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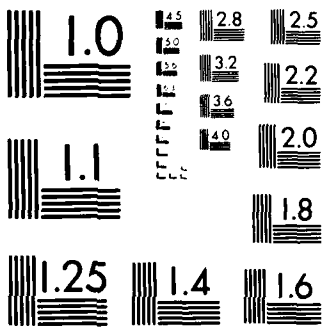

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Stochastic point processes are models of points distributed randomly in some space; these points may represent, for example, locations (or even trajectories) of tracked objects, times and amounts of precipitation events, or failure times and modes of a complex system. This research project is directed toward two principal problems arising in applications of point processes: statistical inference for point processes whose probability law is unknown entirely or in part, and state estimation for partially observed point processes, i.e., minimum mean squared error reconstruction, realization-by-realization, of random variables that are not directly observable. These problems are examined in several (not disjoint) contexts: stationary point processes, Cox processes, multiplicative intensity processes and Poisson processes. Another thrust of the research is inference for stochastic processes based on point process samples, with the particular goal to investigate inference and state estimation for random fields given point process samples. This report documents results in the research for this period.

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I. SUMMARY OF RESEARCH OBJECTIVES

Stochastic point processes are models of points distributed randomly in some space; these points may represent, for example, locations (or even trajectories) of tracked objects, times and amounts of precipitation events, or failure times and modes of a complex system. This research project is directed toward two principal problems arising in applications of point processes: statistical inference for point processes whose probability law is unknown entirely or in part, and state estimation for partially observed point processes, i.e., minimum mean squared error reconstruction, realization-by-realization, of random variables that are not directly observable. These problems are examined in several (not disjoint) contexts: stationary point processes, Cox processes, multiplicative intensity processes and Poisson processes. Another thrust of the research is inference for stochastic processes based on point process samples, with the particular goal to investigate inference and state estimation for random fields given point process samples.

II. RESEARCH ACCOMPLISHMENTS

Research during 1984 has focussed on six major problem areas.

A) Inference for Stationary Point Processes. Let N be a stationary point process on $E=\mathbb{R}^d$, $d\geq 1$, and consider the statistical model $\mathcal{P}$ of probability measures P under which N is *stationary*: $N \overset{d}{=} N\tau_x^{-1}$ for each x, where τ_x is the translation $y \to y-x$. For each $P\in\mathcal{P}$ let P^* be the *Palm measure*, i.e., the unique σ-finite measure such that for each (bounded, measurable) function H,

$$(1)\qquad E\left[\int H(N\tau_x^{-1},x)N(dx)\right] = E^*\left[\int H(N,x)dx\right],$$

where E^* denotes "expectation" with respect to P^*, even though P^* need not be a probability, and dx (and below, λ) denotes Lebesgue measure. Heuristically, $P^*\{N\in(\cdot)\}/P^*(\Omega) = P\{N\in(\cdot)\,|\,N(\{0\})=1\}$, so that P^* is the "law" of N conditional on the null event that there is a point at the origin. Extending work of Krickeberg (1982), a theory of estimation for stationary point processes has been developed in which objects estimated are the Palm measure and integral functionals of it, especially moment and spectral measures. Data are single realizations of N observed over increasing compact sets.

Specifically (cf. Karr, 1985, or Krickeberg, 1982, for background), with $N^k(dx_1,\ldots,dx_k) = N(dx_1)\cdots N(dx_k)$, let $\mu^k = E[N^k]$ be the k-th order moment measure and let μ_*^k, defined on E^{k-1}, be the *reduced moment measure* of order k:

$$\mu^k = \int_{E^{k-1}}\lambda_z\mu_*^k(dz),$$

where λ_z is the image of λ under the mapping $x\to(z_1+x,\ldots,z_{k-1}+x,x)$. The reduced moment measures of orders one and two, to wit $\mu_*^1 = \nu\varepsilon_0$, where ν is the *intensity* of N ($E[N(A)]=\nu\lambda(A)$ for each A) and μ_*^2, along with the reduced covariance measure $\rho_* = \mu_*^2-\nu^2\lambda$, are of central importance, as is the spectral measure F, which satisfies $F(\Psi) = \rho_*(\hat{\Psi})$, where Ψ is a C^∞-function with compact support and $\hat{\Psi}$ is its inverse Fourier transform.

The unbiased estimators

$$\hat{P}^*(H) = \lambda(K)^{-1}\int_K H(N\tau_x^{-1})N(dx)]$$

of the Palm measure, where K is a compact, convex set, are natural "empirical" estimators: for x a point of N -- and only such x contribute to the integral -- $N\tau_x^{-1}$ has a point at the origin, so that $\hat{P}^*(H)$ is simply an average of H-values for translations of N placing each point in K at the origin. Here K should be regarded, albeit loosely, as the sample size; calculation of the estimator requires observation of N over a compact set related to but ordinarily larger than K.

Strong consistency of the $\hat{P}^*(H)$ has been established, uniformly in H over certain classes of functions, under broadly fulfilled conditions on N, as has asymptotic normality of the "standardized" estimation error $\lambda(K)^{1/2}[\hat{P}^*(H)-P^*(H)]$, as the sets K increase to E. Detailed properties of the estimators $\hat{\nu} = N(K)/\lambda(K)$ and $\hat{\mu}_*^2(f) = \lambda(K)^{-1}\int_K N(dx)\int N(dy)f(y-x)$, where f is continuous with compact support, have been worked out; the latter yield properties of substitution estimators $\hat{F}(\Psi) = \hat{\mu}_*^2(\hat{\Psi})$ of the spectral measure. These results appear in

> Karr, A.F., Estimation of Palm measures and spectral measures of stationary point processes. Technical report 403, Department of Mathematical Sciences, The Johns Hopkins University, 1984,

which contains further material concerning the problem of combined inference and linear state estimation, i.e., linear reconstruction of unobserved portions of N when ν and ρ_* -- the only objects required for linear state estimation -- are unknown and must be estimated. This manuscript is currently under revision for *Z. Wahrscheinlich-*

keitstheorie und verw. Geb. to incorporate recently proved results on Poisson approximations that complement the central limit theorem and on estimation of P (the law of N) itself, rather than the Palm measure P^*.

B) State Estimation for Cox Processes. A pair (N,M) comprising a simple point process N and diffuse random measure M on the same space E is a *Cox pair* if conditional on M, N is a Poisson process with mean measure M; N is then termed a *Cox process directed by* M. The most important special case is that of mixed Poisson processes, in which $M = Y\nu^*$, where ν^* is a diffuse measure on E and Y is a nonnegative random variable. In applications, which include signal processing, carcinogenesis and precipitation, only the Cox process N is observable, but the directing measure M, which represents the underlying physical mechanism, is of primary interest, leading to the *state estimation* problem of computation of minimum mean squared error state estimators $E[M|\mathcal{F}^N(A)]$, where $\mathcal{F}^N(A)$ is the σ-algebra corresponding to (uncorrupted) observation of N over the set A. Such computations require (effectively full) knowledge of the law of N, which of course typically is not available.

An optimal solution to the problem of combined nonparametric inference and state estimation for mixed Poisson processes was obtained in Karr (1984). One has as data i.i.d. mixed Poisson processes N_i, with unobservable directing measures $M_i = Y_i\nu^*$, where the Y_i are i.i.d. with unknown distribution F. For each n the true state estimator of M_{n+1} is expressible as

$$E[M_{n+1}|\mathcal{F}^{N_{n+1}}(A)] = H(F,\nu^*,N_{n+1}|_A), \tag{2}$$

where H is a <u>known</u> functional form; in the combined problem it is replaced by a *pseudo-state estimator*

(3) $$\hat{E}[M_{n+1}|\mathcal{F}^{N_{n+1}}(A)] = H(\hat{F},\hat{\nu}^*,N_{n+1}|_A),$$

where $\hat{F}$, $\hat{\nu}^*$ are estimators of F, ν^* based on $N_1,\dots,N_n$.

During 1984 a partial but evidently less-than-optimal solution was obtained for the corresponding problem for completely general Cox processes. Its key components are a representation -- itself significant -- for true state estimators in terms of the *Palm distributions* (see, e.g., Kallenberg, 1983, or Karr, 1985) of the M_i, which are linked in turn to the reduced Palm distributions of the N_i, and a methodology for estimating the latter from observation of the N_i. Estimation is effected with techniques similar conceptually, but very different mathematically, to those discussed in II.A above. The representation theorem yields an analogue for the function H in (2) and (3); the other main results are limit theorems describing asymptotic behavior (as $n\to\infty$) of differences between true state estimators and pseudo-state estimators. They appear in

Karr, A.F., State estimation for Cox processes with unknown probability law. Technical report 379, Department of Mathematical Sciences, The Johns Hopkins University, 1984,

which has been accepted for publication in *Stochastic Processes and their Applications*.

C) <u>Multiplicative Intensity Models</u>. In the theory of intensity-based inference for point processes on $\mathbb{R}_+$, the best compromise between applicability and tractability is the multiplicative intensity model of Aalen (1978) (see Jacobsen, 1982, or Karr, 1985, for general discussion). Let $(\Omega,\mathfrak{F},P)$ be a probability space on which are defined a point process (N_t) and filtration $(\mathcal{H}_t)$. Provided that it exists, the *stochastic intensity* of N is the unique (predictable) process (λ_t) such that $M_t = N_t - \int_0^t \lambda_s ds$ is a martingale. Heuristically, $\lambda_t dt = E[\Delta N_t | \mathcal{H}_{t-}]$, where ΔN_t is the jump (necessarily one or zero) of N at t; the differential form $dM_t = dN_t - \lambda_t dt$ reveals the "innovation" interpretation of the martingale M. In the *multiplicative intensity model* the unknown "parameter" is a positive function $\alpha \in L^1[0,1]$; under the probability P_α, N has stochastic intensity $(\alpha_t \lambda_t)$, where λ is an observable, predictable process. The "martingale method" of inference estimates integrals $\int_0^t \alpha_s ds$ (more precisely, random processes $B_t(\alpha) = \int_0^t 1(\lambda_s>0)\alpha_s ds$) with martingale estimators $\hat{B}_t = \int_0^t 1(\lambda_s>0)\lambda_s^{-1} dN_s$, whose main attractions are computational simplicity and the property that $(\hat{B}_t - B_t(\alpha))$ is a P_α-martingale for each α; the latter permits calculation of variances and application of potent martingale central limit theorems to develop asymptotic properties of martingale estimators.

However, martingale estimators are not derived from optimality considerations. By adapting the *method of sieves* of Grenander (1981) and collaborators, it has been possible -- in the context of i.i.d. multiplicative intensity processes -- to construct maximum likelihood estimators that are strongly consistent in the sense that $\|\hat{\alpha}-\alpha\|_1 \to 0$ almost surely. Further results that identify conditions engendering c_n-consistency, local asymptotic normality of likelihood ratios and asymptotic normality of the $\hat{\alpha}$ have also been obtained. A preliminary report

Karr, A.F., Maximum likelihood estimation in the multiplicative intensity model, via sieves. Technical report 46, Center for Stochastic Processes, University of North Carolina, 1983,

written during a sabbatical visit in Fall, 1983, has been revised and expanded during 1984; this new version has been accepted for publication in *The Annals of Statistics.*

D) <u>Poisson Processes</u>. Consider a Poisson process $N = \Sigma\varepsilon_{(X_i, Z_i)}$ on a product space $E = E_0 \times E_1$ with mean measure $\mu(dx)K(x,dy)$, where K is a <u>known</u> transition probability from E_0 to E_1 and μ is an <u>unknown</u> measure on E_0, and suppose that only the second component $N^1 = \Sigma\varepsilon_{Z_i}$ is observable. The goal is inference concerning μ, the mean measure of the Poisson process $N^0 = \Sigma\varepsilon_{X_i}$, given observation of n i.i.d. copies of N^1. Applications of this model include positron emission tomography (Shepp/Vardi, 1982) and other forms of medical imaging (D.L. Snyder, personal communication). Estimation of μ via the EM algorithm of Dempster/Laird/Rubin (1977) is under investigation; this iterative algorithm (for each fixed value of n) uses a provisional estimate of μ to construct a pseudo-state estimator $\hat{E}[N^0 | \mathcal{F}^{N^1}]$, which is used in turn to update the estimated value of μ. With n fixed, this procedure converges in some cases to a maximum likelihood estimator of μ. The key issue, consistent estimation of μ as $n \to \infty$, remains unresolved despite progress during 1984; completion of this research during 1985 is anticipated.

E) <u>Random Fields</u>. A *random field* is a stochastic process $Y = (Y_x)_{x \in \mathbb{R}^d}$ with a multidimensional (Euclidean) parameter set.

Suppose that Y is a stationary random field on $\mathbb{R}^d$ that can be observed only at the points X_i of a stationary Poisson process N on $\mathbb{R}^d$ that is independent of Y; more precisely, N is also observable, so that observations constitute a marked point process in which each point of the sampling process N is marked by the value of Y at the point. For example, Y may be a precipitation field and the X_i locations of raingages, or Y a petroleum reservoir and the X_i locations of test drillings.

Techniques have been developed for estimation of the mean $m \equiv E[Y_x]$ and the covariance function $R(x) = \mathrm{Cov}(Y_z, Y_{z+x})$ given observation of single realizations over compact, convex sets K, even if the intensity ν of N is unknown. The estimators $\hat{m} = N(K)^{-1}\int_K Y dN$, usable regardless of whether ν is known, are mean square consistent and asymptotically normal under minimal assumptions. Similarly, given that $m = 0$ and ν is known, the nonparametric estimators

$$\hat{R}(x) = [\nu^2\lambda(K)]^{-1}\int_K\int_K w_K(x-x_1+x_2)Y(x_1)Y(x_2)N^{(2)}(dx_1,dx_2),$$

where $N^{(2)}(dx_1,dx_2) = N(dx_1)(N-\varepsilon_{x_1})(dx_2)$ and $w_K(x) = \alpha_K^{-d}w(x/\alpha_K)$ is a "window" function (w is a bounded, isotropic density and the α_K are positive constants converging to zero) are, again under only mild restrictions, mean square consistent, asymptotically normal pointwise in x and asymptotically uncorrelated for $x \neq \pm x'$. These properties, alternative estimators of R via the signed measure $Q(A) = \int_A R(x)dx$ and linear state estimation of unobserved values of Y, are treated in

Karr, A.F., Inference for stationary random fields based on Poisson samples. Technical report xxx, Department of Mathematical Sciences, The Johns Hopkins University, 1985,

which will shortly be submitted for publication in *Advances in Applied Probability*.

Though unrelated to the preceding topics the following research activity synthesizes several lines of inquiry.

F) Stopping Times of Markov Processes. The structure of stopping times of a Markov process is central not only to such theoretical issues as the strong Markov property but also to applications, including optimal stopping, stochastic orderings and "inverse problems"; see, e.g., Karr/Pittenger (1978, 1979). In the manuscript

> Karr, A.F., and Pittenger, A.O., Structural properties of random times. Technical report xxx, Department of Mathematical Sciences, The Johns Hopkins Universit 1985,

an algebraic structure is devised that elucidates the nature of natur… and randomized stopping times, especially randomized terminal times and the new class of randomized quasi-terminal times, extreme randomized terminal times are identified, and a sequential compactness theorem is established. Unfortunately, no "real" applications are apparent or imminent. This manuscript will be submitted to *Z. Wahrscheinlichkeitstheorie und verw. Geb.*

III. PAPERS APPEARING DURING 1984

During 1984 the following papers supported by grants from AFOSR appeared in print:

1) Karr, A.F., Estimation and reconstruction for zero-one Markov processes. *Stochastic Process. Appl.* 16 219-255.

2) Karr, A.F., Combined nonparametric inference and state estimation for mixed Poisson processes. *Z. Wahrscheinlichkeitstheorie und verw. Geb.* 66 81-96.

IV. ADDITIONAL ACTIVITIES

The following activities, even though they either do not receive AFOSR funding or do not result (directly) in publications containing new results, are nevertheless integral to the research program and are evidence of its vitality.

1) Book. The book *Point Processes and their Statistical Inference*, by A.F. Karr, was completed in October, 1984; it is to be published in late 1985 by Marcel Dekker, Inc. Although its writing has not been supported by AFOSR the book contains new results that will not (for various reasons) be published elsewhere. Nearly all of these are outgrowths of AFOSR-supported research and will be so acknowledged.

2) Expository Papers. The papers

Karr, A.F., The martingale method: introductory sketch and access to the literature. *Operations Res. Letters* 3 (1984) 59-63.

Karr, A.F., Point process. In *Encyclopedia of Statistical Sciences* VI, N.L. Johnson and S. Kotz, eds. Wiley, New York (to appear).

Karr, A.F., Poisson process. In *Encyclopedia of Statistical Sciences* VI, N.L. Johnson and S. Kotz, eds. Wiley, New York (to appear).

Karr, A.F., Stationary point process. In *Encyclopedia of Statistical Sciences* VI, N.L. Johnson and S. Kotz, eds. Wiley, New York (to appear),

which are expository presentations of point processes and inference for them, have likewise not been prepared under AFOSR support. However, each is an invited contribution and contains substantial discussion of supported research. That they are invited papers confirms the visibility and strong reputation of the research program.

3) Seminar and Conference Presentations. Invited seminar and conference addresses on the research accomplishments discussed in II were presented during 1984 at The Johns Hopkins University, the Centre for Mathematics and Computer Science (Amsterdam), the University of Copenhagen, the Ecole Polytechnique (Paris), a Mini-conference on Inference for Stochastic Processes (Lexington, KY) and the Fourteenth Conference on Stochastic Processes and their Applications (Göteborg).

4) Visitors to Johns Hopkins. During 1984 visitors to Johns Hopkins for discussion of inference for point processes included D. König (Freiberg, DDR), D.J. Daley (Canberra), P.E. Greenwood (Vancouver), A.N. Shiryayev (Moscow), E. Arjas (Oulu, Finland) and S. Johansen (Copenhagen). None of these visits received grant support (it does not include such funds); the international stature of the visitors and their willingness to visit even without support are further indication of the strength of this research program.

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KARR, A.F. and PITTENGER, A.O. (1978). The inverse balayage problem for Markov chains. *Stochastic Process. Appl.* 7 165-178.

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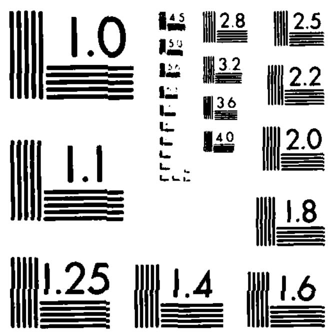
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