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# DEPARTMENT OF DEFENCE

## DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

WEAPONS SYSTEMS RESEARCH LABORATORY

DEFENCE RESEARCH CENTRE SALISBURY SOUTH AUSTRALIA

# **TECHNICAL REPORT**

WSRL-0355-TR

AN ANALYSIS OF THE EFFECT OF MASS UNBALANCES AND ASSEMBLY TOLERANCES ON THE PERFORMANCE OF THE FOUR-AXIS STABILISED DIRECTOR MOUNT

J. HAYWARD

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J. Hayward

# - SUMMARY

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The four-axis, elevation over azimuth over pitch over roll, director mount is analysed to determine the torques generated by mass unbalances about the motor driven axes. The effect of these on the motion of the stabilised platform, and the effect of this motion, as well as that of mechanical tolerances, on the accuracy of direction is examined. The analysis is restricted to the case of the for fully erect system, firmly connected to a stationary, TAI level surface, but the method of approach could be a extended to more complex situations. The results provide meed the basis for the design of small, lightweight, high dation. performance directors. (



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#### 1. INTRODUCTION

A necessary component of many weapon systems is the stabilised mechanical mount, used for directing the boresight of sensors and designators (optical, infrared, radar, etc) and the launching rails of weapons. (guided missiles, guns, etc). It must be able to point the boresight accurately in any direction (within a predetermined sector of three dimensional space) despite the inevitable changes in attitude adopted by the carrying vehicle. Two axes of motion between vehicle and boresight is the minimum number required, but practical considerations (such as the difficulty of providing unlimited motion about the axes, and the desire to reduce the inertia of rapidly slewing structures) often result in the use of three or four axes. Perhaps the most commonly used arrangement is that of the elevation-over-azimuth-over-pitchover-roll, four-axis mount. This report considers only that type.

In many applications (particularly for sensors and designators) it is of great importance that the mount be small in size and light in weight, but nevertheless exhibit high pointing accuracy and rapid slewing of the boresight. These normally conflicting requirements can only be achieved if the design uses accurately fitted, light but strong, components, assembled into well-balanced structures, driven by servo motors of small size and weight and capable of only modest torque. If the requirements for fitting accuracy and static and dynamic balances are tighter than those achievable under normal manufacturing methods, it may be necessary to provide for adjustment-on-test of these parameters. In any case it is essential that the necessary conditions for balance and accuracy are clearly understood. This report is concerned with an analysis of these conditions.

Emphasis is given in this report to the particular configuration of two nominally identical, separately elevatable, sensors mounted symmetrically on a single structure, which rotates at a constant, significant angular rate about the (vertical) azimuth axis. Analysis is restricted to the case of operation on a stationary level vehicle, with small deviations from the vertical of the azimuth axis. This restriction results in mainly linear equations of motion which are directly solvable, but still gives a good in-sight into the problems of balance and accuracy. The approach could be extended, however, to cover other cases, such as the erection process, or operation on a moving vehicle.

The analysis begins in Section 2, where the ideal four-axis mount is described, followed by the definition of a sequence of five Cartesian coordinate systems attached one to each of the five separate solid structures interconnected by the four axes of roll, pitch, azimuth and elevation. Matrix equations are derived relating the coordinates of a single point referred to each of the five coordinate systems.

In Section 3 we derive general relations for the torque components, referred to one coordinate system, caused by the reaction force of a point mass, fixed in a second coordinate system (neither system being inertial), and subjected to gravitational and kinematic accelerations.

In Section 4 we define ten mass distribution parameters (moments) for each of the elevating, rotating and combined elevating and rotating assemblies. We calculate torque components about the elevating and rotating coordinate systems due to each of these parameters under varying angles of roll, pitch, azimuth and elevation. These torques are transformed to the motor driven axes of roll, pitch, azimuth and elevation, and the results are presented in tabular form.

Section 5 is concerned with the conditions of mass distribution for static and dynamic balance of the structure (no torques about the motor driven axes). The azimuth rate is assumed constant, and the elevation rate, roll angle and

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pitch angle are assumed to be zero. Ten separate parameters must be zeroed.

Sections 6, 7 and 8 are concerned with the behaviour of the roll and pitch error correction loops, under the assumption of constant azimuth rate, zero elevation rate and small pitch and roll errors. The servo motor torque demands are assumed to be proportional to error and error rate, with a small unbalance permitted between the two loops.

It is shown in Section 6 that a uniform spiral motion of the azimuth axis about the vertical position generates a reaction torque (via the servo motors and the inertia parameters) which consists of three spiral components. The main component is at the same angular rate, and in the same direction, as the motion. One minor component (at much smaller amplitude) is at the same angular rate, but in opposite direction, and is proportional to the unbalance between the servo loop gains. The second minor component (also at a much smaller amplitude) has an angular rate and direction depending on those of both the spiral motion and of the azimuth scanning rotation, and is proportional to the magnitude of one unbalanced mass distribution parameter. The magnitude of all three components depends on the angular rate of the spiral motion; and that of the main and the second minor component depends on the relative direction of the spiral and scanning motions.

The transient response of the roll and pitch system is motion in the absence of any externally applied forcing torque; and hence motion producing no reaction torque. This is calculated in Section 7 by an iterative process. The spiral motion giving no main component of reaction torque is determined. This solution is then perturbed to correct it for the minor torque components. An infinite series of spiral motions is produced by this method, but, providing the unbalanced mass and servo gains are small, the amplitudes converge rapidly towards zero.

The residual (non-reaction) torques in the roll and pitch system result from mass unbalances interacting with the angular scanning motion about the azimuth axis, and combine to form a single spiral torque at the same rate and direction. Its effect on the system is considered in Section 8. Again an infinite, but rapidly converging series of spiral motions are produced, as forcing torques are matched with reaction torques. The magnification of the forcing torque by the servo loop resonance is considered in terms of the torque demand from the roll and pitch axis motors.

Section 9 considers torques in the azimuth and elevation drives under the same steady conditions postulated for the preceding sections.

Section 10 considers the torques about all four motor-driven axes, due to a varying elevation angle combined with otherwise steady scanning conditions. Section 11 considers the same torques during azimuth axis acceleration.

If the azimuth axis is not truly vertical, due to roll and pitch correction errors, the true azimuth and elevation angles of the boresight differ slightly from the indicated values. The differences are evaluated in Section 12.

Sections 13, 14, 15 and 16 consider the boresight errors and changes in motor torque demands resulting from deviations from the ideal geometrical arrangements of the four motor driven axes.

The results of the analysis, presented in tabular and graphical form, provide a firm basis for the design of the servo drives and of the procedure and facilities for achieving the necessary degree of mass balance.

#### 2. IDEAL ARRANGEMENT OF MOUNT AXES

In this Section we describe and define the ideal arrangement of the axes and assemblies of the elevation-over-azimuth-over-pitch-over-roll, stabilised, four axis aerial mount. Deviations from the ideal arrangement are considered in later sections. A simple extension covers the case of two aerials, back to back, on two separate elevatable assemblies.

The basic concept of the ideal arrangement is that the motor-driven rollcorrection, pitch-correction and azimuth axes always intersect at a common point, 0, irrespective of the roll and pitch correction angles,  $\eta$  and  $\psi$ . Furthermore, the pitch and roll axes are always perpendicular, as are the pitch and azimuth axes.

The roll correction axis is adjusted to be horizontal, and parallel to the rear to front centre line of the carrying vehicle, when the latter is in its nominal level condition. The pitch correction axis is defined to be horizontal under the same vehicular condition and when the roll correction angle,  $\eta$ , is zero. The azimuth axis is defined to be vertical (pointing towards the zenith) when the vehicle is level,  $\eta$  is zero, and the pitch correction angle,  $\psi$ , is zero.

The Cartesian coordinate system with the b-suffix (figure 1(a)) is fixed to the base of the aerial mounting system. Its origin is the point, 0, and its x-axis coincides with the roll correction axis. The y and z-axes coincide with the pitch and azimuth axes respectively when  $\eta$  and  $\psi$  are zero.

The Cartesian system with the g-suffix is fixed to the gimbal ring. Its origin is also 0, its x-axis also coincides with the roll axis, and its y-axis coincides with the pitch axis.

The Cartesian system with the s-suffix is fixed to the stabilised platform (figure 1(b)). Its origin is also 0, its y-axis also coincides with the pitch axis, and its z-axis coincides with the azimuth axis.

The Cartesian system with the r-suffix is fixed to the rotating assembly. Its origin is also 0, its z-axis also coincides with the azimuth axis (figure 1(c)), and its y-axis is defined to be parallel to the motor driven elevation axis. The azimuth angle,  $\theta$ , is defined to be zero when the x and y-axes coincide with those of the s-system.

The elevation axis is parallel to  $OY_r$  (figure 1(d)) and cuts the  $X_rOZ_r$  plane at the point  $O_e$ ,  $(X_1, 0, Z_1)$  in r-system coordinates. The Cartesian system with the e-suffix is fixed to the elevating assembly. Its y-axis coincides with the elevation axis. The x-axis,  $O_e X_e$ , is defined to be the boresight direction of the associated aerial. The elevation angle,  $\phi$ , is defined to be zero when the bore sight, and x-axis, are parallel to the x-axis in the rsystem.

The Cartesian system fixed to the second (back to back) elevating assembly would coincide with that of the first elevating assembly if the latter were to be rotated through  $\pi$  radians about the OZ axis. O would occupy the point  $(-X_1, 0, Z_1)$  in the r-system, and the x and y-axes would be oppositely directed to those in the r-system. An increase in  $\phi$  tilts the boresight of the second assembly upwards.

We define the following rotation matrices and position column vector.

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$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\eta & -\sin\eta \\ 0 & \sin\eta & \cos\eta \end{bmatrix} \qquad B = \begin{bmatrix} \cos\psi & 0 & \sin\psi \\ 0 & 1 & 0 \\ -\sin\psi & 0 & \cos\psi \end{bmatrix}$$

$$C = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & - \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix}$$

$$E = \begin{bmatrix} X_1 & 0 & Z_1 \end{bmatrix}^T$$

$$(1)$$

Then, by inspection of figure 1, the following relations apply to the coordinates of a single point.

$$P_{b} = A.P_{g} \qquad P_{g} = B.P_{s}$$

$$P_{s} = C.P_{r} \qquad P_{r} = E + D.P_{e}$$

$$(2)$$

#### 3. CALCULATION AND TRANSFORMATION OF TORQUE

#### 3.1 General Cartesian axes

Consider a general set of Cartesian axes, OXYZ, moving in inertial space, as shown in figure 1(e). Consider also a point-mass, of magnitude, m, located at the time-dependent coordinates, x, y, z, with respect to these axes. The mass exerts a reaction force as a result of its absolute acceleration and of its presence in the earth's gravitational field. Suppose it is resolved into the components,  $F_x$ ,  $F_y$ ,  $F_z$ , parallel to the x, y and z-axes. We wish to determine the torque components,  $T_x$ ,  $T_y$ ,  $T_z$ , about the same axes.

An examination of the situation, using figure 1(e), shows that the following matrix relationship applies.

 $\begin{bmatrix} \mathbf{T}_{\mathbf{x}} \\ \mathbf{T}_{\mathbf{y}} \\ \mathbf{y} \\ \mathbf{T}_{-} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & -\mathbf{z} & \mathbf{y} \\ \mathbf{z} & \mathbf{0} & -\mathbf{x} \\ -\mathbf{y} & \mathbf{x} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{F}_{\mathbf{x}} \\ \mathbf{F}_{\mathbf{y}} \\ \mathbf{F}_{\mathbf{z}} \end{bmatrix}$ 

We define the column vectors of position, P, force, F, and torque, T, as well as the matrix, N(P), where:

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(3)

 $P = [x \ y \ z ]^{T}$   $F = [F_{x} \ F_{y} \ F_{z}]^{T}$   $T = [T_{x} \ T_{y} \ T_{z}]^{T}$   $N(P) = \begin{bmatrix} 0 & [0 \ 0 \ -1] \ P & [0 \ 1 \ 0] \ P \\ [0 \ 0 \ 1] \ P & 0 & [-1 \ 0 \ 0] \ P \\ [0 \ -1 \ 0] \ P & [1 \ 0 \ 0] \ P & 0 \end{bmatrix}$ 

Then the above relationship becomes:

$$T = N(P) \cdot F \tag{4}$$

#### 3.2 General inertial axes

Consider a general set of Cartesian axes, 0'X'Y'Z', fixed in inertial space with 0'Z' pointing towards the zenith. Suppose the same point-mass is located at the time-dependent coordinates, x', y', z', with respect to these axes. Suppose the same reaction force is resolved into the components,  $F_x'$ ,  $F_y'$ ,  $F_z'$ , parallel to the corresponding inertial axes.

We define the column vectors,  $P^{\,\prime}$  and  $F^{\,\prime},$  and L, and the coordinate rotation matrix, R, such that:

Ρ'	=	[ x' y' z'] <sup>T</sup>		
F'	=	$\begin{bmatrix}F_{x} & F_{y} & F_{z}\end{bmatrix}^{T}$	>	(5)
Ρ'	=	L + R.P		

Here, L defines the time-dependent offset between the origins of the two Cartesian systems, and R defines the time-dependent angles between the corresponding pairs of axes.

If g is the gravitational acceleration, then:

$$F' = -m.(G + \ddot{P}')$$

$$G = [0 \ 0 \ g]^{T}$$
(6)

where the dots denote differentiation with respect to time.

The force-components transform isomorphically with the position coordinates under rotation of axes, and are unchanged under translation of axes. Therefore:

$$\mathbf{F}' = \mathbf{R} \cdot \mathbf{F} \tag{7}$$

Combining the relations (4), (6) and (7), we get the general relationship:

$$T = -m.N(P).R^{-1}.(G + P')$$
 (8)

3.3 Properties of N(P)

It can be shown (by evaluation) that the inverse of the matrices, A, B and C (from equation (1)) are obtained by changing the sign of the angle parameter.

It can also be shown (by evaluation) that, if Q is a matrix of the type A, B or C (ie a coordinate rotation matrix), then for all F:

$$N(Q,P) = Q.N(P).Q^{-1}$$

$$N(P).P = 0$$
(9)

3.4 Expansion of torque relation

Using equations (5) and (9):

$$N(P) \cdot R^{-1} \cdot (P^{\dagger} - L) = R^{-1} \cdot N(R \cdot P) \cdot \frac{d}{dt} \{ \frac{d}{dt} (R \cdot P) \}$$
$$= R^{-1} \cdot \frac{d}{dt} \{ N(R \cdot P) \cdot \frac{d}{dt} (R \cdot P) \}$$
$$= (R^{-1} \cdot \dot{R} + \frac{d}{dt}) \{ N(P) \cdot (R^{-1} \cdot \dot{R} \cdot P + \dot{P}) \}$$

Therefore from equation (8):

$$T = -m.(R^{-1}.R + \frac{d}{dt})\{N(P).(R^{-1}.R.P + P)\} - m.N(P).R^{-1}.(G + L)$$
(10)

#### 3.5 Axis transformation

We wish to determine the torque T", given T and the axis transformation relationship:

$$\mathbf{P}^{\prime\prime} = \mathbf{K} + \mathbf{Q} \cdot \mathbf{P} \tag{11}$$

Then:

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parameters, d and  $c_0$ .

8.3 Magnification of motor torque

The ratio of the magnitude of the servo motor torque, to that of the forcing torque (which equals the total deflection induced reaction torque) is of interest because significant magnification can occur if the forcing angular rate approaches the servo resonance frequency. We consider the simple case of negligible servo unbalance (u and f are zero) and one spiral component of forcing torque.

The expression for motor torque is given in equation (27). If values for U and F are substituted from equation (29), and for  $(\eta+j.\psi)$  from equations (31) and (42), we find that the ratio of motor torque to forcing torque is given by:

$$T_{M}/T_{f} = -e^{SL} \{ \omega_{0}^{2} + g V/(J+Q) + 2c_{0} . \omega_{0} . s \}/G(s)$$
(48)

The magnitude of this expression is plotted in figure 4, parts (a) to (d), for (s = j, 0), as a function of  $w_0/0$ , and for several values of the parameters, d and  $c_0$ . V is assumed to be zero. For (d = 0.5) the ratio is identically unity.

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The parameter, V, appears in equation (48) only because of the definition of  $\omega_0$  in equation (29).

b) For values of d greater than 0.5, the system is in stable equilibrium even without the motor torque, and the magnification is less than unity.

(c) For values of d less than 0.5, the system is unstable without the motor torque, and magnifications of up to 3 dB can occur.

(d) If the servo is to be in control of the system, and particularly if d (5 less than 0.5, the value of  $\omega_0$  must be at least three times the value of  $\dot{\theta}$ .

#### 9. STEADY STATE TORQUES IN AZIMUTH AND ELEVATION

#### 9.1 General

In this Section we consider torques about the azimuth and elevation axes under the same steady-state conditions previously assumed, viz  $(\dot{\theta} = \dot{\phi} = \dot{\phi} = 0, \eta \approx \psi \approx 0)$ . The assumptions imply that these torques are resisted by the respective servo motors with no change in motion (ie infinite stiffness).

9.2 Azimuth torque (T<sub>rz</sub>)

We use the combined rotating and elevating assembly parameters (Subsection 4.5) derived from Table 4, to find that:

 $s = j.\dot{\theta}$   $T_{f} = j.L.\dot{\theta}^{2}.e^{j\vartheta}$ (43)

(b) For the H-term from equation (40):

$$s = j.0$$

$$T_{f} = j.H.g.e^{j\alpha}$$
(44)

(c) For the K-term from equation (35), excited by the spiral deflection defined in equation (31):

$$s' = s^{*} + 2j.\theta$$

$$T_{f}' = -K.A^{*}.e^{-j\beta}.\{(s^{*})^{2} + 2j.\theta.s^{*}\}$$
(45)

(d) For the K-term from equation (35), excited by the main spiral deflection produced by the forcing torque  $T_{\epsilon}$ :

$$s' = s^{*} + 2j.\dot{\theta}$$

$$(T_{f}')^{*} \approx - K.T_{f}.e^{j\beta}.(s^{2}-2j.\dot{\theta}.s)/\{(J+Q).G(s)\}$$

$$(A')^{*} \approx K.T_{f}.e^{j\beta}.(2j.\dot{\theta}.s-s^{2})/\{(J+Q)^{2}.G(s).G^{*}(s^{*}+2j.\dot{\theta})\}$$

$$(46)$$

Note that, if s = j.0, then:

$$s' = j.\hat{\theta}$$

$$(2j.\hat{\theta}.s-s^{2}) = -\hat{\theta}^{2}$$

$$(s''+2j.\hat{\theta}) = j.\hat{\theta}$$

$$(47)$$

and the output angular frequency and direction are the same as the input. This does not apply for  $(s = -j.\theta)$ , for which the output occurs at  $(s'=3j.\theta)$ .

Figure 3, parts (a) to (e), show the value of the magnitude of G(s) (normalised to  $\theta^2$ ) plotted as a function of  $\omega_0$  (normalised to  $\theta$ ) for

 $(s = i, \theta)$ ,  $(s = -i, \theta)$  and  $(s = 3i, \theta)$ , and for several values of the

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#### 7.3 Perturbations

Newton's approximation, or some other iterative method, can be used to correct the value of s, determined in Sub-section 7.2, to make the function (33) zero for finite values of u and f. This converges rapidly for values of the order of 0.2 or less.

If the value of K is small but finite, a spiral torque can be determined from equation (35) for each of the four spiral deflection terms (two values of s, and their conjugates). These are then regarded as forming torques, which produce spiral deflection pairs at the same angular frequency, as discussed in Section 8. These in turn produce further reactions from the linear reaction section. Normally the infinite sequence of terms converges rapidly to negli, ible amplitude. (Note that the terms from equation (35) need a sign reversal, to convert them to forcing torques).

#### 8. STABILISATION SYSTEM FORCED RESPONSE

#### 8.1 General

An examination of Table 6 shows that, under the same conditions assumed in Sections 6 and 7 (ie  $\theta = \phi = \phi = 0$ , and  $\eta \approx \psi \approx 0$ ), there are still some torques not accounted for as deflection induced torques (Section 6). These forcing torques are independent of any (small) spiral deflection motions, and are given by:

$$(T_{roll}+j.T_{pitch}) = j.L.\dot{\theta}^{2}.e^{j\theta}l + j.H.g.e^{j\theta}h$$
(40)

These can be generalised to the form:

$$(T_{roll}^{+j},T_{pitch}) = T_f^{e^{st}}$$
(41)

where, in this case, (s =  $j.\dot{\theta}$ ) and  $T_f$  is the appropriate complex quantity for equivalence of equations (40) and (41).

The forcing torque must always be balanced by a motion induced reaction torque (from equation (33)). Thus s is defined by the forcing torque, and:

$$A = \frac{{}^{T}f}{(J+Q).\{G(s) - f(u,f)\}}$$
(42)

where f(u, f) represents the right-most term in equation (33).

The induced spiral deflection terms are then defined by equation (32). The main term is clearly at the same angular rate, and in the same direction as the forcing torque.

#### 8.2 Special cases

(a) For the L-term from equation (40):

This is applicable independently to the roll and pitch loops, because there is no cross coupling. Critical damping occurs with  $(c_d=1)$ .

(b) If  $\theta \neq 0$ , we substitute (s =  $\alpha + j.\omega$ ) in (36) and equate the real and imaginary parts separately to zero. Thus:

$$(\omega - d \cdot \theta) \cdot (\alpha + c_0 \cdot \omega_0) + c_0 \cdot \omega_0 \cdot d \cdot \theta = 0$$
  
$$-(\omega - d \cdot \theta)^2 + (\alpha + c_0 \cdot \omega_0)^2 + d^2 \cdot \theta^2 + \omega_0^2 \cdot (1 - c_0^2) = 0$$

Note that  $(\omega = d.\theta)$  is not a solution, else  $(c_0.\omega_0 = 0)$  and  $\alpha$  is imaginary. Therefore:

$$\alpha = -c_n . \omega_n . \omega / (\omega - d . \theta)$$
(38)

and, multiplying by  $-(\omega-d.\theta)^2$  and rearranging:

$$2(\omega - d.\dot{\theta})^{2} = d^{2}.\dot{\theta}^{2} + \omega_{0}^{2}.(1 - c_{0}^{2}) + (d^{2}.\dot{\theta}^{2} + \omega_{0}^{2}.(1 - c_{0}^{2}))^{2} + 4c_{0}^{2}.\omega_{0}^{2}.d^{2}.\dot{\theta}^{2})^{\frac{1}{2}}$$

$$= 2d^{2}.\dot{\theta}^{2} + \omega_{0}^{2}.(1 - c_{0}^{2}) - d^{2}.\dot{\theta}^{2} + (\omega_{0}^{2}.(1 - c_{0}^{2}) - d^{2}.\dot{\theta}^{2})^{2} + (\omega_{0}^{2}.(1 - c_{0}^{2}) - d^{2}.\dot{\theta}^{2})^{2} + (\omega_{0}^{2}.d^{2}.\dot{\theta}^{2})^{\frac{1}{2}}$$
(39)

Note that, from equations (39), (38):

(1)  $(\omega - d.\theta)^2 > d^2.\theta^2$ 

Thus:  $\omega > 2d.\theta$  or  $\omega < 0$ 

(2) In both cases,  $\alpha < 0,$  and transients decay to zero with increase in time.

(3)  $\left|\frac{\omega}{\alpha}\right| = \left|\frac{\omega - d \cdot \theta}{c_0 \cdot \omega_0}\right|$  takes the same value for both solutions, so the spirals are symmetrical, although one is executed much faster than the other.

Figure 2 shows the magnitude of  $(\omega - d.\theta)$  from equation (39), plotted in a normalised form, as a function of  $\omega_0/d.\theta$  and  $c_0$ . It can be seen that, for large values of  $c_0$ , there is a forward spiral response at an angular rate of just greater than 2 d. $\theta$ , together with a reverse spiral response at an angular rate close to zero. The directions are unchanged but the rates increase considerably as the damping factor,  $c_0$ , is reduced, depending also on the stiffness of the servo loops ( $\omega_0$ ). - 16 -

Then equation (30) becomes:

$$(T_{roll}^{+j}.T_{pitch})/(J+Q)$$

$$\simeq A.e^{St}.\{G(s) - (u.\omega_0^2 + 2f.c_0.\omega_0.s)^2/(G(s) + 4j.d.\theta.s)\}$$
(33)

#### 6.4 Non-linear reaction to spiral deflection

We now consider the contribution of the K-term to the spiral torque. From Table 6:

$$-(T_{roll}+j.T_{pitch}) = K.e^{2j.\theta}k.\{(\eta-j.\psi) + 2j.\theta.(\eta-j.\psi)\}$$
(34)

$$= K.A^{*}.e^{(s^{*}t+2j.\theta_{k})}.\{(s^{*})^{2} + 2j.\theta.s^{*}\}$$
(35)

under the single spiral deflection (equation (31)).

The significance of this component will be considered later.

#### 7. STABILISATION SYSTEM TRANSIENT RESPONSE

#### 7.1 General

The transient response of any system is obtained by equating to zero, the total deflection-induced torque. In this case it is the sum of the functions (30) and (34). A closed solution can not be obtained directly, However we can obtain a solution by assuming that K, u and f are zero, and then perturbing it to take into account the small finite values of these parameters.

7.2 Unperturbed solution

We assume the form of the solution is as defined in equation (32). Then, with (K = u = f = 0), from equations (33) and (35), the torque is zero if:

$$G(s) = 0$$

This can be rearranged algebraically to give:

$$(s-j.d.\theta+c_{c}.\omega_{0})^{2} + 2j.c_{0}.\omega_{0}.d.\theta + d^{2}.\theta^{2} + \omega_{0}^{2}.(1-c_{0}^{2}) = 0$$
(36)

(a) If  $\theta = 0$ , we get the well known result:

$$s = -c_{a} \cdot \omega_{a} \pm \omega_{a} \cdot (c_{a}^{2} - 1)^{\frac{1}{2}}$$
(37)

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$$\omega_0^2 = \frac{U-g.V}{J+Q}$$

$$c_0 = \frac{F}{2\{(U-g.V).(J+Q)\}^{\frac{1}{2}}}$$

$$d = \frac{J}{J+Q}$$
(29)

and assuming that  $\frac{g.V}{U} \approx 0$ , equation (28) becomes:

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$$-(T_{roll}^{+j},T_{pitch}^{-})/(J+Q) \approx (\ddot{\eta}+j.\ddot{\psi}) + 2(c_{0}.\omega_{0}-j.d.\dot{\theta}).(\dot{\eta}+j.\dot{\psi}) + \omega_{0}^{2}.(\eta+j.\psi) + \omega_{0}^{2}.(\eta+j.\psi) + u.\omega_{0}^{2}.(\eta-j.\psi) + 2f.c_{0}.\omega_{0}.(\dot{\eta}-j.\dot{\psi})$$
(30)

6.3 Linear reaction to spiral deflection

Suppose: 
$$\eta + j \cdot \psi = A \cdot e^{st}$$
  
 $s = \alpha + j \cdot \omega$ 
(31)

From parts (a) and (b) of figure 1, this is an anti-clockwise spiral motion of the point  $Z_r$  (looking down towards the origin) about the point,  $Z_b$ , with angular frequency,  $\omega$ , and logarithmic amplitude increment,  $\alpha$ . A should be considered as complex, to represent the amplitude and phase angle at t = 0. Replacing s with its complex conjugate reverses the direction of the spiral.

If we substitute equation (31) into equation (30) we obtain a major term in  $e^{st}$  (being a spiral torque in the same direction as the spiral deflection, and with the same angular frequency and increment) and a minor term in  $e^{st}$  (being a spiral torque in the reverse direction with the same angular frequency and increment) due to the unbalanced servo motor gains u and f. It is more useful to add a small spiral deflection in the reverse direction to cancel the reverse spiral torque. This is achieved if:

$$(\eta + j.\psi) = A.e^{st} - B^{*}.e^{s^{*}t}$$

$$G(s) = s^{2} + 2(c_{0}.\omega_{0} - j.d.\theta).s + \omega_{0}^{2}$$

$$B = A.(u.\omega_{0}^{2} + 2f.c_{0}.\omega_{0}.s)/(G(s) + 4j.d.\theta.s)$$
(32)

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$$2K_{e} = \sum_{e'} m.(z_{e'})^{2} - \sum_{e'} m.(x_{e'})^{2}$$
(26)

Thus  $K_e$  is proportional to the difference between the principal moments of inertia in the  $X_e Z_e$ - plane. For  $K_e = 0$ , the two principal moments of inertia must be equal (and the e'-system axes are principal for all values of  $\phi$ ).

#### 6. DEFLECTION INDUCED TORQUES IN THE STABILISATION SYSTEM

#### 6.1 General

In this Section we examine the torques present in the roll and pitch correction system due to the deflection of the azimuth axis from the vertical position  $(n+j.\psi)$  and its time derivatives, under the assumptions  $(\theta = \phi = \phi = 0)$  and  $(n \approx \psi \approx 0)$ . These consist of gravitational and inertial reactions (already determined, and listed in Table 6) and servo motor supplied torques.

We assume that the roll and pitch correction servo motors produce torques, proportional to the deflection error and its first time derivative, and in the direction to reduce that error. We wish also to examine the effect of small differences in the constants of proportion, between the roll and pitch systems. We therefore postulate the following relations for motor torque:

$$T_{roll} = -U.(1+u).\eta - F.(1+f).\dot{\eta}$$

$$T_{pitch} = -U.(1-u).\psi - F.(1-f).\dot{\psi}$$

$$-(T_{roll} + j.T_{pitch}) = U.(\eta+j.\psi) + F.(\dot{\eta}+j.\dot{\psi})$$

$$+ U.u.(\eta-j.\psi) + F.f.(\dot{\eta}-j.\dot{\psi})$$
(27)

#### 6.2 Linear reaction torque

From Table 6, and the assumption ( $\theta = 0$ ) all reaction terms are linear except those in K. For this sub-section we assume that K = 0. Then the total reaction torque is given by:

$$-(T_{roll}+j,T_{pitch}) = (J+Q), (\ddot{n}+j,\ddot{\psi}) + (F-2j,J,\theta), (\ddot{n}+j,\dot{\psi}) + (U-g,V), (n+j,\psi) + (U-g,V), (n+j,\psi)$$

$$+ u U, (n-j,\psi) + f,F, (\ddot{n}-j,\psi)$$
(28)

Making the normalising substatutions:

(2) The conditions involving  $H_e$  and  $H_r$  are the obvious ones that the centre of mass of the elevating assemblies be on the elevation axis, and that of the rotating plus both elevating assemblies be on the azimuth axis. The structure is then statically balanced. These conditions can be easily met in practice, with proper mechanical design and with adjustable weights for fine trimming.

(3) The conditions involving  $L_e$  and  $L_r$  are those for dynamic balance of the elevating, and total rotating assemblies, about the elevation and azimuth axes, respectively. These conditions are less easily met, and require careful attention.

(4) The condition involving  $K_e$  is very difficult to achieve in practice. Its phy\_ical implication is discussed in detail in Subsection 5.6. With two elevating assemblies, the effect about the roll and pitch axes tends to cancel, if the two values of  $\phi$  are not too dissimilar. This may not be a reasonable constraint, however.

(5) If torque balance is required for small constant values of  $\psi$  and  $\eta$  (as well as at the zero values) then the Tables 6, 1, 3, 4 and 2 show the extra requirement to be:

$$V_r + M_o Z_1 = 0$$
 (25)

The e-suffix term implies the difference between the two elevating assemblies.

5.6 Physical significance of K

The Cartesian system with the e'-suffix is defined by:

$$P_e' = D.P_e$$

Then, by evaluation:

$$\sum_{e} m.(x_e')^2 = J_e - K_e.\cos(2\phi - \beta_e)$$

$$\sum_{e} m.(z_e')^2 = J_e + K_e.\cos(2\phi - \beta_e)$$

$$\sum_{e} m.x_e'.z_e' = - K_e.\sin(2\phi - \beta_e)$$

We choose  $2\phi = \beta_e$ , making the cross product function, zero. Hence the e'-system becomes a principal set of axes, and the other two functions take the maximum and minimum possible values. Then:

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5.2 Roll and pitch torques

$$(T_{roll} + j.T_{pitch}) = j.\theta^{2}.e^{j\theta}.\{(L_{r}.\cos x_{r} - K_{e}.\sin 2\phi_{k} + M_{e}.X_{1}.Z_{1} + X_{1}.H_{e}.\sin \phi_{h} + Z_{1}.H_{e}.\cos \phi_{h}) + j.(L_{r}.\sin x_{r} + L_{e}.\sin \phi_{1} + Z_{1}.V_{e})\} + j.g.e^{j\theta}.\{(H_{r}.\cos \alpha_{r} + H_{e}.\cos \phi_{h} + M_{e}.X_{1}) + j.(H_{r}.\sin \alpha_{r} + V_{e})\}$$

$$(21)$$

For this expression to be identically zero for arbitrary values of  $\phi$ , the requirements are:

$$H_{e} = K_{e} = L_{e} = 0$$

$$H_{r} \cdot \cos \alpha_{r} + M_{e} \cdot X_{1} = 0$$

$$H_{r} \cdot \sin \alpha_{r} + V_{e} = 0$$

$$L_{r} \cdot \cos \delta_{r} + M_{e} \cdot X_{1} \cdot Z_{1} = 0$$

$$L_{r} \cdot \sin \delta_{r} + Z_{1} \cdot V_{e} = 0$$

$$(22)$$

5.3 Azimuth torque  $(T_{rz})$ 

This torque is automatically zero, so no special requirements apply.

5.4 Elevation torque  $(T_{ey})$ 

$$T_{elev} = \theta^2 \left\{ X_1 \cdot H_e \cdot \sin\phi_h - K_e \cdot \sin2\phi_k \right\} + g \cdot H_e \cdot \cos\phi_h$$
(23)

For this expression to be identically zero for arbitrary values of  $\phi$ , the requirements are:

$$H_{\mu} = K_{\rho} = 0 \tag{24}$$

#### 5.5 Discussion of results

(1) The conditions for e-suffix terms in equation (22) imply the difference between the two elevating assemblies, because  $e^{j\theta}$  terms tend to cancel (Sub-section 4.6). But if different values of  $\phi$  are allowed for the two units, cancellation is not possible and the zero condition for H<sub>e</sub>, K<sub>e</sub> and L<sub>e</sub> apply to each unit individually. The conditions in equation (24) apply individually in either case.

$$T_{roll} = T_{gx} = [1 \ 0 \ 0] . B.C.T_{r}$$
  
 $T_{pitch} = T_{sy} = [0 \ 1 \ 0] . C.T_{r}$ 

Both  $T_{roll}$  and  $T_{pitch}$  are real variables because B,C and  $T_r$  are real. (Note that complex terms in  $T_r$  occur as the sum of conjugate pairs). Therefore:

$$T_{roll} = Re{T_{roll} + j.T_{pitch}}$$

$$T_{pitch} = Im{T_{roll} + j.T_{pitch}}$$

$$(20)$$

$$(T_{roll} + j.T_{pitch}) = [e^{j\theta} j.e^{j\theta} \psi].T_{r}$$

Table 5 lists the contribution of the elevating assembly to  $(T_{roll}^{+j},T_{pitch})$ , evaluated from Table 3, using equation (20). Table 6 lists the contribution of the combined assemblies, at fixed elevation, and is derived from Table 4 by deleting the r-suffix from the mass distribution parameters (as discussed in Sub-section 4.5).

#### 5. CONDITIONS FOR BALANCE WITH STEADY OPERATION

#### 5.1 Steady operation

For a continuously scanning radar, it is normally very desirable that all torques about the four motor driven axes (roll, pitch, azimuth and elevation) should be zero when:

- (a) the aerial mount is fixed to a level surface
- (b) the rotating assembly is fully erect (ie  $\psi = n = 0$ )
- (c) there is a fixed, but arbitrary elevation angle  $(\phi)$
- (d) there is a constant (or zero) scan rate  $(\theta)$ .

Under these conditions, the torques, obtained from Tables 6, 1, 3, 4 and 2 become as shown in the next sub-sections.

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of torque in the  $\theta$ -column requires that:

$$L.\cos \delta = L_r \cdot \cos \delta_r + M_e \cdot X_1 \cdot Z_1 - K_e \cdot \sin 2\phi_k + H_e \cdot (X_1 \cdot \sin \phi_h + Z_1 \cdot \cos \phi_h)$$

All such relationships are listed in Table 1.

Note that most of the non-suffixed parameters are functions of  $\phi$ .

4.6 Contribution of second elevating assembly

As discussed in Section 2, the second elevating assembly is (ideally) equivalent to the first assembly rotated by  $\pi$  radians about the z-axis of the r-system. Its contribution to its own T is therefore given by the terms in Table 2, except that each  $\theta$  (including any in the column heading) is replaced by  $(\theta+\pi)$ . The effect of this is to change the sign of entries in those columns containing  $\exp(\overline{+j}, \theta)$  in the heading.

The contribution to  $T_r$  is determined indirectly. Clearly, the contribution to  $T_s$  is obtained by replacing  $\theta$  by  $(\theta+\pi)$  wherever it occurs. Thus the contribution to  $T_r$  is given by:

$$C^{-1}.(C'.T_{r}') = \begin{bmatrix} -T_{rx}' \\ -T_{ry}' \\ T_{rz}' \end{bmatrix}$$
(19)

where the prime denotes replacement of  $\theta$  by  $(\theta+\pi)$ . Thus the rule for Table 3 becomes: change the sign of elements in the x and y components of torque; then change the sign of elements in the columns containing  $\exp(\mp j.\theta)$  in the heading.

Note that this algorithm for  $T_r$  is equivalent to changing the sign for each occurrence (in element and row and column heading) of  $H_e$ ,  $V_e$ ,  $X_1$ ,  $\phi$ ,  $\phi_h$ ,  $\phi_1$ ,  $\phi_k$ . Executing this latter algorithm on the relationships for the non-suffixed mass distribution parameters in Table 1, results in a change of sign for contributions to those parameters which change sign when both  $x_r$  and  $y_r$  are changed in sign, ie H.cos $\alpha$ , H.sin $\alpha$ , L.cosi and L.sini. This is as would be expected, since the x and y directions in the r-system are reversed for the second elevating assembly.

Final sign reversals for contributions to  $T_r$  and  $T_s$  reflect the balancing effect of the second assembly upon the first, due to the symmetrical structure and location about the azimuth axis.

4.7 Torque about roll and pitch axes

From figure 1 and equation (12) it follows that:

From equations (2) and (11):

$$K = E$$

$$Q = D$$

$$(17)$$

Items (k) to (u) of Appendix I are additional key results in the evaluation of equation (16). The final results are shown in Table 3 (in the same format as used in Table 2), except that terms in  $M_e$  (ie those which consider the elevating assembly as a point mass at its origin) have been included in Table 4, together with the contributions of the rotating assembly.

Note that the z-component of  $T_r$  is the torque (due to the elevating assembly) to be resisted by the azimuth servo motor.

4.4 Contribution of rotating assembly to T\_

The torques generated about the axes of the r-system due to the reactions from the masses forming the rotating (but not elevating) assembly are given by equation (14), where in this case from equations (2) and (5):

$$P' = P_{b} = A.B.C.P_{r}$$

$$P = P_{r}$$

$$L = 0$$

$$R = A.B.C$$
(18)

Items (v) to (y) of Appendix I are additional key results in the evaluation, which also uses the set of ten mass distribution parameters for the rotating assembly (indicated by the r-suffix) listed in Table 1. The final results are shown in Table 4, which includes in addition, the terms in  $M_{\rm o}$  from the elevating assembly, derived in Sub-section 4.3.

Note that columns involving  $\phi$  and its derivatives have been deleted from the table. The mass distribution of the rotating assembly (alone) is obviously independent of the elevation angle of the elevating assembly.

Note also that the z-component of  $T_r$  is the torque (due to the rotating assembly alone) to be resisted by the azimuth servo motor.

4.5 Combined contribution to T\_

It is obvious that, at a fixed elevation angle ( $\phi$ ), the elevating assembly can be considered as an extension of the rotating assembly. Therefore we can define a combined set of mass distribution parameters, M, J, Q, K, L, H, V,  $\alpha$ ,  $\beta$  and  $\delta$ , which behave isomorphically with the r-suffixed equivalents, but which include the contribution from the elevating assembly at a fixed, but arbitrary, elevation.

The relationship between the non-suffixed parameters and the r and esuffixed ones can be determined from the data in Tables 3 and 4, by equating total torques in each axis-column. For example, the x-component

$$T_{e} = -(R^{-1}.\dot{R} + \frac{d}{dt}) \left\{ \sum_{e} m.N(P).(R^{-1}.\dot{R}.P+\dot{P}) \right\} - \sum_{e} m.N(P).R^{-1}.(G+\dot{L}) \quad (14)$$

where from equations (2) and (5):

$$P' = P_b = A.B.C.(E + D.P_e)$$

$$P = P_e$$

$$L = A.B.C.E$$

$$R = A.B.C.D$$
(15)

This can be evaluated directly, using the mass-distribution parameters defined in Table 1. Between them, the ten parameters associated with the elevating assembly (indicated by the e-suffix) cover the entire range of zero order, first order and second order moments of mass, measured in the coordinate system fixed to that assembly. The form of the parameters has been chosen to suit the evaluation of the torques.

Items (a) to (j) of Appendix I are key results in the step by step evaluation of equation (14). The final results are presented, in Table 2, in the form of a matrix. Each entry in the matrix is to be multiplied by its row and column headers, before being accumulated into the total component of the torque.

The first five columns contain dynamic entries due to the motions of rotation ( $\theta$ ) and elevation ( $\phi$ ). The ninth and tenth columns contain static entries due to gravity. Columns 6, 7 and 8 contain dynamic entries due to the changing values of roll ( $\eta$ ) and pitch ( $\psi$ ) correction. Note that each of columns 6, 7, 8 and 10, represent two separate sets of entries, denoted by the upper and lower sign (where a choice is provided). The upper sign is used for one set; the lower sign for the other. Wherever only one sign is given, it applies to both sets.

The complex variable form of column headings simplifies evaluation and presentation, and will be found to be useful in later Sections.

Note that the y-component of  $T_e$  (as listed in Table 2) is the torque to be resisted by the elevation servo motor.

4.3 Contribution of elevating assembly to  $T_{\mu}$ 

Consider the torques generated by the reactions of the masses located on the elevating assembly, about the axes of the r-system. Evaluation is easier if we use the transformation relationship (12), rather than the direct relationship (10). Thus:

$$T_r = Q.T_e - \sum_{e} m.N(K).Q.R^{-1}.(G + P')$$
 (16)

where the variables are defined in equations (15) and (17).

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 $P' = L + R.Q^{-1}.(P'' - K)$ 

and from equation (8):

$$\Gamma'' = -m.N(P'').(R.Q^{-1})^{-1}.(G + P')$$
  
= -m.N(K + Q.P).Q.R<sup>-1</sup>.(G + P')  
= Q.T - m.N(K).Q.R<sup>-1</sup>.(G + P') (12)

Note that the second term of equation (12) contains no second order terms in the elements of a position vector. All second order terms are transformed within the first term of equation (12).

#### 4. TORQUES DUE TO MASS UNBALANCE

4.1 General

In this Section we consider the torques acting about various axes, with the aerial base levelled on firm ground, and with the azimuth axis approximately vertical. In this case, the roll and pitch correction angles,  $\eta$  and  $\psi$ , are small enough that second order terms in these quantities can be neglected. Then:

$$A \cdot B \approx I + X$$

$$X = \begin{bmatrix} 0 & 0 & \psi \\ 0 & 0 & -\eta \\ -\psi & \eta & 0 \end{bmatrix}$$

$$(A \cdot B)^{-1} \approx I - X$$

$$\frac{1}{4t}(A \cdot B) \approx \dot{X}$$

$$(13)$$

The torques considered are caused by mass unbalances within the structure of the rotating and elevating assemblies. To calculate them, we must sum the effect of reaction forces over the entire rotating and elevating assemblies.

On the assumption that the b-system of axes is inertial, with the z-axis pointing towards the zenith, this system is isomorphic with the singly primed inertial system discussed in Section 3, and the torques can be evaluated using the relationships derived therein.

4.2 Contribution of elevating assembly to  $T_{p}$ 

Consider the torques generated about the axes of the e-system due to reactions from the masses forming the normal elevating assembly. From equation (10):

$$T_{az} = \frac{1}{2}L.\left\{e^{-j\theta}I.\left(\ddot{\eta}+j.\ddot{\psi}\right) + e^{j\theta}I.\left(\ddot{\eta}-j.\ddot{\psi}\right)\right\}$$
$$- \frac{1}{2}g.H.\left\{e^{-j\theta}h.\left(\eta+j.\psi\right) + e^{j\theta}h.\left(\eta-j.\psi\right)\right\}$$
$$= Re\left\{L.e^{-j\theta}I.\left(\ddot{\eta}+j.\ddot{\psi}\right) - g.H.e^{-j\theta}h.\left(\eta+j.\psi\right)\right\}$$
(49)

where Re denotes "the real part of".

Considering a single spiral deflection component, as defined in equation (31):

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$$T_{az} = |A|.Re\{L.s^{2}.e^{\{st-j(\theta_{1}-\angle A\}\}} - g.H.e^{\{st-j(\theta_{h}-\angle A)\}}\}$$
(50)

In the common case  $(s = j.\theta)$ :

$$T_{az} = -|A| \cdot L \cdot \dot{\theta}^{2} \cdot \cos(\langle A - \mathcal{X} \rangle) - |A| \cdot g \cdot H \cdot \cos(\langle A - \alpha \rangle)$$
(51)

Both terms consist of a steady torque, of magnitude dependent on the phase relationship between the relevant mass-unbalance and the deflection component.

If  $(s = -j.\theta)$  or  $(s = 3j.\theta)$  the torque components are sinusoidal, and at an angular frequency of  $2\theta$ .

9.3 Elevation torque  $(T_{ey})$ 

From Table 2, and assuming the same single spiral deflection (equation (31)):

$$T_{elev} = \theta^{2} \cdot (X_{1} \cdot H_{e} \cdot \sin\phi_{h} - K_{e} \cdot \sin2\phi_{k}) + g \cdot H_{e} \cdot \cos\phi_{h}$$

$$+ \operatorname{Re}[A \cdot e^{(st - j\theta)} \cdot (j \cdot s^{2} \cdot (2J_{e} + X_{1} \cdot H_{e} \cdot \cos\phi_{h} + Z_{1} \cdot H_{e} \cdot \sin\phi_{h} - j \cdot L_{e} \cdot \cos\phi_{1})$$

$$+ 2\theta \cdot s \cdot (J_{e} + X_{1} \cdot H_{e} \cdot \cos\phi_{h} - K_{e} \cdot \cos2\phi_{k} - j \cdot L_{e} \cdot \cos\phi_{1}) - j \cdot g \cdot H_{e} \cdot \sin\phi_{h}\}] \quad (52)$$

For  $(s = j.\theta)$ :

$$T_{elev} = \hat{\theta}^{2} \cdot (X_{1} \cdot H_{e} \cdot \sin\phi_{h} - K_{e} \cdot \sin2\phi_{k}) + g \cdot H_{e} \cdot \cos\phi_{h}$$
  
+  $\hat{\theta}^{2} \cdot |A| \cdot (Z_{1} \cdot H_{e} \cdot \sin\phi_{h} - X_{1} \cdot H_{e} \cdot \cos\phi_{h} + 2K_{e} \cdot \cos2\phi_{k}) \cdot \sin \Delta A$   
+  $\hat{\theta}^{2} \cdot |A| \cdot L_{e} \cdot \cos\phi_{1} \cdot \cos \Delta A + |A| \cdot g \cdot H_{e} \cdot \sin\phi_{h} \cdot \sin \Delta A$  (53)

For  $(s = j.\omega)$ , the term in  $J_e$  becomes:

$$2|A|.\omega.(\omega-\theta).\sin(\omega.t-\theta+A).J_e$$
(54)

Note that, for the second elevating assembly,  $\theta$  is incremented by  $\pi$  in equation (52), and all terms in A take a reversed sign.

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#### 10. TORQUES DUE TO VARYING ELEVATION ANGLE

Tables 5, 3 and 2 list all significant torques due to  $\dot{\phi}$  and  $\ddot{\phi}$ , about the roll and pitch, azimuth and elevation axes, respectively. Major terms, and those of first order of smallness, are given below.

$$(T_{roll} + j.T_{pitch})$$

$$\simeq j.e^{j\theta}.(\overset{\cdots}{\phi}.(2J_{e} + X_{1}.H_{e}.\cos\phi_{h} + Z_{1}.H_{e}.\sin\phi_{h} + j.L_{e}.\cos\phi_{1})$$

$$- \overset{\cdot}{\phi^{2}}.(X_{1}.H_{e}.\sin\phi_{h} - Z_{1}.H_{e}.\cos\phi_{h} + j.L_{e}.\sin\phi_{1})$$

$$+ 2j.\overset{\cdot}{\theta}.\overset{\cdot}{\phi}.(J_{e} + Z_{1}.H_{e}.\sin\phi_{h} + K_{e}.\cos2\phi_{k})) \qquad (55)$$

$$T_{az} \approx -\dot{\phi} \cdot L_{e} \cdot \sin\phi_{1} \cdot \dot{\phi}^{2} \cdot L_{e} \cdot \cos\phi_{1}$$

$$+ 2\dot{\theta} \cdot \dot{\phi} \cdot (X_{1} \cdot H_{e} \cdot \sin\phi_{h} - K_{e} \cdot \sin2\phi_{k})$$

$$+ 2\dot{\phi} \cdot |A| \cdot J_{a} \cdot \operatorname{Re}[s \cdot e^{(st - j\theta + j \angle A)}] \qquad (56)$$

$$T_{elev} = 2J_{e} \cdot \dot{\phi}$$
 (57)

Note that all terms in equation (55) tend to cancel when two elevating assemblies have the same variation of elevation angle. Only the term in |A| does so, in equation (56).

#### 11. TORQUES DUE TO AZIMUTH ACCELERATION

Tables 6, 4 and 2 list the significant torques due to  $\theta$ . Major terms, and those of first order of smallness, are given below.

$$(^{T}_{roll} + j.T_{pitch})$$

$$\simeq L.\theta.e^{j\theta} 1 - 2J.\theta.|A|.Im[e^{(st+j\Delta)}]$$
(58)

. .:

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$$T_{az} \simeq -2J.\dot{\theta}$$
 (59)

$$\stackrel{T}{\operatorname{elev}} \simeq \stackrel{\cdots}{\theta} \cdot \mathbf{L}_{e} \cdot \sin \phi_{1} \tag{60}$$

#### 12. EFFECT OF ROLL AND PITCH ERRORS ON BORESIGHT

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12.1 Boresight error

As discussed in Section 2, the aerial boresight is assumed to be the axis,  $0_e X_e$ , which is normal to the elevation axis at point,  $0_e$ , and parallel to the XY-plane of the r-system when  $\phi = 0$  (see figure 1(d)).

Suppose the boresight passes through a fixed target, located at the point,  $P_b$ , under two separate conditions. Firstly, with no roll and pitch errors:

A.B = I C = C D = D P<sub>e</sub> = r.  $\begin{bmatrix} 1\\0\\0 \end{bmatrix}$  = r.Z

Secondly, with finite roll and pitch errors:

$$A \cdot B = A \cdot B$$

$$C = C + \Delta C$$

$$D = D + \Delta D$$

$$P_{e} = (r + \Delta r) \cdot Z$$

Using equation (2) to determine  $P_b$  for each case, and then equating the results:

A.B.  $(C+\Delta C)$ .  $(E + (D+\Delta D) \cdot (r+\Delta r) \cdot Z) = C \cdot (E + D \cdot r \cdot Z)$ 

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Therefore:

$$D^{-1} \cdot (I + C^{-1} \cdot \Delta C) \cdot D \cdot \{(I + D^{-1} \cdot \Delta D) \cdot (r + \Delta r) \cdot Z + D^{-1} \cdot E\}$$
  
=  $D^{-1} \cdot C^{-1} \cdot (A \cdot B)^{-1} \cdot C \cdot D \cdot (r \cdot Z + D^{-1} \cdot E)$  (61)

Assuming that  $\Delta r/r$  and the elements of  $D^{-1}$ . E/r are negligibly small compared with unity, then equation (61) reduces to:

$$D^{-1}.C^{-1}.\Delta C.D.Z + D^{-1}.\Delta D.Z + D^{-1}.C^{-1}.\Delta C.D.D^{-1}.\Delta D.Z$$

$$\simeq D^{-1}.C^{-1}.\{(A.B)^{-1}-I\}.C.D.Z$$
(62)

Noting that:

$$\cos(\theta + \Delta \theta) = \cos\Delta\theta \cdot \cos\theta + \sin\Delta\theta \cdot \frac{d}{d\theta}(\cos\theta)$$
  

$$\sin(\theta + \Delta \theta) = \cos\Delta\theta \cdot \sin\theta + \sin\Delta\theta \cdot \frac{d}{d\theta}(\sin\theta)$$
  

$$C + \Delta C = \cos\Delta\theta \cdot C + \sin\Delta\theta \cdot \frac{dC}{d\theta}$$
  

$$C^{-1} \cdot \Delta C = (\cos\Delta\theta - 1) \cdot I + \sin\Delta\theta \cdot C^{-1} \cdot \frac{dC}{d\theta}$$

and using equation (1), the left hand side of equation (62) becomes:

$$cos\Delta\theta.cos\Delta\phi - 1$$

$$sin\Delta\theta.cos(\phi+\Delta\phi)$$

$$sin\Delta\phi.cos\Delta\theta$$
(63)

Making the substitutions:

$$\eta = \mu . \cos \beta$$

$$\psi = \mu . \sin \beta$$

$$\mu = (\eta^{2} + \psi^{2})^{\frac{1}{2}}$$

$$(64)$$

and including terms of second order in  $\eta$  and  $\psi$  from relation (1), the right hand side of equation (62) becomes:

 $\frac{1}{2}\mu^{2} \cdot (\cos^{2}\phi \cdot \cos^{2}(\theta - \beta) - 1)$   $\mu \cdot \sin\phi \cdot \cos(\theta - \beta) - \frac{1}{4}\mu^{2} \cdot \cos\phi \cdot (\sin^{2}(\theta - \beta) + \sin^{2}\beta)$   $-\mu \cdot \sin(\theta - \beta) - \frac{1}{4}\mu^{2} \cdot \sin^{2}\phi \cdot \cos^{2}(\theta - \beta)$ (65)

Equating the second and third elements of (63) and (65), and using the approximations:

$$\cos(\phi + \Delta \phi) = \cos \phi / (1 + \tan \phi \cdot \sin \Delta \phi)$$

- 25 -

$$\cos\Delta\theta = 1$$

we find approximate expressions for:

$$\sin\Delta\theta \approx \mu \cdot \tan\phi \cdot \cos(\theta - \beta) - \frac{1}{4}\mu^2 \cdot \{(1 + 2\tan^2\phi) \cdot \sin2(\theta - \beta) + \sin2\beta\}$$
(66)  
$$\sin\Delta\phi \approx -\mu \cdot \sin(\theta - \beta) - \frac{1}{4}\mu^2 \cdot \sin2\phi \cdot \cos^2(\theta - \beta)$$

#### 12.2 Maximum value of boresight errors

For a fixed value of  $\mu$ , and for positive values of  $\phi$ , we find, by differentiation with respect to  $\theta$ , and subsequent inspection, that:

(a)  $\sin\Delta\phi$  has its maximum positive value when:

$$(\theta - \beta) = -\pi/2$$

 $\beta$  = any value

(b)  $\sin\Delta\theta$  has its maximum positive value when:

$$\beta = -\pi/4 \quad \text{or} \quad 3\pi/4$$

$$\sin(\theta - \beta) = -\frac{\tan \phi}{2\mu \cdot (1 + 2\tan^2 \phi)} \cdot \left\{ \left( 1 + \frac{2\mu^2 \cdot (1 + 2\tan^2 \phi)^2}{\tan^2 \phi} \right)^2 - 1 \right\} \quad (67)$$

Figure 5 shows  $\Delta\theta$  and  $\Delta\phi$  plotted as functions of  $\mu$ , for various values of  $\phi$ , and with these values of  $\beta$  and of  $(\theta - \beta)$ . The latter is also plotted in the figure.

Note that:

(1)  $\beta$  is the angle of the resultant axis of tilt, due to the combined

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action of  $\eta$  and  $\psi$ .

(2) for a fixed value of  $(\eta^2 + \psi^2)$ , the maximum value of  $\Delta\theta$  occurs when  $\eta$  and  $\psi$  are equal in magnitude; that is  $\beta = -\pi/4$  or  $3\pi/4$ .

(3) for a fixed value of  $(\eta^2 + \psi^2)$ , the maximum value of  $\Delta \phi$  is  $(\eta^2 + \psi^2)^{\frac{1}{2}}$ , and is independent of  $\beta$  or  $\phi$ .

(4) the maximum value of  $\Delta\phi$  occurs when  $\theta = \beta - \pi/2$ ; that is, the boresight is normal to the effective tilt axis.

(5) the maximum value of  $\Delta\theta$  occurs when  $\theta$  is in the range  $(\beta \text{ to } (\beta - \pi/4))$ , depending on the values of  $\mu$  and  $\phi$  (figure 5(b)).

(6) if the tilt of the azimuth axis is due to a single spiral deflection, then  $\mu = A = \eta_{max} = \psi_{max}$ .

(7) if the pitch and roll errors are uncorrelated sinusoidal motions, then  $\psi_{\text{max}}$  and  $\eta_{\text{max}}$  can occur simultaneously and the value of  $\mu$  is increased by  $2^{\frac{1}{2}}$ ; that is  $\mu = 2^{\frac{1}{2}} \cdot \eta_{\text{max}} = 2^{\frac{1}{2}} \cdot \psi_{\text{max}}$ .

(8)  $\Delta \theta$  is not zero when  $\phi$  is zero, due to second order effects.

#### 13. EFFECT OF TILTED ELEVATION AXIS

13.1 General

We wish to examine the first order effect of incorrect alignment of the elevation axis on the boresight errors and on the motor axis torques.

If  $0_e$ , the origin of the e-system coordinates fixed to the elevating assembly, is displaced from its nominal position, the effect is equivalent to a variation in the coordinates,  $X_1$ ,  $Z_1$  and  $\theta$ , and can be evaluated accordingly.

In the remainder of this Section, we consider the misalignment in the direction of the elevation axis,  $0_{\text{e}}Y_{\text{e}}$ .

13.2 Misalignment matrix

Suppose the elevation axis,  $0_e Y_e$ , is rotated through the small angle, D0, about the normal through  $0_e$ , parallel to  $0Z_r$ ; followed by rotation through the small angle, Dn, about the normal through  $0_e$ , parallel to the  $X_r Y_r$ -plane. This is illustrated in figures 1(d) and 6. Then, to a first order approximation in D0 and Dn:

$$P_{r} = E + M.D.P_{e}$$

$$M \approx \begin{bmatrix} 1 & -D\theta & 0 \\ D\theta & 1 & -D\eta \\ 0 & D\eta & 1 \end{bmatrix}$$
(68)

Subsequent analysis is based on the assumption that roll and pitch correction errors are also small. In this case, to a first order approximation:

$$A.B = I$$

$$P_{b} = P_{s} = C.(E + M.D.P_{e})$$

$$(69)$$

13.3 Boresight error

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Using the same argument as for Section 12.1, it follows that:

$$(C+\Delta C) \cdot (E + M \cdot (D+\Delta D) \cdot (r+\Delta r) \cdot Z) = C \cdot (E + D \cdot r \cdot Z)$$
(70)

Neglecting  $D^{-1}$ . E/r and  $\Delta r/r$ , as in Section 12.1, and multiplying on the left by  $D^{-1}$ .  $C^{-1}$ :

$$(I + D^{-1}.C^{-1}.\Delta C.D).(I + D^{-1}.(M-I).D).(I + D^{-1}.\Delta D).Z \simeq Z$$

Neglecting second order product terms between  $\Delta C$ , (M-I) and  $\Delta D$ :

$$(D^{-1}, C^{-1}, \Delta C, D + D^{-1}, (M-I), D + D^{-1}, \Delta D), Z \simeq 0$$

By evaluation:

$$\sin\Delta\phi \simeq 0$$
  
 $\sin\Delta\theta \simeq D\eta. \tan\phi - D\theta$  (71)

#### 13.4 Elevation torque changes

Changes in elevation torque due to the inclusion of the misalignment matrix, follow from equation (14):

$$\Delta \mathbf{T}_{\mathbf{e}} \approx -\Delta \left(\mathbf{R}^{-1}, \mathbf{\hat{R}}\right), \qquad \forall \mathbf{k} \in \mathbf{k} \in \mathbf{k} \in \mathbf{\hat{k}} \in \mathbf{\hat{k}}$$

$$= -\mathbf{R}^{-1}, \mathbf{\hat{R}} + \frac{\mathbf{d}}{\mathbf{dt}} \qquad = -\mathbf{n} \exp \left\{\mathbf{h} \cdot \mathbf{A} \left(\mathbf{R}^{-1}, \mathbf{\hat{R}}\right), \mathbf{P}\right\}$$

$$= -\frac{1}{2} \sum_{\mathbf{e}} \mathbf{m} \cdot \mathbf{N}(\mathbf{P}), \mathbf{D}^{-1}, \mathbf{D}, \Delta \left(\mathbf{R}^{-1} \in (\mathbf{G} + \mathbf{\hat{L}})\right) \qquad (72)$$

Key results in the evaluation are given in items (a) to (c) of Appendix II, and the final results are listed in the left hand side of Table 7. Note that the y-component is the change in torque required of the elevation servo motor.

13.5 Pitch, roll and azimuth torque changes

Using equation (16) to determine the torque changes in the r-system:

$$\Delta T_{r} \simeq \Delta(Q) \cdot T_{e} + Q \cdot \Delta T_{e} - \sum_{e} m \cdot N(K) \cdot \Delta(Q \cdot R^{-1}) \cdot G$$
  
$$- \sum_{e} m \cdot N(K) \cdot \Delta(Q \cdot R^{-1} \cdot \ddot{P}^{*}) \qquad (73)$$

Key results in this evaluation are given in items (f) to (k) of Appendix II. Final results are listed in the right hand side of Table 7. Note that the z-component is the change in torque required of the azimuth serve motor.

Equation (20) is used to derive  $\Delta(T_{roll}+j,T_{pitch})$  from  $\Delta T_r$ , and this is listed in Table 8.

13.6 Discussion of results

From equation (71), both Dn and D0 contribute a significant error in azimuth boresight, but the contribution of D0 is constant, and can be calibrated out.

From Tables 8 and 5, most changes in roll and pitch torques are small compared with components existing with no axis tilt. The only changes not so swamped, are the terms in  $D\eta \cdot (J_e - Q_e)$  in the first two columns of Table 8. These may be of significant magnitude in some applications.

A similar comparison between Tables 7, 3 and 2, shows that none of the changes in azimuth and elevation torque are swamped by non-perturbed torques. The most significant terms are those in  $20\eta$ , J<sub>e</sub> coupled with  $\dot{\theta}$  and

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Thus D $\eta$  is more significant in its effects than D $\theta$ , and requires closer control.

#### 14. EFFECT OF NON-INTERSECTION OF AZIMUTH, PITCH AND ROLL AXES

14.1 Misalignment matrix

We consider the case of translation (without rotation) of the pitch correction, and azimuth axes so that they do not intersect each other, or the roll correction axis.

Figure 7(a) shows the pitch correction axis,  $\begin{array}{c} 0 \\ g \\ g \\ s \end{array}$ , in a plane normal to the roll correction axis,  $\begin{array}{c} 0 \\ b \\ b \\ b \\ \end{array}$ .  $\begin{array}{c} 0 \\ b \\ b \\ \end{array}$  is normal to both axes, and is of length,  $\begin{array}{c} Z_2 \\ Z_2 \end{array}$ . The coordinates of a single point, P, are related by:

$$P_{b} = A.(P_{g} + [0 \ 0 \ Z_{2}]^{T})$$

Figure 7(b) shows the azimuth axis,  $0_{s}Z_{s}$ , in a plane normal to the pitch correction axis,  $0_{g}Y_{g}$ .  $0_{g}O_{g}'$  is normal to both axes, is of length,  $X_{2}$ , and is collinear with the axis,  $0_{s}X_{s}$ .  $0_{g}O_{g}'$  is of length,  $Y_{2}$ . Then:

$$P_{g} = B.(P_{s} + [X_{2} \ Y_{2} \ 0]^{T})$$

Combining these relations with the approximation of equation (13):

$$P_{b} \approx (I + X) \cdot (P_{s} + E_{2}) - Y \cdot E_{2}$$

$$E_{2} = [X_{2} Y_{2} Z_{2}]^{T}$$

$$Y = \begin{bmatrix} 0 & 0 & \psi \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
(74)

14.2 Boresight error

Using the method of Sub-section 12.1:

$$A.B.(C+\Delta C).(E + (D+\Delta D).(r+\Delta r).Z) + (A.B-Y).E_2 = A.B.C.(E + D.r.Z.)$$
(75)

After multiplication by  $B^{-1}.A^{-1}/r$ , and neglecting terms of second order of smallness:

$$\Delta C.D.Z + C.\Delta D.Z \simeq -(I-Y).E_2/r$$
(76)

Thus for small values of the elements of  $E_2$  (compared with r, the distance to the target), the boresight error is negligible.

### 14.3 Torque changes in the s-system

Using relations (10) and (77) to determine the change in  $T_s$  due to a non-zero value of  $E_2$ :

$$P = P_{s} = C \cdot P_{r}$$

$$P' = P_{b} = (I+X) \cdot P_{s} + (I+X-Y) \cdot E_{2}$$

$$R = I + X$$

$$L = (I+X-Y) \cdot E_{2}$$
(77)

and:

$$\Delta T_{s} = -\sum_{r,e}^{\prime} m.N(P).R^{-1}.L$$

$$= -N(C.\sum_{r,e}^{\prime} m.P_{r}).(X-Y).E_{2}$$
(78)

Clearly, all terms involve  $\eta$  or  $\psi$ , and none involve time-derivatives of  $\phi$ . Key, and final, results are given in items (a) to (c) of Appendix III. The z-component of  $\Delta T_s$  is the change in torque required of the azimuth motor.

14.4 Torque changes about the elevation axis

Using the relations (12) and (79) to determine the change in the y-component of  $T_p$  due to the non-zero value of  $E_2$ :

79)

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$$P = P_{s} = C.(E + D.P_{e})$$

$$P_{e} = D^{-1}.C^{-1}.P_{s} - D^{-1}.E$$

$$Q = D^{-1}.C^{-1}$$

$$K = -D^{-1}.E$$

$$P', R \text{ from equation (77)}$$

$$(C + D.P_{e})$$

and:

$$\Delta T_{e} = Q.\Delta T_{s} - \sum_{e} m.N(K).Q.R^{-1}.\frac{d^{2}}{dt^{2}} \{ (I+X-Y).E_{2} \}$$

$$\approx D^{-1}.C^{-1}.\Delta T_{s} + M_{e}.D^{-1}.N(E).C^{-1}.(I-X).(X-Y).E_{2}$$

$$\approx D^{-1}.C^{-1}.(\Delta T_{s} + M_{e}.N(C.E).(X-Y).E_{2})$$
(80)

Again, all terms involve  $\ddot{\eta}$  or  $\ddot{\psi}$ , and none involve time-derivatives of  $\phi$ . The y-component of  $\Delta T_e$  is the change in torque required of the elevation motor. Key, and final, results in its evaluation are given in items (d) to (g) of Appendix III.

14.5 Torque changes in roll and pitch

Using relations (12) and (81):

$$Q_{b} = A \cdot B = (I+X)$$

$$K_{b} = A \cdot [0 \ 0 \ Z_{2}]^{T} + A \cdot B \cdot [X_{2} \ Y_{2} \ 0]^{T}$$

$$\approx A \cdot B \cdot [(X_{2} - \psi \cdot Z_{2}) \ Y_{2} \ Z_{2}]^{T}$$

$$P' = P_{b} = (I+X) \cdot P_{s} + K_{b}$$
(81)

then:

$$\Delta T_{b} = (I+X) \cdot \Delta T_{s} - \sum_{r,e} m \cdot N(K_{b}) \cdot \left(G + K_{b} + \frac{d^{2}}{dt^{2}}\left((I+X) \cdot P_{s}\right)\right)$$

$$= \Delta T_{s} - N(K_{b}) \cdot M \cdot G - N(E_{2}) \cdot \left\{M \cdot K_{b} + \sum_{r,e} m \cdot (2X \cdot P_{s} + X \cdot P_{s})\right\}$$

$$- (I+X) \cdot N\{(I-X) \cdot K_{b}\} \cdot \sum_{r,e} m \cdot P_{s} \qquad (82)$$

Similarly, using relations (12) and (83):

$$Q_{g} = B$$

$$K_{g} = B [X_{2} Y_{2} 0]^{T}$$

$$P' = P_{b} = (I+X) \cdot P_{s} + K_{b}$$
(83)

then:

$$\Delta T_{g} = B \cdot \Delta T_{s} - \sum_{\mathbf{r}, \mathbf{e}} m \cdot N(K_{g}) \cdot \left[ G + \ddot{K}_{b} + \frac{d^{2}}{dt^{2}} \{ (\mathbf{I} + \mathbf{X}) \cdot P_{s} \} \right]$$

$$\approx \Delta T_{s} - N(K_{g}) \cdot M \cdot G - N(B^{-1} \cdot K_{g}) \cdot \left[ M \cdot \ddot{K}_{b} + \sum_{\mathbf{r}, \mathbf{e}} m \cdot (2\dot{\mathbf{X}} \cdot \dot{P}_{s} + \ddot{\mathbf{X}} \cdot P_{s}) \right]$$

$$- (\mathbf{I} + \mathbf{X}) \cdot N\{ (\mathbf{I} - \mathbf{X}) \cdot K_{g} \} \cdot \sum_{\mathbf{r}, \mathbf{e}} m \cdot \ddot{P}_{s}$$
(84)

The x-component of  $\Delta T_b$ , and the y-component of  $\Delta T_g$ , are the roll and pitch axis torque variations. Key results of the evaluation of equations (82) and (84) are given in items (h) to (q) of Appendix III. Table 9 lists the results (as well as those for azimuth and elevation, derived previously) except that terms in  $\eta$  and  $\psi$  have been ignored wherever they are swamped by larger terms in the same cell.

#### 14.6 Discussion of torque changes

From Table 1 and relations (22) and (25) we see that H/M, V/M and  $H_e/M$  are small distances in most applications. Also, from equation (31):

- 33 -

$$|\dot{\eta}/\dot{\theta}^{2}|_{max} \approx |\dot{\psi}/\dot{\theta}^{2}|_{max} \approx (\omega^{2}/\dot{\theta}^{2}).\eta_{max}$$
  
 $|\dot{\eta}/\dot{\theta}|_{max} \approx |\dot{\psi}/\dot{\theta}|_{max} \approx (\omega/\dot{\theta}).\eta_{max}$ 

are small quantities in most applications. Hence the terms in Table 9 can be divided into groups of first, second and third order of smallness.

Only the terms in g are of first order and these will probably determine the maximum values of  $X_2$  and  $Y_2$ . Those terms independent of  $\eta$  and  $\psi$ , together with the elevation terms in  $M_e$ , comprise the second order group. These may be significant. The remaining (third order) terms are most likely negligible for small values of  $\eta_{max}$ .

#### 15. NON-NORMALITY OF PITCH AND ROLL AXES

#### 15.1 Misalignment matrix

Consider the Cartesian coordinate system, suffixed g', derived by rotation of the g-system (fixed to the gimbal ring) through the small angle,  $D\theta$ , about the common z-axis. Then:

$$P_{g} \approx M.P_{g}'$$

$$M \approx \begin{bmatrix} 1 & -D\theta & 0 \\ D\theta & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
(85)

Suppose the pitch-correction axis is now transferred from the y-axis of the g-system, to the y-axis of the g'-system. The pitch and roll axes are still coplanar, and intersect at a common point, but this intersection departs from normal by the small angle, D0. Clearly now:

$$P_{\rm b} = A.M.B.P_{\rm s}$$
(86)

and A.B is replaced by A.M.B wherever it occurs.

By evaluation, neglecting terms of second order of smallness:

$$A.M.B \approx \begin{bmatrix} 1 & -D\theta & \psi \\ D\theta & 1 & \eta \\ -\psi & \eta & 1 \end{bmatrix}$$
(87)

Similarly, by evaluation, we find that:

$$A.M.B.C \simeq (A.B.C)'$$
(88)

where the slash denotes replacement of  $\theta$  by ( $\theta$ +D $\theta$ ) wherever it occurs.

15.2 Boresight error

Following the argument discussed in Sub-section 12.1, it is clear that:

$$A.M.B.(C+\Delta C).(D+\Delta D).Z = A.B.C.D.Z$$

if:

$$\Delta \theta \simeq -D \theta$$

$$\Delta \phi \simeq 0$$
(89)

Because  $\Delta \theta$  is constant it can be calibrated out.

15.3 Torque changes

It follows from relations (15), (17) and (18) that replacement of (A.B.C) by (A.B.C)' (ie replacement of  $\theta$  by ( $\theta$ +D $\theta$ ) wherever it occurs) accounts for the changes in T<sub>e</sub>, T<sub>r</sub>, T<sub>elev</sub> and T<sub>az</sub>.

The changes in roll and pitch axis torques follow from:

$$T_{roll} = T_{gx} = [1 \ 0 \ 0] . M. B. C. T_{r}' \simeq T_{roll}'$$
 (90)

$$T_{pitch} = T_{sy} = \{0 \ 1 \ 0\} . C. T_{r}'$$

$$= [0 \ 1 \ 0] . (I + (M^{-1} - I)) . M. C. T_{r}'$$

$$\approx T_{pitch}' - D0 . [1 \ 0 \ 0] . M. B. C. T_{r}'$$

$$\approx T_{pitch}' - D0 . T_{r} (91)$$

15.4 Discussion

Thus the effect of the non-normality is accounted for by replacement of  $\theta$  by ( $\theta$ +D $\theta$ ) in all boresight error and motor torque relations. In addition, the pitch axis receives a component of the roll axis torque, proportional to the angular deviation from normality. This crosscoupling of torque is the only significant effect of the non-normality, and cannot be eliminated by calibration of  $\theta$ .

TABLE 7. CHANGES IN T<sub>e</sub> AND T<sub>r</sub> DUP TO THITED ELEVATION AXIS

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J	- 2¥		-2		4 j		
Q			-2				
к				$-2e^{j2\theta}k$		j2θ −4je <sup>k</sup>	
L	e <sup>jθ</sup> L	j <sup>θ</sup> L je					
н							j <sup>j θ</sup> h j e
V							η+jψ

TABLE 6. COMBINED VALUES OF (T<sub>rol1</sub>+j.T<sub>pitch</sub>) AT FIXED ELEVATION

(T <sub>roll+j.T<sub>pitc</sub></sub>
1.0
ASSEMBLY
ELEVATING
:10
CONTRIBUTION
TABLE 5.

	:0	ت_0 •	:0	ė,	0.0	$\frac{1}{\sqrt{1+n}}$	: <u>.) (- n</u> : <u>.) (</u> - n	$\frac{1}{2}$ , $\frac{1}{2}$ , $\frac{1}{2}$ , $\frac{1}{2}$	$\frac{\eta - i\dot{\psi}}{2}$ , $\dot{\theta}$	$\phi$ $\frac{1}{2}$	$\frac{\dot{n}-j}{2}$ , $\dot{\phi}$	cs.
- <sup>3</sup>	· .		2 j c <sup>j</sup> 0		- 2e <sup>j0</sup>	د	e j 20	ā	2 j e <sup>j</sup> 20			
່ ອ ເ		· · · · · · · · · · · · · · · · · · ·			:	: 	-e <sup>j</sup> 20		-2je <sup>j</sup> 20		• •	
Ke.crs20k					-2ej0		- e j _0	- <sup>2</sup> j	- 2j e <sup>j</sup> 20			
ke.sin20k		- j e j <del>0</del>			32-					^1	2c.j.20	
Le.cosoL			-ejθ	·>			2je <sup>j</sup> 20		-4e <sup>J</sup> 20			
$L_{e}.sin\phi_{L}$	jejθ	-e <sup>j0</sup>	-2 -	e j <i>θ</i>						5	-2jej20	
H <sub>e</sub> .cosoh	_1.e <sup>jθ</sup> -2X1.Ψ	jī ej	jXı.e <sup>j0</sup>	jΞı.e <sup>jθ</sup>		- 2X1	2X <sub>1</sub> e <sup>j</sup> 20	4 j X1	4 j X <sub>1</sub> e <sup>j</sup> 20	- - -		jej0
H <sub>e</sub> .sim¢h	Xı.e <sup>jθ</sup>	jΧ <sub>1</sub> .e <sup>jθ</sup>	jī.ej	-јХı .е <sup>ј0</sup>	-22 <sub>1</sub> .e <sup>jθ</sup> +2X1.ψ	-421				2X1	-2X1.e <sup>j20</sup>	a+jċ
e V e	jī.ej	$-2_1 \cdot e^{j\theta}$					2jX1.e <sup>j20</sup>		-4X1 .e <sup>j</sup> 20			- c <sup>j0</sup>
ž	X1.21.e <sup>jθ</sup> -X1 <sup>2</sup> .ψ	jXı.ï.e <sup>jθ</sup>				$-X_{1}^{2} - 2Z_{1}^{2}$	x <sub>1</sub> <sup>2</sup> . c <sup>-j</sup> 2θ	2 j X1 <sup>2</sup>	2j X <sub>1</sub> <sup>2</sup> . e <sup>j 20</sup>			$jX_1 \cdot e^{j\theta}$ + $\hat{z}_1 (\eta + j\psi)$

# TABLE 4. CONTRIBUTION OF ROTATING ASSEMBLY TO $T_r$

		 Ө	$\dot{\theta}^2$	$\frac{\ddot{\eta}\pm j\ddot{\psi}}{2}.e^{\mp}j\theta$	$\frac{\dot{\eta}\pm j\dot{\psi}}{2}.\dot{\theta}.e^{\mp j\theta}$	g	$\frac{\eta \pm j \psi}{2}$ .g.e <sup>∓</sup> j $\theta$
	J <sub>r</sub>			-1	±2j		
	Q <sub>r</sub>			-1			
Y	ĸŗ			$-e^{\pm j\beta}r$	<sup>±jβ</sup> r ±2je		
Å	<sup>L</sup> r	cosy <sub>r</sub>	-siny <sub>r</sub>				
	Hr					-sina r	
	V <sub>r</sub>						1
	Me	X <sub>1</sub> .Z <sub>1</sub>		-Z1 <sup>2</sup>			Zı
	<sup>J</sup> r			±j	2		
	Q <sub>r</sub>			±j			
v	K <sub>r</sub>			±jβ ∓je <sup>r</sup>	<sup>±jβ</sup> r -2e		
1	L <sub>r</sub>	siny <sub>r</sub>	cosy <sub>r</sub>				
	<sup>H</sup> r					cosa <sub>r</sub>	
	v <sub>r</sub>						Ŧj
	м <sub>е</sub>		X1 .Z1	$\pm j(X_1^2+Z_1^2)$	2X <sub>1</sub> <sup>2</sup>	Xı	∓jZı
	<sup>J</sup> r	-2		·			
	Q <sub>r</sub>						
z	K <sub>r</sub>					· · · · · · · · · · · · · · · · · · ·	
	L <sub>r</sub>			<sup>∓</sup> jγ <sub>r</sub> e			
	H <sub>r</sub>						<sup>∓</sup> ja <sub>r</sub> -e
	v <sub>r</sub>						
	Me	- X1 <sup>2</sup>		X <sub>1</sub> .Z <sub>1</sub>			-X1

TABLE 5. CONTREDUTION OF ELEVATING ASSEMBLY TO T

n* 10		,		I	sinda						-†jsin¢ <sub>h</sub>						- cos¢h	+1
54									1 - - -		h deso							
$\theta_{1}^{\dagger} \stackrel{\circ}{,} $			2sin20k	±2jsim¢ <sub>l</sub>	<sup>1</sup> ;¢soɔ, اב2-						$\pm 2j(z_1 \cdot \cos \phi_h - v_1 \cdot \sin \phi_h)$		5		-2cos2¢ <sub>k</sub>	∓2jcos¢L	2Χ <sub>1</sub> .cosφ <sub>h</sub>	
$a(\cdot, \cdot, \cdot, \cdot, \cdot, \cdot, \cdot)$		[c •		- Jeosop		- 2X1	<b>C</b> 1	               	-2cos20k	72jcosøL	$4X_1 \cdot \cos \phi_h$	∓ 2 j X1						
n' jč. ()	-	7	× 0.7 × 0.7 -	<sup>1</sup> ¢soot <sub>±</sub>	- 21, si <b>nó</b> <sub>h</sub>	- 1 Χ <sup>1</sup>	(7+			cos¢l	$\pm 2j(X_1,\cos\phi_{h}\pm \overline{z}_1,\sin\phi_{h})$	Xı			-sin2¢ <sub>k</sub>	Ŧjsin¢[	Xı.sin¢ <sub>h</sub> +Zı.cos¢ <sub>h</sub>	Ŧj2ı
0. E	- 1 5	•	- 2005 &	· · · · · · · · · · · · · · · · · · ·	- 221 . S1 mg h				1						-2sir.2ø <sub>k</sub>		2X1.sin¢h	
6		• • • • • • •		sind		•					-X <sub>1</sub> .simoh +Z <sub>1</sub> .cosoh					-1 - cos¢		
0				losoo-			~ 1	i i t	1		X <sub>1</sub> .ros¢ <sub>h</sub> +2 <sub>1</sub> .sim¢ <sub>h</sub>					-sim¢L		
				outs-		11			-sin2¢k		X1 .sindh +21 .cosdh							
=		•		• • • •	Å, sind Åesind Åesind	• • •			* 	sind		11	-		205.20 K		معمی. الا <sup>2</sup> - الا	
		نے <del>ا</del> نے ا	ير بر بر	ني _ي	+ =		-°	e O	ي بر بر	۔ س	т <sup>е</sup>	e K	ب م	و م	يو ب	بە 1	πů	<u>ه</u>

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TABLE 2. CONTRIBUTION OF ELEVATING ASSEMBLY TO T<sub>c</sub>

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	:0	θ	.0	φ.	0. ¢	$\frac{\overline{\eta}\pm j\overline{\psi}}{2}, e^{\mp}j\theta$	$\frac{\underline{\dot{n}^{\pm}j\dot{\psi}}}{2}.\dot{\theta}.e^{\mp}j\theta}.$	$\frac{\dot{n}^{\pm}j\dot{\psi}}{2}.\dot{\phi}.e^{\mp}j\theta$	50	$\frac{\eta^{\pm}j\psi}{2}.g.e^{\mp}j\theta$
J.	-sim¢				- 2cos¢	- cos¢		2sin¢		
ھی	-sim¢					- cos¢	±2jcos¢			
پ ج	-sin(\$\vartheta_c)				$-2\cos(\phi-\beta_e)$	- cos (φ-β <sub>e</sub> )		$2\sin(\phi - \beta_e)$		
ں بور ا ×		-sim¢L.cos¢	- cost <sub>e</sub>	sinne		<sup>∓</sup> jcosγ <sub>e</sub>	$-2\cos\phi_{\mathrm{L}},\cos\phi$	±2jsim <sub>e</sub>		
<sup>ب</sup> ا	X <sub>1</sub> .sinde					-Z <sub>1</sub> .sinae				sinde
پ ح		-X <sub>1</sub> .simol				∓j(Z₁.sir¢+X₁.cos¢)	-2Χ <sub>1</sub> .cosφ		-cos¢	±jsim¢
ۍ م			2			±2j	2			
س م										
ب م		-sin2¢k					- 2cos 2¢ k			
و ا ~	sin¢L					<sup>1</sup> øsoo	∓2j.cos¢L			
_e 		Xı.sim¢ <sub>h</sub>				±j(Z <sub>1</sub> .simφ <sub>h</sub> +X <sub>1</sub> cosφ <sub>h</sub> )	2Χι.cosφ <sub>h</sub>		cos¢h	∓jsin¢ <sub>h</sub>
ہ د										
r, e	-cos¢				2sim¢	sim¢		2cos¢		
سي ا	- cos¢					sin¢	∓2jsim¢			
ч К С	$\cos(\phi - \beta_e)$				$-2\sin(\phi-\beta_e)$	-sin(φ-β <sub>e</sub> )		-2cos(φ-β <sub>e</sub> )		
പ്		sim¢ <sub>L</sub> .sim¢	-sim <sub>e</sub>	-cosye		∓jsim∕ <sub>e</sub>	2cosφ <sub>L</sub> .simΦ	∓2jcosγ <sub>e</sub>		
• <sub>±</sub>	-X1 .cosae					Z1.cosa <sub>e</sub>				- cosa
_0 _		-X <sub>1</sub> .cos¢				±j(X <sub>1</sub> .sínφ-Z <sub>1</sub> .cosφ)	2Χ, sinφ		simot	± j cos¢

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Symbol	Section First Used	Defined	Remarks						
α	4	T1	angle associated with H						
α	6	E31	log. increment of spiral motion						
β	4	T1	angle associated with K						
β	12	E64	angle associated with $\mu$						
r	4	T1	angle associated with L						
η	1	-	roll error angle						
θ	1	-	azimuth rotation angle						
μ	12	E64	amplitude of spiral motion						
φ	1	-	elevation angle						
ψ	1	-	pitch error angle						
ω	6	E31	angular frequency of spiral motion						
ωο	6	E29	natural frequency of roll and pitch servo						
*	4	-	complex conjugate operator						
T	3	-	indicates inertial coordinate system						
t	4	-	indicates replacement of $\theta$ by ( $\theta$ + $\pi$ )						
•	15	-	indicates replacement of $\theta$ by ( $\theta$ +D $\theta$ )						
11	3	-	indicates transformed coordinate system						

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Symbol	Section First Used	Defined	Remarks		
C o	6	-	servo damping factor		
d	6	E29	ratio of inertias		
е	2	-	suffix for e-coordinate-system		
f	6	E27	damping unbalance factor		
f	8	-	suffix denoting forcing		
f	8	E42	servo unbalance function		
g	2	-	suffix for g-coordinate-system		
g	3	-	acceleration of gravity		
h	4	T1	suffix for reference to H		
k	4	T1	suffix for reference to K		
1	4	T1	suffix for reference to L		
m	3	-	mass of point quantity		
r	2	-	suffix for r-coordinate-system		
r	12	-	range to target		
s	2	_	suffix for s-coordinate-system		
s	6	E31	complex variable		
t	6	-	time		
u	6	E27	servo stiffness unbalance factor		
x	3	-	Cartesian coordinate		
у	3	-	Cartesian coordinate Cartesian coordinate		
Z	3	-	Cartesian coordinate Cartesian coordinate		
Δ	12	-	Cartesian coordinate operator specifying a change in argume		
Δ0	12	-	azimuth error		
Δφ	12	-	elevation error		
Σe	4	-	summation operator over elevating assembly		
Σ <sub>r</sub>	4	-	summation over rotating assembly		
۲ <sub>.</sub> г,е	4	-	summation over rotating and all elevating assemblies		

Symbol	Section First Used	Defined	Remarks
0	2	-	origin of Cartesian axes
Р	2	-	point coordinate vector
Q	3	E 1 1	transformation matrix
Q	4	Τ1	mass distribution parameter
R	3	E5	transformation matrix
Т	3	-	torque
Т	2	-	superfix denoting matrix transposition
$T_{az}$	9	-	torque about azimuth axis
Telev	8	-	torque about elevation axis
т <sub>м</sub>	8	-	motor torque
T pitch	4	-	torque about pitch axis
T <sub>rol1</sub>	4	-	torque about roll axis
U	6	E27	servo stiffness factor
v	4	T1	mass distribution parameter
x	2	-	Cartesian axis
х	4	E13	pitch-roll matrix approximation
X1	2	-	elevation axis offset component
X <sub>2</sub>	14	-	pitch-roll axis offset
Y	2	-	Cartesian axis
Y	14	E74	matrix approximation
Y <sub>2</sub>	14	-	pitch-roll axis offset
Z	2	-	Cartesian axis
Z	12	-	[1 0 0] <sup>T</sup>
Z <sub>1</sub>	2	-	elevation axis offset component
Z <sub>2</sub>	14	-	pitch-roll axis offset
b	2	-	suffix for b-coordinate-system

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## NOTATION

Symbol	Section First Used	Defined	Remarks
A	2	E 1	roll correction matrix
A	6	E31	complex amplitude of spiral deflection
В	2	E 1	pitch correction matrix
В	6	E32	complex amplitude of spiral deflection
С	2	E 1	azimuth rotation matrix
D	2	E 1	elevation matrix
Dŋ	13	-	elevation axis deflection angle
Dŋ	16	-	azimuth axis deflection angle
D0	13	-	elevation axis deflection angle
D <b>0</b>	15	-	deflection angle of pitch axis
E	2	E 1	elevation offset vector
E <sub>2</sub>	14	E74	pitch-roll offset vector
F	3	-	point mass force vector
F	6	E27	servo damping factor
G	3	E6	gravitation vector
G	6	E32	servo transfer function
н	4	T 1	mass distribution parameter
I	4	-	unit matrix
J	4	Τ1	mass distribution parameter
К	3	E11	offset vector for transformation
К	4	Τ1	mass distribution parameter
L	3	E5	offset vector
L	4	Τ1	mass distribution parameter
Μ	4	Τ1	mass of structure
M	13	E68	elevation axis deflection matrix
М	15	E85	pitch-roll deflection matrix
М	16	E92	pitch-azimuth deflection matrix
N	3	E3	torque generation matrix

to back about the axis of azimuth rotation, has been examined. Clear rules are given for determining whether the torques about the azimuth, pitch and roll axes, due to mass unbalance parameters of the nominally identical structures, cancel each other, or are additive. For many of the parameters, cancellation occurs only if the two assemblies are elevated identically and simultaneously.

The results of the analysis, presented in tabular and graphical form, provide a firm basis for the design of the servo drives and of the procedure and facilities for achieving the necessary degree of mass balance.

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The motion of the stabilised platform can be described in terms of a sequence of small amplitude spiral nutations of the azimuth axis about the true vertical direction.

Two of the main components consist of the transient decay of an initial deflection of the azimuth axis from vertical. One of these is in the forward direction (the same direction as the azimuth scanning rotation) and the other is in the reverse direction. They are mirror images, although executed at widely different rates. The actual angular velocity of the nutations depends on the stiffness and damping factor of the pitch and roll servo loops, the rate of azimuth rotation, and the distribution of mass of the combined rotating and elevating assemblies.

The third main component is a non-decaying (circular) nutation in the forward direction, and at the same rate as the azimuth scanning rotation. Its amplitude depends on the degree of unbalance in the distribution of mass, as well as on the servo stiffness and damping, and the azimuth rate.

Each of the three main components spawns two minor components proportional to its amplitude. One, at the same rate and opposite direction to the parent nutation, is proportional to the unbalance in the stiffness and damping of the pitch and roll servo loops. The other, at a rate and direction depending upon both that of the parent nutation and of the scanning rotation, is proportional to another measure of the unbalance in the mass distribution.

Each of the minor components, in turn, spawns two more minor components, ad infinitum, but the amplitudes of all these are negligible for modest amounts of unbalance.

Many imperfections of mass balance and axis geometry have an effect on the accuracy of the boresight direction, but most are negligibly small, or are cancelled out during the calibration process. Those that have a significant residual effect (and therefore require appropriate control) are:

(a) the instantaneous amplitude of the total nutation of the azimuth axis about the vertical

(b) rotation of the elevation axis from its nominal position, in the vertical plane

(c) non-normality of the pitch and azimuth axes.

$$T_{roll} = T_{gx} = [1 \ 0 \ 0] . B.M.C.T_{r}'$$
$$= [1 \ 0 \ 0] . A.B.M.C.T_{r}'$$
$$= T_{roll}'$$
(95)

$$T_{pitch} = T_{sy} = [0 \ 1 \ 0] . M.C.T_{r}'$$
  
= [0 \ 1 \ -Dn] . C.T\_{r}'  
= T\_{pitch}' - Dn . T\_{az}' (96)

#### 16.4 Discussion

Thus the effect of the non-normality is accounted for by replacing  $\eta$  by  $(\eta+D\eta)$  in all boresight error and motor torque relations. In addition, the pitch axis receives a component of the azimuth axis torque proportional to the angular deviation from normality. This cross coupling of torque is the only significant effect of the non-normality, as it cannot be eliminated by the calibration of  $\eta$ .

#### 17. CONCLUSION

The four-axis stabilised director system, while mounted in the normal erect position on a firm foundation, has been analysed to determine the driving torques, boresight errors, and motion of the stabilised platform caused by a small:

(a) deflection of the stabilised platform from the horizontal position

(b) unbalanced mass distribution on the rotating and elevating assemblies

(c) unbalance between the gain parameters of the roll and pitch correction servo loops

(d) deviations from the ideal geometrical relationships between the axes.

Techniques are presented for factorising the complex matrix relationships in such a way as to greatly simplify their evaluation.

The approximation of trigonometric functions of the roll and pitch angles by first order functions of the angles themselves (justified by the assumption that these angles are always small) greatly simplifies the interpretation of the results of the analysis by reducing significantly the number of terms contributing to each relationship. Conclusions drawn in respect of a particular application should be carefully checked for validity; if necessary using the accurate relationships. This is particularly important if the analysis were to be extended to the case of the director system mounted on a pitching and rolling vehicle.

The case of two separately elevatable assemblies, mounted symmetrically back

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#### 16. NON-NORMALITY OF PITCH AND AZIMUTH AXES

#### 16.1 Misalignment matrix

Consider the Cartesian coordinate system, suffixed s', derived by rotation of the s-system (fixed to the stabilised platform) through the small angle, Dn, about the common x-axis. Then:

$$P_{s} = M.P_{s}'$$

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -D\eta \\ 0 & D\eta & 1 \end{bmatrix}$$
(92)

Suppose the azimuth axis is now transferred from the z-axis of the s-system to the z-axis of the s'-system. Then the azimuth and pitch axes intersect at an angle departing from normality by the small angle,  $D\eta$ . Clearly now:

$$P_{\rm b} = A.B.M.C.P_{\rm r} \tag{93}$$

By evaluation, neglecting terms of second order of smallness:

A.B.M 
$$\simeq$$
  $\begin{bmatrix} 1 & 0 & \psi \\ 0 & 1 & -(\eta + D\eta) \\ \psi & (\eta + D\eta) & 1 \end{bmatrix}$  (94)  
 $\simeq$  (A.B)'

where the slash denotes replacement of  $\eta$  by  $(\eta+D\eta)$  wherever it occurs.

16.2 Boresight error

From equation (94), the effect of D $\eta$  is clearly the same as for the same magnitude of  $\eta$ , as determined in Sub-section 12.1.

16.3 Torque changes

It follows from relations (15), (17) and (18) that replacement of (A.B) by (A.B)' (ie replacement of  $\eta$  by  $(\eta+D\eta)$  wherever it occurs) accounts for the changes in  $T_e$ ,  $T_r$ ,  $T_{elev}$  and  $T_{az}$ . The changes in roll and pitch axis torques follow from:

TABLE 8. CHANGES IN (Troll+j.T<sub>pitch</sub>) DUE TO TILTED ELEVATION AXIS

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g.ejθ					Dm.sin¢ <sub>h</sub> -D0.cos¢ <sub>h</sub>	6 <b>0</b> .j-
$\dot{\theta}$ . $\dot{\phi}$ . $\dot{\theta}$	- 2jD0		2jDM.sin2¢ <sub>k</sub> -2jD0.cos2¢ <sub>k</sub>	-2Dm.sin¢ <sub>L</sub>	-2jDm.Z <sub>1</sub> .cos¢ <sub>h</sub> -2jD0.Z <sub>1</sub> .sin¢ <sub>h</sub>	
$\dot{\phi}^2$ . $e^{j\theta}$				jDn.cos¢ <sub>L</sub> +jD0.sin¢ <sub>L</sub>	Dm.Z <sub>1</sub> .sinφ <sub>h</sub> -D0.Z <sub>1</sub> .cosφ <sub>h</sub>	
	-2109			jD <i>m</i> .sin¢ <sub>L</sub> -jD0.cos¢ <sub>L</sub>	-D <i>n</i> .Zı.cos¢ <sub>h</sub> -D0.Zı.sin¢ <sub>h</sub>	
$\dot{\theta}^2$ , $e^{j\theta}$	μ	- Dn	D <i>n</i> .cos2\$ +D0.sin2\$_k	jDm.cos¢ <sub>L</sub> -jD0.sin¢ <sub>L</sub>	D <i>n</i> .Z <sub>1</sub> .sin¢ <sub>h</sub> -D0.Z <sub>1</sub> .cos¢ <sub>h</sub>	j Dm.Xı -j D0.Zı
$\theta$ .ej $\theta$	- j Dn	jDn	-jDM.cos2¢ <sub>k</sub> -jDØ.sin2¢ <sub>k</sub>	Dn.cos¢ <sub>L</sub> -D0.sin¢ <sub>L</sub>	-jDn.Zı.sin¢ <sub>h</sub> +jD0.Zı.cos¢ <sub>h</sub>	Dη.X <sub>1</sub> -Dθ.Z <sub>1</sub>
	Je	о <sup>е</sup>	к е	e Le	н <sup>ө</sup>	e C

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ROLL
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PITCH
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\$ ``				2simon.	$(x \cdot \sin\theta)$				2Ns.cosθ. simφ			   	
n. ¢								• • •	-2Xe.sin0. sin0h			• • •	
6.7		-2Y2.sin0h -2X2.cos0h					2X2.sin <sup>0</sup> h	1				-	
0.1							$2\chi_2 \cdot \cos\theta_{\rm h}$						
:->	X2 . Υ2	$\gamma_2$ , $\cos\theta_h$				- X <sub>2</sub> <sup>2</sup>	$-2X_2 \cdot \cos\theta_h$				-X1.cosa		X1 .X2
u	$-(2^{2}_{2}+Y^{2}_{2})$	$-\gamma_2$ . sin $\theta_h$	י קי י י			X2 .Y2	$Y_2$ .cos $\theta_h$ + $\chi_2$ .sin $\theta_h$			$z_2 \cdot \cos \theta_h$	Y2.cosa	Zz.sim0.	-Z <sub>1</sub> .Z <sub>2</sub> .sin <del>0</del> -X <sub>1</sub> .Y <sub>2</sub>
0.0				-222. cosθ.	sindh				-2π.Χ <sub>2</sub> .cosθ. sim¢h				
\$				-2sind.	ц cosф				-n.Xz.sin0. cos¢ <sub>h</sub>				
: <del>0</del>		· · · · · · · · · · · · · · · · · · ·		-22.sin0.	sin¢				- <i>n.X</i> z.sin <sup>g</sup> . sin¢ <sub>h</sub>				
θ2		-22 . sind	· · · · · · · · · · · · · · · · · · ·				-n.X2.sin <sup>0</sup> h						
θ		ی. ت ا					n.Xz.cos <sup>0</sup> h						
	7.	H		Ξ°		×	Ξ	2	т <sup>о</sup>	н	Ξ	~	×e
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#### APPENDIX I

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# KEY RESULTS IN THE EVALUATION OF $\mathbf{T}_{\mathbf{e}}$ and $\mathbf{T}_{\mathbf{r}}$

Making the substitution:

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-	\$110
=	cosθ
=	sin¢

$$d = \cos\phi$$

(a) 
$$C^{-1}.X.C = \begin{bmatrix} 0 & 0 & -(a.\eta-b.\psi) \\ 0 & 0 & -(b.\eta+a.\psi) \\ (a.\eta-b.\psi) & (b.\eta+a.\psi) & 0 \end{bmatrix}$$
  
$$= \frac{1}{2}(\eta+j.\psi).e^{-j\theta} \cdot \begin{bmatrix} 0 & 0 & -j \\ 0 & 0 & -1 \\ j & 1 & 0 \end{bmatrix} + \frac{1}{2}(\eta-j.\psi).e^{j\theta} \cdot \begin{bmatrix} 0 & 0 & j \\ 0 & 0 & -1 \\ -j & 1 & 0 \end{bmatrix}$$

Note that the second term is the complex conjugate of the first term.

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(b) 
$$C^{-1}.X.C = \frac{1}{2}(\eta+j.\psi).e^{-j\theta}.\theta.\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & -j & 0 \end{bmatrix} + Conjugate$$

(c) 
$$C^{-1} \cdot \dot{C} = \dot{\theta} \cdot \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
  
(d)  $C^{-1} \cdot \ddot{C} = \ddot{\theta} \cdot \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \dot{\theta}^{2} \cdot \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ 

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$$(e) \quad p^{-1} \cdot \dot{p} = \dot{\phi} \cdot \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$(f) \quad R^{-1} \cdot \dot{R} = p^{-1} \cdot C^{-1} \cdot (I - X) \cdot (\dot{X} \cdot C \cdot D + (I + X) \cdot (\dot{C} \cdot D + C \cdot \dot{D}))$$

$$= p^{-1} \cdot C^{-1} \cdot \dot{X} \cdot C \cdot D + p^{-1} \cdot C^{-1} \cdot \dot{C} \cdot D + p^{-1} \cdot \dot{D}$$

$$= \frac{1}{2} (\dot{n} + j \cdot \dot{\psi}) \cdot e^{-j\theta} \cdot \begin{bmatrix} 0 & c & -j \\ -c & 0 & -d \\ j & d & 0 \end{bmatrix} + \dot{\phi} \cdot \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$+ \dot{\theta} \cdot \begin{bmatrix} 0 & -d & 0 \\ d & 0 & -c \\ 0 & c & 0 \end{bmatrix} + \dot{\phi} \cdot \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$(g) \quad \sum_{e} m \cdot N(P) \cdot R^{-1} \cdot \dot{R} \cdot P =$$

$$\frac{1}{2} (\dot{n} + j \cdot \dot{\psi}) \cdot e^{-j\theta} \cdot \begin{bmatrix} (J_{e} + Q_{e}) \cdot d + K_{e} \cdot \cos(\phi - \beta_{e}) + j \cdot L_{e} \cdot \cos \theta_{e} \\ -L_{e} \cdot \cos(\phi + \chi_{e}) - 2j \cdot J_{e} \\ -(J_{e} + Q_{e}) \cdot c + K_{e} \cdot \sin(\phi - \beta_{e}) + j \cdot L_{e} \cdot \sin \chi_{e} \end{bmatrix} + Conjugate$$

$$+ \dot{\theta} \cdot \begin{bmatrix} (J_{e} + Q_{e}) \cdot c + K_{e} \cdot \sin(\phi - \beta_{e}) \\ -L_{e} \cdot \sin(\phi + \chi_{e}) \\ (J_{e} + Q_{e}) \cdot d - K_{e} \cdot \cos(\phi - \beta_{e}) \end{bmatrix} + \dot{\phi} \cdot \begin{bmatrix} L_{e} \cdot \cos \chi_{e} \\ -2J_{e} \\ L_{e} \cdot \sin \chi_{e} \end{bmatrix}$$

$$(h) \quad \sum_{e} m \cdot N(P) \cdot D^{-1} = \begin{bmatrix} -c \cdot V_{e} & -H_{e} \cdot \sin \alpha_{e} & d \cdot V_{e} \\ H_{e} \cdot \sin(\phi + \alpha_{e}) & 0 & -H_{e} \cdot \cos(\phi + \alpha_{e}) \\ -d \cdot V_{e} & H_{e} \cdot \cos \alpha_{e} & -c \cdot V_{e} \end{bmatrix}$$

(i) 
$$D.R^{-1}.G = C^{-1}.(I-X).G$$

$$= \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} + \frac{1}{2}(\eta+j.\psi).e^{-j\theta} \cdot \begin{bmatrix} j.g \\ g \\ 0 \end{bmatrix} + Conjugate$$

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(j) 
$$D.R^{-1}.\ddot{L} = C^{-1}.(I-X).(\ddot{X}.C.E + 2\dot{X}.\dot{C}.E + (I+X).E)$$
  

$$= \{C^{-1}.\ddot{X}.C + 2C^{-1}.\dot{X}.\dot{C} + C^{-1}.\ddot{C}\}.E$$

$$= \frac{1}{2}(\ddot{n}+j.\ddot{\psi}).e^{-j\theta}.\left[-j.Z_{1}\right] + Conjugate$$

$$+ (\dot{n}+j.\dot{\psi}).e^{-j\theta}.\dot{\theta}.\left[0\right] + Conjugate$$

$$+ \ddot{\theta}.\left[0\right] + \dot{\theta}^{2}.\left[-X_{1}\right]$$

$$+ \ddot{\theta}.\left[0\right] + \dot{\theta}^{2}.\left[-X_{1}\right]$$

$$0$$

(k) N(E) = 
$$\begin{bmatrix} 0 & -Z_1 & 0 \\ Z_1 & 0 & -X_1 \\ 0 & X_1 & 0 \end{bmatrix}$$

(1) 
$$M_e.E + \sum_{e} m.D.P_e = M_e.X_1 + H_e.\cos\phi_h$$
  
 $V_e$   
 $M_e.Z_1 + H_e.\sin\phi_h$ 

(m) 
$$\sum_{\mathbf{e}} \mathbf{m} \cdot \mathbf{D} \cdot \mathbf{P}_{\mathbf{e}} = \phi \cdot \begin{bmatrix} -\mathbf{H}_{\mathbf{e}} \cdot \sin\phi_{\mathbf{h}} \\ 0 \\ \mathbf{H}_{\mathbf{e}} \cdot \cos\phi_{\mathbf{h}} \end{bmatrix}$$

(o) N(E).C<sup>-1</sup>.X.C = 
$$\frac{1}{2}(\eta+j.\psi).e^{-j\theta}$$
.  
 $\begin{bmatrix} 0 & 0 & Z_1 \\ -j.X_1 & -X_1 & -j.Z_1 \\ 0 & 0 & -X_1 \end{bmatrix}$  + Conjugate

(p) N(E).C<sup>-1</sup>.X.C. = 
$$\frac{1}{2}(n+j.\psi).\theta.e^{-j\theta}$$
.  $\begin{bmatrix} 0 & 0 & 0 \\ -X_1 & j.X_1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$  + Conjugate

(r) N(E).C<sup>-1</sup>.X.C = 
$$\frac{1}{2}(\eta+j.\psi).e^{-j\theta}$$
.  $\begin{bmatrix} 0 & 0 & Z_1 \\ -j.X_1 & -X_1 & -j.Z_1 \\ 0 & 0 & -X_1 \end{bmatrix}$  + Conjugate

(s) N(E). $C^{-1}.C = \theta$ .  $\begin{bmatrix} -Z_1 & 0 & 0 \\ 0 & -Z_1 & 0 \\ X_1 & 0 & 0 \end{bmatrix}$ 

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(t) 
$$m.N(K).Q.R^{-1}.G = M_e.N(E).C^{-1}.(I-X).G$$

$$(\mathbf{u}) \sum_{\mathbf{e}} \mathbf{m}.\mathbf{N}(\mathbf{K}).\mathbf{Q}.\mathbf{R}^{-1}.\mathbf{P}^{\dagger} =$$

$$\mathbf{N}(\mathbf{E}).(\mathbf{C}^{-1}.\mathbf{X}.\mathbf{C} + 2\mathbf{C}^{-1}.\mathbf{X}.\mathbf{C} + \mathbf{C}^{-1}.\mathbf{C}).\left(\mathbf{M}_{\mathbf{e}}.\mathbf{E} + \sum_{\mathbf{e}} \mathbf{m}.\mathbf{D}.\mathbf{P}_{\mathbf{e}}\right)$$

$$+ 2\mathbf{N}(\mathbf{E}).(\mathbf{C}^{-1}.\mathbf{X}.\mathbf{C} + \mathbf{C}^{-1}.\mathbf{C}).\sum_{\mathbf{e}} \mathbf{m}.\mathbf{D}.\mathbf{P}_{\mathbf{e}} + \mathbf{N}(\mathbf{E}).\sum_{\mathbf{e}} \mathbf{m}.\mathbf{D}.\mathbf{P}_{\mathbf{e}}$$

(v) 
$$R^{-1}.R = C^{-1}.X.C + C^{-1}.C$$

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$$(\mathbf{w}) \sum_{\mathbf{r}} \mathbf{m} \cdot \mathbf{N}(\mathbf{P}) \cdot (\mathbf{R}^{-1} \cdot \mathbf{R} \cdot \mathbf{P} + \mathbf{P}) =$$

$$\stackrel{\dot{\theta}}{\overset{-L_{\mathbf{r}} \cdot \cos\delta_{\mathbf{r}}}{\overset{-L_{\mathbf{r}} \cdot \sin\delta_{\mathbf{r}}}{\overset{-L_{\mathbf{r}} \cdot \sin\delta_{\mathbf{r}}}{\overset{-L_{\mathbf{r}} \cdot \sin\delta_{\mathbf{r}}}{\overset{-L_{\mathbf{r}} \cdot \sin\delta_{\mathbf{r}}}{\overset{-L_{\mathbf{r}} \cdot \sin\delta_{\mathbf{r}}}{\overset{-L_{\mathbf{r}} \cdot \sin\delta_{\mathbf{r}}}}} = \underbrace{\int_{\mathbf{J}_{\mathbf{r}}}^{J_{\mathbf{r}} + Q_{\mathbf{r}} + K_{\mathbf{r}} \cdot \cos\delta_{\mathbf{r}} + j \cdot K_{\mathbf{r}} \cdot \sin\delta_{\mathbf{r}}}{\overset{-j(J_{\mathbf{r}} + Q_{\mathbf{r}}) + j \cdot K_{\mathbf{r}} \cdot \cos\delta_{\mathbf{r}} - K_{\mathbf{r}} \cdot \sin\delta_{\mathbf{r}}}{\overset{j \cdot L_{\mathbf{r}} \cdot \sin\delta_{\mathbf{r}} - L_{\mathbf{r}} \cdot \cos\delta_{\mathbf{r}}}}$$

## + Conjugate

$$(\mathbf{x}) \sum_{\mathbf{r}} \mathbf{m} \cdot \mathbf{N}(\mathbf{P}) = \begin{bmatrix} \mathbf{0} & -\mathbf{V}_{\mathbf{r}} & \mathbf{H}_{\mathbf{r}} \cdot \sin \alpha_{\mathbf{r}} \\ \mathbf{V}_{\mathbf{r}} & \mathbf{0} & -\mathbf{H}_{\mathbf{r}} \cdot \cos \alpha_{\mathbf{r}} \\ -\mathbf{H}_{\mathbf{r}} \cdot \sin \alpha_{\mathbf{r}} & \mathbf{H}_{\mathbf{r}} \cdot \cos \alpha_{\mathbf{r}} & \mathbf{0} \end{bmatrix}$$

(y) 
$$R^{-1}.G = C^{-1}.(I-X).G$$

$$\begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -L_{e} \cdot \cos x_{e} \\ 2J_{e} \\ -L_{e} \cdot \sin x_{e} \end{bmatrix}$$

$$(c) \quad D. \Delta (R^{-1}) = D. D^{-1} \cdot M^{-1} \cdot C^{-1} - D. D^{-1} \cdot C^{-1}$$

$$\approx - (M-I) \cdot C^{-1}$$

$$(d) \quad D. \Delta (R^{-1}) \cdot G \approx D\eta \cdot g \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$(e) \quad D. \Delta (R^{-1}) \cdot \ddot{L} \approx - (M-I) \cdot C^{-1} \cdot \ddot{C} \cdot E$$

$$\approx \ddot{\theta} \cdot X_{1} \cdot \begin{bmatrix} D\theta \\ 0 \\ -D\eta \end{bmatrix} + \dot{\theta}^{2} \cdot X_{1} \cdot \begin{bmatrix} 0 \\ D\theta \\ 0 \end{bmatrix}$$

$$\simeq D^{-1} \cdot \{C^{-1} \cdot C \cdot (M-I) - (M-I) \cdot C^{-1} \cdot C\} \cdot D$$
$$\simeq D\eta \cdot \dot{\theta} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

(a)  $\Delta(R^{-1},\dot{R}) = D^{-1}.M^{-1}.C^{-1}.(\dot{C}.M.D + C.M.\dot{D}) - D^{-1}.C^{-1}.(\dot{C}.D + C.\dot{D})$ 

KEY RESULTS IN THE EVALUATION OF EQUATIONS (72) AND (73)

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$$(f) \quad \Delta(Q) = (M-I).D$$

$$= D\eta \cdot \begin{bmatrix} 0 & 0 & 0 \\ -c & 0 & -d \\ 0 & 1 & 0 \end{bmatrix} + D\theta \cdot \begin{bmatrix} 0 & -1 & 0 \\ d & 0 & -c \\ 0 & 0 & 0 \end{bmatrix}$$

$$(g) Q = D = \begin{bmatrix} d & 0 & -c \\ 0 & 1 & 0 \\ c & 0 & d \end{bmatrix}$$

(h) 
$$\Delta(Q.R^{-1}) = M.D.D^{-1}.M^{-1}.C^{-1} - D.D^{-1}.C^{-1} = 0$$

(i) 
$$\Delta(Q.R^{1}.P') = \Delta(C^{1}.\frac{d^{2}}{dt^{2}}\{C.E + C.M.D.P_{e}\})$$
  
=  $C^{1}.\frac{d^{2}}{dt^{2}}\{C.(M-I).D.P_{e}\}$ 

$$= C^{-1} \cdot \frac{d}{dt} \{ (\dot{C} + C \cdot \frac{d}{dt}) ((M-I) \cdot D \cdot P_{e}) \}$$
  
=  $(C^{-1} \cdot \dot{C} + \frac{d}{dt}) \{ (C^{-1} \cdot \dot{C} + \frac{d}{dt}) \{ (M-I) \cdot D \cdot P_{e} \} \}$ 

$$(j) \sum_{e} m.(M-I).D.P_{e}$$

$$= D\eta. \begin{bmatrix} 0\\ -H_{e}.\sin\phi_{h}\\ V_{e} \end{bmatrix} + D\theta. \begin{bmatrix} -V_{e}\\ H_{e}.\cos\phi_{h}\\ 0 \end{bmatrix}$$

$$(k) N(K) = N(E) = \begin{bmatrix} 0 & -Z_{1} & 0\\ Z_{1} & 0 & -X_{1}\\ 0 & X_{1} & 0 \end{bmatrix}$$

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## APPENDIX III

## KEY RESULTS IN EVALUATION OF SECTION 14

(a) 
$$C.\sum_{r,e} m.P_r = [H.\cos\theta H.\sin\theta V]^T$$

(b) 
$$(X-Y) \cdot E_2 = [0 - \eta \cdot Z_2 (\eta \cdot Y_2 - \psi \cdot X_2)]^T$$

(c) 
$$\Delta \Gamma_{s} = \eta \cdot \begin{bmatrix} -V \cdot Z_{2} - Y_{2} \cdot H \cdot \sin \theta_{h} \\ Y_{2} \cdot H \cdot \cos \theta_{h} \\ Z_{2} \cdot H \cdot \cos \theta_{h} \end{bmatrix} + \psi \cdot \begin{bmatrix} X_{2} \cdot H \cdot \sin \theta_{h} \\ -X_{2} \cdot H \cdot \cos \theta_{h} \\ 0 \end{bmatrix}$$

(d)  $[0\ 1\ 0].D^{-1}.C^{-1} = [-\sin\theta\ \cos\theta\ 0]$ 

(e) C.E = 
$$[X_1.\cos\theta X_1.\sin\theta Z_1]^T$$

(f) 
$$[0\ 1\ 0].D^{-1}.C^{-1}.N(C.E) = [Z_1.\cos\theta\ Z_1.\sin\theta\ -X_1]$$

(g) 
$$\Delta T_{elev} = \eta \cdot \{ (V - M_e \cdot Z_1) \cdot Z_2 \cdot \sin \theta - Y_2 \cdot (M_e \cdot X_1 - H \cdot \cos \alpha) \}$$
  
+  $\psi \cdot X_2 \cdot (M_e \cdot X_1 - H \cdot \cos \alpha)$ 

(h) 
$$\nabla m.P_{s} = \begin{bmatrix} H.\cos\theta_{h} \\ H.\sin\theta_{h} \\ V \end{bmatrix}$$
  
(i)  $\int m.\dot{P}_{s} = \begin{bmatrix} -\dot{\theta}.H.\sin\theta_{h} - \dot{\phi}.H_{e}.\cos\theta.\sin\phi_{h} \\ \dot{\theta}.H.\cos\theta_{h} - \dot{\phi}.H_{e}.\sin\theta.\sin\phi_{h} \\ 0 \end{bmatrix}$ 

$$(\mathbf{j}) \sum_{\mathbf{r},\mathbf{e}} \mathbf{m}.\ddot{\mathbf{P}}_{\mathbf{s}} = \begin{bmatrix} -\ddot{\theta}.\mathrm{H.sin}\theta_{\mathbf{h}} - \dot{\theta}^{2}.\mathrm{H.cos}\theta_{\mathbf{h}} + 2\dot{\theta}.\dot{\phi}.\mathrm{H}_{\mathbf{e}}.\mathrm{sin}\theta.\mathrm{sin}\phi_{\mathbf{h}} \\ - \ddot{\phi}.\mathrm{H}_{\mathbf{e}}.\mathrm{cos}\theta.\mathrm{sin}\phi_{\mathbf{h}} - \dot{\phi}^{2}.\mathrm{H}_{\mathbf{e}}.\mathrm{cos}\theta.\mathrm{cos}\phi_{\mathbf{h}} \end{bmatrix}$$
$$\ddot{\theta}.\mathrm{H.cos}\theta_{\mathbf{h}} - \dot{\theta}^{2}.\mathrm{H.sin}\theta_{\mathbf{h}} - 2\dot{\theta}.\dot{\phi}.\mathrm{H}_{\mathbf{e}}.\mathrm{cos}\theta.\mathrm{sin}\phi_{\mathbf{h}} \\ - \ddot{\phi}.\mathrm{H}_{\mathbf{e}}.\mathrm{sin}\theta.\mathrm{sin}\phi_{\mathbf{h}} - \dot{\phi}^{2}.\mathrm{H}_{\mathbf{e}}.\mathrm{sin}\theta.\mathrm{cos}\phi_{\mathbf{h}} \end{bmatrix}$$

(k) 
$$K_{b} = \begin{bmatrix} X_{2} \\ Y_{2} - \eta \cdot Z_{2} \\ Z_{2} + \eta \cdot Y_{2} - \psi \cdot X_{2} \end{bmatrix}$$

(1)	[1 0 0].N(K <sub>b</sub> )	$= [0 - (Z_{2}+\eta, Y_{2}-\psi, X_{2}) (Y_{2}-\eta, Z_{2})]$
(m)	[1 0 0].N(E <sub>2</sub> ).X	$= [-\psi \cdot Y_2 - (Z_2 - \psi \cdot X_2) Y_2]$
(n)	[1 0 0].(I+X).N{(I-X).K <sub>b</sub> }	$= [-\psi . Y_{2} - (Z_{2} - \psi . X_{2}) Y_{2}]$
(0)	[0 1 0].N(K <sub>g</sub> )	$= \{-\psi X_2  0  -X_2\}$
(p)	[0 1 0].N(B <sup>-1</sup> .K <sub>g</sub> ).X	$= [\psi \cdot X_2 - \eta \cdot X_2 - X_2]$
(q)	[0 1 0].(I+X).N{(I-X).K <sub>g</sub> }	$= [0 - \eta X_2 - X_2]$

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(c)











Figure 2. Properties of transient solution of stabilisation system

WSRL-0355-TR Figures 3(a) & (b)





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### 16 SUMMARY OR ABSTRACT:

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The four-axis, elevation over azimuth over pitch over roll, director mount is analysed to determine the torques generated by mass unbalances about the motor driven axes. The effect of these on the motion of the stabilised platform, and the effect of this motion, as well as that of mechanical tolerances, on the accuracy of direction is examined. The analysis is restricted to the case of the fully erect system, firmly connected to a stationary, level surface, but the method of approach could be extended to more complex situations. The results provide the basis for the design of small, lightweight, high performance directors.
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WSRL-0355-TR Figure 4

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WSRL-0355-TR Figures 3(d) & (e)



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WSRL-0355-TR Figures 3(d) & (e)



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WSRL-0355-TR Figure 3(c)

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Figure 3(Contd.).

