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THE UNIVERSITY OF NEW MEXICO  
COLLEGE OF ENGINEERING

# BUREAU OF ENGINEERING RESEARCH

ELASTIC-WORKHARDENING SDF SYSTEM SUBJECTED  
TO RANDOM BLAST EXCITATIONS

by

Frederick D. Ju  
Thomas L. Paez  
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## NOMENCLATURE

### Symbol

A	excitation amplitude
$A_7$	post-yield stiffness
$E_d$	energy of dissipation
g	a function of random variables
h	permanent set
k	stiffness in the elastic range
m	system mass
R	maximum displacement, $z_{\max}$
$R(z)$	restoring force
t	time
$t_{\max}$	the time the system reaches its maximum displacement, $z_{\max}$
$t_7$	the time the system first reaches its yield displacement, $u_7$
$u_7$	yield displacement
z	response displacement
$z_{\max}$	maximum displacement response, R
c	excitation decay rate
$\zeta$	damping ratio
u	mean
Y	random variable
$\rho$	correlation coefficient
$\sigma$	standard deviation
B	random variable
$\omega_n$	natural frequency (small displacement)
$\omega_d$	damped frequency (small displacement)
$\omega_y$	post-yield frequency ( $= \sqrt{A_7/m}$ )

## 1.0 INTRODUCTION

An above ground explosion generates a shock wave in air, or airblast, and is accompanied by some duration of strong wind. Especially, the airblast from a nuclear detonation can cause extremely high air pressures and has a relatively long period of duration. When structures under design may be subjected to this sort of loading condition, it is necessary to analyze the behaviors of the structures when subjected to this kind of input to determine whether the structures can survive or not. → p. 1473

When an airblast produces a high air pressure and has a long period of duration, it will cause many structures to have an extreme response. If the structure can execute a response that has plastic deformation, the designer must have some technique to analyze inelastic response in the design process. The designer needs the capability to predict such measures of structural response as the plastic deformation and maximum displacement.

When a structure executes an extreme response, it may behave inelastically, and during the response, offsets from the initial elastic configuration may occur. When these offsets are large, residual deformation in the structure may exist at the completion of structural motion, and the magnitudes of these residual deformations may be important. When they are important, it is desirable to include the potential for these residual deformations in the mathematical model of the structure so that they may be predicted in the structural analysis. The models used in References [1], [2], [3] do account for some features of inelastic response, but do not permit permanent offset (residual deformation). The model used in the current investigation does account for permanent offset.

The properties of nuclear and high explosive airblasts are random. The reason is that the material and geometric properties of blast sources are random. In view of this, an airblast measurement is never repeated even when nominally identical blast sources are used. An airblast signal usually consists of a rapidly increasing pressure followed by a gradual decay. Two properties of the pressure wave are of great concern. The first is the amplitude of the airblast; the second is the decay rate. The first property describes the highest air pressure the airblast produces

throughout the time history. In general, the rise time is so brief that the peak amplitude is assumed to occur at the beginning of explosion. The second property deals with the rate at which the airblast dies out. In this report, these two properties are considered random.

In most applications the structure itself should also be considered random. The material properties of the structure determine the elastic stiffness, yield stiffness, yield displacement, etc. Consequently, the natural frequency is affected strongly by the material properties. When the material of the structure is considered to be random, then the properties of the structure must also be considered as random. In this report, the material properties considered as random are natural frequency, yield stiffness, yield displacement and damping ratio. These four properties are considered random because they affect the response significantly.

The previous discussion points out that both the excitation and structural system can be regarded as random; in this study both sources of randomness are considered. The response of the system, therefore, can be regarded as a response random process, and there is a relationship that characterizes the response in terms of the excitation and system random variables.

These random variables include input random variables which are peak amplitude of air pressure and decay rate, and system random variables which are natural frequency, yield stiffness, yield displacement, and damping ratio. It is desirable to develop a method to establish the relationship among the input characteristics, the system characteristics, and the response characteristics. The response random process is then characterized in terms of the relationship.

Many measures of structural response concern the designer. Some measures of special importance are plastic deformation, maximum displacement response, and energy dissipated by the system spring, or by the damper. The plastic deformation, which is the offset from the static configuration, must be controlled in structures. The maximum displacement response must be less than the design value. The energy dissipated must be less than an allowable value since it is related to the fatigue and failure.



of a structure. When the probabilistic character of the input excitation and system random variables are known, and when the relationship between measures of response and the input and system variables has been established, then it is possible to specify the response random process. Certain probabilistic measures of structural response are obtained. Specifically, the mean and variance of permanent offset, peak response, and energy dissipated in an inelastic structure are obtained. The results can be used to determine whether design criteria are satisfied.

In past investigations, several mathematical models have been used with structural identification procedures to study structural behavior. Lutes and Hsieh [1] used linear, third and fourth order equations to approximately simulate the response of an SDF hysteretic system. They found that the spectral density, or the autocorrelation function of the linear and hysteretic systems can be matched. Toussi and Yao [2] used the response of a ten-story reinforced concrete test structure as a means to study the feasibility and practicality of a hysteresis identification technique. They considered the hysteresis to be a measure of damage. Yao, Toussi and Sozen presented the state-of-the art on damage assessment of existing structures in Reference [3]. Morrison and Paez [4] proposed a procedure for predicting the probability of survival of inelastic structures excited by blast. In this study the failure criterion was related to peak strain in the structural response. A method for establishing the probabilistic character of nonlinear structural response was developed. Rohani [5] presented a probabilistic solution of one-dimensional wave propagation phenomenon in earth media by using a deterministic model. Bennett and Paez [6] provided a means for relating uncertainties of input parameters to the uncertainty in response in a reinforced concrete slab. The method used in the last three references is adopted in this investigation to characterize nonlinear measures of inelastic structure response.

The objectives of this investigation are, first, to develop a model to define the permanent offset in an inelastic structure. Next, we develop the relationship among input characteristic, system characteristic and response characteristic through which the response random process is established. Based on this response random process, and given the moments

of input and system variables, the response characteristics are computed. This provides a means to analyze the structural response when the structure is subjected to blast type loading and both the input and the system are considered random.

The structure considered here is a single-degree-of-freedom (SDF) bilinear hysteretic system which possesses an elastic stiffness and yield stiffness. The natural frequency and yield natural frequency are therefore defined. The hysteresis loop of a spring, strongly influences the energy dissipated in a structure; this will be discussed in detail. The airblast input is modeled by a decaying exponential function. The value of peak air pressure and decay rate of the input are obtained from the Air Force Design Manual [7].

## 2.0 THE ANALYSIS OF MAXIMUM RESPONSE

The maximum response of a structure when subjected to blast type load can be predicted through basic structure response analysis. Intuitively, the maximum response must relate to the input parameters, which are peak air pressures and decay rate, and system parameters, which are natural frequency, damping ratio, etc. All these properties affect the response. Specifically, we can establish a mathematical function which governs the maximum response. The response random process can be simply expressed. Given the statistical moments of the input and system parameters, the statistical moments of the responses can be computed. The response considered here can be inelastic.

It is assumed that the system under consideration has significant elastic and yield stiffness so that the bilinear hysteretic system can be used as the structural model. The offset function for blast type input will behave as in Figure 2-1.

Again consider an SDF, bilinear hysteretic system, with small damping ratio,  $\zeta$ , system mass,  $m$ , elastic stiffness,  $k$ . The equation of motion is given by

$$\ddot{z} + 2\zeta\omega_n\dot{z} + \frac{1}{m}R(z) = Ae^{-\alpha t} \quad t \geq 0 \quad (2-1)$$

Assume that the system starts from rest,  $z(0) = 0$  and  $z'(0) = 0$ . Let  $t_7$  denote the time the system first reaches its yield displacement,  $u_7$ .

Within time  $t < t_7$ , the system remains linear, so that Equation (2-1) can be written as

$$\ddot{z} + 2\zeta\omega_n\dot{z} + \omega_n^2 z = Ae^{-\alpha t} \quad \text{for } t < t_7 \quad (2-2)$$

Because the system remains linear during the initial portion of response, the above equation can be solved immediately by using convolution integral

$$z(t) = \int_0^t h(\tau) \times (t - \tau) d\tau \quad (2-3)$$

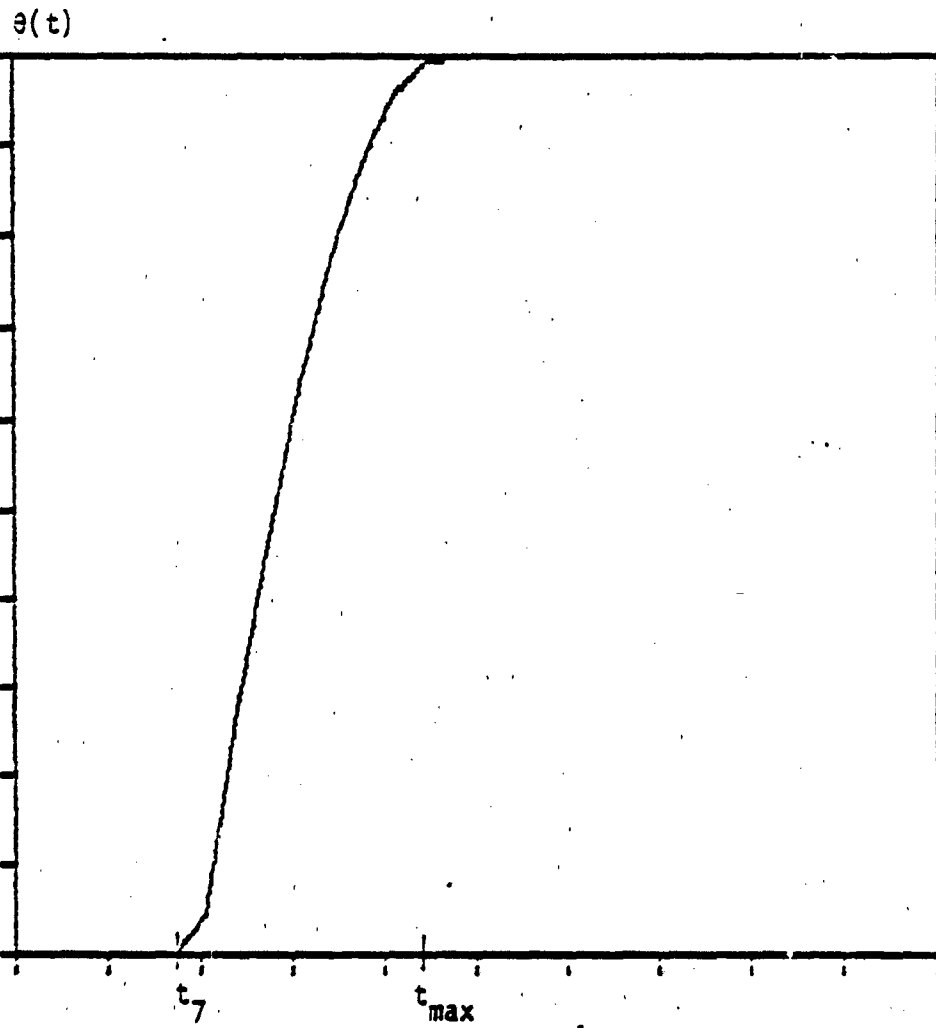


Figure 2-1. The Offset Function in Time Domain.

where  $h(t)$  is the linear system impulse response function and  $x(t)$  is the input forcing function. The impulse response function is

$$h(t) = \frac{e^{-\zeta\omega_n t}}{\omega_d} \cdot \sin(\omega_d t) \quad (2-4)$$

where  $\omega_d = \omega_n \sqrt{1 - \zeta^2}$  and  $\ddot{x}(t) = Ae^{-\alpha t}$ . Carry out the integral; then

$$z(t) = \frac{A}{\omega_d [(\alpha - \zeta\omega_n)^2 + \omega_d^2]} \left\{ e^{-\zeta\omega_n t} [(\alpha - \zeta\omega_n) \sin(\omega_d t) - \omega_d \cos(\omega_d t)] + \omega_d e^{-\alpha t} \right\}$$

for  $0 \leq t \leq t_7$  (2-5)

At time  $t = t_7$ , the displacement response should be equal to  $u_7$ .

$$z(t_7) = \frac{A}{\omega_d [(\alpha - \zeta\omega_n)^2 + \omega_d^2]} \left\{ e^{-\zeta\omega_n t_7} [(\alpha - \zeta\omega_n) \sin(\omega_d t_7) - \omega_d \cos(\omega_d t_7)] + \omega_d e^{-\alpha t_7} \right\} \quad (2-6)$$

The time  $t_7$  at which yielding first occurs must be determined by solving the above equation for  $t_7$ . Since  $\zeta$  and  $t_7$  are both small;

$e^{-\zeta\omega_n t_7} \approx 1$ . Simplify the notation by letting  $\alpha - \zeta\omega_n = p$ ;  $\omega_d [p^2 + \omega_d^2] = x_1$ ;  $A/x_1 = x_2$ . Rewrite Equation (2-6) using the above notation.

$$x_2 \cdot \left[ p \sin(\omega_d t_7) - \omega_d \cos(\omega_d t_7) + \omega_d e^{-\alpha t_7} \right] = u_7 \quad (2-7)$$

The value of  $\alpha$  is about from 0.5 - 6.0. This is usually much smaller than the value of  $\omega_n$ ; hence,

$$p \cdot \sin(\omega_d t_7) \ll \omega_d \cos(\omega_d t_7)$$

and the quantity on the left is negligible.

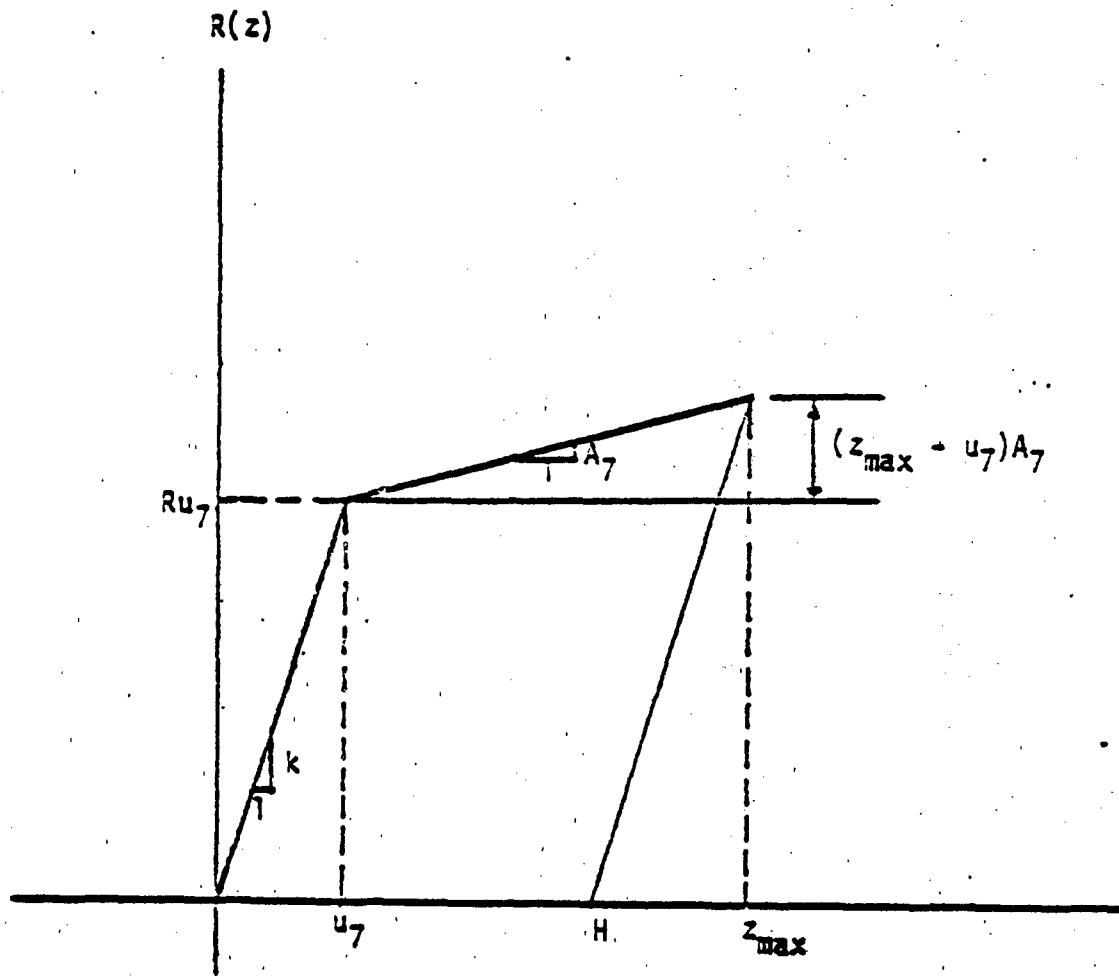


Figure 2-2. The Behavior of Bilinear Hysteretic Restoring Force.

Using Taylor's expansion to express  $e^{-\alpha t_7}$  and  $\cos(\omega_d t_7)$  and neglecting the high order term,  $t_7$  can finally be evaluated

$$t_7 = \frac{1}{\alpha} (B + \sqrt{B^2 + 4C}) \text{ where } B = 2\alpha/(\omega_0^2 + \alpha^2), C = 2(u_7)/A. \quad (2-8)$$

After solving for  $t_7$ , the velocity at  $t = t_7$  can be computed immediately. Let  $\dot{z}(t_7) = \dot{z}_7$ .

To this point, the only assumption used in determining the time of first yield and the velocity at that time is that the system viscous damping factor is relatively small. No assumptions regarding the input level or yield level have been used.

Next, we consider the phase of the response when yielding starts to occur. Let  $t_{max}$  be the time the response reaches its maximum displacement value. During the time interval  $(t_7, t_{max})$  the system has entered the plastic region and permanent offset starts to occur. If we let  $A_7$  denote the yield stiffness, then for time  $t_{max} < t_7$ , the restoring force function can be written as (refer to Fig. 2-2):

$$z + 2\zeta\omega_n \dot{z} + \frac{1}{m}[A_7 z + (k - A_7) u_7] = Ae^{-\alpha t}, \quad t_{max} \geq t \geq t_7 \quad (2-9)$$

or

$$z + 2\zeta\omega_n \dot{z} + \omega_y^2 z = Ae^{-\alpha t} - (\omega_n^2 - \omega_y^2)u_7, \quad t_{max} > t > t_7 \quad (2-10)$$

where  $\omega_y^2 = A_7/m$

The solution for  $z(t)$  can be expressed

$$z(t) = e^{-\zeta\omega_n t} (c_1 \cos(\bar{\omega}_d t) + c_2 \sin \bar{\omega}_d t) + z_p, \quad t_y \leq t \leq t_{max} \quad (2-11)$$

where  $\bar{\omega}_d = \sqrt{(k_y/m) - (\zeta\omega_n)^2}$

$$z_p = q_1 + q_2 e^{-\alpha t}$$

$$q_1 = [(k_y/m - \omega_n^2) z_y + \dot{z}_y m/k_y]$$

$$q_2 = A/[\alpha^2 - 2\zeta\omega_n\alpha + (k_y/m)]$$

By using  $q_1$  and  $q_2$  in Equation 2-11, the constants  $C_1$  and  $C_2$  can be solved by using as initial condition,  $z(t_7) = u_7$  and  $\dot{z}(t_7) = \dot{z}_7$  which were obtained in the previous discussion.

Since the displacement response has been determined in Equation (2-11), the maximum value can be obtained by maximizing Equation (2-11). The maximum occurs when

$$\dot{z}(t_{\max}) = 0 \quad (2-12)$$

The  $t_{\max}$  can be obtained from Equation (2-12). The corresponding  $z_{\max}$  then can be evaluated by using  $t_{\max}$  in Equation (2-11).



### 3.0 PROBABILISTIC ANALYSIS OF RESPONSE

In the previous section, the relationship among the maximum response and input parameters and system parameters was established. The response random process is then characterized in terms of the established relationship. In many cases, the established function may be complicated. In order to analyze some measures of the response, a method using Taylor's expansion is available and has been widely used. A brief discussion of that method is presented here.

Let  $R$  denote a random variable that is a measure of the structural response random process. It is assumed that  $R$  is a function of  $n$  random variables. These  $n$  random variables are the parameters of the input and the structural system. Let the functional relation be denoted

$$R = g(\gamma_1, \gamma_2, \dots, \gamma_n) \quad (3.1)$$

The  $\gamma_i$ ,  $i=1, \dots, n$  are the random variable with mean values  $\mu_{\gamma_i}$ ,  $i=1, \dots, n$ .

The function  $g$  can be expanded in a Taylor series about the means of the parameters:

$$R = g(\mu_{\gamma_1}, \mu_{\gamma_2}, \dots, \mu_{\gamma_n}) + \sum_{i=1}^n (\gamma_i - \mu_{\gamma_i}) \left. \frac{\partial g}{\partial \gamma_i} \right|_{\{\mu\}} + \dots \quad (3-2)$$

where  $\{\mu_{\gamma_i}\}$  is the vector of mean values of the  $\gamma_i$ .

The moments of  $R$  can be evaluated by using some portion of the series in Equation (3-2). For example, the series can be truncated following the linear terms. When this is done, the mean of  $R$  is

$$E[R] = g(\mu_{\gamma_1}, \mu_{\gamma_2}, \dots, \mu_{\gamma_n}) \quad (3-3)$$

This mathematical expression shows that the mean value of the dependent random variable,  $R$ , is closely related to the mean values of the underlying random variables,  $\mu_{\gamma_i}$ , and the function expression  $g(\mu_{\gamma_1}, \mu_{\gamma_2}, \dots, \mu_{\gamma_n})$ .

The variance of  $R$  can be computed as:

$$\begin{aligned}
 \text{Var}[R] &= E[R^2] - E[R]^2 \\
 &= E\left[g^2(\mu_{\gamma_1}, \mu_{\gamma_2}, \dots, \mu_{\gamma_n})\right] + E\left[2g(\mu_{\gamma_1}, \mu_{\gamma_2}, \dots, \mu_{\gamma_n}) \cdot \sum_{i=1}^n (\gamma_i - \mu_{\gamma_i}) \frac{\partial g}{\partial \gamma_i} \Big|_{\{\mu_{\gamma_j}\}}\right] \\
 &\quad + E\left[\sum_{j=1}^n \sum_{i=1}^n (\gamma_i - \mu_{\gamma_i}) \frac{\partial g}{\partial \gamma_i} \Big|_{\{\mu_{\gamma_j}\}} \cdot (\gamma_j - \mu_{\gamma_j}) \frac{\partial g}{\partial \gamma_j} \Big|_{\{\mu_{\gamma_j}\}}\right] - E[R]^2 \\
 &= \sum_{i=1}^n \sum_{j=1}^n \rho_{ij} \sigma_i \sigma_j \frac{\partial g}{\partial \gamma_i} \Big|_{\{\mu_{\gamma_j}\}} \frac{\partial g}{\partial \gamma_j} \Big|_{\{\mu_{\gamma_j}\}} \quad (3-4)
 \end{aligned}$$

The  $\sigma_i$  and  $\sigma_j$  represent the standard deviations of  $\gamma_i$  and  $\gamma_j$  and  $\rho_{ij}$  here represents the coefficient of correlation between  $\gamma_i$  and  $\gamma_j$ .

The analysis technique outlined above can be used in a wide variety of cases; especially, it is useful in the characteristics of the response of a nonlinear system. When the moments of the underlying random variables are known (means, variances, and covariances), then it is easy to apply. But there are some cases where the random variable  $R$  depends on some underlying random variables whose characteristics are unknown, a priori, as well as random variables whose characteristics are known. For example, we may be interested in establishing the moments of energy dissipated during inelastical response. This quantity can be expressed as a function of

input parameters, system parameters and the peak response. The moments of the input and system parameters are probably known, a priori, but the moments of peak response certainly are not. To account for this, the first phase of an investigation can be aimed at analyzing peak response and its statistical relation to the input and system parameters. After this is done, the energy dissipated can be analyzed using the general approach outlined above. In general, any sequence of analyses like this one can be executed. At each step, a new random variable characterizing the response can be introduced.

For example, the analysis referred to above can be accomplished in the following way. Let  $R$  be a function of  $n$  random variables,  $\beta_i$ ,  $i=1,2,\dots,m$ ,  $m+1,m+2,\dots,n$ . The moments and cross moments of the  $\beta_i$ ,  $i=1,2,\dots,m$ , are known, a priori, and the moments and cross moments of the  $\beta_i$ ,  $i=m+1,\dots,n$ , are not known. However, the moments of the  $\beta_i$ ,  $i=m+1,\dots,n$ , can be determined from the  $\beta_i$ ,  $i=1,\dots,m$ . Let  $\beta_i$ ,  $i=m+1,\dots,n$  be expressed as follows:

$$\beta_i = h_i(\beta_1, \beta_2, \dots, \beta_m), \quad i=m+1, \dots, n \quad (3-5)$$

The Taylor expansion for the above expression can be written as in Equation (3-2). The series can be truncated at some point and the result can be used to approximate the moments and cross moments for all the  $\beta_i$ ,  $i=m+1,\dots,n$ . For example, when the series are truncated following the linear term, the covariance between two of the  $\beta_i$ 's, one from the group  $i=1,\dots,m$ , and the other from the group,  $i=m+1,\dots,n$ , can be expressed as

$$\begin{aligned} \text{Cov}(\beta_p, \beta_q) &= E \left[ \beta_p \left\{ h_q(\mu_{\beta_1}, \mu_{\beta_2}, \dots, \mu_{\beta_m}) + \sum_{j=1}^r (\beta_j - \mu_{\beta_j}) \frac{\partial h_q}{\partial \beta_j} \Big|_{\{\mu_{\beta_j}\}} + \dots \right\} \right] \\ &= E[\beta_p] E[h_q] + \sum_{j=1}^r \left\{ E[\beta_p \beta_j] - E[\beta_p] \mu_{\beta_j} \right\} \frac{\partial h_q}{\partial \beta_j} \Big|_{\{\mu_{\beta_j}\}} \end{aligned}$$

$$= \sum_{j=1}^r \rho_{pj} \sigma_{\beta_p} \sigma_{\beta_j} \left. \frac{\partial h_q}{\partial \beta_j} \right|_{\mu_{\beta_j}}, \quad \text{for } 1 \leq p \leq m, \quad m+1 \leq q \leq n, \quad r \leq m \quad (3-6)$$

The covariance between two of the  $\beta_i$ 's, for which  $i=m+1, \dots, n$ , can be expressed as

$$\begin{aligned} \text{Cov} [\beta_p \beta_q] &= E \left\{ h_p(\mu_{\beta_1}, \dots, \mu_{\beta_r}) + \sum_{j=1}^r (\beta_j - \mu_{\beta_j}) \left. \frac{\partial h_p}{\partial \beta_j} \right|_{\mu_{\beta_j}} + \dots \right\} \\ &\quad \left\{ h_q(\mu_{\beta_1}, \dots, \mu_{\beta_s}) + \sum_{i=1}^s (\beta_i - \mu_{\beta_i}) \left. \frac{\partial h_q}{\partial \beta_i} \right|_{\mu_{\beta_i}} + \dots \right\} - E[h_p] E[h_q] \\ &= \sum_{j=1}^r \sum_{i=1}^s \rho_{ij} \sigma_{\beta_j} \sigma_{\beta_i} \left. \frac{\partial h_p}{\partial \beta_j} \right|_{\mu_{\beta_j}} \left. \frac{\partial h_q}{\partial \beta_i} \right|_{\mu_{\beta_i}}, \quad \text{for } m+1 \leq p, \quad q \leq n, \quad r, s \leq m \end{aligned} \quad (3-7)$$

The means and variances of the  $\beta_i$ ,  $i=m+1, \dots, n$  are obtained in the same way as Equations (3-3) and (3-4). Given these moments and cross moments, the mean and variance of  $R$  can be obtained using Equation (3-3) and (3-4), as they are written.

The response random process can be accurately characterized using three measures: the permanent offset, the maximum response, and the energy dissipated. Using the techniques described above, the means and variances of each random variable can be computed. The following section will present more detail about those random variables.

### 3.1 The Maximum Response Random Process

From Section 2.0, the maximum displacement response is governed by 6 random parameters and can be represented as:

$$R = g(\gamma_1, \gamma_2, \dots, \gamma_6)$$

where  $R$  = maximum displacement response

$\gamma_1$  = input peak air pressures ( $A$ ),  $\gamma_2$  = decay rate ( $\alpha$ )

$\gamma_3$  = natural frequency ( $\omega_n$ ),  $\gamma_4 = \sqrt{A_7/m}$

$\gamma_5$  = yield displacement  $u_7$ ,  $\gamma_6$  = damping ratio ( $\zeta$ )

Using Equations (3-3), (3-4), the mean and variance of  $R$  can then be computed.

### 3.2 The Permanent Offset Random Process

The relation between permanent offset,  $H$ , and maximum displacement,  $R$ , is:

$$H = (R - u_7) \left(1 - \frac{A_7}{k}\right) \quad (3-8)$$

So we can let

$$H = h(\beta_1, \beta_2, \beta_3, \beta_4)$$

where  $\beta_1 = \omega_n$ ,  $\beta_2 = \omega_y$ ,  $\beta_3 = u_7$ ,  $\beta_4 = R$ . Then the mean and variance of  $H$  can be computed using the technique as stated above.

### 3.3 The Energy Dissipated Random Process

The energy dissipated by a bilinear hysteretic system includes two parts. The first part is the energy dissipated by the spring. The second part is the energy dissipated by the damper. The system will reach its maximum offset within the first cycle. Then the energy is dissipated by the spring only during this time. After the system reaches its maximum offset, then the system remains linear in a new equilibrium configuration. The energy is then dissipated by the damper only. The energy dissipated by a linear system is related to the damper only and has already been studied (Reference 8). In this report, only energy dissipated by the spring is considered.

Refer to Figure (2-2). The energy dissipated by the spring can be computed as:

$$E_d = .5 \cdot k \cdot u_7^2 + .5 (z_{\max} - u_7)A_7 + k \cdot u_7(z_{\max} - u_7) - \frac{1}{2} k(z_{\max} - H)^2 \quad (3-9)$$

therefore  $E_d = m\left(\frac{1}{2} \omega_n^2 u_7^2 + \frac{1}{2} \omega_y^2 (z_{\max} - u_7)^2 + \omega_n^2 u_7 (z_{\max} - u_7) - \frac{1}{2} [(z_{\max} - u_7)\omega_y/\omega_n + u_7\omega_n]^2\right)$

In other words, the energy dissipated can be expressed as:

$$E_d = s(n_1, n_2, n_3, n_4), \quad (3-10)$$

where we denote  $n_1 = \omega_n$ ,  $n_2 = \omega_y$ ,  $n_3 = u_7$ ,  $n_4 = z_{\max}$ . Again using Equation (3-3), (3-4), and (3-6), the mean and variance of energy dissipated can be computed.

#### 4.0 NUMERICAL EXAMPLE

In Section 2.0 an approach for computing the maximum displacement for a bilinear hysteretic system subjected to blast type load was developed. In Section 3.0 an approach for the probabilistic analysis of the response was developed. A computer program, called VARE.F is used in this section to calculate the means and variances of the response, which includes maximum displacement, permanent offset and energy dissipated by the spring.

In Section 3.0 the technique used to evaluate the variance of a response random variable, which depends on  $n$  underlying random variables and whose statistical moments are known, refers to the partial derivatives with respect to the underlying random variables. In order to calculate the partial derivatives with respect to the underlying variables, an approximated method is used here. Let  $R$  be a random variable which depends on  $n$  underlying variables  $\gamma_i$ ,  $i=1, \dots, n$  (for example, maximum displacement which is expressed in Equation 3-1.). Then

$$R = g(\gamma_1, \gamma_2, \dots, \gamma_n) \quad (4-1)$$

and the partial derivative of  $R$  with respect to  $\gamma_i$  can be approximated

$$\frac{\partial g}{\partial \gamma_i} = \frac{g(\gamma_1, \gamma_2, \dots, \gamma_i + \Delta\gamma_i, \dots, \gamma_n) - g(\gamma_1, \gamma_2, \dots, \gamma_i - \Delta\gamma_i, \dots, \gamma_n)}{2\Delta\gamma_i} \quad (4-2)$$

This approximation can be computed even though  $g(\gamma_1, \gamma_2, \dots, \gamma_n)$  may not be a function that can be written explicitly. Writing  $g(\gamma_1, \gamma_2, \dots, \gamma_n)$  here implies that a computer program can be run using the  $\gamma_i$ ,  $i=1, \dots, n$ , as inputs to obtain a result  $R$ . The program can be run twice to obtain the above approximation. Using above approximation in the program VARE.F the first order partial derivative with respect to any given underlying random variable can be calculated. The maximum displacement, permanent offset and energy dissipated are considered in the following section.

##### 4.1 The Maximum Displacement

It is mentioned in Section 3.1 that the maximum displacement is related to six random parameters. The functional expression is

$$R = g(\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6)$$

where  $R$  is maximum displacement,  $\gamma_1$  = input peak air pressure  
 $\gamma_2$  = decay rate of input force,  $\gamma_3$  = natural frequency,  $\omega_n$   
 $\gamma_4 = \sqrt{A_7/m}$   $\gamma_5$  = damping ratio,  
 $\gamma_6$  = yield displacement,  $u_y$

The means and standard deviations of the above six random variables are given in Table 4.1.

Table 4.1

The Means and Standard Deviation of  $\gamma_i$ ,  $i=1,6$

$\gamma_i$	Mean Value, $\mu_{\gamma_i}$	Standard Deviation, $\sigma_{\gamma_i}$
$i=1$	0.121428e+04	.1 $\mu_{\gamma_1}$
$i=2$	0.120000e+01	.1 $\mu_{\gamma_2}$
$i=3$	0.358568e+02	.1 $\mu_{\gamma_3}$
$i=4$	0.113389e+02	.1 $\mu_{\gamma_4}$
$i=5$	0.119523e-01	.2 $\mu_{\gamma_5}$
$i=6$	0.100000e+01	.2 $\mu$

The covariances between random variable pairs in  $\gamma_i$ ,  $i=1,6$  are given in Table 4.2.



Table 4.2  
The Covariance Matrix

$.01 \mu_1^2$	$.001 \mu_1 \mu_2$	0.	0.	0.	0.
	$.01 \mu_2^2$	0.	0.	0.	0.
		$.01 \mu_3^2$	$.001 \mu_3 \mu_4$	$.001 \mu_3 \mu_5$	$.0001 \mu_3 \mu_6$
(symmetric)			$.01 \mu_4^2$	0.	0.
				$.04 \mu_5^2$	0
					$.04 \mu_6^2$

The  $\partial R / \partial \gamma_i$ ,  $i=1,6$  are computed using Equation and the computer program VARE.F. The results are shown in Table 4.3.

TABLE 4.3  
Partial Derivative of Peak Response

i	$\partial R / \partial \gamma_i$
i=1	0.57023e-02
i=2	-0.36344e+00
i=3	-0.17445e+00
i=4	-0.34095e-01
i=5	-0.13495e+02
i=6	-0.42679e+01

Using Equation 3-3 and the computer program VARE.F, the mean of maximum displacement is computed. The results is:

$$E[R] = 2.65 \text{ (in)}$$

Using Equation 3-4 and the computer program VARE.F, the variance of maximum displacement is computed. Each term in Equation 3-4 is listed in Table 4.4.

Table 4.4.  
The Terms in Variance R

.47943+00	-.302e-02	0.	0.	0.	0.
	.1902e-02	0.	0.	0.	0.
		.3913e+00	-.2418e-02	.2017e-02	.5339e-02
			.1494e-02	0.	0.
(symmetric)				.1041e-02	0.
					.7268e+00

The variance of maximum displacement is the sum of every term as shown in Table 4.4. The result is

$$\text{Var}[R] = 1.617 \text{ (in}^2\text{)}$$

Table 4.4 shows that only the variance of input peak air pressure, natural frequency and yield displacement have significant effect on the variance of maximum displacement. The rest of the terms are almost negligible.

#### 4.2 The Permanent Offset

In Section 3.2, the permanent offset is expressed using Equation 3-8. Let  $H$  represent the permanent offset; then

$$H = h(s_1, s_2, s_3, s_4)$$

where  $H$  = permanent offset,

$s_1$  = natural frequency,  $\omega_n$

$s_2$  =  $\sqrt{A_7/m}$

$s_3$  = yield displacement,  $u_7$

$s_4$  = maximum displacement,  $R$

The means and standard deviations of  $s_i$ ,  $i=1,4$  are given in Table 4.5.

Table 4.5  
The Means and Standard Deviations of  $\beta_i, i=1,4$

$i$	Mean $\mu_{\beta_i}$	Standard Deviation $\sigma_{\beta_i}$
i=1	0.358569e+02	.1 $\mu_{\beta_1}$
i=2	0.113389e+02	.1 $\beta_{\beta_2}$
i=3	0.100000e+01	.2 $\mu_{\beta_3}$
i=4	0.265600e+01	.12717

Again using Equation 3-3 and the computer program VARE.F, the mean value of H is evaluated and the result is

$$E[H] = 1.4907 \text{ (in)}$$

The first order derivatives of H with respect to  $\beta_i, i=1,4$  are computed directly from Equation (3-8) and are shown in Table 4.6.

Table 4.6  
Partial Derivative of H

$i$	i=1	i=2	i=3	i=4
$\partial H / \partial \beta_i$	0.92387e-02	-0.29215e-01	-0.90000e+00	0.90000e+00

The covariance between  $\beta_i$  and  $\beta_j, i=1,3$  is computed using Equation (3-6). The covariance between two  $\beta_i, i=1,3$  is a known quantity. Therefore, when the variance of permanent offset is expressed by the form of Equation (3-4), in which g represents permanent offset at this stage, each term in Equation (3-4) is computed and is listed in Table 4.7.

Table 4.7  
The Terms in Variance H

.0011	- .00011	-.00006	0.0191
	.0011	0	.00302
		.0324	.1393
(symmetric)			1.13100

The variance of H is the sum of every term of Table 4.7; the result is  
 $\text{Var} (H) = 1.5906 \text{ (in}^2\text{)}$

From Table 4.7, it is observed that the maximum displacement has great influence on the variance of permanent offset. The covariance between maximum displacement and yield displacement has less influence, and the rest of the terms are almost negligible.

#### 4.3 The Energy Dissipated

In Section 3.3, the energy dissipated in a bilinear hysteretic system is separated into two parts. The first part is energy dissipated in the spring and the second part is energy dissipated in the damper. Here, only the energy dissipated by the spring is considered. From Equation 3-9, the energy dissipated by the spring can be expressed as

$$E_d = S (n_1, n_2, n_3, n_4)$$

where  $n_1 = \omega_n$ ,  $n_2 = \sqrt{A_7}/m$ ,  $n_3 = u_7$ ,  $n_4 = R$ .

The means and variances of the  $n_i$ ,  $i=1,4$  are the same as given in Table 4.5. The first order derivatives of  $E_d$  with respect to  $n_i$ ,  $i=1,4$  are obtained directly from the expression of  $E_d$  which is given in Equation (3-10). The results are

Table 4.8  
 The Derivatives of  $E_d$

$i$	$i=1$	$i=2$	$i=3$	$i=4$
$\partial E_d / \partial n_i$	.838372e+03	- .887303e+02	.56783e+03	.944165e+04

The mean value of  $E_d$  is computed by using Equation (3-3) and computer program VARE.F and the result is  $E(E_d) = .161578e+05 \text{ (lb-in)}$ . When using Equation (3-4) to compute the variance of  $E_d$ , the variable  $g$  in Equation (3-4) represents  $E_d$  at this stage. Each element in Equation 3-4 can be obtained using the approach of Section 4-2. Table 4.9 lists each term of Equation (3-4).

Table 4.9  
The Terms in Variance  $E_d$

.903687e+07	-.302450e+05	.341400e+04	-.181982e+08
	.101225e+05	0	.961467e+05
		.128976e+05	-.921978e+06
			.144175e+09

The variance of  $E_d$  is the sum of every element of Table 4.9. The result is

$$\text{Var}(E_d) = 0.115133e+09 \text{ (lb}^2\text{-in}^2\text{)}$$

From Table 4.9, it is observed that the maximum displacement has great influence on the variance of  $E_d$ , and the rest of the terms are almost negligible.

## 5.0 SUMMARY

The objectives of this study were, first, to develop a model that characterizes the permanent offset of an SDF, bilinear hysteretic system subjected to blast type load; and second, to probabilistically characterize the features of the inelastic structural response. In this report, the response characteristics considered were: (1) maximum displacement response (2) permanent offset (3) energy dissipated by the inelastic spring. The statistical properties characterizing these measures of response are the mean and variance.

In Section 2, the maximum displacement response was computed. The response random process was then established. Section 3 discussed the techniques of probabilistic analysis of a complicated function. Since the response random process which was established in Section 2 could not be expressed in a closed form, the computer program VARE.F was developed to compute the moments of critical measures of inelastic response.

Several numerical results were shown in Section 4. One of the important results showed that the maximum displacement response has great influence on the variance of permanent offset and energy dissipated. The results developed in this study are restricted to blast type inputs, and bilinear hysteretic, SDF systems.

The results developed in this report can be used in the probabilistic design process. For example, the computer program VARE.F may be used to determine the mean and variance of some critical response measures and the designer may determine if the response satisfies certain design criteria. Or, the response may be restricted to some extreme level, then the system parameters may be determined by using computer program VARE.F with a trial and error method. Also we can predict the mean and variance of the response in an assessment of an existing building.

Future study might apply the present techniques to multi-degree-of-freedom systems. Or response models that characterize response to other than blast inputs might be sought.

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APPENDIX

COMPUTER PROGRAM VARE.F



```

C*****
C
C
C
C this program compute the var [ED] and E [ED]
C
C Equation of motion :
C  $m_7 y'' + c_7 y' + k_7 (y - 0) = f_m \exp(-\alpha t)$ 
C or
C  $y'' + 2 \zeta \omega_n y' + \omega_n^2 (y - 0) = A \exp(-\alpha t)$ 
C where :
C  $g$  is  $z_{\text{-max}} = G(\text{bata1}, \text{bata2}, \dots, \text{bata6})$ 
C  $\text{bata1} = A$ , and  $x_{\mu}(1) = E[\text{bata1}]$ 
C  $\text{bata2} = \alpha$ , and  $x_{\mu}(2) = E[\text{bata2}]$ 
C  $\text{bata3} = \omega_n$ , and  $x_{\mu}(3) = E[\text{bata3}]$ 
C  $\text{bata4} = \omega_{n1}$ , and  $x_{\mu}(4) = E[\text{bata4}]$ 
C  $\text{bata5} = \zeta$ , and  $x_{\mu}(5) = E[\text{bata5}]$ 
C  $\text{bata6} = v_7$ , and  $x_{\mu}(6) = E[\text{bata6}]$ 
C
C and :
C  $x_{\rho}(i, j)$  are coefficient of correlation between  $\text{bata}(i)$ 
C  $x_{\sigma}(i)$  are corresponding STANDARD DEVIATION
C  $x_{\text{var}}(i, j)$  are variance matrix
C  $pg_{wb}(i) = dG / d\text{bata}(i)$ 
C
C offset is the max offset of the response, ie:
C  $\text{offset} = H(\text{gamma1}, \text{gamma2}, \text{gamma3}, \text{gamma4})$ 
C  $\text{gamma1} = G$  and  $z_{\mu 1} = E[\text{gamma1}]$ 
C  $\text{gamma2} = \omega_{y1}$  and  $z_{\mu 2} = E[\text{gamma2}]$ 
C  $\text{gamma3} = \omega_n$  and  $z_{\mu 3} = E[\text{gamma3}]$ 
C  $\text{gamma4} = v_7$  and  $z_{\mu 4} = E[\text{gamma4}]$ 
C and  $z_{\rho}(i, j)$ ,  $z_{\sigma}(i)$ ,  $z_{\text{var}}(i, j)$ ,  $ph(i)$  represent the same
C characteristic as in  $x_{\mu}$ 
C
C In energy dissipation part ,
C  $smu(i)$  are related to the spring
C therefore :
C  $smu(1) = E[\omega_n]$ 
C  $smu(2) = E[\omega_{y1}]$ 
C  $smu(3) = E[v_7]$ 
C  $smu(4) = E[z_{\text{max}}]$ 
C The  $s_{\text{var}}(i, j)$ ,  $s_{\rho}(i, j)$ ,  $pes(i)$  are corresponding
C variance matrix, coeff. of correlation, and first order
C derivative w.r.p to those parameter.
C
C  $dmu(i)$  are related to the damping
C Therefore :
C  $dmu(1) = a$ 
C  $dmu(2) = \alpha$ 
C  $dmu(3) = \omega_n$ 
C  $dmu(4) = \zeta$ 
C  $dmu(5) = \theta$ 
C

```

```

c
c *****
c
c dimension xmu(6),xsigma(6),xrho(6,6),xvar(6,6),pgwb(6)
c dimension zmu(4),zsigma(4),zrho(4,4),zvar(4,4),ph(4)
c dimension smu(4),ssigma(4),srho(4,4),svar(4,4),pes(4)
c dimension dmu(5),dsigma(5)
c dimension ff(1024),offset(1024),try1(1000),qq1(1000),v(1024)
c real m7, m7
c
c define input parameter
c   fm=8500.
c   c7=6.
c   m7=7.
c   k7=9000.
c   a7=.1*k7
c   v7=1.
c   dt=0.01
c   nb=1024
c   a=fm/m7
c   alpha=1.2
c   wn=(k7/m7)**.5
c   wyl=(a7/m7)**.5
c   zita=c7/(2.*m7*wn)
c
c
c   do 80 i=1,nb
c     tx=(i-1)*dt
c     ff(i)=fm*exp(-alpha*tx)
30 continue
c
c define the moment value of random parameter
c   xmu(1)=a
c   xmu(2)=alpha
c   xmu(3)=wn
c   xmu(4)=wyl
c   xmu(5)=zita
c   xmu(6)=v7
c
c   xsigma(1)=.1*a
c   xsigma(2)=0.1*xmu(2)
c   xsigma(3)=0.1*xmu(3)
c   xsigma(4)=0.1*xmu(4)
c   xsigma(5)=.2*zita
c   xsigma(6)=.2*v7
c
c   xrho(1,2)=.1
c   xrho(3,4)=.1
c   xrho(3,5)=.1
c   xrho(3,6)=.01
c   do 90 i=1,6
c     do 90 j=1,6
c       if (i.eq.j) go to 91

```

```

        xrho(j,i)=xrho(i,j)
        go to 90
91  xrho(i,j)=1.
90  continue
C
C
C
C  find the value of q which is the parameter to be identify
C  in offset
        call sbilin (FF,c7,m7,k7,a7,v7,dt,nb,v,offset,ened)
        thcon=offset(nb)
        write (6,10) ened, thcon
10  format ('Energy dissipated and offset from sbilin is ',
C      e13.6,2x,f9.5)
C
C
C  In order to prevent offset oscillating, check if the max offset
C  equal to perment offset , if not, it means that the forcinG
C  function , or the system itself , does not practical.
C
        offmax=0.
        nbl=nb-1
        do 75 i=1,nbl
        offmax=amax1 (offmax,offset(i))
75  continue
        if (offmax.eq.0.) go to 3000
        if (offmax.ne.thcon) go to 3001
C
C
C  identify the parameter q1 and q2
        q1=1000.
        qx=q1
        p=thcon*2./3.1415926
        diff=1.
        cycle=1.
C
1001 call search (offset,p,q1,nb,dt,epsiln)
        try1(1)=epsiln
        ku=1
        qql(ku)=q1
C
        dq1=qx/(diff*10.7)
C
1021 q1=q1+dq1
        call search (offset,p,q1,nb,dt,epsiln)
        ku=ku+1
        qql(ku)=q1
        try1(ku)=epsiln
        if (try1(ku).lt.try1(ku-1)) go to 1021
        if (ku.ge.3) go to 2000
C
        q1=q1-dq1
1010 q1=q1-dq1

```

```
t,p,ql,nb,dt,epsiln)
```

```
y1(ku-1)) go to 2000
```

```
DO TO 1001
```

```
1.6)
```

```
*****  
var[G]
```

```
*****  
2),xmu(3),xmu(4),xmu(5),xmu(6),  
,thir)
```

```
(i),xsigma(i),i=1,6)
```

```
! = zmax is ',f16.9)  
!s :',6x,'xsigma[i] is '
```

```
!f,k7,a7,dt,nb,pmox,pg)
```

```
o(i),i=1,6)  
!ta is :')
```

```
xsigma(iqq)  
xrho(ipp,iqq).eq.0.) go to 99  
o to 98  
o to 97
```



```

ph(3)=(pmr-v7)*2.*wyl**2/wn**3
ph(4)=-1.*(1.-(wyl/wn)**2)
c
zmu(1)=pmr
zmu(2)=wyl
zmu(3)=wn
zmu(4)=v7
c
zsigma(1)=varx**.5
zsigma(2)=xsigma(4)
zsigma(3)=xsigma(3)
zsigma(4)=xsigma(6)
write (6,55)
write (6,56)(zmu(i),zsigma(i),i=1,4)
write (6,27) off
27 format ('E [offset] = ',f15.8)
55 format (' zmu(i) is :',6x,'zsigma is :')
56 format (e14.6,3x,e14.6)
c
c
c
c
define zrho(i,j)
zrho(2,3)=xrho(3,4)
zrho(3,4)=xrho(3,6)
do 320 i=1,4
do 320 j=1,4
if (i.eq.j) go to 321
zrho(j,i)=zrho(i,j)
go to 320
321 zrho(i,j)=1.
320 continue
c
c
c compute cov[G wyl], cov[G wn], cov[G v7], E[G**2]
do 350 i=1,4
if (i.eq.1) go to 351
call egr (xmu,xsigma,xrho,pgwb,zmu(i),epara)
zvar(1,i)=epara*ph(1)*ph(i)
go to 350
351 zvar(1,i)=(zsigma(1)*ph(1))**2
350 continue
c
c
do 400 i=2,4
do 400 j=2,4
if (i.gt.j) go to 400
zvar(i,j)=zrho(i,j)*zsigma(i)*zsigma(j)*ph(i)*ph(j)
400 continue
do 410 i=1,4
do 410 j=1,4
zvar(j,i)=zvar(i,j)
410 continue
write (6,28)

```

```

write (6,29) (ph(i),i=1,4)
write (6,30)
write (6,31) ((zvar(i,j),j=1,4),i=1,4)
28 format (/ 'ph(i) is :')
29 format (e16.7)
30 format (/ 'Z-variance matrix is :')
31 format (4f15.6)
c
varof=0.
do 500 i=1,4
do 500 j=1,4
varof=varof+zvar(i,j)
500 continue
c
write (6,32) varof
32 format (/ 'Var [offset] = ',f16.8)
c
c
c*****
c
c
c
c
c compute E [ED]
c ED : ENERGY DIssipated
c ED1= dissipate by spring
c ED2= dissipate by damping
c
c*****
c
c
c define mean value and standard deviation
smu(1)=wn
smu(2)=wyl
smu(3)=v7
smu(4)=pmr
c
ssigma(1)=xsigma(3)
ssigma(2)=xsigma(4)
ssigma(3)=xsigma(6)
ssigma(4)=varx**.5
c
dmu(1)=a
dmu(2)=alpha
dmu(3)=wn
dmu(4)=zita
dmu(5)=off
c
dsigma(1)=xsigma(1)
dsigma(2)=xsigma(2)
dsigma(3)=xsigma(3)
dsigma(4)=xsigma(5)
dsigma(5)=varof**.5
c

```

```

call eng1(smu,m7,ed1)
call eng2(dmu,nb,q,m7,ed2)
write (6,60)
write (6,61) (smu(i),dmu(i),i=1,4)
write (6,62) dmu(5)
write (6,33) ed1,ed2
60 format (' smu(i) is ',8x,'dmu(i) is ')
61 format (2e15.6)
62 format (15x,e15.6)
33 format (/'E[Ed1] is ',e15.6,' E[Ed2] is ',e13.6)

c
c
c
c
c
c
compute var [Ed]
do 600 cn=1,4
call peng1(cn,smu,m7,ped1)
icn=cn
pes(icn)=ped1
600 continue
write (6,34)
write (6,35) (pes(i),i=1,4)
34 format (/'pes(i) is :')
35 format (e13.6)

c
c
c
define srho(i,j),j=1,3 i=1,3
c where srho(i,j) is the coefficient of correlation
c in spring energy dissipate case
c
srho(1,2)=xrho(3,4)
srho(1,3)=xrho(3,6)
do 650 i=1,4
do 650 j=1,4
if (i.eq.j) go to 651
srho(j,i)=srho(i,j)
go to 650
651 srho(i,j)=1.
650 continue

c
c
do 700 i=1,3
do 700 j=1,3
if (i.gt.j) go to 700
svar(i,j)=srho(i,j)*ssigma(i)*ssigma(j)*pes(i)*pes(j)
700 continue

c
svar(4,4)=varx*pes(4)**2

c
do 800 i=1,3
call egr(xmu,xsigma,xrho,pgwb,smu(i),covhs)
svar(i,4)=covhs*pes(i)*pes(4)

```



```

800 continue
c
c
do 900 i=1,5
do 900 j=1,5
svar(j,i)=svar(i,j)
900 continue
write (6,36)
write (6,37)((svar(i,j),j=1,4),i=1,4)
36 format ('S-variance matrix is :')
37 format (4e15.6)
c
do 920 i=1,4
do 920 j=1,4
svar1=svar1+svar(i,j)
920 continue
c
write (6,38) svar1
38 format ('var [ED1] IS ',e13.6)
go to 6000
3000 WRITE (6,18)
18 FORMAT (' NO YIELDING OCCUR')
GO TO 6000
3001 WRITE (6,19)
19 FORMAT (' THIS CASE DOEN NOT PRACTICAL')
c
6000 STOP
END

c
c
c
subroutine pgl(icn,xmu,ff,k7,a7,dt,nb,pmo,pg)
c This subroutine compute dG/db, denote it as pg ;
c Where b is random parameter.
c cn = control number ;
c if :
c cn=1. : compute dG/dA
c =2. : compute dG/dalpha
c =3. : compute dG/dwn
c =4. : compute dG/dwyl
c =5. : compute dG/dzita
c =6. : compute dG/dv7
c
dimension ff(1024),xmu(6)
real K7,M7,k71,k72
c
a=xmu(1)
alpha=xmu(2)
wn=xmu(3)
wyl=xmu(4)
zita=xmu(5)

```

```

v7=xmu(6)
c
c   read (5,*) error
error=500.
c
if (icn.eq.6) go to 6
if (icn.eq.5) go to 5
if (icn.eq.4) go to 4
if (icn.eq.3) go to 3
if (icn.eq.2) go to 2
if (icn.eq.1) go to 1
go to 4999
c
1 m7=k7/(wn**2)
fm=a*m7
dfm=fm/error
da=dfm/m7
al=a+da
a2=a-da
call g(al,alpha,wn,wyl,zita,v7,ff,k7,a7,dt,nb,spmo,thir)
c
write (6,22) thir
c
gl=spmo
call g(a2,alpha,wn,wyl,zita,v7,ff,k7,a7,dt,nb,spmo,thir)
g2=spmo
pg=(gl-g2)/(2.*da)
pmo=gl
go to 5000
c
2 dalpha=alpha/error
alphan=alpha+dalpha
alpha2=alpha-dalpha
call g(a,alphan,wn,wyl,zita,v7,ff,k7,a7,dt,nb,spmo,thir)
c
write (6,22) thir
c
gl=spmo
call g(a,alpha2,wn,wyl,zita,v7,ff,k7,a7,dt,nb,spmo,thir)
g2=spmo
pg=(gl-g2)/(2.*dalpha)
pmo=gl
go to 5000
c
3 dk7=k7/error
k71=k7+dk7
k72=k7-dk7
m7=a7/(wyl**2)
wn1=(k71/m7)**.5
wn2=(k72/m7)**.5
dwn=wn1-wn2
call g(a,alpha,wn1,wyl,zita,v7,ff,k71,a7,dt,nb,spmo,thir)
c
write (6,22) thir
c
gl=spmo

```

```

call g(a, alpha, wn2, wyl, zita, v7, ff, k72, a7, dt, nb, spmo, thir)
g2=spmo
pg=(g1-g2)/(2.*dwn)
pmo=g1
go to 5000
c
4 da7=a7/error
a71=a7+da7
a72=a7-da7
m7=k7/(wn**2)
wyl1=(a71/m7)**.5
wyl2=(a72/m7)**.5
dwyl=wyl1-wyl2
call g(a, alpha, wn, wyl1, zita, v7, ff, k7, a71, dt, nb, spmo, thir)
c
c write (6,22) thir

g1=spmo
call g(a, alpha, wn, wyl2, zita, v7, ff, k7, a72, dt, nb, spmo, thir)
g2=spmo
pg=(g1-g2)/(2.*dwyl)
pmo=g1
go to 5000
c
5 dzita=zita/error
zital=zita+dzita
zita2=zita-dzita
call g(a, alpha, wn, wyl, zital, v7, ff, k7, a7, dt, nb, spmo, thir)
c
c write (6,22) thir

g1=spmo
call g(a, alpha, wn, wyl, zita2, v7, ff, k7, a7, dt, nb, spmo, thir)
g2=spmo
pg=(g1-g2)/(2.*dzita)
pmo=g1
go to 5000
c
6 dv7=v7/error
v71=v7+dv7
v72=v7-dv7
call g(a, alpha, wn, wyl, zita, v71, ff, k7, a7, dt, nb, spmo, thir)
c
c write (6,22) thir

g1=spmo
call g(a, alpha, wn, wyl, zita, v72, ff, k7, a7, dt, nb, spmo, thir)
g2=spmo
pg=(g1-g2)/(2.*dv7)
pmo=g1
go to 5000
c
22 format ('thita ratio is ',f13.7)
4999 write (6,19)
19 format ('Control Number Wrong')
c

```

```
5000 return
      end
```

```
C
C
```

```
      subroutine g(a,alpha,wn,wyl,zita,v7,ff,k7,a7,dt,nb,pmr,thir)
C This subroutine is to predict the max offset for blast load
C pmo=predicted max response
C pmr=100: represent jmm expired (in do loop)
C pmr=101: represent something trouble in finding tmax
C
```

```
      dimension ff(1024),VV(1024),OFFSET(1024),SLOPE(30)
      REAL K7,M7
      WD=((1.-ZITA**2)**.5)*WN
      m7=k7/(wn**2)
      c7=2.*zita*wn*m7
      V=-ZITA*WN
      F=ALPHA+V
```

```
C
C
```

```
      wy=(wyl**2-(zita*wn)**2)**.5
      D=(WN**2-WY1**2)*V7
```

```
C
C
```

```
      call sbilin (ff,c7,m7,k7,a7,v7,dt,nb,vv,offset,ened)
```

```
C
C
```

```
C
```

```
      CALCULATE T7
```

```
      B=2.*ALPHA/(WD**2+ALPHA**2)
      C=2.*V7/A
      T7=(B+(B**2+4.*C)**.5)/2.
      T71=T7
```

```
C
```

```
      WRITE (6,1) T7
```

```
C
```

```
1      FORMAT ('THE TEMPERATORY T7 = ',F11.7)
```

```
C
```

```
C
```

```
      CALCULATE EXACT T7
```

```
      DO 700 JII=1,2000
      XX=JII*.00001
      T7=T71+XX
      X1=((ALPHA-ZITA*WN)**2+WD**2)*WD
      X2=A/X1
      X3=EXP (-ZITA*WN*T7)
      X4=(ALPHA-ZITA*WN)*SIN(WD*T7)
      X5=WD*COS(WD*T7)
      X6=WD*EXP (-ALPHA*T7)
```

```
C
```

```
C
```

```
      CHECK Z AT T7 IF IT IS EQUAL TO V7
```

```
      ZT7=X2*(X3*(X4-X5)+X6)
      ERR1=ABS(1.-ZT7/V7)
      IF (ERR1.LE.0.0005) GO TO 701
700 CONTINUE
```

```
C
```

```
C
```

```
      compute the z'(t7)
```

```
701 X7=WD*(ALPHA-ZITA*WN)*COS(WD*T7)
```

```

X8=WD**2*SIN(WD*T7)
X9=-ZITA*WN*X3*(X4-X5)
ZDOT7=X2*(X9+X3*(X7+X8)-ALPHA*X6)

```

C  
C

C calculate tmax (using approached method)

```

rx=2.*zita*wn
g1=a*(1.-exp(-rx*t7))
g2=(wn**2)*v7*(exp(-rx*t7)-1.)
gx=-(g1+g2)+zdot7*rx

```

C

```

h1=a*exp(alpha*t7)
h2=a*rx/(alpha-rx)+(wn**2)*v7
h3=a*alpha*exp(t7*(alpha-rx))/(alpha-rx)
h4=gx*exp(rx*t7)
h5=h1-h3
h6=h2+h4

```

C

```

p1=h5*alpha**2+h6*(rx**2)
p2=h5*alpha+h6*rx
p3=h5+h6-(wn**2)*v7

```

C

```

ux=p2**2-4.*p1*p3
if(ux.le.0.) go to 2700
uu=ux**.5
tmax1=(p2+uu)/(2.*p1)
tmax2=(p2-uu)/(2.*p1)
if (tmax1.le.0.17.and.tmax1.ge.0.) go to 70
tmax=tmax2
go to 71

```

70 tmax=tmax1

71 tmaxt=tmax

C

C

C compute c1 and c2

```

q1=-d/(wyl**2)

```

C

```

qx1=alpha**2
qx2=-2.*zita*wn*alpha
qx3=wyl**2
q2=a/(qx1+qx2+qx3)

```

C

```

xal=exp(-zita*wn*t7)
xa2=-zita*wn*cos(wy*t7)
xa3=-wy*sin(wy*t7)
xat7=xal*(xa2+xa3)

```

C

```

xbl=-zita*wn*sin(wy*t7)
xb2=wy*cos(wy*t7)
xbt7=xal*(xbl+xb2)

```

C

```

xct7=-alpha*q2*exp(-alpha*t7)

```

C

```

xdt7=xal*cos(wy*t7)
c
xet7=xal*sin(wy*t7)
c
xft7=q1+q2*exp(-alpha*t7)
c
xgt7=v7-xft7
c
xht7=zdot7-xct7
c
deno=xat7*xet7-xdt7*xbt7
c1=(xht7*xet7-xbt7*xgt7)/deno
c2=(xat7*xgt7-xdt7*xht7)/deno
c
c compute exact tmax, since z'=0 when t=tmax
tmx=tmaxt
dtx=tmaxt
880 tx=dtx/30.
do 900 jmm=1,30
tmx=tmx-tx
xal=exp(-zita*wn*tmx)
xa2=-zita*wn*cos(wy*tmx)
xa3=-wy*sin(wy*tmx)
xatmx=xal*(xa2+xa3)
c
c
xbl=-zita*wn*sin(wy*tmx)
xb2=wy*cos(wy*tmx)
xbtmx=xal*(xbl+xb2)
c
xctmx=-alpha*q2*exp(-alpha*tmx)
c
zdtmx=xatmx*c1+xbtmx*c2+xctmx
slope(jmm)=zdtmx
if (jmm.eq.1.and.slope(jmm).gt.0.) go to 899
if (jmm.eq.1) go to 900
prod=slope(jmm)*slope(jmm-1)
if (prod.le.0.) go to 902
900 continue
write (6,10)
10 format ('something WRONG in t-max[subG]')
9C2 if (slope(jmm-1).gt.-.0001) go to 901
dtx=dtx/30.
tmx=tmx+tx
go to 880
c
899 do 920 jmm=1,1000
tx=(jmm-1)*0.0001
tmx=tmaxt+tx
xal=exp(-zita*wn*tmx)
xa2=-zita*wn*cos(wy*tmx)
xa3=-wy*sin(wy*tmx)

```

```

      xatmx=xal*(xa2+xa3)
C
C
      xbl=-zita*wn*sin(wy*tmx)
      xb2=wy*cos(wy*tmx)
      xbtmx=xal*(xbl+xb2)
C
      xctmx=-alpha*q2*exp(-alpha*tmx)
C
      zdtmx=xatmx*c1+xbtmx*c2+xctmx
      if (zdtmx.le.0.01) go to 901
920  continue
      go to 2100
C
C compute exact z(at exact tmax)
901  xdtmx=xal*cos(wy*tmx)
      xetmx=xal*sin(wy*tmx)
      xftmx=q1+q2*exp(-alpha*tmx)
      ztmx=xdtmx*c1+xetmx*c2+xftmx
C
C calculate predicted max-offset
      pmo=(ztmx-v7)*(1.-a7/k7)
      thir=pmo/offset(nb)
      pmr=ztmx
C
      go to 3000
C
2100 write (6,98)
      98 format ('fmm expire')
      pmr=100.
      go to 3000
2700 write (6,99)
      99 format ('U less than zero')
      pmr=101.
3000 return
      END
C
      SUBROUTINE SBILIN(F,C7,M7,K7,A7,V7,D9,N,V,VO,ened)
      DIMENSION F(1024),VO(1024),V(1024)
      DIMENSION V1(1024),V2(1024)
      REAL M7,K7,K8,K9
      U7=K7*V7
      K9=1.-A7/K7
C
C INITIALIZE VARIABLES
      V(1)=0.
      VO(1)=0.
      V1(1)=0.
      V2(1)=F(1)/M7
C START THE RESPONSE CYCLE
      Q1=6.*M7/D9**2
      Q2=3.*C7/D9
      Q3=6.*M7/D9

```

```

Q4=3.*M7
Q5=3.*C7
Q6=0.5*D9*C7
Q7=3./D9
Q8=3.
Q9=.5*D9
K8=K7
NM=N-1
ENED=0.
DO 1199 I=1,NM
  I1=I+1
  U1=Q1+Q2+K8
  U2=Q3*V1(I)+Q4*V2(I)
  U3=Q5*V1(I)+Q6*V2(I)
  V5=(F(I1)-F(I)+U2+U3)/U1
  V6=Q7*V5-Q8*V1(I)-Q9*V2(I)
  V(I1)=V(I)+V5
  V1(I1)=V1(I)+V6
C COMPUTE THE STIFFNESS AT T+DT
  XO=K7*(V(I1)-VO(I))
  X1=A7*(V(I1)-V7)+U7
  X2=A7*(V(I1)+V7)-U7
  IF (XO.GT.X1) GO TO 1150
  IF (XO.LT.X2) GO TO 1160
  K8=K7
  VO(I1)=VO(I)
  GO TO 1170
1150 IF (V1(I1).GT.O.) K8=A7
  IF (V1(I1).LE.O.) K8=K7
  VO(I1)=(V(I1)-V7)*K9
  GO TO 1170
1160 IF (V1(I1).LT.O.) K8=A7
  IF (V1(I1).GE.O.) K8=K7
  VO(I1)=(V(I1)+V7)*K9
1170 V2(I1)=(F(I1)-C7*V1(I1)-K7*(V(I1)-VO(I1)))/M7
  ened=ened+d9*.5*(v1(i)*(f(i)-m7*v2(i))+
c      v1(i1)*(f(i1)-m7*v2(i1)))
1199 CONTINUE
  RETURN
  END

```

```

c
c
c      subroutine fft (a,n,nb,isgn,dt)
c      complex a(nb),u,w,t
c
c      dividing all element by nb
c      do 1 j=1,nb
1  a(j)=a(j)/nb
c
c      reorder sequencer
c      nbd2=nb/2
c      nbm1=nb-1
c      j=1

```



```

do 4 l=1,nbml
if (l.ge.j) go to 2
t=a(j)
a(j)=a(l)
a(l)=t
2 k=nbdl
3 if (k.ge.j) go to 4
j=j-k
k=k/2
go to 3
4 j=j+k
c calculate fft
pi=3.1415926
do 6 m=1,n
u=(1.0,0.0)
ME=2**M
k=me/2
w=cplx(cos(pi/k), isgn*sin(pi/k))
do 6 j=1,k
do 5 l=j,nb,me
lpk=l+k
t=a(lpk)*u
a(lpk)=a(l)-t
5 a(l)=a(l)+t
6 u=u*w
c
tt=(nb-1)*dt
c
if (isgn.eq.1) go to 99
do 110 i=1,nb
a(i)=a(i)*tt
110 continue
go to 100
c
99 do 120 i=1,nb
a(i)=a(i)/dt
120 continue
100 return
end
c
c
c
subroutine engl (smu,m7,endi)
c this subroutine compute the E [ED1]
dimension smu(4)
real k7,m7
k7=(smu(1)**2)*m7
a7=(smu(2)**2)*m7
v7=smu(3)
zmax=smu(4)
thita=(1.-a7/k7)*(zmax-v7)
c
xl=.5*k7*v7**2

```

```
x2=a7*(zmax-v7)**2/2.  
x3=k7*v7*(zmax-v7)  
x4=k7*(zmax-thita)**2/2.
```

```
c  
end1=x1+x2+x3+x4  
return  
end
```

```
c  
c  
c
```

```
subroutine eng2 (dmu,nb,q,m7,ed2)  
c this subroutine compute E [ED2]  
dimension dmu(5), f1(1024), f2(1024), w(1024), deno(512)  
complex ff1(1024), ff2(1024), ff(512), hh(512)  
real m7
```

```
c  
a=dmu(1)  
alpha=dmu(2)  
wn=dmu(3)  
zita=dmu(4)  
thita=dmu(5)  
p=thita*2./3.1415926
```

```
c  
dt=0.01  
tt=(nb-1)*dt
```

```
c  
do 100 i=1,1024  
tx=(i-1)*dt  
f1(i)=a*exp(-alpha*tx)  
f2(i)=wn**2*p*atan(q*tx**4)  
ff1(i)=cplx (f1(i),0.)  
ff2(i)=cplx (f2(i),0.)  
w(i)=(i-1)*2.*3.1415926/tt  
100 continue
```

```
c  
call fft(ff1,10,nb,-1,dt)  
call fft(ff2,10,nb,-1,dt)
```

```
c  
nb2=nb/2  
do 200 i=1,nb2  
ff(i)=ff1(i)+ff2(i)  
re=wn**2-w(i)**2  
xim=-2.*zita*wn*w(i)  
deno(i)=re**2+xim**2  
hh(i)=cplx (re,xim)/deno(i)  
200 continue
```

```
c  
end2=0.  
do 400 i=1,nb2  
xe1=cabs(hh(i))  
xe2=cabs(ff(i))  
ed2=ed2+(xe1*xe2*w(i))**2  
400 continue
```

```
ed2=2.*ed2
return
end
```

```
c
c
c
```

```
subroutine pengl(cn,smu,m7,ped1)
c this subroutine compute d ED1/d bata
c cn =1 d ED1/d wn
c cn =2 d ED1/d wyl
c cn =3 d ED1/d v7
c cn =4 d ED1/d zmax
c
```

```
dimension smu(4)
real m7
wn=smu(1)
wyl=smu(2)
v7=smu(3)
zmax=smu(4)
const=(zmax-v7)*wyl**2/wn+v7*wn.
```

```
c
```

```
if (cn.eq.1.) go to 100
if (cn.eq.2.) go to 200
if (cn.eq.3.) go to 300
if (cn.eq.4.) go to 400
go to 600
```

```
c
```

```
100 const1=const*(v7-(zmax-v7)*(wyl/wn)**2)
ped1=(wn*v7**2+2.*wn*v7*(zmax-v7)-const1)*m7
go to 700
200 ped1=(wyl*(zmax-v7)**2-2.*(zmax-v7)*wyl*const/wn)*m7
go to 700
300 const2=const*(wn**2-wyl**2)/wn
ped1=(zmax-v7)*(wn**2-wyl**2)-const2
go to 700
400 ped1=((zmax-v7)*wyl**2+v7*wn**2-const*wyl**2/wn)*m7
go to 700
600 write (6,1)
1 format (' control number wrong')
700 return
end
```

```
c
c
c
c
c
c
c
```

```
subroutine egr(xmu,xsigma,xrho,pgwb,gamma,epara)
c this subroutine compute E [R,gamma]
c where R=zmax
c xmu: mean value of bata
c where bata see main program
```

```

c pgwb: d g/d bata(i)
c
c dimension xmu(6),xsigma(6),xrho(6,6),pgwb(6)
c
do 100 i=1,6
if (xmu(i).eq.gamma) go to 101
100 continue
write (6,1)
1 format ('gamma is not equal to xmu(i) in subroutine egr,WRONG!!')
101 nj=i
c
epara=0.
do 200 j=1,6
ex1=xrho(nj,j)*xsigma(nj)*xsigma(j)
ex2=(ex1)*pgwb(j)
epara=epara+ex2
200 continue
return
end

c
c
c
subroutine search (thita,p,a,nb,dt,epsiln)
dimension thita(nb)
sum1=0.
do 100 i=1,nb
tx=(i-1)*dt
ty=p*atan(a*(tx**4))
sum1=sum1+(thita(i)-ty)**2
100 continue
epsiln=sum1
return
end

```

**END**

**FILMED**

5-85

**DTIC**