74619E-02	.60891E-02	.10085E+04
113300.	-.37938E+02	.74266E-02	.60624E-02	.10087E+04
113400.	-.37853E+02	.73915E-02	.60360E-02	.10089E+04
113500.	-.37769E+02	.73565E-02	.60095E-02	.10091E+04
113600.	-.37684E+02	.73218E-02	.59834E-02	.10093E+04
113700.	-.37600E+02	.72872E-02	.59572E-02	.10094E+04
113800.	-.37516E+02	.72529E-02	.59313E-02	.10096E+04
113900.	-.37431E+02	.72186E-02	.59054E-02	.10098E+04
114000.	-.37347E+02	.71845E-02	.58797E-02	.10100E+04
114100.	-.37262E+02	.71507E-02	.58540E-02	.10102E+04
114200.	-.37178E+02	.71170E-02	.58285E-02	.10103E+04
114300.	-.37094E+02	.70835E-02	.58032E-02	.10105E+04
114400.	-.37009E+02	.70502E-02	.57779E-02	.10107E+04
114500.	-.36925E+02	.70170E-02	.57528E-02	.10109E+04
114600.	-.36840E+02	.69840E-02	.57277E-02	.10111E+04
114700.	-.36756E+02	.69511E-02	.57028E-02	.10112E+04
114800.	-.36671E+02	.69184E-02	.56780E-02	.10114E+04
114900.	-.36587E+02	.68858E-02	.56534E-02	.10116E+04
115000.	-.36503E+02	.68535E-02	.56289E-02	.10118E+04

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1962 U.S. STANDARD ATMOSPHERE

PRESSURE ALT (FT)	TEMPERATURE (DEG.CENTIGRADE)	SIGMA (RHO/S.L.RHO)	DELTA (PRESS/S.L.PRESS)	SPEED OF SOUND (FT/SEC)
115100.	-.36418E+02	.68214E-02	.56044E-02	.10120E+04
115200.	-.36334E+02	.67893E-02	.55801E-02	.10121E+04
115300.	-.36249E+02	.67575E-02	.55558E-02	.10123E+04
115400.	-.36165E+02	.67259E-02	.55318E-02	.10125E+04
115500.	-.36081E+02	.66942E-02	.55078E-02	.10127E+04
115600.	-.35996E+02	.66628E-02	.54839E-02	.10129E+04
115700.	-.35912E+02	.66315E-02	.54601E-02	.10131E+04
115800.	-.35827E+02	.66005E-02	.54365E-02	.10132E+04
115900.	-.35743E+02	.65695E-02	.54130E-02	.10134E+04
116000.	-.35659E+02	.65389E-02	.53895E-02	.10136E+04
116100.	-.35574E+02	.65083E-02	.53662E-02	.10138E+04
116200.	-.35490E+02	.64778E-02	.53430E-02	.10140E+04
116300.	-.35405E+02	.64475E-02	.53199E-02	.10141E+04
116400.	-.35321E+02	.64174E-02	.52969E-02	.10143E+04
116500.	-.35237E+02	.63874E-02	.52741E-02	.10145E+04
116600.	-.35152E+02	.63575E-02	.52513E-02	.10147E+04
116700.	-.35068E+02	.63279E-02	.52285E-02	.10149E+04
116800.	-.34983E+02	.62984E-02	.52061E-02	.10150E+04
116900.	-.34899E+02	.62690E-02	.51836E-02	.10152E+04
117000.	-.34815E+02	.62397E-02	.51612E-02	.10154E+04
117100.	-.34730E+02	.62105E-02	.51390E-02	.10156E+04
117200.	-.34646E+02	.61817E-02	.51169E-02	.10158E+04
117300.	-.34561E+02	.61529E-02	.50948E-02	.10159E+04
117400.	-.34477E+02	.61242E-02	.50729E-02	.10161E+04
117500.	-.34393E+02	.60957E-02	.50511E-02	.10163E+04
117600.	-.34308E+02	.60674E-02	.50293E-02	.10165E+04
117700.	-.34224E+02	.60391E-02	.50077E-02	.10166E+04
117800.	-.34139E+02	.60110E-02	.49862E-02	.10168E+04
117900.	-.34055E+02	.59831E-02	.49647E-02	.10170E+04
118000.	-.33971E+02	.59553E-02	.49434E-02	.10172E+04
118100.	-.33886E+02	.59275E-02	.49222E-02	.10174E+04
118200.	-.33802E+02	.59001E-02	.49010E-02	.10175E+04
118300.	-.33718E+02	.58727E-02	.48800E-02	.10177E+04
118400.	-.33633E+02	.58455E-02	.48591E-02	.10179E+04
118500.	-.33549E+02	.58183E-02	.48382E-02	.10181E+04
118600.	-.33464E+02	.57914E-02	.48175E-02	.10183E+04
118700.	-.33380E+02	.57645E-02	.47969E-02	.10184E+04
118800.	-.33296E+02	.57378E-02	.47763E-02	.10186E+04
118900.	-.33211E+02	.57112E-02	.47559E-02	.10188E+04
119000.	-.33127E+02	.56848E-02	.47355E-02	.10190E+04
119100.	-.33042E+02	.56585E-02	.47152E-02	.10192E+04
119200.	-.32958E+02	.56323E-02	.46951E-02	.10193E+04
119300.	-.32874E+02	.56062E-02	.46750E-02	.10195E+04
119400.	-.32789E+02	.55803E-02	.46550E-02	.10197E+04
119500.	-.32705E+02	.55545E-02	.46351E-02	.10199E+04
119600.	-.32621E+02	.55288E-02	.46153E-02	.10201E+04
119700.	-.32536E+02	.55033E-02	.45956E-02	.10202E+04
119800.	-.32452E+02	.54779E-02	.45760E-02	.10204E+04
119900.	-.32367E+02	.54525E-02	.45565E-02	.10206E+04
120000.	-.32283E+02	.54274E-02	.45370E-02	.10208E+04

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