

NATIONAL BUREAU OF STANDARDS
MICROCOPY RESOLUTION TEST CHART

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO. AD-A156	3. RECIPIENT'S CATALOG NUMBER 463
4. TITLE (and Subtitle) Use of the Wavenumber Technique With the Lloyds Mirror For an Acoustic Doublet		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis March 1985
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Portia Baird King		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93943		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93943		12. REPORT DATE March 1985
		13. NUMBER OF PAGES 112
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report)
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for Public Release; Distribution is Unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Lloyds Mirror; Underwater Acoustics; Fast Fourier Transform; Wavenumber Technique; Pressure Spectrum; Source Depth Determination; Acoustic Propagation; ASW, Isospeed Environment.		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This thesis examines a method to determine the depth of a point source in an isospeed ocean environment. Using the Fourier Transform on the acoustic pressure field in the range domain results in the attainment of the acoustic pressure spectrum in the wavenumber domain and a characteristic nodal spacing unique to the source-receiver depths. Quantitative examination of a magnitude plot of the spectrum and use of simple mathematical formulae yield the source depth. The debilitating effects of narrowband		

DD FORM 1 JAN 73 1473

EDITION OF 1 NOV 65 IS OBSOLETE
S N 0102-LF-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

noise and surface roughness on the pressure spectrum are also examined. The pressure spectrum is recognizable in noise after the pressure field in the range domain has been lost in the noise field. The effect of surface gravity waves on the pressure spectrum is similar to that on the pressure field in the range domain: the characteristic nodal spacing is suppressed as the height of the surface waves increases.

W. J. ...

S-N 0102-LF-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Approved for public release; distribution is unlimited.

**Use of the Wavenumber Technique
With the Lloyds Mirror
For an Acoustic Doublet**

by

Portia B. King
Lieutenant Commander, United States Navy
B.S., Louisiana State University in New Orleans, 1971

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN SYSTEMS TECHNOLOGY
(Antisubmarine Warfare)

from the

NAVAL POSTGRADUATE SCHOOL
March 1985

Author:

Portia B. King
Portia B. King

Approved by:

A. B. Coppens
A. B. Coppens, Thesis Advisor

C. R. Dunlap
C. R. Dunlap, Thesis Advisor

R. N. Forrest
R. N. Forrest, Chairman,
Antisubmarine Warfare Academic Group

David A. Schrad
David A. Schrad,
Academic Dean

ABSTRACT

This thesis examines a method to determine the depth of a point source in an isospeed ocean environment. Using the Fourier Transform on the acoustic pressure field in the range domain results in the attainment of the acoustic pressure spectrum in the wavenumber domain and a characteristic nodal spacing unique to the source-receiver depths. Quantitative examination of a magnitude plot of the spectrum and use of simple mathematical formulae yield the source depth. The debilitating effects of narrowband noise and surface roughness on the pressure spectrum are also examined. The pressure spectrum is recognizable in noise after the pressure field in the range domain has been lost in the noise field. The effect of surface gravity waves on the pressure spectrum is similar to that on the pressure field in the range domain: the characteristic nodal spacing is suppressed as the height of the surface waves increases.

TABLE OF CONTENTS

I.	HISTORY AND INTRODUCTION	14
II.	THEORY	18
	A. THE LLOYDS MIRROR PHENOMENON	18
	B. THE RELATIONSHIP BETWEEN k , γ , AND β	20
	C. THE WAVENUMBER TECHNIQUE AND THE LLOYDS MIRROR	20
	D. THE EFFECTS OF SURFACE ROUGHNESS	27
	E. THE EFFECT OF ADDING NOISE	28
III.	RESULTS AND CONCLUSIONS.	33
	A. THE FFT ALGORITHM	33
	B. SOURCE DEPTH DETERMINATION	38
	C. THE EFFECT OF SURFACE ROUGHNESS	42
	D. THE EFFECT OF NOISE	43
	E. SUMMATION	44
	APPENDIX A: LLOYDS MIRROR PRESSURE FIELD SOURCE CODE . .	93
	LIST OF REFERENCES	109
	BIBLIOGRAPHY	110
	INITIAL DISTRIBUTION LIST	111

LIST OF TABLES

I Critical Values Used in the Research 34
II Results of Source Depth Determination Runs 39

LIST OF FIGURES

2.1	The Geometry of the Lloyds Mirror Effect	30
2.2	A Classic $ P(R) $ vs. R Curve	31
2.3	The Relationship Between K, γ and β	32
3.1	Pressure Spectrum Using FFT2C	46
3.2	Pressure Spectrum Using FFTCC	47
3.3	Pressure Spectrum Using Cooley-Tukey FFT	48
3.4	Non-smoothed Pressure Field	49
3.5	Pressure Spectrum Showing Gibbs Phenomenon	50
3.6	Pressure Field Combined With a Hanning Window	51
3.7	Pressure Spectrum Combined With a Hanning Window	52
3.8	Theoretical Source Depth Determination Curve	53
3.9	Graph of Pressure Spectrum, Source at 22.0 Meters	54
3.10	Graph of Pressure Spectrum, Source at 31.4 Meters	55
3.11	Graph of Pressure Spectrum, Source at 15.7 Meters	56
3.12	Graph of Pressure Spectrum, Receiver at 22.0 Meters	57
3.13	Graph of Pressure Spectrum, Receiver at 31.4 Meters	58
3.14	Graph of Pressure Spectrum, Receiver at 15.7 Meters	59
3.15	Pressure Spectrum, Range Window Set at 47.1 Meters	60
3.16	Pressure Spectrum, Range Window Set at 50.3 Meters	61

3.17	Pressure Spectrum, Range Window Set at 62.8 Meters	62
3.18	Pressure Spectrum, Sea State 0	63
3.19	Pressure Spectrum, Sea State 2	64
3.20	Pressure Spectrum, Sea State 3	65
3.21	Pressure Spectrum, Sea State 5	66
3.22	Pressure Spectrum, Sea State 0, $K = 1.0$	67
3.23	Pressure Spectrum, Sea State 3, $K = 1.0$	68
3.24	Pressure Spectrum, Sea State 0, $K = 2.0$	69
3.25	Pressure Spectrum, Sea State 3, $K = 2.0$	70
3.26	Pressure Spectrum, SS 2, Range Window Set at 50 Meters	71
3.27	Pressure Spectrum, SS 2, Range Window Set at 100 Meters	72
3.28	Pressure Spectrum, SS 2, Range Window Set at 200 Meters	73
3.29	Pressure Spectrum, SS 2, Range Window Set at 300 Meters	74
3.30	Pressure Field, $K = 1.0$, $\mu = 0.005$	75
3.31	Pressure Spectrum vs. Gamma	76
3.32	Pressure Spectrum vs. Beta	77
3.33	Pressure Field, $K = 1.0$, $\mu = 0.01$	78
3.34	Pressure Spectrum vs. Gamma	79
3.35	Pressure Spectrum vs. Beta	80
3.36	Pressure Field, $K = 2.0$, $\mu = 0.0001$	81
3.37	Pressure Spectrum vs. Gamma	82
3.38	Pressure Spectrum vs. Beta	83
3.39	Pressure Field, $K = 2.0$, $\mu = 0.001$	84
3.40	Pressure Spectrum vs. Gamma	85
3.41	Pressure Spectrum vs. Beta	86
3.42	Pressure Field, $K = 2.0$, $\mu = 0.005$	87
3.43	Pressure Spectrum vs. Gamma	88
3.44	Pressure Spectrum vs. Beta	89

3.45	Pressure Field, $K = 2.0$, $\mu = 0.01$	90
3.46	Pressure Spectrum vs. Gamma	91
3.47	Pressure Spectrum vs. Beta	92

LIST OF SYMBOLS

A	Amplitude
c	Sound Speed
e	2.718281828...
FFT	Fast Fourier Transform
FFT ⁻¹	Inverse Fast Fourier Transform
f	Frequency
$\underline{f}(r)$	Generic Function of Range
$\underline{g}(\gamma)$	Spectral Density of $\underline{f}(r)$
H ₀	Hankel Function
i	Square Root of -1
J ₀	Bessel Function of the First Kind
k	Wavenumber
M	Surface Roughness Factor
m	Index of Calculations or Null Number in the Range Domain
N	Number of Points in the Wavenumber Spectrum

n	Index of Calculations or Null Number in the Wavenumber Spectrum
\tilde{p}	Complex Narrowband Noise Field
\tilde{p}	Time Independent Factor of Complex Pressure
\tilde{p}_s	Complex Pressure of the Source
\tilde{p}_i	Complex Pressure of the Reflection
\tilde{p}_N	Complex Pressure in the Presence of Noise
$\tilde{p}(k)$	Pressure Field in the Wavenumber Spectrum
p	Time Dependent Factor of Complex Pressure
R_1	Distance of the Direct Wave Path
R_2	Distance to the Image (Reflected Path)
RCVR	Receiver
r	Range Between Source and Receiver
SRC	Source
t	time factor
W	Hanning Window
Y_0	Bessel Function of the Second Kind
Z_s	Source Depth

z_r	Receiver Depth
Δr	Range Increment
β	Vertical Component of the Wavenumber
$\Delta\beta$	Vertical Wavenumber Increment
γ	Horizontal Component of the Wavenumber
$\Delta\gamma$	Horizontal Wavenumber Increment
θ	Angle of Grazing Incidence at the Surface
λ	Wavelength
π	3.14159265...
φ	Phase Angle
ψ_R	Surface Reflection Coefficient
ω	Angular Frequency
$\sqrt{\quad}$	Square Root Operator
\int	Integration Operator

ACKNOWLEDGEMENTS

Contrary to popular opinion, this part of the thesis was the hardest to write because, for several reasons, it is the most important part of this entire undertaking. I encountered many difficulties during the course of this research; solutions to all the myriad problems were found eventually. Let me express heartfelt gratitude for the help and guidance of three experts in the field of acoustics: Dr. A. E. Coppens, Dr. T. Gabrielson, and Dr. Suk Wang Yoon, for the support of Professor C. R. Dunlap, for the help of Dr. DeWayne White of N.O.R.D.A in understanding the FFT, and the relatively minor but ever-so-important helpful hints and good wishes of my peers and fellow officers in the class of IX-33. Most of the mathematics and much of the direction for the research came from Professors Coppens and Gabrielson. Indeed, this thesis would not have been written without Professor Coppens' patience and leadership. Each person played a role, whether major or minor, in getting this thesis done. Without them, this paper would remain just another unfulfilled wish.

And, last but not least, I wish to thank a very special person for his support, cajolery, laughter, faith and other expressions of his love for me - my husband, Dr. David Benton King. Whenever I look back at what went into the writing of this paper, I shall always remember that I met him for the first time at the start of all the research that has gone into it. I shall also remember that, if I had never come to the Naval Postgraduate School, I would never have met him. And for that, I shall always be grateful to the United States Navy.

I. HISTORY AND INTRODUCTION

This thesis is the third in a series of investigations into a proposal put forth by R. Lauer of Naval Underwater Systems Center, New London, CT, in 1979. In his memorandum [Ref. 1], Lauer describes a "new" way to analyze sound propagation in the ocean and two applications of the method, which he named the "Wavenumber Technique," or W.T. In his proposal, Lauer stated that a source of sound in the ocean could be pinpointed in both depth and range by a single omnidirectional hydrophone provided the source is generating a continuous wave tone.

The W.T. was initially described by F. DiNapoli [Ref. 2] as an intermediary step in his development of a speedier computer algorithm used in analyzing sound propagation as a function of range. DiNapoli and Lauer proposed converting the pressure as a function of range $p(r)$ to the pressure spectrum as a function of the wavenumber $p(k)$. The conversion is accomplished by taking a weighted Fourier transform of the pressure field $p(r)$. Once the pressure spectrum is obtained, analysis of the pressure field can be accomplished in a manner analogous to that presently used in signal processing.

Lauer proposed two uses for his "Wavenumber Technique:"

1. determination of the depth of the acoustic source, an ability which has obvious tactical applications;
2. use of the W.T. to evaluate the accuracy of existing, and future, acoustic models such as P.E. and F.A.C.T., by breaking the generated pressure field down into ray path "families," such as bottom-bounce, refracted-surface-reflected, and surface-ducted families, thus giving a quantitative read-out of what

proportion of the total acoustic energy is being channelled through the various ray paths.

For both uses, a knowledge of the acoustic environment is necessary.

It is the intended purpose of this thesis to investigate the validity of the first application of Lauer's wavenumber technique by using it in conjunction with an acoustic phenomenon that is well-known and for which accurate results can be calculated with precision; this phenomenon is the LLOYDs Mirror for an Acoustic Doublet. Using several source/receiver combinations, this investigator intends to compare actual source depth with that predicted by the W.T. If Lauer's technique is cogent, the source depths calculated via the two methods should be equal, or very close within a statistically acceptable degree.

As mentioned in the beginning of this thesis, two prior investigators, B. Stamey and J. Blanchard, looked into applying the W.T. to determining source depth. Stamey's investigation [Ref. 3] utilized a parabolic equation computer model developed by H.K. Brock, the Split Step Fast Fourier Transform, or SSFFT, to generate the pressure fields. He qualitatively compared the model to a Normal Mode model and a P. E. Finite Difference model, both range dependent, and to DiNapoli's Fast Field Program model, a range independent model. Stamey concluded that the W.T. showed promise as an analysis tool and that further investigations were warranted.

J. Blanchard carried Stamey's researches one step further [Ref. 4]. Using two Parabolic Equation computer algorithms, Brock's Split Step Fast Fourier Transform and Jaeger's Implicit Finite Difference, as his pressure field generators, he examined the use of the spacing between nulls of the pressure spectrum to determine source depth. His results are interesting and support the need for further investigation.

The author was not satisfied with Stamey's and Blanchard's findings. Both individuals acknowledged shortcomings in their respective studies, especially with regard to the mutual presence of the "U-shaped phenomenon" encountered in the pressure spectrum. One should remember, however, that their theses were preliminary studies only and were produced within a highly restricted time frame. Also, they used existing computer algorithms specifically designed to approximate acoustic propagation in a velocity-variant medium. However, the environmental settings used by both gentlemen were that of the Lloyds Mirror for an Acoustic Doublet which requires a constant sound speed throughout the water column; thus, the approximations, assumptions and "fudge factors" used by these models make their results for an isospeed medium highly artificial and somewhat suspect. Fortunately, however, there exists for the Lloyds Mirror a simple, geometric solution specifically designed for an isospeed environment [Ref. 5]. Since the pressure fields for varying source and receiver depths and, therefore, the corresponding pressure spectra, can be precisely calculated, it was thought this model would be a good check on the operational applicability of Lauer's proposed technique.

Secondary considerations of this study were to examine what effect, if any, the introductions of, firstly, surface waves and, secondly, noise would have on the pressure spectrum. The first objective was simulated by use of mathematical formulae given in reference 5; for the second objective, it was thought that complex noise, random in both amplitude and phase and similar to a Rician distribution [Ref. 6: p. 189], would provide a simple but reasonable approximation of an ocean noise field. This investigator intends to gradually intensify the surface waves and the noise field, separately, until the original pressure spectrum is no longer recognizable. In this manner, it should

be possible to make a qualitative assessment of their respective debilitating effects under the carefully controlled conditions found in the Lloyds Mirror phenomenon.

Section II sets forth in more detail the theoretical development of each point described in this introduction. Section III summarizes the investigator's results and the conclusions drawn from those results. Section IV contains a listing of the computer algorithms utilized.

II. THEORY

This chapter presents the mathematical basis of the wavenumber technique.

A. THE LLOYDS MIRROR PHENOMENON

A diverging monofrequency spherical pressure wave [Ref. 5: p.112] can be written in complex form as

$$p(R,t) = \underline{p}(R) e^{-i\omega t} = \frac{A}{R} e^{i(kR - \omega t)} \quad (\text{eqn 2.1})$$

where $\underline{p}(R)$ is the spatial factor. Referring to Figure 2.1, R_1 is the range from the source to the receiver and R_2 is the range from the image to the receiver. For convenience in all that follows, we shall set A to unit magnitude. The following equations apply to the acoustic waves propagating directly out from the source \underline{p}_s and from the apparent image \underline{p}_i (which is actually the wave reflected from the surface):

$$\begin{aligned} \underline{p}_s(R,t) &= \underline{p}_s(R) e^{-i\omega t} = \frac{1}{R_1} e^{i(kR_1 - \omega t)} \\ \underline{p}_i(R,t) &= \underline{p}_i(R) e^{-i\omega t} = \frac{-1}{R_2} e^{i(kR_2 - \omega t)} \end{aligned} \quad (\text{eqn 2.2})$$

where \underline{p}_s , \underline{p}_s , and \underline{p}_i are complex functions of horizontal range r , vertical depth Z and time t ; the minus sign in the equation for \underline{p}_i is derived from the surface reflection coefficient, $\psi_R = -1$, for a smooth surface. Inspection of Figure 2.1 reveals that

$$R_1 = \sqrt{(z_r - z_s)^2 + r^2}, \quad R_2 = \sqrt{(z_r + z_s)^2 + r^2},$$

$$\text{and } \Delta r = |R - R_{1,2}|$$

And so the total field can be written as

$$p_i - p_o = \underline{P}(R) e^{-i\omega t} = \frac{e^{i(kR_1 - \omega t)}}{R_1} - \frac{e^{i(kR_2 - \omega t)}}{R_2} \quad (\text{eqn 2.3})$$

Since \underline{p}_s and \underline{p}_i both have the same time factor, $\exp(-i\omega t)$, we can retain just the spatial factors and equations 2.3 reduce to

$$\underline{P}(R) = \left[\frac{e^{ikR_1}}{R_1} - \frac{e^{ikR_2}}{R_2} \right] \quad (\text{eqn 2.4})$$

Equation 2.4 is a form of the complex pressure as a function of range used in the computer algorithm shown in Appendix I.

Inspection of Figure 2.1 reveals that, for $R \gg Z_s$ and θ very small,

$$r \approx R$$

or

$$\sin \theta \approx \frac{Z_r}{R}$$

or

$$\Delta r \approx \frac{Z_r Z_s}{R} \quad (\text{eqn 2.5})$$

and the pressure amplitude can be approximated by

$$\underline{P}(R) \approx \frac{2}{R} \left| \sin \left(\frac{k Z_r Z_s}{R} \right) \right| \quad (\text{eqn 2.6})$$

Looking at just the formula for the pressure amplitude, we can see that as

$$\left(\frac{kz z_s}{R}\right) \rightarrow n\pi, \quad n=0,1,2,3,\dots \quad (\text{eqn 2.7})$$

the pressure amplitude goes to zero, producing the classic $|P(R)|$ vs R curve shown in Figure 2.2

B. THE RELATIONSHIP BETWEEN k , γ , AND β

In his description of the Wavenumber Technique [Ref. 1: p. 5-6], Lauer utilizes the horizontal and vertical components of the wavenumber, k . The general relationship among these three terms is illustrated in Figure 2.3 and can be written mathematically as

$$k = \text{the wavenumber} = 2\pi f/c$$

$$\gamma = \text{the horizontal component of } k = k \cos \varphi$$

$$\beta = \text{the vertical component of } k = k \sin \varphi$$

or

$$k = \sqrt{\gamma^2 + \beta^2} \quad (\text{eqn 2.8})$$

C. THE WAVENUMBER TECHNIQUE AND THE LLOYDS MIRROR

Given a point source in free space, the monofrequency pressure field at the receiver, $p(R)$, can be expressed as a spherical wave (time factor suppressed),

$$P(R) = \frac{e^{ikR}}{R} \quad (\text{eqn 2.9})$$

where, from Figure 2.1,

$$R^2 = r^2 + z_r^2, \text{ for } z_r > z_s$$

or

$$R^2 = r^2 + z_s^2, \text{ for } z_r < z_s \quad (\text{eqn 2.10})$$

In integral form, $P(R)$ can be written as [Ref. 7: p. 127]

$$P(R) = \frac{e^{ikR}}{R} = \int_0^\infty \frac{J_0(\gamma r) e^{\pm i\beta|z_r - z_s|}}{i\beta} \gamma d\gamma \quad (\text{eqn 2.11})$$

which is taken from the Fourier-Bessel Transform Pair [Ref. 7: p. 126],

$$f(r) = \int_0^\infty q(\gamma) J_0(\gamma r) \gamma d\gamma$$

$$q(\gamma) = \int_0^\infty f(r) J_0(\gamma r) r dr \quad (\text{eqn 2.12})$$

where $f(r)$ represents the acoustic pressure function in the range domain, and $q(\gamma)$ is the acoustic pressure spectrum in the wavenumber domain. The sign of the exponential function in the integral is based on which of z_s and z_r is greater. For the case of receiver depth being greater than source depth so that waves from both image and source are traveling downward at the receiver depth, the total pressure at the receiver can be expressed as

$$P(R) = \int_0^\infty \frac{e^{i\beta(z_r - z_s)} - e^{i\beta(z_r + z_s)}}{i\beta} J_0(\gamma r) \gamma d\gamma \quad (\text{eqn 2.13})$$

Use of Euler's Identity reduces Equation 2.13 to

$$L(R) = -2 \int_0^{\infty} \frac{e^{i\beta z r} \sin(\beta z r)}{\beta} J_0(\gamma r) \gamma dr \quad (\text{eqn 2.14})$$

Using the relationship between the Hankel functions [Ref. 5: p. 449],

$$H_0^{(1)}(\gamma r) = J_0(\gamma r) + iY_0(\gamma r)$$

and

$$H_0^{(2)}(\gamma r) = J_0(\gamma r) - iY_0(\gamma r) \quad (\text{eqn 2.15})$$

one can re-write the Bessel function in equation 2.14 as

$$J_0(\gamma r) = \frac{1}{2} [H_0^{(1)}(\gamma r) + H_0^{(2)}(\gamma r)] \quad (\text{eqn 2.16})$$

so that

$$2q(\gamma) = \int_0^{\infty} \underline{f}(r) H_0^{(1)}(\gamma r) r dr + \int_0^{\infty} \underline{f}(r) H_0^{(2)}(\gamma r) r dr \quad (\text{eqn 2.17})$$

Letting $r' = -r$ and looking at the second term on the right hand side of equation 2.17,

$$\int_0^{\infty} \underline{f}(-r') H_0^{(2)}(-\gamma r') r' dr' = \int_{-\infty}^0 \underline{f}(-r) H_0^{(1)}(\gamma r) r dr \quad (\text{eqn 2.18})$$

Now, assuming that $\underline{f}(r) = \underline{f}(-r)$, then

$$2q(\gamma) = \int_{-\infty}^{\infty} \underline{f}(r) H_0^{(1)}(\gamma r) r dr \quad (\text{eqn 2.19})$$

Therefore,

$$P(R) = - \int_{-\infty}^{\infty} \frac{\gamma H_0^{(1)}(\gamma r)}{\beta} e^{i\beta z_r} \sin(\beta z_s) d\gamma \quad (\text{eqn 2.20})$$

For $\gamma r > 2\pi$, the asymptotic approximation of the Hankel function for large argument can be used:

$$H_0^{(1)}(\gamma r) \approx \sqrt{\frac{2}{\pi \gamma r}} e^{i(\gamma r - \frac{\pi}{4})} \quad (\text{eqn 2.21})$$

assuming $|r \underline{f}(r)|$ goes to zero faster than $(\ln r)$,

$$P(R) \approx - \sqrt{\frac{2}{\pi r}} e^{-i\frac{\pi}{4}} \int_{-\infty}^{\infty} \left[\frac{\sqrt{\gamma}}{\beta} e^{i\beta z_r} \sin(\beta z_s) \right] e^{i\gamma r} d\gamma \quad (\text{eqn 2.22})$$

Let the terms inside the brackets in equation 2.22 be defined as $g(\gamma)$, then

$$P(R) = - \sqrt{\frac{2}{\pi r}} e^{-i\frac{\pi}{4}} \int_{-\infty}^{\infty} g(\gamma) e^{i\gamma r} d\gamma$$

and

$$\int_{-\infty}^{\infty} g(\gamma) e^{i\gamma r} d\gamma = - \sqrt{\frac{\pi r}{2}} \sqrt{r} P(R) \quad (\text{eqn 2.23})$$

or, using equation 2.12,

$$\xi(\gamma) = - \left[e^{i\frac{\pi}{4}} \right] \sqrt{\frac{\pi}{2}} \frac{1}{2\pi} \int_{-\infty}^{\infty} \sqrt{r} P(R) e^{-i\gamma r} dr \quad (\text{eqn 2.24})$$

Note that equation 2.24 has the form of the Fourier transform of the pressure function if we define

$$\underline{f}(r) \equiv \sqrt{r} P(R) \quad (\text{eqn 2.25})$$

so that $\sqrt{r} p(r)$ and $g(\gamma)$ are Fourier Transform pairs.

This equation can be easily evaluated [Ref. 8],

$$g(\gamma) = - \left[e^{i \frac{\pi}{4}} \right] \frac{1}{\sqrt{2\pi}} \left[\frac{\sqrt{\gamma}}{\beta} e^{i\beta z_r} \sin(\beta z_s) \right] \quad (\text{eqn 2.26})$$

for $z_r > z_s$. And the magnitude of $g(\gamma)$ is

$$|g(\gamma)| = \frac{\sqrt{\gamma}}{\beta} \sin(\beta z_s) \quad (\text{eqn 2.27})$$

For the case where $z_s > z_r$, it can be shown that

$$g(\gamma) = - \frac{e^{i \frac{\pi}{4}}}{\sqrt{2\pi}} \left[\frac{\sqrt{\gamma}}{\beta} e^{i\beta z_s} \sin(\beta z_r) \right] \quad (\text{eqn 2.28})$$

and the magnitude of $g(\gamma)$ is

$$|g(\gamma)| = \frac{\sqrt{\gamma}}{\beta} \sin(\beta z_r) \quad (\text{eqn 2.29})$$

A close look at the magnitude of the pressure spectrum function in equation 2.27 reveals that nodes occur for values of

$$\beta z_s = n\pi, \quad n = 0, 1, 2, 3, \dots \quad (\text{eqn 2.30})$$

If the magnitude of the pressure spectrum is plotted as a function of the vertical wavenumber then the spacing between nodes $\Delta\beta$ is uniform and the depth of the source can be derived from the relationship given by

$$z_s = \frac{\pi}{\Delta\beta} \quad (\text{eqn 2.31})$$

In the event the source is deeper than the receiver, looking at equation 2.29, we can see that nodes now occur for values of

$$2z_r = n\pi, \quad n = 0, 1, 2, 3, \dots \quad (\text{eqn 2.32})$$

Plotting equation 2.29 as a function of the vertical wavenumber, the spacing between nodes will now reveal the receiver depth based on

$$z_r = \frac{\pi}{\Delta\beta} \quad (\text{eqn 2.33})$$

Notice in equations 2.31 and 2.33 the exchange of source and receiver depths. By placing the receiver at a shallower depth than the source, no new information (namely the source depth) is to be found.

In summary, there is a way to calculate the complex pressure amplitude as a function of range using the complex pressure spectrum. Conversely, if the complex pressure $\underline{p}(r)$ has been measured at the receiver, then the complex pressure spectrum $\underline{p}(k)$ can be derived by making use of the relationship between the Fourier-Bessel Transform Pairs (see equation 2.12), and the depth of the source can be found from the magnitude of that spectrum (see equation 2.31).

The foregoing is the mathematics involved in deriving the theoretical pressure spectrum. One purpose of this thesis was to compare the theoretical value of the pressure spectrum as derived with the Fourier-Bessel Transform with the pressure spectrum obtained by use of the computerized Discrete Fourier Transform (DFT), otherwise known as the Fast Fourier Transform, or FFT. At this point, it will be helpful to review what happens in the FFT.

Starting with the Fourier-Bessel transform pair shown in equation 2.12, $\underline{f}(r)$ and $\underline{g}(\gamma)$ can be re-written in terms of a discrete sum as is done in the DFT:

$$\text{FFT}^{-1} \left\{ \underline{g}(\gamma) \right\} = \sum_{n=0}^{N-1} \underline{g}(n\Delta\gamma) e^{i \frac{2\pi mn}{N}} = \underline{f}(m\Delta r) \quad (\text{eqn 2.34})$$

where $N = 1, 2, 3, \dots$, up to some large positive integer, and represents the number of transform points, and

$$\gamma r = (n\Delta\gamma)(m\Delta r) = \frac{2\pi mn}{N} \quad (\text{eqn 2.35})$$

based on the relationship between Δr and $\Delta\gamma$ as described in sampling theory [Ref. 2: p. 2],

$$\Delta\gamma\Delta r = \frac{2\pi}{N}$$

or

$$\Delta r = \frac{2\pi}{N\Delta\gamma} \quad (\text{eqn 2.36})$$

By convention, the sign of the exponential function in equation 2.34 is taken as negative when performing the "forward" transform,

$$\text{FFT} \left\{ \underline{f}(m\Delta r) \right\} = \underline{g}(n\Delta\gamma) \quad (\text{eqn 2.37})$$

and is taken as positive when performing the "inverse" transform,

$$FFT^{-1} \left\{ g(n\Delta\gamma) \right\} = f(m\Delta r) \quad (\text{eqn 2.38})$$

Then,
$$\int_{-\infty}^{\infty} g(\gamma) e^{i\gamma r} d\gamma \approx \left[\sum_{n=1}^{N-1} g(n\Delta\gamma) e^{i \frac{2\pi mn}{N}} \right] \Delta\gamma$$

where

$$g(n\Delta\gamma) = \frac{\sqrt{\gamma}}{\beta} e^{i\beta z_r} \sin(\beta z_s)$$

Therefore

and
$$P(m\Delta r) = - \sqrt{\frac{2}{\pi r}} e^{-i \frac{\pi}{4}} \left[\Delta\gamma FFT^{-1} \left\{ g(n\Delta\gamma) \right\} \right]$$

$$\left| g(n\Delta\gamma) \right| = \frac{\sqrt{\gamma}}{\beta} \sin(\beta z_s) \quad (\text{eqn 2.39})$$

D. THE EFFECTS OF SURFACE ROUGHNESS

Surface roughness [Ref. 5: p.409] reduces the Lloyds Mirror effect. The rougher the surface, the greater the effect for angles increasingly closer to grazing incidence. If M represents the surface roughness factor, then equation 2.3 can be re-written as

$$P(R) = \frac{e^{ikR}}{R} - e^{-M^2} \frac{e^{ikR}}{R}$$

where

$$M \equiv \frac{4H \sin\phi}{\lambda} \quad (\text{eqn 2.40})$$

ϕ is the grazing angle of incidence, H is the average height of the surface gravity waves, and λ is the wavelength of the narrowband continuous wave acoustic signal. When M is less than 1, the surface is said to be smooth; when M is greater than 1, the surface is said to be rough.

Looking at Figure 2.1, we can see that the overall effect of increasing surface roughness is to reduce the contribution of the surface reflection or "image" to the interference pattern at the receiver. In other words, as H increases, the energy detected by the receiver is increasingly only the energy coming directly from the source. When the effect of surface roughness is a maximum such that the contribution to the pressure field of the "image" is completely suppressed, then

$$\begin{aligned} g_s(\gamma) &= \int_0^{\infty} \frac{e^{ikR_1}}{R_1} J_0(\gamma r) r dr \\ &= \frac{e^{i\beta|z_s - z_r|}}{i\beta} \end{aligned} \quad (\text{eqn 2.41})$$

and the magnitude of the spectrum is

$$|g(\gamma)| = \frac{1}{\beta} \quad (\text{eqn 2.42})$$

E. THE EFFECT OF ADDING NOISE

In an effort to simulate a realistic ocean environment, a normally distributed narrowband noise field based on a Rician distribution [Ref. 6: p. 189] was adapted. The resulting acoustic pressure field can be expressed as

$$P_N(R) = P(R) + \underline{n} \quad (\text{eqn 2.43})$$

where \underline{n} is a normally-distributed random function with a mean of zero, a standard deviation of one and possesses both amplitude and phase. The noise operates independently on both the amplitude and phase components of the pressure field. Since \underline{n} is independent of range and wavenumber, it is treated as a constant by the Fourier Transform.

A fourth intention of this thesis was to discover the degree to which noise degrades the wavenumber spectrum compared with the corresponding degradation of the range domain. To produce the random noise fields used in this research, the routine listed as "GGNML" in the IMSL library [Ref. 9] was called upon twice to generate independent, pseudo-random functions which interact separately and simultaneously with both the amplitude and phase components of $P(R)$. While not an elaborate scheme, owing to time and resource constraints, it was felt that this modest simulacrum of narrowband noise would give a fairly accurate, first cut "feel" for the effects of noise on the wavenumber spectrum. The reader should be aware, however, that no direct consideration of coherency along the range path was taken into account by this method.

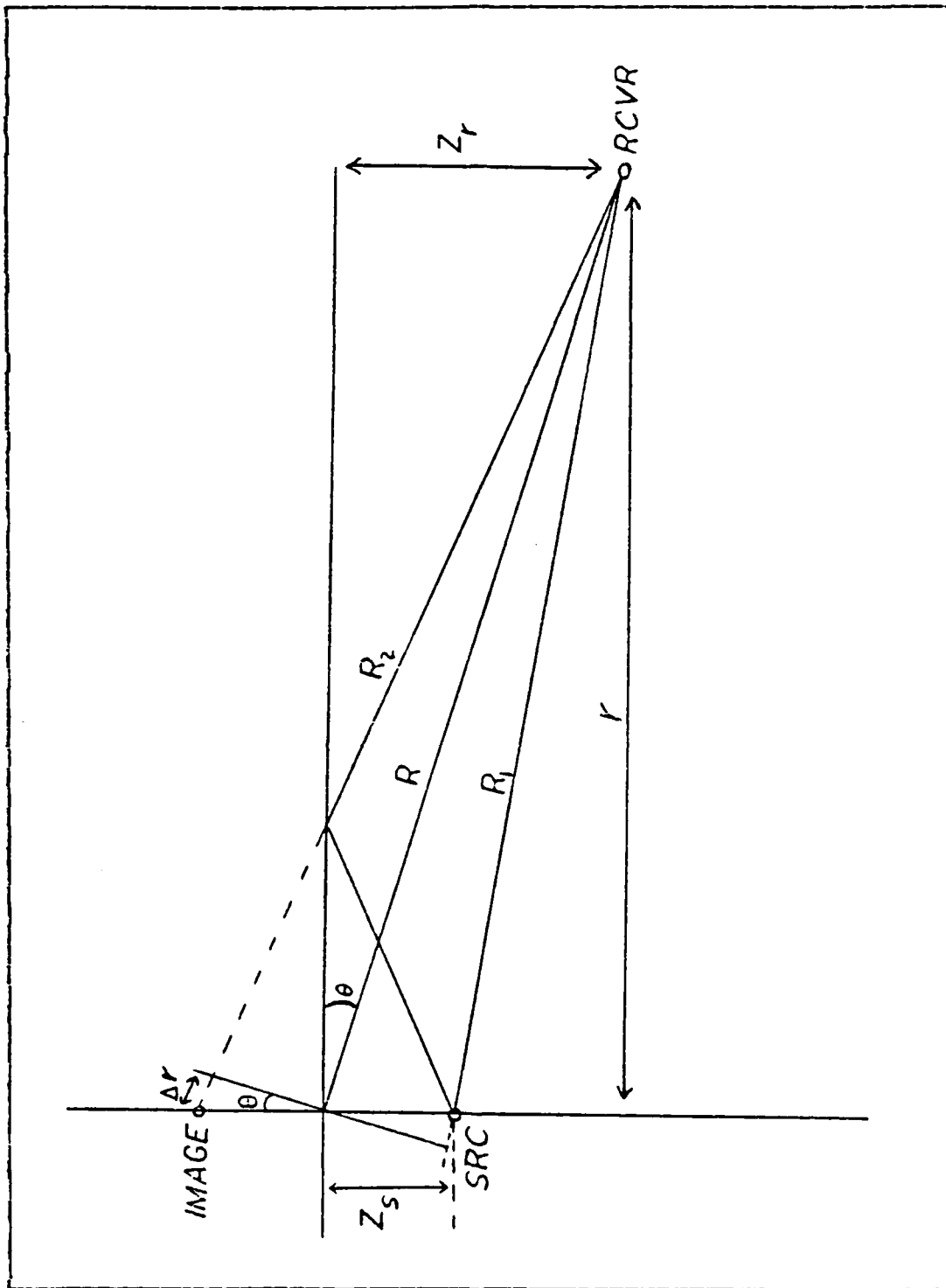


Figure 2.1 The Geometry of the Lloyds Mirror Effect

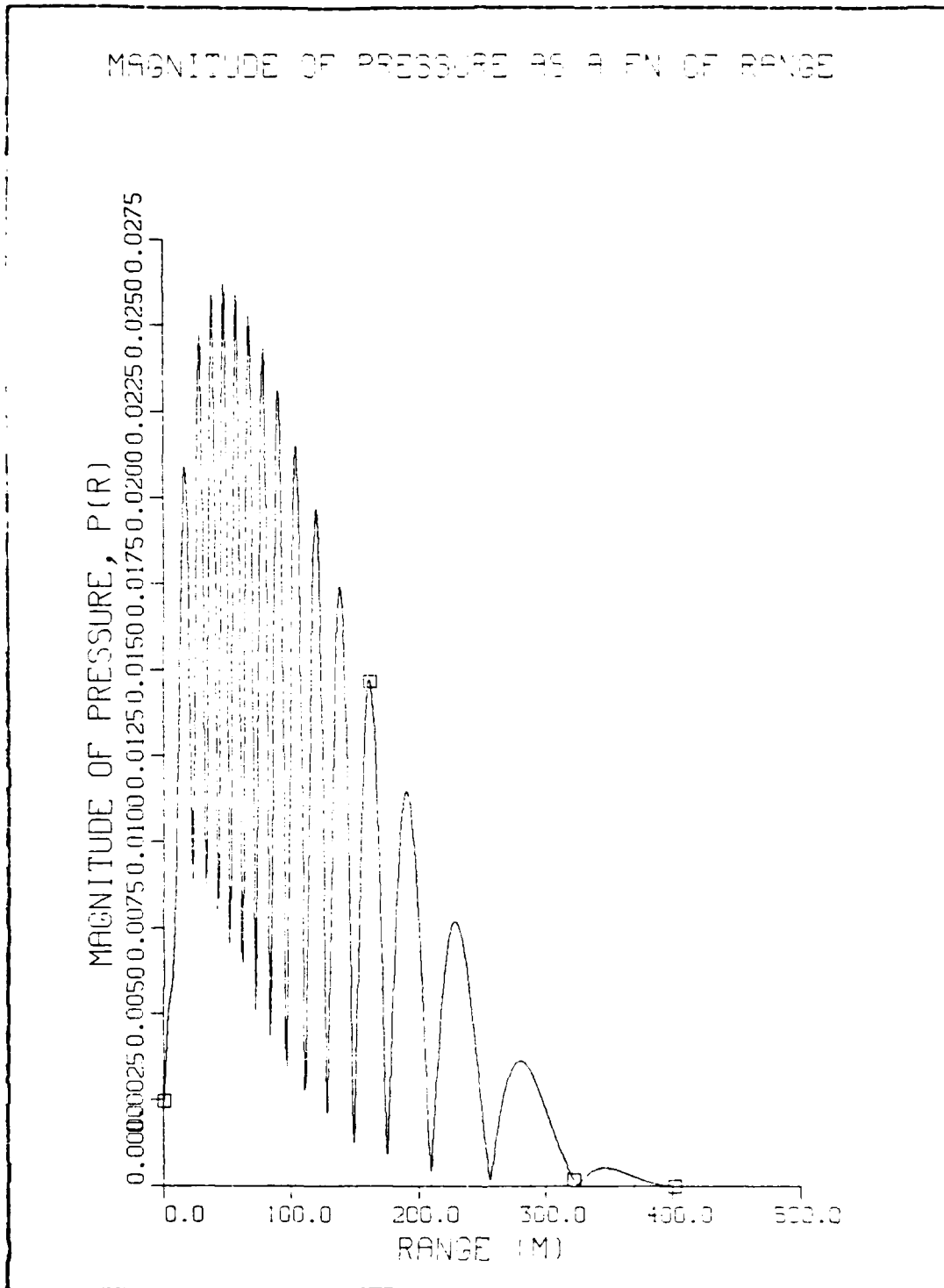


Figure 2.2 A Classic $|P(R)|$ vs. R Curve

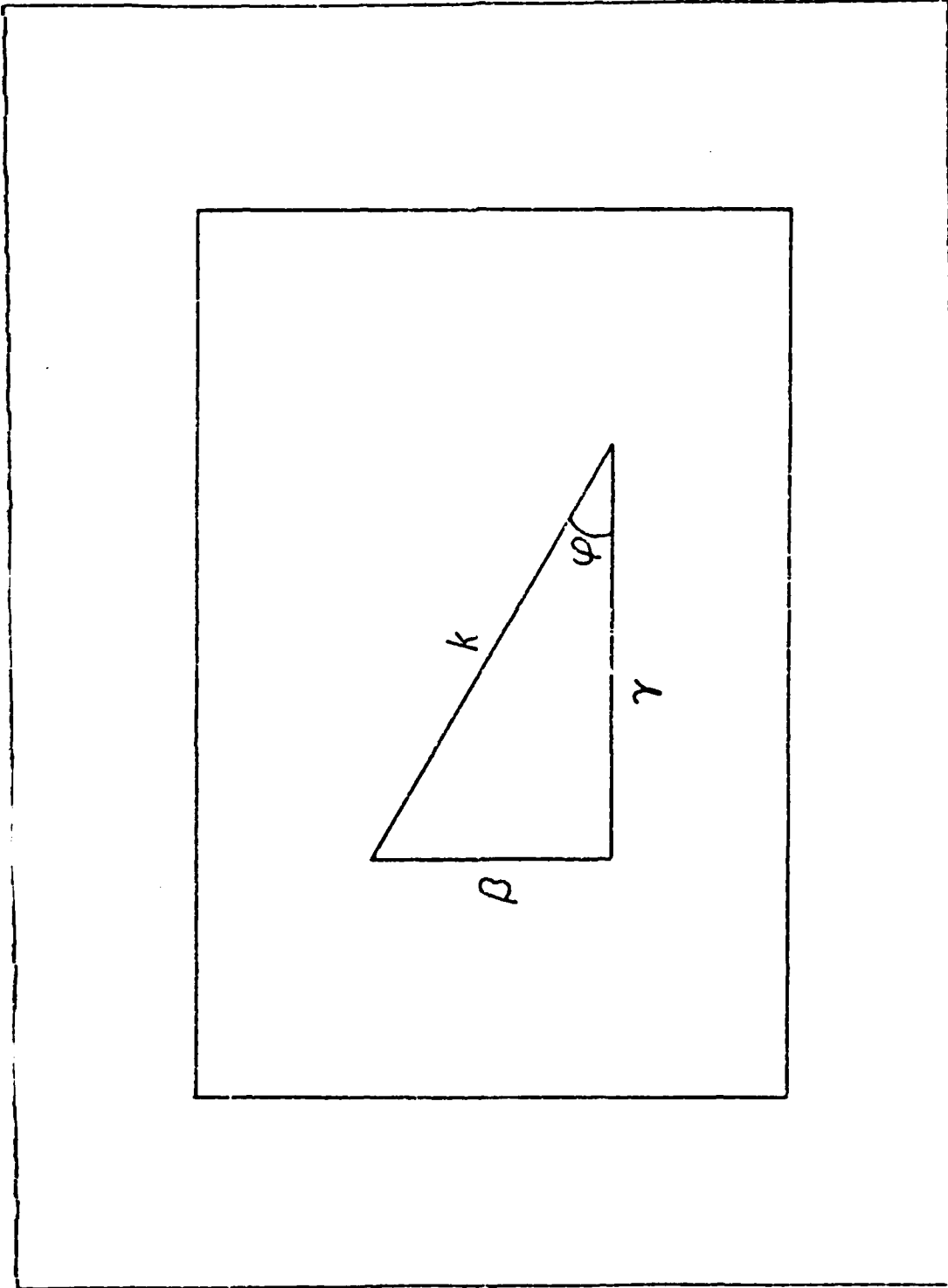


Figure 2.3 The Relationship Between k , γ and β

III. RESULTS AND CONCLUSIONS

A. THE FFT ALGORITHM

The research for this thesis was conducted entirely with computer algorithms to model the Lloyds Mirror, the Fast Fourier Transform, the range windows, waves, and the narrow-band noise. This was necessary since the Lloyds Mirror is an idealized representation of a situation that occurs only rarely in nature, and at that is limited to isospeed surface ducts. Also, time and money constraints were such that the use of computer models was an absolute necessity. For these reasons and because the FFT is the heart of the Wavenumber Technique, finding a reliable, easy-to-use computer algorithm to perform the FFT on the complex pressure function was deemed very important.

Initially, the IMSL library routines, FFT2C and FFTCC [Ref. 9: p. 232], were used to generate the pressure spectra. Although the author was unable to ascertain just how Stamey performed the FFT on his data, it is highly probable that he used an IMSL library routine. While in his thesis, Blanchard stated that he used the FFTCC routine. The magnitude of a typical spectrum generated by the routine FFT2C is shown in Figure 3.1.

The same scenario used to generate Figure 3.1 was used to produce the spectrum shown in Figure 3.2, but the routine FFTCC was utilized. As can be seen, the resulting graphs of the pressure spectrum magnitude are practically identical. Indeed, the set-up of the pressure fields for insertion into each routine differ only in that FFT2C requires a data set consisting of an integral power of two number of points, whereas FFTCC will handle any number of points. And, when

the Cooley-Tukey FFT listed in Appendix I was used to transform the same pressure field as was utilized to produce Figures 3.1 and 3.2, Figure 3.3 was the result. This algorithm differs somewhat from the IMSL routines by the manner in which the pressure field data is treated. Both FFT2C and FFTCC are designed to handle a complex array. The Cooley-Tukey FFT, on the other hand, is designed to transform real data; but the pressure field is complex. To get around this apparent conundrum, the pressure data is defined as either real or imaginary by its placement in the array. As can be seen, the resultant graph of the spectrum magnitude is almost indistinguishable from that produced by either IMSL routine. In conclusion, it does not matter which FFT routine is used to generate the pressure spectrum, as long as the pressure field in the range domain is "fitted" into the array in a manner suitable to the particular algorithm used.

TABLE I
Critical Values Used in the Research

$K \text{ (m}^{-1}\text{)}$	N	$r \text{ (m)}$	$\text{(m}^{-1}\text{)}$
3.0	1024	0.524	1.172×10^{-2}
2.0	1024	0.785	7.816×10^{-3}
1.0	1024	1.571	3.906×10^{-3}

It is important to note four things about the FFT at this point.

1. Since the FFT is given only a finite sample of a function while the theoretical Fourier Transform

looks at the complete function, the computer algorithm is designed to overcome this necessary shortcoming by assuming the submitted finite sample repeats its pattern an infinite number of times. This can present problems. If the right hand side of one pattern does not flow smoothly into the left hand side of the next repetition, as is shown in Figure 3.4, then high frequency oscillations known as the Gibbs Phenomenon can be introduced into the spectrum. (See Figure 3.5) This is a very real danger where acoustic pressure fields are concerned. For example: if the sample size does not include the entire pattern (i.e., the pressure magnitude has not been completely attenuated prior to the endpoints of the sample), this unwanted "jitter" in the spectrum can result. To avoid this situation, not only was a pressure field symmetric about the origin (the source) used, but also a Hanning Window was constructed to reduce both the amplitude and the phase components of the pressure field to zero at the endpoints, thus avoiding the Gibbs Phenomenon. Figure 3.6 shows the same pressure field depicted in Figure 3.4 combined with a Hanning Window. Its spectrum is shown in Figure 3.7. The Gibbs Phenomenon is absent. The form of the window is given below. The reader's attention is directed to Appendix I for a view of the manner in which it is combined with the pressure field.

The Hanning Window:

$$W = 0.5 \left[1 + \cos \left(\frac{(T-1)\pi}{NPTS} \right) \right] \quad (\text{eqn 3.1})$$

where "NPTS" is the number of data points,
and $I = 1, 2, 3, \dots, NPTS$.

The Hanning Window does not affect the acoustic pressure field in any other way; the interference pattern in the range domain characteristic of the Lloyds Mirror for an Acoustic Doublet remains the same except for its amplitude near the end points. As shall be shown later in this chapter, it is the pattern of the entire pressure field, not simply the amplitude of the pattern, on which the W.T. performs its legerdemain.

2. Because the vertical wavenumber β is real and has physical meaning for values of the horizontal wavenumber less than k , only those values of the magnitude of the acoustic pressure spectrum -- corresponding to $\gamma < k$ were retained.
3. In the Wavenumber Technique, the relationship between the horizontal range step size Δr and the horizontal wavenumber step size $\Delta \gamma$ is extremely important. Recalling equation 2.36,

$$\Delta \gamma \Delta r = \frac{2\pi}{N} \quad (\text{eqn 3.2})$$

for a given "N," the term on the right hand side, $2\pi/N$, is a constant. Consequently,

$$\Delta r \propto (\Delta \gamma)^{-1} \quad (\text{eqn 3.3})$$

Simultaneously, in accordance with sampling theory, in order to prevent aliasing in the pressure spectrum, there is an upper limit on the size of Δr :

$$\Delta r \approx \frac{\lambda}{2} \quad (\text{eqn 3.4})$$

Normally, Δr is chosen such that it is much less than half a wavelength; specifically, $\Delta r < \lambda/5$ or $\lambda/6$, thus a good sample of the range domain is ensured. However, from equation 2.36, the smaller Δr becomes, the larger $\Delta \gamma$ becomes and the coarser the sample sizes of the spectrum become. Also, to ensure adequate samples in the spectrum, examination of equations 2.25 and 2.36 and [Ref. 10] reveals that

$$\Delta \gamma \approx \frac{1}{2k} \left(\frac{\pi}{z} \right)^2 \quad (\text{eqn 3.5})$$

where Z is the deeper of the two depths, whether source or receiver. So there is a delicate balance to maintain between the sizes of Δr and $\Delta \gamma$. In the spirit of compromise, the investigator chose to use a range step size equal to a quarter of a wavelength where

$$\lambda \approx \frac{2\pi}{k} \quad (\text{eqn 3.6})$$

For most cases investigated, values for k , N , Δr and $\Delta \gamma$ are summarized in Table I. As can be seen, using these values and equation 3.5, for a "Z" value of 25 wavelengths, the maximum size $\Delta \gamma$ can assume and still ensure an adequate sample size is $\Delta \gamma = 0.0004$ /m. Clearly the sizes of $\Delta \gamma$ shown in Table I are too coarse by a factor of approximately 20. Only by increasing N , the number of sample points, can this discrepancy be corrected.

However, as mentioned earlier, computer resources were limited. The effect on the research results is not significant; nevertheless, it is recommended the reader bear this limitation in mind during the succeeding pages.

4. Looking again at Figures 3.1 through 3.3, the FFT is the curve marked by the solid line, while the theoretical Fourier Transform is the curve marked by the dotted line. Notice the large discrepancy between the theoretical values and the actual values in the region corresponding to small vertical wavenumbers. This happens to correspond to a region of the spectrum characterized by more rapid fluctuations in the spectrum. However, the size of beta is constant. Consequently, fewer samples of this region of the spectrum are available as compared with that region corresponding to high beta values. Since the plotting routine utilized is interpolating between sampled points of the spectrum, the graph at this end is less smooth and results in a marked difference between the curves. Where the curves are determined by more points and are smoother, the discrepancy between theory and computerized reality is much less noticeable.

B. SOURCE DEPTH DETERMINATION

Theory predicts that, for a source shallower than the receiver, when the magnitude of the pressure spectrum is plotted as a function of the vertical wavenumber, the resultant nodal spacing can be used to determine source depth by either consulting a chart (see Figure 3.8) or by performing a simple calculation (see equation 2.31). To test this prediction, several scenarios were designed. Figures 3.9

through 3.11 are typical of the many cases run. In each, source depth is less than receiver depth. For each case, careful measurement of the $\Delta\beta$ spacing coupled with use of equation 2.31 yielded the (already known) source depth with less than a two per cent error, which can be attributed to human error in the measurement of $\Delta\beta$.

TABLE II
Results of Source Depth Determination Runs

Figure Nr.	ZS (m)	ZR (M)	(Meas'd)	Z(Calc'd)
3.9	21.998	47.124	0.14 m ⁻¹	22.4 m
3.10	31.416	47.124	0.10 "	31.4 "
3.11	15.708	47.124	0.20 "	15.7 "
3.12	47.124	21.998	0.14 "	22.4 "
3.13	47.124	31.416	0.10 "	31.4 "
3.14	47.124	15.708	0.20 "	15.7 "

To test what happens when the receiver depth is shallower than the source depth, the cases depicted in Figures 3.12 through 3.14 were run. The scenario is the same as for Figures 3.9 through 3.11; only the source and receiver depths have been exchanged. As can be seen, the $\Delta\beta$ spacing corresponds with the receiver depth. This is again as predicted by theory. Table II summarizes the findings of Figures 3.9 through 3.14.

The scenarios shown in Figures 3.9 through 3.14 all start with a pressure field measured horizontally from the source; i.e., the exact range to the source is known. What if the range to the source is not known, or only minimal information concerning the range is known? In this case, only a portion of the pressure field starting at some initial range R_0 can be sampled. What is the effect on the spectrum?

To answer these questions, a range window was constructed and placed at varying initial ranges from the source. As Figure 2.2 illustrates, as range from the source increases, the nodes of the Lloyds Mirror interference pattern in the range domain become more widely spaced. Given fewer nodes to sample, will the spectrum be the same? In accordance with theory, the spectrum is independent of range (see equation 2.39) and so should be the same wherever the window is placed.

Figures 3.15 through 3.17 illustrate that the spectrum is not entirely independent of range. The figures utilize the same scenario as was used above, differing only in the range from the source at which the sampling of the pressure field begins. As can be seen, beginning at the right hand side which corresponds to high vertical wavenumber values, or the low horizontal wavenumber values (see equation 2.8), a "washing out" of the spectrum occurs, increasing as the range window is moved further from the source. Again, the theoretical curve is marked by the dotted line.

When the magnitude of the pressure spectrum is plotted as a function of the vertical wavenumber, the useful portion of the resulting graph is the last two-thirds of the beta range. For $K = 2.0/m$, this is the range $0.67/m < \beta < 2.0/m$. This phenomenon is explained more fully in Section III A 4. With this caveat in mind and based upon multiple source, receiver, range window

combinations run through the model, the following criterion was established: the nodal spacing of the spectral plots was no longer determinable once the initial range R_0 was increased to approximately three times the source depth. In other words, minimum knowledge of the range from receiver to source must be available to ensure adequate sampling of the acoustic pressure field in the range domain. This, in turn, will produce a pressure spectrum of such a quality that source depth determination can be made.

What causes this not entirely unexpected result? As the range window is moved further from the source, for $Z_s \neq Z_r$, those samples of the pressure field corresponding to low gamma values are lost to the spectrum first. Low gamma values correspond to waves arriving at high angles with respect to the horizontal. At greater receiver ranges, R more closely approximates r and the arrival angles become closer to the horizontal (see Figure 2.1).

As the window is moved further from the source and fewer nodes of the interference pattern are encountered, if a wider window were used, could this "washing out" effect in the spectrum be minimized or even eliminated? In the author's opinion, it would be minimized; because of the reasoning in the preceding paragraph, it probably would not be eliminated. However, a wider window would require more sample points; because of equation 2.36, one cannot merely make Δr larger. Time and the computer resources available to this investigator were limited, and the test cases run were already using the maximum available array sizes, thus precluding a quantitative illustration of this hypothesis. Succeeding research into the W.T. might include ways to test this point.

In conclusion, the Wavenumber Technique has the potential for being a valuable operational tool in determining the depth of an acoustic source provided the receiver is at

a greater depth than the source. It is strongly recommended that further research into use of the W.T. in a realistic ocean environment be done.

C. THE EFFECT OF SURFACE ROUGHNESS

This and the succeeding section were purely qualitative portions of the research. Therefore, in the figures the spectra were plotted as functions of the horizontal wavenumber for ease of discussion except where a particular point about the vertical wavenumber was being illustrated, and the theoretical curve was omitted.

As described in Chapter II, the effect of surface roughness is to suppress the contribution of the image to the dipole interference pattern. Moreover, by increasing M , the effect is to suppress the image's contribution to the pressure field in the range domain and reduce the pressure spectrum to the contribution of the direct path wave only (see equation 2.41); the resultant magnitude of the spectrum is inversely proportional to the vertical wavenumber (see equation 2.42). As Figures 3.18 through 3.21 illustrate, this is exactly what happens in the FFT. In Figure 3.21, an inverse beta curve has been manually superimposed onto the actual curve to reflect this point.

Looking at the pressure field in the range domain $\underline{p}(R)$ the effect of surface roughness is inversely proportional to the wavelength of the signal. For longer wavelengths, the Lloyds Mirror effect is more tolerant of surface roughness than it is for shorter wavelengths. Does the same relationship hold for the pressure spectrum? Comparison of the respective sea states in Figures 3.22 and 3.23 with Figures 3.24 and 3.25 illustrate that it does.

This investigator also looked at the effect of varying the range window in the presence of waves. This is similar

to what was looked at in Section B of this chapter. Results should be similar and for the same reasons. For a given sea state, source and receiver geometry and wavelength, the range window was moved successively further from the source (see Figures 3.26 through 3.29). As can be seen, the "washing out" effect of the spectral nodes begins to effect the lower horizontal wavenumber values first.

D. THE EFFECT OF NOISE

Random noise was simulated in this research. Being independent of range and frequency, it was treated as a constant by both the Fourier Transform and the FFT. The investigator wished to compare qualitatively the effects of equal amounts of noise on the pressure field in the range domain and in the wavenumber domain. To do this, successively larger values of the "noise factor" were used in computing \tilde{p} (see equation 2.43). Values of the noise factor used in the research were limited to the maximum amplitude of the pressure field $p(R)$. Since dipole radiation is predicated, as range from the source increases, the maxima of the interference pattern decrease as the inverse square of the range (i.e., R^{-2}). Hence, even small noise factors can have a devastating effect on the pressure field as the receiver is stepped out in range. The destructive effect on the spectrum of successively more intense amounts of noise at varying wavelengths is illustrated in Figures 3.30 through 3.47

Several conclusions concerning the pressure spectra can be drawn from these results:

1. For equal amounts of noise, longer wavelengths are affected less than shorter wavelengths. Compare Figures 3.31 and 3.32 with Figures 3.43 and 3.44, and Figures 3.34 and 3.35 with Figures 3.46 and 3.47.

This was not a surprising result since the same principle holds for the range domain.

2. The spectrum is affected less by noise than is the range domain. Compare Figures 3.30 through 3.35 and Figures 3.36 through 3.47 for an illustration of this point at two different wavelengths and various noise levels. This was a rather welcome surprise.
3. If the magnitude of the pressure spectrum is plotted as a function of beta, the $\Delta\beta$ spacing can still be detected even after the pressure function in the range domain has been "swallowed up" by the noise field. Compare Figures 3.33 through 3.35 and Figures 3.42 through 3.44. This was not surprising in view of 2. above.

E. SUMMATION

All of the foregoing results are based specifically on the Lloyds Mirror for an Acoustic Doublet. This is a highly idealized and artificial environment. Any thought of immediately applying conclusions reached in this paper to a real world situation is premature. However, the results are still significant if only for supporting the statement made by each preceding investigator that the Wavenumber Technique should be examined very closely for a future operational role, especially in view of the current trends in source levels and ambient noise levels.

Certain fundamental conclusions regarding the author's research into the Wavenumber Technique can now be made within the confines of the above restriction:

1. Source depth can be determined quickly and easily from the acoustic pressure spectrum provided
 - a) the receiver is deeper than the source,

- b) some knowledge of the range from receiver to source is available, and
 - c) the magnitude of the pressure spectrum is plotted as a function of the vertical wavenumber.
2. Source depth determination in even a noisy environment is possible. While the introduction of increasing amounts of narrow band noise adversely affects both the range domain and the wavenumber domain, the pressure spectrum seems able to withstand the chaotic destruction longer than does the pressure field in the range domain.
 3. The ability to determine source depth is adversely affected by the height of surface gravity waves since surface roughness reduces the Lloyds Mirror as the sea state increases.

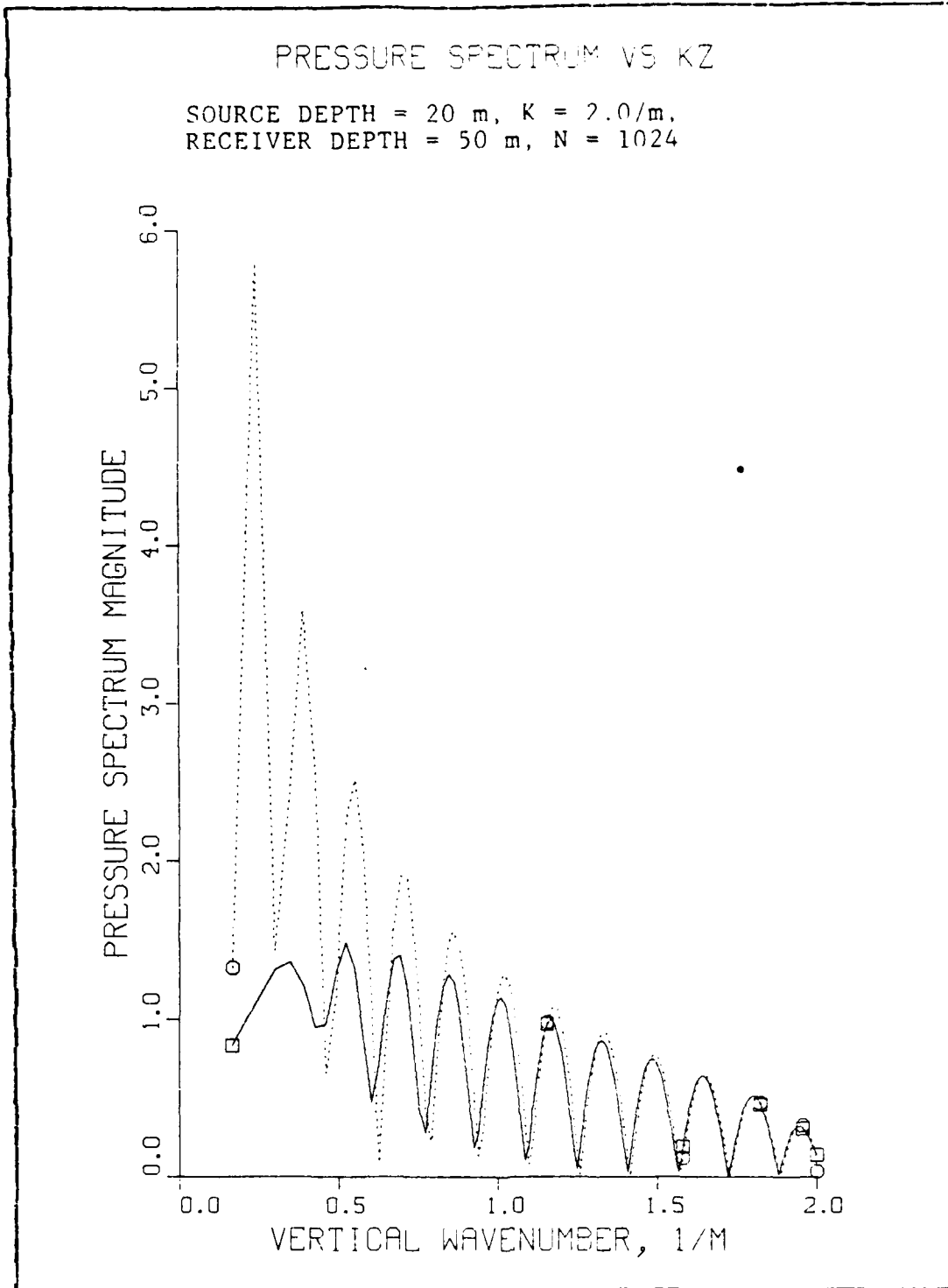


Figure 3.1 Pressure Spectrum Using FFT2C

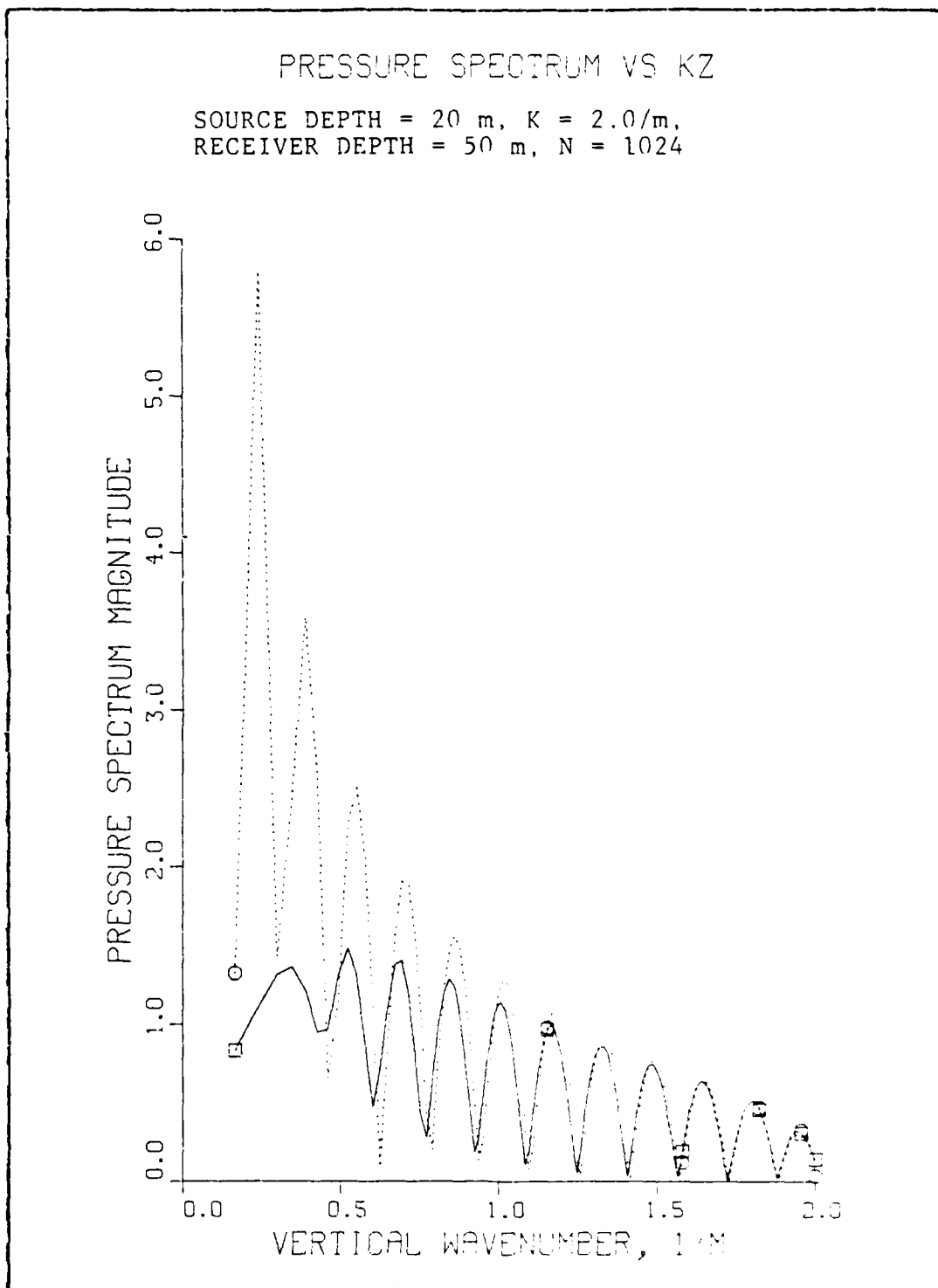


Figure 3.2 Pressure Spectrum Using FFTCC

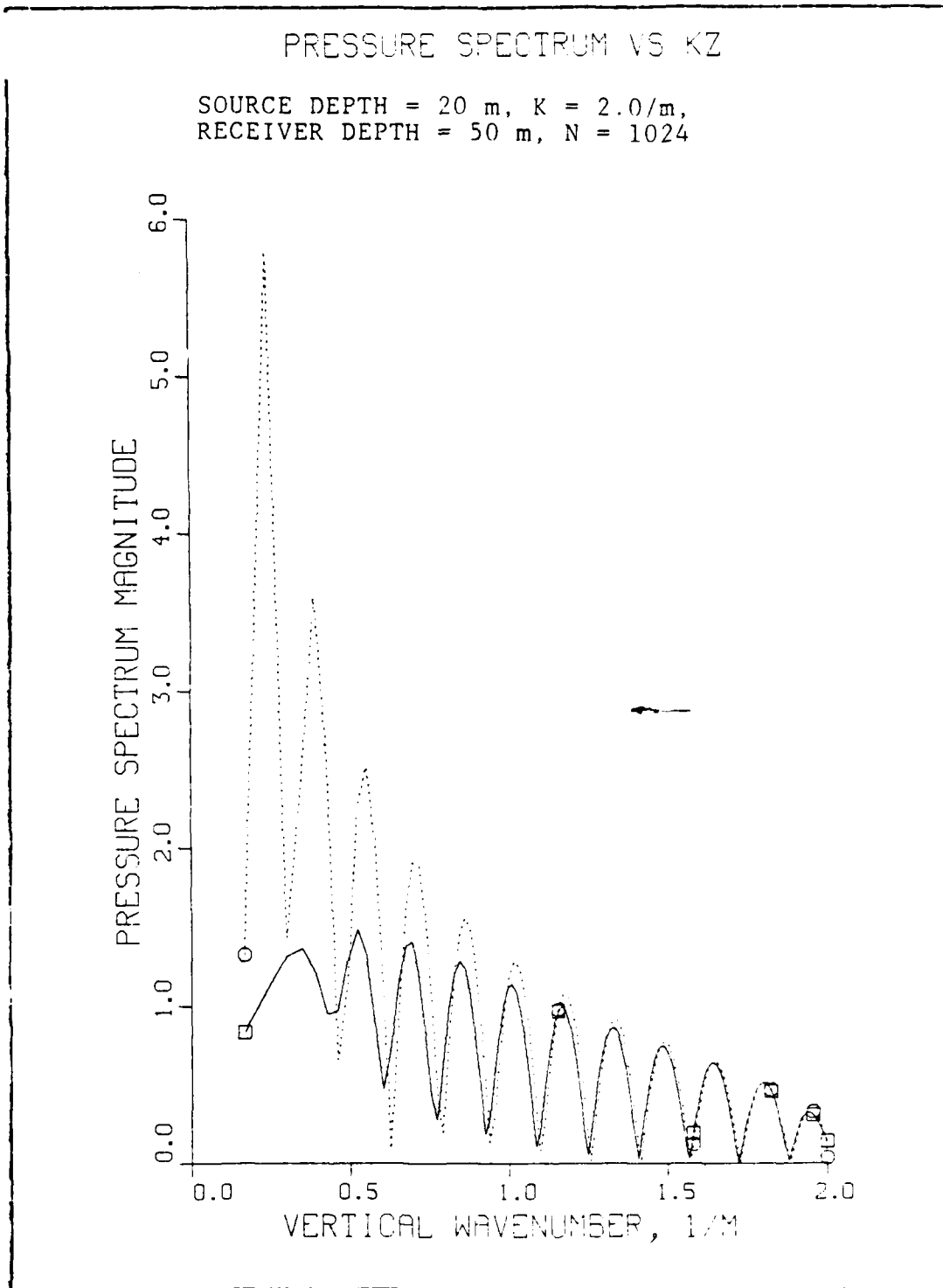


Figure 3.3 Pressure Spectrum Using Cooley-Tukey FFT

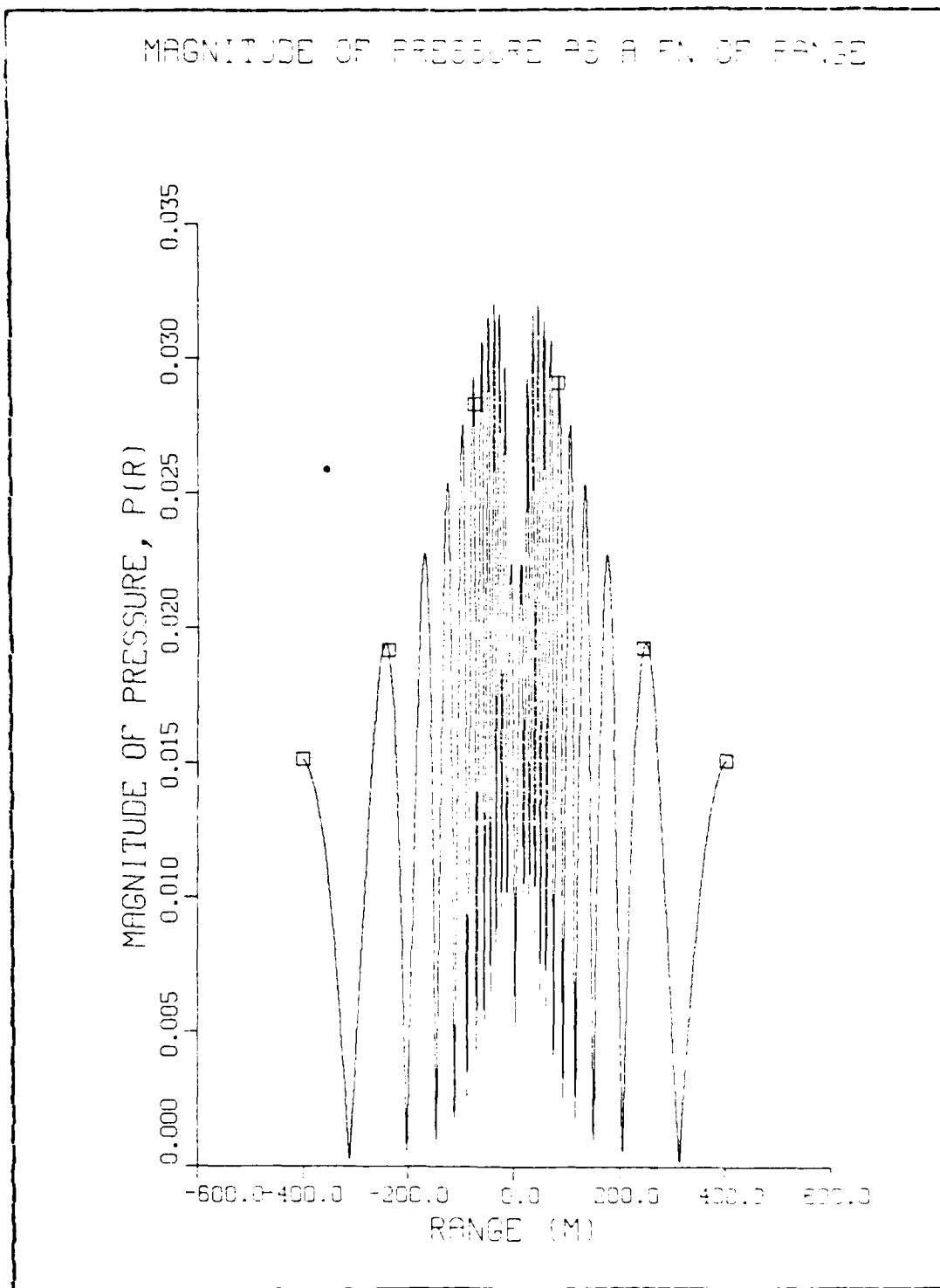


Figure 3.4 Non-smoothed Pressure Field

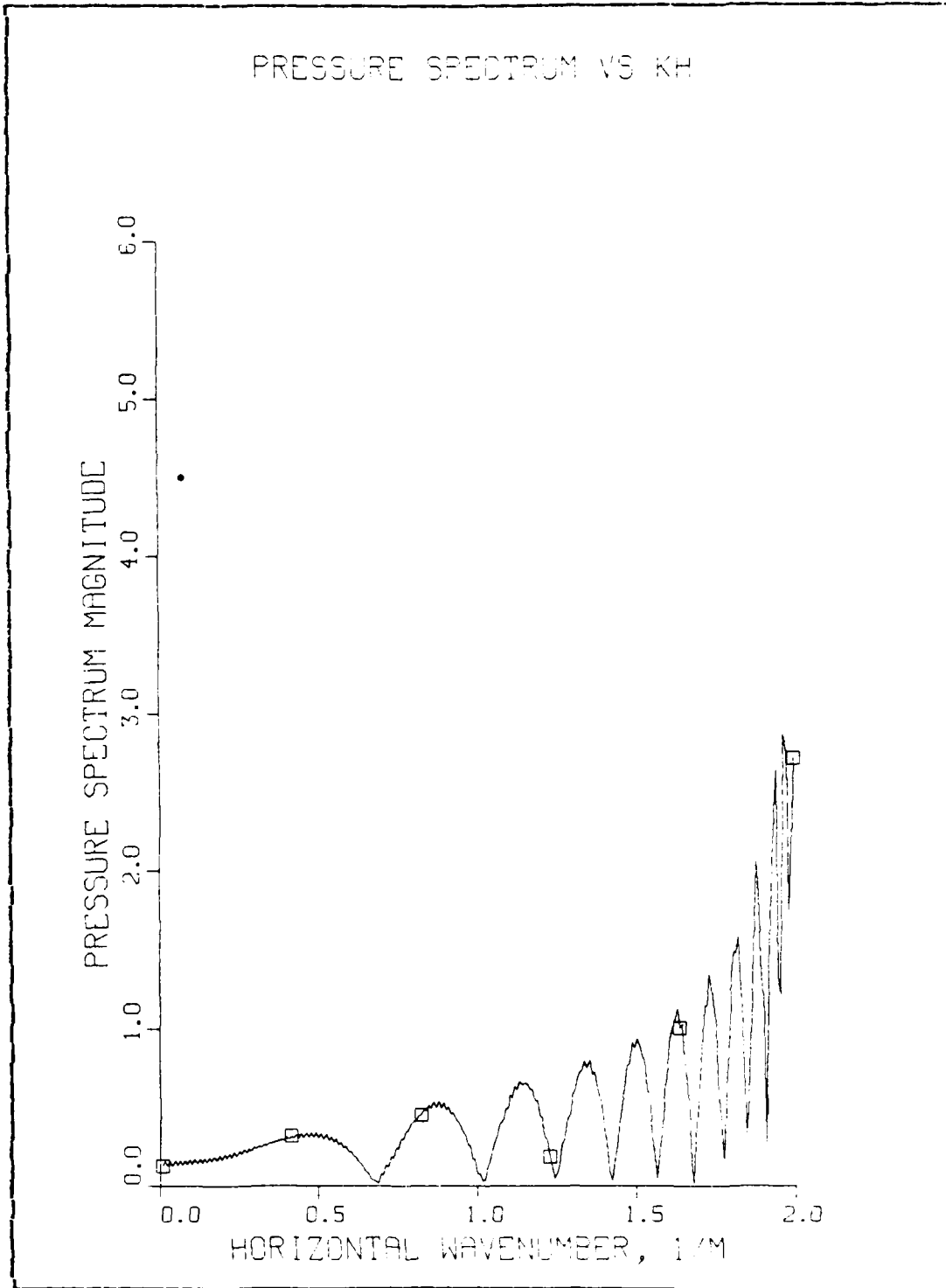


Figure 3.5 Pressure Spectrum Showing Gibbs Phenomenon

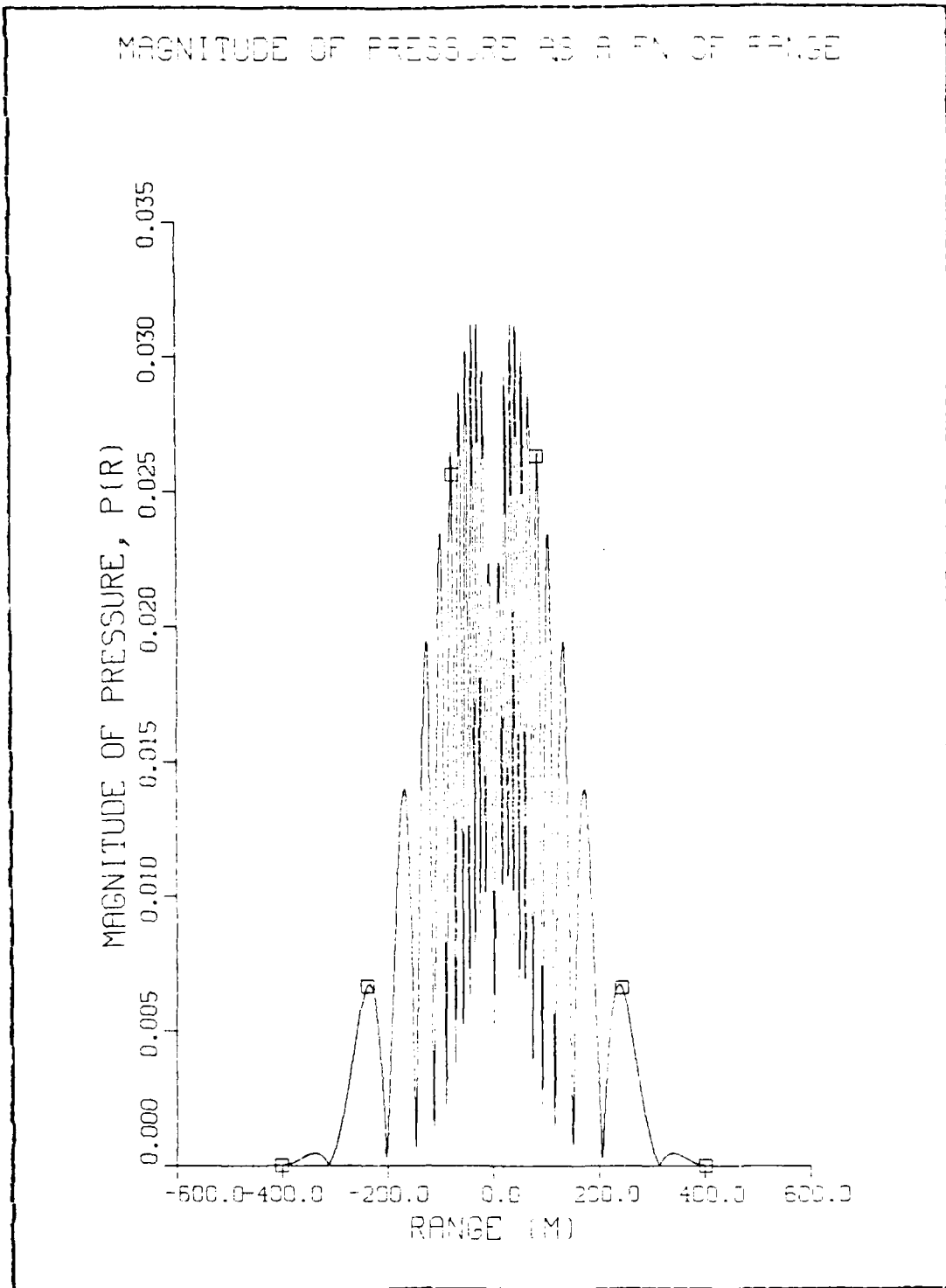


Figure 3.6 Pressure Field Combined With a Hanning Window

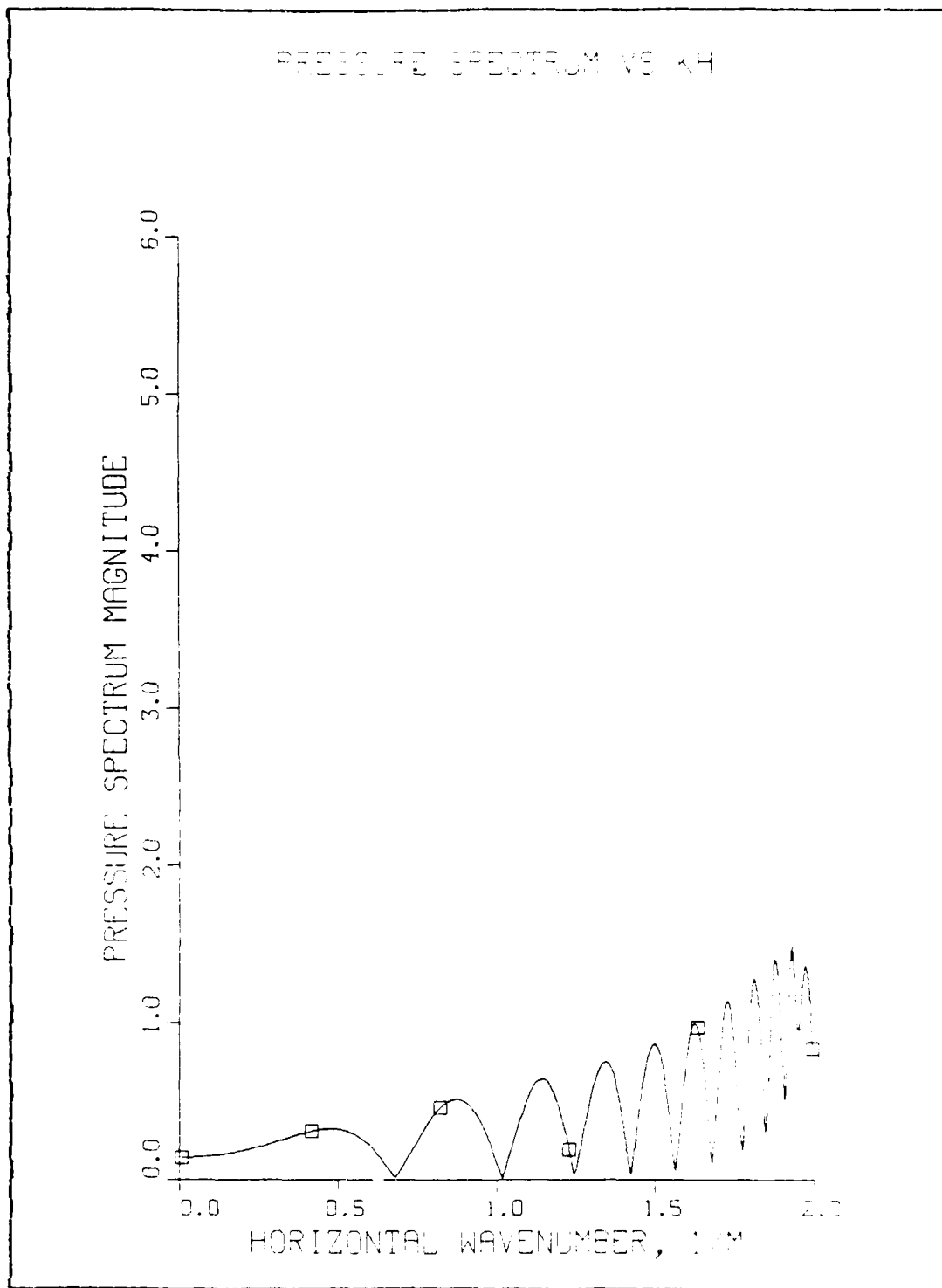


Figure 3.7 Pressure Spectrum Combined With a Hanning Window

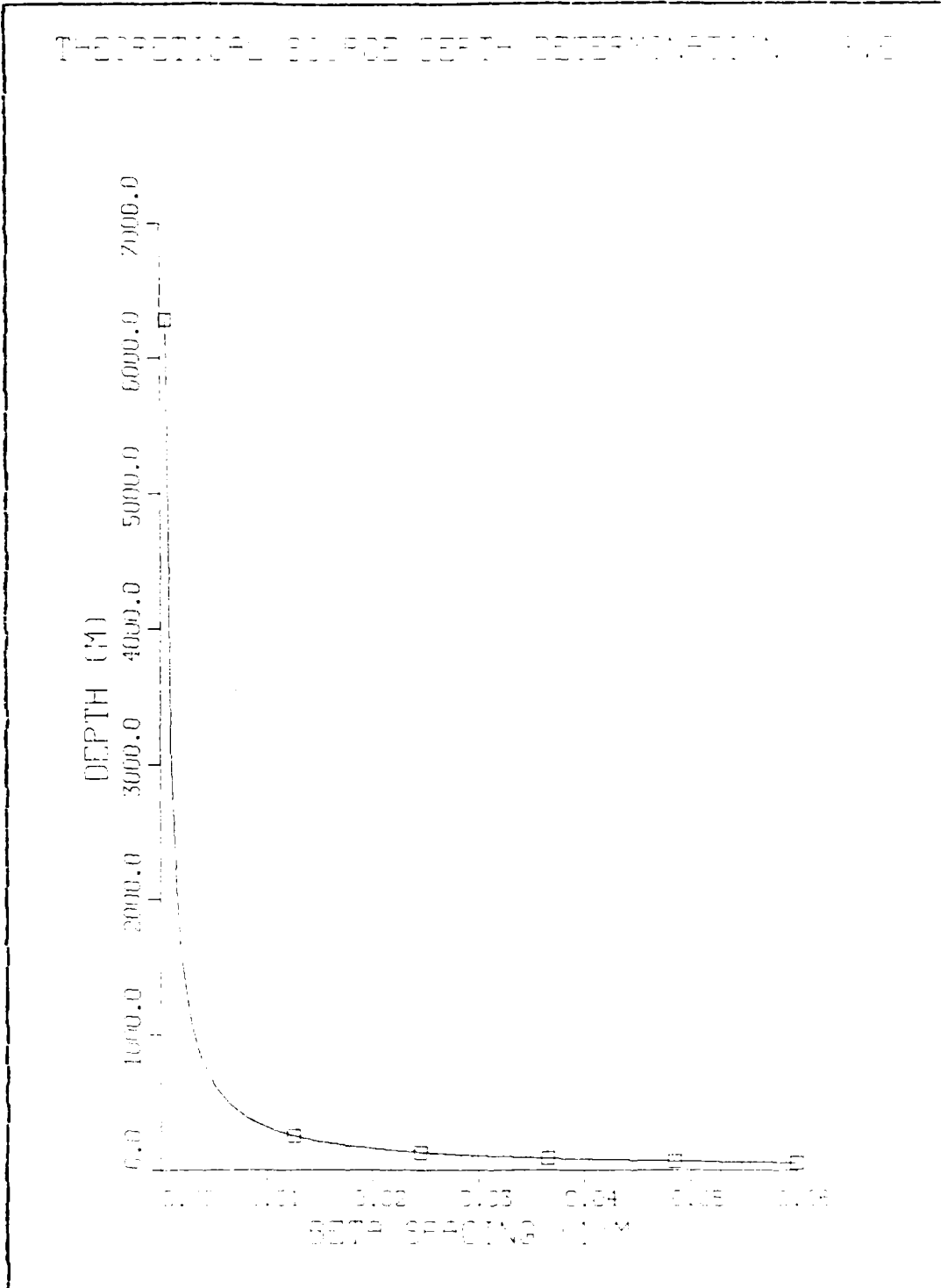


Figure 3.8 Theoretical Source Depth Determination Curve

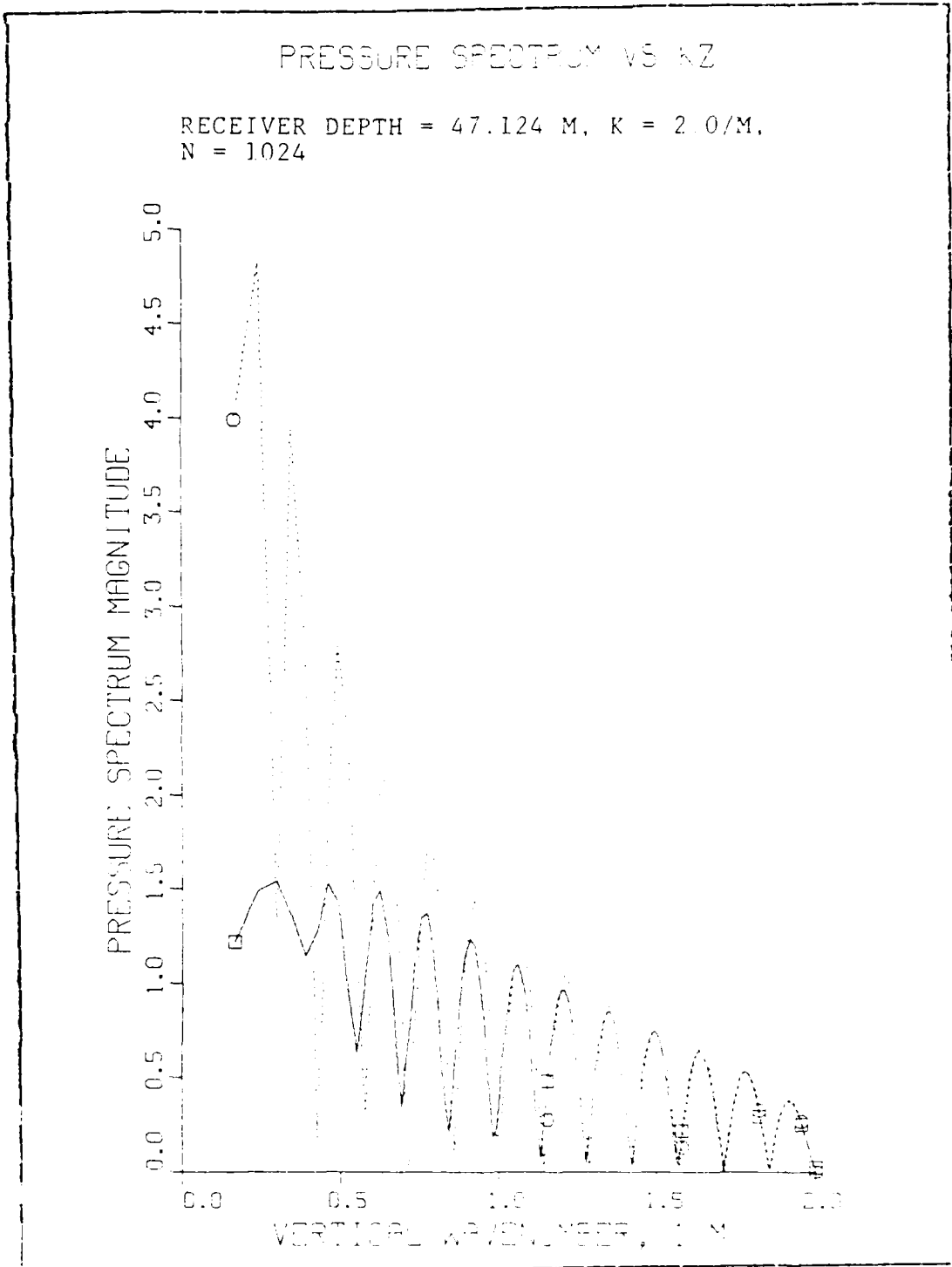


Figure 3.9 Graph of Pressure Spectrum,
Source at 22.0 Meters

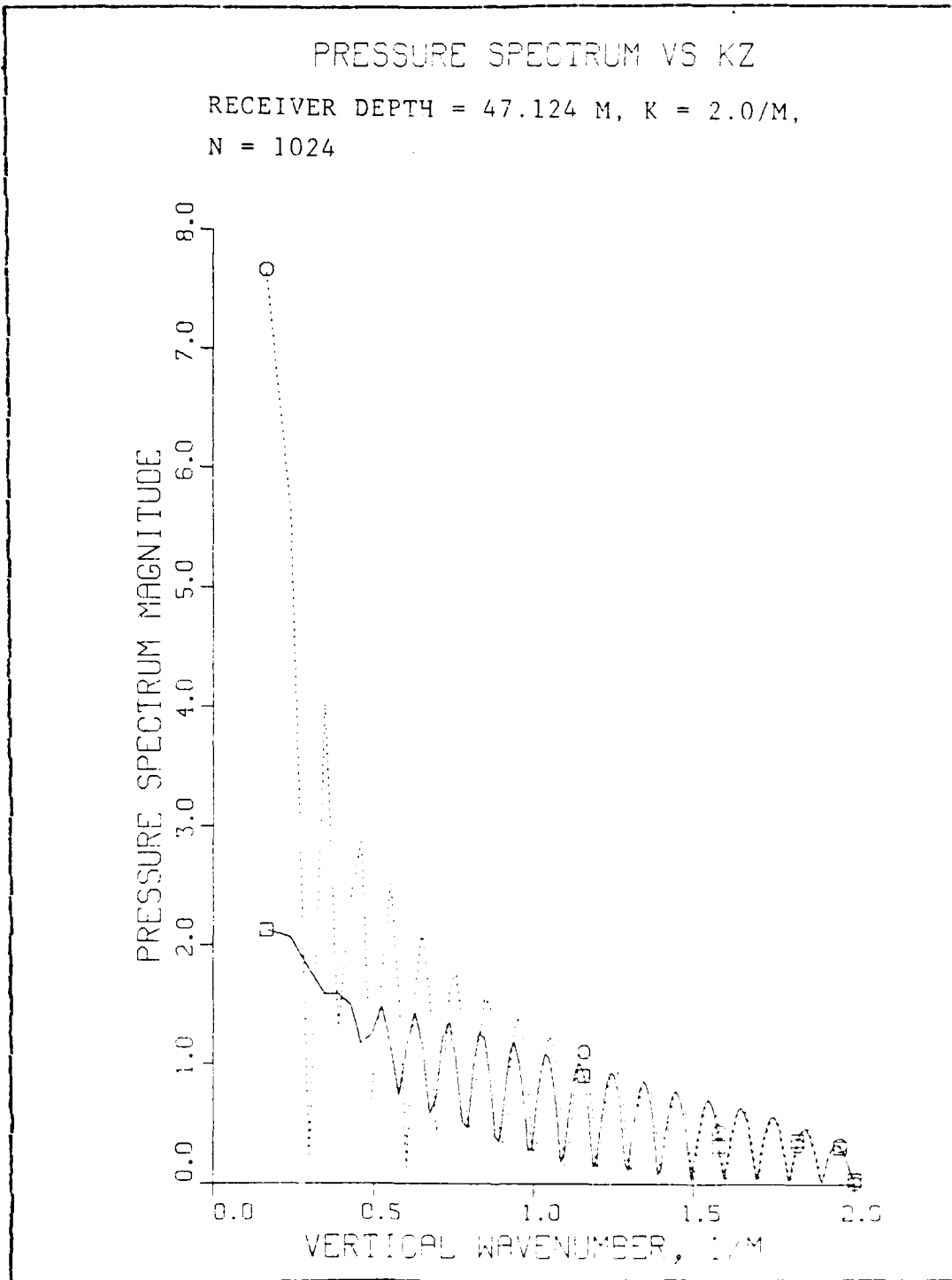


Figure 3.10 Graph of Pressure Spectrum,
Source at 31.4 Meters

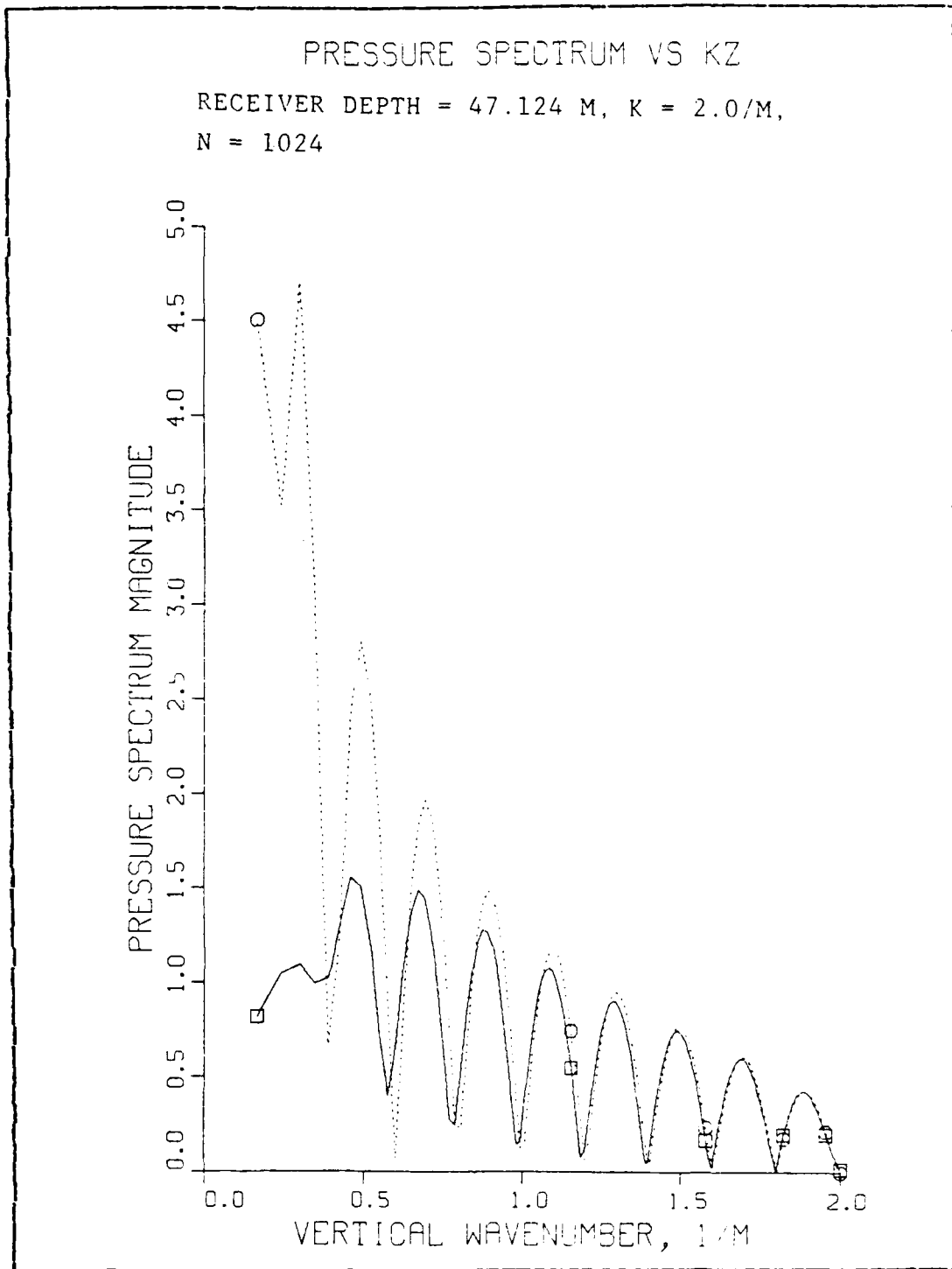


Figure 3.11 Graph of Pressure Spectrum,
Source at 15.7 Meters

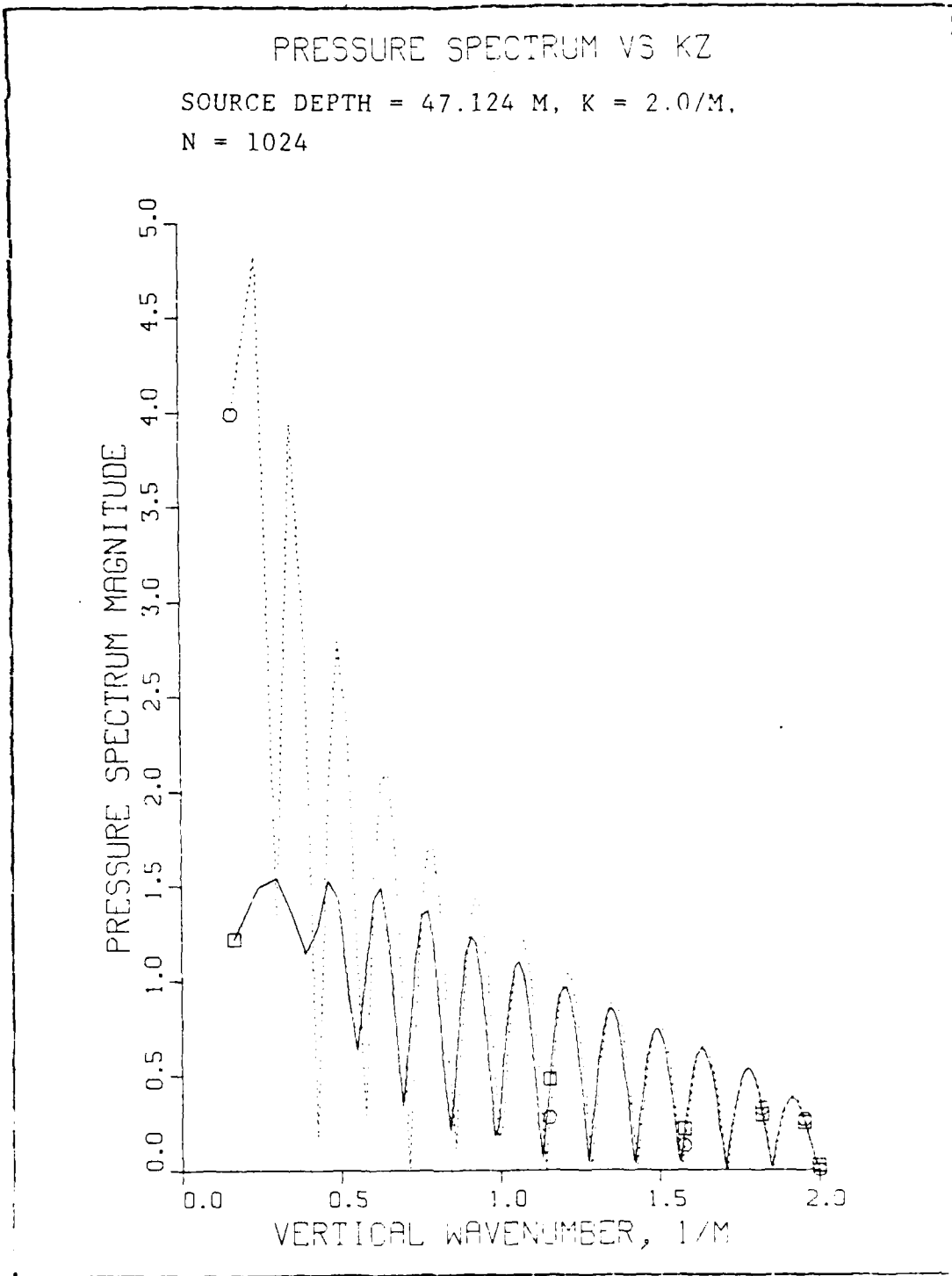


Figure 3.12 Graph of Pressure Spectrum,
Receiver at 22.0 Meters

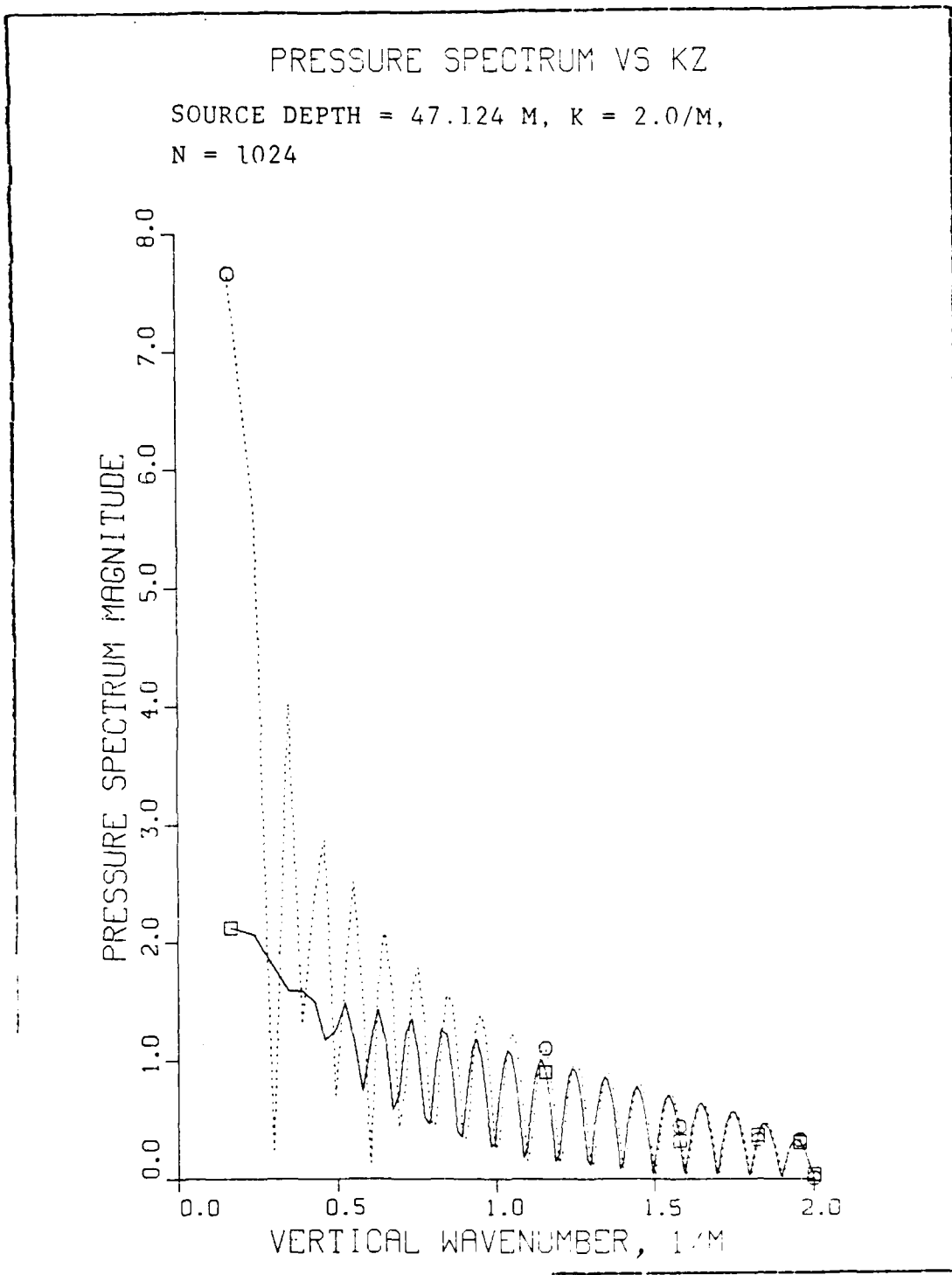


Figure 3.13 Graph of Pressure Spectrum, Receiver at 31.4 Meters

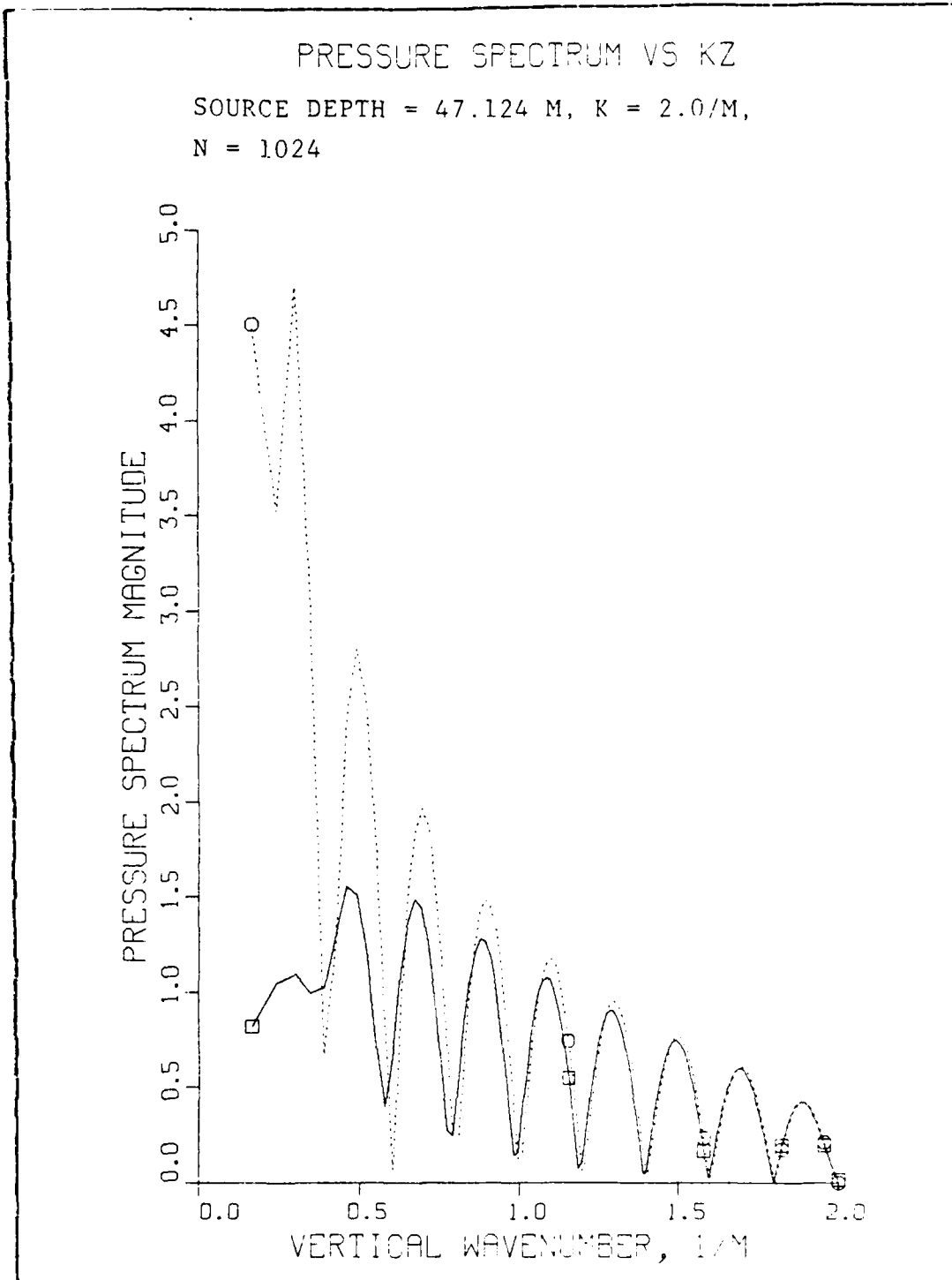


Figure 3.14 Graph of Pressure Spectrum, Receiver at 15.7 Meters

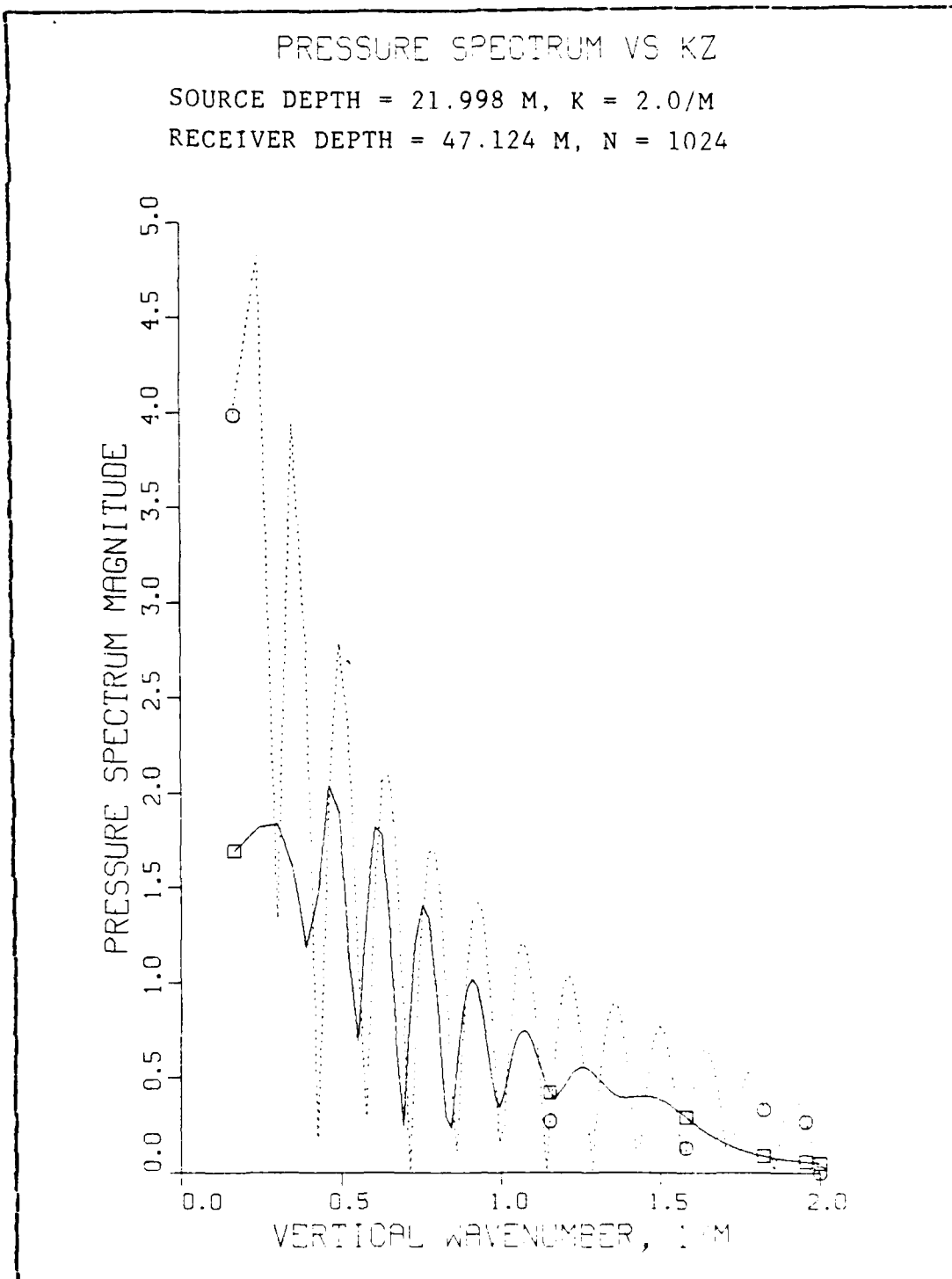


Figure 3.15 Pressure Spectrum,
Range Window Set at 47.1 Meters

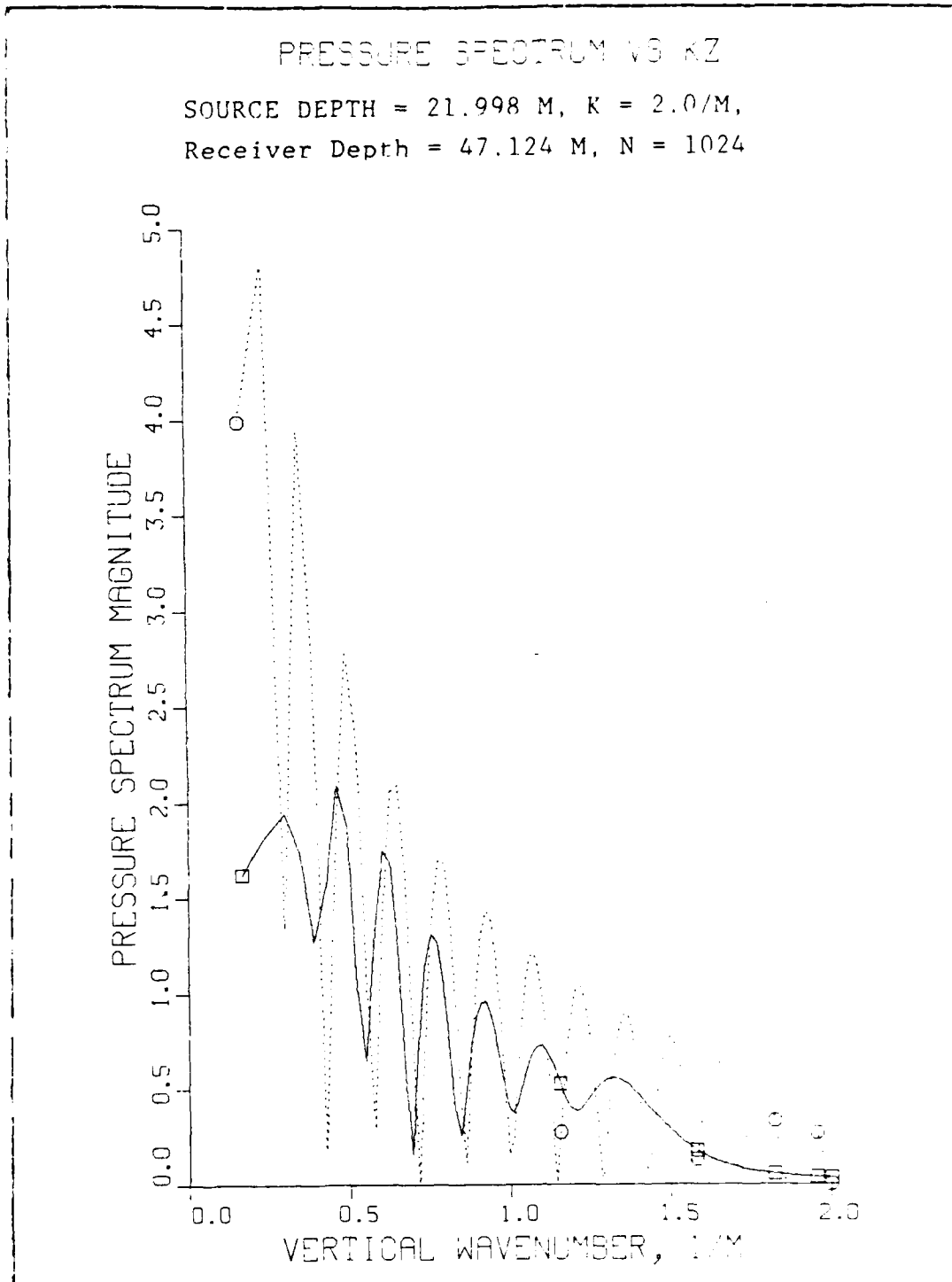


Figure 3.16 Pressure Spectrum,
Range Window Set at 50.3 Meters

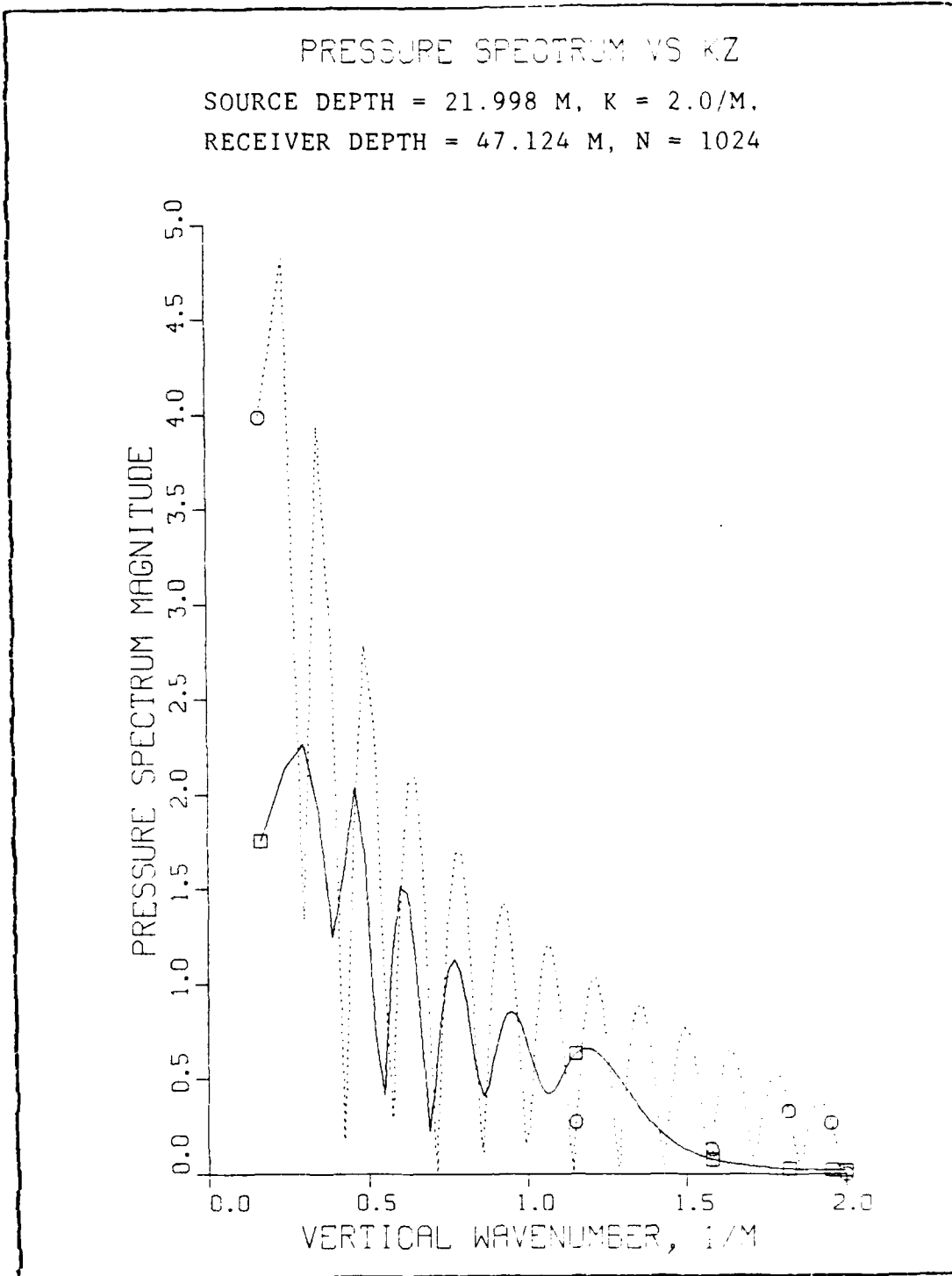


Figure 3.17 Pressure Spectrum,
Range Window Set at 62.8 Meters

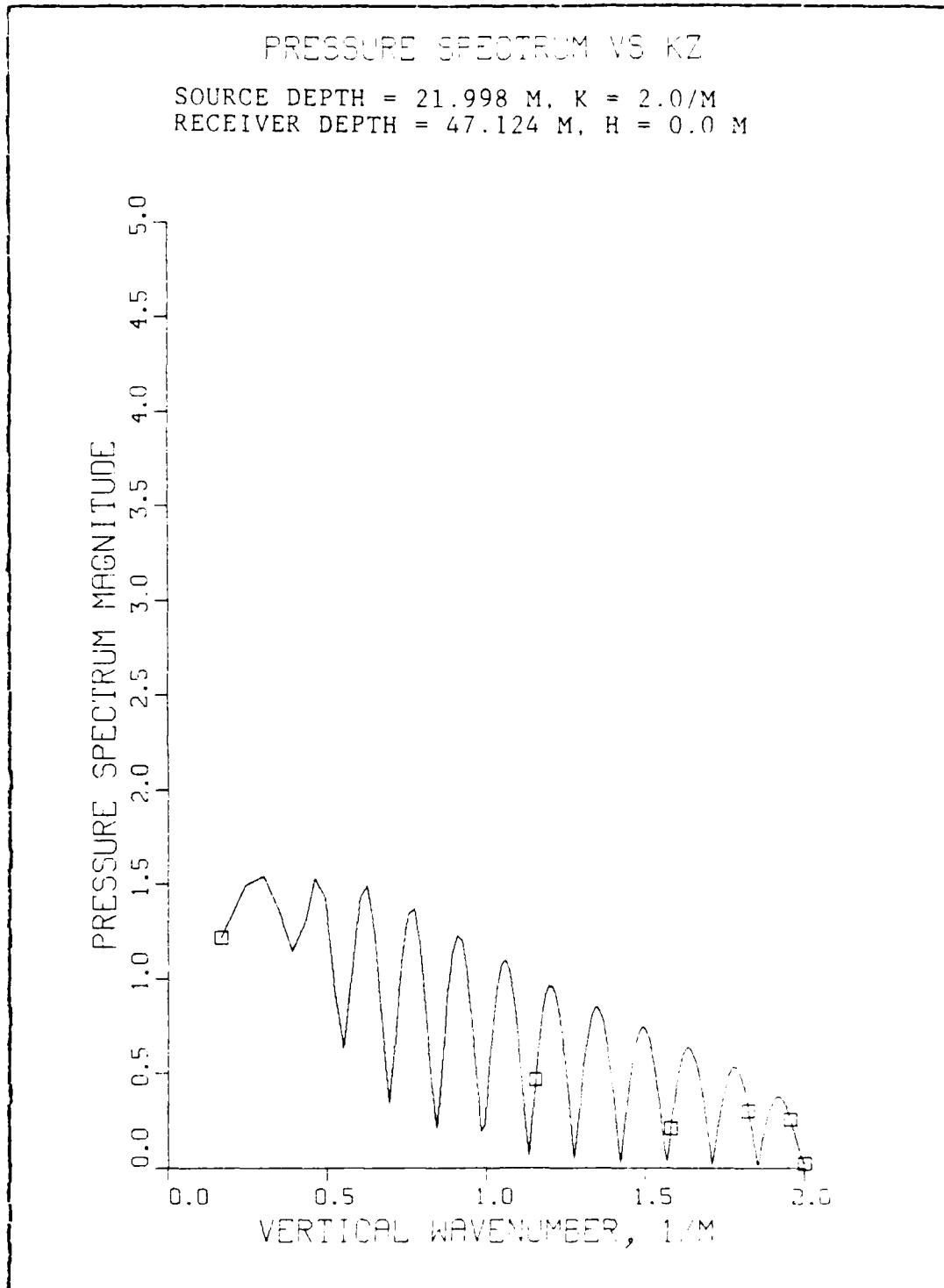


Figure 3.18 Pressure Spectrum, Sea State 0

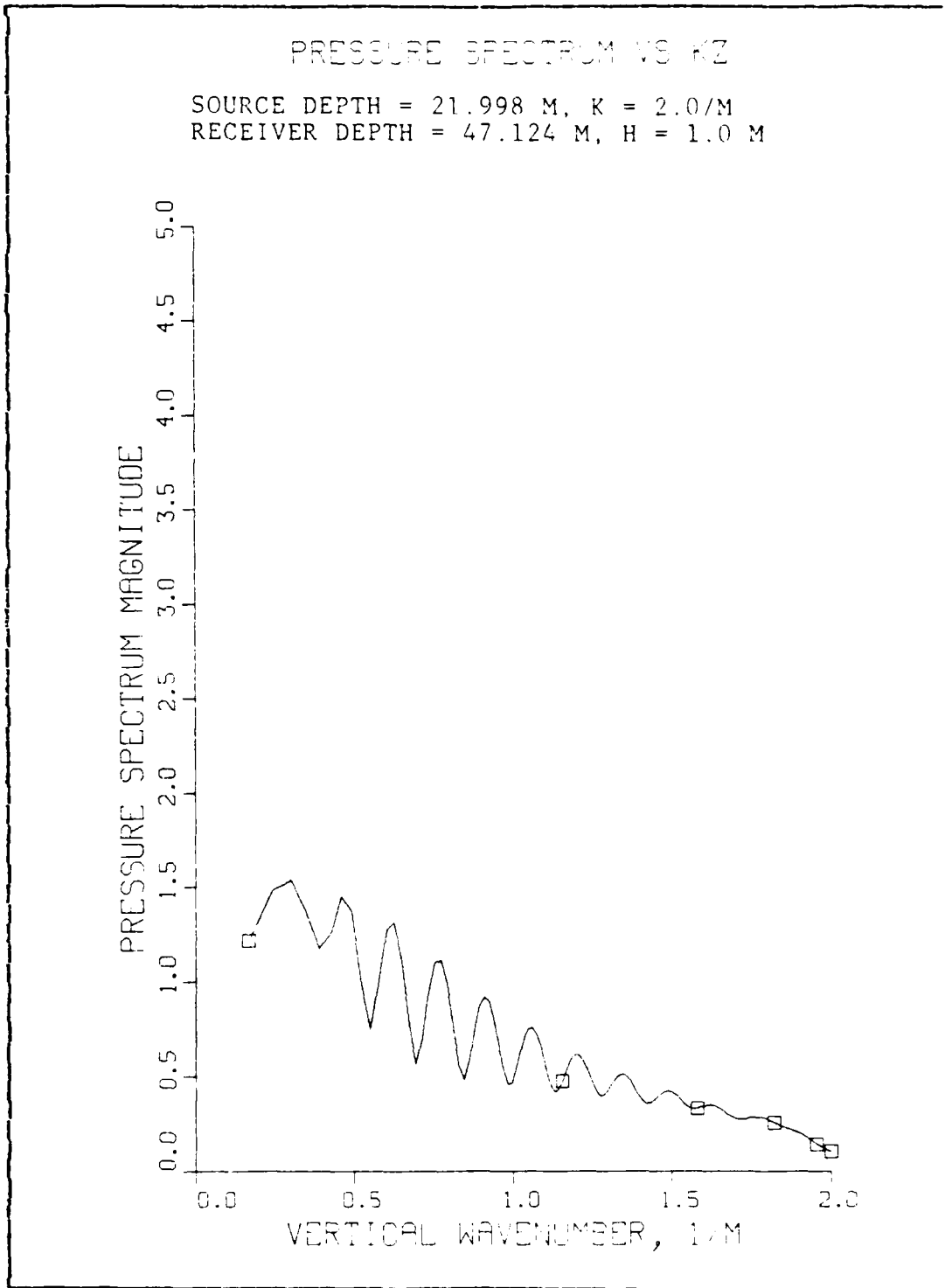


Figure 3.19 Pressure Spectrum, Sea State 2

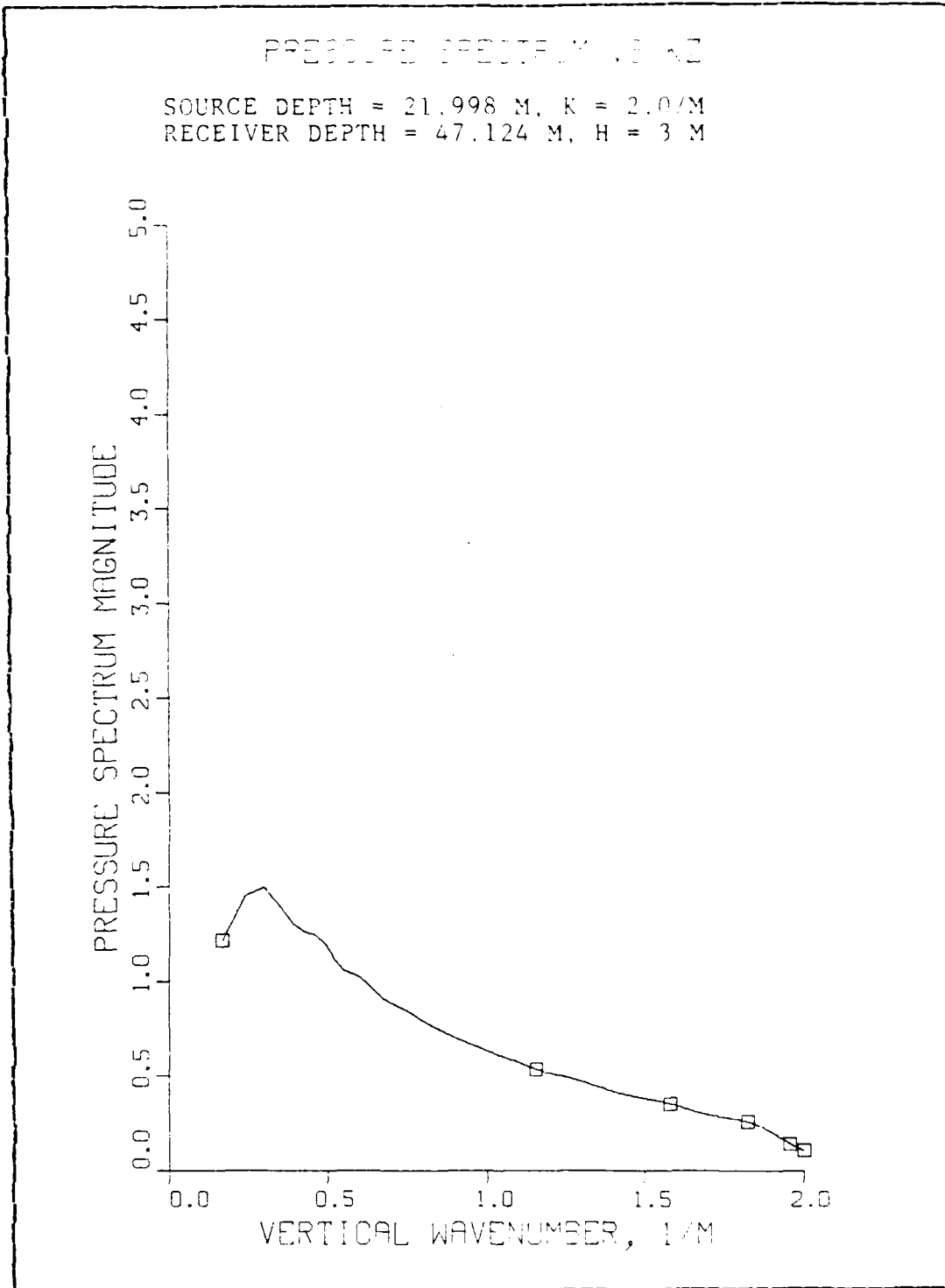


Figure 3.20 Pressure Spectrum, Sea State 3

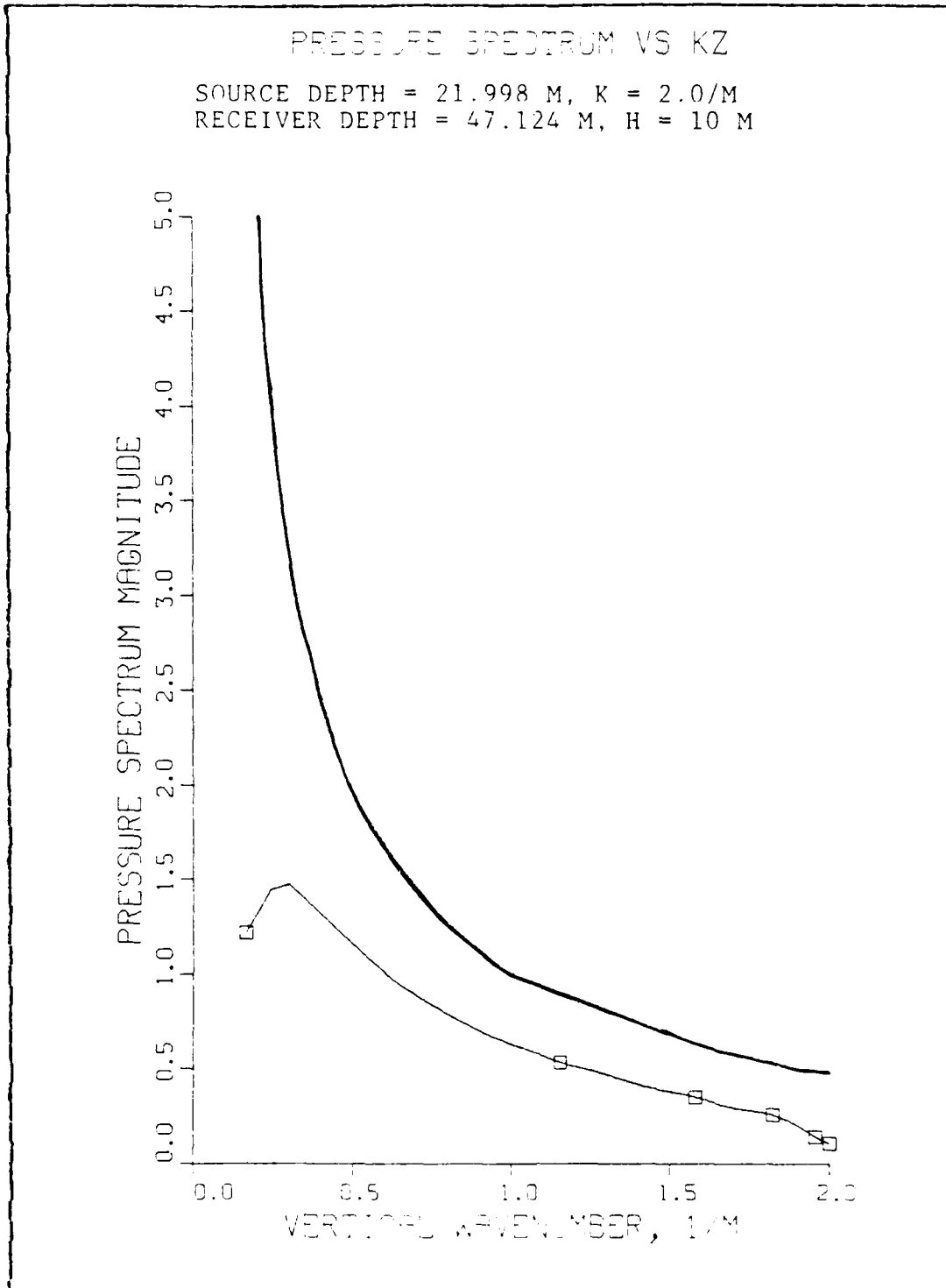


Figure 3.21 Pressure Spectrum, Sea State 5

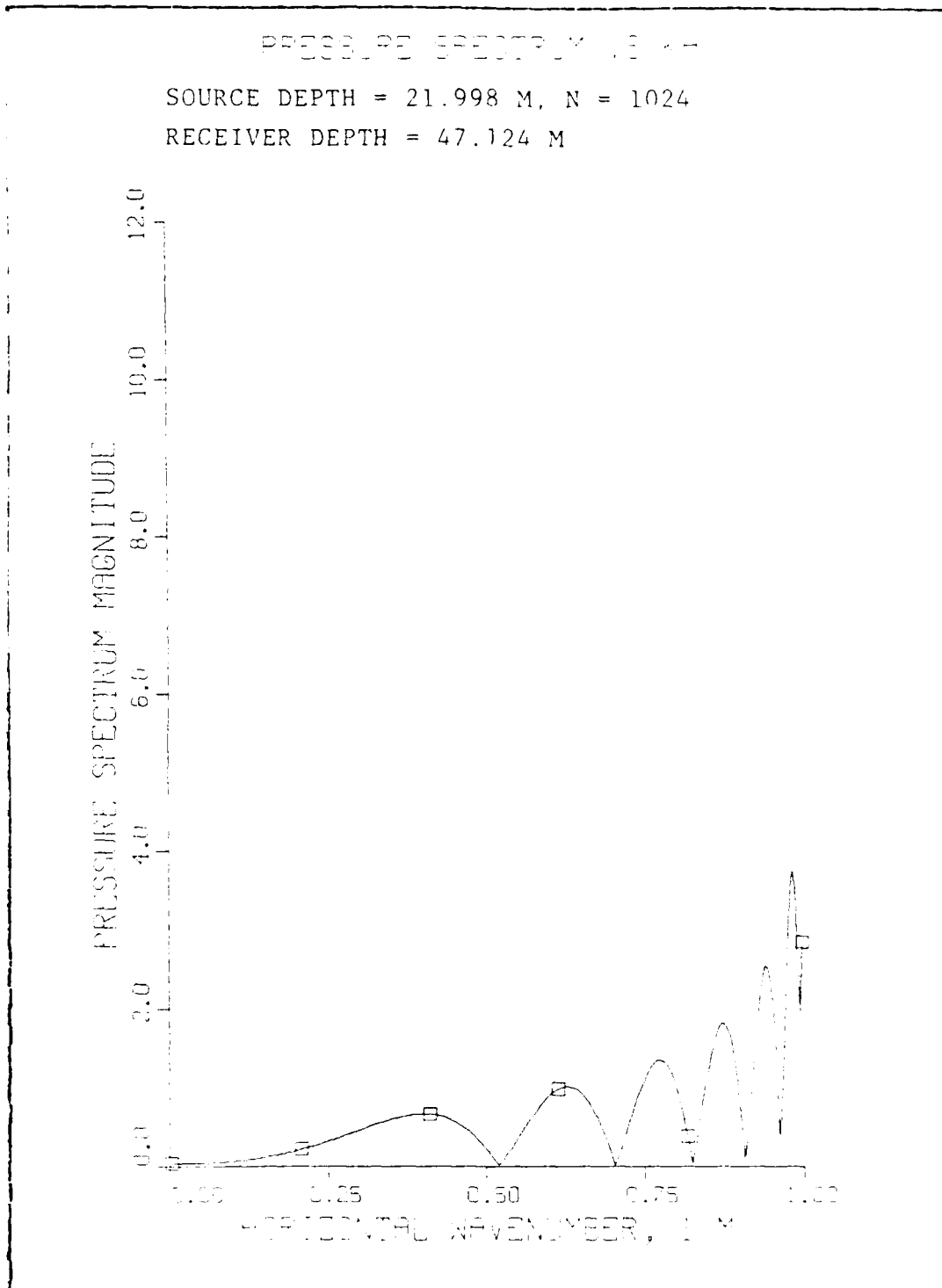


Figure 3.22 Pressure Spectrum, Sea State 0, K = 1.0

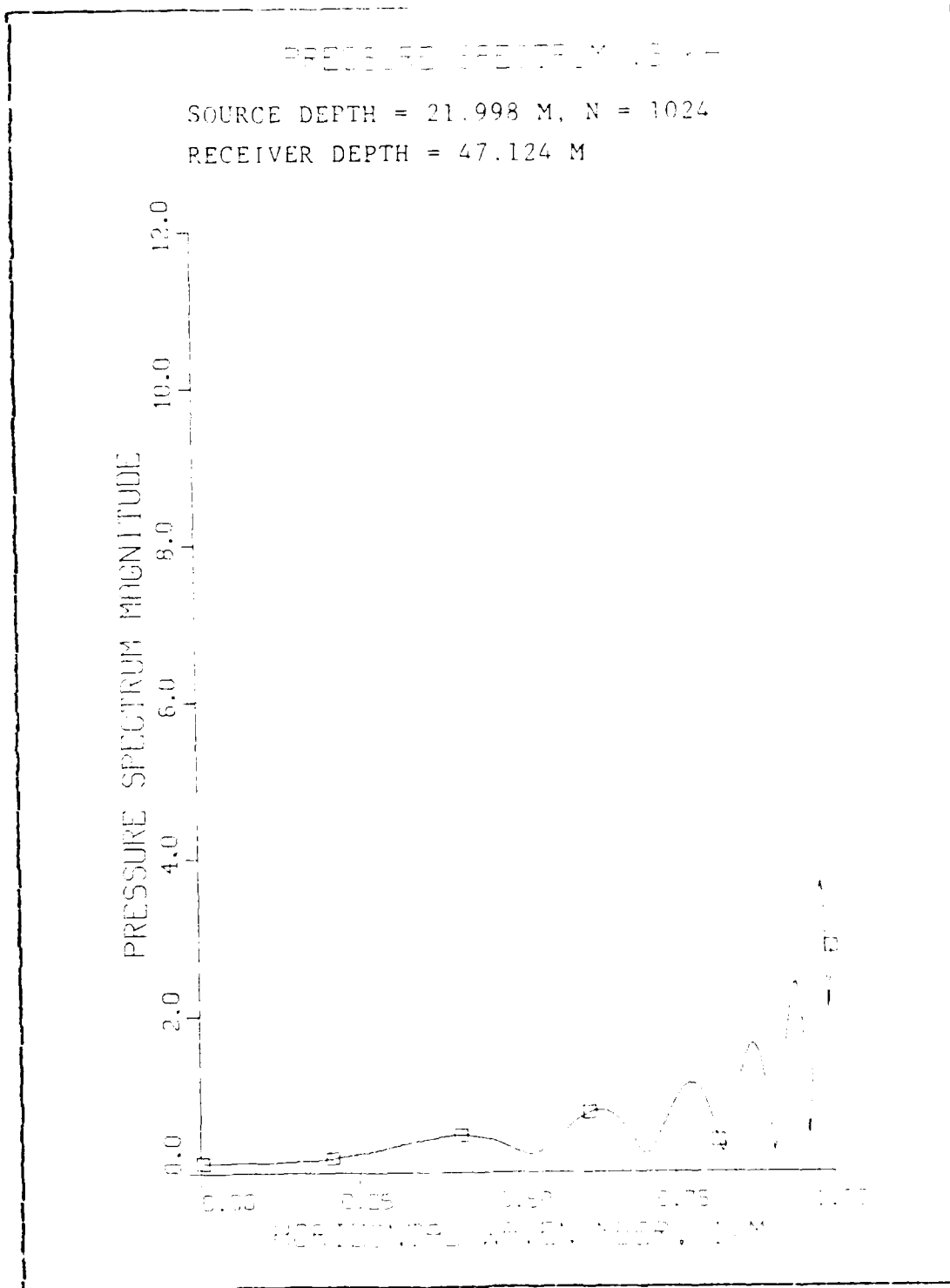


Figure 3.23 Pressure Spectrum, Sea State 3, K = 1.0

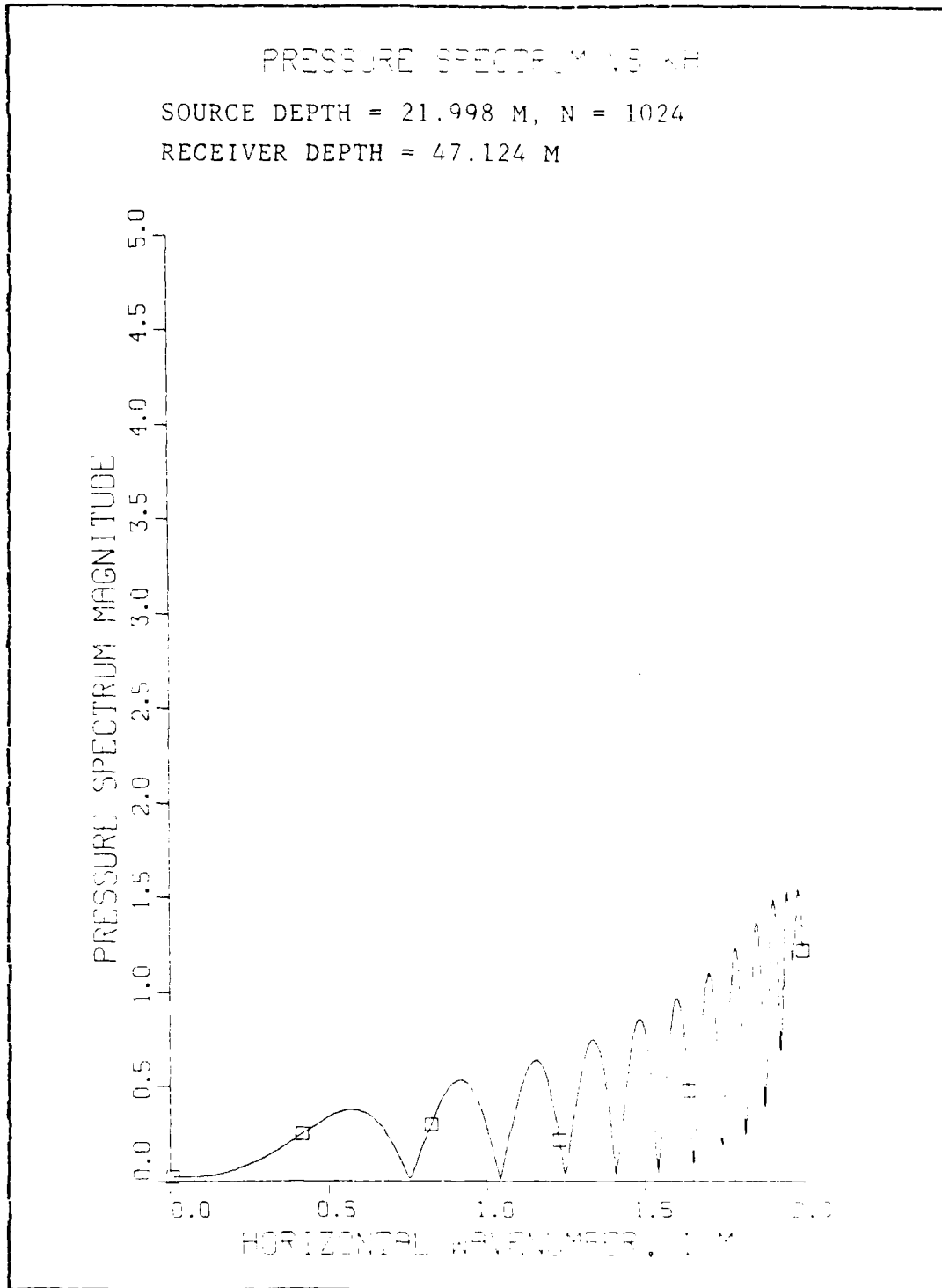


Figure 3.24 Pressure Spectrum, Sea State 0, $K = 2.0$

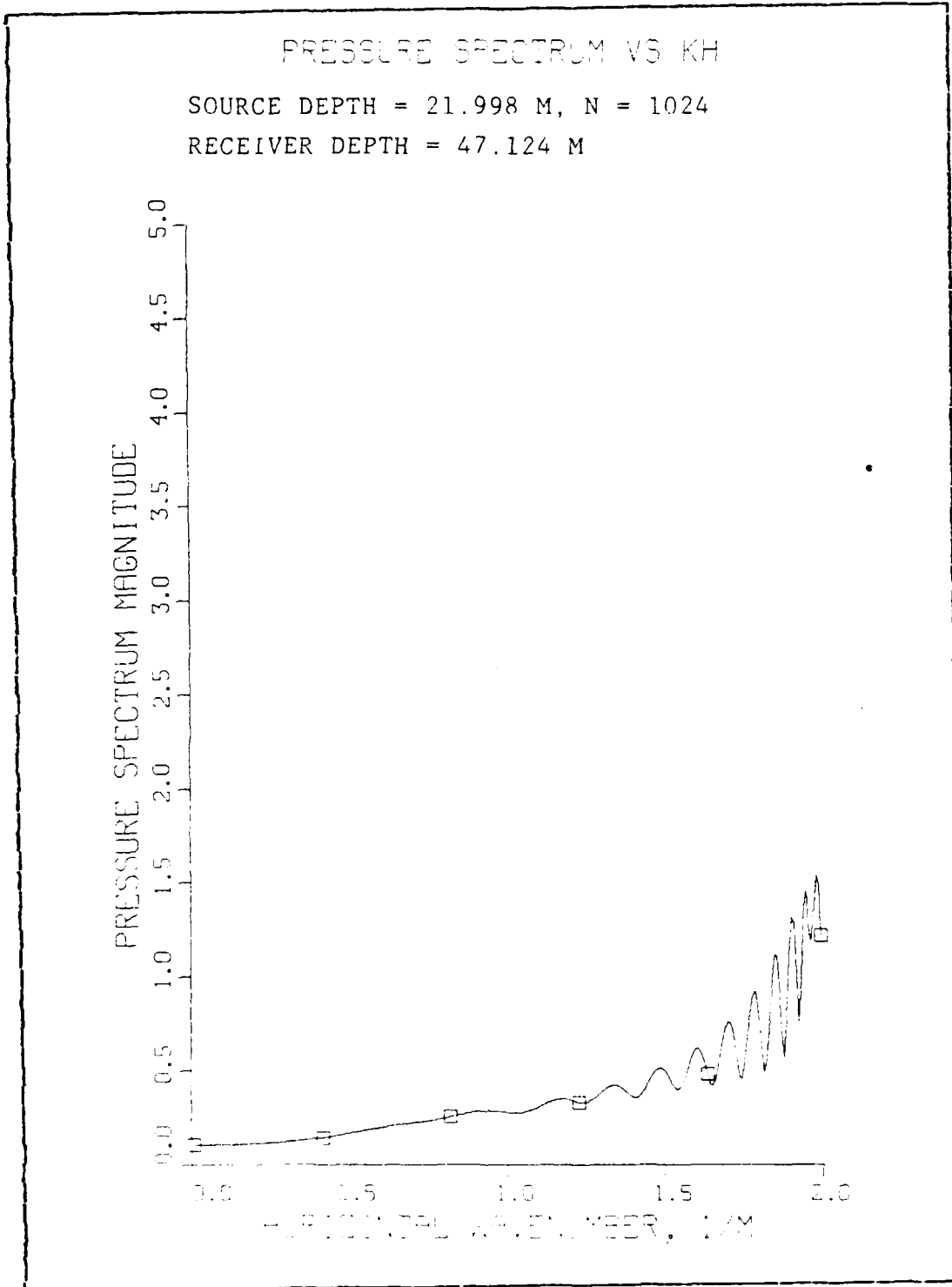


Figure 3.25 Pressure Spectrum, Sea State 3, $K = 2.0$

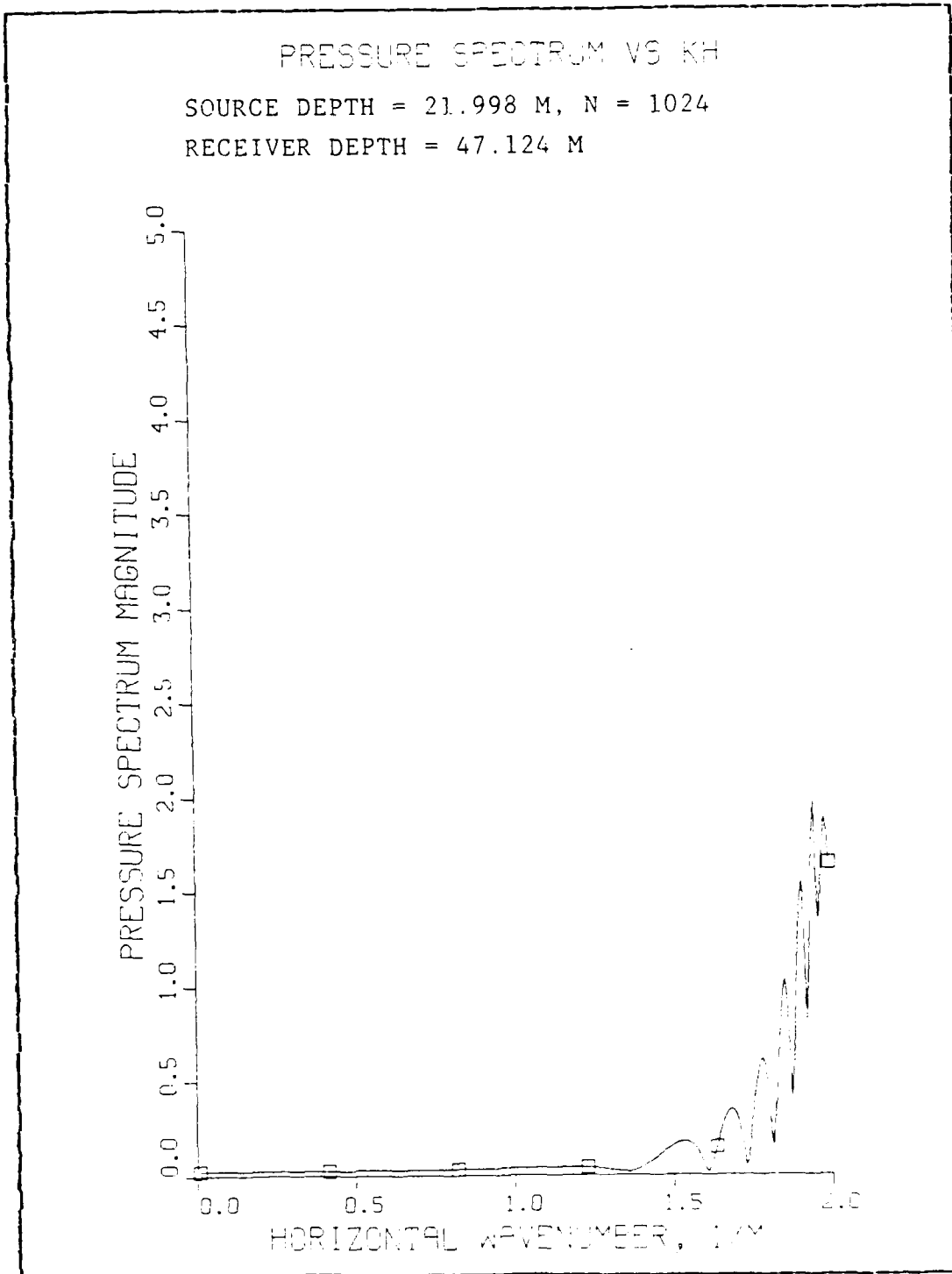


Figure 3.26 Pressure Spectrum,
SS 2, Range Window Set at 50 Meters

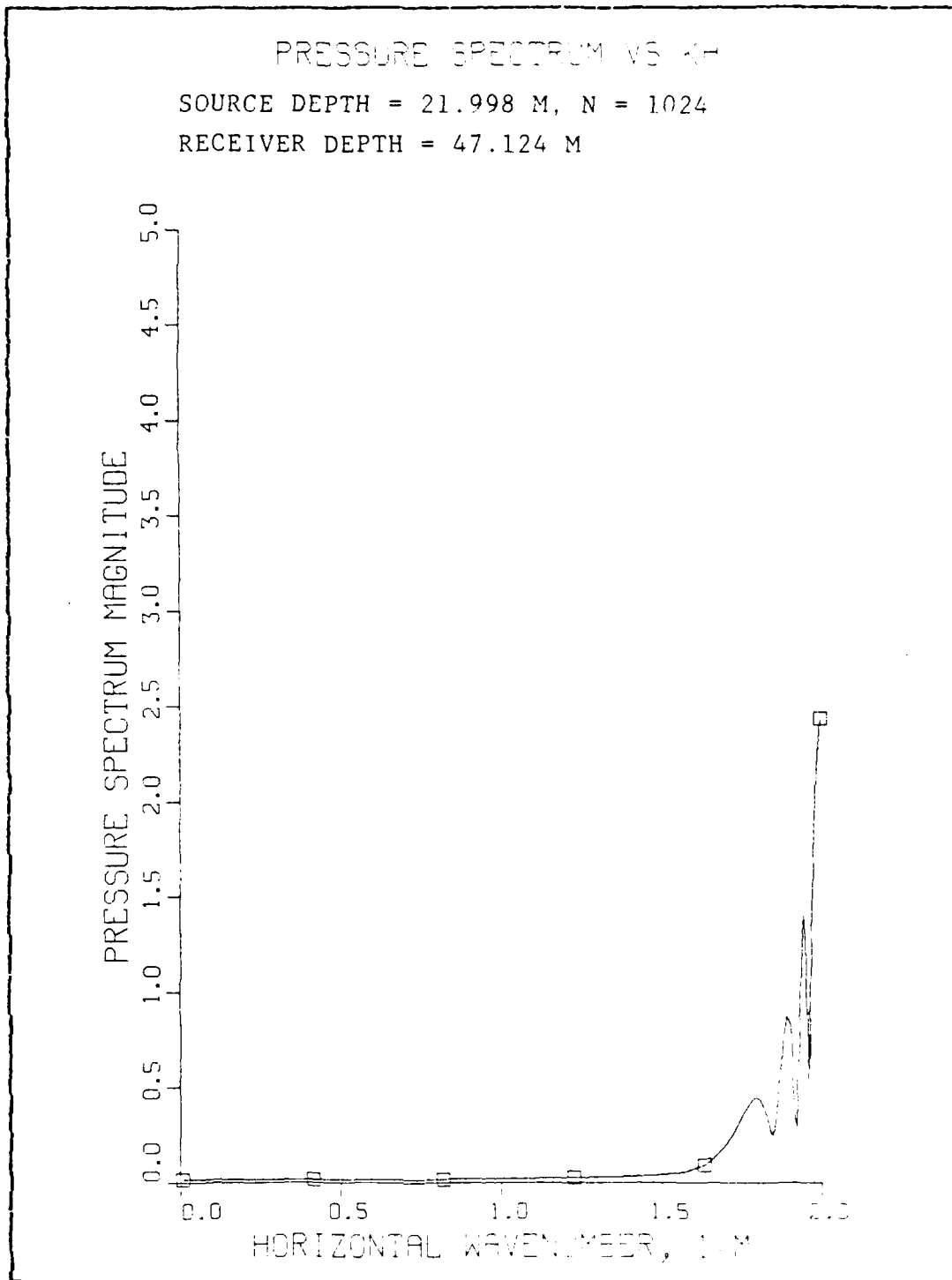


Figure 3.27 Pressure Spectrum,
SS 2, Range Window Set at 100 Meters

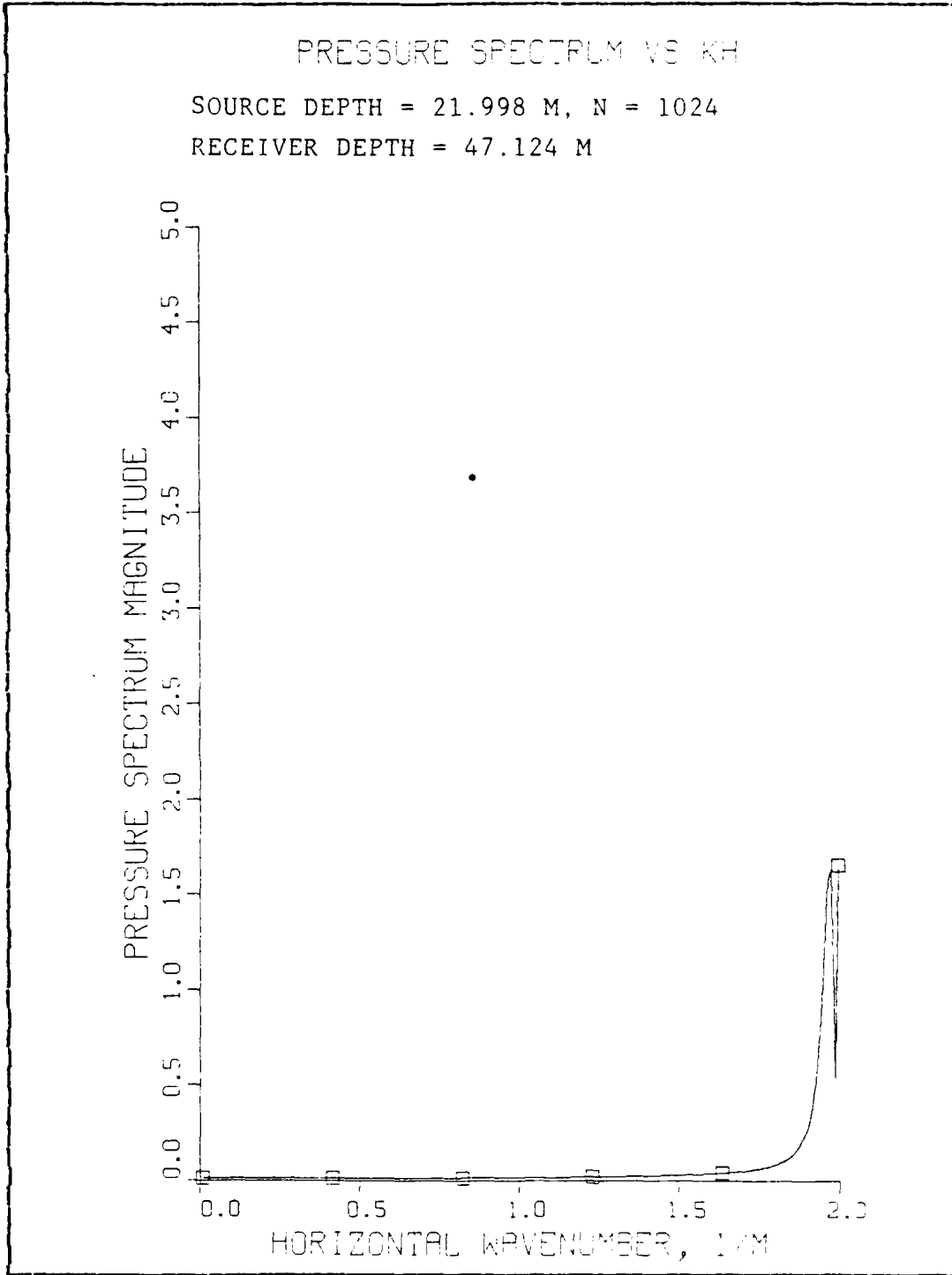


Figure 3.28 Pressure Spectrum,
SS 2, Range Window Set at 200 Meters

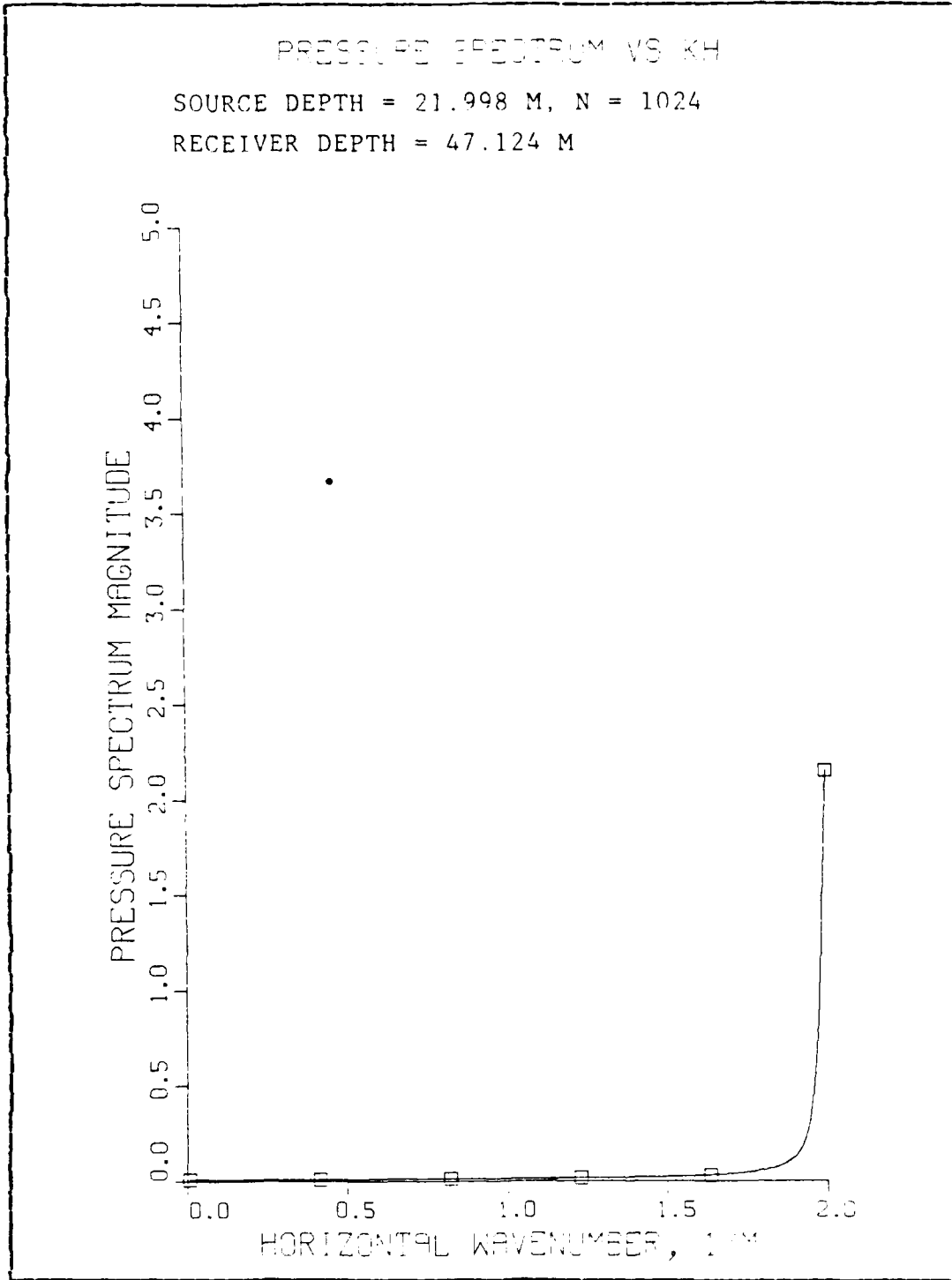


Figure 3.29 Pressure Spectrum,
SS 2, Range Window Set at 300 Meters

MAGNITUDE OF PRESSURE AS A FN OF RANGE

SOURCE DEPTH = 21.998 M, RECEIVER DEPTH = 47.124 M,
RANGE STEP SIZE = 1.57 M, N = 1024

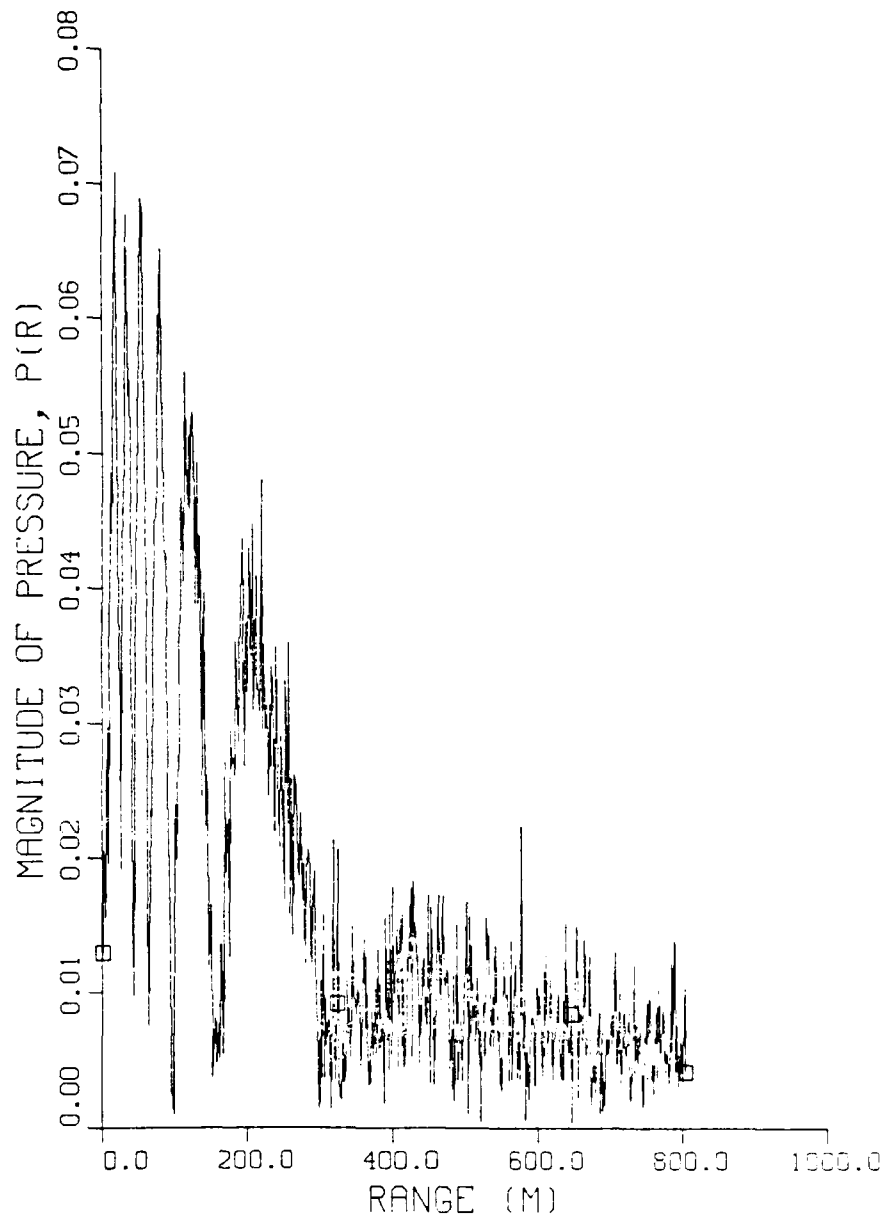


Figure 3.30 Pressure Field, $K = 1.0$, $\mu = 0.005$

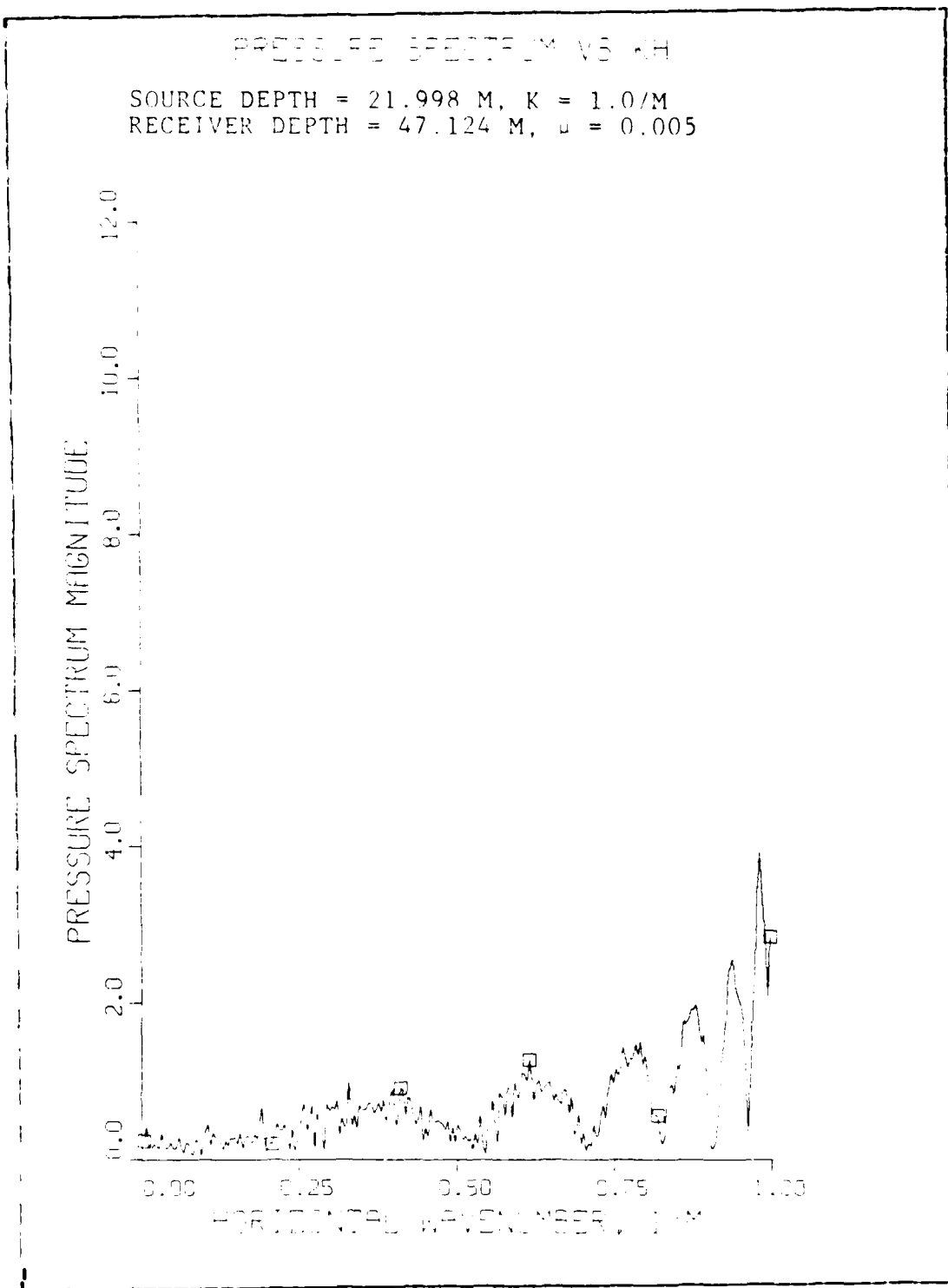


Figure 3.31 Pressure Spectrum vs. Gamma

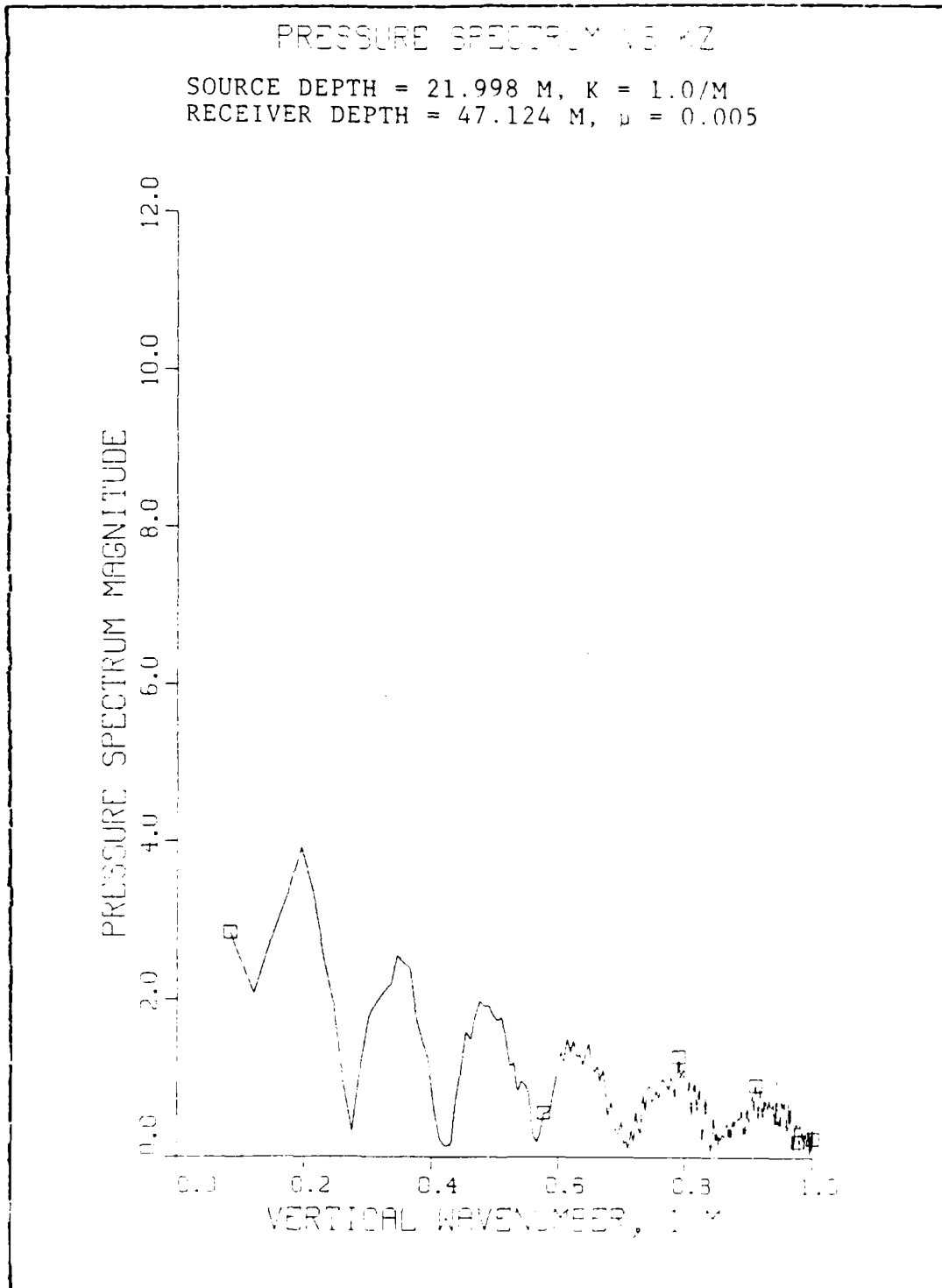


Figure 3.32 Pressure Spectrum vs. Beta

MAGNITUDE OF PRESSURE AS A FN OF RANGE

SOURCE DEPTH = 21.998 M, RECEIVER DEPTH = 47.124 M,
RANGE STEP SIZE = 1.57 M, N = 1024

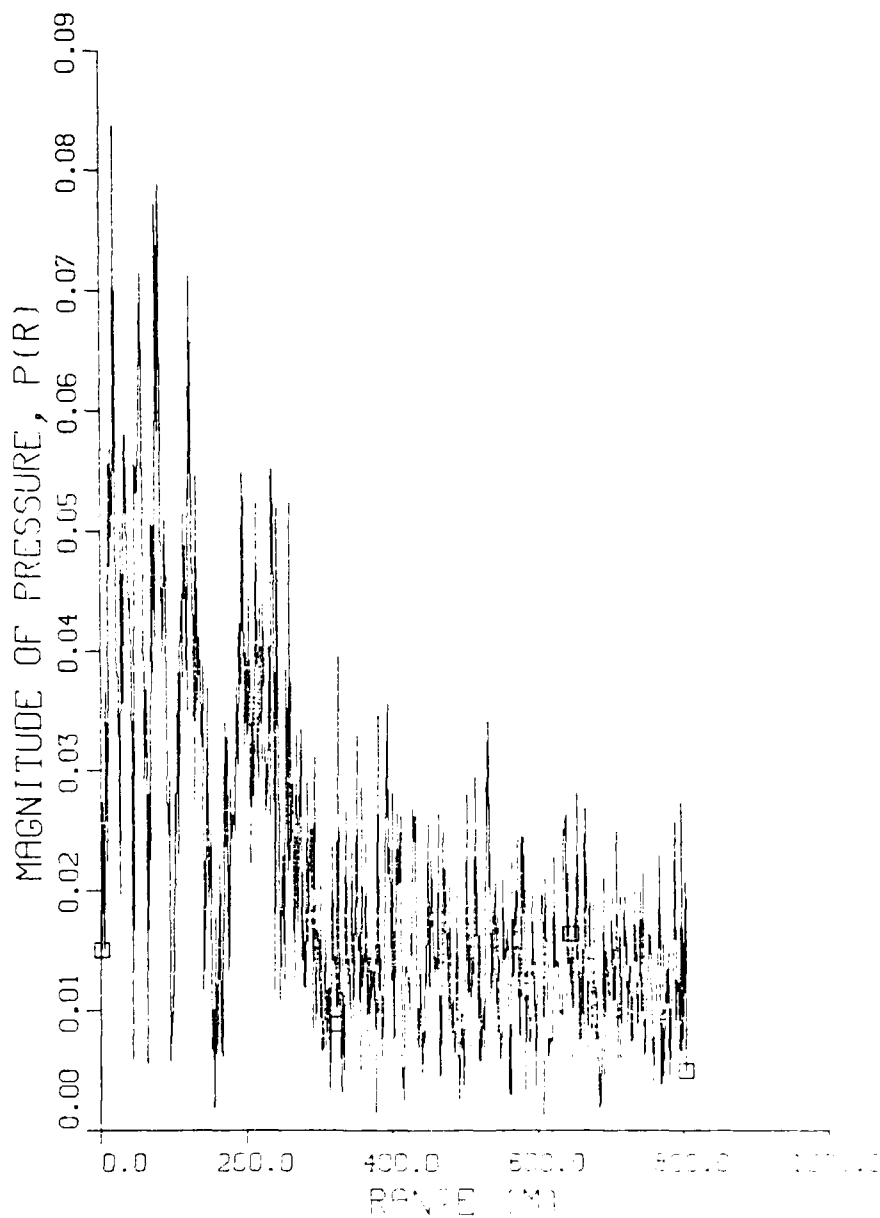


Figure 3.33 Pressure Field, $K = 1.0$, $\mu = 0.01$

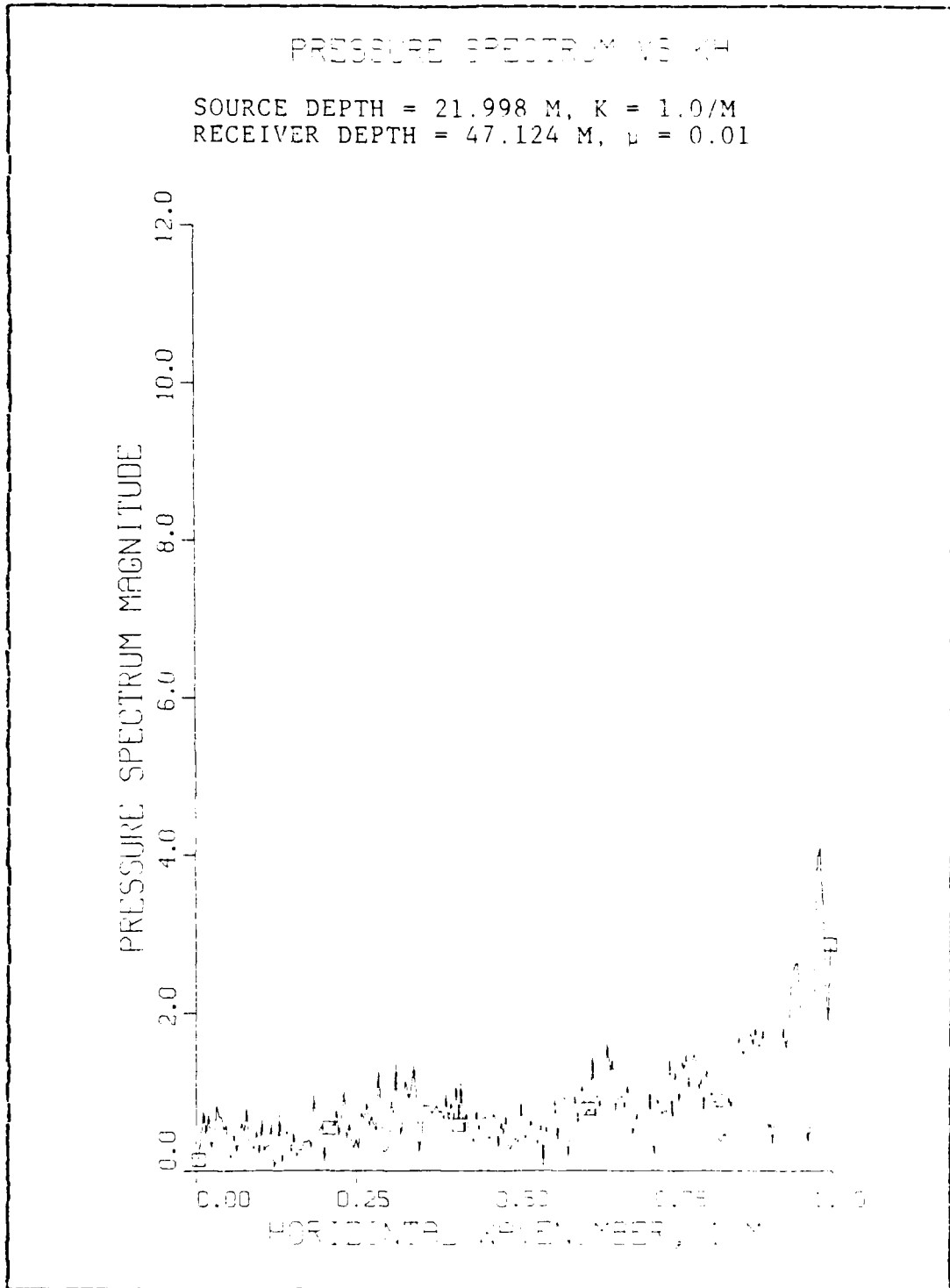


Figure 3.34 Pressure Spectrum vs. Gamma

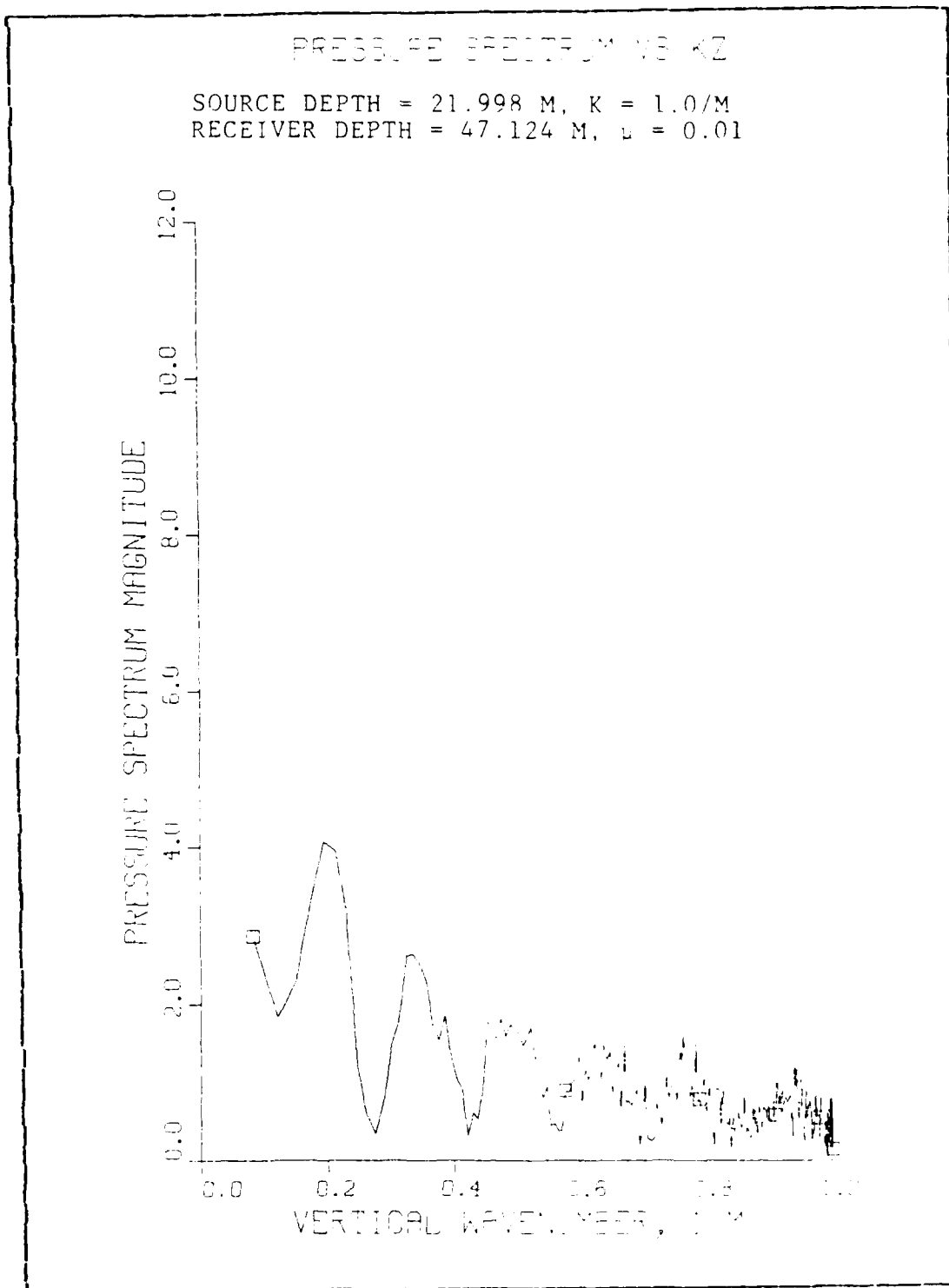


Figure 3.35 Pressure Spectrum vs. Beta

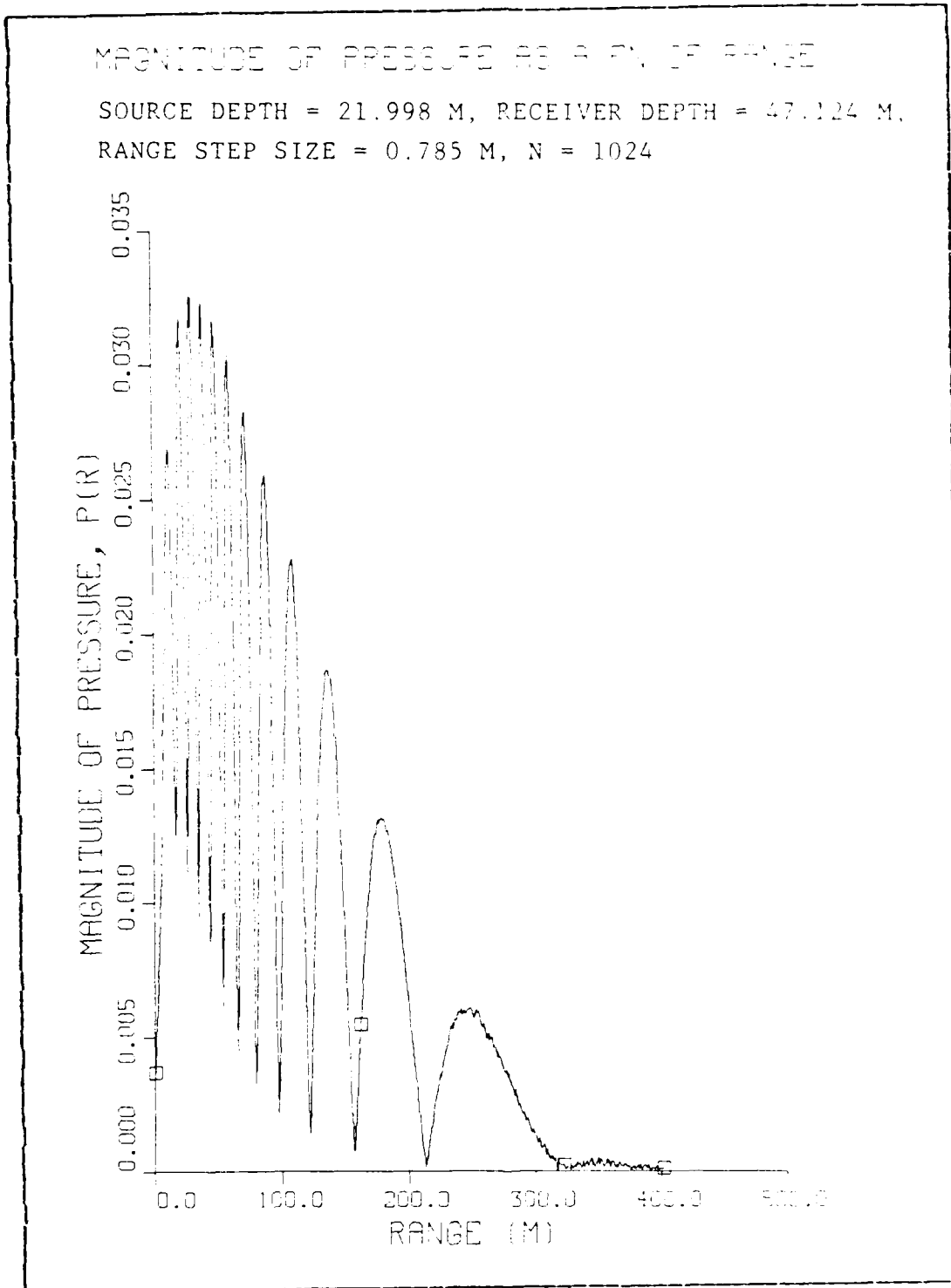


Figure 3.36 Pressure Field, $K = 2.0$, $\mu = 0.0001$

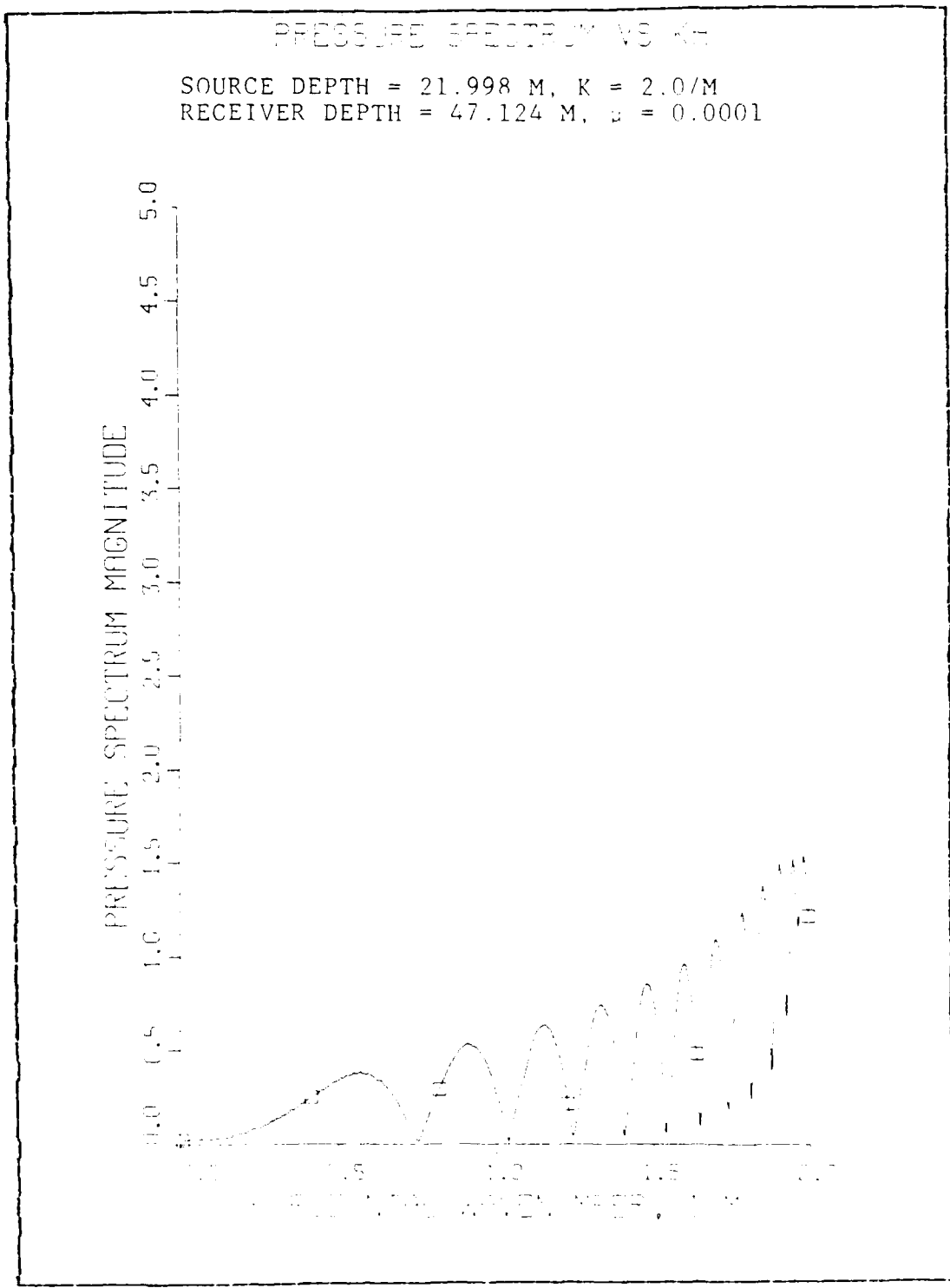


Figure 3.37 Pressure Spectrum vs. Gamma

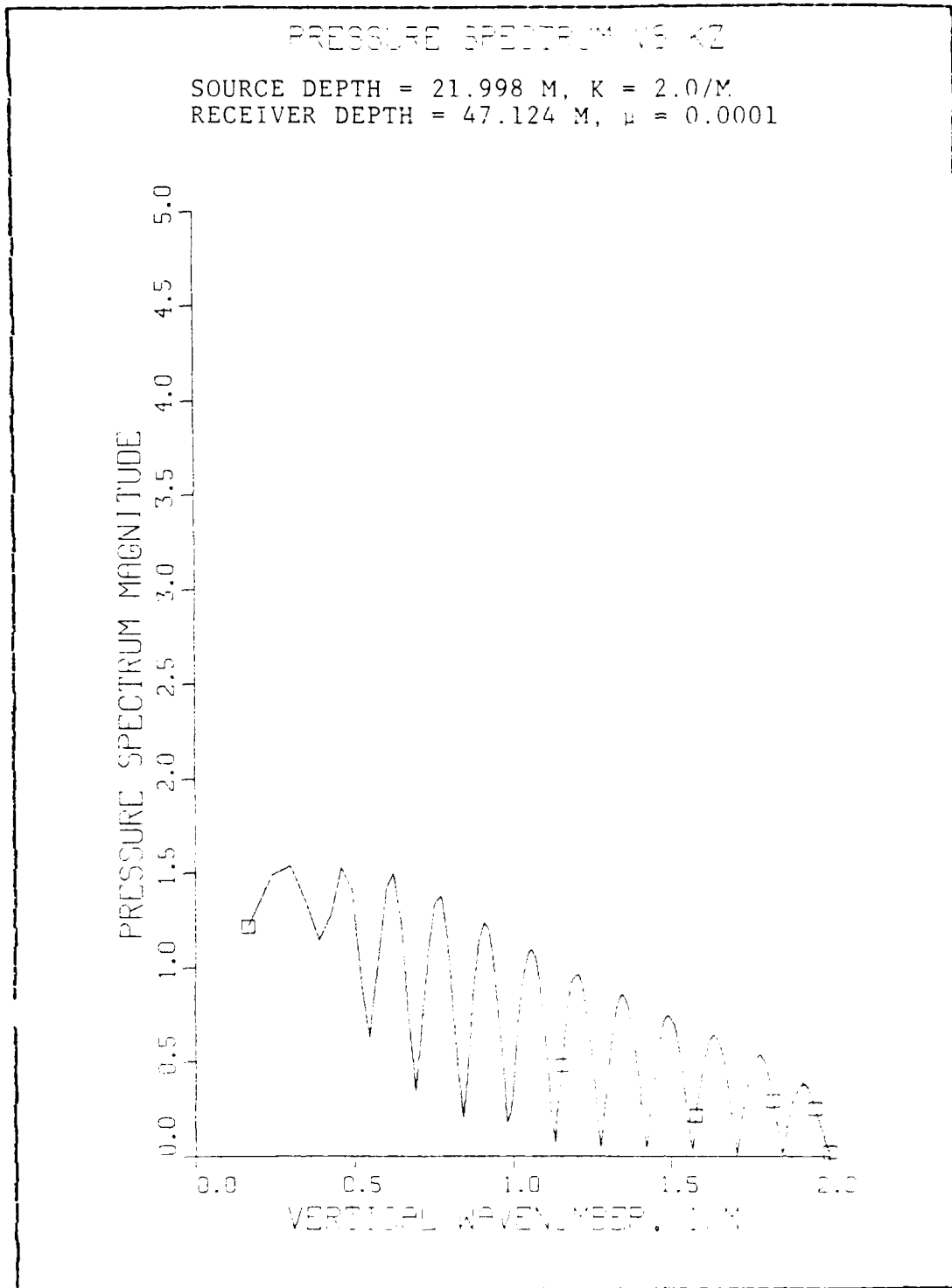


Figure 3.38 Pressure Spectrum vs. Beta

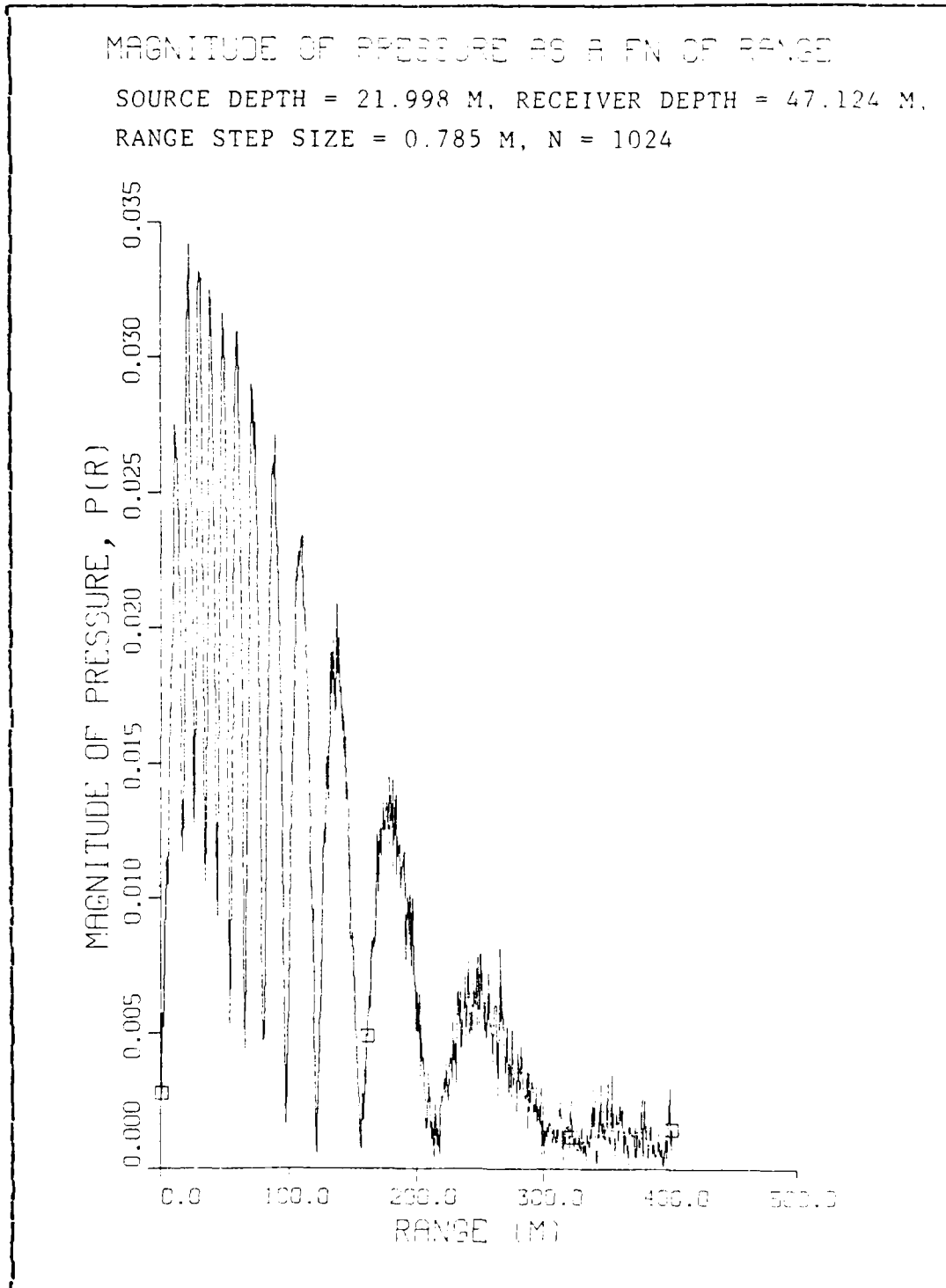


Figure 3.39 Pressure Field, $K = 2.0$, $\mu = 0.001$

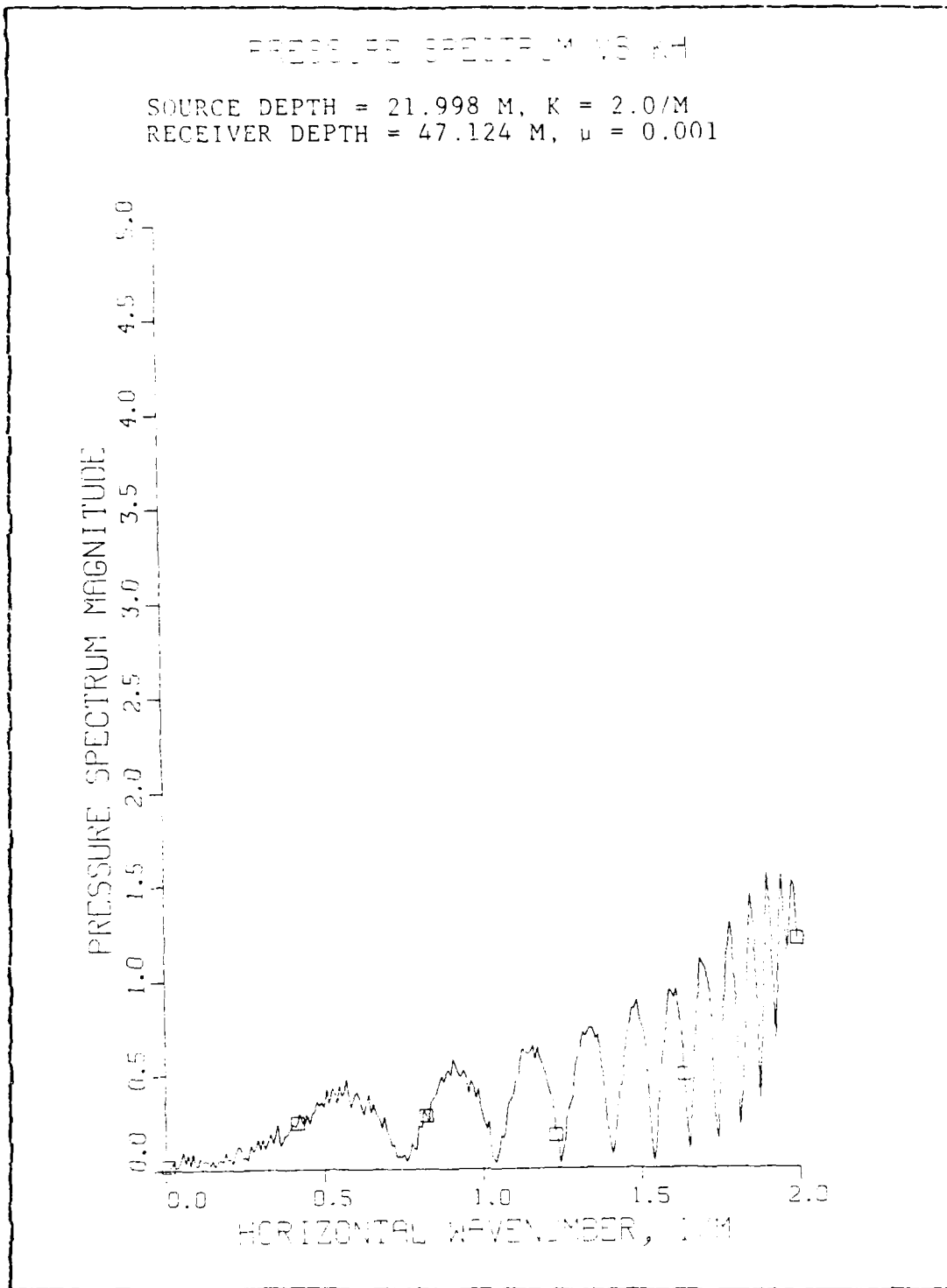


Figure 3.40 Pressure Spectrum vs. Gamma

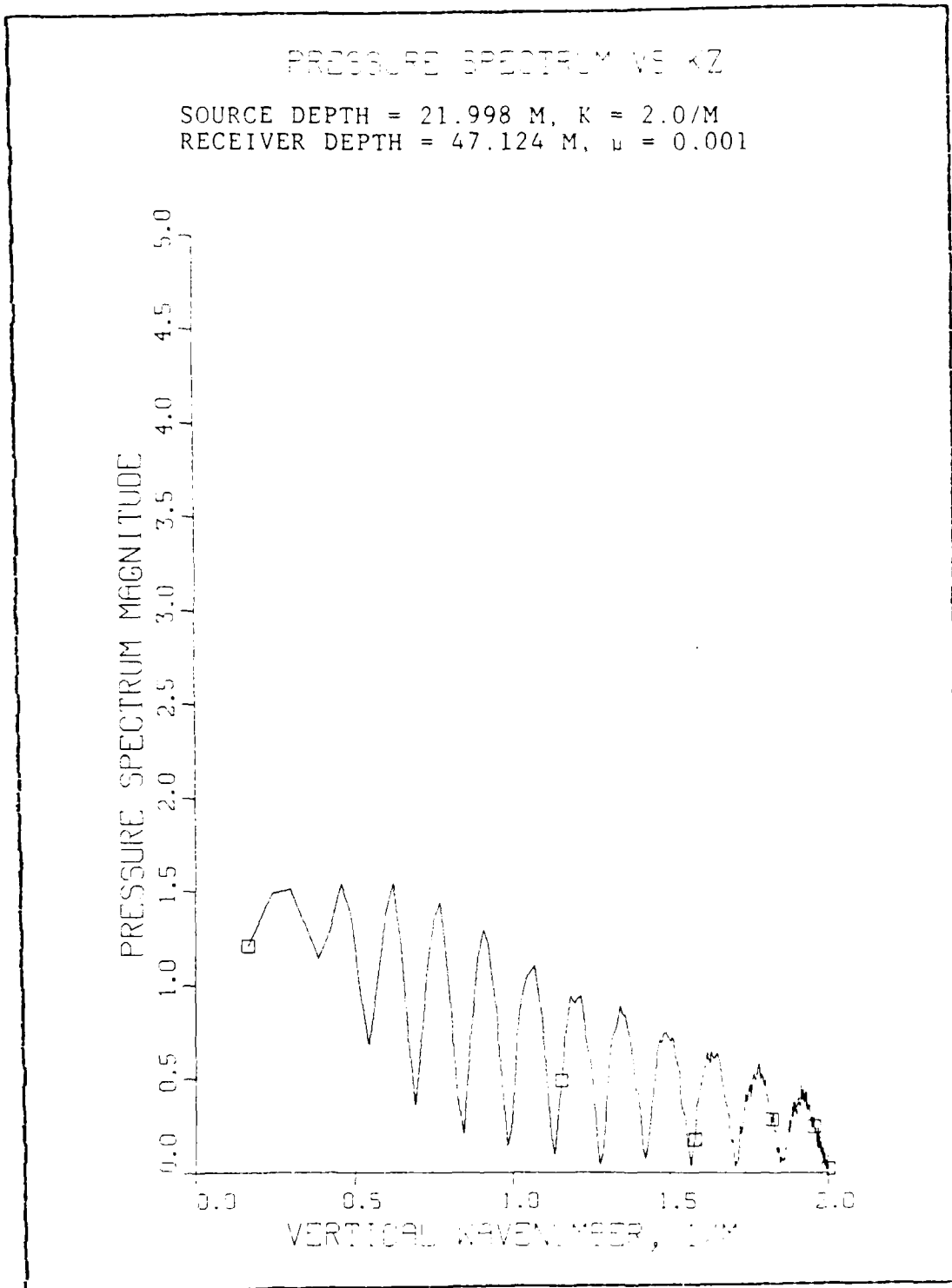


Figure 3.41 Pressure Spectrum vs. Beta

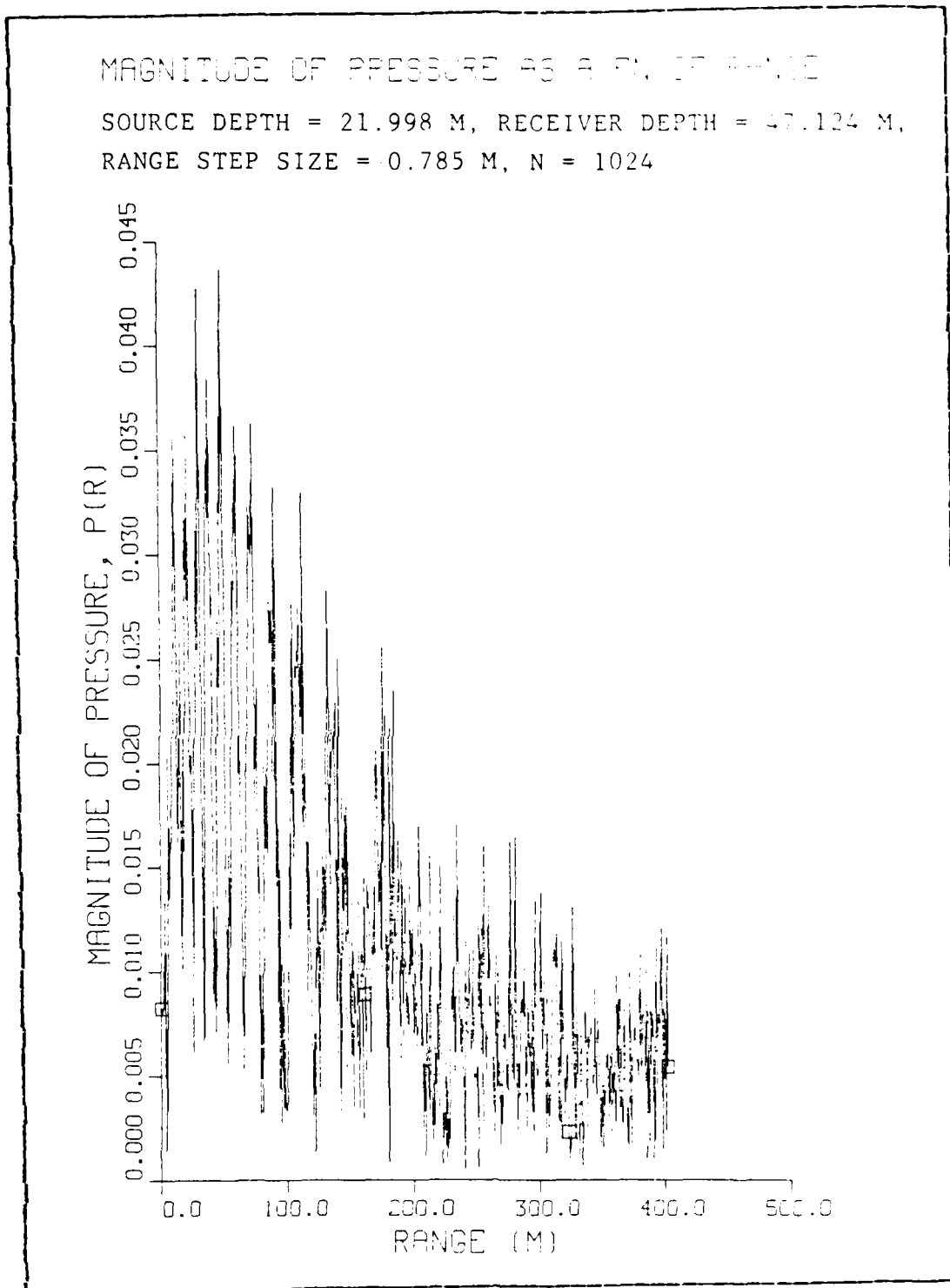


Figure 3.42 Pressure Field, $K = 2.0$, $\mu = 0.005$

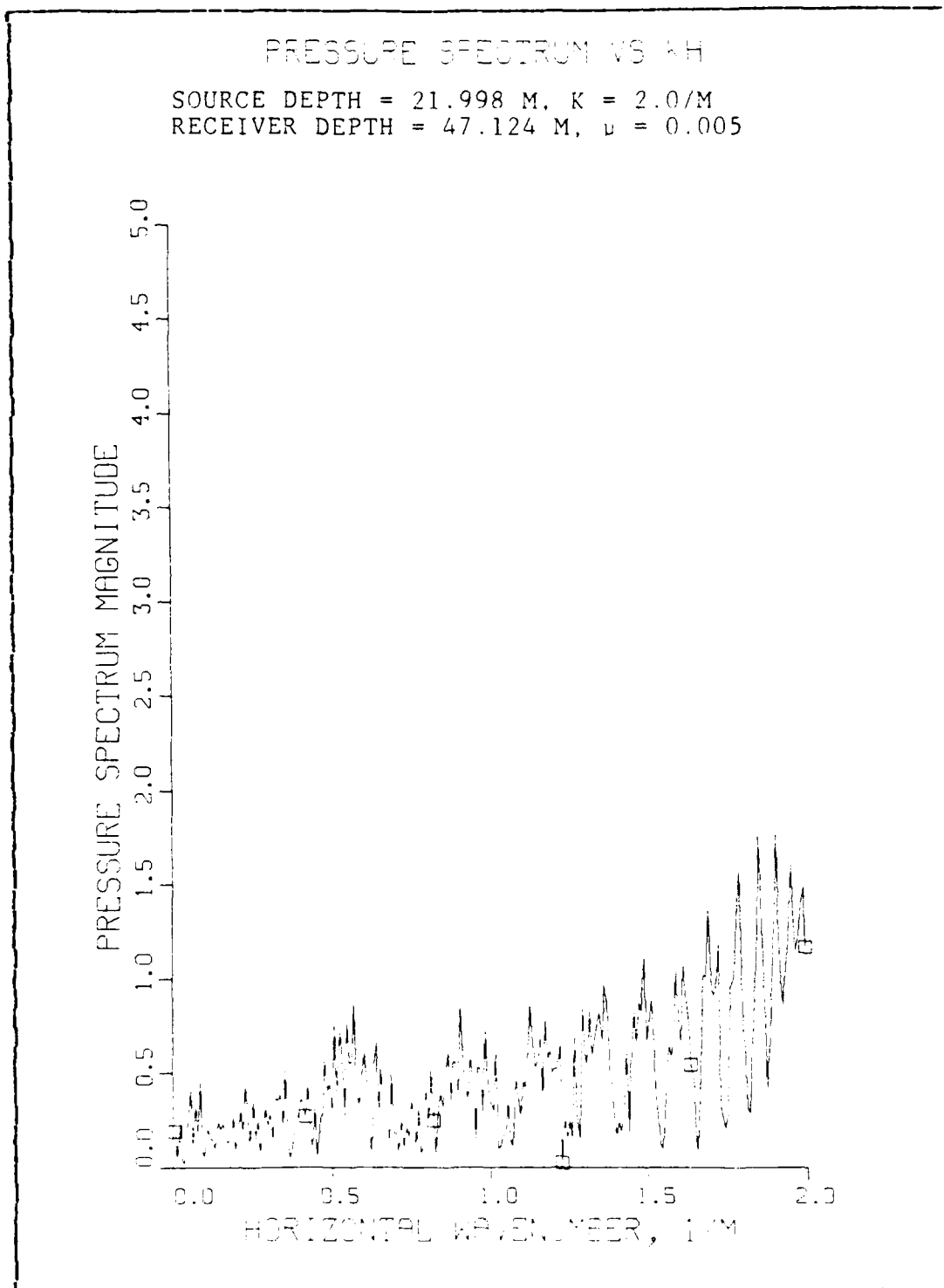


Figure 3.43 Pressure Spectrum vs. Gamma

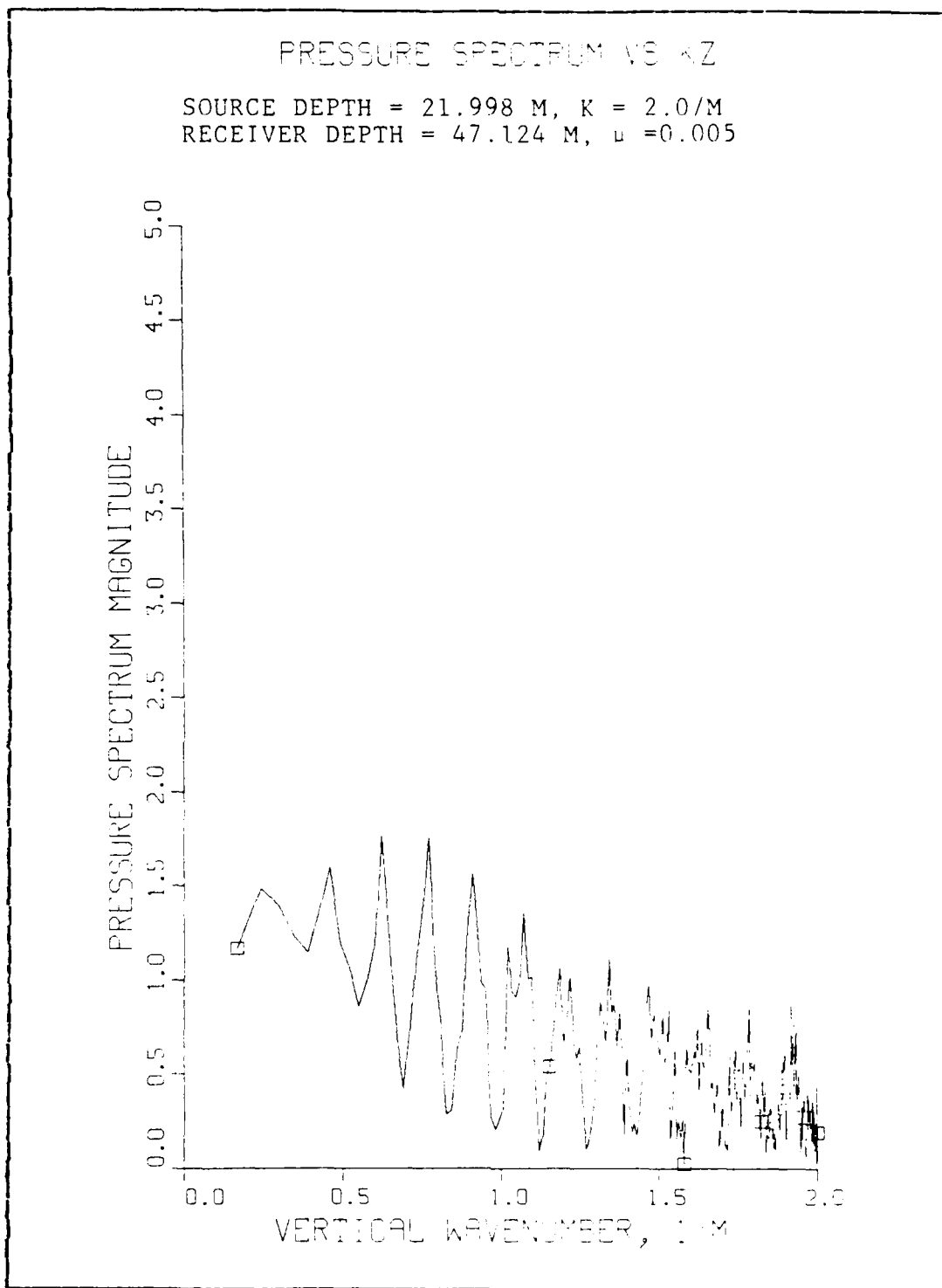


Figure 3.44 Pressure Spectrum vs. Beta

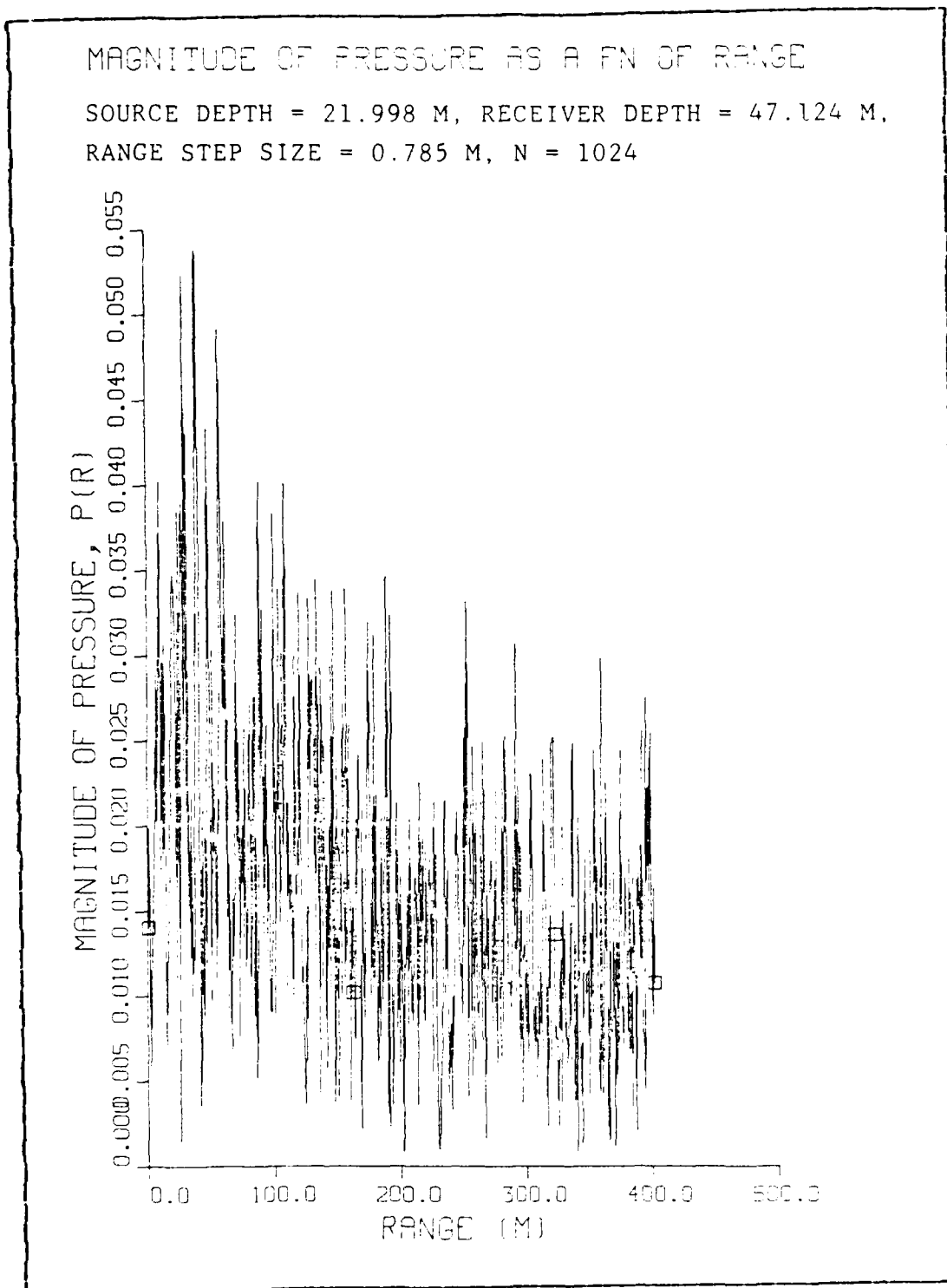


Figure 3.45 Pressure Field, $K = 2.0$, $\mu = 0.01$

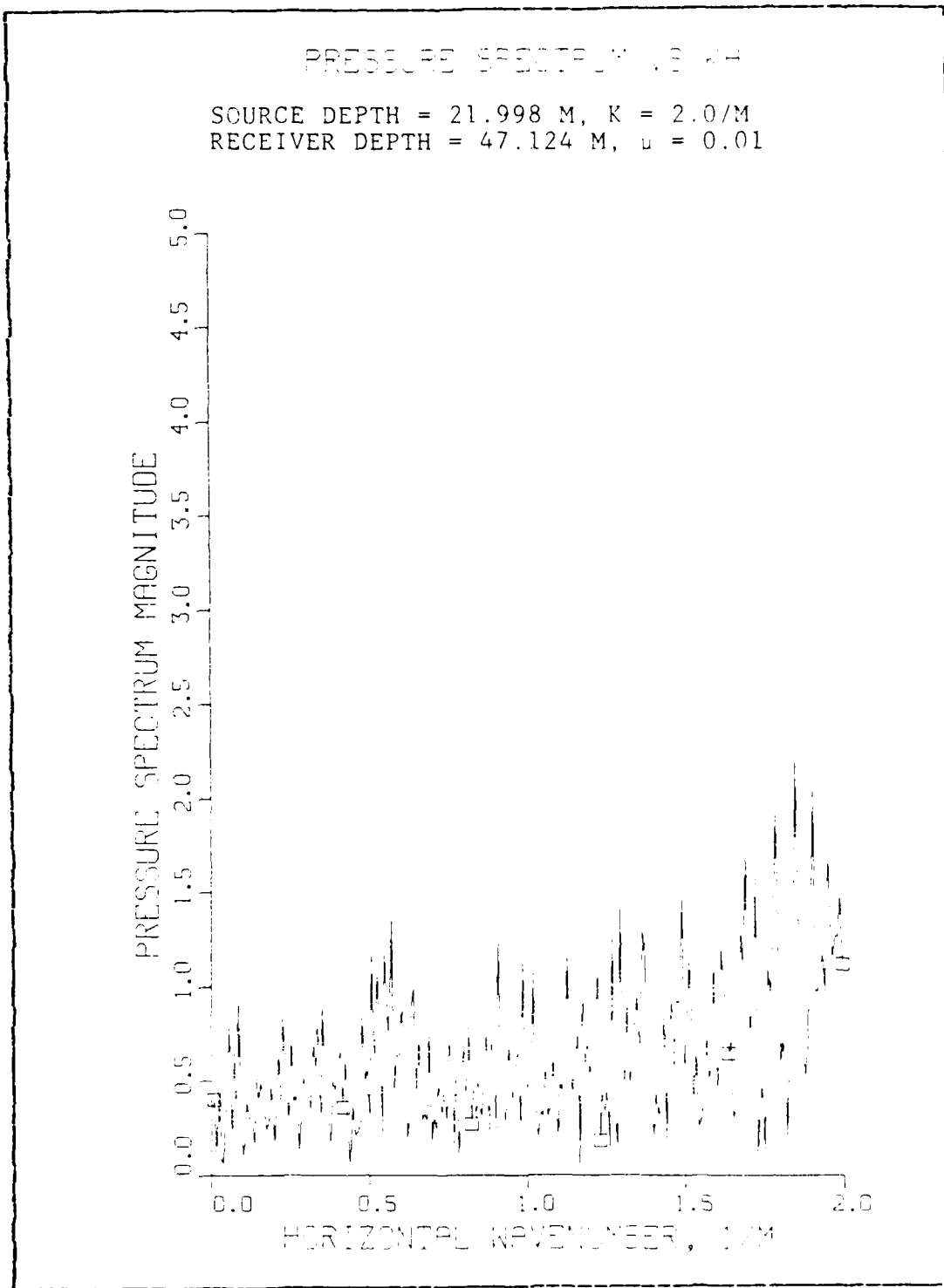


Figure 3.46 Pressure Spectrum vs. Gamma

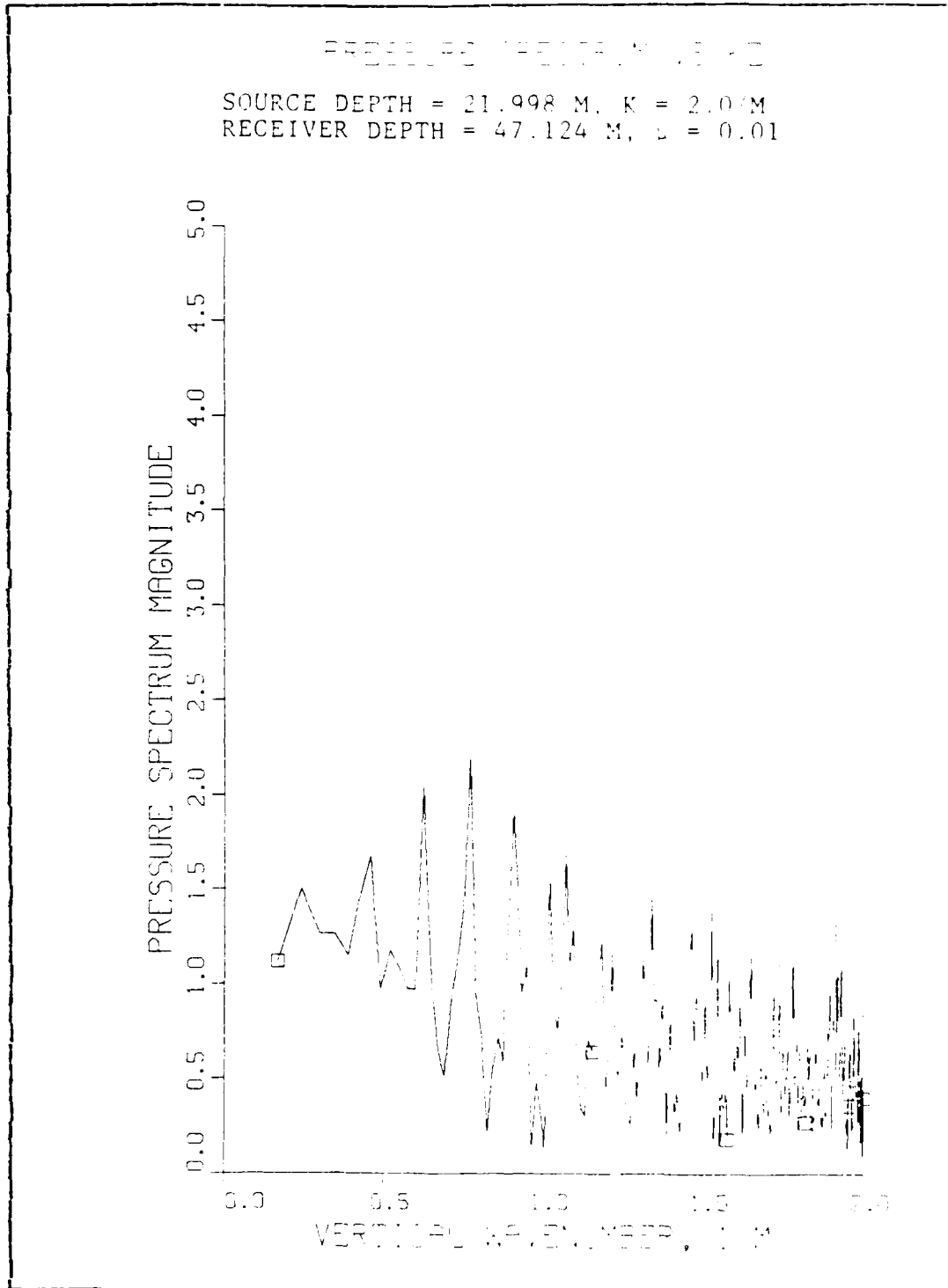


Figure 3.47 Pressure Spectrum vs. Beta


```

360 IF (ZC,3E,25) GO TO 360
270 IX = 20
175 GO TO 370
280 ZC = 23
290 CALL E(C,1.5) ZX
300 PCHRY(,IX,THE SHALLOWEST DEPTH = ,FB.2)
310 REAL = FICAI(LMAX)
320 REALM = ALOS1((REALK)/ALOG1(2.0) + .9)
330 I32 = INT(CFAL4)
340 JMAX4 = IMAX/2
350 KMAX = IMAX/2 + 1
360 LMAX = IMAX - 1
370 MMAX = IMAX + 1
380 NMAX = IMAX + 2
390 LEAY2 = IMAX * 2
400 IMAX3 = IMAX2 + 1
410 F1LMAX = FICAI(LMAX)
420 FJMAX = FICAI(JMAX)
430 IMAX3 = INT(F1LMAX/3.0)
440 WRITE(9) C, F1LMAX, FJMAX, LMAX, IMAX3, JMAX, KMAX, LMAX, MMAX, NMAX
450 FORMAL(IX,Z(ZX,FE,Z),7(2X,I4))
460 WRTT(0) ZS,ZE,PAR,ARCH
470 FCPAR(IV,THE SELECTED PARAMETERS ARE: ,/,10X, SOURCE DEPTH IN METERS,
480 1 FEWS: ,FB.2,/,10X, RCVT DEPTH IN METERS: ,FB.2,/,10X, VALUE CF,
490 2 THE WAVELENGTH: ,FB.5,/,10X, THE WAVELENGTH IS: ,FB.2,/,/,5X,
500 3 DO YOU DESIRE TO MAKE ANY CHANGES? (YES/NO)')
510 READ(5,121)ANS
520 FCPAR(41)
530 IF (ANS.EQ.ON) GO TO 122
540 IF (ANS.NE.Y) GO TO 11
550 GO TO 1
560 LAMBDA = ZC * PI E/AK
570 WRITE(9) LAMBDA
580 FOFVAL(IX,THE WAVELENGTH IN METERS IS: ,F12.4,/,5X, THE RANGE
590 INCREMENT MUST BE LESS THAN -LAMBDA/(2*PI E) - TO PREVENT ALIASING.
600 ENTER THE RANGE INCREMENT.))
610 REAL(5,* ) DE
620 DO = 9)
630 IMAX = 50 + DO * JMAX
640 LMAX = (-1.0) * IMAX
650 FAC = (-1.0) * DE / (2.0 * PI E)
660 CALL E(,10) F1LMAX,DS,FAC
670 FCPAR(18,CHANGE SELECTIONS ARE: ,/,10X, MINIMUM RANGE IN METERS,
680 1: ,F10.7,/,10X, IMAX, LMAX, MMAX, NMAX,
690 2: ,F10.7,/,10X, FACTOR HERE IS: ,F10.7,/,/
700 TES00490
710 TES00500
720 TES00510
730 TES00520
740 TES00530
750 TES00540
760 TES00550
770 TES00560
780 TES00570
790 TES00580
800 TES00590
810 TES00600
820 TES00610
830 TES00620
840 TES00630
850 TES00640
860 TES00650
870 TES00660
880 TES00670
890 TES00680
900 TES00690
910 TES00700
920 TES00710
930 TES00720
940 TES00730
950 TES00740
960 TES00750
970 TES00760
980 TES00770
990 TES00780
1000 TES00790
1010 TES00800
1020 TES00810
1030 TES00820
1040 TES00830
1050 TES00840
1060 TES00850
1070 TES00860
1080 TES00870
1090 TES00880
1100 TES00890
1110 TES00900
1120 TES00910
1130 TES00920
1140 TES00930
1150 TES00940
1160 TES00950
1170 TES00960

```


AD-A156 463

USE OF THE WAVENUMBER TECHNIQUE WITH THE LLOYDS MIRROR
FOR AN ACOUSTIC DOUBLET(U) NAVAL POSTGRADUATE SCHOOL
MONTEREY CA P B KING MAR 85

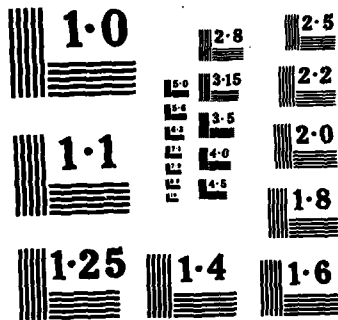
2/2

UNCLASSIFIED

F/G 17/1

NL





NATIONAL BUREAU OF STANDARDS
MICROCOPY RESOLUTION TEST CHART

```

37      DATA(2*I) = DATA(IMAX + (2*I))
      CONTINUE
339     CALL ERPLOT(DADA,DE,IMAX,JMAX,RO)
340
341     CALL FOUR2(DATA,IMAX,-1,1)
342
343     WRITE(6,352)
344     FORMAT(/,5X,'I',4X,'SPEC DATA REAL',3X,'SPEC DATA IMAG',/)
345
346     DO 18 I = 1,IMAX
347     WRITE(6,353) I,DATA(2*I - 1),DATA(2*I)
348     CONTINUE
349     FORMAT(2X,I4,2(4X,F10.7))
350
351     DS = 2.0 * PI / (FLIMAX * DS)
352     WRITE(6,300) DS
353     FORMAT(/,2X,'DS = ',F12.8,/)
354
355     WRITE(6,201)
356     FORMAT(/,5X,'I',6X,'CKM',10X,'Q',10X,'KZ',/)
357
358     DO 50 I=1,IMAX4
359     FLY = FLYAI(I)
360     GM(I) = FLY * DS
361     IF (GM(I).GE.AK) GO TO 420
362     GO TO 420
363     CONTINUE
364
365     *****
366     IF GM(I) < AK, THEN BE(I) IS REAL AND REDUCES TO A FORM OF
367     THE SIN(A)/X FUNCTION, APPROXIMATELY. ACTUAL FORM IS GIVEN BELOW.
368     *****
369
370     BE(I) = SQRT(AK**2 - GM(I)**2) * EPS
371     X(-I) = ABS(SIN(BE(I)*ZX)*SQRT(GM(I))/BE(I))
372     GO TO 430
373     CONTINUE
374
375     *****
376     IF GM(I) > AK, THEN BE(I) = SQRT OF A NEGATIVE NUMBER, AND THE
377     HYPERBOLICAL FUNCTION, P(X), REDUCES TO THE HYPERBOLIC SIN(SINH).
378     *****

```

```

TES01930
TES01940
TES01950
TES01960
TES01970
TES01980
TES01990
TES02000
TES02010
TES02020
TES02030
TES02040
TES02050
TES02060
TES02070
TES02080
TES02090
TES02100
TES02110
TES02120
TES02130
TES02140
TES02150
TES02160
TES02170
TES02180
TES02190
TES02200
TES02210
TES02220
TES02230
TES02240
TES02250
TES02260
TES02270
TES02280
TES02290
TES02300
TES02310
TES02320
TES02330
TES02340
TES02350
TES02360
TES02370
TES02380
TES02390

```



```

C C
      10  WRITTEN BY NORMAN BRENNER OF MIT LINCOLN LABORATORY, JANUARY 1969.
      20  SEE -- IEEE AUDIO TRANSACCTIONS (JUNE 1967), SPECIAL ISSUE ON FFT.
      30  DIMENSION DATA(1), N(1)
      40  NDIM = 1
      50  NTOCT=1
      60  DO 10 IDIM=1,NDIM
      70  NPOT=NCCL1*N(IDIM)
      80  IF (IFCFH) 70,20,20
      90  REEM=NTCT
      100  DO 60 IDIM=1,NDIM
      110  NREM=NREM/N(IDIM)
      120  NPREV=NTOCT/N(IDIM)*NFEA1
      130  NCURR=N(IDIM)
      140  IF (IDIM-1)IFORN) 30,30,40
      150  NCURR=NCURR/2
      160  CALL BITREV (DATA, NPREV, NCURR, NREM)
      170  CALL CCCL2 (DATA, NPREV, NCURR, NREM, ISIGN)
      180  IF (IDIM-1)IFORN) 50,50,60
      190  CALL FLYRL (DATA, N(1), NREM, ISIGN, IFORN)
      200  NTOCT=(NTOCT/N(1))*(N(1)/2+1)
      210  CONTINUE
      220  RETURN
      230  NTOCT=(NTOCT/N(1))*(N(1)/2+1)
      240  DO 100 JDIM=1,NDIM
      250  IDIM=NDIM+1-JDIM
      260  NCOEF=N(IDIM)
      270  IF (IDIM-1) 30,80,90
      280  NCURR=NCURR/2
      290  CALL FIXRL (DATA, N(1), NPEA, ISIGN, IFORN)
      300  NTOCT=NTOCT/(N(1)/2+1)*N(1)
      310  NPREV=NTOCT/(N(IDIM)*NREM)
      320  CALL BITREV (DATA, NPREV, NCURR, NREM)
      330  CALL CCCL2 (DATA, NPREV, NREM, NREM, ISIGN)
      340  NREM=NREM*N(IDIM)
      350  END
      360  SUBROUTINE BITREV (DATA, NPREV, N(NREM)
      370  SUBROUTINE THE DATA BY BIT REVERSAL.
      380  DIMENSION DATA(NPREV, N, NREM)
      390  COMPLEX DATA
      400  EACH LINE OF DATA (J1, JPREV, J2) WITH DATA (J1, J+J5) FOR ALL J1 FROM 1
      410  TO NPREV, ALL J4 FROM 1 TO N (WHICH MUST BE A POWER OF TWO), AND
      420  ALL J5 FROM 1 TO NREM. J4REV-1 IS THE BIT REVERSAL OF J4-1. E.G.
      430  DIMENSION N = 32. THEN FOR J4-1 = 10011, J4REV-1 = 11001, ETC.
      440  DIMENSION DATA(1)
      450  DO 2=

```

TESO3370
 TESO3380
 TESO3390
 TESO3400
 TESO3410
 TESO3420
 TESO3430
 TESO3440
 TESO3450
 TESO3460
 TESO3470
 TESO3480
 TESO3490
 TESO3500
 TESO3510
 TESO3520
 TESO3530
 TESO3540
 TESO3550
 TESO3560
 TESO3570
 TESO3580
 TESO3590
 TESO3600
 TESO3610
 TESO3620
 TESO3630
 TESO3640
 TESO3650
 TESO3660
 TESO3670
 TESO3680
 TESO3690
 TESO3700
 TESO3710
 TESO3720
 TESO3730
 TESO3740
 TESO3750
 TESO3760
 TESO3770
 TESO3780
 TESO3790
 TESO3800
 TESO3810
 TESO3820
 TESO3830
 TESO3840

```

IP1=IP0*NPREV
IP4=IP1*N
IP5=IP4*NREM
I4REV=1
I4REV = 1+(J4REV-1)*IP1
DO 6 I4=1,IP4,IP1
  I4 = 1+(J4-1)*IP1
  IF (I4-I4REV) 10,30,30
  I1MAX=I4+IP1-IPO
  DO 20 I1=I4-I1MAX,IP0
    I1 = 1+(J1-1)*IP0+(J4-1)*IP1
  DC 20 I5=I1,IP5,IP4
  I5 = 1+(J1-1)*IP0+(J4-1)*IP1+(J5-1)*IP4
  I5REV=I4REV+I5-I4
  I5REV = 1+(J1-1)*IP0+(J4REV-1)*IP1+(J5-1)*IP4
  TEMPI=DATA(I5)
  TEMPI=DATA(I5+1)
  DATA(I5)=DATA(I5REV)
  DATA(I5+1)=DATA(I5REV+1)
  DATA(I5REV)=TEMPI
  DATA(I5REV+1)=TEMPI
  ADD ONE WITH DOWNWARD CARRY TO THE HIGH CRDEE BIT OF J4REV-1.
  IP2=IP4/2
  IF (I4REV-IP2) 60,60,50
  I4REV=I4REV-IP2
  IP2=IP2/2
  IF (IP2-IP1) 60,40,40
  I4REV=I4REV+IP2
  RETURN
END

SUBROUTINE COOL2 (DATA,NPREV,N,NREM,ISIGN)
DISCRETE CARRIER TRANSFORM OF LENGTH N. IN-PLACE COOLEY-TUKEY
ALGORITHM, BIT-REVERSED TO NORMAL CRDEE, SANDY-TUKEY PHASE SHIFTS.
DIMENSION DATA(NPREV,N,NREM)
COMPLEX DATA
DATA(J1,K4,J5) = SUM(DATA(J1,J4,J5)*EXP(ISIGN*2*PI*I*(J4-1)*
(K4-1)/N)), SUMMED OVER J4 = 1 TO N FOR ALL J1 FROM 1 TO NPREV
K4 FROM 1 TO N AND J5 FROM 1 TO NREM. 2 CR 4, 4 CR 8, WHETHER N IS
METHOD--LET IPREV TAKE THE VALUES 1, 2 OR 4, ETC., DEPENDS ON WHETHER N IS
N/4 OR 4. THE CHOICE BETWEEN 2 OR 4, OR 4, THE NEXT FACTOR THAT
A POWER OF FACTOR. AND IREM = N/(IFACT*IPREV). THEN--
IPREV MUST DATA
DIMENSION DATA
COMPLEX DATA
DATA(J1,J2,K3,J4,J5) = SUM(DATA(J1,J2,J3,J4,J5)*EXP(ISIGN*2*PI*I*
(K3-1)*I/(J3-1)/IFACT+(J2-1)/IFACT*IPREV)}), SUMMED OVER J3 = 1
TO IFACT FOR ALL J1 FROM 1 TO NPREV, J2 FROM 1 TO IPREV, K3 FROM
  
```


TERS03850
 TERS03860
 TERS03870
 TERS03880
 TERS03890
 TERS03900
 TERS03910
 TERS03920
 TERS03930
 TERS03940
 TERS03950
 TERS03960
 TERS03970
 TERS03980
 TERS03990
 TERS04000
 TERS04010
 TERS04020
 TERS04030
 TERS04040
 TERS04050
 TERS04060
 TERS04070
 TERS04080
 TERS04090
 TERS04100
 TERS04110
 TERS04120
 TERS04130
 TERS04140
 TERS04150
 TERS04160
 TERS04170
 TERS04180
 TERS04190
 TERS04200
 TERS04210
 TERS04220
 TERS04230
 TERS04240
 TERS04250
 TERS04260
 TERS04270
 TERS04280
 TERS04290
 TERS04300
 TERS04310
 TERS04320

```

1 TO IFACT, J4 FROM 1 TO IKEM AND J5 FROM 1 TO NREM. THIS IS
A PHASE-SHIFTED DISCRETE FOURIER TRANSFORM OF LENGTH IFACT.
FACTORS IN BY FOURS SAVES ABOUT TWENTY FIVE PERCENT OVER FACTOR-
ING BY TWOS. DATA MUST BE BIT-REVERSED INITIALLY.
IT IS NOT NECESSARY TO REWRITE THIS SUBROUTINE INTO COMPLEX
ROTATION SO LONG AS THE FORTHAN COMPILER USED STORES REAL AND
IMAGINARY PARTS IN ADJACENT STORAGE LOCATIONS. IT MUST ALSO
STORE ARRAYS WITH THE FIRST SUBSCRIPT INCREASING FASTEST.
DIMENSION DATA(1)
THOPI=6.2831853072*FLOAT(ISIGN)
IP0=2
IP1=IP0*NPREV
IP4=IP1*N
IP5=IP4*NREM
IP2=IP1
IP2=IP1*IPKOD
NPART=1
IF (NPART-2) 00,30,20
NPART=NEARST/4
GO TO 10
DO A FOURIER TRANSFORM OF LENGTH TWO
IF (IP2-IP4) 40,160,160
IP3=IP2*2
IP3=IP2*IFACT
DC 50 I1=1,IP1,IP0
I1 = 1+J1-1)*IP1
DC 50 I2=1,1,25,IP3
I2 = 1+(J1-1)*IP0+(J4-1)*IP3+(J5-1)*IP4
I3A=I5
I3B=I3A+IP2
I3 = 1+(J1-1)*IP0+(J2-1)*IP1+(J3-1)*IP2+(J4-1)*IP3+(J5-1)*IP4
TEMPK=DATA(I3B)
TEMPI=DATA(I3B+1)
DATA(I3E)=DATA(I3A)-TEMPK
DATA(I3H+1)=DATA(I3A+1)-TEMPI
DATA(I3A)=DATA(I3A)+TEMPR
DATA(I3A+1)=DATA(I3A+1)+TEMPI
IF 2=1
DO A FOURIER TRANSFORM OF LENGTH FOUR (FROM EIT REVERSED ORDER)
IF (IP2-IP4) 70,160,160
I23=IP2+4
IP3=IP2*IFACT
COMPUTE THOPI THRU NE AND #I IN DOUBLE PRECISION, IF AVAILABLE.
I3H=1+CP1/FLOAT(115/101)
I3H=SIGN(ABS(I3H),1/2)
NPREV=-2.*SIGN(I3H)
NPREV=SIGN(I3H)
NPREV=1.
  
```

C 10
 C 20
 C 30
 C 40
 C 50
 C 60
 C 70

TIES04810
 TIES04820
 TIES04830
 TIES04840
 TIES04850
 TIES04860
 TIES04870
 TIES04880
 TIES04890
 TIES04900
 TIES04910
 TIES04920
 TIES04930
 TIES04940
 TIES04950
 TIES04960
 TIES04970
 TIES04980
 TIES04990
 TIES05000
 TIES05010
 TIES05020
 TIES05030
 TIES05040
 TIES05050
 TIES05060
 TIES05070
 TIES05080
 TIES05090
 TIES05100
 TIES05110
 TIES05120
 TIES05130
 TIES05140
 TIES05150
 TIES05160
 TIES05170
 TIES05180
 TIES05190
 TIES05200
 TIES05210
 TIES05220
 TIES05230
 TIES05240
 TIES05250
 TIES05260
 TIES05270
 TIES05280

```

TEMPR=WF
WP=NSTPR*TEMPR-WSIPI*WI+TEMPR
WI=NSTPR*WI+WSIEI*TEMPR+WI
IE2=IP3
GC TO 60
RETURN
END
SUBROUTINE PIXRL (DATA, N, NREM, ILSIGN, IFORM)
FOR IFORM=0 CCNVERT TO THE TRANSFORM OF A DOUBLED-UP REAL ARRAY,
CONSIDERED COMPLEX INTO ITS TRUE TRANSFORM. SUPPLY ONLY THE
FIRST HALF OF THE COMPLEX TRANSFORM, AS THE SECOND HALF HAS
CONJUGATE SYMMETRY. FOR IFORM=-1, CCNVERT THE FIRST HALF
OF THE TRUE TRANSFORM INTO THE TRANSFORM OF A DOUBLED-UP REAL
ARRAY. N MUST BE EVEN.
USING COMPLEX NOTATION AND SUBSCRIPTS STARTING AT ZERO, THE
TRANSFORMATION IS--
DIMENSION DATA(N,NREM)
ZSTP = EXP( ILSIGN*2*PI*I/N)
DO 10 I2=0, NREM-1
DATA(0,I2) = CONJ (DATA(0,I2)) *(1+I)
DO 10 I1=1, N/4
Z = (1+(2*IFORM+1)*ZSTP**I1)/2
I1CNJ = N/2-I1
DIF = DATA(I1,I2)-CONJ(DATA(I1CNJ,I2))
TEMP = 2*DIF
DATA(I1CNJ,I2) = (DATA(I1,I2)-TEMP)*(1-IFORM)
IF IFORM=1 I1CNJ, I2) = (DATA(I1CNJ,I2)+CONJ(TEMP))*(1-IFORM)
A SIMPLE CCNJECTION FOR THAT VALUE COLLAPSES INTO
DIMENSION DATA(I)
TEMP1=6.283185307*FLOAT( ILSIGN)
IP0=2
IP1=IP0*(N/2)
IP2=IP1*NREM
IF (IFORM) 19,70,70
PACK (THE REAL INPT VALUES (TWO PER COLUMN)
JI=IP1+1
DATA(2)=DATA(J1)
IF (N%4-1) 70,70,20
JI=JI+IE0
L2MIN=IP1+1
DO 60 I2=12MIN,IP2,IE1
DATA(I2)=DATA(J1)
JI=JI+IE0
IF (I-2) 50,50,30
L1MIN=I2+IE0
L1MAX=I2+IE1-IE0
  
```

150

160

10

C 10

20

30

5290
 5300
 5310
 5320
 5330
 5340
 5350
 5360
 5370
 5380
 5390
 5400
 5410
 5420
 5430
 5440
 5450
 5460
 5470
 5480
 5490
 5500
 5510
 5520
 5530
 5540
 5550
 5560
 5570
 5580
 5590
 5600
 5610
 5620
 5630
 5640
 5650
 5660
 5670
 5680
 5690
 5700
 5710
 5720
 5730
 5740
 5750
 5760

```

DC 40 I1=I1MIN, I1MAX, IPO
DATA (I1)=DATA(J1)
DATA (I1+1)=DATA(J1+1)
J1=J1+I20
DATA (I2+1)=DATA(J1)
J1=J1+I20
DO 80 I2=1, IP2, IP1
TEMPR=DATA(I2)
DATA (I2)=DATA(I2)+DATA(I2+1)
DATA (I2+1)=LE*PR-DATA(I2+1)
IF (N-2) 200, 200, 90
THETA=TWOP/ELOAF(N)
SINH= SIN(THETA/2.)
ZSTPI= -2.*SINTH*SINTH
ZSTPI= SIN(THETA)
ZR= (1.-ZSTPI)/2.
ZI= (1.+ZSTPI)/2.
IF (IFORM) 100, 110, 110
ZR= -ZR
ZI= -ZI
I1MIN=IPO*(N/4)+1
I1MAX=IPO*(N/4)+1
DO 190 I1=I1MIN, I1MAX, IPO
DO 180 I2=I1, IP2, IP1
I2CNJ=IPO*(N/2+1)-2*I1+I2
IF (ISIGN*(2*IFORM+1)) 130, 140, 140
DATA (I2+1)=-DATA(I2+1)
IF (IFORM) 170, 180, 180
DIFR=DATA(I2)-DATA(I2CNJ)
DIFI=DATA(I2+1)+DATA(I2CNJ)
TEMPR=DIFR*ZR-DIFI*ZI
TEMPI=DIFR*ZI+DIFI*ZR
DATA (I2)=DATA(I2)-TEMPR
DATA (I2+1)=DATA(I2+1)-TEMPI
DATA (I2CNJ)=DATA(I2CNJ)+TEMPR
DATA (I2CNJ+1)=DATA(I2CNJ+1)-TEMPI
IF (IFORM) 160, 180, 180
DATA (I2CNJ)=DATA(I2CNJ)+DATA(I2CNJ)
DATA (I2CNJ+1)=DATA(I2CNJ+1)+DATA(I2CNJ+1)
DATA (I2)=DATA(I2)+DATA(I2)
DATA (I2+1)=DATA(I2+1)+DATA(I2+1)
CCNT=CCNT+5
TEMPR=ZF-5
ZR=ZSTPR*TEMPR-ZSTPI*ZI+ZR
ZI=ZSTPR*ZI+ZSTPI*TEMPR+ZI
RECJESICH SAVES TIME, AT A SLIGHT LOSS IN ACCURACY. IF AVAILABLE,
USE DOUBLE PRECISION IC COMPITE ZK AND ZI.
  
```

40
50
60
70

80
90

100
110

120
130
140
150

160
170
180

190
C
C


```

100  FORMAT(/, 3X, 'I', 6X, 'RANGE', 7X, 'REALP', 7X, 'IMAGP', 5X, 'MODULUS OF P',
110  1,/)
120  DC 10 I = 1, JMAX
130  PMOD(I) = SQRT((DATA(2*I-1))**2 + (DATA(2*I))**2)
140  WRITE(6, 200) I, R(I), DATA(2*I-1), DATA(2*I), PMOD(I)
150  CCONTINUE
160  CFORMAT(2X, 14, 2X, F10.2, 3(2X, F10.7))
170  C
180  CPMAX = -1.0
190  CDO 20 I = 1, JMAX
200  CIF (PMOD(I) .GE. PMAX) PMAX = PMOD(I)
210  CCONTINUE
220  C
230  CCALL COMPLES
240  CCALL AREA2D(4.0, 6.0)
250  CCALL XNAME('RANGE (4)', $, 100)
260  CCALL YNAME('MAGNITUDE OF PRESSURE, P(R)', $, 100)
270  CCALL LINESP(0.5)
280  CCALL HEADIN('MAGNITUDE OF PRESSURE AS A FN OF RANGE $', 100, 1.0, 1)
290  CCALL NCCHER
300  CCALL GRAF(RMIN, 'SCALE', RYAX, 0.0, 'SCALE', PMAX)
310  CCALL CURVE(RP, PHCP, JMAX, IMARK)
320  CCALL ENDPL(1)
330  CRETURN
340  CEND
350  C
360  CSUBROUTINE NTHETA(IMAX, G, CKM, Q, AK, BE)
370  C
380  C*****
390  CTHIS SUBROUTINE PLOTS THE ACOUSTIC PRESSURE SPECTRUM AS A FUNCTION
400  COF THE HORIZONTAL WAVELENGTH
410  C*****
420  CDIMENSION GM(4096), CKM(4096), C(4096), BE(4096)
430  C
440  CIMARK = INT(IMAX/5.) + 1
450  CGMIN = GM(1)
460  CGMAX = AK
470  CPMAX = -1.0
480  CSMAX = 0.0
490  CDO 10 I = 1, IMAX
500  CIF (CKM(I) .GT. PMAX) PMAX = CKM(I)
510  CCONTINUE
520  CDO 15 I = 1, IMAX

```

```

TES06250
TES06260
TES06270
TES06280
TES06290
TES06300
TES06310
TES06320
TES06330
TES06340
TES06350
TES06360
TES06370
TES06380
TES06390
TES06400
TES06410
TES06420
TES06430
TES06440
TES06450
TES06460
TES06470
TES06480
TES06490
TES06500
TES06510
TES06520
TES06530
TES06540
TES06550
TES06560
TES06570
TES06580
TES06590
TES06600
TES06610
TES06620
TES06630
TES06640
TES06650
TES06660
TES06670
TES06680
TES06690
TES06700
TES06710
TES06720

```

```

15 IF(Q(I),GT,QMAX)QMAX = Q(I)
C CONTINUE
C
C IF(PMAX,GF,QMAX)SMAX = PMAX
C SMAX = QMAX
C
C *****
C GMAX IS THE MAXIMUM VALUE KH CAN TAKE ON BASED ON THE EQUATION
C ((DELTA K) * (DELTA R)) = (2 * PI/IMAX)
C
C GMAX IS THE MAXIMUM VALUE KH CAN TAKE ON AND STILL PRODUCE "REAL"
C THEORETICAL Q AND DELTA VALUES. KZ < SQRT(AK**2 + KH**2) => Q REAL.
C
C SCALE THE HORIZONTAL AXIS IN KH (GM)
C *****
C CALL COMPRS
C CALL AREA2D(4.0,6.0)
C CALL XNAME('HORIZONTAL WAVENUMBER, 1/M$, 100)
C CALL YNAME('PRESSURE SPECTRUM MAGNITUDE$, 100)
C CALL LINESP(0.5)
C CALL HEADIN('PRESSURE SPECTRUM VS KH$, 100,1.0,1)
C CALL HCCHK
C CALL GRAF(GMIN,SCALE',GMAX,0.0,SCALE',SMAX)
C CALL CURVE(GM,CX,IMAX,IMARK)
C CALL DCI
C CALL CURVE(GM,Q,IMAX,IMARK)
C CALL FSET('DGI)
C CALL ENDPL(2)
C RETURN
C END
C
C SUBROUTINE WTVPLI(IMAX,CX,AK,BE)
C *****
C THIS SUBROUTINE PLOTS THE PRESSURE SPECTRUM AS A FUNCTION OF THE
C VERTICAL WAVENUMBER (EE).
C *****
C DIMENSION C(K(4096)),C(4096),PF(4096)
C IMARK = INT(IMAX/5.) + 1
C BMAX = EE(1)
C BMIN = EE(IMAX)
C PMAX = -1.0
C QMAX = -1.0

```

```

TES06730
TES06740
TES06750
TES06760
TES06770
TES06780
TES06790
TES06800
TES06810
TES06820
TES06830
TES06840
TES06850
TES06860
TES06870
TES06880
TES06890
TES06900
TES06910
TES06920
TES06930
TES06940
TES06950
TES06960
TES06970
TES06980
TES07000
TES07010
TES07020
TES07030
TES07040
TES07050
TES07060
TES07070
TES07080
TES07090
TES07100
TES07110
TES07120
TES07130
TES07140
TES07150
TES07160
TES07170
TES07180
TES07190
TES07200

```

0-108 Blank

LIST OF REFERENCES

1. Lauer, R. B., Signal Transmission in the Wavenumber Domain, Technical Memorandum, Naval Underwater Systems Center, New London Laboratory, New London, CT, 5 June 1979.
2. Naval Underwater Systems Center Report 4103, Fast Field Program for Multilayered Media, by F. R. DiNapoli, 26 August 1977.
3. Stamey, B. B., Preliminary Investigation of the Environmental Sensitivity of Acoustic Signal Transmission in the Wavenumber Domain with Respect to Source Depth Determination, Masters Thesis, Naval Postgraduate School, Monterey, CA, 1982.
4. Blanchard, J., A Comparison of Two Acoustic Parabolic Equation Transmission Loss Models for Compatibility with the Wavenumber Technique in the Determination of Source Depth, Masters Thesis, Naval Postgraduate School, Monterey, CA, 1984.
5. Kinsler, L., and others, Fundamentals of Acoustics, 3rd ed., John Wiley and Sons, 1982.
6. Urick, R. J., Principles of Underwater Sound, 3rd ed., McGraw-Hill, 1983.
7. Officer, C. B., Introduction to the Theory of Sound Transmission, McGraw-Hill, 1958.
8. Coppens, A. B., Mathematical Derivation of the Acoustic Pressure Spectrum of the Lloyds' Mirror, Naval Postgraduate School, Monterey, CA, January 1985.
9. IMSL Library Reference Manual, 9th ed., v. 2, IMSL, 1982.
10. Gabrielson, T., Mathematical Derivation of the Acoustic Pressure Spectrum for the Lloyds' Mirror, Naval Postgraduate School, Monterey, CA, February 1985.



BIBLIOGRAPHY

Bergland, G. D., "A Guided Tour of the Fast Fourier Transform," IEEE Spectrum, v.6, July 1969.

Blackman, R. B. and Tukey, J. W., The Measurement of Power Spectra, Dover Publications, Inc., 1958.

Brigham, E. O., The Fast Fourier Transform, Prentice-Hall, Inc., 1974.

DiNapoli, F. R. and Deavenport, R. L., "Numerical Models for Underwater Acoustic Propagation," Ocean Acoustics, v. 1, Springer-Verlag, New York, NY, 1979.

DiNapoli, F. R. and Deavenport, R. L., "Theoretical and Numerical Green's Function Field Solution in a Plane Multilayered Media," Journal of the Acoustic Society of America, v. 67(1), January 1980.

International Series of Monographs on Electronics and Instrumentation, v.9, Pergamon Press, Inc., 1960.

Naval Surface Weapons Center Report 75-18, A Statistical Model for the Fluctuation of Sound Transmission in the Sea, Urick, R. J., 11 February 1975.

Naval Underwater Systems Center Technical Memorandum 781054, A Simplified Derivation of a Quadrature Subsampling Technique, Lackoff, M. R., 14 March 1978.

Stanton, T. K. and Beyer, R. T., "The Interaction of Sound with Noise in Water," Journal of the Acoustic Society of America, v.64(6), December 1978.

Thomas, G. B. and Finney, R. L., Calculus and Analytic Geometry, 5th ed., Addison-Wesley Publishing Company, 1979.

Urick, R.J., "Models for the Amplitude Fluctuations of Narrow-Band Signals and Noise in the Sea," Journal of the Acoustic Society of America, v. 64(6), December 1978.

INITIAL DISTRIBUTION LIST

	No.	Copies
1. Defense Technical Information Center Cameron Station Alexander, VA 22314	2	2
2. Library, Code 0142 Naval Postgraduate School Monterey, CA 93943	2	2
3. Chairman (Code 55Fo) ASW Academic Group Naval Postgraduate School Monterey, CA 93943	1	1
4. Prof. A. B. Coppens (Code 61Cz) Department of Physics Naval Postgraduate School Monterey, CA 93943	4	4
5. Prof. C. R. Dunlap (Code 68Du) Department of Oceanography Naval Postgraduate School Monterey, CA 93943	3	3
6. Prof. Suk Wang Yoon (Code 61Yo) Department of Physics Naval Postgraduate School Monterey, CA 93943	3	3
7. Prof. T. Gabrielson (Code 61Gt) Department of Physics Naval Postgraduate School Monterey, CA 93943	1	1
8. LCDR P. B. King 425 Pine Shadows Drive Slidell, LA 70458	3	3
9. Commanding Officer Naval Ocean Research and Development Activity NSTL, MS 39522	1	1
10. Commanding Officer Naval Ocean Research and Development Activity (ATTN: Code 223) NSTL, MS 39522	1	1
11. Office of Naval Research ATTN: Mr. R. B. Lauer Naval Ocean Research and Development Activity 800 N. Quincy Street Arlington, VA 22217	1	1
12. Commanding Officer Naval Eastern Oceanography Center ATTN: LCDR J. Blanchard McAdie Bldg. (U-117) Naval Air Station Norfolk, VA 23511	1	1

13. ICDF B. Stamey 1
5508 Buskirk Street
North Charleston, SC 29406
14. Ms Catherine Smith 2
P. O. Box 58
Carmel Valley, CA 93924

END

FILMED

8-85

DTIC