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ASYMPTOTIC METHODS ESPECIALLY IN COMBUSTION

FINAL REPORT

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Modern asymptotic methods have been applied to a wide range of problems in combustion science as well as certain magnetohydrodynamic and fluid mechanical questions. Details are contained in the 49 Technical Reports and 4 Ph.D. theses listed. A list of participating scientists is also given. Topics covered under this contract included			

(Deflagration to detonation Transition)

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Section 19 continued

flames, radicals, surface equilibrium, monopropellant, reactive atmosphere, stagnation-point flow, galloping detonations, flame bubbles, solidification fronts, Hartmann layers, intumescent paints, flickering, Karlovitz stretch, chambered diffusion flames, Lewis-number effect, fast-time stability.

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## Objectives and Results

Asymptotic methods were applied to a variety of combustion and related problems, including:

- (i) detonations and fast deflagrations, including the deflagration-to-detonation transition (DDT);
- (ii) near-stoichiometric behavior of two-reactant mixtures;
- (iii) stability of both premixed and diffusion flames, including flame bubbles and polyhedral flames;
- (iv) burning of droplets;
- (v) fluid-mechanical effects;
- (vi) effects of complex chemistry, in particular stretch resistance;
- (vii) effects of strain and pressure variations.

The most important results may be summarized as follows.

- (a) Determination of dissociation effects on burning rate of non-dilute mixtures.
- (b) Calculation of the shape of wrinkled flames.
- (c) Stability analysis of burner flames.
- (d) Effect of heat loss on quenching of strained flames.
- (e) Suppression of ignition and extinction in fuel-drop burning.
- (f) Demonstration that deflagration waves can be accelerated beyond the Chapman-Jouget limit.
- (g) Proof that deflagration and detonation waves cannot propagate steadily into the same fresh mixture.
- (h) Explanation of polyhedral flames as a bifurcation phenomenon.
- (i) Determination of the effect of ambient pressure variations on flame propagation.
- (j) Determination of the effect of radicals on flame propagation and hence stability.
- (k) Discovery of the effect of surface equilibrium on fuel-drop combustion.
- (l) Full description of the response for a monopropellant droplet burning in a reactive atmosphere.
- (m) An analytical treatment of diffusion-flame stability.

- (n) Explanation of the N-shaped velocity profiles obtained in experiments on flames held by a stagnation-point flow.
- (o) Development of the highly accurate numerical schemes for galloping detonation, a well-known experimental phenomenon.
- (p) Description of flame bubbles (an inevitable feature of hydrogen burn-off after accidents such as occurred at Three-Mile Island).
- (q) A theory of the structure of solidification fronts in molten metals and its effect on Hartmann layers.
- (r) Criterion for DDT.
- (s) Theory of intumescent paints.
- (t) The discovering of a universal frequency for the flickering of diffusion flames.
- (u) Correct theory of stretch-resistant flames, which vindicates Karlovitz.
- (v) Numerical investigation of the Lewis-number effect on the stability of chambered diffusion flames.
- (w) Analytical treatment of galloping detonations.
- (x) Discovery of slow final decay in the extinction of burning fuel drops.
- (y) Investigation of the fast-time stability of the reaction zone of a premixed flame.

Many of these results were incorporated into a 1983 monograph entitled "Lectures on Mathematical Combustion", written by J. Buckmaster and G.S.S. Ludford for SIAM as part of the CBMS-NSF Regional Conference Series in Applied Mathematics. (The lectures were given at Colorado State University in the Summer of 1982.) This followed their 1982 book entitled "Theory of Laminar Flames" that appeared in the series of Cambridge Monographs on Mechanics and Applied Mathematics.

Details of all the results are contained in Progress Reports Nos. 32-39.

#### Publications

49 Technical Reports were written, as follows.

- 129. D. Mikolaitis and J. Buckmaster: Blow-off of a flame located in a rear stagnation point flow. Transactions of the 27th Conference of Army Mathematicians, West Point (NY) 1981, p. 181. (Published by ARO, Report 82-1.)

130. D.S. Stewart and G.S.S. Ludford: The equation governing the propagation of fast deflagration waves for small heat release. Transactions of the 27th Conference of Army Mathematicians, West Point (NY) 1981, p. 185 (Published by ARO, Report 82-1.)
131. G.S.S. Ludford and D.S. Stewart: Deflagration to detonation transition. Transactions of the 27th Conference of Army Mathematicians, West Point (NY) 1981, p. 563. (Published by ARO, Report 82-1.) Solicited paper.
132. J. Buckmaster: Free boundary problems in combustion. Research Notes in Mathematics 79 (1983), p. 483.
133. J. Buckmaster: Stability of the porous plug burner flame. SIAM Journal on Applied Mathematics 43 (1983), p. 1335.
134. G.S.S. Ludford and A.K. Sen: The effect of dissociation on the near-stoichiometric burning of non-dilute mixtures. To appear in the Proceedings of the Ninth International Colloquium on Dynamics of Explosions and Reactive Systems, held at Poitiers (France) in July 1983.
135. A.K. Sen and G.S.S. Ludford: Maximum flame temperature and burning rate of combustible mixtures. Nineteenth Symposium (International) on Combustion, p. 267. Pittsburgh: The Combustion Institute, 1983.
136. H. Daneshyar, J.M.C. Mendes-Lopes, and G.S.S. Ludford: Effect of strain fields on burning rate. Nineteenth Symposium (International) on Combustion, p. 413. Pittsburgh: The Combustion Institute, 1983.
137. R.D. Janssen and G.S.S. Ludford: Burning rate response of a methanol drop to ambient air pressure. Nineteenth Symposium (International) on Combustion, p. 999. Pittsburgh: The Combustion Institute, 1983.
138. G.S.S. Ludford: Activation-energy asymptotics of the plane premixed flame. Notes on Numerical Fluid Mechanics 5 (1982), 15. Invited paper.
139. J. Buckmaster: Polyhedral flames - an exercise in biomodal bifurcation analysis. SIAM Journal on Applied Mathematics 44 (1984), p. 40.
140. R.D. Janssen and G.S.S. Ludford: The response to ambient pressure of fuel drop combustion with surface equilibrium. International Journal of Engineering Science 22 (1984), p. 403.
141. D.S. Stewart and G.S.S. Ludford: Evolution of near Chapman-Jouget deflagrations. Transactions of the Twenty-Eighth Conference of Army Mathematicians, Bethesda (MD) 1982, p. 133. (Published by ARO, Report 83-1.)
142. H. Daneshyar, J.M.C. Mendes-Lopes, G.S.S. Ludford, and P.S. Tromans: The influence of straining on a premixed flame and its relevance to combustion in SI engines. Proceedings of the Conference on Combustion in Engineering, Oxford (UK) 1983, p. 191. London: Institute of Mechanical Engineers.



143. N. Peters and G.S.S. Ludford: The effect of pressure variations on premixed flames. Combustion Science and Technology 34 (1983) p. 311. Invited paper.
144. D.S. Stewart and G.S.S. Ludford: The acceleration of fast deflagration waves. Zeitschrift fur Angewandte Mathematik und Mechanik 63 (1983), p. 291. Invited paper.
145. H.V. McConnaughey, G.S.S. Ludford, and G.I. Sivashinsky: A calculation of wrinkled flames. Combustion Science and Technology 33 (1983), 103.
146. G.S.S. Ludford and N. Peters: Slowly varying flames with chain-branching/chain-breaking kinetics. To appear in the Proceedings of the Ninth International Colloquium on Dynamics of Explosions and Reactive Systems, held at Poitiers (France) in July 1983.
147. J.D. Buckmaster and G.S.S. Ludford: Lectures on mathematical combustion. Lecture 1: Pre-asymptotic combustion revisited. In "Lectures on Mathematical Combustion," CBMS Regional Conference Series. Philadelphia: SIAM Publications. Invited lecture.
148. J.D. Buckmaster and G.S.S. Ludford: Lectures on mathematical combustion. Lecture 2: Governing equations, asymptotics, and deflagrations. In "Lectures on Mathematical Combustion," CBMS Regional Conference Series. Philadelphia: SIAM Publications. Invited lecture.
149. J.D. Buckmaster and G.S.S. Ludford: Lectures on mathematical combustion. Lecture 3: General deflagrations. In "Lectures on Mathematical Combustion," CBMS Regional Conference Series. Philadelphia: SIAM Publications. Invited lecture.
150. J.D. Buckmaster and G.S.S. Ludford: Lectures on mathematical combustion. Lecture 4: SVFs and NEFs. In "Lectures on Mathematical Combustion," CBMS Regional Conference Series. Philadelphia: SIAM Publications. Invited lecture.
151. J.D. Buckmaster and G.S.S. Ludford: Lectures on mathematical combustion. Lecture 5: Stability of the plane deflagration wave. In "Lectures on Mathematical Combustion," CBMS Regional Conference Series. Philadelphia: SIAM Publications. Invited lecture.
152. J.D. Buckmaster and G.S.S. Ludford: Lectures on mathematical combustion. Lecture 6: Cellular flames. In "Lectures on Mathematical Combustion," CBMS Regional Conference Series. Philadelphia: SIAM Publications. Invited lecture.
153. J.D. Buckmaster and G.S.S. Ludford: Lectures on mathematical combustion. Lecture 7: Pulsating flames. In "Lectures on Mathematical Combustion," CBMS Regional Conference Series. Philadelphia: SIAM Publications. Invited lecture.

154. J.D. Buckmaster and G.S.S. Ludford: Lectures on mathematical combustion. Lecture 8: Counterflow diffusion flames. In "Lectures on Mathematical Combustion," CBMS Regional Conference Series. Philadelphia: SIAM Publications. Invited lecture.
155. J.D. Buckmaster and G.S.S. Ludford: Lectures on mathematical combustion. Lecture 9: Spherical diffusion flames. In "Lectures on Mathematical Combustion," CBMS Regional Conference Series. Philadelphia: SIAM Publications. Invited lecture.
156. J.D. Buckmaster and G.S.S. Ludford: Lectures on mathematical combustion. Lecture 10: Free-boundary problems. In "Lectures on Mathematical Combustion," CBMS Regional Conference Series. Philadelphia: SIAM Publications. invited lecture.
157. H.V. McConnaughey and G.S.S. Ludford: Droplet decomposition in a reactive atmosphere: complete responses for large activation energies. Physica D 9 (1983), 209. Solicited paper.
158. J.D. Buckmaster, A. Nachman, and S. Taliaferro: The fast-time instability of diffusion flames. Physica D 9 (1983), 408. Solicited paper.
159. J.D. Buckmaster and S. Weeratunga: The stability and structure of flame-bubbles. Combustion Science and Technology 35 (1984), 287.
160. J.D. Buckmaster and A. Crowley: The fluid mechanics of flame tips. Journal of Fluid Mechanics 31 (1983), 341.
161. J.D. Buckmaster: Fronts and patterns in low Mach. no. combustion - A review. To appear in Physica D as Proceedings of the Conference on Fronts, Interfaces and Patterns at Los Alamos (N.M.) in May '83. Invited paper.
162. G.S.S. Ludford and B. Scheurer: Problèmes mathématiques de la combustion. In "Problèmes Non Linéaires Appliqués. Textes des Exposés Présentés à la Session Combustion 7 au 13 Décembre 1982. Ecoles CEA-INRIA-EDF 82-83", p. 5. (Edité par l'Institut National de Recherche en Informatique et en Automatique)
163. A.A. Oyediran and G.S.S. Ludford: Are detonations steady?. Transactions of the 1st Army Conference on Applied Mathematics and Computing, Washington (DC) 1983, p. 813. (Published by ARO, Report 84-1.)
164. H.V. McConnaughey and G.S.S. Ludford: Wrinkled-flame calculations revisited. Transactions of the First Army Conference on Applied Mathematics and Computing, Washington (DC) 1983, p. 791. (Published by ARO, Report 84-1.)
165. D.S. Stewart and G.S.S. Ludford: Near Chapman-Jouget detonations. Transactions of the First Army Conference on Applied Mathematics and Computing, Washington (DC) 1983, p. 801. (Published by ARO, Report 84-1.)
166. D.S. Stewart: Transition to detonation in a model problem. Submitted for publication.

167. A.K. Sen and G.S.S. Ludford: Near-stoichiometric burning. In "Mathematical Methods in Energy Research," (Proceedings of the Special Year in the Mathematics of Energy at the University of Wyoming '82-'83), p. 139. Philadelphia: SIAM Publications. Invited paper.
168. K.A. Ames: On the comparison of solutions of related properly and improperly posed Cauchy problems for first order operator equations. SIAM Journal on Mathematical Analysis 13 (1982), 594.
169. R. Tam and G.S.S. Ludford: Comment on the stretch-resistant flames of Seshadri and Peters. Combustion Science and Technology 40 (1984), 303.
170. H.V. McConnaughey and G.S.S. Ludford: Two-step sequential reactions revisited. Submitted for publication.
171. F.S. Hall and G.S.S. Ludford: Hartmann layers in slowly solidifying liquids. To appear in Progress in Aeronautics and Astronautics. (Proceedings of the 4th International Seminar on MHD-Flows and Turbulence, Beersheva and Eilat 1984.)
172. D.S. Stewart: On the stability of the reaction zone of the plane deflagration. Submitted for publication.
173. R. Tam and G.S.S. Ludford: The stretch-resistant flames of Seshadri and Peters. To appear in Combustion Science and Technology.
174. G. Joulin, A. Liñán, G.S.S. Ludford, N. Peters, and C. Schmidt-Lainé: Flames With Chain-Breaking/Chain-Branching Kinetics. To appear in SIAM Journal on Applied Mathematics.
175. J. Buckmaster, C. Anderson, and A. Nachman: A model for intumescent paints. To appear in the International Journal of Engineering Science.
176. G.S.S. Ludford: Saw-tooth evolution in a Stefan problem. To appear in Lectures in Applied Mathematics: Nonlinear Systems of PDE in Applied Mathematics. (Proceedings of the 1984 AMS-SIAM Summer Seminar at Santa Fe.). Invited paper.
177. J. Buckmaster, C. Anderson, and A. Nachman: The response of intumescent paints to heat. Submitted for publication.

4 Ph.D. theses were completed, as follows.

Guey-Chia Lu: Diffusion flames in a chamber and plane steady detonations for large activation energy, ix + 141 pp., including 21 figures and 1 table, January 1982.

Robert Daniel Janssen: Drop combustion, vii + 100 pp., including 34 figures and 3 tables, January 1983.

Helen Verna McConnaughey: Three topics in combustion theory, viii + 252 pp., including 72 figures and 4 tables, January 1983.

Edward Kazimir Buratynski: A lower-upper factored implicit scheme for the numerical solution of the Euler equations applied to arbitrary cascades, viii + 182 pp., including 37 figures, May 1983.

#### Participating Scientists

Y.S. Choi, E. Eteng, F. Hall, R.D. Janssen (Ph.D.), F. Mancini, H.V. McConnaughey (Ph.D.), A.A. Oyediran, R. Tam, and M.G. Williams were supported for various periods as Research Assitants. R.D. Janssen was also an Instructor for one semester, partly supported by the contract. In addition to those indicated, E.K. Buratynski and G.C. Lu (who was supported at an earlier time) earned Ph.D.'s during the report period.

Prof. J.D. Buckmaster of the University of Illinois - Urbana was consultant and then Visiting Professor for various periods. Prof. D.S. Stewart of the University of Illinois - Urbana was a Visiting Assistant Professor. Dr. C. Lainé of the Ecole Centrale de Lyon was at Cornell twice as Visiting Scientist.

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