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MODELS OF EXPLOSIVELY DRIVEN METAL

James T. Dehn

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US ARMY BALLISTIC RESEARCH LABORATORY
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A comprehensive discussion is given of various simple models of explosively driven metal. These are compared with each other and with experiments, wherever possible, and their relation to more complicated models is discussed as needed. Certain types enable us to describe detailed space-time trajectories, while others are more suitable for asymptotic cases, such as estimating launch velocities. All are simple enough to use in initial design work where insight and suggestions for experiment are more important than great precision.

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I. INTRODUCTION

There is a continuing need for and interest in simple models of explosively driven metal. Military applications have received the most attention, beginning with the work of Taylor[1] and Gurney[2] on the launch speeds of fragments from explosively driven metal shells. Taylor also applied his ideas to the formation of shaped charge jets[3] and later published his results.[4] Variations of these basic ideas are still being used today in design and analysis work.[5,6,7] These ideas have also proven to be useful in the general field of shock physics,[8] and several recent papers have appeared on applications of

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[1] G. I. Taylor, "Analysis of the Explosion of a Long Cylindrical Bomb Detonated at One End," paper written for the Civil Defence Research Committee, Ministry of Home Security in 1941, available in The Scientific Papers of Sir Geoffrey Ingram Taylor, Vol. III, G. K. Batchelor, ed., The University Press, Cambridge, 1963, p. 277.

[2] R. W. Gurney, "The Initial Velocities of Fragments from Bombs, Shell, Grenades," Ballistic Research Laboratory R405, 1943. (AD ATI 36218)

[3] G. I. Taylor, "A Formulation of Mr. Tuck's Conception of Munroe Jets," written in 1943 and available in The Scientific Papers of Sir Geoffrey Ingram Taylor, Vol. III, ed. by G. K. Batchelor, The University Press, Cambridge, 1963, p. 358.

[4] G. Birkhoff, D. P. MacDougall, E. M. Pugh, and G. I. Taylor, "Explosives with Lined Cavities," Journal of Applied Physics, 19, 1948, p. 563.

[5] J. T. Harrison, "BASC, An Analytical Code for Calculating Shaped Charge Properties," Sixth International Symposium on Ballistics sponsored by the American Defense Preparedness Association (ADPA), 1981, p. 253.

[6] G. Randers-Pehrson, R. R. Karpp, C. E. Anderson, Jr., and H. J. Blische, "SHORTFRAG Users Guide," Ballistic Research Laboratory Memorandum Report 3007, 1980. (ADB 046644L)

[7] L. R. Kruse, "Theoretically Determined Fragmentation Characteristics and Lethality Estimates for a Newly Proposed Tungsten Alloy Cased BLV-97 Combined Effects Bomblet," Ballistic Research Laboratory Memorandum Report 3294, 1983. (ADC 032497L)

[8] M. A. Meyers and L. E. Murr, eds, Appendix E, "Nomograph for Determination of Flyer-Plate Velocity," Shock Waves and High-Strain-Rate Phenomena in Metals, proceedings of an International Conference on Metallurgical Effects of High-Strain-Rate Deformation and Fabrication, Plenum Press, NY and London, 1981, p. 1057.

Gurney's analysis.[9-14] Other references to work on this subject can be found in these papers. However, a bibliography compiled from these sources alone is far from complete, as will become evident below.

Here we will express Taylor's analysis[1] in simple formulas and extend it to cases of planar as well as cylindrical symmetry. Taylor's numerical analysis was presented in tabular form and has been virtually ignored. By contrast, Gurney's results were presented in simple formulas which have been widely used and extended. An exception is Taylor's simple formula for the projection angle. This small part of Taylor's model has also been widely used in a somewhat obscure combination with Gurney formulas to obtain a projection velocity (magnitude and direction). Here we will develop time-dependent formulas for projection positions and velocities as simple as Gurney formulas. First, we will treat side-on cases in which the detonation front propagates parallel to the metal surface (surface normal perpendicular to the propagation vector). Cases in which the propagation vector and metal surface normal make an angle between zero and ninety degrees can be treated in the manner of References 3 and 4. Finally, we will give some simple formulas for head-on projection velocities (propagation vector parallel to surface normal vector).

In Section II of this report we will establish the validity of using the exponent $\gamma = 3$ in the entropic equation of state. This is required for the simplification of Taylor's theory and its extension to planar geometry as discussed in Section V. A description and comparison of Taylor's and Gurney's models is given in Sections III and IV by way of preparation. Thomas' model is simplified and compared with other models in Section VI. Comparisons with experimental data are also made wherever possible. In Section VII we describe Sterne's extensions of the models of his co-workers, Gurney and Thomas. Finally, we will generalize Sterne's method in order to discuss more than one layer of metal in contact with more than one layer of explosive in three geometries: spherical, cylindrical, and planar. In particular, we will develop simple design methods for optimizing certain types of performance.

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[9] G. E. Jones, J. E. Kennedy, and L. D. Bertholf, "Ballistic Calculations of R. W. Gurney," American Journal of Physics, 48. 1980, p.264.

[10] G. E. Jones, "The Gurney Equations for Multilayered Fragments," Journal of Applied Physics, 50, 1979, p. 3746.

[11] M. J. Kamlet and M. Finger, "An Alternative Method for Calculating Gurney Velocities," Combustion and Flame, 34, 1979, p. 213.

[12] E. W. LaRocca, "A Simplified Method of Calculating the Gurney Constant of Common Explosives," presented at the meeting of the Pyrotechnics and Explosives Applications Section of the American Defense Preparedness Association, 1978.

[13] E. W. LaRocca, "Advances in Predicting the Relative Power of High Explosives and the Correlation with Fragment Velocity," presented at the meeting of the Pyrotechnics and Explosives Applications Section of the American Defense Preparedness Association, 1980.

[14] D. R. Hardesty and J. E. Kennedy, "Thermochemical Estimation of Explosive Energy Output," Combustion and Flame, 28, 1977, p. 45.

II. THE ADIABATIC EXPANSION OF DETONATION PRODUCT GASES

A key element in Taylor's analysis was provided by H. Jones who numerically evaluated one of Taylor's integrals. Jones worked with Taylor on the Research Committee (R.C.) of the Ministry of Home Security and wrote several papers with him.[15,16,17] Jones gave an account of his methods in several Research Committee papers which are no longer available.[18,19,20] Fortunately, his work was later published.[21,22]

In Reference 21, Jones gives as his Equation (27)

$$p = B\rho^{\gamma} \tag{1}$$

relating the pressure p and density ρ of the product gas during its expansion. Here B and γ are constants. Equation (1) is called the entropic equation of state and applies to polytropic gases for which the internal energy depends only on the temperature.[23] For ideal gases at moderate pressures and temperatures, γ is not much larger than unity. Jones estimates that γ should be close to three near the rear of the reaction zone of a typical solid explosive loading and remarks that such a high value occurs because the products initially form a very imperfect gas. He derives the value $\gamma = 3$ by assuming that the ratio of the densities at the front (ρ_o) and rear (ρ_1) of the reaction zone is $\rho_o/\rho_1 = 3/4$.

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[15] G. I. Taylor and H. Jones, "Note on the Lateral Expansion behind a Detonation Wave," The Scientific Papers of Sir Geoffrey Ingram Taylor, Vol. III, G. K. Batchelor, ed., The University Press, Cambridge, 1963, p. 309.

[16] G. I. Taylor and H. Jones, "Blast Impulse and Fragment Velocities from Cased Charges," The Scientific Papers of Sir Geoffrey Ingram Taylor, Vol. III, G. K. Batchelor, ed., The University Press, Cambridge, 1963, p. 363.

[17] G. I. Taylor and H. Jones, "The Bursting of Cylindrical Cased Charges," The Scientific Papers of Sir Geoffrey Ingram Taylor, Vol. III, G. K. Batchelor, ed., The University Press, Cambridge, 1963, p.379.

[18] H. Jones, Research Committee Paper 166, no longer available.

[19] H. Jones, Research Committee Paper 212, no longer available.

[20] H. Jones and A. R. Miller, Research Committee Paper 306, no longer available.

[21] H. Jones, "A Theory of the Dependence of the Rate of Detonation of Solid Explosives on the Diameter of the Charge," Proceedings of the Royal Society of London, 189A, 1946, p. 415.

[22] H. Jones and A. R. Miller, "The Detonation of Solid Explosives: the Equilibrium Conditions in the Detonation Wave-Front and the Adiabatic Expansion of the Products of Detonation," Proceedings of the Royal Society of London, 194A, 1948, p. 480.

[23] R. Courant and K. O. Friedrichs, Supersonic Flow and Shock Waves, Interscience, NY, 1948, p. 6.

In Reference 22 Jones and Miller used the following virial equation of state

$$pV'/N = RT + bp + cp^2 + dp^3 \qquad (2)$$

to calculate pressure and volume (V') of N moles of gas at temperature T resulting from the detonation of one mole of explosive. They also calculated product species for TNT and predicted an approximately linear dependence of both detonation velocity and detonation pressure on loading density. In Part B of their paper, they give numerical examples for TNT at a high (1.5 g/cm^3) loading density and a low (1.0 g/cm^3) loading density. For the high density loading, their Table III gives the pressure, temperature, inverse density, and effective γ value calculated for the entire expansion process. Here we are particularly interested in the γ values. At the detonation pressure achieved at the rear of the reaction zone ($p_1 = 15.88 \times 10^{10}$ dyne/cm^2) they find γ = 3.36 (and $\rho_1 = 1.95$ g/cm^3). By the time the pressure has dropped to 6.25×10^{10} dyne/cm^2, $\rho = 0.65$ g/cm^3 and $\gamma = 2.39$. Between these two points we note that a γ value obtained by taking half of (3.36 + 2.39) is 2.9. Their Table continues down to $p = 2.818 \times 10^6$ dyne/cm^2 and $\rho = 0.003$ g/cm^3 with $\gamma = 1.27$. Their Table V gives similar results for the lower loading density with γ ranging from 2.43 at the detonation pressure down to 1.20 near atmospheric conditions. Normally cast TNT for military applications has a loading density slightly greater than 1.6 g/cm^3 so that a suitable effective value over the range of interest to metal acceleration ($2.1 > \rho > 0.6$ g/cm^3) ought to be γ = 3. Jones and Miller go on to compare their calculated species concentrations favorably with experiment, implying that their p, T, ρ and γ estimates ought to be approximately correct also, even though no direct comparison of these quantities with experimental measurements is possible even today. These results of Jones and Miller were again used by Taylor in a later paper[24] in which he demonstrated for the first time the possibility of a spherical detonation wave, contradicting an opinion previously expressed by Jouget.[25]

Many other authors have followed Jones by using $\gamma = 3$ to describe the high pressure gases which result from the detonation of condensed explosives near their maximum loading density. Jacobs[26] has pointed out that both the Kistiakowsky-Wilson equation of state[27] and that used by Jones and Miller

[24] G. I. Taylor, "The Dynamics of the Combustion Products behind Plane and Spherical Detonation Fronts in Explosives," Proceedings of the Royal Society of London, 200A, 1950, p. 235.

[25] M. Jouget, C. R. Academy of Science, 144, Paris, 1907, p. 633.

[26] S. J. Jacobs, "Recent Advances in Condensed Media Detonations," American Rocket Society Journal, 30, 1960, p. 151.

[27] G. B. Kistiakowsky and E. B. Wilson, Jr., OSRD Report 114, US National Defense and Research Committee of the Office of Scientific Research and Development, 1941.

(Equation (2) above) lead to calculated results which can be expressed by Equation (1) above with $\gamma = 3$ down to one per cent of the detonation pressure, with an accuracy acceptable for hydrodynamic usage.

In 1968, Jacobs, together with Kamlet, proposed a simple method of calculating detonation properties, that is, properties at the rear of the reaction zone.[28] This was the first in a long series of papers which has not yet been completed.[29-35] The sixth paper[33] in this series gives an empirical expression for γ in terms of the loading density with γ again approximately equal to three for military explosives at typical loading densities. In connection with this paper, we also note a recent paper by Andersen.[36]

The fifth paper[32] in the above series provides us with a table of pressures and inverse densities (specific volumes) for various conditions during the expansion of the products from the detonation of twelve common explosive loadings, calculated by using a Kistiakowsky-Wilson equation of state:

$$pV'/(NRT) = 1 + Xe^{\beta X}, \tag{3}$$

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[28] M. J. Kamlet and S. J. Jacobs, "Chemistry of Detonations, I: A Simple Method for Calculating Detonation Properties of C-H-N-O Explosives," Journal of Chemical Physics, 48, 1968, p. 23.

[29] M. J. Kamlet and J. E. Ablard, "Chemistry of Detonations, II: Buffered Equilibria," Journal of Chemical Physics, 48, 1968, p. 36.

[30] M. J. Kamlet and C. Dickinson, "Chemistry of Detonations, III: Evaluation of the Simplified Calculational Method for Chapman-Jouget Detonation Pressures on the Basis of the Available Experimental Information," Journal of Chemical Physics, 48, 1968, p. 43.

[31] M. J. Kamlet and H. Hurwitz, "Chemistry of Detonations, IV: Evaluation of a Simple Predictional Method for Detonation Velocities of C-H-N-O Explosives," Journal of Chemical Physics, 48, 1968, p. 3685.

[32] M. J. Kamlet and H. Hurwitz, "The Chemistry of Detonations, V: Pressures of C-H-N-O Explosives at Various Stages of the Isentropic Expansion," NOLTR 68-44, 1968 or Israel Journal of Technology, 7, 1968, p. 431.

[33] M. J. Kamlet and J. M. Short, "The Chemistry of Detonations, VI: A 'Rule for Gamma' as a Criterion for Choice among Conflicting Detonation Pressure Measurements," Combustion and Flame, 38, 1980, p. 221.

[34] J. M. Short, F. H. Helm, M. Finger, and M. J. Kamlet, "The Chemistry of Detonations, VII: A Simplified Method for Predicting Explosive Performance in the Cylinder Test," Combustion and Flame, 43, 1981, p. 99.

[35] M. J. Kamlet, J. M. Short, M. Finger, F. Helm, R. R. McGuire, and I. B. Akst, "The Chemistry of Detonations, VIII: Energetics Relationships on the Detonation Isentrope," Combustion and Flame, 51, 1983, p. 325.

[36] W. H. Andersen, "Comments on 'The Chemistry of Detonations'," Combustion and Flame, 45, 1982, p. 309.

where $X = K\sum_i x_i k_i/[V'(T+\Theta)^{\alpha'}]$ with x_i the mole fraction, k_i the covolume factor, and α', β, K and θ empirical constants. If we use this table to plot p versus ρ^3, we see that Equation (1) with $\gamma = 3$ can be fitted very closely to these calculated results, at least over the range of ρ of interest to metal acceleration, confirming Jacobs' remark.[26] Figure 1 illustrates this for two explosive loadings which are not very different from ones used in military applications. The values marked X and O were calculated from their table. Kamlet and Hurwitz also quote values for TNT at $\rho_0 = 1.55$ g/cm^3 which are quite close to those calculated for $\rho_0 = 1.50$ g/cm^3 by Jones and Miller,[22] using a very different equation of state (Equation (2) versus Equation (3)). We conclude that Equation (1) with $\gamma = 3$ is a satisfactory approximation for most applications. In fact it is much better than we might expect from the estimates given by Jones and Miller, quoted above. At present we have no satisfactory explanation for this remarkable observation. The fact that the range $.6 < \rho < 2$ is centered near $\rho = 1$ where p is independent of γ may be of some help, but this does not explain the agreement over the range $0.2 < \rho^3 < 8$.

If we adjusted γ as well as B in Equation (1) we might obtain slightly better agreement. However, there is little justification for doing this since the values being fitted can be calculated from various equations of state chosen for convenience rather than theoretical reasons. Since almost any reasonable form will do, we should not place much weight on the fact that a particular form gives us reasonable results. As Jones and Miller remarked long ago,[22] we have little information to guide us in choosing a correct form for the equation of state. There is no virtue in using complicated empirical equations which are designed to cover critical phenomena controlled by weak molecular attractions. The best that can be done is to choose the simplest equation which is adequate for one's purpose, as they did and as we shall do.

If we let $\gamma = 3$ in Equation (1), analytical simplifications become possible, as we shall see presently. One example of such simplification has already been given by Aziz and co-workers.[37] who compared the results of an exact solution for rigid piston loading (possible if $\gamma = 3$) with numerical solutions in the parameter range $2.5 \leq \gamma \leq 3.5$. They concluded that only a one percent error would be made in calculating the energy transmitted to the piston, in spite of variations in the detonation pressure and velocity of about fifty per cent. Similar conclusions were reached earlier by Gurney in his discussion of gas leakage.[38]

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[37] A. K. Aziz, H. Hurwitz, and H. M. Sternberg, "Energy Transfer to a Rigid Piston under Detonation Loading," The Physics of Fluids, 4, 1961, p. 380.

[38] R. W. Gurney, "Fragmentation of Bombs, Shells and Grenades," Ballistic Research Laboratory Report 635, 1947. (ADB 800451)

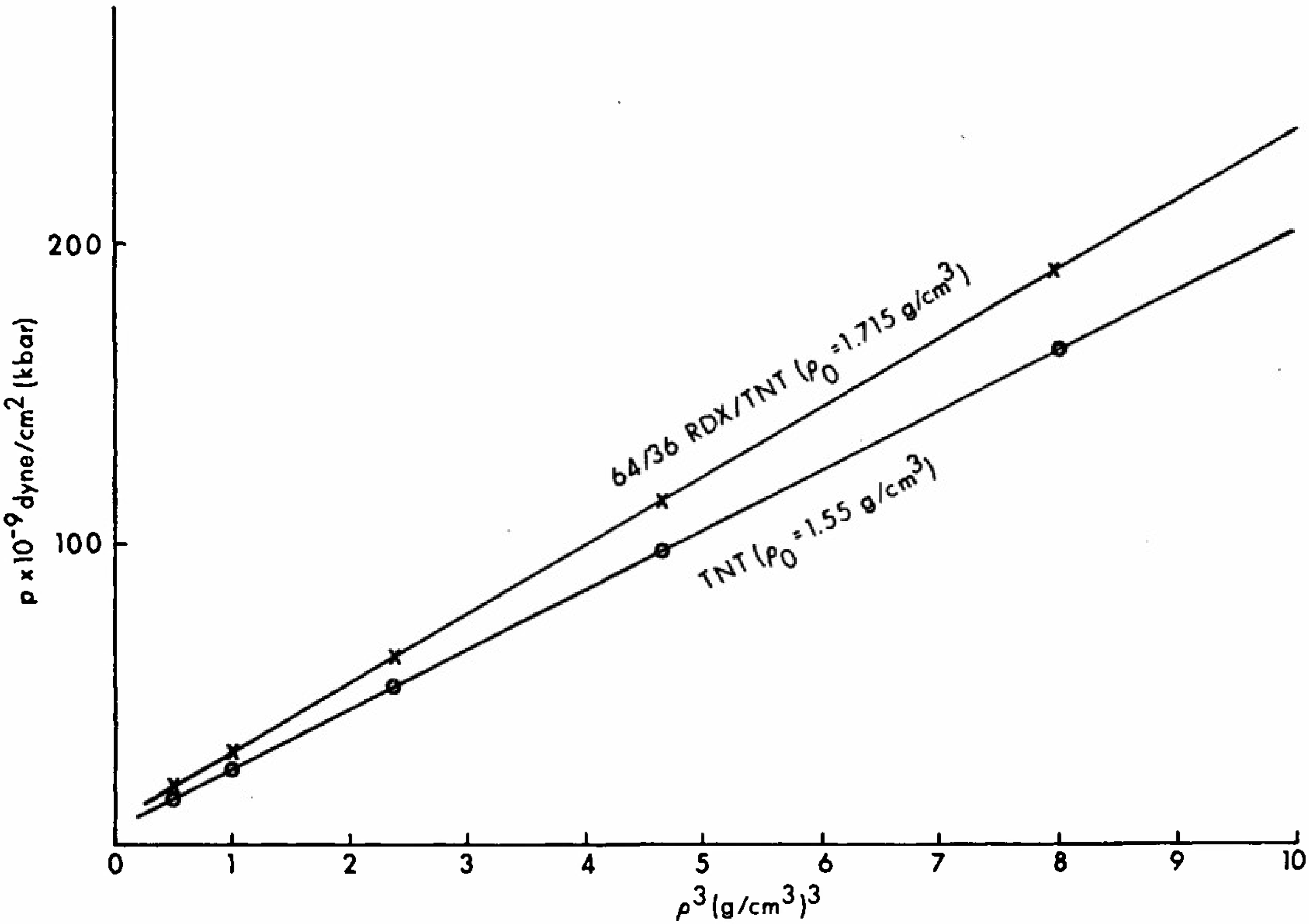

Figure 1. Linear Relation between Pressure and Density Cubed.

III. TAYLOR'S ANALYSIS OF A TUBULAR BOMB

Since the details of this model[1] are not widely known, we will summarize them here. Figure 2 shows a longitudinal cross section of an infinitely long tubular bomb in the neighborhood of a planar detonation front (solid vertical line) which is moving to the right at constant speed, D. To an observer in the laboratory, the process of gas expansion and metal acceleration appears to be complicated, since the gas first moves forward following the detonation wave front and later moves backward away from it so that it acts twice on the same element of the case, as we shall see. However, to an observer who travels with the detonation front and looks back at the expansion process, these complexities disappear, since the gas pressure and velocity at a given distance behind the front (dashed vertical line) do not change with time (steady state condition). Moreover, the velocity of a case element along its length ds is constant and equal to D, since there is no stretching along ds. The transformation between the laboratory coordinate X and the coordinate x used by the moving observer is

$$X = -(x - Dt) + X_o, \tag{4}$$

as shown in Figure 2. Here t is the time, which is taken to be the same for both observers, since detonation wave speeds are negligible compared to the speed of light. The radial coordinates $R = r$ are also the same for both observers as is the (ignorable) equatorial angle measured about the x-axis. The time derivative of Equation (4) gives us the relation between the speeds $U = \dot{X}$ and $u = \dot{x}$, using the dot convention for time derivative,

$$U = D - u. \tag{5}$$

Let M be the constant mass per unit length of the cylindrical shell casing and r be the radial position of the center of a ringlike element of the case wall, namely, the average of the inner and outer radii of the casing at any time t with r much larger than the wall thickness. If we take the ring to be of unit length, then the area acted on by the gas pressure is $2\pi r$ times unity and the mass per unit area is $M/(2\pi r)$. The pressure always acts normal to the metal surface and balances the centrifugal force $[M/(2\pi r)]\ (D^2/R_c)$, where R_c is the radius of curvature (see Figure 2), the inverse of the curvature of the arc ds. Thus,

$$\frac{M}{2\pi r} D^2 \frac{\frac{d}{dx}\left(\frac{dr}{dx}\right)}{\left[1 + \left(\frac{dr}{dx}\right)^2\right]^{3/2}} = p \tag{6}$$

where we have retained the exact expression for the curvature. For a sufficiently heavy case, $\tan^2 \phi = \left(\frac{dr}{dx}\right)^2$ will be much less than unity (see Figure 2), approaching unity only as ϕ approaches 45^o. Thus it is a good approximation to neglect it in the denominator of Equation (6) as Taylor implicitly did. However, there is no difficulty introduced by retaining it, at least for a while, so we will.

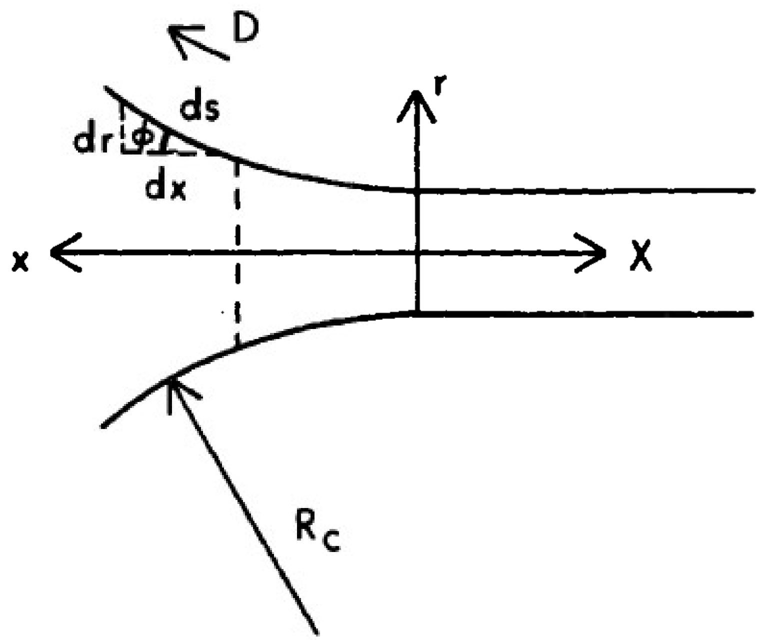

Figure 2. Tubular Case Expansion Geometry.

If we multiply both sides of Equation (6) by $(4\pi r dr)/MD^2) = (4\pi r \tan\phi\, dx)/(MD^2)$, we find

$$2\,\frac{\tan\phi\; d(\tan\phi)}{[1 + \tan^2\phi]^{3/2}} = 2 \sin\phi\, d\phi = \frac{2\pi}{MD^2}\, p\, d(r^2) \tag{7}$$

where we have used the identities $d(\tan\phi) = \sec^2\phi\, d\phi$, $1 + \tan^2\phi = \sec^2\phi$ and $2\, r dr = d(r^2)$. The left side of Equation (7) is readily integrated from $\phi = 0$ to obtain

$$2(1 - \cos\phi) = 4 \sin^2(\phi/2) \tag{8}$$

by another identity. If we approximate Equation (8) by retaining only the first order term in a binomial expansion,

$$\begin{aligned} 2(1 - \cos\phi) &= 2\left(1 - \frac{1}{\sec\phi}\right) = 2\left[1 - (1 + \tan^2\phi)^{-\frac{1}{2}}\right] \\ &= 2\left[1 - (1 - \tfrac{1}{2}\tan^2\phi + \ldots)\right] \approx \tan^2\phi \quad , \end{aligned} \tag{9}$$

we recover Taylor's approximation. The same thing can be accomplished by retaining only first order terms in series expressions of either side of Equation (8). Thus $2\,[1 - \cos\phi] \approx 2\,[1 - (1 - \phi^2/2)] = \phi^2 \approx \tan^2\phi$, or $4 \sin^2(\phi/2) \approx 4(\phi/2)^2 = \phi^2 \approx \tan^2\phi$ for small ϕ.

We are interested in the square root of Equation (8). In moving coordinates, the components of the case velocity are $v_r = dr/dt = D \sin\phi$ and $v_x = dx/dt = D \cos\phi$ from Figure 2. In laboratory coordinates the components are $V_r = v_r$ and $V_x = D(1 - \cos\phi)$, so the magnitude of the case velocity in laboratory coordinates is

$$\begin{aligned} V = \sqrt{V_r^{\,2} + V_x^{\,2}} &= D\sqrt{\sin^2\phi + (1 - \cos\phi)^2} = D\sqrt{2(1 - \cos\phi)} \\ &= 2D \sin(\phi/2) \quad , \end{aligned} \tag{10}$$

where we have used the identity $\sin^2\phi + \cos^2\phi = 1$ and Equation (8). Equation (10) is the only formula in Taylor's analysis which has received wide use, clearly because of its simplicity. We also note by another identity that

$$\frac{V_x}{V_r} = \frac{1 - \cos\phi}{\sin\phi} = \tan(\phi/2) \quad , \tag{11}$$

so that the angle of projection in laboratory coordinates is $\phi/2$. Equations (10) and (11) are exact. If we follow Taylor and use $\tan\phi \approx 2 \sin(\phi/2)$ for the integral of the left side of Equation (7), we make less than a 5% error for $\phi \le 20^\circ$. The exact expression in Equation (8) has already been used by others. In the journal literature it was used by Allison and co-workers[39,40] who wrote $d\phi/ds = (dr/ds)(d\phi/dr) = \sin\phi(d\phi/dr)$ for the curvature in Equation (7). They did not point out that this is equivalent to retaining the exact curvature instead of approximating the curvature.

In order to integrate the right side of Equation (7) let us write down a few more useful relations. For an explosive gas mixture in a rigid cylinder, such as a hydrogen/oxygen mixture in a shock tube experiment, the equation of mass conservation in moving coordinates is

$$dm = \rho_o A_o D dt = \rho A u dt \quad , \tag{12}$$

where Ddt and udt are the original and compressed length of a gas volume element of mass dm and constant cross sectional area $A = A_o$. Here ρ_o is the uncompressed gas density, while ρ is its compressed density. Since the tube is rigid, A is constant and $\rho_o D = \rho u$. However, for the gaseous detonation products of a solid explosive high density loading expanding in a metal tube, A will not remain constant. For a tube with a circular cross section, the area before expansion is $A_o = \pi r_o^2$, while after some expansion it becomes $A = \pi r^2 > A_o$. If we use these expressions in Equation (12) and cancel π, we find

$$\rho_o D r_o^2 = \rho u r^2 \tag{13}$$

for mass conservation. Now ρ_o is the loading density of the solid explosive before detonation. Since steady state flow conditions hold in moving coordinates, the equation of continuity is

$$\frac{1}{r}\frac{\partial}{\partial r}(r\rho v) + \frac{\partial}{\partial x}(\rho u) = 0 \quad , \tag{14}$$

where v is the radial component of the gas velocity (as contrasted with v_r, the radial component of the metal velocity). In Table 1 below we will note that $v << u \approx D$, so that the first term in Equation (14) might be neglected on these grounds for r values of interest. This would make (ρu) independent of x as it is in Equation (13). However, it is more interesting to conclude from Equation (13) that (ρu) is independent of x, so that $(r\rho v)$ in Equation (14) must be independent of r. Since ρ is inversely proportional to r^2 from Equation (13) when $u \approx D$, then v must be proportional to r under these conditions. Thus we may write

$$v = r(v_B/r_B), \tag{15}$$

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[39] F. E. Allison and J. T. Schriempf, "Explosively Loaded Metallic Cylinders, II," Journal of Applied Physics, 31, 1960, p. 846.

[40] F. E. Allison and R. W. Watson, "Explosively Loaded Metallic Cylinders, I," Journal of Applied Physics, 31, 1960, p. 842.

Table 1. Taylor's Results for TNT

1	2	3	4	5	6	7	8	9	10
						for $\alpha = 2$		for $\alpha = 2.67$	
ℓ	ρ g/cm^3	$p \times 10^{-10}$ $dyne/cm^2$	u mm/μs	$\sqrt{\frac{\alpha}{2}}(V/D)$	$\sqrt{\frac{2}{\alpha}}(x/r_o)$	ϕ exact	ϕ approx	V_{exp} mm/μs	V mm/μs
1.000	2.00	15.00	4.83	0	0	0	0		0
1.044	1.52	5.59	5.84	.119	.615	6.82	6.79		.66
1.074	1.39	3.75	6.04	.134	.842	8.54	8.48		.74
1.159	1.14	1.67	6.31	.159	1.419	10.13	10.04		.88
(1.217)	(1.02)	(1.15)	(6.38)	(.166)	(1.900)	(10.58)	(10.47)		(.92)
1.371	.78	.60	6.48	.186	2.638	11.86	11.71	(1.13)	1.03
1.538	.62	.40	6.53	.194	3.527	12.37	12.20	1.20	1.07
1.667	.53	.32	6.55	.200	4.181	12.75	12.57	1.25	1.11
1.883	.41	.20	6.59	.211	5.233	13.46	13.24	1.30	1.17
2.372	.26	.10	6.64	.223	7.488	14.23	13.97	1.35	1.23
3.060	.15	.05	6.67	.230	10.53	14.67	14.39	(1.37)	1.27
4.340	.08	.02	6.70	.239	15.99	15.25	14.94		1.32
5.680	.04	.01	6.72	.245	21.53	15.64	15.30		1.35
∞	0	0	6.914	6.291	∞	-	89.96		34.74

where v_B and r_B are constants which we will define later. Equation (15) was introduced by Gurney[2] as a postulate, although it follows from the equation of continuity under the conditions mentioned. Gurney gave no indication that he ever noticed this fact. Inatead, his assumption had quite a different motivation as we shall see.

In the Chapman-Jouget model of detonation, the approximation is made that all chemical reaction is completed in such a short time that the width of the reaction zone is negligible. Thus we may approximate changes in variables like p, ρ and u by discontinuous jumps instead of using derivatives with very large values. If we use the subscript unity to denote values just behind this very narrow reaction zone and the approximation $r_1 = r_o$ (no expansion in the reaction zone), then Equation (13) becomes

$$\rho_o D = \rho_1 u_1 \tag{16}$$

as in a rigid tube. Similarly, we can use Newton's second law in its impulse-momentum form to derive another jump condition. The impulse delivered across the reaction zone is $(p_1 - p_o)Adt$ which is equal to the change in momentum across this zone, namely, $dm(D - u_1)$ where dm is given by Equation (12). If we neglect the (ambient) pressure p_o at the front of the reaction zone compared to the detonation pressure p_1 at its rear and cancel Adt, we obtain

$$p_1 = \rho_o D(D - u_1). \tag{17}$$

We need one other relation in order to integrate the right side of Equation (7). This ia the strong form of Bernoulli's law for steady flow[41]

$$\int_{p_1}^{p} \frac{dp}{\rho} = \tfrac{1}{2} u_1^2 - \tfrac{1}{2} (u^2 + v^2) \approx \tfrac{1}{2} (u_1^2 - u^2) \tag{18}$$

since $v^2 << u^2 \approx D^2$ as mentioned above. The approximate differential form of Equation (18) is then

$$dp = - \rho u du. \tag{19}$$

Every science student is familiar with the integral of Equation (19) in the case of an incompressible fluid for which $\rho = \rho_o$ for any p and u (in the absence of external fields like gravity), namely, $p + \tfrac{1}{2}\rho_o u^2 = p_1 + \tfrac{1}{2}\rho_o u_1^2$. In the present case, of course, $\rho_1 > \rho_o$.

The variable on the right side of Equation (7) was chosen to be r^2 so that we may use Equation (13) to introduce the new variable $1/(\rho u)$. Then the right side of Equation (7) becomes

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[41] R. Courant and K. O. Friedrichs, Supersonic Flow and Shock Waves, Interscience, NY, 1948, p. 22.

$$
\begin{aligned}
\frac{2\pi}{MD^2}\int_{r_o^2}^{r^2} p\,d(r^2) &= \frac{2\pi\rho_o r_o^2}{MD}\int_1 p\,d\left(\frac{1}{\rho u}\right) \\
&= \frac{2}{\alpha D}\left[\frac{p}{\rho u} - \frac{p_1}{\rho_1 u_1} - \int_1 \frac{dp}{\rho u}\right] \\
&= \frac{2}{\alpha D}\left[\frac{p}{\rho u} - \frac{p_1}{\rho_1 u_1} + u - u_1\right] \\
&= \frac{2}{\alpha D}\left[\frac{p}{\rho u} + u - D\right] \\
&= 4\sin^2(\phi/2) \\
&= (V/D)^2 \approx \tan^2\phi
\end{aligned}
\tag{20}
$$

which is Taylor's[1] Equation (11) when $\tan^2\phi$ is used. In the second line of Equation (20), we have introduced Taylor's notation for the metal case mass to explosive charge mass ratio

$$\alpha = \frac{M}{(\rho_o \pi r_o^2)} = \frac{M}{C} \quad , \tag{21}$$

where C is the explosive mass per unit length of cylinder. We have also integrated by parts from the highly compressed state 1 to an expanded state some distance x behind the detonation front (zero width reaction zone). In the third line of Equation (20), we have used Equation (19) to evaluate the remaining integral. In the fourth line, we have eliminated $\rho_o D$ in Equation (17) by using Equation (16) to obtain

$$\frac{p_1}{\rho_o D} + u_1 = \frac{p_1}{\rho_1 u_1} + u_1 = D \quad , \tag{22}$$

then used Equation (22) in Equation (20). In the last two lines we have used Equations (8), (9), and (10) to complete the derivation. Equation (20) gives V as a function of the variables p, ρ and u and the parameters D and α. Taylor went on to find the shape of the case by numerically integrating the equation

$$x = \int_{r_o}^{r} dr/\tan\phi \tag{23}$$

which is derived from $\tan\phi = dr/dx$.

Equation (20) is rarely cited in the literature since p and u do not appear as simple functions of r, although (ρu) has a simple form in Equation (13). Instead, u must be found from Equation (18) with u_1, given by Equation (16). As was mentioned above, H. Jones evaluated the integral in Equation

(18) for Taylor. Once this was done (using $\rho_o = 1.51$ g/cm^3 and D = 6.38×10^5 cm/s), Taylor presented his solution in tabular form. Table 1 here is an abbreviated version of his results with one row added (for u = D) and some additional columns--two giving ϕ in degrees for the exact and approximate expressions in Equation (20), using the example $\alpha = 2$. From this table we see that by the time $\ell = r/r_o = 1.5$, V has already achieved 90% of the value it would achieve if the case were ductile enough to avoid fragmentation until $r/r_o = 2$. Calculations for large values of r/r_o are purely academic, since no casing material will stretch indefinitely. This is emphasized by the fact that the angle ϕ found by using the exact curvature becomes imaginary as soon as $\sqrt{\frac{\alpha}{2}}\,(V/D) > 2$. This corresponds to the physical fact that V cannot exceed D. The value $\phi = 80.97^o$, found by using the approximation $\tan^2\phi$, is without significance since the approximation ceased to be valid long before $r/r_o \rightarrow \infty$. Since practical metal cases usually burst near $r/r_o = 1.5$, we see that the range of pressure and density which can be adequately represented by Equation (1) with $\gamma = 3$ is covered quite well. As can be seen, approximate values are very close to exact values in this range. Finally, we note that u is initially somewhat smaller than D, then becomes slightly larger than D for $r/r_o > 1.2$. If we recall Equation (5), it becomes clear that to a laboratory observer the gas first appears to move forward after the detonation front (U > 0), then backward (U < 0) for any given X position. The approximation $v^2 \ll u^2 \approx D^2$ also becomes clear, since $v \leq V \approx 0.2D$ for reasonable $\alpha \approx 2$ in the fifth column.

Taylor did not have cased TNT observations to compare with his calculations when he wrote his paper, so he compared his V values with observations made for tetryl explosive cased in steel and found rough agreement. Tetryl is similar to TNT since it too has an hexagonal trinitro ring. However, it has an extra nitro group bonded to its methyl group via a nitrogen atom, so we might expect some disagreement. Taylor later collaborated with H. Jones[16,17] and compared calculated case shapes and velocities with experimental values for composition B explosive (60/40 RDX/TNT) encased in steel, using Jones' calculations for this mixture. They found agreement over the range observed, namely, $0.6 < \alpha < 13.5$, which corresponds to $26.5^o > \phi > 6.5^o$ near bursting.

In recent years cylinder tests have become standardized as explained for example by Short,[34] who gives measured cylinder wall velocities as a function of case expansion for two densities of TNT ($\rho_o = 1.45$ g/cm^3 and $\rho_o = 1.63$ g/cm^3), which bracket the densities being used by Taylor in his calculations ($\rho_o = 1.51$ g/cm^3). For the standard copper cylinder, $\alpha = M/C = (\rho_M / \rho_o)[(r_{out}/r_{in})^2 - 1] = 4.026/\rho_o$, since $\rho_{Cu} = 8.92$ g/cm^3, the outer cylinder radius $r_{out} = 1.53$ cm $= r_o$ and the inner radius $r_{in} = 1.27$ cm. For $\rho_o = 1.51$ g/cm^3 we find $\alpha = 2.67$. We can use this value together with D = 6.38 mm/μs to find V (mm/μs) = 5.52 times the values in column 5 of Table 1. These values are about 10% lower than the experimental values in column 10, consistent with Taylor's neglect of the radial gas velocity component, an approximation which becomes worse as ℓ increases. The experimental values were obtained by plotting Short's values[34] versus $r/r_o = 1 + (r - r_o)/r_o$, since his expansion variable is $(r - r_o)$, followed by a linear interpolation between the cases he gives for $\rho_o = 1.45$ g/cm^3 and $\rho_o = 1.63$ g/cm^3. The method of interpolation is unimportant, since the values in column 9 lie below the lower curve for $\rho_o = 1.45$ g/cm^3 instead of between the two curves. We can imagine a variety of corrections for both the calculated and measured values. For example, we might use a more modern value of D closer to 7 mm/μs, as indicated in the survey by Kamlet and Hurwitz.[31] We might also use the higher pressure values given by Jones and Miller[22] and adjust u accordingly. On the experimental side we might correct for the neglect of Taylor's angle in Short's values (which are V_r, not V) by eliminating ϕ between Equations (10) and (11) to obtain

$$V = D\left[2\left\{1 - \sqrt{1 - (V_r/D)^2}\right\}\right]^{\frac{1}{2}} \quad . \tag{24}$$

However, this leads to corrections only in the third decimal place. Even corrections for observing the outside instead of the center of the thinning case wall are beyond the accuracy of the experiment, since the data reduction process includes numerical differentiation of displacement versus time curves. Altogether, the agreement is remarkable.

IV. GURNEY'S MODEL AND THOMAS' SYNTHESIS

It is clear from Section 3 of his report[2] that Gurney wanted to explain an experimental observation; namely, the fact that the fragment launch speeds of very different size weapons containing the same explosive seemed to depend mainly on the ratio of the mass of the explosive to the mass of the metal; that is, $(C/M)^y$, where y is a power near 0.22 for large bombs, but closer to 0.50 for small projectiles. This led him to his basic assumption which he stated in his abstract and repeated in Section 4 of his report: namely, the contribution made to the total kinetic energy by the detonation of each unit mass of explosive is independent of the size of the projectile. He used the symbol E to denote this constant energy contribution per unit mass and $C = \rho_o \pi r_o^2$ for the explosive mass per unit cylinder length. Thus, EC is the energy contribution per unit length. Initially, the energy released by the detonation appears entirely as the internal energy of the highly compressed detonation product gases. However, it is rapidly converted into kinetic energies of gas and metal as the case expands until, at the moment of bursting, most of it is in this form. Since Gurney knew from experiments that radial fragment motion completely dominates axial components, he partitioned EC into radial energies of metal and gas, integrating over $1/2\rho v^2$ for the latter. He took Equation (15) above to be true, using the symbol v_o for v_B, the case velocity at the moment of bursting (equal to the gas velocity at the case), and the symbol a instead of r_B for the case radius at this time. He then wrote his Equation (1):

$$EC = \tfrac{1}{2}Mv_B^{\,2} + \tfrac{1}{2}\, v_B^{\,2}\int_0^{r_B} 2\pi r\rho \,\frac{r^2}{r_B^{\,2}}\, dr \quad . \tag{25}$$

Here we are using M for the case mass per unit length, a sum over mass elements which form a ring. Since his basic assumption requires EC to be independent of r_B, the integral in Equation (25) must somehow be independent of r_B. Near the top of page 5 in his report, Gurney states that he assumed ρ to be constant. Clearly, he took $\rho = C/(\pi r_B^{\,2}) = \rho_o r_o^{\,2}/r_B^{\,2}$, a constant which can be taken outside the integral sign, putting $r_B^{\,4}$ in the denominator. Gurney's reason for assuming a linear dependence of v on r now becomes clear. Equation (15) puts $r^2/r_B^{\,2}$ in the integral. Since the integral of r^3 is $r^4/4$, $r_B^{\,4}$ appears in the numerator to cancel $r_B^{\,4}$ in the denominator and make EC independent of the projectile size. The integral becomes $Cv_B^{\,2}/4$ and a solution for v_B gives us

$$v_B = \sqrt{2E/\left[(M/C) + 1/2\right]} \tag{26}$$

which is Gurney's widely used cylinder formula.

Gurney was also interested in small projectile warheads and grenades which more closely resemble spheres than long cylinders. His Equation (2) for a cased spherical charge is analogous to his Equation (1):

$$EC = \tfrac{1}{2}\, Mv_B^{\,2} + \tfrac{1}{2}\, v_B^{\,2}\int_0^{r_B} 4\pi r^2\rho \,\frac{r^2}{r_B^{\,2}}\, dr \quad . \tag{27}$$

where M is now the case mass and $C = \rho_o(4/3\pi r_o^{\,3})$ is the charge mass with r the spherical radial coordinate. Again he took $\rho = 3C/(4\pi r_B^{\,3})$ to be constant which put $r_B^{\,5}$ in the denominator. Of course the integral of r^4 is $r^5/5$ which puts $r_B^{\,5}$ in the numerator, making EC independent of projectile size in this geometry also. The integral becomes $3Cv_B^{\,2}/10$, and a solution for v_B gives us

$$v_B = \sqrt{2E \,/\, [(M/C) + 3/5]} \tag{28}$$

for a sphere. Gurney noted that Equations (26) and (28) make v_B vary as $(C/M)^{.5}$ for small C/M and as $(C/M)^{.25}$ for large C/M in agreement with the observations he set out to explain as simply as he could. Of course both equations tend to asymptotic values as $(C/M)\rightarrow\infty$ (bare charge).

In his discussion in Section IV of his report, Gurney suggests a slightly lower value than 1/2 in Equation (26). He also comments that his equations are intended to express the fact that a fraction of the chemical energy released is converted into kinetic energy, other details being unimportant for his purpose. (His report contains the word "important" instead of unimportant. This is an obvious typographical error since the context demands the latter.) In addition, he remarks that his taking r_B as the upper limit of his integral suggests a dependence on metal strength. He goes on to assert that he did this to simplify the calculation, since the evidence available to him at the time gave no hint of such a dependence. Later experiments have confirmed the fact that there is a small dependence of launch speed on metal strength. For example, Famiglietti[42] found that for identical geometries and explosive fills, the launch speed of Hadfield manganese steel fragments was about 15% greater than that of stainless steel fragments in spite of the fact that C/M was virtually the same in both cases. This implies that Gurney's $\sqrt{2E}$ is not strictly a characteristic of the explosive alone, although it has usually been treated as if it were.[11-14]

In Section V of his report, Gurney notes that the value $\sqrt{2E} = 2.44 \times 10^5$ cm/s gave a good fit for TNT-filled shells over the range $0.18 < M/C < 16.67$, which is a bit larger than the range examined by Taylor and Jones.[17] He also notes that this fitted value gives an E value which is only 80% of the energy per unit mass which ought to be released in a TNT detonation. This is, of course, expected since E is only the contribution to the total kinetic energy, the rest of the energy still remaining as internal energies of gas and metal after fragmentation (neglecting minor factors, such as light emission which occurs later anyway).

In the last paragraph of his report, Gurney mentions Taylor's paper[1] and correctly states that Taylor neglected radial gas motion compared to longitudinal. Most likely he had not read Taylor's paper at this time and had this on hearsay since he continues by saying that Taylor's results "were not intended to apply to a projectile from which the end sprays are feeble compared with the side spray." Gurney was obviously not aware that Taylor neglected v compared to u in moving coordinates but found $V_r \gg V_x$ in laboratory coordinates (small angle ϕ in Equation (11)). Gurney was a physicist with remarkable insight who could grasp the essentials of a problem without much mathematical analysis. For an appreciation of Gurney, especially his insights into quantum mechanical tunneling, see Condon.[43] As we have seen in our discussion of Equations (14) and (15) above, Gurney was making an excellent approximation in Equation (15). In effect, he was letting u = D in

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[42] M. Famiglietti, "Fragmentation of Ring Type Cylindrical Shell Made of Various Metals," Ballistic Research Laboratory Memorandum Report 597, 1953. (AD 486744)

[43] E. U. Condon, "Tunneling - How It All Started," American Journal of Physics, 46, 1978, p. 319.

Equation (13) when he let $\rho = \rho_B = C/(\pi r_B^2)$, which is very nearly true for bursts which occur for $r/r_o > 1.16$ (see Table 1). As Taylor pointed out, the gas density is nearly a constant over an entire plane at any distance x from a moving observer, that is, for any given r/r_o. For a sphere in which only radial motion occurs, the analog of Equation (14) is

$$\frac{1}{r^2}\frac{d}{dr}(r^2\rho v) = 0 \tag{29}$$

which can be considered together with mass conservation

$$C = \rho_o \frac{4}{3}\pi r_o^3 = \rho \frac{4}{3}\pi r^3 \tag{30}$$

to arrive at Equation (15) in this geometry also. If y is the cartesian coordinate normal to an initially planar explosive/metal interface and the detonation propagates along the x-axis, the analog of Equation (14) is

$$\frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial x}(\rho u) = 0 \quad , \tag{31}$$

where each term in Equation (31) vanishes. Mass conservation requires $\rho uy = \rho_o D y_o$ where y_o is the initial explosive sheet thickness, so u is not a function of x. For $u \approx D$, $\rho \approx 1/y$ so $v \sim y$ to make ρv independent of y.

Thomas,[44] a co-worker of Gurney, compared Gurney's and Taylor's models. The year before,[45] he originated the method of mentally dividing a shell or bomb into sections by planes perpendicular to its longitudinal axis in order to apply Gurney's method of calculating launch speeds, taking into account the curvature and variable wall thickness of real shells. His method is still used today and has been re-invented several times, most recently by Burman and Bedford.[46] In his synthesis,[44] Thomas began with a brief description of Gurney's model, generalizing Equations (26) and (28) to include the planar case

$$V = v_B = \sqrt{2E \,/\, [(M/C) + n/(n+2)]}, \tag{32}$$

where n = 1, 2 and 3 for planar, cylindrical and spherical symmetries respectively. Following this, he gave a brief account of Taylor's model, but

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[44] L. H. Thomas, "Theory of the Explosion of Cased Charges of Simple Shape," Ballistic Research Laboratory Report 475, 1944. (AD 491945)

[45] L. H. Thomas, "Analysis of the Distribution in Mass, in Speed, and in Direction of Motion of the Fragments of the M71 (90 mm) A. A. Shell, When Filled with TNT, and When Filled with Ednatol," Ballistic Research Laboratory Report 434, 1943. (ADB 493515)

[46] N.M. Burman and A.J. Bedford, "A Concept for the Prediction of Fragment Mass/Number Distributions of Fragmenting Munitions," Proceedings of the Sixth International Symposium on Ballistics, 1981, p. 245.

used the exact curvature $d\phi/ds = \sin\phi(d\phi/dr)$ as Allison and Shriempf[39] did sixteen years later. Thomas went on to solve the Lagrangian equations of motion by a series of successive approximations, using a parameter inverse to the detonation velocity. He obtained Gurney's result as the asymptotic limit of the zeroth order solution in which detonation is imagined to occur simultaneously everywhere in the explosive ($D = \infty$). Taylor's model emerged in higher order solutions. He concluded that Taylor's model should be an excellent approximation. Thomas also considered shocks and concluded that their effect on the energy is slight. After two or three brief reverberations, the motion settles down to an asymptotic form. For very thin shells stepwise shock acceleration at very early times has been observed by Allison and Shriempf;[39] Eden and Wright[47] have observed a similar effect for thin plates. Theoretical studies of shock acceleration have also been carried out in recent years, using the method of characteristics.[48,49] Neither of these studies changes Thomas' conclusion that there is no need to consider shock effects when considering the motion of most practical devices.

Thomas went on to consider planar, cylindrical and spherical cased charges initiated simultaneously at all points on a central plane, on an axis of symmetry or at the center respectively. For the planar case, he pointed out that the problem is equivalent to Lagrange's problem in internal ballistics which was treated in detail by Love and Pidduck.[50] This theme was later taken up by Jacobs,[51] Sterne[52,53] later extended Gurney's work to explosive sandwiches as well as cored cylinders and spheres with a fuze cavity. He also took up Thomas' model of symmetrical initiation as did Gurney.[38]

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[47]G. Eden and P.W. Wright, "A Technique for the Precise Measurement of the Motion of a Plane Free Surface," Fourth Symposium on Detonation, Naval Ordnance Laboratory ACR-126, 1965, p. 573.

[48]B. D. Lambourn and J. E. Hartley, "The Calculation of the Hydrodynamic Behavior of Plane One Dimensional Explosive/Metal Systems," Fourth Symposium on Detonation, 1965, p. 538.

[49]N. E. Hoskin and B. D. Lambourn, "The Acceleration of Two Metal Plates in an HE-Metal Sandwich," Seventh Symposium on Detonation, Naval Surface Weapons Center MP 82-334, 1981, p. 811.

[50]Love and Pidduck, "Lagrange Ballistic Problem," Phil. Trans. Royal Society of London, 222, 1922, p. 167.

[51]S. J. Jacobs, "The Gurney Formula: Variations on a Theme by Lagrange," NOLTR 74-86, 1974.

[52]T. E. Sterne, "A Note on the Initial Velocities of Fragments from Warheads," Ballistic Research Laboratory Report 648, 1947. (AD 898680)

[53]T. E. Sterne, "The Fragment Velocity of a Spherical Shell Containing an Inert Core," Ballistic Research Laboratory Report 753, 1951. (ADB 377181)

Let us compare Gurney's model with Taylor's. Gurney's model is more elementary since it relies on an energy partition at a particular moment, while Taylor solves an equation of motion. With Gurney's model, we can only estimate a speed at a given time. With Taylor's model, we can calculate the position (case shape) and velocity (speed and direction) of each case element at any time. For many purposes, a Gurney speed with or without the addition of a Taylor angle is sufficient. If metal trajectories are required, Taylor's model might be preferred. Thomas' solution of the Lagrangian equations could be used, of course, but this is usually too complicated an approach for design work where the aim is insight with minimum effort rather than great precision. Sometimes it is useful or even necessary to include shock effects, as when one metal plate is used to accelerate and fragment another.[54]

More detailed modeling might include microscopic defects, crack propagation or even molecular interactions.[55-58]

V. TAYLOR'S MODEL SIMPLIFIED AND EXTENDED

A. Cylinder Struck Side-on

Since the publication of Taylor's papers by Batchelor in 1963,[1] his model has been more readily accessible to a wider audience than Gurney's report,[2] which was, practically speaking, an internal laboratory memorandum which relatively few people have actually read. In spite of this, Gurney's model has been widely used and extended, while Taylor's model has been virtually ignored, except for one equation. Clearly, this is because of the relative simplicity of Gurney's model. In this section we will simplify Taylor's model in order to facilitate its use in problems for which it is appropriate.

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[54] J. F. Mescall and P. V. Riffin, "Slapper Concept in Fragmentation," AMMRC Technical Report 76-8, 1976.

[55] N. F. Mott, "Fragmentation of Shell Cases," Proceedings of the Royal Society of London, 189. 1947, p. 300.

[56] R. Curran, L. Seaman, and D. A. Shockey, "Dynamic Failure in Solids," Physics Today, January 1977, p. 46.

[57] F. E. Walker, A. M. Karo, and J. R. Hardy, "Comparison of Molecular Dynamics Calculations with Observed Initiation Phenomena," Seventh Symposium on Detonation, 1981, p. 777.

[58] A. M. Karo, F. E. Walker, W. G. Cunningham, and J. R. Hardy, "Theoretical Studies of Shock Dynamics in Two Dimensional Structures," Shock Waves in Condensed Matter - 1981, W. J. Nellis, L. Seaman, and R. A. Graham, eds., AIP Conference Proceedings 78, American Institute of Physics, NY, 1982, p. 92.

As we mentioned after Equation (25) above, Gurney took ρ to be uniform inside the case for any $\ell = r/r_o$ value. In effect, he took

$$\rho = \rho_o/\ell^2 \quad , \tag{33}$$

where $r = r_B$ (late in the expansion). Equation (13) can be written

$$\rho = (\rho_o D)/\ell^2 u) = \rho_o/(\ell^2 f) \quad , \tag{34}$$

where $f = u/D \approx 1$. Equations (34) and (33) are the same for $f = 1$. If we put Equation (34) for ρ in Equation (1) with $\gamma = 3$ and Equation (34) for (ρu) in the first term of Equation (20), we find

$$\frac{p}{\rho u} = [(B\rho_o{}^2)/(Df^3)]/\ell^4 = U_B/\ell^4 \tag{35}$$

which defines U_B. From Figure 1 for a TNT loading density near that discussed by Taylor, we see that $B = 2 \times 10^{10}(\text{dyne/cm}^2)/\text{g/cm}^3)^3 = 2(\text{mm}/\mu\text{s})^2/(\text{g/cm}^3)^2$. Consequently, for $\rho_o = 1.51\ \text{g/cm}^3$ and $D = 6.38\ \text{mm}/\mu\text{s}$, $U_B = .7/f^3 = 1\ \text{mm}/\mu\text{s}$ if $f = 0.89$. This value of f can be taken as an effective value during the acceleration period and enables us to deal with U_B as a constant dependent only on explosive properties.

Next let us approximate u by the formula

$$u = U_\infty - U_B(1/\ell^2 + 1/\ell^4) \quad , \tag{36}$$

where $U_\infty = 6.914\ \text{mm}/\mu\text{s}$ from Table 1. If $U_B = 1$ mm/ s, Equation (36) can be used in Equation (34) to find

$$\rho = 9.634/[6.914\ell^2 - (1 + 1/\ell^2)] \quad , \tag{34a}$$

where $\rho_o D = (1.51\ \text{g/cm}^3) \times (6.38\ \text{mm}/\mu\text{s}) = 9.634\ (\text{g/cm}^3)(\text{mm}/\mu\text{s})$.

In Figure 3 we plot ρ versus ℓ from Table 1 as the solid line and Equation (34a) as the long dash line which deviates from the solid line by at most 15% (near $\ell = 1.1$) and is indistinguishable from it on the scale shown for $\ell >$ 1.5. A somewhat smaller U_B would improve the agreement except near $\ell = 1$. The short dash line in Figure 3 plots $\rho = \rho_o/\ell^2 = 1.51/\ell^2$, showing why Gurney's cylinder formula based on a radial energy partition can be applied during the later stages of the expansion of devices initiated at one end. A plot of $\rho = \rho_1/\ell^2 = 2/\ell^2$ would lie considerably higher.

Figure 4 plots u versus ℓ from Table 1 as a solid line and Equation (36) with the values mentioned as the dashed line. Smaller U_B would make the dashed line lie higher. The per cent deviation is about the same as in Figure 3, reaching a maximum of about 15% near $\ell = 1.1$. Disagreement with Taylor's calculations during the early stages of the motion is not necessarily bad. Because he used a model in which the width of the reaction zone is neglected, we expect his results to be in error at early times. This point will be discussed further in Section C below.

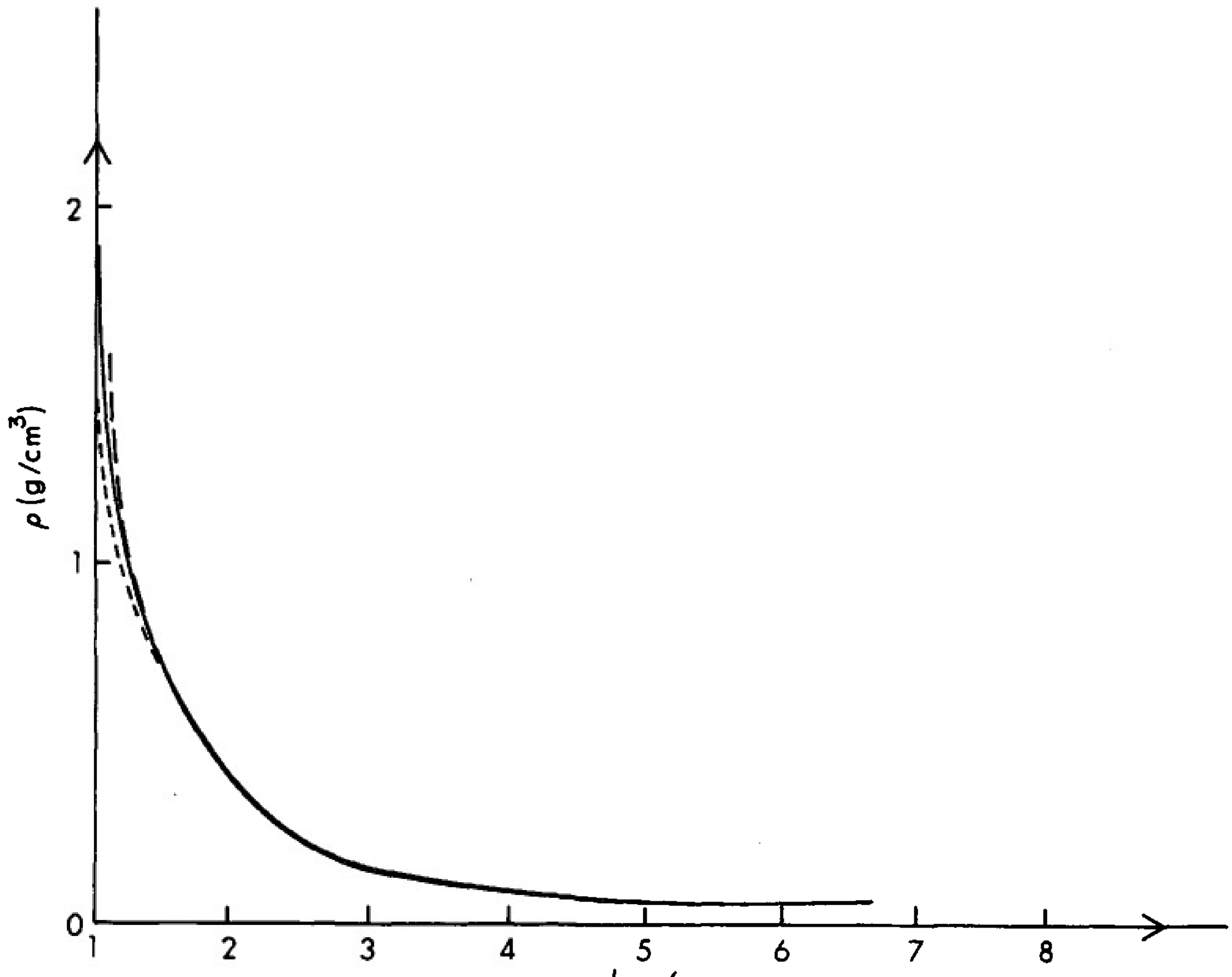

Figure 3. Gas Density in an Expanding Cylinder.

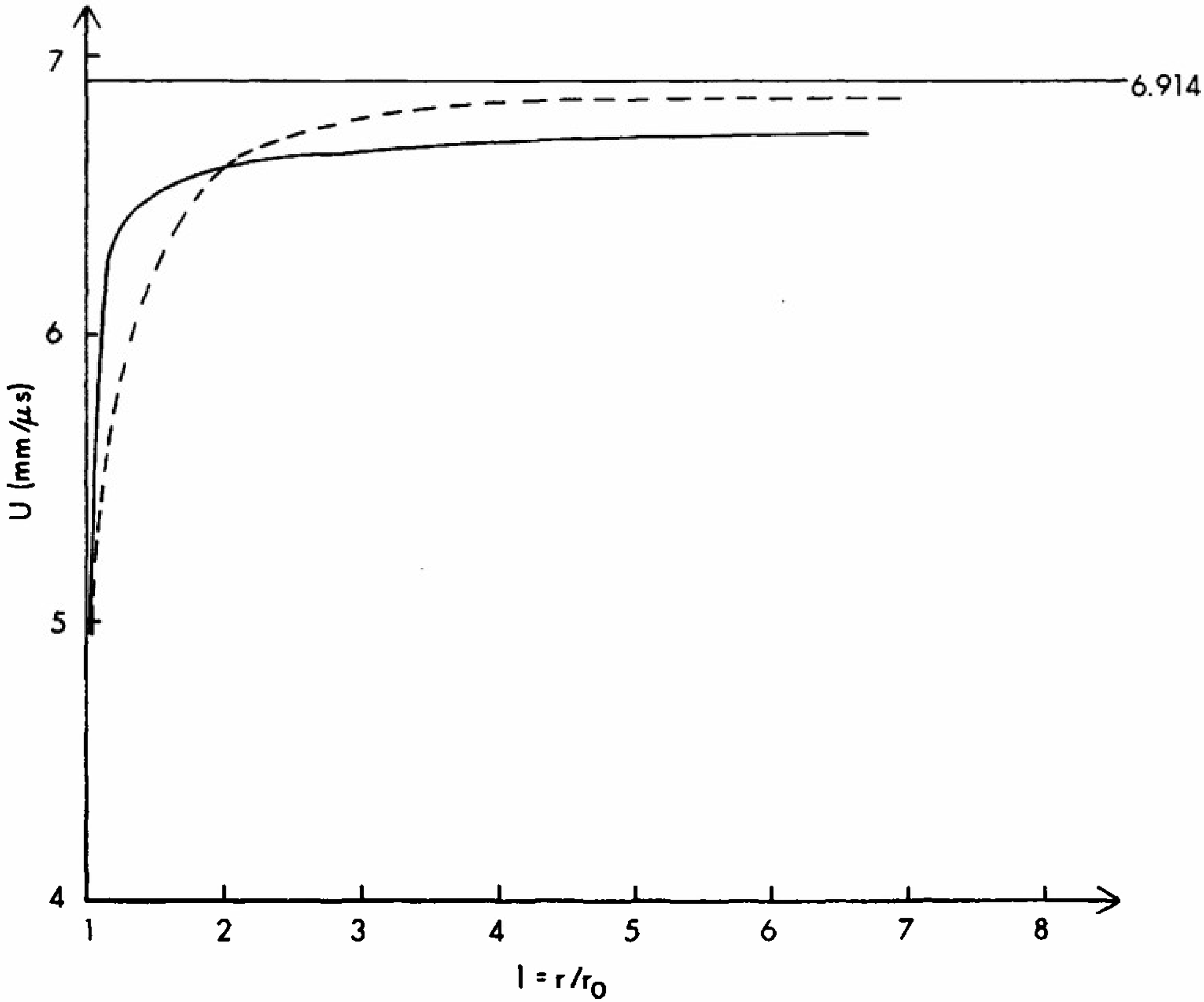

Figure 4. Moving Coordinate Axial Gas Velocity in an Expanding Cylinder.

Now let us put Equations (35) and (36) in Equation (20) and require $V = 0$ for $\ell = 1$. We find

$$U_\infty = D + U_B , \tag{37}$$

so that U_∞ can be estimated for any explosive loading from D, ρ_o, B and f -- knowledge which is available for most cases of interest. We may assume $f = 0.89$ or use Taylor's $f = u/D$ as a universal function. Equation (20) becomes

$$V = \rho_o f^{-3/2} \sqrt{(2B/\alpha)(1-1/\ell^2)} \tag{38}$$

which does not depend on D explicitly as in Taylor's model (only through ρ_o and B). The parameter $\alpha = M/C$ appears in the denominator under a square root sign as in Gurney's formula, although there is no constant added to since Taylor neglects v compared to u. This will be discussed below in Section VI. As $\ell \to \infty$ and $f \approx 1$, $V = \rho_o \sqrt{(2B)/\alpha}$ and $\sqrt{2B\rho_o^2}$ plays a similar role to $\sqrt{2E}$ in Gurney's formula. Here, however, there is an explicit dependence on ρ_o, as well as that implicit in B and α. It would be desirable to tabulate values of $\sqrt{2B\rho_o^2}$ for cases of practical interest. However, to do a proper job, we should make a critical comparison of p versus ρ curves calculated by various methods. Alternatively, we can tabulate $\sqrt{2B\rho_o^2}$ from experimental information.

If we use Taylor's values of f in Equation (38) and $B = 2(\mathrm{mm}/\mu\mathrm{s})^2/(\mathrm{g/cm}^3)^2$ with $\alpha = 2.67$ for the standard cylinder, we find the values given in column 3 of Table 2. These are about 15% higher than the experimental values which have been repeated in column 2. If we let B = 1.5 instead of 2, we find the values in column 4 which exhibit a root mean square error of 0.04 mm/μs relative to the six experimental values. Here we are lowering B by 25% much as Gurney lowered E from the value he expected.

Since $V \approx V_r = dr/dt = r_o d\ell/dt$, we can integrate Equation (38) with f constant (U_B constant) to find

$$Dt \approx r_o \sqrt{[(\alpha D)/(2U_B)](\ell^2-1)} \approx x \tag{39}$$

since $v_x = dx/dt = D \cos\phi \approx D$ as was mentioned above in connection with Equation (10). If we put Equation (39) into Equation (4), it is clear that a case element which is seen by a laboratory observer to have very little axial motion ($X \approx X_o$) is said by the moving observer to recede axially at about the detonation speed. Of course, both observers see the same radial motion. We can rewrite Equation (39) as

$$r^2/r_o^2 - x^2/[r_o^2 \alpha D/(2U_B)] = 1 , \tag{40}$$

which exhibits the approximately hyperbolic form of the case shape in moving coordinates.

Since $V_x/V_r = dX/dr = \tan(\phi/2)$ from Equation (11) and $\tan\phi \approx 2 \sin(\phi/2) \approx 2 \tan(\phi/2)$, we can integrate Equation (38) with U_B conatant, using $V = 2D \sin(\phi/2)$ from Equation (20) to find the X, r path of a case element in laboratory coordinates, namely,

$$X - X_o = r_o \int_1^{\ell} \tan(\phi/2) d\ell = r_o \sqrt{\frac{U_B}{2\alpha D}} \left[\sqrt{\ell^2 - 1} - \cos^{-1}(1/\ell)\right] \qquad (41)$$

which also gives us X(t) since $\ell(t)$ is known from Equation (39). Accelerations as functions of time may also be found. High speed cinematography or a sequential series of still photographs as used by Taylor and Jones[17] could be used to check some of these relationships.

Column 5 of Table 2 gives a more complete listing of Taylor's values of $\frac{x}{r_o}\sqrt{\frac{2}{\alpha}}$ than column 6 of Table 1, while column 6 of Table 2 gives values calculated from Equation (39) using $D = 6.38$ mm/μs and $U_B = .75$ mm/μs corresponding to $B = 1.5$ instead of 2. Column 7 gives values for the same case shape function $x(\ell)$ as will be explained in part C below.

Column 8 of Table 2 gives experimental values of the time $t(\ell)$ observed by Lee and co-workers[59] for TNT with $\rho_o = 1.63$ g/cm^3 ($\alpha = 2.47$) in the standard copper cylinder. Columns 9 and 10 give values of t calculated from Equation (39) using $r_o = 15.3$ mm for this test, $D = 6.38$ mm/μs and $U_B = 1$ or $.75$ mm/μs corresponding to $B = 2$ or 1.5 (mm/μs)^2/(g/cm^3)^2. During the later stages of the expansion, the calculated values tend to bracket the experimental values but are higher in the early stages.

Column 11 of Table 2 gives experimental values[34] for 64/36 RDX/TNT with $\rho_o = 1.717$ g/cm^3 ($\alpha = 2.34$) in the standard cylinder. From Figure 1 we see that $B = 2.4$ (mm/μs)^2/(g/cm^3)^2 for this loading. Column 12 gives V calculated from Equation (38) for $B = 2.4$, while column 13 uses $B = .7(2.4) = 1.68$. This illustrates the fact that Equation (38) and presumably Equations (39)-(41) and others related to them can be applied to other explosive loadings. Comparisons with experimental values covering a larger range of α values would also be desirable, but little time-dependent data is available. In Section VI we will make some comparisons of Equation (38) with launch speeds for a range of α values.

[59] E.L. Lee, H.C. Hornig, and J.W. Kury, "Adiabatic Expansion of High Explosive Detonation Products," UCRL-50422, Lawrence Radiation Labortory, 1968.

Table 2. Comparisons of Simplified Taylor Model with Experiment

TNT										COMP B		
1	2	3	4	5	6	7	8	9	10	11	12	13
ℓ	V_{EXP}	V	V	$\frac{x}{r_c}\sqrt{\frac{2}{\alpha}}$	$2.92\sqrt{\ell^2-1}$	$2\cosh^{-1}\ell$	t_{EXP}	t	t	V_{EXP}	V	V
		(B = 2)	(B = 1.5)					(B = 2)	(B = 1.5)		(B = 2.4)	(B = 1.68)
	mm/μs	mm/μs	mm/μs				μs	μs	μs	mm/μs	mm/μs	mm/μs
1.000		0	0	0	0	0		0	0		0	0
1.005		.03	.02	.16	.29	.06		.67	.78		.04	.03
1.009		.34	.29	.25	.39	.09		.90	1.04		.45	.38
1.014		.41	.35	.31	.49	.33		1.13	1.31		.55	.46
1.020		.47	.40	.38	.58	.40		1.35	1.56		.62	.52
1.044		.60	.52	.62	.88	.59	1.14	2.02	2.33		.81	.68
1.074		.73	.63	.84	1.15	.76	1.72	2.64	3.05		.97	.82
1.159		.95	.82	1.42	1.71	1.12	3.09	3.94	4.55		1.26	1.06
1.217		1.05	.91	1.90	2.02	1.30	3.91	4.67	5.39		1.40	1.17
1.371	(1.13)	1.22	1.06	2.64	2.74		5.98	6.31	7.29	(1.43)	1.64	1.38
1.538	1.20	1.34	1.16	3.53	3.42		8.03	7.87	9.08	1.49	1.80	1.51
1.667	1.25	1.41	1.22	4.18	3.89		9.56	8.98	10.4	1.53	1.89	1.58
1.883	1.30	1.48	1.28	5.23	4.66		12.1	10.7	12.4	1.58	1.98	1.66
2.372	1.35	1.56	1.35	7.49	6.28			14.5	16.7	1.66	2.10	1.76
3.060	(1.37)	1.62	1.40	10.5	8.45			19.5	22.5	(1.68)	2.17	1.82
4.34		1.80	1.55	16.0	12.4			28.4	32.8		2.40	2.01
5.68		1.80	1.55	21.5	16.4			37.6	43.4		2.40	2.01

B. Plate Struck Side-On

Now let us apply Taylor's model to a planar rather than a cylindrical geometry. The situation is similar to that in Figure 2 except that we will use the Cartesian coordinate y instead of r, so $\tan\phi = dy/dx$. We will allow the metal plates to have different thicknesses h_1 and h_0. We will let the thickness of the explosive sheet be $W_0 = 2y_0$. The mass per unit area of each metal plate will be designated by $M_i = \rho_{Mi} h_i$ with $i = 0, 1$ where we allow the plates to have different densities ρ_{Mi} as well as different thicknesses. Equation (6) is now replaced by

$$M_i D^2 \frac{\frac{d}{dx}\left(\frac{dy}{dx}\right)}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}} = p \,. \tag{42}$$

We multiply each side of this equation by $2dy/(M_i D^2) = 2 \tan\phi \; dx/(M_i D^2)$ and obtain

$$2 \sin\phi \; d\phi = \frac{2}{M_i D^2} \; pdy \quad , \tag{43}$$

where the left side is readily integrated as in Equation (8). Equations (10) and (11) will have V_y instead of V_r but are otherwise unchanged. Equation (13) is replaced by

$$\rho_0 D y_0 = \rho u y \quad , \tag{44}$$

and Equation (14) by Equation (31) which we have already discussed. Equations (16) through (19) are unchanged, so the analog of Equation (20) is

$$\begin{aligned} \frac{2}{M_i D^2} \int_{y_0}^{y} pdy &= \frac{2\rho_0 D y_0}{M_i D^2} \int_1 pd\left(\frac{1}{\rho u}\right) \\ &= \frac{1}{\alpha_i D} \left[\frac{p}{\rho u} + u - D\right] \\ &= 4 \sin^2 (\phi/2) \approx \tan^2 \phi \\ &= (V/D)^2 \quad , \end{aligned} \tag{45}$$

where

$$\alpha_i = M_i/(2y_0\rho_0) = M_i/C \tag{46}$$

and C is the explosive mass per unit area. We have not repeated several steps here since they are the same as before. The analog of Equation (23) is

$$x = \int_{y_0}^{y} dy/\tan\phi \quad . \tag{47}$$

Now let us simplify this model. Since u is a function of y by Equation (44) as is $\rho = \rho_o/(f\ell)$ where $\ell = y/y_o$ and $f = u/D$, we can write the analog of Equation (35)

$$\frac{p}{\rho u} = \left[B\rho_o^{\ 2}/(Df^3)\right] /\ell^2 = U_B/\ell^2 \ , \tag{48}$$

where U_B has the same form as in Equation (35).

Now let us approximate u by the formula

$$u = U_\infty - U_B\left(\frac{1}{\ell} + \frac{1}{\ell^2}\right) \tag{49}$$

which is the analog of Equation (36). Here the inverse powers of ℓ are n and 2n with n = 1, just as they were in Equation (36) with n = 2. The spherical analog might consist of a disk-shaped detonation front rotating about a radius, s configuration very difficult to realize experimentally. If we use Equation (49) in Equation (44), we find

$$\rho = \rho_o D/\left[U_\infty \ell - U_B\left(1 + \frac{1}{\ell}\right)\right] \ . \tag{50}$$

If we use $\rho_o D = 9.634(g/cm^3)(mm/\mu s)$, $U_\infty = 6.914$ mm/μs and $U_B = 1$ mm/μs, we obtain the decreasing long dash line in Figure 5 which diminishes more slowly than the analogous curve in Figure 3. The short dash line just above it corresponds to $\rho = \rho_1/\ell = 2/\ell$ for the head-on case, while the dotted line below it is for $\rho = \rho_o/\ell = 1.51/\ell$, a Gurney-type assumption. The increasing dashed line plots Equation (49) for the same U_∞ and U_B. This line increases more slowly than its analog in Figure 4.

Next, let us put Equations (48) and (49) in Equation (45) and require V = 0 for $\ell = 1$, giving Equation (37) again. Equation (45) becomes

$$V = \rho_o f^{-3/2}\sqrt{\left(\frac{B}{\alpha_i}\right)\left(1 - \frac{1}{\ell}\right)} \tag{51}$$

which is the analog of Equation (38) with n = 1 replacing n = 2 as the coefficient of B and the inverse power of ℓ. As $\ell \to \infty$ and $f \sim 1$ we can compare Equation (51) with Equation (32) above with n = 1. For two equal mass metal plates in a symmetric sandwich, $\alpha = 2\,\alpha_1 = 2M/C$ in Equation (46), so Equation (51) becomes $V = \rho_o\sqrt{2B/\alpha}$ in this limit. We will compare these formulas for a range of α values in Section VII below.

Since $V \approx V_y = dy/dt = y_o d\ell/dt$, we can integrate Equation (51) with f constant to find the analog of Equation (39)

$$Dt \approx y_o\sqrt{\frac{\alpha_i D}{2U_B}}\left\{\ell\sqrt{1 - 1/\ell} + \tfrac{1}{2}\ln\left[\ell\sqrt{1 - 1/\ell} + \ell - \tfrac{1}{2}\right] - \tfrac{1}{2}\ln\tfrac{1}{2}\right\} \tag{52}$$

$$\approx x \ ,$$

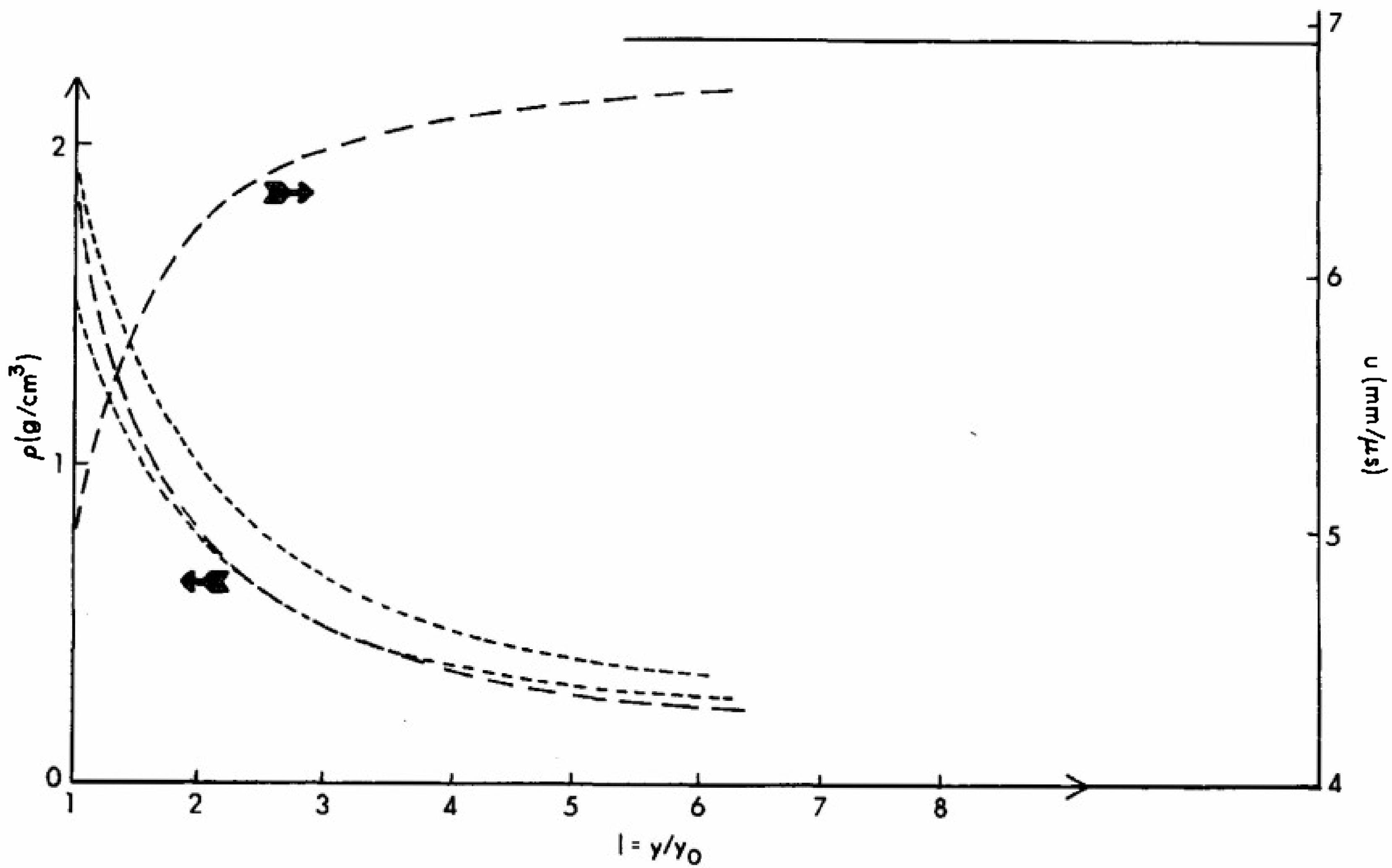

Figure 5. Gas Density and x Velocity Component for an Expanding Sheet.

Clearly the case shape (y versus x) is not approximately hyperbolic as it was for the cylinder. Since $V_x/V_y = dX/dy = \tan(\phi/2)$ by the analog of Equation (11) and $\tan\phi \sim 2\sin(\phi/2) \sim 2\tan(\phi/2)$, we can integrate Equation (45) with U_B (or f) constant to find the X, y path of a plate element in laboratory coordinates,

$$X - X_o = y_o\sqrt{U_B/(4\alpha_i D)}\left\{\ell\sqrt{1 - 1/\ell} - \tfrac{1}{2}\ln\left[\ell\sqrt{1 - 1/\ell} + \ell - \tfrac{1}{2}\right] + \tfrac{1}{2}\ln\tfrac{1}{2}\right\} \tag{53}$$

which is the analog of Equation (41). Use of Equation (52) gives X(t). Accelerations may also be found.

C. Expansion in the Reaction Zone

As we noted above, Taylor used a model which assumes a zero width for the reaction zone. H. Jones[21] has pointed out that case expansion occurs inside the reaction zone, leading to a slight decrease in detonation velocity compared to the idealized Chapman-Jouget value. He also derived an expression for the shape of an expanding cylinder valid inside the reaction zone:

$$\sqrt{\frac{2}{\alpha}}\,\frac{x}{r_o} = 2\cosh^{-1}(\ell) \tag{54}$$

for $1 \leq \ell = r/r_o \leq r_1/r_o$. Here we have used α as defined in Equation (21) above instead of Jones' mass per unit area $\sigma = M/(2\pi r_o)$. At the rear of the reaction zone we can rewrite Equation (54) as

$$\ell_1 = r_1/r_o = \cosh\frac{x_1}{2r_o}\sqrt{\frac{2}{\alpha}} = 1 + \lambda \quad . \tag{55}$$

Jones gave a numerical example in which he took $x_1/r_o = 2/\alpha = 1/2$, assuming that the width of the reaction zone, x_1, might be as large as half the charge radius for $\alpha = 4$. Thus, $\lambda = 0.015$ in Equation (55) so ℓ_1 is only slightly larger than unity. This would lead to a 6.25% reduction in detonation velocity compared to the idealized value. For a bare charge (the extreme departure from a rigid tube), Jones estimates a 9.6% reduction for the same reaction zone. For solid or liquid explosives, the idealization of a rigid tube cannot be realized experimentally since at least the inner radius of even a very heavy-walled tube of metal will expand somewhat under the pressures produced by condensed explosives (unlike a shock tube filled with a detonable gas mixture).

Column 7 in Table 2 exhibits Equation (54) as a function of ℓ and compares it with Taylor's values in column 5, as well as the present model values in column 6. According to Jones there is less expansion near the front of the reaction zone than Taylor estimates (or a given amount of expansion occurs farther behind the detonation front). However, the two curves agree rather closely even in the reaction zone (say $1 < \ell < 1.03$) and deviate appreciably only well behind the reaction zone where Jones' formula is no longer applicable. This is illustrated in Figure 6.

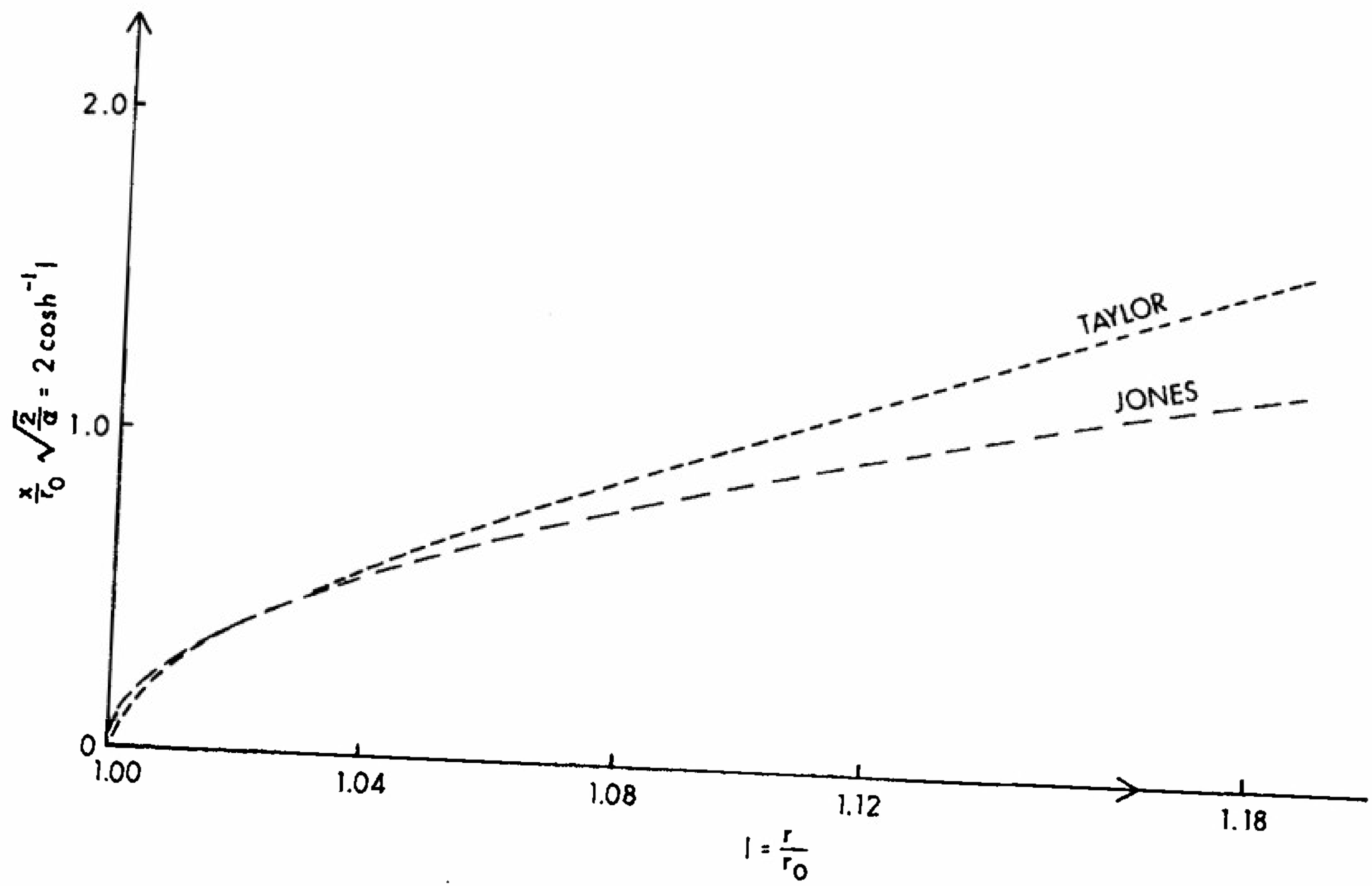

Figure 6. Jones' and Taylor's Cylinder Case Shapes Compared.

VI. THOMAS' HEAD-ON MODEL SIMPLIFIED

Thomas[44] developed his own model for plane, cylindrical and spherical cased charges initiated on a symmetry plane, axis or point. In these cases the detonation front will strike all points of the metal surface simultaneously, "head-on" (propagation vector parallel to surface normal). If dm' is the mass of product gas in a volume element of thickness dr (per unit area for the plane, per unit length and angle for the cylinder and per unit solid angle for the sphere), then this specific mass element is

$$dm' = \rho r^{n-1} dr \quad , \tag{56}$$

where n = 1, 2 and 3 for plane, cylinder and sphere respectively. The equation of gas motion is

$$\frac{\partial^2 r}{\partial t^2} = -\frac{1}{\rho}\frac{\partial p}{\partial r} = -r^{n-1}\frac{\partial r}{\partial m'}\frac{\partial p}{\partial r} = -r^{n-1}\frac{\partial p}{\partial m'} \quad . \tag{57}$$

Thomas used Equation (1) for p but kept γ arbitrary and concentrated on cylindrical symmetry. He was able to obtain a first integral but could not completely solve the problem. He assumed an initial uniform distribution of mass and outward velocity when the detonation wave reached the metal surface with boundary conditions $m' = 0$ for $r = 0$ and

$$M\frac{\partial^2 r}{\partial t^2} = r^{n-1} p \tag{58}$$

for $m' = C$, the total specific charge mass, at the moving metal surface. As before, M is the specific metal mass. After a brief period of shock reverberation, Thomas argued, the solution should approach a form in which the method of separation of variables can be applied. Letting r be a product of a function of time and a function of mass, namely,

$$r = \ell(t)g(m') \quad . \tag{59}$$

Equation (57) separates into

$$\frac{d^2\ell}{dt^2} = \nu^2 \ell^{n-1-n\gamma} \tag{60}$$

and

$$\nu^2 g = -Bg^{n-1}\frac{d}{dm'}\left[\left(g^{n-1}\frac{dg}{dm'}\right)^{-\gamma}\right] \quad , \tag{61}$$

where we have used ν^2 for the separation constant instead of letting it be unity as Thomas did. Dimensionally speaking, ν is an inverse relaxation time. The boundary conditions become $g(0) = 0$ and

$$M\nu^2 g = Bg^{n-1}\left(g^{n-1}\frac{dg}{dm'}\right)^{-\gamma} \tag{62}$$

for $m' = C$ where we have used Equations (59) and (60) in Equation (58).

If we multiply Equation (60) by $d\ell = \dot{\ell}dt$ and integrate, we obtain the first integral of Equation (60)

$$\dot{\ell} = \nu \sqrt{\frac{2}{n(\gamma - 1)} \left[1 - \ell^{-n(\gamma-1)}\right]} = V/r_o \quad , \tag{63}$$

where we have taken $\dot{\ell} = 0$ for $\ell = 1$. The second equality comes from taking $g(C) = r_o$ in Equation (59) so $V = \dot{r} = r_o\dot{\ell}$ for the case velocity. If $\gamma = 3$ and $\ell \to \infty$ in Equation (63), $V = r_o\nu/\sqrt{n}$. As $\ell \to \infty$ and $f \sim 1$, Equation (38) for a cylinder and Equation (51) for a symmetric sandwich ($\alpha = 2\alpha_1$) both give $V = \rho_o\sqrt{(2B)/\alpha}$. If we assume that the head-on and side-on formulas are equal under these conditions (same launch velocity),

$$\nu = \sqrt{n}\,(\rho_o/r_o)\sqrt{(2B)/\alpha} \quad , \tag{64}$$

and Equation (63) becomes for $\gamma = 3$

$$V = \rho_o\sqrt{(2B/\alpha)(1 - \ell^{-2n})} \quad . \tag{65}$$

Requiring approximate equality for large ℓ corresponds to the approximate equalities of the gas densities illustrated in Figure 3 as ℓ increases. For a cylinder ($n = 2$) the head-on velocity in Equation (65) may be expressed as

$$V = \rho_o\sqrt{(2B/\alpha)(1 + \ell^{-2})(1 - \ell^{-2})} \tag{66}$$

and compared with Equation (38) for the side-on velocity

$$V = \rho_o\sqrt{(2B/\alpha)f^{-3}(1 - \ell^{-2})} \quad . \tag{38}$$

In both cases $V = 0$ for $\ell = 1$. They differ by the factors $\sqrt{1 + \ell^{-2}}$ and $f^{-3/2}$, where f is given by u in Taylor's Table I or Equation (36) divided by D. Similarly for a symmetric sandwich ($n = 1$), the head-on velocity in Equation (65) is

$$V = \rho_o\sqrt{(2B/\alpha)(1 + \ell^{-1})(1 - \ell^{-1})} \quad , \tag{67}$$

while the side-on velocity in Equation (51) is

$$V = \rho_o\sqrt{(2B/\alpha)f^{-3}(1 - \ell^{-1})} \quad . \tag{51}$$

These formulas differ by the factors $\sqrt{1 + \ell^{-1}}$ and $f^{-3/2}$, where f is given by Equation (49) divided by D.

Thomas did not let $\gamma = 3$ or make such interpretations of ℓ and g and of course set $\nu = 1$. However, he did write down the first integral of Equation (61) for arbitrary n and γ, namely, with K constant,

$$\tfrac{1}{2}\nu^2 g^2 + \frac{\gamma B}{(\gamma - 1)} \left(g^{n-1} \frac{dg}{dm'}\right)^{-(\gamma - 1)} = K . \tag{68}$$

For the special case of a cylinder (n = 2), he also found a second integral

$$m' = (B/\nu^2)\left(\frac{\gamma - 1}{\gamma B}\right)^{\frac{\gamma}{\gamma - 1}} \left[K^{\frac{\gamma}{\gamma - 1}} - (K - \tfrac{1}{2}\nu^2 g^2)^{\frac{\gamma}{\gamma - 1}}\right] . \tag{69}$$

However, for n = 2 and arbitrary γ, he could not integrate Equation (63) exactly. Instead, he integrated numerically with γ near 3 as a parameter. Finally he found the kinetic energy of the gas in the expanding cylinder and added this to the kinetic energy of the case, equating the sum to EC in order to compare with Gurney's model. In the limit $\ell \to \infty$ he wrote a formula for V which depended on γ, as well as $\sqrt{2E}$ and α. Even in this limit his formula for arbitrary γ is rather complicated. However, for $\gamma = 3$ it simplifies to

$$V = \sqrt{[(2E)/(2\alpha/5)]\ [(1 + 1/\alpha)^{2/3} - 1] / [(1 + 1/\alpha)^{5/3} - 1]} \tag{70}$$

which gives almost the same launch speed as Gurney's much simpler cylinder formula,

$$V = \sqrt{(2E)/[\alpha + \tfrac{1}{2}]} \ . \tag{26}$$

These two formulas are compared as curves (a) and (b) in Figure 7. As $\alpha \to 0$, Thomas' $V \to \sqrt{5E}$ while Gurney's $V \to \sqrt{4E}$. Both formulas approach zero as $\alpha \to \infty$ and are practically indistinguishable for $\alpha > 0.1$. Of course V will reach zero for large finite α when the case fails to break and the model has ceased to apply.

Equation (38) for $\ell \to \infty$, $f \approx 1$ gives $V = \rho_o\sqrt{2B/\alpha}$ which approaches zero as $\alpha \to \infty$ but increases without limit as $\alpha \to 0$ since Taylor's model takes no account of the gas energy and assumes a "sufficiently heavy" case ($\alpha > 0$). One way to compare this form of V with Gurney's formula might be to assume that some fraction, say 0.8, of the available energy appears as case kinetic energy with the rest appearing mainly as gas energy as $\ell \to \infty$. Then

$$\tfrac{1}{2} MV^2 = \tfrac{1}{2} M(2B\rho_o^{\,2}/\alpha) = 0.8\ (EC), \tag{71}$$

and the limit form of Equation (38) becomes

$$V = \sqrt{(2E)/(\alpha/0.8)}\ , \tag{72}$$

which is plotted as curve (c) in Figure 7. If we wish to avoid the large divergence near $\alpha = 0$, we might add a constant like n/(n + 2) from Equation (32) to α in Equations (64) - (67), (38) and (51). Then for n = 2, the limit form of Equation (38) becomes

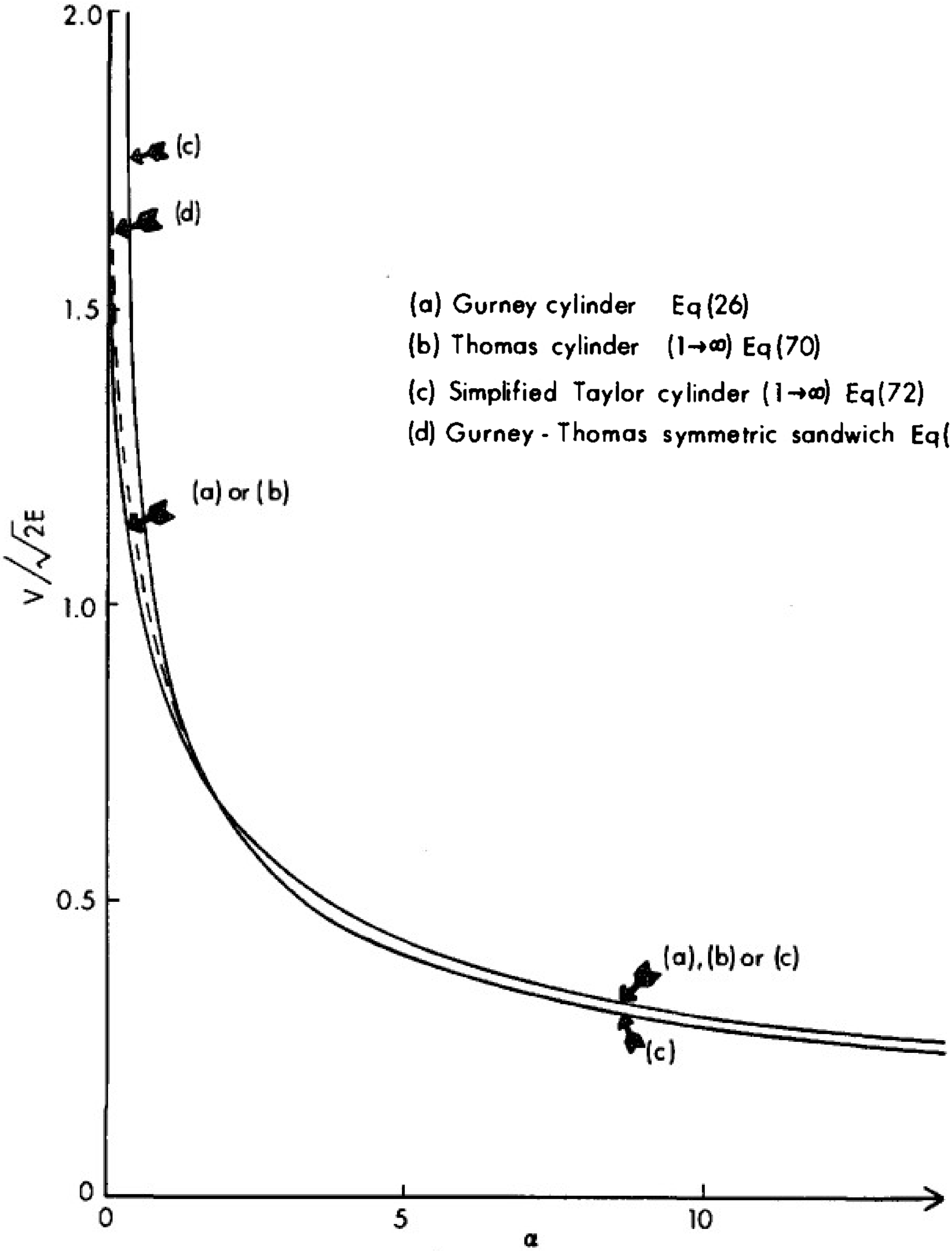

Figure 7. Comparison of Launch Velocity Formulas Versus α.

$$V = \rho_o \sqrt{(2B)/(\alpha + 1/2)} \ . \tag{73}$$

If we require Equation (73) to reduoe to Gurney's formula as $\alpha \to 0$, then $V = \sqrt{4B\rho_o^2} = \sqrt{4E}$ or

$$B\rho_o^2 = E \ , \tag{74}$$

completing the analogy between B and E. If we add 1/3 to α for n = 1 and consider the limit form ($\ell \to \infty$; $f \approx 1$), Equation (51) becomes for $\alpha_1 = \alpha/2$

$$V = \sqrt{2B\rho_o^2/(\alpha + 1/3)} = \sqrt{2E/(\alpha + 1/3)} \tag{75}$$

by Equation (74). This is the same as the Gurney-Thomas formula for a symmetric sandwich, Equation (32), and is also plotted in Figure 7. Equation (32) with n = 3 for a sphere might also be plotted but has been omitted to avoid further confusing the figure. For typical values of α which generally exceed unity, all of these formulas predict similar launch velocities independent of geometry (plane, cylinder or sphere) or direction of travel of the detonation wave (head-on or side-on). They differ mainly for small ℓ. None of them are really intended for a bare charge ($\alpha = 0$) or for very large values of $\alpha = M/C$. A finite value for V in the limit $\alpha \to 0$ might be thought of as corresponding to the case in which a finite mass of metal is accelerated by an unlimited amount of explosive. As is well known, beyond a certain point the addition of more explosive does almost nothing to increase launch velocity, even in a vacuum where air resistance is not a limiting factor. Similarly, if the limit $\alpha \to \infty$ is approached by decreasing the amount of explosive, we reach a situation in which a detonation wave will not propagate because the dimensions are too small. Again the model does not apply.

Thomas did not consider the case $\gamma = 3$, nor did he pursue a solution for planar symmetry (n = 1). However, this is a case for which his formulation of the problem may be solved exactly. For $\gamma = 3$, n = 1, the integral of Equation (63) is

$$\ell = \sqrt{1 + \nu^2 t^2} \ , \tag{76}$$

while the integral of Equation (68) is

$$\sqrt{\frac{3B}{\nu^2}} \ m' = \tfrac{1}{2}\left[g\sqrt{\frac{2K}{\nu^2} - g^2} + \frac{2K}{\nu^2}\sin^{-1}\left(g/\sqrt{\frac{2K}{\nu^2}}\right)\right] \ . \tag{77}$$

Since Equation (77) gives $g(m')$ implicitly while Equation (76) gives $\ell(t)$ explicitly, the problem of finding r in Equation (59) is solved. The constant K can be found from Equation (68) by using $\gamma = 3$, n = 1 and the initial values $g = r_o$ and $dm'/dg = \rho_1$, the density at the rear of the reaction zone.

We may use Equation (76) and $r = r_o\ell$ for $\ell \geq 1$ in Equation (58) with n = 1 to find

$$Mr_o\nu^2/\ell^3 = p = B\rho^3 = B(\rho_1/\ell)^3 \ . \tag{78}$$

Equation (64) with n = 1 and $\alpha = 2M/C = 2M/(2r_o\rho_o)$ for a symmetric sandwich may then be used in Equation (78) to find

$$(\rho_1/\ \rho_o) = 2^{1/3} = 1.26 \tag{79}$$

close to the value 4/3 = 1.33 suggested by H. Jones.[21]

VII. STERNE'S EXTENSIONS OF EARLIER WORK

In 1947, Sterne[52] considered methods of extending the ideas of Gurney and Thomas to asymmetric sandwiches, including the extreme case of a single plate (sometimes called an open-face sandwich). He also considered cylinders with solid metal cores, representing fuzing devices. For two plates with specific masses (per unit area) M_1 and M_o separated by a sheet of explosive of specific mass C, he wrote for the radial energy partition at launch

$$EC = 1/2(M_1V_1^2 + M_oV_o^2) + (1/6)C(V_1^2 + V_o^2 + V_1V_o)\ ; \tag{80}$$

and for the conservation of momentum (initially zero),

$$0 = M_1V_1 + M_oV_o + (1/2)C(V_1 + V_o)\ , \tag{81}$$

where $V_1 > 0$ and $V_o < 0$ in the chosen coordinate system. Sterne mentioned that he was following Gurney's procedure, assuming a linear variation of gas velocity v with distance and uniformity of ρ at all points between the plates at launch time. He did not bother to derive Equations (80) and (81), probably thinking it to be sufficiently obvious. We will give a derivation below as part of a more general treatment.

If we eliminate V_o between Equations (80) and (81) we obtain the asymmetric sandwich formula of Sterne:

$$V_1 = \sqrt{(2E)/[\alpha_1 + \alpha_oq^2 + 1/3\ (1 - q + q^2)]} \tag{82}$$

with $V_o = -qV_1$ and $q = (1 + 2\ \alpha_1)/(1 + 2\alpha_o)$ with $\alpha_1 = M_1/C$ and $\alpha_o = M_o/C$. If $M_1 = M_o$, $\alpha_1 = \alpha_o = \alpha/2$ and q = 1, then

$$V_1 = \sqrt{(2E)/(\alpha + 1/3)} = -V_o \tag{83}$$

for the symmetric sandwich, the same as Equation (32) with n = 1. If $\alpha_o = 0$,

$$V_1 = \sqrt{(2E)/[1/3\ (1 + 5\alpha_1 + 4\alpha_1^2)]} \tag{84}$$

for a single plate. In this limit the model predicts that $V_o = -(1 + 2\alpha_1)V_1$, so $|V_o| > V_1$ as $M_o \rightarrow 0$. We can interpret V_o in the limit $M_o \rightarrow 0$ as the velocity of the free surface of the gas.

Twenty years later, Henry[60] cited Sterne's report and repeated his results. Unlike Gurney's report, Sterne's was widely circulated and had Hughes Aircraft among its recipients. Henry's report has been frequently cited in the literature while Sterne's has been forgotten. Probably Henry's report has been the main instrument through which most interested parties have become acquainted with Gurney's and Thomas' work, since almost no one cites Sterne or Thomas. It also seems likely that Henry never read Gurney's report since he does not seem to have a clear understanding of Gurney's method. In his own derivation of Equation (84), Henry arrives at the more cumbersome but equivalent form

$$V_1 = \sqrt{(2E)/[\alpha + 1/6\{1 + (1 + 2\alpha)^3\}/(1 + \alpha)]} \;. \qquad (85)$$

The equivalence is easily seen by expanding the cubic, factoring 2 from the curly brackets, dividing by $(1 + \alpha)$ and collecting terms. Similar considerations hold for Henry's form of Equation (82) which appears again more recently in the paper by G. Jones and co-workers.[9]

About five years after Henry's report was published, Défourneaux[61] repeated Equation (84) as his Equation (18), citing Gurney's report,[2] but not Sterne's. Of course Gurney's report does not contain Equation (84), although other references did by this time. Défourneaux pointed out that the kinetic energy per unit explosive mass for a single plate, namely $\phi_1=(1/2)\alpha_1 V_1^2$ where V_1 is given by Equation (84), has a maximum (equal to E/3) for $\alpha_1 = M_1/C = 1/2$. He did not pursue this, but it is interesting to investigate the asymmetric sandwich as well. We wish to optimize ϕ_1, using Equation (82) for V_1, subject to the constraints $\alpha_1 \geq 0$, $\alpha_o \geq 0$. If we let $\partial\phi_1/\partial\alpha_o = 0$, we find

$$\alpha_1 + \alpha_o + 6\alpha_1\alpha_o = 0.5 \;, \qquad (86)$$

which is plotted as the dashed curve in Figure 8 and gives the locus of the inflection points in the curves $\phi_1(\alpha_1)$ for $\alpha_o < 0.5$. Values of ϕ_1 in units of E, namely, ϕ_1/E, are given in parentheses at some points in the figure. Clearly ϕ_1 increases with α_o. If we let $\partial\phi_1/\partial\alpha_1 = 0$, we find

$$\alpha_1 = 0.5\sqrt{(1 + 5\alpha_o + 4\alpha_o^2)/(1 + 3\alpha_o)} \;, \qquad (87)$$

which is plotted as the solid curve in Figure 8 and gives a maximum $\phi_1(\alpha_1)$ for any α_o. The straight line at 45° is for $\alpha_o = \alpha_1$ (symmetric sandwich) while the horizontal straight line is for $\alpha_1 = 0.5$. As α_1 and α_o increase without

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[60] I.G. Henry, "The Gurney Formula and Related Approximations for the High-Explosive Detonation Products," Hughes Aircraft Report PUB-189, 1967.

[61] M. Défourneaux, "Transferts d'Energie Dans les Combustions et Detonations avec Confinement," Astronautica Acta, 17, 1972, p. 609.

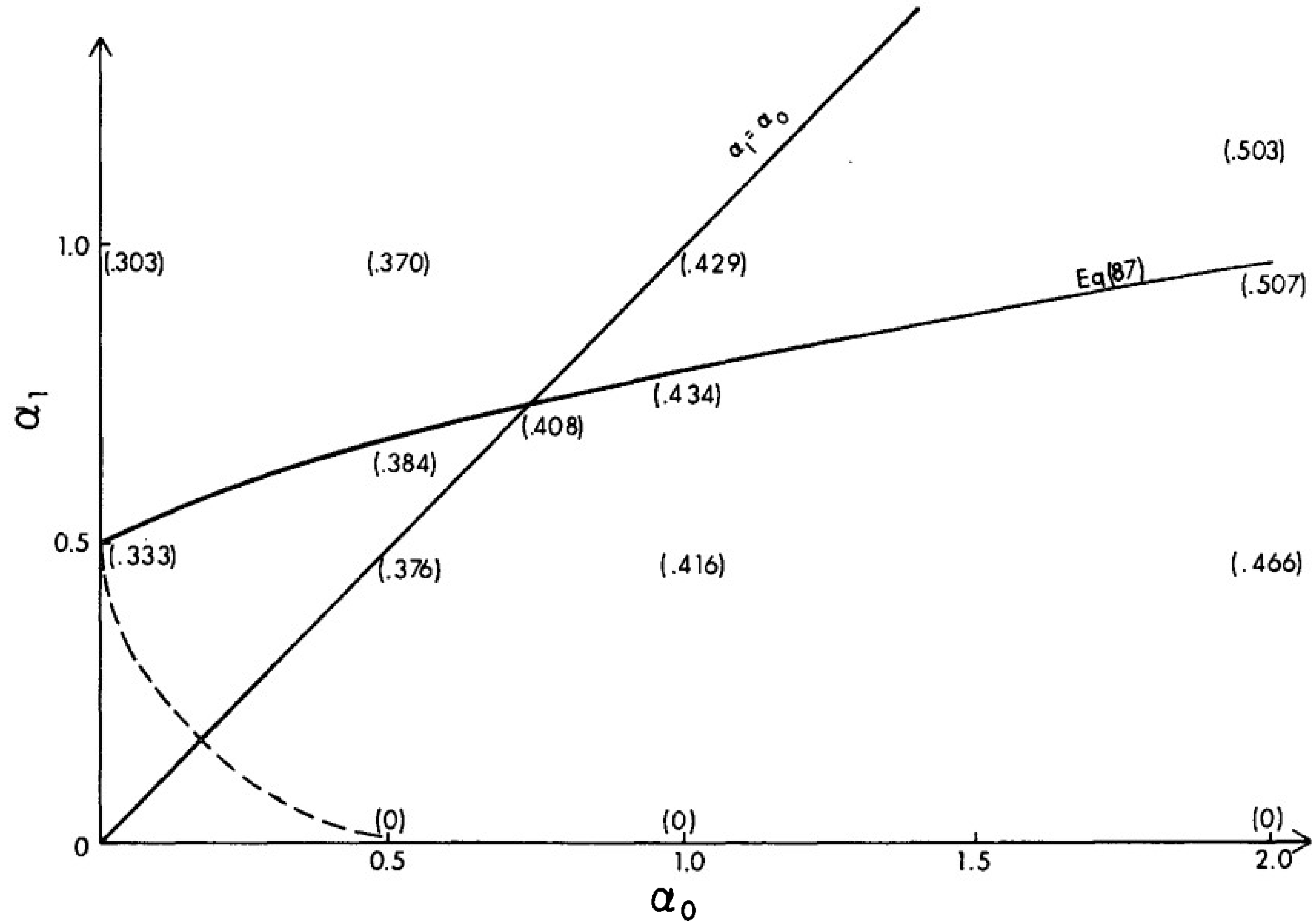


Figure 8. Kinetic Energy Per Unit Explosive Mass for One Plate in an Asymmetric Sandwich.

limit, ϕ_1/E approaches unity. This incorrect prediction says that we achieve the greatest possible launch energy per unit explosive mass when we sandwich the explosive between two infinitely heavy plates. The model fails for large α_1 since it does not account for energy absorbed as heat by relatively immobile masses. This will be discussed further below after Equation (104). For the present, it is sufficient to note that errors should be less than 10% if $\alpha_1 < 7$. For typical α_1, the model should correctly predict that the addition of a backing plate ($\alpha_0 > 0$) will increase the specific launch energy of plate 1. Hoskin and Lambourn[49] arrived at a similar conclusion by using the method of reflected shock characteristics.

In units of the Gurney velocity, $\sqrt{2E}$, the launch velocity for $\alpha_1 = 0.5$, $\alpha_0 = 0$ is $V_1/\sqrt{2E} = 2/3 = 0.816$, while the optimized launch energy is $\phi_1/E = 1/3 = 0.333$. At $\alpha_1 = \alpha_0 = 0.729$ where the optimized sandwich is symmetric, $V_1/\sqrt{2E} = 0.748$ (8% less). However, $\phi_1/E = 0.408$ (22.5% greater). The combined energy of both plates is of course still larger. For the optimal values $\alpha_1 = 0.98$, $\alpha_0 = 2$, we find $V_1/\sqrt{2E} = 0.72$ (down 28%), but $\phi_1/E = 0.507$ (up 52%),

and so on.

It is interesting to note that Sewell[62] assumed an equivalence between warhead effectiveness and launch kinetic energy (or momentum) per unit explosive mass and optimized the design of cylindrical and spherical configurations, using Gurney's formulas. Of course these geometries are necessarily symmetrical. His cylinder calculations were later refined by Zulkoski,[63] who took account of finite end effects.

G. Jones and co-workers,[9] who recently rederived Equations (82)-(84), referring to Henry's report[60] but not to Sterne's,[52] also derived a formula for the velocity of a plate in a symmetric sandwich based on Equation (83) and Equation (1):

$$V_1 = \sqrt{\frac{2E}{(\alpha + 1/3)}\left[1 - \ell^{-(\gamma-1)(1+1/3\alpha)}\right]} \qquad (88)$$

If $\gamma = 3$ in Equation (88) and $3\alpha \gg 1$, this formula reduces to Equation (65) with $n = 1$, $B\rho_0^{\,2} = E$ from Equation (74). For an asymmetric sandwich they obtained

[62] R.G.S. Sewell, "Fixed-weight and Fixed-volume Constraints on Optimum Charge-to-Metal Ratios in Warhead Design," NAVWEPS Report 8471, NOTS TP 3430, China Lake, CA, 1964.

[63] T. Zulkoski, "Development of Optimum Theoretical Warhead Design Criteria," Naval Weapons Center TP 5892, 1976.

$$V_1 = \sqrt{\left(\frac{2E}{Q}\right)\left\{1 - [(q+1)\ell - q]^{-(\gamma - 1)(q+1)Q/\alpha_1}\right\}} \qquad (89)$$

with

$$Q = \alpha_1 + \alpha_0 q^2 + 1/3(1 - q + q^2) \ . \qquad (90)$$

For a single plate or open-face sandwich ($\alpha_0 = 0$), $q = 1 + 2\alpha_1$, and $Q = 1/3(1 + 5\alpha_1 + 4\alpha_1^2)$ in Equation (89). Of course γ should be 3 as we have shown. Sterne derived similar formulas for $V(\ell)$ in the case of sandwiches. In the same report, he discussed "cored" cylinders, that is, a Gurney cylinder with a solid metal core or fuze cavity. This will be discussed further in the next sections, as will his treatment of a "cored" sphere.[53]

Chanteret[64] follows a method similar to that of Jones and co-workers,[9] but treats the "side-on" acceleration of plates and cylinders explicitly in his extension of the Gurney model. For symmetric cases he again arrives at equation (32), the generalization first made by Thomas.[42] However, his expression for E is the square brackets in equation (20) multipled by D. His notation differs from ours since his u is our U, defined in equation (5) above. His calculation of V still involves finite time or space steps, so it is not completely analytical, but numerical. For an asymmetric sandwich he employs the usual technique of a zero velocity plane in the explosive. He also applies this technique to obtain a Gurney type formula for the launch velocity of a hollow cylinder, namely, a tube of explosive between two tubes of metal, which he looks upon as an asymmetric sandwich rolled about an axis above or below the sandwich parallel to an edge until two edges meet. His expression for the location of the zero velocity cylindrical surface in the explosive involves a cubic equation with the Chapman-Jouget density of the explosive products as one of the parameters. His expression for E still involves finite space or time steps, except, of course, in the limit $\ell \to \infty$.

In 1965 Hoskin and co-workers[65] presented experimental data for single plate launch velocities in terms of C/M, the inverse of our $\alpha = M/C$. They compared their data with their version of Equation (84), namely

$$V_1 = \sqrt{(2E)/[1/3(1 + 4\alpha_1 + 4\alpha_1^2)]} = \sqrt{6E}/(1 + 2\alpha_1) \ , \qquad (91)$$

[64] P.Y. Chanteret, "An Analytical Model for Metal Acceleration by Grazing Detonation," Seventh International Symposium on Ballistics, The Hague-The Netherlands, 1983, p. 515.

[65] N.E. Hoskin, J.W.S. Allan, W.A. Bailey, J.W. Lethaby, and I.C. Skidmore, "The Motion of Plates and Cylinders Driven by Detonation Waves at Tangential Incidence," Forth International Symposium on Detonation, ONR ACR-126, 1965, p. 14.

a simplification achieved by using 4α instead of 5α in the denominator with little variation in α-dependence. Their analysis was based on a method of shock characteristics previously discussed by Hill and Pack.[66] Related theoretical work appears in References 48 and 49 mentioned above. Equation (91) is for the side-on case while Equation (84) is head-on since it is based on an energy partition for motion normal to the initial plane of the plate surface. Again we see that launch velocity formulas ($\ell \to \infty$), in this case for single plates, do not depend strongly on the direction of motion of the detonation wave.

Figure 9 shows Hoskin's data as the solid line. Here we are taking his normal velocity component and V_1 to be approximately the same as we did with similar cylinder data. Our α_1 is the inverse of his. Since he gave no numerical values, only a graph, the values we show for large α_1 are somewhat uncertain,and there may be better agreement than shown between model and experiment. Equation (91) is shown as the dashed line, using $\sqrt{2E}$ = 2.575 mm/μs = $4.46/\sqrt{3}$ as suggested by Hoskins. On the scale of the figure, it is indistinguishable from Equation (84). Of course Equation (89) with $\ell \to \infty, \gamma = 3$ and $\alpha_o = 0$ is the same as Equation (84).

It is interesting to note that $\phi_1 = (1/2)\alpha_1 V_1^2$ has a maximum for $\alpha_1 = 1/2$ whether we use 4 or 5 as the coefficient of α_1 in the denominator of Equations (91) or (84).

VIII. SIMPLE GEOMETRIES WITH AN ARBITRARY NUMBER OF LAYERS

A. Introduction

Henry[60] also repeated Sterne's modeling of a cored cylinder[52] and mentioned a cored sphere which Sterne also modeled. Neither author mentioned the planar analog, a sandwich with a metal center, which implies a minimum of five layers instead of three. Henry did, however, discuss the problem of a cylindrical configuration with more than two or three alternating tubes (cylindrical layers) of metal and explosive, an arrangement which he called a jelly roll. If we ignore the lack of layer closure in the pastry analog, this is a descriptive name. The spherical analog might be called an onion, if we ignore the fact that the alternate layers are of different materials. Finally, the Dagwood seems to be a suitable name for a multilayered planar sandwich.

Henry[60] wrote down a very simple model for a many-layered jelly roll in which he envisioned all metal layers to be of the same composition and

[66] R. Hill and D.C. Pack, "An Investigation, by the Method of Characteristics, of the Lateral Expansion of the Gases behind a Detonating Slab of Explosive," Proceedings of the Royal Society of London, 191, 1947, p. 524.

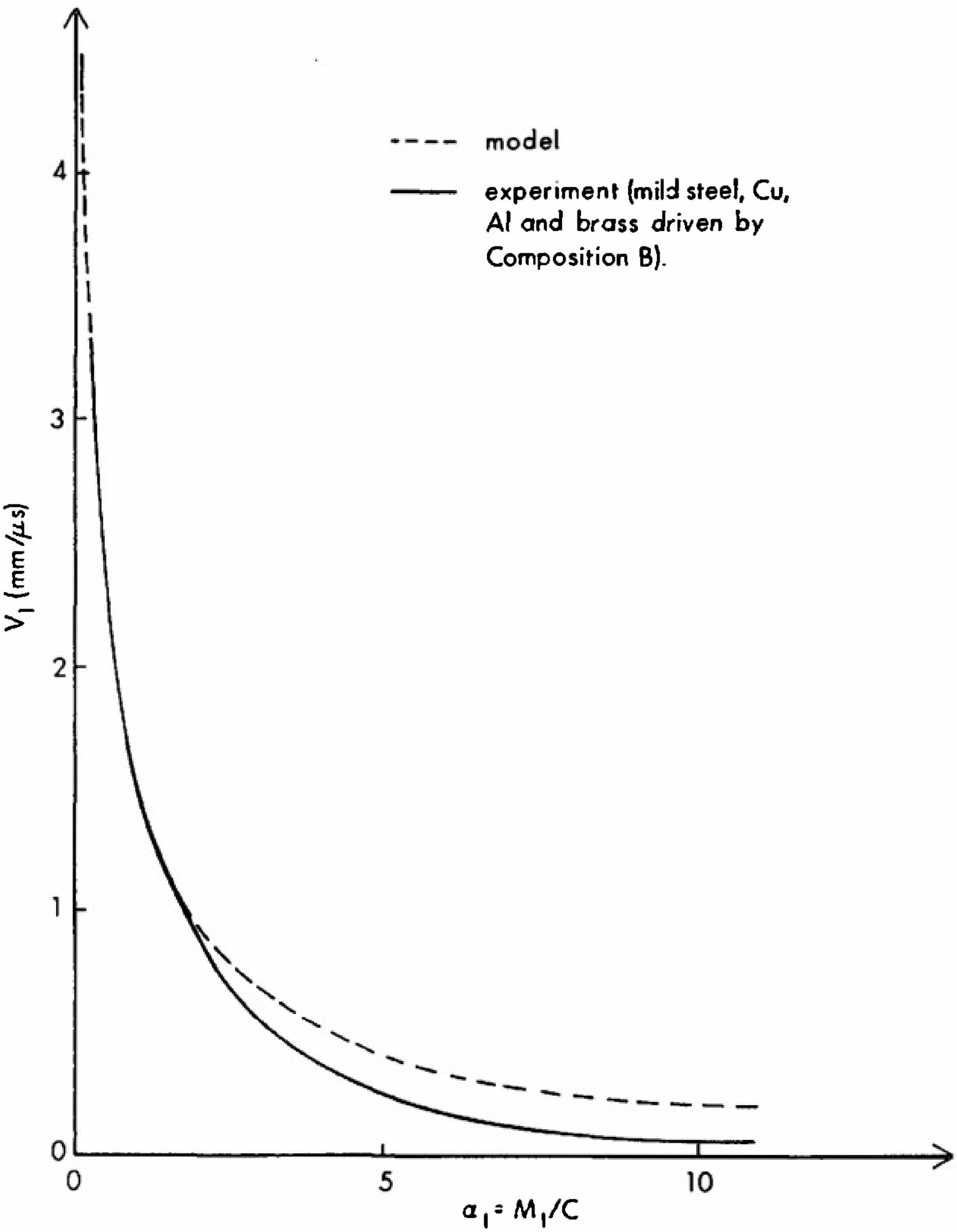


Figure 9. Experimental Single Plate Launch Speeds Compared with Model Predictions.

thickness with metal on the exterior. He also envisioned all explosive layers of the same composition and thickness. He then wrote $(C + M)/(\pi R^2)$ for the average initial density of the array, where C and M have their usual meanings for a cylinder and R is the outer radius before detonation. His energy balance became

$$EC = \tfrac{1}{2}\int_0^R \left[(C + M)/(\pi R^2)\right]\left[V^2\left(\frac{r}{R}\right)^2\right]2\pi r dr$$

$$= 1/4\ (C + M)\ V^2 \quad . \tag{92}$$

Here he used the initial radius instead of the radius at bursting, apparently not having Gurney's report to aid his understanding of the model. Equation (92) gives

$$V = \sqrt{(4E)/(1 + \alpha)} \quad , \tag{93}$$

where $\alpha = M/C$ for the entire device. He compared Equation (93) graphically with Gurney's simple cylinder formula, Equation (26)

$$V_G = \sqrt{(4E)/(1 + 2\alpha_G)} \quad , \tag{26}$$

where we have used the subscript G for Gurney. Since for $\alpha = \alpha_G > 0$, $V > V_G$, Henry concluded that the jelly roll ought to give a higher launch speed for the outermost metal layer than the simple cylinder for the same α. Or, to achieve the same speed, $V = V_G$, we need $\alpha = M/C = 2\alpha_G = 2M/C_G$, that is $C = C_G/2$, or half the explosive. This kind of prediction is traceable to his integrating over the metal layers as if they were expanding gas, leading to $1/4\ MV^2$ instead of $1/2\ MV^2$. His use of R instead of r_B has no practical consequences since it cancels in Gurney's scheme anyway. If Henry's formula were correct, it should reduce to Gurney's when all the metal is placed outside of the explosive, since he specifies nothing about ordering in Equation (92). Obviously it does not. Although Henry's model is incorrect, he has raised an interesting question about the potential advantages of many alternating layers. For example, instead of the usual expanding tube with some spread in wall thickness as air resistance slows the smaller fragments more than the larger ones, we might obtain different launch speeds initially which (together with spreading due to air resistance) could fill more of space over a longer time than a simple cylinder can. Of course, the slower inner-layer fragments might be less effective.

B. The Jelly Roll and Onion

The first practical problem in making such devices is that of detonation. For a head-on encounter between detonation wave and metal surface, we would probably have to drill holea of sufficient size and frequency in the metal layers and fill them with explosive in order to insure propagation to the outermost layer. Initiation in a laboratory device should not be difficult. A small spherical detonator or an exploding wire or foil might be used. However, inadequate electrical sources might prevent the use of such techniques in field devices. Initiation by a disk or sheet of explosive covering one or both ends of a jelly roll or Dagwood should be practical for side-on encounter. End initiation of a uniform explosive loading should lead to an approximately plane wave consisting of the sum of the waves in each layer of a jelly-roll or Dagwood. Consequently, the pressure and density at any distance behind this front should be approximately uniform over the gaseous portions of the front and depend only on the state of expansion. If the metal layers are significantly different in their bending properties, this picture might have to be modified. For the present, let us assume an arrangement for which this picture is sufficiently accurate.

First, let us consider the jelly roll and adopt the layer numbering scheme shown in Figure 10. The specific mass (per unit length) for the i^{th} layer of explosive is

$$C_i = \rho_{oi}\pi\left(R_{2i}^2 - R_{2i-1}^2\right) \tag{94}$$

while that for the metal is

$$M_i = \rho_{Mi}\pi\left(R_{2i+1}^2 - R_{2i}^2\right) . \tag{95}$$

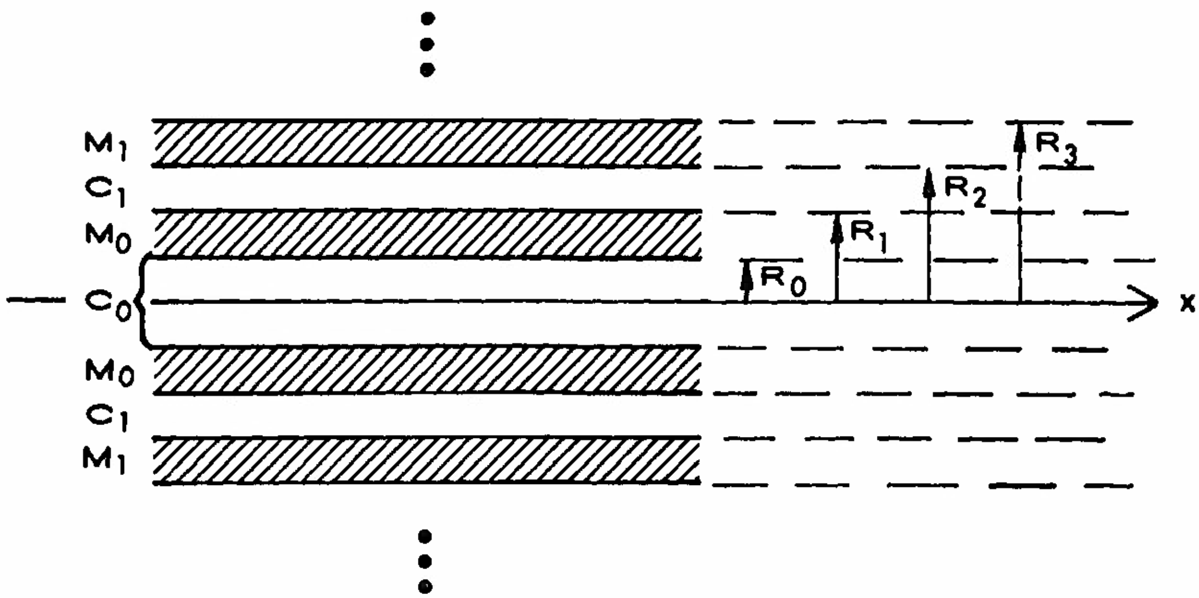
Figure 10 gives some examples for $i = 0, 1, \ldots.$ Because of cylindrical symmetry, all R_i are positive ($R_i = 0$ if $i < 0$) and are measured from the symmetry axis. Capital R_i are initial values. As the device expands, $r_i > R_i$ are the radii, and the decreasing density of each gas layer is

$$\rho_i = (C_i/\pi)/\left(r_{2i}^2 - r_{2i-1}^2\right) . \tag{96}$$

If we assume that the radial gas velocity is a simple linear form in r, we obtain an energy balance (and thus a launch speed) which is rather complicated. Thus, if

$$v = A_i r + B_i = \left(\frac{V_i - V_{i-1}}{r_{2i} - r_{2i-1}}\right) r + \left(\frac{V_{i-1}r_{2i} - V_i r_{2i-1}}{r_{2i} - r_{2i-1}}\right) \tag{97}$$

for the i^{th} layer of gas, then $v = V_i$ when $r = r_{2i}$ and $v = V_{i-1}$ when $r = r_{2i-1}$. That is, the gas moves at the metal speed V at each interface. If $i = 0$ is the only value of i, we have $v = V_o(r/r_o) = v_B(r/r_B)$ since $r_{-1} = V_{-1} = 0$, and Equation (97) reduces to Gurney's assumption for the simple cylinder, Equation (15). The energy balance becomes



$$C_0 = \rho_{00}\,\pi\left[R_0^2 - 0\right]$$

$$C_1 = \rho_{01}\,\pi\left[R_2^2 - R_1^2\right]$$

$$M_0 = \rho_{M0}\,\pi\left[R_1^2 - R_0^2\right]$$

$$M_1 = \rho_{M1}\,\pi\left[R_3^2 - R_2^2\right]$$

Figure 10. Layer Numbering Scheme for the Jelly Roll and Onion.

$$\sum_i E_i C_i = \tfrac{1}{2} \sum_i M_i V_i^{\,2} + \tfrac{1}{2} \sum_i \int_{r_{2i-1}}^{r_{2i}} \rho_i v^2 2\pi r dr$$

$$= \tfrac{1}{2} \sum_i M_i V_i^{\,2} + \sum_i \frac{C_i}{\left(r_{2i}^2 - r_{2i-1}^2\right)} \int_{r_{2i-1}}^{r_{2i}} v^2 r dr \quad . \tag{98}$$

If we use Equation (97) in Equation (98), we find

$$\sum_i E_i C_i = \tfrac{1}{2} \sum_i M_i V_i^{\,2} + \frac{1}{4} \sum_i C_i \left[F V_i^{\,2} + \frac{2}{3} V_i V_{i-1} + G V_{i-1}^2\right] \tag{99}$$

$$F = \left[1 + \frac{1}{3} x(x^2 + x - 5)\right] / \left[1 + x(x^2 - x - 1)\right] \tag{100}$$

$$G = \frac{1}{3}\left[1 + x(3x^2 - 5x + 1)\right] / \left[1 + x(x^2 - x - 1)\right] \tag{101}$$

with $x = r_{2i-1}/r_{2i}$. Since x is a ratio of radii at launch, we still have no dependence on device size. If $i = 0$ is the only i value, Equation (99) reduces to Equation (25).

If we want to simplify our result, we might follow Gurney's method of choosing a form for v designed for this purpose. Thus, if

$$v^2 = a_i r^2 + b_i = \left(\frac{V_i^{\,2} - V_{i-1}^2}{r_{2i}^2 - r_{2i-1}^2}\right) r^2 + \left(\frac{V_{i-1}^2 r_{2i}^2 - V_i^{\,2} r_{2i-1}^2}{r_{2i}^2 - r_{2i-1}^2}\right) \quad , \tag{102}$$

we still have $v = V_i$ for $r = r_{2i}$, $v = V_{i-1}$ for $r = r_{2i-1}$ and $v = V_o(r/r_o)$ if $i = 0$ is the only value of i. Since $vdv = a_i rdr$, the integral in Equation (98) is simplified if we change the variable to v with limits V_{i-1} and V_i. By using Equation (102) in Equation (98) we find

$$\sum_i E_i C_i = \tfrac{1}{2} \sum_i M_i V_i^{\,2} + \frac{1}{4} \sum_i C_i \left(V_i^{\,2} + V_{i-1}^2\right) \quad , \tag{103}$$

which also reduces to Equation (25) if $i = 0$ is the only value of i.

Now let us consider the case of a metal-cored cylinder with $C_o = V_o = 0$, $M_o \neq 0$ and $i = 0, 1$. Equation (103) gives us

$$V_1 = \sqrt{(2E_1)/(\alpha_1 + 1/2)} \quad . \tag{104}$$

In other words, this model predicts that a cored cylinder will have the same velocity as a simple cylinder, if E_1 and α_1 are the same. Of course, if we simply replace explosive by a metal core, we increase α_1 and decrease V_1. However, if we alter the device dimensions in order to keep α_1 the same, the launch velocity ought to be lower when an inert core is present. Sterne[52] obtained essentially the same result by a different method and compared this prediction with experiment. For a steel core with a radius about 3/4 the initial radius of an outer steel case, using composition C3 explosive and α_1 = 2.33, the measured launch velocity was down 13% compared to that measured for a simple cylinder with the same α_1 and E_1. He attributed this to energy absorption and conversion to heat by the immobile core. This hypothesis was approximately confirmed by a calorimeter measurement of the heat content of the recovered core right after the experiment. He suggested lowering E_1 to about $E_1 - 10\ \alpha_o$, with E_1 expressed in calories per unit specific explosive mass. Here 10 calories per gram of specific metal core mass was approximately the measured value. Thus, for example, if $E_1 \approx 700$ cal/gm (as for TNT), then $\alpha_o = 7$ would mean about a 10% reduction in available energy. However, if α_o = 1, there would only be a 1.5% reduction. For $\alpha_o = 1$, $\alpha_1 = 0.5$ in Figure 8, ϕ_1/E would be (0.410) instead of (0.416), still larger than its value (0.333) without a backing plate ($\alpha_o = 0$). Sterne also noted that there was some permanent deformation of the core which he took to be negligible by comparison to the above correction. The energy absorbed by heating and fracturing the outer metal tube has already been accounted for in the original reduction of E_1 from its expected chemical value to its Gurney value. Even for a large core like that used, the loss in velocity is fairly small, probably because there is little time for energy transfer by shock transmission and heat conduction.

As we have noted, Equation (99) is more complicated than Equation (103). In the case of a cored cylinder, Equation (99) gives

$$V_1 = \sqrt{(2E_1)/(\alpha_1 + .5F)} \quad , \tag{105}$$

with $x = r_1/r_2 = R_1/r_2$ if the radius of the core does not change during the expansion. Of course for the inner radius of the case at launch $r_2 > R_2 > R_1$ so $x < 1$. If $x = 0$ because $R_1 = 0$ (no core), $F = 1$ by Equation (100) and Equation (105) is that of a simple cylinder. Typically x might be near 0.5. As x increases from zero to unity in Equation (100), F decreases from 1 to 2/3, being about 0.8 near $x = 1/2$. Thus for the same E_1 and α_1, Equation (105) predicts that V_1 for a cored cylinder is greater than for a simple cylinder, with larger cores giving greater V_1. This is contrary to the experimental results discussed by Sterne. Consequently, Equation (99) and Equation (97) upon which it is based are suspect. Equation (103) and Equation (102) on which it is based are preferred in this instance, not only for reasons of simplicity, but also because the prediction is at least closer to experiment and gives no misleading trends.

Now consider Equation (103) for the simplest jelly roll which is illustrated in Figure 10 ($i = 0, 1$):

$$E_oC_o + E_1C_1 = 1/2\ M_oV_o^{\,2} + 1/2\ M_1V_1^{\,2} + 1/4\ [C_oV_o^{\,2} + C_1(V_o^{\,2} + V_1^{\,2})] \ . \qquad (106)$$

Equation (106) of course reduces to the metal-cored cylinder if $C_o = V_o = 0$, $M_o \neq 0$ and the simple cylinder if $C_o = V_o = M_o = 0$. If only $C_1 = 0$, $V_o = V_1$, provided we weld the two metal tubes together and again we have a simple cylinder. If the metal tubes are in loose contact initially, the shock rarefaction wave returning from the outer metal/air interface will tend to separate the tubes, while the gas pressure inside will tend to keep them together. Because of the high gas pressure during the acceleration phase of the motion, we expect the tubes to remain in contact until they break in most cases.

In designing a warhead, it is desirable to have $V_o < V_1$ in Equation (106) since this would mean that the inner layer of fragments would travel more slowly than the outer layer and there would be a greater spread of metal in space for a longer time. This would increase the chance of hitting a moving target and (provided V_o and V_1 are not too small) would increase the probability of damage.

It is possible but not desirable to select the E_i, C_i and M_i in Equation (106) so that the launch speeds are the same, namely, $V_o = V_1 = V$. In this case, with $E_o = E_1 = E$,

$$V = \sqrt{2E/[\alpha + 1/2(1 + Z)]} \quad , \qquad (107)$$

where $\alpha = M/C = (M_o + M_1)/(C_o + C_1)$ and $Z = C_1/C$. If we divide this equation by the simple cylinder formula,

$$V/V_c = \sqrt{(\alpha + 1/2)/[\alpha + 1/2(1 + Z)]} \qquad (108)$$

for the same α and E. For $Z > 0$ we have $V < V_c$ and neither an increase in metal distribution is achieved nor an increase in launch velocity. For α much above unity $V/V_c \rightarrow 1$. In the extreme $Z \rightarrow 1$, ($C_o \rightarrow 0$, $V_o \rightarrow 0$), which is the metal-cored cylinder, we can no longer assume $E_o = E$, since the inner metal tube absorbs a significant fraction of the energy imparted to it and converts it to heat rather than to its diminishing kinetic energy.

In our comparisons with a simple cylinder, we always fill the simple cylinder with the most energetic explosive available to avoid making the jelly roll appear better because it has a more energetic fill. We can, of course, use a less energetic explosive in either layer of the simple jelly roll. However, the practical range of E is rather limited. The most energetic solid explosive commonly available is HMX with $E = (3)^2/2$, while the least energetic is TNT with $E = (2.4)^2/2$. The values in parentheses are the $\sqrt{2E}$ (mm/μs) Gurney velocities given by Kamlet and Finger.[11] Roughly speaking then, $2/3 < E_o/E_1 < 1$ when $E_o < E_1$. If $V_o = V_1 = V$ and $\sigma = E_o/E_1 < 1$, then Equation (108) is replaced by

$$V/V_c = \sqrt{(\alpha + \tfrac{1}{2})\ [\sigma + Z(1 - \sigma)]/[\alpha + \tfrac{1}{2}(1 + Z)]} \tag{109}$$

so that V will be even less than in Equation (108), as expected. If the launch speeds are approximately equal, then there is not much decrease in launch velocity compared to a simple cylinder. However, there is not much increase in metal distribution either, since the inner layer will tend to maintain its original separation from the outer layer at least after launch time. Most likely, the outer layer will break first and tend to slow down first, emphasizing the tendency for both layers to fly together.

There are two ways to achieve a significant increase in separation of the two layers compared to their initial separation. One way is to have $M_o/C_o \gg M_1/C_1$, approaching the case of the metal-cored cylinder for which $V_o = 0$. The other way is to use a less energetic explosive in the core so that the inner pressure is lower (at least initially). A combination of both techniques should enable us to design a device where the inner layer flies with half the speed of the outer layer, giving an increasing gap between the layers, a configuration which may be desirable for some purposes. An estimate of inner layer launch speed might be made by treating the layers M_o, C_1 and M_1 as a single inertial layer, so that

$$V_o = \sqrt{(2E_o)/[(M_o + C_1 + M_1)/C_o + 1/2]} \quad . \tag{110}$$

More complicated jelly rolls can be discussed in the same way.

The spherical analog of the jelly roll will also lead to fairly complicated expressions if we use Equation (97) for v. However, if we use the cubic analog of Equation (102) and relate v^3 to r^3, we find

$$\sum_i E_i C_i = \tfrac{1}{2} \sum_i M_i V_i^{\,2} + \frac{3}{10} \sum_i C_i \left(V_i^{\,5} - V_{i-1}^5\right) / \left(V_i^{\,3} - V_{i-1}^3\right) \tag{111}$$

since
$$\rho_i = \left[C_i/(\tfrac{4}{3}\pi)\right] / \left(r_{2i}^3 - r_{2i-1}^3\right) \tag{112}$$

$$C_i = \rho_{oi}\left(\tfrac{4}{3}\pi\right)\left(R_{2i}^3 - R_{2i-1}^3\right) \tag{113}$$

and
$$M_i = \rho_{Mi}\left(\tfrac{4}{3}\pi\right)\left(R_{2i+1}^3 - R_{2i}^3\right) \quad , \tag{114}$$

using the layer numbering scheme of Figure 10. Thus Equation (111) is independent of device size and reasonably simple. If $i = 0$ is the only value of i, we obtain Gurney's sphere formula, Equation (28). The same design goals and limitations apply to the onion as to the jelly roll, and an entirely analogous discussion could be carried out. In the limit of equal launch speeds, the ratio in the last term of Equation (111) is 5/3 V^2 and Equation (111) becomes

$$\sum_i E_i C_i = \tfrac{1}{2}\left(\sum_i M_i + \sum_i C_i\right)V^2 = \tfrac{1}{2}(M+C)V^2 \ . \tag{115}$$

For a uniform explosive fill of the same type as in a simple sphere with the same $\alpha = M/C$

$$V/V_s = \sqrt{(\alpha + 3/5)/(\alpha + 1)} \ , \tag{116}$$

where $V < V_s$, the launch velocity of a simple sphere. A discussion of metal spreading by controlling the E_i and α_i analogous to that given for the cylinder can also be carried out.

C. The Dagwood

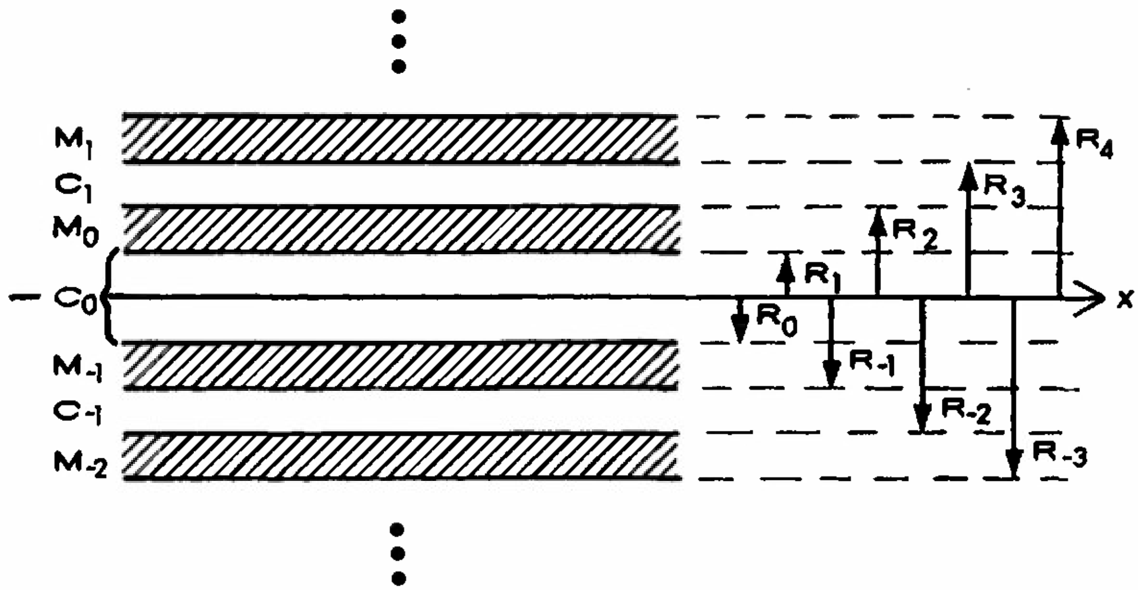
Here the possibility of asymmetry about the central plane exists, and the momentum balance equation is not identically zero except in the symmetrical limit. Similar considerations hold for the Dagwood and jelly roll with respect to detonation wave initiation and propagation as well as pressure and velocity equalization by launch time. The layer numbering scheme in Figure 11 enables us to write

$$C_i = \rho_{oi}\left[R_{2i+1} - R_{2i}\right] \ , \tag{117}$$

$$M_i = \rho_{Mi}\left[R_{2i+2} - R_{2i+1}\right] , \tag{118}$$

and
$$\rho_i = C_i / \left[r_{2i+1} - r_{2i}\right] \ , \tag{119}$$

where $R_i = Y_i$ are initial values and $r_i = y_i$ are values during the expansion. In the planar case, using a linear relation between v and r enables us to preserve Gurney's basic assumption. We let



$$C_0 = (\rho_0)_0 \left[R_1 - R_0\right] = (\rho_0)_0 \left[R_1 + |R_0|\right]$$

$$C_1 = (\rho_0)_1 \left[R_3 - R_2\right]$$

$$C_{-1} = (\rho_0)_{-1} \left[R_{-1} - R_{-2}\right] = (\rho_0)_{-1} \left[|R_{-2}| - |R_{-1}|\right]$$

$$M_0 = (\rho_M)_0 \left[R_2 - R_1\right]$$

$$M_1 = (\rho_M)_1 \left[R_4 - R_3\right]$$

$$M_{-1} = (\rho_M)_{-1} \left[R_0 - R_{-1}\right] = (\rho_M)_{-1} \left[|R_{-1}| - |R_0|\right]$$

$$R_i \gtrless 0 \quad \text{for } i \gtrless 0 \quad \text{and} \quad R_0 < 0$$

Figure 11. Layer Numbering Scheme for the Dagwood.

$$v = a_i r + b_i = \left[\frac{V_i - V_{i-1}}{r_{2i+1} - r_{2i}}\right] r + \left[\frac{V_{i-1} r_{2i+1} - V_i r_{2i}}{r_{2i+1} - r_{2i}}\right] \tag{120}$$

since each plate has a single V_i at launch time which may be specified by either the outer or inner surface or some point between. Now we find

$$\sum_i E_i C_i = \tfrac{1}{2} \sum_i M_i V_i^2 + \frac{1}{6} \sum_i C_i \left(V_i^2 + V_i V_{i-1} + V_{i-1}^2 \right) \tag{121}$$

when we use Equations (119) and (120) to find $\int 1/2\ \rho_i v^2 dr$ and divide $V_i^{\ 3} - V_{i-1}^{\ 3}$ by $a_i(r_{2i+1} - r_{2i}) = (V_i - V_{i-1})$. A similar procedure involving $\int \rho_i v dr$ gives the conservation of momentum equation

$$0 = \sum_i M_i V_i + \tfrac{1}{2} \sum_i C_i \left(V_i + V_{i-1} \right), \tag{122}$$

which is identically zero only for a symmetric Dagwood. In general, $V_i > 0$ for $i \geq 0$ and $V_i < 0$ for $i < 0$. For the simple asymmetric sandwich these equations reduce to Equations (80) and (81), namely,

$$E_1 C_1 = \tfrac{1}{2} \left[M_1 V_1^{\ 2} + M_o V_o^{\ 2} \right] + \frac{1}{6} C_1 \left[V_1^{\ 2} + V_1 V_o + V_o^{\ 2} \right] \tag{123}$$

and

$$0 = \left[M_1 V_1 + M_o V_o \right] + \tfrac{1}{2} C_1 \left[V_1 + V_o \right] \tag{124}$$

if we identify $E_1 = E$ and $C_1 = C$ (with $C_o = M_{-1} = C_{-1} = M_{-2} = 0$ in Figure 11). Since for given M_i, C_i and E we have two equations in two unknowns, we can solve for V_o and V_1 as before.

For the asymmetric Dagwood illustrated in Figure 11, Equations (121) and (122) become

$$\begin{aligned} E_{-1}C_{-1} + E_o C_o + E_1 C_1 = \tfrac{1}{2} & \left[M_{-2} V_{-2}^{\ 2} + M_{-1} V_{-1}^{\ 2} + M_o V_o^{\ 2} + M_1 V_1^{\ 2} \right] \\ & + \frac{1}{6} \Big\{ C_{-1} \left(V_{-1}^{\ 2} + V_{-1} V_{-2} + V_{-2}^{\ 2} \right) \\ & + C_o \left(V_o^{\ 2} + V_o V_{-1} + V_{-1}^{\ 2} \right) \\ & + C_1 \left(V_1^{\ 2} + V_1 V_o + V_o^{\ 2} \right) \Big\} \end{aligned} \tag{125}$$

and

$$\begin{aligned} 0 = & \left[M_{-2} V_{-2} + M_{-1} V_{-1} + M_o V_o + M_1 V_1 \right] \\ & + \tfrac{1}{2} \left\{ C_{-1} \left(V_{-1} + V_{-2} \right) + C_o \left(V_o + V_{-1} \right) + C_1 \left(V_1 + V_o \right) \right\} \end{aligned} \tag{126}$$

Now for given E_i, C_i and M_i, we have two equations in four unknown V_i. For the symmetric case when Equation (126) is identically zero, we have one equation in two unknowns, $V_0 = -V_1$ and $V_1 = -V_2$.

Let us consider the symmetric case, assuming equality of velocities at launch time and a single explosive loading. With $\alpha = 2(M_0 + M_1)/(C_0 + C_1) = M/C$, we find that Equation (125) with $E_{-1} = E_0 = E_1 = E$ becomes

$$2E = (\alpha + 1)V^2 \qquad (127)$$

for the simplest symmetric Dagwood with explosive in the center. This may be compared with Equation (83) and V_p for the simple symmetric planar case to obtain (for the same E and α)

$$V/V_p = \sqrt{(\alpha + 1/3)/(\alpha + 1)} \qquad (128)$$

so that the symmetric Dagwood launch speed is somewhat less than for the simple symmetric sandwich. For typical α values the loss in launch speed will not be great, but for velocity equalization the increase in metal spreading will not be great either. For the ideal planar configuration (unlike the cylinder) there will be no metal layer thinning or fragmentation during flight. A launch in air instead of vacuum will, however, tend to slow the outer plates first. Again we may increase the spread of metal at the sacrifice of inner layer speed by controlling the ratios E_0/E_1, M_0/C_0 and M_1/C_1. In addition, an estimate of V_0 can be made by treating the layers exterior to C_0 as a single inertial mass.

If $M_{-2} = C_{-1} = V_0 = 0$, $M_{-1} = M_1$, $C_0 = C_1$, $\alpha = (2M_1)/2C_1)$ and $V_{-1}^2 = V_1^2 = V^2$ in Figure 11, we have a symmetric sandwich with the metal plate M_0 at its center. For $E_0 = E_1 = E$, Equation (125) becomes

$$V = \sqrt{(2E)/(\alpha + 1/3)} \quad , \qquad (129)$$

and the model predicts the same launch speed as if $M_0 = 0$ (the simple symmetric sandwich). Again we should account for the fact that the effective values $E_0 = E_1 < E$ since some energy is absorbed by the inert stationary core as in the case of the metal-cored cylinder.

If $C_{-1} = C_1 = 0$ in Figure 11 and Equations (125) and (126), we have four plates but only one layer of explosive in the middle. Again we have a simple asymmetric sandwich, provided we bond the plates together so they will not trap momentum and separate as soon as rarefaction waves reflected from the free surfaces return to the interfaces. This configuration is a generalization of the case discussed by G. Jones[10] which consists of more than one plate, but only on one side of a single sheet of explosive.

If asymmetries are not great, and we assume approximate equalization of velocities at launch, Equation (127) holds for any number of explosive sheets sandwiched between metal plates.

For significant asymmetries, we encounter a problem as soon as we add a sheet of explosive to one side of a simple asymmetric sandwich. If we let $E_o = E_1 = E$, $M_{-2} = M_{-1} = C_{-1} = 0$ in Figure 11, Equations (125) and (126) become

$$EC = 1/2[M_oV_o^2 + M_1V_1^2] + 1/6[C_o(V_o^2 + V_oV_{-1} + V^2_{-1}) + C_1(V_1^2 + V_1V_o + V_o^2)] \tag{130}$$

and

$$0 = [M_oV_o + M_1V_1] + 1/2[C_o(V_o + V_{-1}) + C_1(V_1 + V_o)] \quad , \tag{131}$$

where V_{-1} is the launch velocity of the free gas surface, which cannot be zero if $C_o > 0$ in the present configuration $(M_1/C_1/M_o/C_o)$. Since we only have two equations for the three unknowns V_1, V_o and V_{-1}, we cannot give a general solution within the limitations of this model. If for some reason we want to fix the V_i, then we would have two linear equations in some M_i, C_j pair (for given E) and some physically interesting solutions would exist.

The assumptions of uniform density and linear spatial distribution of gas velocity at any instant imply the existence of a stationary plane (or surface) of zero gas velocity. This was mentioned above in the discussion after Equation (90). For the simple asymmetric sandwich $(M_1/C_1/M_o)$ the location of this plane may be found by letting $v = 0$, $r = R$, and $i = 1$ in Equation (120), giving $R = (V_1R_2 - V_oR_3)/(V_1-V_o)$ with $V_o = -qV_1$ and V_1 given by Equation (82). Thus we can find R from α_1, α_o, R_2, and R_3 since $\sqrt{2E}$ cancels. For example, when $\alpha_o = \alpha_1$, $q = 1$, and $R = (R_2 + R_3)/2$. That is, the plane lies in the middle of the layer C_1. If we add the layer C_o below the sandwich $(M_1/C_1/M_o)$, we shift the plane downward. Sufficiently large C_o will move the plane through M_o and into C_o. When $V_o = 0$, we have

$$V_1 = \sqrt{2E / \left\{ \alpha_1 + 1/3 \left[\beta_1 + \beta_1 + 2\alpha_1)^2/(1 - \beta_1)\right]\right\}} \tag{132}$$

and

$$V_{-1} = - [(\beta_1 + 2\alpha_1)/(1 - \beta_1)]\, V_1 , \tag{133}$$

where $\beta_1 = C_1/C = C_1/(C_o + C_1) = 1 - \beta_o = 1 - C_o/C$. This case is equivalent to the simple open-face sandwich (M_1/C) with a reduction in E dependent on the size of M_o as explained above for other "cored" configurations. Usually this correction is negligible. If we neglect it and set Equation (132) equal to Equation (84), we find $\beta_1 = 1/[2(1 + \alpha_1)]$. Obviously for $\alpha_1 = 0$, $\beta_1 = \beta_o = 1/2$. For $\alpha_1 = 1$, $\beta_1 = 1/4$ and M_o must divide C so $C_1 = C/4$ and $C_o = 3C/4$ to make $V_o = 0$, etc.

If we wish to launch two plates in the same direction but with $V_o \leq V_1$ so there will be a greater spread of metal, our model is too simple to enable us to analyze all possibilities. However, for certain fixed relations, we can estimate what is needed. For example, to obtain $V_o = sV_1$ with $s < 1$, using $M_o = kM_1$ with $k > 1$, Equations (131) and (130) become

$$V_{-1} = -\{[(s + \beta_1) + 2\alpha_1(1 + ks)]/(1 - \beta_1)]V_1 \tag{134}$$

and

$$V_1 = \sqrt{(2E)/\left\{\left(1 + ks^2\right)\alpha_1 + \frac{1}{3}\left[\beta_1 + \beta_1 + s^2 - \left\{S - \frac{1}{(1-\beta_1)}\right\}\left\{\beta_1 + S + 2\alpha_1\left(1 + k_s\right)\right\}\right]\right\}} . \tag{135}$$

For example, if $s = 1/2$, $k = 8$, $\alpha_1 = 1/2$ and $\beta_1 = 1/4$, we find $V_1/\sqrt{2E} = 0.55$ from Equation (135) with $V_o/\sqrt{2E} = 0.275$. Of course we can also use different types of explosive in the two sheets ($E_o \neq E_1$) and vary the design further. The principles are clear and will not be elaborated here.

The next, more complicated case is $(M_1/C_1/M_o/C_o/M_{-1})$ in Figure 11. This requires the addition of $1/2M_{-1}V^2_{-1}$ to the right side of Equation (130) and of $M_{-1}V_{-1}$ to the right side of Equation (131). There is one more dimensionless parameter, $\alpha_{-1} = M_{-1}/C$, but still only three launch speeds. Let us close this section by giving some simple examples of how we might estimate launch speeds. Suppose we have $M_{-1} = M_o = M_1$, $C_o = 2C_1$, and $E_o = E_1$. We can solve the momentum equation for

$$V_{-1} = a_o V_o + a_1 V_1 , \tag{136}$$

where

$$a_o = -(3+6\alpha_1)/(2+6\alpha_1), \tag{137}$$

and

$$a_1 = -(1+6\alpha_1)/(2+6\alpha_1), \tag{138}$$

and use Equation (136) to eliminate V_{-1} in the energy equation to obtain

$$2E = XV_o^2 + YV_oV_1 + ZV_1^2 , \tag{139}$$

where

$$X = a_o^2(\alpha_1+2/9) + 2/9a_o + (\alpha_1+1/3) \tag{140}$$

$$Y = 2a_oa_1(\alpha_1+2/9) + 2/9a_o + 1/9 \tag{141}$$

$$Z = a_1^2(\alpha_1+2/9) + (\alpha_1+1/9) . \tag{142}$$

Thus the V_i depend only on $\alpha_1 = M_1/C$, where $C = C_0+C_1$. Suppose we let $\alpha_1=1$, that is, $\alpha = M/C = 3M_1/C = 3$. If $2E = (2.4)^2 = 5.76$ (mm/μs)^2 as for TNT at a typical loading density, then Equation (139) becomes

$$5.76 = 2.63\ V_0^{\ 2} + 2.27\ V_0V_1 + 2.05\ V_1^{\ 2}\ , \tag{143}$$

while Equation (136) becomes

$$V_{-1} = -(1.125V_0 + .875V_1). \tag{144}$$

If we consider two related cases which are known, we can restrict the range of allowed values. First, consider $C_0 = C_1 = C/2$, a case for which $V_0 = 0$ and $V_1 = 2.4/\sqrt{2M_1/C + 1/3} = 1.57$ mm/μs $= -V_{-1}$. If we move M_0 so $C_1<C/2$ and $C_0>C/2$, then $V_0>0$, $V_1<1.57$ mm/μs and $V_{-1}<-1.57$ mm/μs. Second, consider $C_0 = C_1 = C/3$, for which $V_0 = 0$ and $V_1 = 2.4/\sqrt{2M_1/(2C/3) + 1/3} = 1.315$ mm/μs $= -V_{-1}$. Increasing C_0 to $2C/3 = 2C_1$ will make $V_1>1.315$ mm/μs and $V_0>0$. The range of V_1 values is thus fairly narrow, namely, $1.315<V_1<1.57$ mm/μs). These extreme values may be used in Equation (143) to find the range of V_0, namely $.512>Vo>.176$ and from Equation (144) we find $-1.727>V_{-1}>-1.572$. If we take the mid-points of these ranges as our estimate, we see that the model predicts $V_1 = 1.44\pm.13$ mm/μs, $V_0 = .34 \pm.17$ mm/μs and $V_{-1} = -1.64\pm.08$ mm/μs.

Suppose we wish to make $V_1 = 2V_0>0$. Of course there are many choices of M_i and C_i which will accomplish this goal. For example, $\alpha_0 = \alpha_1 = 1/2$, $\alpha_{-1} = 1$ and $\alpha = M/C = (\alpha_1+\alpha_0+\alpha_{-1}) = 2$. In this case $V_1 = 1.54$ mm/μs, $V_0 = .77$ mm/μs and $V_{-1} =-1.76$ mm/μs for $\sqrt{2E} = 2.4$ mm/μs. In this and the previous example we are neglecting any reduction in $\sqrt{2E}$.

The procedure illustrated here is not completely straightforward since it requires some ingenuity to narrow the range of values by comparisons with known theoretical or experimental cases. However, it can give us useful estimates for fairly complicated Dagwoods, jelly rolls and onions.

SUMMARY

In this report we have seen that the exponent γ in the entropic equation of state may be taken equal to three in practical applications involving the expansion of solid explosive detonation products. This fact has been used to simplify, extend and unify the ideas of Taylor, Gurney, Thomas and Sterne concerning the acceleration of metal by explosives. The modified theories which have been developed here can be used in the design of lined cavity charges and fragmentation warheads as well as flying plate experiments like those which have been used to study the shock properties of solids.

The author wishes to acknowledge the support of his supervisors and the director of the Ballistics Research Laboratory who advocate a return to fundamentals as a means of obtaining fresh insights and of stimulating new approaches to problems. The author shares this view and has attempted to give it substance in this report.

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REFERENCES

1. G. I. Taylor, "Analysis of the Explosion of a Long Cylindrical Bomb Detonated at One End," paper written for the Civil Defence Research Committee, Ministry of Home Security in 1941, available in The Scientific Papers of Sir Geoffrey Ingram Taylor, Vol. III, G. K. Batchelor, ed., The University Press, Cambridge, 1963, p. 277.

2. R. W. Gurney, "The Initial Velocities of Fragments from Bombs, Shell, Grenades," Ballistic Research Laboratory R405, 1943. (AD ATI36218)

3. G. I. Taylor, "A Formulation of Mr. Tuck's Conception of Munroe Jets," written in 1943 and available in The Scientific Papers of Sir Geoffrey Ingram Taylor, Vol. III, ed. by G. K. Batchelor, The University Press, Cambridge, 1963, p. 358.

4. G. Birkhoff, D. P. MacDougall, E. M. Pugh, and G. I. Taylor, "Explosives with Lined Cavities," Journal of Applied Physics, 19, 1948, p. 563.

5. J. T. Harrison, "BASC, An Analytical Code for Calculating Shaped Charge Properties," Sixth Internstional Symposium on Ballistics sponsored by the American Defense Preparedness Association (ADPA), 1981, p. 253.

6. G. Randers-Pehrson, R. R. Karpp, C. E. Anderson, Jr., snd H. J. Blische, "SHORTFRAG Users Guide," Ballistic Research Laboratory Memorandum Report 3007, 1980. (ADB 046644L)

7. L. R. Kruse, "Theoretically Determined Fragmentation Characteristics and Lethality Estimstes for a Newly Proposed Tungsten Alloy Cased BLV-97 Combined Effects Bomblet," Ballistic Research Laboratory Memorsndum Report 3294, 1983. (ADC 032497L)

8. M. A. Meyers and L. E. Murr, eds, Appendix E, "Nomograph for Determination of Flyer-Plate Velocity," Shock Waves and High-Strain-Rate Phenomena in Metals, proceedings of an International Conference on Metallurgicsl Effects of High-Strain-Rate Deformstion and Fabrication, Plenum Press, NY and London, 1981, p. 1057.

9. G. E. Jones, J. E. Kennedy, and L. D. Bertholf, "Ballistic Calculations of R. W. Gurney," American Journal of Physics, 48. 1980, p.264.

10. G. E. Jones, "The Gurney Equations for Multilayered Fragments," Journal of Applied Physics, 50, 1979, p. 3746.

11. M. J. Kamlet and M. Finger, "An Alternative Method for Calculating Gurney Velocities," Combustion and Flame, 34, 1979, p. 213.

12. E. W. LaRocca, "A Simplified Method of Calculating the Gurney Constant of Common Explosives," presented at the meeting of the Pyrotechnics and Explosives Applications Section of the American Defense Preparedness Association, 1978.

13. E. W. LaRocca, "Advances in Predicting the Relative Power of High Explosives and the Correlation with Fragment Velocity," presented at the meeting of the Pyrotechnics and Explosives Applications Section of the American Defense Preparedness Association, 1980.

14. D. R. Hardesty and J. E. Kennedy, "Thermochemical Estimation of Explosive Energy Output," Combustion and Flame, 28, 1977, p. 45.

15. G. I. Taylor and H. Jones, "Note on the Lateral Expansion behind a Detonation Wave," The Scientific Papers of Sir Geoffrey Ingram Taylor, Vol. III, G. K. Batchelor, ed., The University Press, Cambridge, 1963, p. 309.

16. G. I. Taylor and H. Jones, "Blast Impulse and Fragment Velocities from Cased Charges," The Scientific Papers of Sir Geoffrey Ingram Taylor, Vol. III, G. K. Batchelor, ed., The University Press, Cambridge, 1963, p. 363.

17. G. I. Taylor and H. Jones, "The Bursting of Cylindrical Cased Charges," The Scientific Papers of Sir Geoffrey Ingram Taylor, Vol. III, G. K. Batchelor, ed., The University Press, Cambridge, 1963, p.379.

18. H. Jones, Research Committee Paper 166, no longer available.

19. H. Jones, Research Committee Paper 212, no longer available.

20. H. Jones and A. R. Miller, Research Committee Paper 306, no longer available.

21. H. Jones, "A Theory of the Dependence of the Rate of Detonation of Solid Explosives on the Diameter of the Charge," Proceedings of the Royal Society of London, 189A, 1946, p. 415.

22. H. Jones and A. R. Miller, "The Detonation of Solid Explosives: the Equilibrium Conditions in the Detonation Wave-Front and the Adiabatic Expansion of the Products of Detonation," Proceedings of the Royal Society of London, 194A, 1948, p. 480.

23. R. Courant and K. O. Friedrichs, Supersonic Flow and Shock Waves, Interscience, NY, 1948, p. 6.

24. G. I. Taylor, "The Dynamics of the Combustion Products behind Plane and Spherical Detonation Fronts in Explosives," Proceedings of the Royal Society of London, 200A, 1950, p. 235.

25. M. Jouget, C. R. Academy of Science, 144, Paris, 1907, p. 633.

26. S. J. Jacobs, "Recent Advances in Condensed Media Detonations," American Rocket Society Journal, 30, 1960, p. 151.

27. G. B. Kistiakowsky and E. B. Wilson, Jr., OSRD Report 114, US National Defense and Research Committee of the Office of Scientific Research and Development, 1941.

28. M. J. Kamlet and S. J. Jacobs, "Chemistry of Detonations, I: A Simple Method for Calculating Detonation Properties of C-H-N-O Explosives," Journal of Chemical Physica, 48, 1968, p. 23.

29. M. J. Kamlet and J. E. Ablard, "Chemistry of Detonations, II: Buffered Equilibria," Journal of Chemical Physics, 48, 1968, p. 36.

30. M. J. Kamlet and C. Dickinson, "Chemistry of Detonations, III: Evaluation of the Simplified Calculational Method for Chapman-Jouget Detonation Pressures on the Basis of the Available Experimental Information," Journal of Chemical Physics, 48, 1968, p. 43.

31. M. J. Kamlet and H. Hurwitz, "Chemistry of Detonations, IV: Evaluation of a Simple Predictional Method for Detonation Velocities of C-H-N-O Explosives," Journal of Chemical Physics, 48, 1968, p. 3685.

32. M. J. Kamlet and H. Hurwitz, "The Chemistry of Detonations, V: Pressures of C-H-N-O Explosives at Various Stages of the Isentropic Expansion," NOLTR 68-44, 1968, or Israel Journal of Technology, 7, 1968, p. 431.

33. M. J. Kamlet and J. M. Short, "The Chemistry of Detonations, VI: A 'Rule for Gamma' as a Criterion for Choice among Conflicting Detonation Pressure Measurementa," Combustion and Flame, 38, 1980, p. 221.

34. J. M. Short, F. H. Helm, M. Finger, and M. J. Kamlet, "The Chemistry of Detonations, VII: A Simplified Method for Predicting Explosive Performance in the Cylinder Test," Combustion and Flame, 43, 1981, p. 99.

35. M. J. Kamlet, J. M. Short, M. Finger, F. Helm, R. R. McGuire, and I. B. Akst, "The Chemistry of Detonations, VIII: Energetics Relationships on the Detonation Isentrope," Combustion and Flame., 51, 1983, p. 325.

36. W. H. Andersen, "Comments on 'The Chemistry of Detonations'," Combustion and Flame, 45. 1982, p. 309.

37. A. K. Aziz, H. Hurwitz, and H. M. Sternberg, "Energy Transfer to a Rigid Piston under Detonation Loading," The Physics of Fluids, 4, 1961, p. 380.

38. R. W. Gurney, "Fragmentation of Bombs, Shells and Grenades," Ballistic Research Laboratory Report 635, 1947. (ADB 800451)

39. F. E. Allison and J. T. Schriempf, "Explosively Loaded Metallic Cylinders, II," Journal of Applied Physics, 31, 1960, p. 846.

40. F. E. Allison and R. W. Watson, "Explosively Loaded Metallic Cylinders, I," Journal of Applied Physics, 31, 1960, p. 842.

41. R. Courant and K. O. Friedrichs, Supersonic Flow and Shock Waves, Interscience, NY, 1948, p. 22.

42. M. Famiglietti, "Fragmentation of Ring Type Cylindrical Shell Made of Various Metals," Ballistic Research Laboratory Memorandum Report 597, 1952. (AD 486744)

43. E. U. Condon, "Tunneling - How It All Started," American Journal of Physics, 46, 1978, p. 319.

44. L. H. Thomas, "Theory of the Explosion of Cased Charges of Simple Shape," Ballistic Research Laboratory Report 475, 1944. (AD 491945)

45. L. H. Thomas, "Analysis of the Distribution in Mass, in Speed, and in Direction of Motion of the Fragments of the M71 (90 mm) A. A. Shell, When Filled with TNT, and When Filled with Ednatol," Ballistic Research Laboratory Report 434, 1943. (ADB 493515)

46. N. M. Burman and A. J. Bedford, "A Concept for the Prediction of Fragment Mass/Number Distributions of Fragmenting Munitions," Proceedings of the Sixth International Symposium on Ballistics, 1981, p. 245.

47. G. Eden and P. W. Wright, "A Technique for the Precise Measurement of the Motion of a Plane Free Surface," Fourth Symposium on Detonation, Naval Ordnance Laboratory ACR-126, 1965, p. 573.

48. B. D. Lambourn and J. E. Hartley, "The Calculation of the Hydrodynamic Behavior of Plane One Dimensional Explosive/Metal Systems," Fourth Symposium on Detonation, 1965, p. 538.

49. N. E. Hoskin and B. D. Lambourn, "The Acceleration of Two Metal Plates in an HE-Metal Sandwich," Seventh Symposium on Detonation, Naval Surface Weapons Center MP 82-334, 1981, p. 811.

50. Love and Pidduck, "Lagrange Ballistic Problem," Phil. Trans. Royal Society of London, 222, 1922, p. 167.

51. S. J. Jacobs, "The Gurney Formula: Variations on a Theme by Lagrange," NOLTR 74-86, 1974.

52. T. E. Sterne, "A Note on the Initial Velocities of Fragments from Warheads," Ballistic Research Laboratory Report 648, 1947. (AD 898680)

53. T. E. Sterne, "The Fragment Velocity of a Spherical Shell Containing an Inert Core," Ballistic Research Laboratory Report 753, 1951. (ADB 377187)

54. J. F. Mescall and P. V. Riffin, "Slapper Concept in Fragmentation," AMMRC Technical Report 76-8, 1976.

55. N. F. Mott, "Fragmentation of Shell Cases," Proceedings of the Royal Society of London, 189, 1947, p. 300.

56. R. Curran, L. Seaman, and D. A. Shockey, "Dynamic Failure in Solids," Physics Today, January 1977, p. 46.

57. F. E. Walker, A. M. Karo, and J. R. Hardy, "Comparison of Molecular Dynamics Calculations with Observed Initiation Phenomena," Seventh Symposium on Detonation, 1981, p. 777.

58. A. M. Karo, F. E. Walker, W. G. Cunningham, and J. R. Hardy, "Theoretical Studies of Shock Dynamics in Two Dimensional Structures," Shock Waves in Condensed Matter - 1981, W. J. Nellis, L. Seaman, and R. A. Graham, eds., AIP Conference Proceedings 78, American Inetitute of Physics, NY, 1982, p. 92.

59. E. L. Lee, H. C. Hornig, and J. W. Kury, "Adiabatic Expansion of High Explosive Detonation Products," UCRL-50422, Lawrence Radiation Laboratory, 1968.

60. I. G. Henry, "The Gurney Formula and Related Approximations for the High-Explosive Deployment of Fragments," Hughes Aircraft Report PUB-189, 1967.

61. M. Défourneaux, "Transferts d'Energie Dans les Combustions et Detonations avec Confinement," Astronautica Acta, 17, 1972, p. 609.

62. R. G. S. Sewell, "Fixed-weight and Fixed-volume Constraints on Optimum Charge-to-Metal Ratios in Warhead Design," NAVWEPS Report 8471, NOTS TP 3430, China Lake, CA, 1964.

63. T. Zulkoski, "Development of Optimum Theoretical Warhead Design Criteria," Naval Weapons Center TP 5892, 1976.

64. P. Y. Chanteret, "An Analytical Model for Metal Acceleration by Grazing Detonation," Seventh International Symposium on Ballistics, The Hague-The Netherlands, 1983, p. 515.

65. N. E. Hoskin, J. W. S. Allan, W. A. Bailey, J. W. Lethaby, and I. C. Skidmore, "The Motion of Plates and Cylinders Driven by Detonation Waves at Tangential Incidence," Fourth International Symposium on Detonation, ONR ACR-126, 1965, p. 14.

66. R. Hill and D. C. Pack, "An Investigation, by the Method of Characteristics, of the Lateral Expansion of the Gases behind a Detonating Slab of Explosive," Proceedings of the Royal Society of London, 191, 1947, p. 524.

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